BFKL resummation effects in exclusive production of rho meson pairs at the ILC
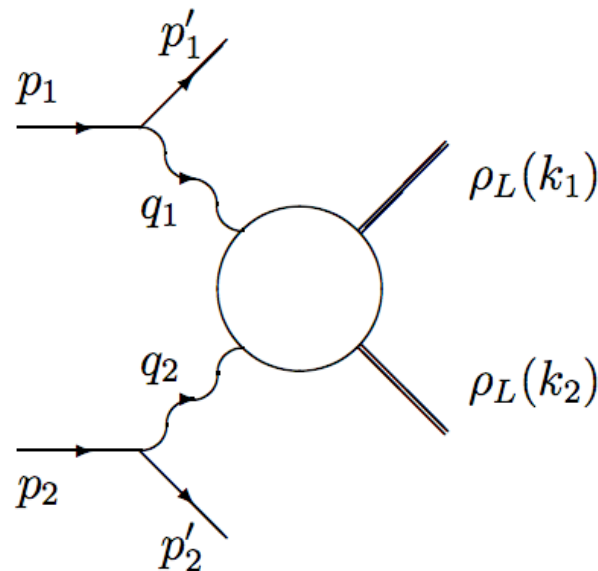
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in a collaboration with :
B.Pire ,
L.Szymanowski and
S.Wallon

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Diffractive mesons production with leptons tagging for studying the BFKL Pomeron

- We consider the process $e^+e^- \to e^+e^- \rho_L \rho_L$

In the Regge limit, we expect to 'observe' an exchange of a BFKL Pomeron in the t-channel.

We compute the scattering amplitude in a complete analytical way at the Born order.

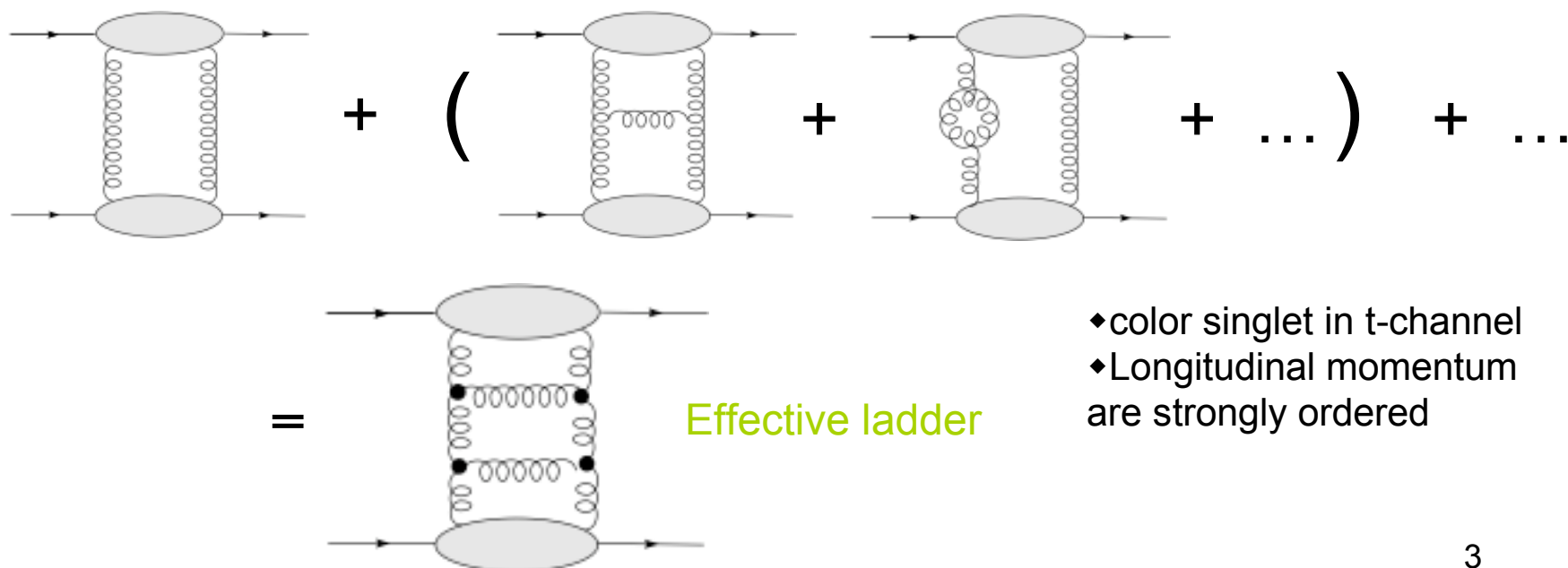This process has already been studied until NLO but only in the forward case.

D.Ivanov, A.Papa
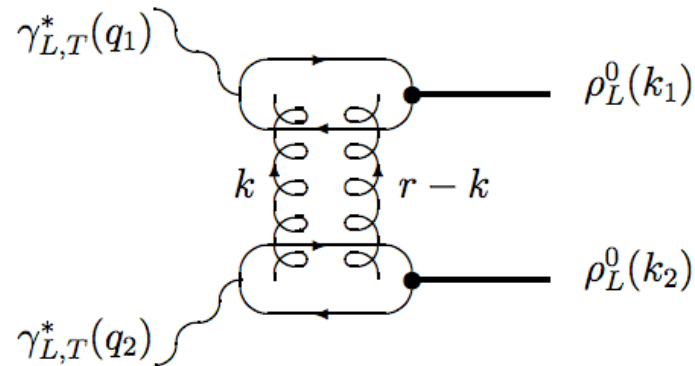
The BFKL Pomeron in QCD

- IR divergencies (lns) emerge in high energy scattering

- But small values of α_s at high energy can be compensated by large lns
$\longrightarrow$ resummation of the terms $\alpha_s lns$ at each order in a infinite serie $\longrightarrow$ BFKL equation
Leading Log Approximation (LLA) in the Regge limit

= Effective ladder

◆color singlet in t-channel
◆Longitudinal momentum
are strongly ordered

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Study of the process $\gamma_L^*(q_1)\, \gamma_L^*(q_2) \to \rho_L(k_1)\, \rho_L(k_2)$

Selection of events in which two vectors ρ mesons with longitudinal polarization are produced in the final state with a big gap in rapidity.

IR safe probes: double tagging of final leptons (➔photons polarizations) and Cut off over soft photons. The highly virtual photons $Q_1^2, Q_2^2 \gg \Lambda_{QCD}^2$ give the hard scales on both sides of the *t*-channel exchanged state ➔ fully perturbative process (except for the final mesons)

$Q_1^2 \sim Q_2^2$ ➔ neglect DGLAP partonic evolution

In the Regge limit $s \gg -t,\, Q_1^2, Q_2^2$, the process is dominated by BFKL evolution.

kinematics

q_1

$\rho_L^0(k_1)$

M

$\rho_L^0(k_2)$

q_2

Q_1 , Q_2 $\longrightarrow$ hard scales
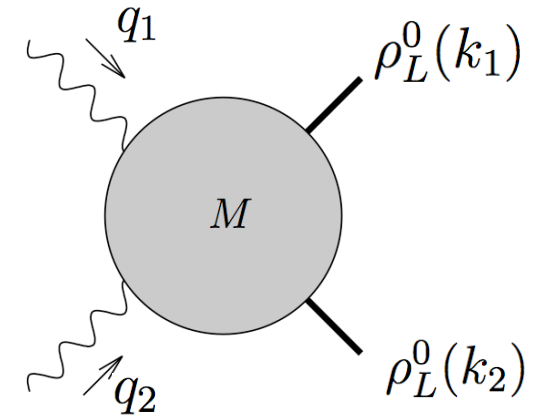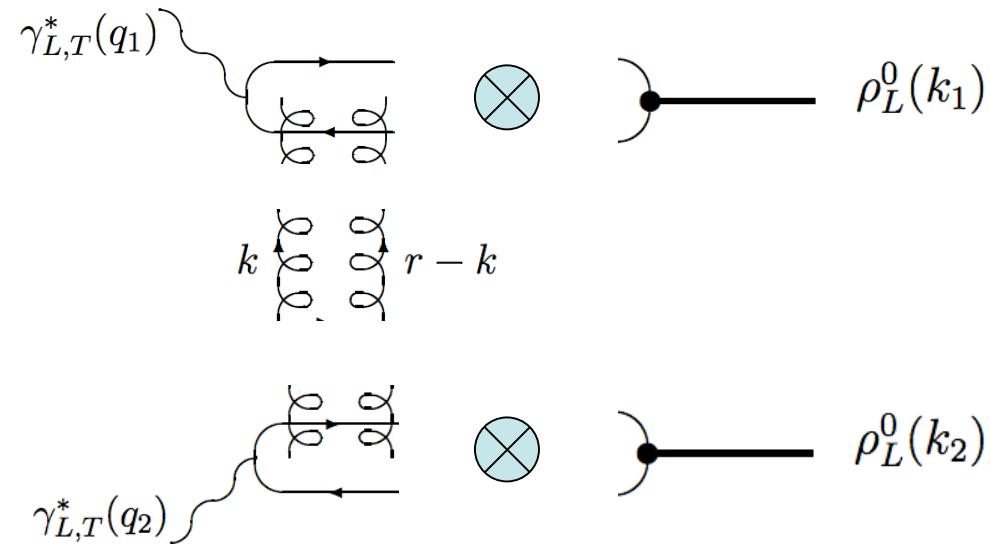
- Sudakov decomposition : two light-cone vectors

momentum transfer

$$t \sim -\frac{Q_1^2 Q_2^2}{s} - \underline{r}^2 \left(1 + \frac{Q_1^2}{s} + \frac{Q_2^2}{s} + \frac{\underline{r}^2}{s} \right)$$

photons momenta

$$q_1 = q_1' - \frac{Q_1^2}{s} q_2' \qquad q_2 = q_2' - \frac{Q_2^2}{s} q_1'$$

mesons momenta

$$k_1 = \alpha(k_1)\, q_1' + \frac{\underline{r}^2}{\alpha(k_1)\, s} q_2' + r_\perp$$

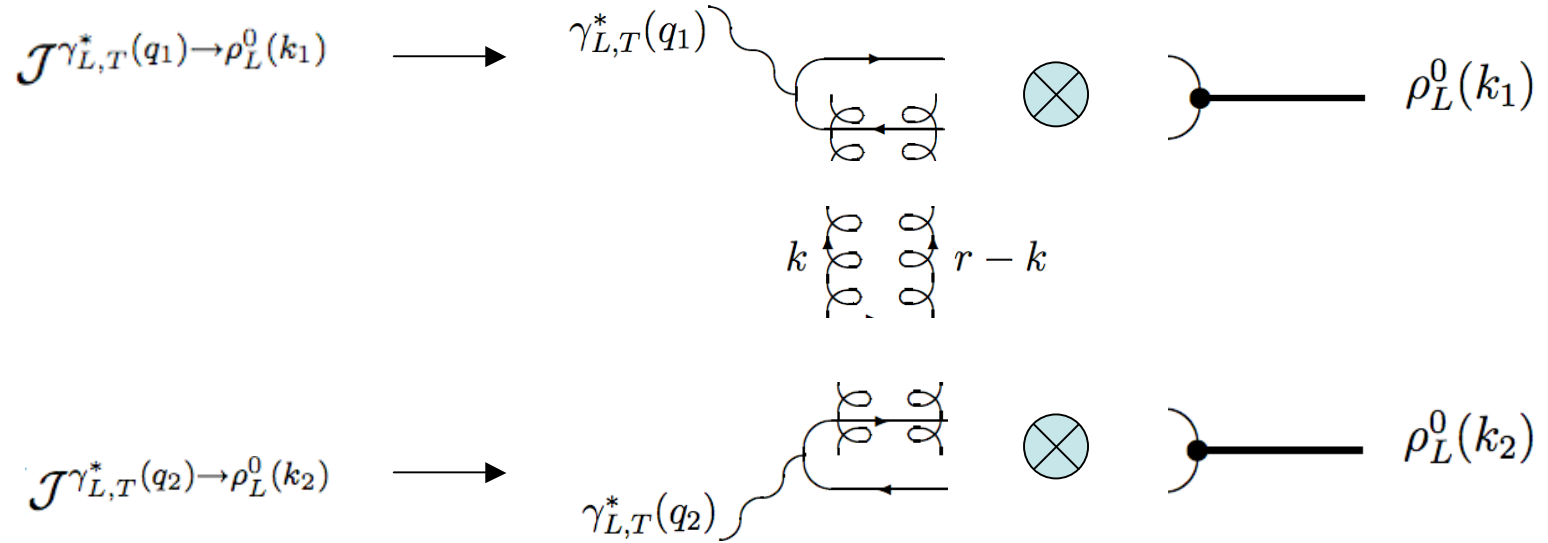$$k_2 = \beta(k_1)\, q_2' + \frac{\underline{r}^2}{\beta(k_1)\, s} q_1' - r_\perp$$

Amplitude of the process at the Born order

Integration over the internal moments :

♦ Sudakov basis $\qquad k = \alpha q_1' + \beta q_2' + k_\perp \qquad q_1'^2 = q_2'^2 = 0$

♦ In the BFKL dynamics the longitudinal momenta of the gluons are strongly ordered.

➔ kT-factorization in transverse momentum $\qquad$ cf. $\qquad \int d^4k = \int d\alpha d\beta dk^2$

♦ Collinear approximation ➔ we neglect transverse relative momentum of quark inside the mesons.

Impact representation of the amplitude

$$\mathcal{J}^{\gamma^*_{L,T}(q_1) \to \rho^0_L(k_1)} \longrightarrow$$

$$\mathcal{J}^{\gamma^*_{L,T}(q_2) \to \rho^0_L(k_2)} \longrightarrow$$

$$\mathcal{M} = is \int \frac{d^2\underline{k}}{(2\pi)^4 \underline{k}^2 (\underline{r} - \underline{k})^2} \mathcal{J}^{\gamma^*_{L,T}(q_1) \to \rho^0_L(k_1)}(\underline{k}, \underline{r} - \underline{k}) \; \mathcal{J}^{\gamma^*_{L,T}(q_2) \to \rho^0_L(k_2)}(-\underline{k}, -\underline{r} + \underline{k})$$

Every impact factor $\mathcal{J}^{\gamma^*_{L,T}(q_1) \to \rho^0_L(k_1)}$ is written as a convolution of the DA of the meson with the more simple impact factor corresponding to the quark-antiquark opened pair production from one polarized photon with two reggeized gluons exchanged in the t channel.

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as :

$$\langle \rho(k_2) | \bar{q}(-\frac{z}{2}) \, \gamma^\mu \, q(\frac{z}{2}) | 0 \rangle = f_\rho \, k_2^\mu \int_0^1 du e^{i(1-2u)(k_2 \frac{z}{2})} \phi(u)$$

- In the case of longitudinaly polarized photons, they read :

$$\mathcal{J}^{\gamma_L^*(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k})$$

$$= 8\pi^2\alpha_s \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} Q_i\, f_\rho \alpha(k_i) \int\limits_0^1 dz_i z_i\, \bar{z}_i\, \phi(z_i) \mathrm{P_P}(z_i,\underline{k},\underline{r},\mu_i)$$

with $\quad \mathrm{P_P}(z_i,\underline{k},\underline{r},\mu_i) = \dfrac{1}{z_i^2\underline{r}^2+\mu_i^2} + \dfrac{1}{\bar{z}_i^2\underline{r}^2+\mu_i^2} - \dfrac{1}{(z_i\underline{r}-\underline{k})^2+\mu_i^2} - \dfrac{1}{(\bar{z}_i\underline{r}-\underline{k})^2+\mu_i^2}$

where $\quad \mu_i^2 = Q_i^2\, z_i\, \bar{z}_i + m^2$

- For transversely polarized photons, one obtains :

$$\mathcal{J}^{\gamma_T^*(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k})$$

$$= 4\pi^2\alpha_s \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} f_\rho \alpha(k_i) \int\limits_0^1 dz_i\,(z_i-\bar{z}_i)\,\phi(z_i)\,\underline{\epsilon}\cdot\underline{Q}(z_i,\underline{k},\underline{r},\mu_i)$$

with $\quad \underline{Q}(z_i,\underline{k},\underline{r},\mu_i) = \dfrac{z_i\,\underline{r}}{z_i^2\underline{r}^2+\mu_i^2} - \dfrac{\bar{z}_i\,\underline{r}}{\bar{z}_i^2\underline{r}^2+\mu_i^2} + \dfrac{\underline{k}-z_i\,\underline{r}}{(z_i\underline{r}-\underline{k})^2+\mu_i^2} - \dfrac{\underline{k}-\bar{z}_i\underline{r}}{(\bar{z}_i\,\underline{r}-\underline{k})^2+\mu_i^2}$

Both Impact factor vanish when $\underline{k}\to 0$ or $\underline{r}-\underline{k}\to 0$ due to QCD gauge invariance (probes are colorless)

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To compute the scattering amplitude $M_{\lambda_1 \lambda_2}$ we have to perform analytically the 2D integration over the transverse momentum.

Analytical computation of the 2D integrals involved is performed after the use of conformal transformations in the transverse momentum space. (method inspired by Vassiliev in 2-d coordinate space)

This reduces the number of propagators .

For example , we have to compute this kind of integrals with 3 propagators (1 massive) :

$$J_{3\mu}(a) = \int \frac{d^2 \underline{k}}{\underline{k}^2 (\underline{k} - \underline{r})^2} \left[\frac{1}{(\underline{k} - \underline{r}a)^2 + \mu^2} - \frac{1}{a^2 \underline{r}^2 + \mu^2} + (a \leftrightarrow \bar{a}) \right]$$

Inversion on the integration variable and vector parameter

$$\underline{k} \to \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \to \frac{\underline{R}}{\underline{R}^2}, \quad m \to \frac{1}{M}$$

$$= R^2 \int \frac{d^2 \underline{K}}{(\underline{K} - \underline{R})^2} \left(\frac{K^2 R^2}{(\underline{R} - a\underline{K})^2 + \frac{K^2 R^2}{M^2}} - \frac{1}{a^2 r^2 + m^2} + (a \leftrightarrow \bar{a}) \right)$$

Then we perform the shift of variable $\qquad \underline{K} = \underline{R} + \underline{k}'$

And an other inversion

And we obtain an integral with 3 propagators (1 massive) :

$$J_{3m} = \frac{1}{r^2} \int \frac{d^2\underline{k}}{\underline{k}^2} \left[\frac{(\underline{r}+\underline{k})^2}{(r^2 a^2 + m^2)\left(\left(\underline{k} - \underline{r}\frac{r^2 a\,\underline{a} - m^2}{r^2\underline{a}^2 + m^2}\right)^2 + \frac{m^2 r^4}{(r^2\underline{a}^2 + m^2)^2}\right)} - \frac{1}{a^2 r^2 + m^2} + (a \leftrightarrow \bar{a}) \right]$$
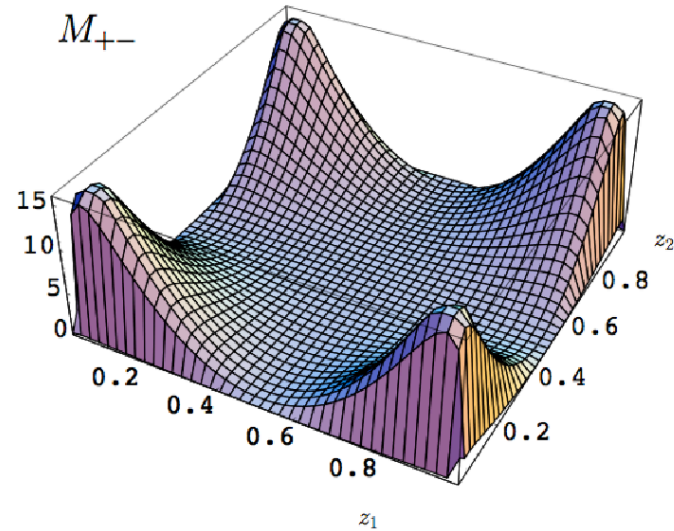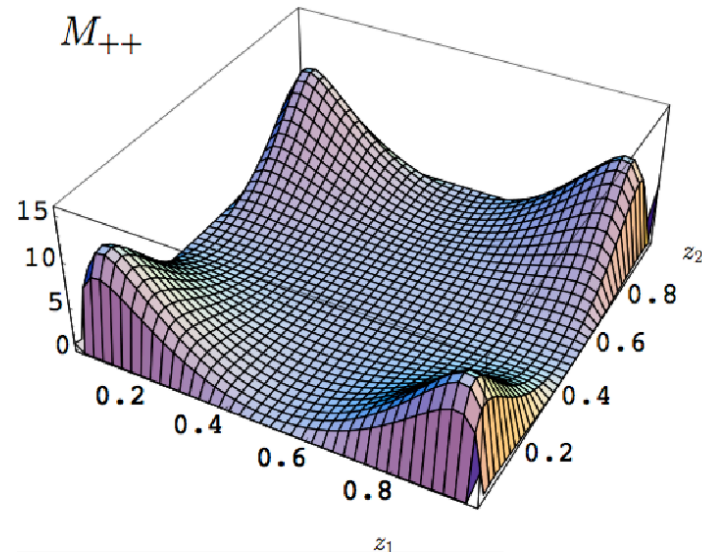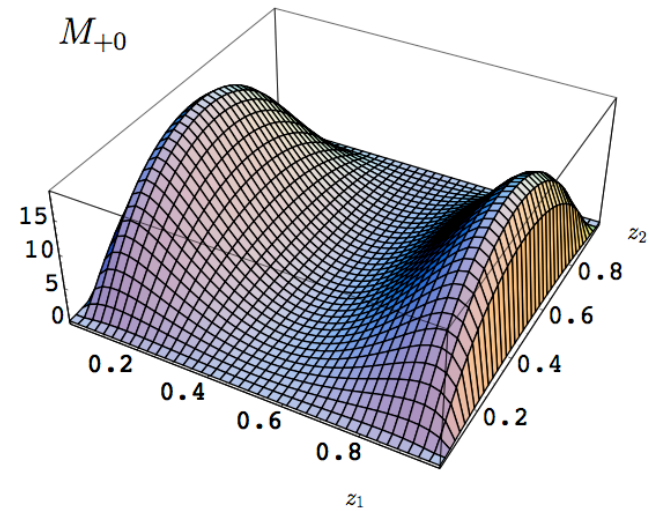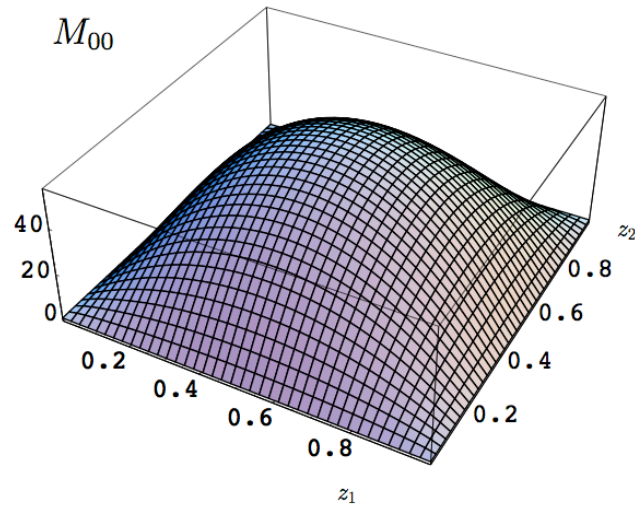
UV and IR finite

It is now possible to compute this integral by using standard technique

$$J_{3m} = \frac{2\pi}{r^2} \left\{ \left(\frac{1}{r^2 a^2 + m^2} - \frac{1}{r^2 \bar{a}^2 + m^2} \right) \ln \frac{r^2 a^2 + m^2}{r^2 \bar{a}^2 + m^2} \right.$$
$$\left. + \left(\frac{1}{r^2 a^2 + m^2} + \frac{1}{r^2 \bar{a}^2 + m^2} + \frac{2}{r^2 a\bar{a} - m^2} \right) \ln \frac{(r^2 a^2 + m^2)(r^2 \bar{a}^2 + m^2)}{m^2 r^2} \right\}$$

We deduce the differential cross-section in the large s limit :

$$\frac{d\sigma^{\gamma^*_{\lambda_1}\gamma^*_{\lambda_2} \to \rho^0_L \rho^0_L}}{dt} = \frac{|\mathcal{M}_{\lambda_1\lambda_2}|^2}{16\,\pi\,s^2}$$

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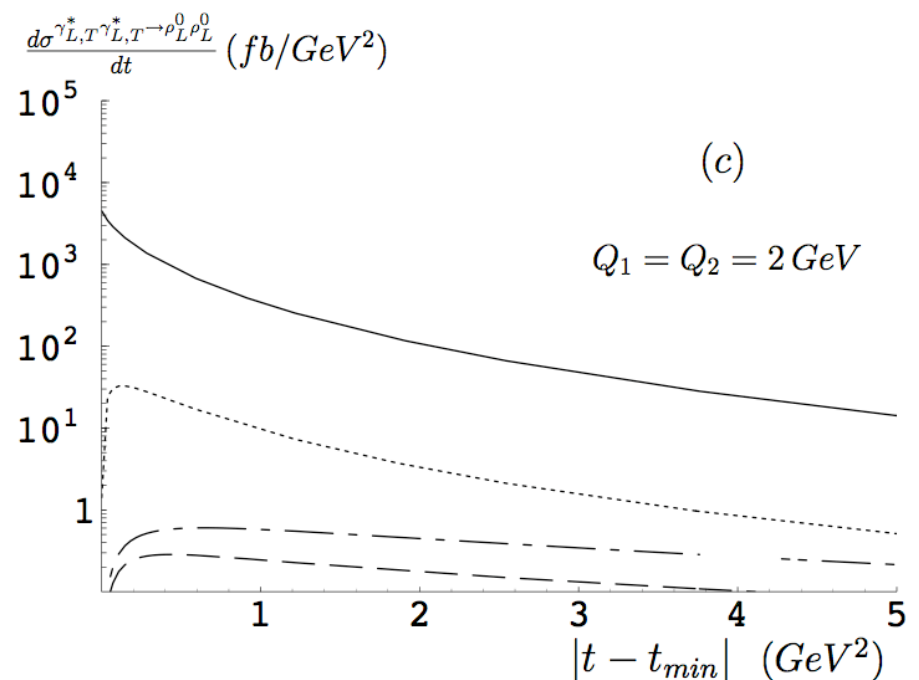# Shape of the k_T-integrated amplitudes in the z_i plane
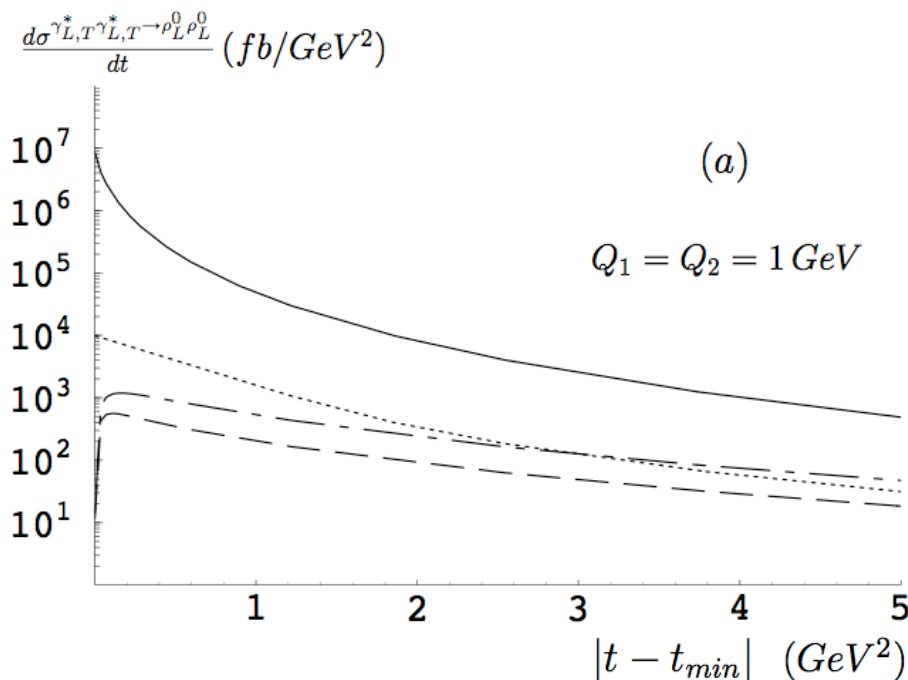

$$\overline{M}_{00} = z_1 \, \bar{z}_1 \, \phi(z_1) \, z_2 \, \bar{z}_2 \, \phi(z_2) \, M_{00}(z_1, \, z_2)$$

$$\overline{M}_{\lambda_1 0} = (z_1 - \bar{z}_1) \, \phi(z_1) \, z_2 \, \bar{z}_2 \, \phi(z_2) \, M_{\lambda_1 0}(z_1, \, z_2)$$

$$\overline{M}_{\lambda_1 \lambda_2} = (z_1 - \bar{z}_1) \, \phi(z_1) \, (z_2 - \bar{z}_2) \, \phi(z_2) \, M_{\lambda_1 \lambda_2}(z_1, \, z_2)$$

Differential cross section for the different polarizations
of the virtual photons

The integration over momentum fractions z_1 and z_2 are performed numerically
we use Q_1Q_2 as a scale for $\alpha_s(\sqrt{Q_1Q_2})$ running at 3 loops

$\frac{d\sigma^{\gamma_{L,T}^*\gamma_{L,T}^*\to\rho_L^0\rho_L^0}}{dt}\,(fb/GeV^2)$

(a)

$Q_1 = Q_2 = 1\,GeV$

$|t - t_{min}|\;(GeV^2)$

$\frac{d\sigma^{\gamma_{L,T}^*\gamma_{L,T}^*\to\rho_L^0\rho_L^0}}{dt}\,(fb/GeV^2)$
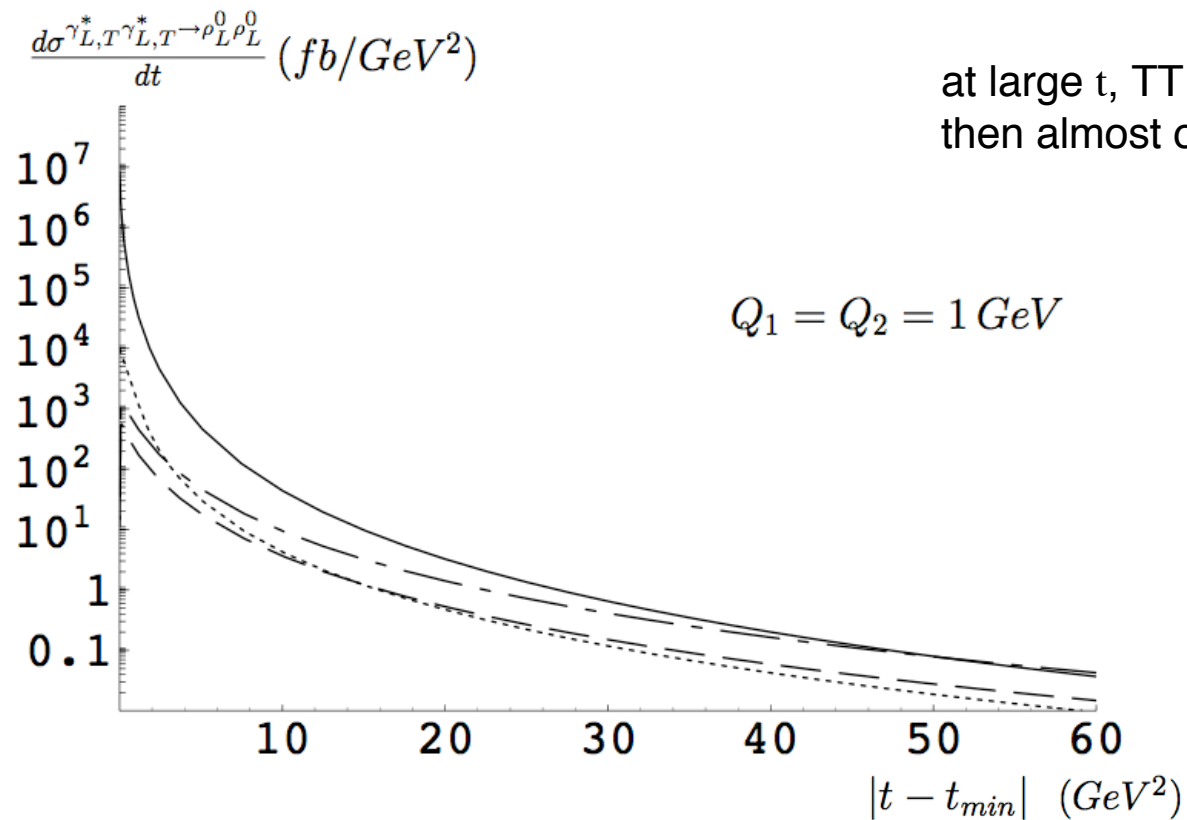
(c)

$Q_1 = Q_2 = 2\,GeV$

$|t - t_{min}|\;(GeV^2)$

solid curve : LL mode
dotted curve : LT mode
dashed and dashed-dotted curves :TT mode

strong decrease with Q

any cross-section with at least one tranverse
photon vanishes in the forward case

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Differential cross-sections for different polarizations of the virtual photons up to asymptotically large t

$$\frac{d\sigma^{\gamma^*_{L,T}\gamma^*_{L,T}\to\rho^0_L\rho^0_L}}{dt}\ (fb/GeV^2)$$

at large t, TT mode dominates (photon are then almost on-shell with respect to t)

$$Q_1 = Q_2 = 1\,GeV$$

solid curve : LL mode
dotted curve : LT mode
dashed and dashed-dotted curves :TT mode

Non-forward cross-sections at ILC for

$$e^+ e^- \to e^+ e^- \rho_L^0 \, \rho_L^0$$

We use the same Sudakov basis

and the equivalent photon approximation

$$\frac{d\sigma(e^+ e^- \to e^+ e^- \rho_L \rho_L)}{dy_1 dy_2 dQ_1^2 dQ_2^2}$$

$$= \frac{1}{y_1 y_2 \, Q_1^2 Q_2^2} \left(\frac{\alpha}{\pi}\right)^2 [l_1(y_1)\, l_2(y_2) \sigma(\gamma_L^* \gamma_L^* \to \rho_L \rho_L) + t_1(y_1)\, l_2(y_2) \, \sigma(\gamma_T^* \gamma_L^* \to \rho_L \rho_L)$$

$$+ \; l_1(y_1)\, t_2(y_2)\, \sigma(\gamma_L^* \gamma_T^* \to \rho_L \rho_L) + t_1(y_1)\, t_2(y_2)\, \sigma(\gamma_T^* \gamma_T^* \to \rho_L \rho_L)] \; .$$

with the usual photons flux factors given by $\quad t_i = \dfrac{1 + (1 - y_i)^2}{2}, \quad l_i = 1 - y_i$

$y_i \, (i = 1, 2)$ are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the incoming leptons

$$s_{\gamma^* \gamma^*} \sim y_1 \, y_2 \, s_{e^+ e^-}$$

$\Rightarrow \sigma^{e^+ e^- \to e^+ e^- \rho_L \rho_L}$ is peaked in the low y and Q^2 region

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Kinematical constraints coming from experimental features of the ILC collider are used to perform the phase-space integration.

Photons momentum fractions $y_i = \dfrac{E - E_i' \cos^2(\theta_i/2)}{E}$ In the cms frame

and virtualities $Q_i^2 = 4EE_i' \sin^2(\theta_i/2)$

kinematical constraints coming from the minimal detection angle around the beampipe and from the conditions on the energies of the scattered leptons and the Regge limit.

$$y_{i\,max} = 1 - \frac{E_{min}}{E}$$

$$y_{1\,min} = \max\left(f(Q_1), 1 - \frac{E_{max}}{E}\right)$$ with $f(Q_i) = 1 - \dfrac{Q_i^2}{s \tan^2(\theta_{min}/2)}$

$$y_{2\,min} = \max\left(f(Q_2), 1 - \frac{E_{max}}{E}, \frac{c\,Q_1 Q_2}{s\,y_1}\right)$$

$$\frac{d\sigma^{e^+e^- \to e^+e^-\,\rho_L\rho_L}}{dt} = \int_{Q_{1min}^2}^{Q_{1max}^2} dQ_1^2 \int_{Q_{2min}^2}^{Q_{2max}^2} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1Q_2}{sy_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^- \to e^+e^-\,\rho_L\rho_L}}{dt\,dy_1\,dy_2\,dQ_1^2\,dQ_2^2}$$

Experimental features of the ILC collider design of the detector

Each design of detector for ILC project involves a very forward electromagnetic calorimeter for luminosity measurement, with tagging angle for outgoing leptons

down to 5 mrad which is an ideal tool for diffractive physics whose cross-sections are sharply peaked in the very forward region

The high luminosity will allow to obtain sufficient statistics to measure exclusive events

European LDC collaboration
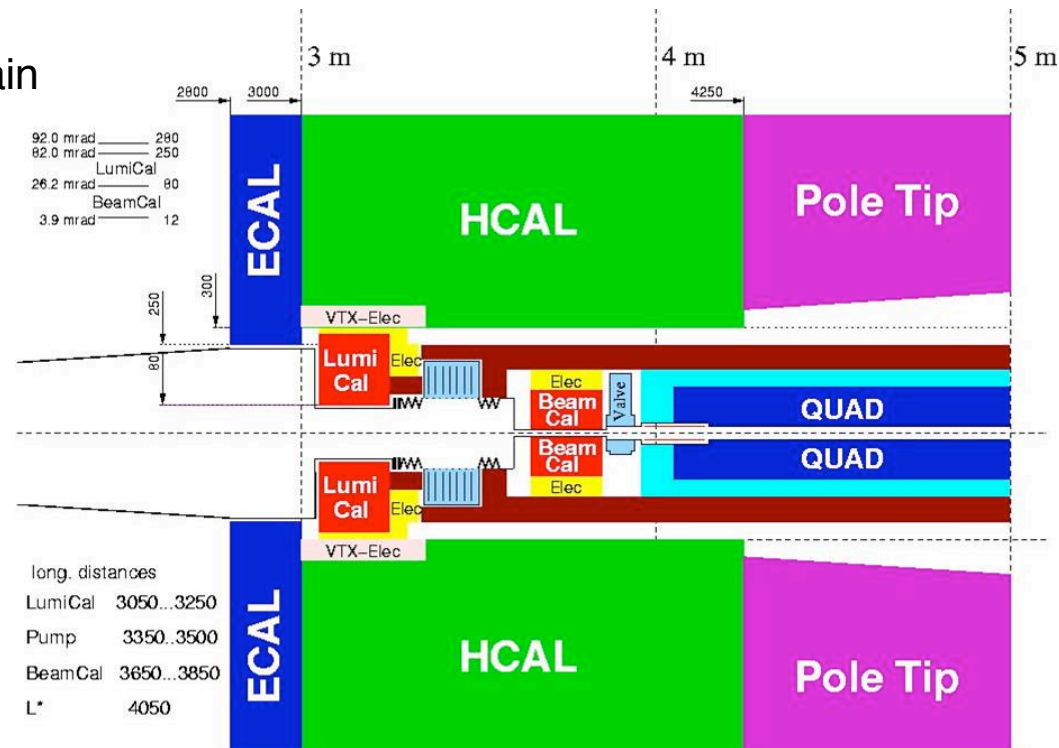
ECAL, HCAL : hadron calorimeters

LumiCal, BeamCal : electromagnetic calorimeters

Experimental features of the ILC collider

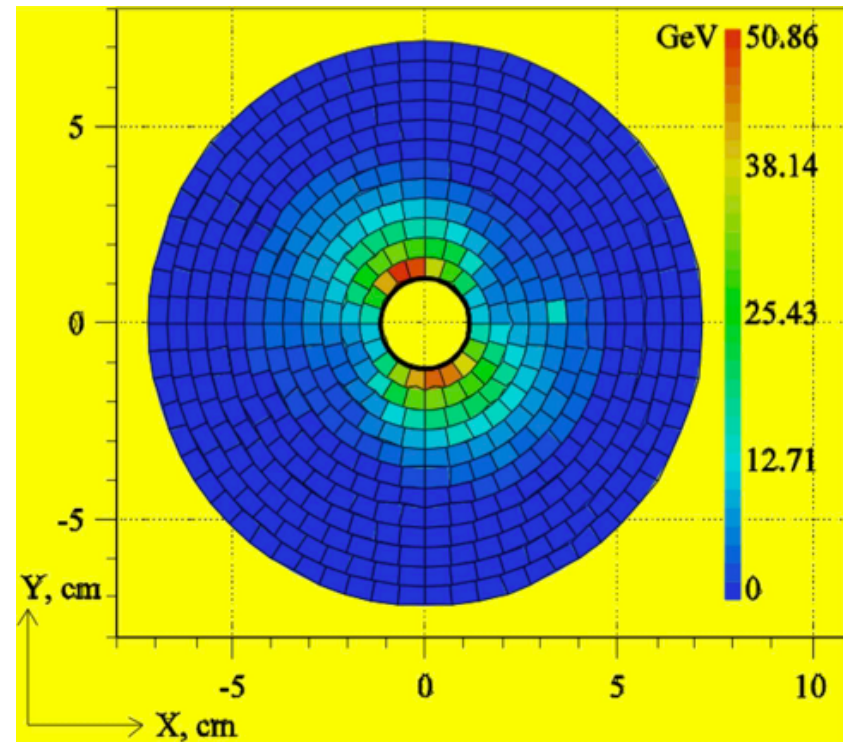Foreseen cms energy $\sqrt{s} = 2E = 500\,GeV$

LDC detector :
emc BeamCal around the beampipe at 3.65 m
from the vertex and devoted to the luminosity
measurement

It can be used for diffractive physics

Simulation of the energy density of
beamstrahlung remnants (photons..) per bunch
crossing at the front face of the BeamCal

We cut-off the cells for leptons tagging with

$\longrightarrow E_{min} = 100\,GeV$

$\longrightarrow \theta_{min} = 4\text{ mrad}$ $\Rightarrow$ access to the (very) forward region
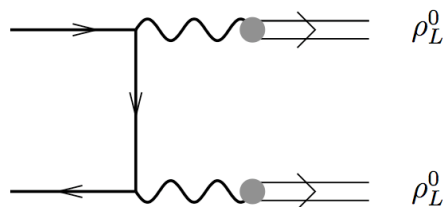
$$\frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1 Q_2}{s y_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt\, dy_1\, dy_2\, dQ_1^2\, dQ_2^2}$$

Background in the detector
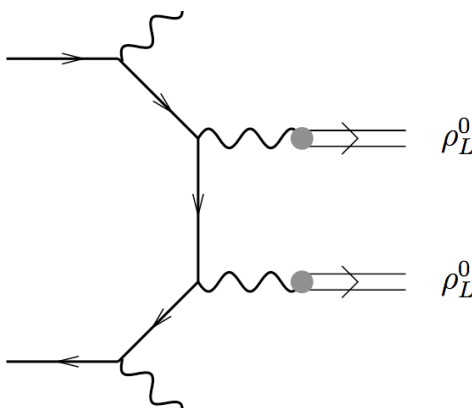
starting with :

$$e^+e^- \rightarrow \rho_L^0 \, \rho_L^0$$

$$\frac{d\sigma}{dt} = \frac{\alpha_{em}^4 f_\rho^4}{s^2 m_\rho^4}$$

we consider the competitor process :

$$e^+e^- \rightarrow \gamma\gamma\rho_L^0 \, \rho_L^0$$

$$\frac{d\sigma^{\,e^+e^- \rightarrow \gamma\gamma\rho_L\rho_L}}{dt \, dy_1 \, dy_2 \, dQ_1^2 \, dQ_2^2} \Big/ \frac{d\sigma^{\,e^+e^- \rightarrow e^+e^-\rho_L\rho_L}}{dt \, dy_1 \, dy_2 \, dQ_1^2 \, dQ_2^2}$$

$$\simeq \frac{\alpha_{em}^2 Q_1^4 Q_2^4}{\alpha_s^4 s^2 m_\rho^4}$$
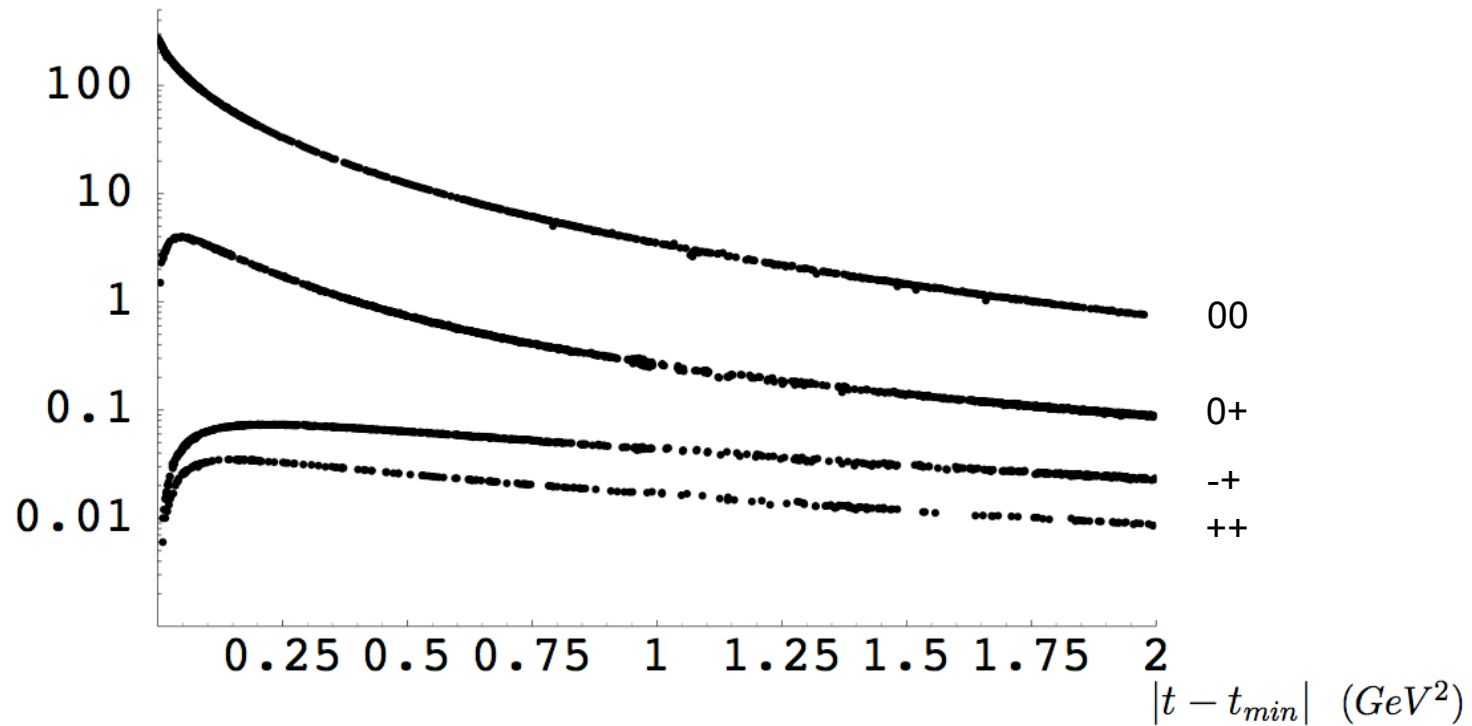
M.Davier, M.Peskin

G.T.Bodwin, E.Braaten

the background (dominated by photons which would be misidentified in BeamCal
as leptons) is completely negligible at $\sqrt{s}$ = 500 GeV

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$$e^+e^- \to e^+e^- \rho_L^0 \, \rho_L^0$$

$\frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt}(fb/GeV^2)$

00

0+

-+

++

$|t - t_{min}|$ (GeV^2)

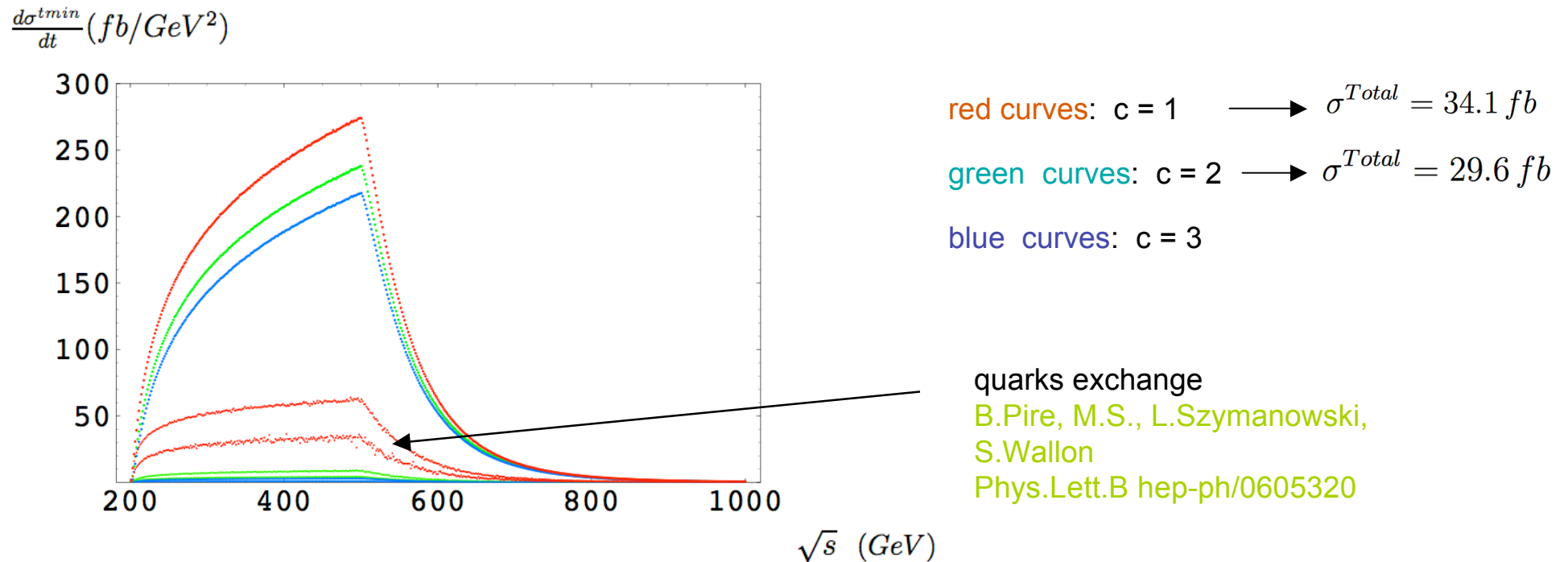$$\sigma^{LL} = 32.4\,fb$$
$$\sigma^{LT} = 1.5\,fb$$
$$\sigma^{TT} = 0.2\,fb$$

$$\sigma^{Total} = 34.1\,fb$$

with $\quad \alpha_s(\sqrt{Q_1 Q_2})$ running at three loops

$\sqrt{s} = 500 \text{GeV}$

$c = 1$

$\longrightarrow$ $4.26\,10^3$ events per year with foreseen luminosity

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Effects of parameters and quark exchange contribution to the non-forward cross-sections for $e^+e^- \to e^+e^- \rho_L^0 \rho_L^0$ at t_{min}

$\frac{d\sigma^{tmin}}{dt}(fb/GeV^2)$

red curves: c = 1 $\longrightarrow$ $\sigma^{Total} = 34.1\, fb$

green curves: c = 2 $\longrightarrow$ $\sigma^{Total} = 29.6\, fb$

blue curves: c = 3

quarks exchange
B.Pire, M.S., L.Szymanowski,
S.Wallon
Phys.Lett.B hep-ph/0605320

quarks contribution are indeed negligible. This is related to c through $s_{\gamma^*\gamma^*} > c\, Q_1\, Q_2$

a more drastic Regge limit with c=10 reduces the total cross-section by 40% which is still measurable

effects of radiative corrections (order of loops) for $\alpha_s(\sqrt{Q_1 Q_2})$ are negligible
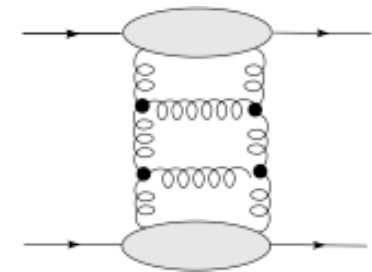
The strong suppression beyond 500 GeV comes from the detector and kinematical constraints

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Effects of parameters on the non-forward cross-sections for
$$e^+e^- \to e^+e^- \rho_L^0 \, \rho_L^0 \quad \text{with LO BFKL evolution at } t_{min}$$

$\frac{d\sigma^{tmin}}{dt}(fb/GeV^2)$
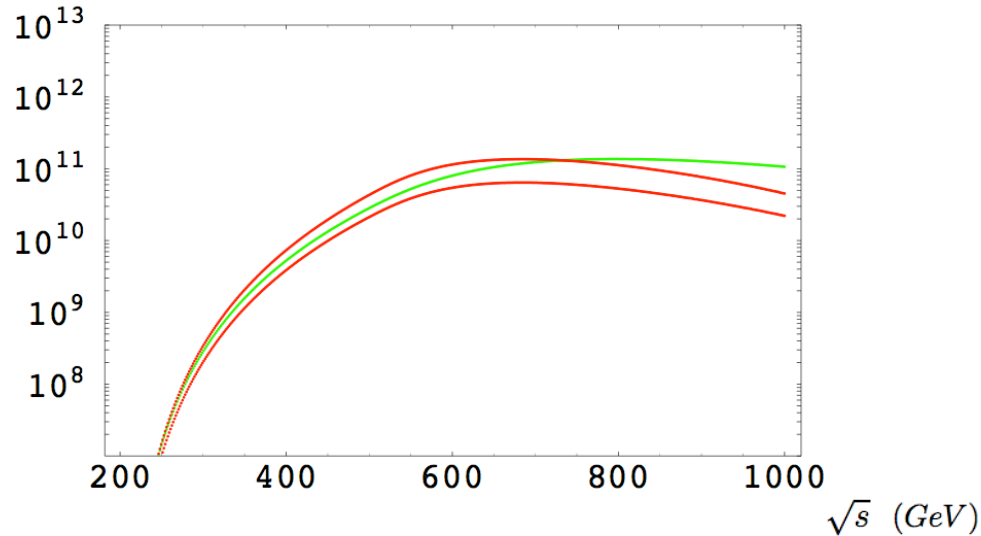
upper red curve: $\alpha_s(\sqrt{Q_1 Q_2})$ running at one loop

lower red curve: $\alpha_s(\sqrt{Q_1 Q_2})$ running at three loops

green curve: fixed value of $\alpha_s = 0.46$

forward case BFKL amplitude in the saddle-point approximation:

$$A(s, t=t_{min}, Q_1, Q_2) \sim is \, \frac{\alpha_s^2 \alpha_{em} f_\rho^2}{Q_1^2 Q_2^2} \frac{e^{4 \ln 2 \, \bar{\alpha}_s Y}}{\sqrt{14 \bar{\alpha}_s \zeta(3) Y}} \exp\left(-\frac{\ln^2 R}{14 \bar{\alpha}_s \zeta(3) Y}\right)$$

$$Y = \ln\left(\frac{c' \, s \, y_1 \, y_2}{Q_1 Q_2}\right)$$

with: $\bar{\alpha}_s = \frac{N_c}{\pi} \alpha_s(\sqrt{Q_1 Q_2})$ and $R = \frac{Q_1}{Q_2}$

flat curve to be compared with strongly decreasing curve at Born level, between 500GeV and 1TeV

fig enhancement compared to the Born order $\longrightarrow$ the NLL BFKL prediction will be between LL and Born

work based on resummed BFKL (Khoze, Martin, Ryskin, Stirling) with LL impact factor and BLM scale fixing (Enberg, Pire, Szymanowski, Wallon) is in progress

Conclusions

We gave a precise estimation of the two gluons t-channel exchange which dominates at HE, in the exclusive production of rho meson pairs at the ILC.

This evaluation corresponds to the BFKL background.

Since the impact factor are completely known in a pertubative way, not only the behaviour with energy but the complete amplitude can be analytically computed.

$\longrightarrow$ Clean test of the BFKL resummation scheme at ILC.

We demonstated the measurability of this process at the level of $e^+e^- \to e^+e^- \rho_L \rho_L$ within LDC detector and with a emc located in the forward region.

Born order evaluation $\longrightarrow$ resummed BFKL or NLO BFKL evolution fot any t.

Possibility of entering in the saturation regime when increasing the cms energy from 500 GeV ($\longrightarrow Q_{sat} \sim 1.1$ GeV) to 1 TeV ($\longrightarrow Q_{sat} \sim 1.4$ GeV)