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Probing CP Properties of the Higgs Boson via $e^-e^+ \to t\bar{t}\phi$

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in collaboration with

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- To study the New Physics effects beyond SM, we need to establish the CP eigenvalues for the Higgs states if CP is conserved, and measure the mixing between CP-even and CP-odd states if it is not.
- CP violation in the Higgs sector can be an alternative source of CP violation beyond the SM, required to explain the observed baryon asymmetry in our universe.

  [See e.g. Accomando et al., *CERN 2006-009* (2006)]

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- $t\bar{t}$ couples 'democratically' to both the CP-even and CP-odd state as opposed to a $VV(V = W/Z)$ pair.

Anomalous $t\bar{t}\phi$ Couplings

- The most general Lorentz invariant form of the $t\bar{t}\phi$ coupling is

$$g_{t\bar{t}\phi} = -ig_2 \frac{m_t}{2m_W}(a + ib\gamma_5) \ ,$$

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- We take the $ZZ\phi$ Coupling to be similar to the SM case:

$$(g_{ZZ\phi})_{\mu\nu} = -ic\frac{g_2 m_Z}{\cos\theta_W} g_{\mu\nu}$$

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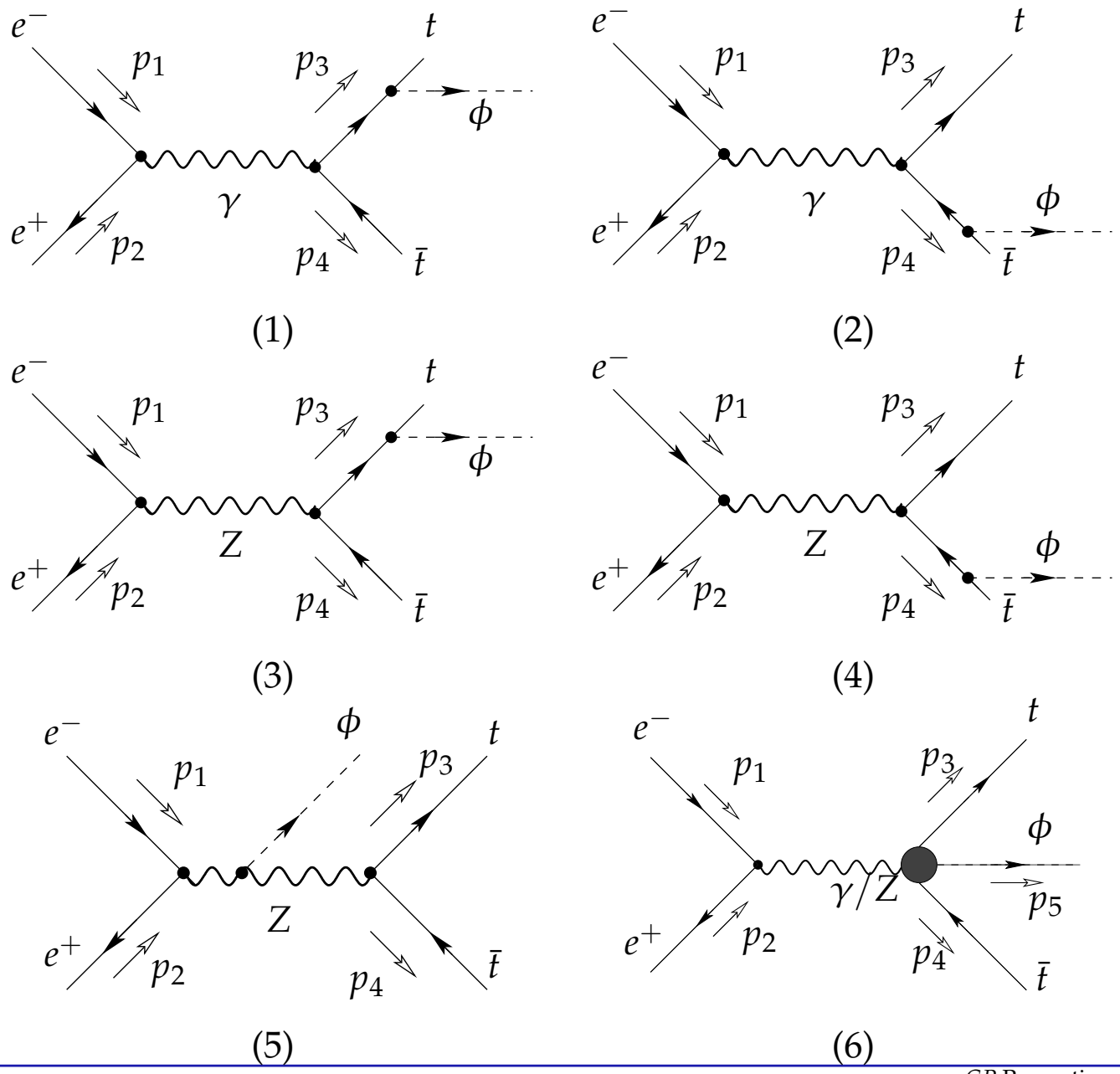
- Moreover, we treat $a,\ b,\ c$ to be all real.
- Hence only one CP-violating term ab and only independent parameter b.
- In principle, in a specifc model we may have predictions for a, b, c: e.g. THDM and CP-violating MSSM.

The Process $e^-e^+ \longrightarrow t\bar{t}\phi$: Feynman diagrams



Matrix Elements

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- The $f\bar{f}\gamma/Z$ couplings $G^{\mu}_{f_n}$ (with $n = 1, ...5$) can be written as

$$G^{\mu}_{e_n} = \gamma^{\mu}[l_{e_n} P_L + r_{e_n} P_R] \ , \ G^{\mu}_{t_n} = \gamma^{\mu}[l_{t_n} P_L + r_{t_n} P_R]$$

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- The matrix elements for the process $e^-(p_1)e^+(p_2) \to t(p_3)\bar{t}(p_4)\phi(p_5)$ are

$$\begin{aligned} \mathcal{M}_m &= C_m (J^{\mu}_{e_n}) g_{\mu\nu} (J^{\nu}_{t_n}) \ , \quad (m = 1, ..., 4) \\ \mathcal{M}_5 &= C_5 (J^{\mu}_{e_5}) g_{\mu\alpha} g^{\alpha\beta} \left(g_{\beta\nu} - \frac{k'_{\beta} k'_{\nu}}{m_Z^2} \right) (J^{\nu}_{t_5}) \end{aligned}$$

with the fermion-current structures

$$\begin{aligned} J^{\mu}_{e_n} &= \overline{v}(p_2) G^{\mu}_{e_n} u(p_1) \qquad (n = 1, ..., 5), \\ J^{\mu}_{t_{1(3)}} &= \overline{u}(p_3)(a + ib\gamma_5)(\not{q}_1 + m_t) G^{\mu}_{t_{1(3)}} v(p_4), \\ J^{\mu}_{t_{2(4)}} &= \overline{u}(p_3) G^{\mu}_{t_{2(4)}} (\not{q}_2 + m_t)(a + ib\gamma_5) v(p_4) \end{aligned}$$

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  only 8 will contribute to the Feynman amplitudes as the electron-current J_{e_n} vanishes unless e^- and e^+ have *opposite* helicity, or equivalently, unless their spinors have the *same* helicity.

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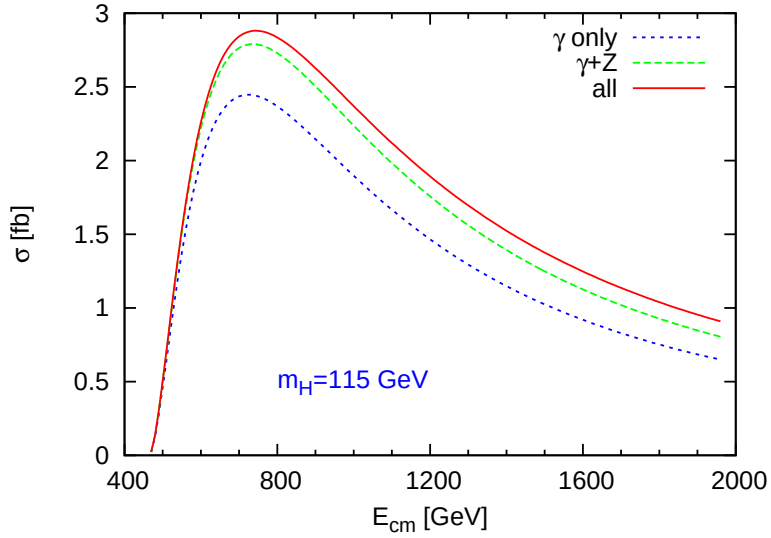
  - Bouchiat-Michel method in which the squared amplitudes $|\mathcal{M}|^2$ are calculated using the trace technique.

Total Production Cross Section

$$\sigma_{\rm tot} = \frac{1}{2^9\pi^4}\int_0^{2\pi} d\phi_4' \int_{-1}^{+1} d(\cos\theta_3) \int_{-1}^{+1} d(\cos\theta_4') \int_{(m_H+m_t)^2}^{(s-m_t)^2} d(K^2) \left(\frac{b_s}{s}\right) |\overline{\mathcal{M}}_{fi}|^2,$$

$$\text{where } b_s = \frac{|\mathbf{p}_3|}{\sqrt{s}}\frac{|\mathbf{p}_4'|}{m_K} \text{ is the phase-space volume.}$$

We first verify some of the SM results already existing in literature [Gaemers and Gounaris, *Phys. Lett.* **77B**, 379 (1978); Djouadi, Kalinowski and Zerwas, *Z. Phys.* **C 54**, 255 (1992)]:



Justifies choice of the $ZZ\phi$ coupling.

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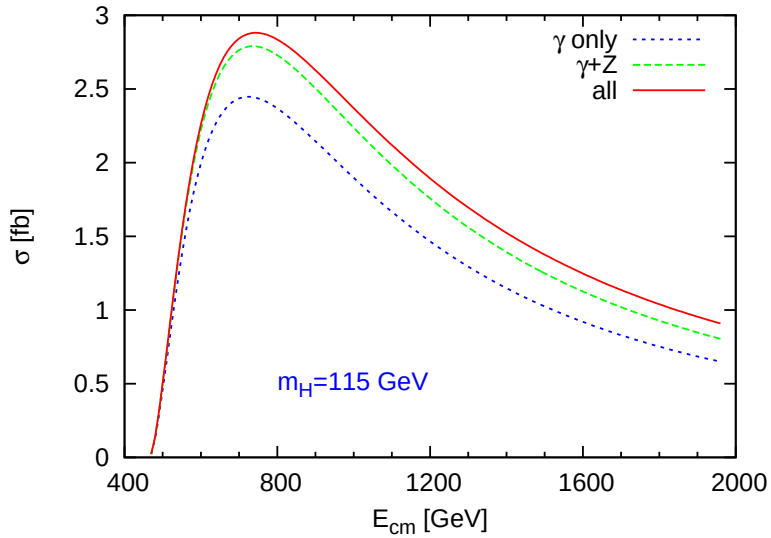

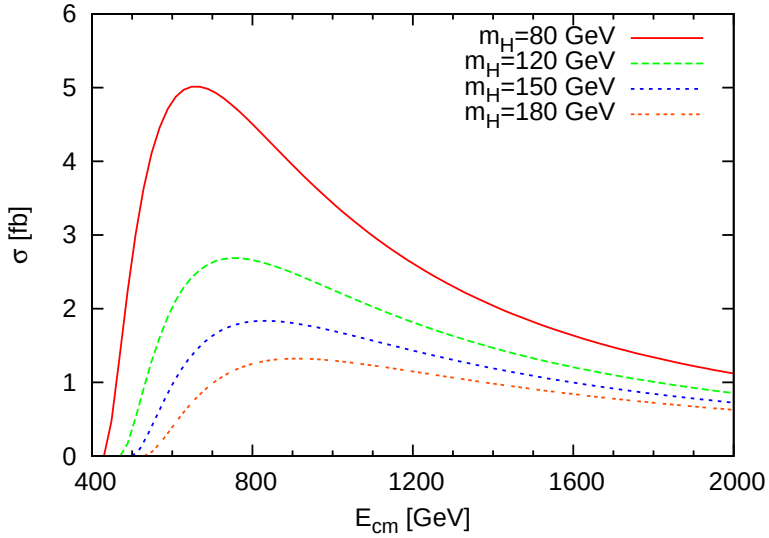

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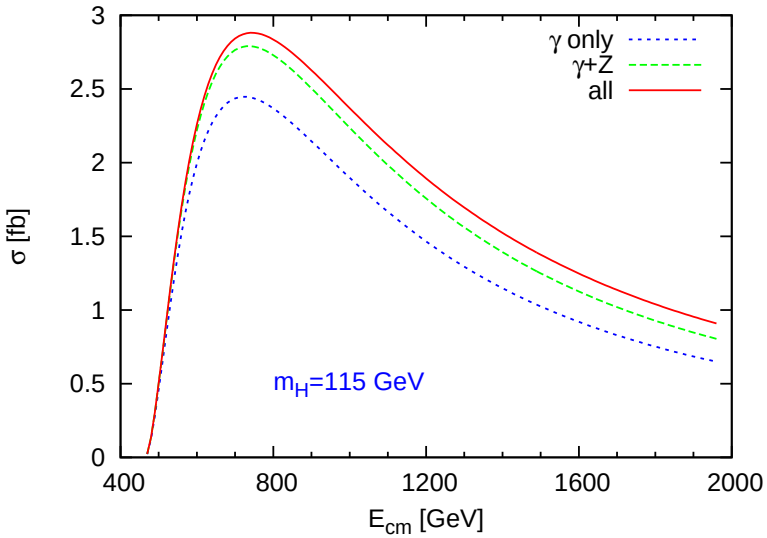
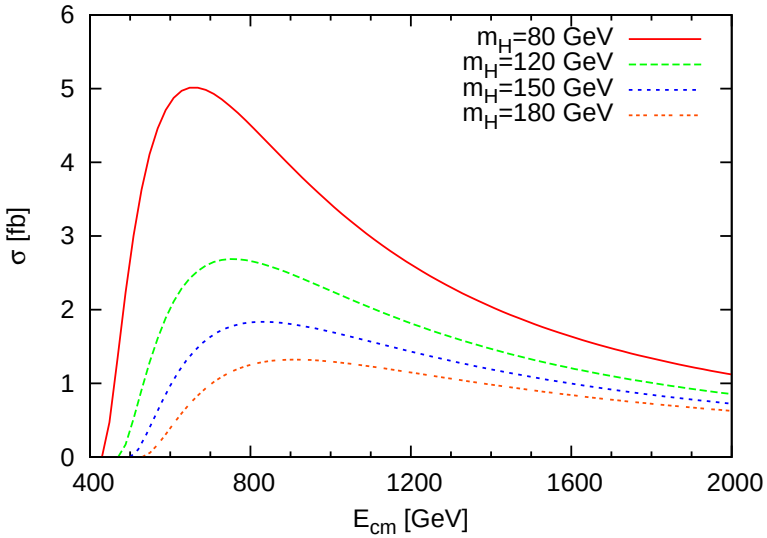
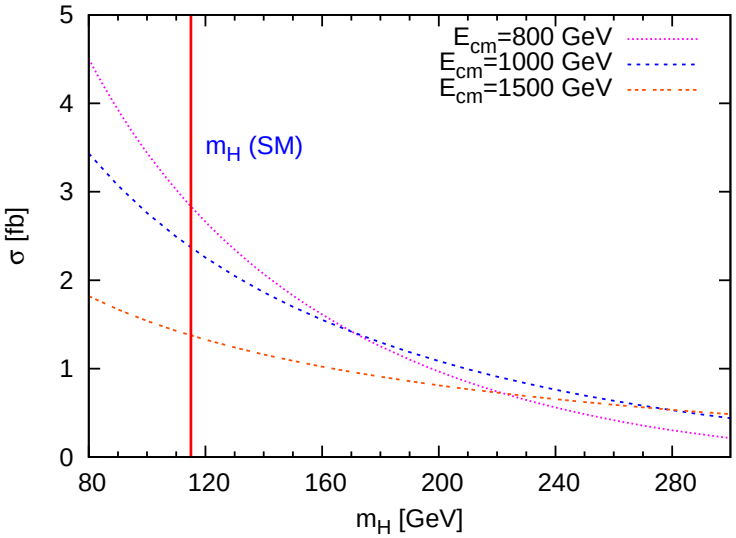
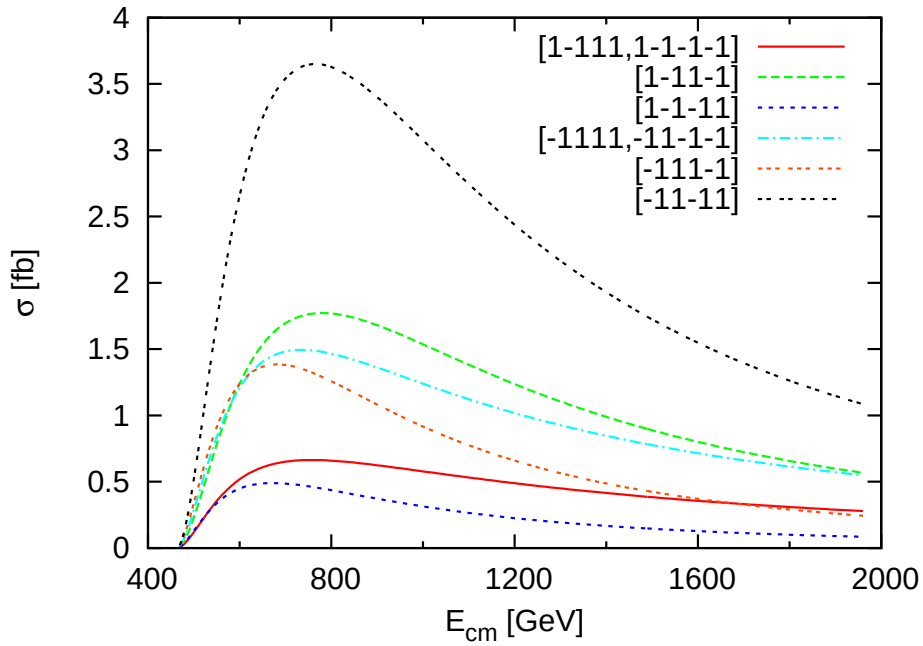






Contributions of various Helicity States to $\sigma_{\rm tot}$

$$\sigma(h_1,h_2,h_3,h_4) = \frac{1}{2^9\pi^4}\int_0^{2\pi} d\phi_4' \int_{-1}^{+1} d(\cos\theta_3) \int_{-1}^{+1} d(\cos\theta_4') \int_{(m_H+m_t)^2}^{(s-m_t)^2} d(K^2) \left(\frac{b_s}{s}\right) \times |\overline{\mathcal{M}}_{fi}(h_1,h_2,h_3,h_4)|^2$$



- The contributions from various states differ in magnitude because of the fact that the Z boson couples to left- and right-handed fermions with different strengths ($l_e = -1 + 2\sin^2\theta_W,\ r_e = 2\sin^2\theta_W;\ l_t = 1 - \frac{4}{3}\sin^2\theta_W,\ r_t = -\frac{4}{3}\sin^2\theta_W$).

Probe of the Pseudo-scalar $t\bar{t}\phi$ Coupling

- Recall the generalized $t\bar{t}\phi$ and $ZZ\phi$ couplings:

$$g_{t\bar{t}\phi} = -ig_2 \frac{m_t}{2m_W}(a + ib\gamma_5) \ ,$$

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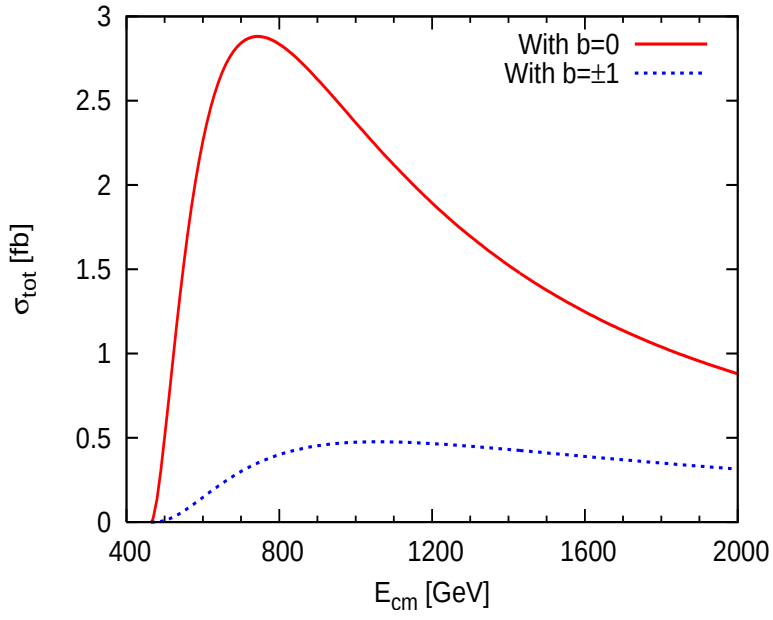
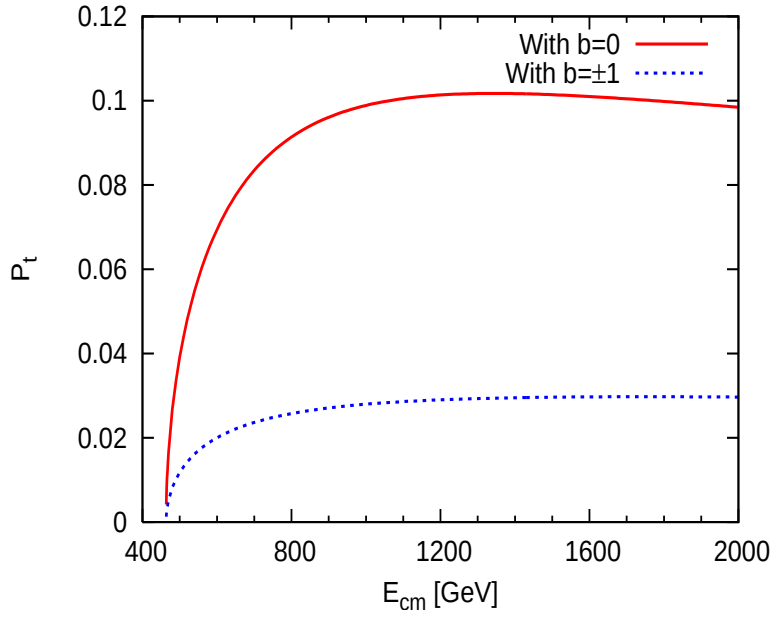
- The Polarization Asymmetry for top-quark is given by

$$\begin{aligned}
P_t &= \frac{\sigma(t_L) - \sigma(t_R)}{\sigma(t_L) + \sigma(t_R)} \qquad \text{(with unpolarized initial beams)}, \\
P_t^e &= \frac{\sigma_t^e(t_L) - \sigma_t^e(t_R)}{\sigma_t^e(t_L) + \sigma_t^e(t_R)} \qquad \text{(with polarized initial beams)}, \\
\text{with } \sigma_{\text{tot}}(\text{unpolarized}) &= \frac{1}{4}\,[\sigma_{RL} + \sigma_{LR}], \\
\text{and } \sigma_t^e(\text{polarized}) &= \frac{1+P_{e^-}}{2}\,\frac{1-P_{e^+}}{2}\,\sigma_{RL} + \frac{1-P_{e^-}}{2}\,\frac{1+P_{e^+}}{2}\,\sigma_{LR}
\end{aligned}$$

($\sigma_{RL(LR)}$ corresponds to the completely polarized $e^-_{R(L)}e^+_{L(R)}$ beams)

Variation with $E_{\rm cm}$ for $b = 0$ and $b = 1$





- The cases $b = 0$ and $b = 1$ are clearly distinguishable from each other in both the measurements.

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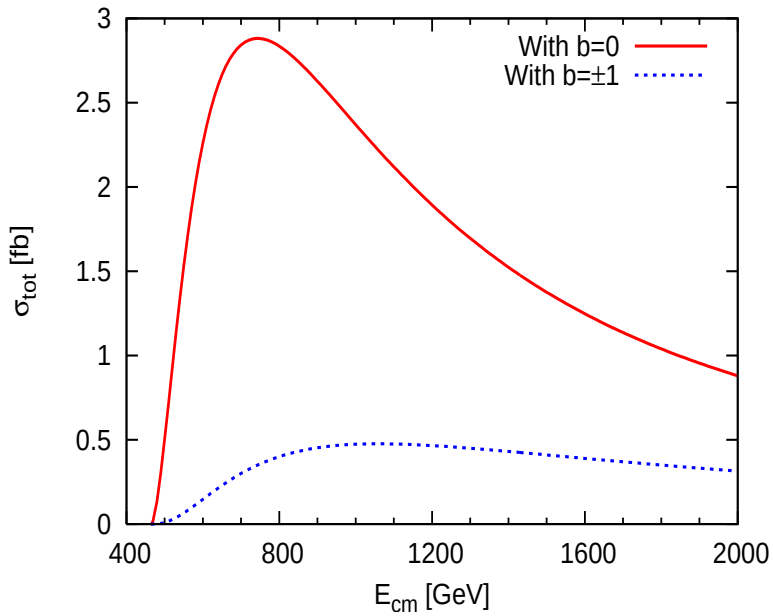

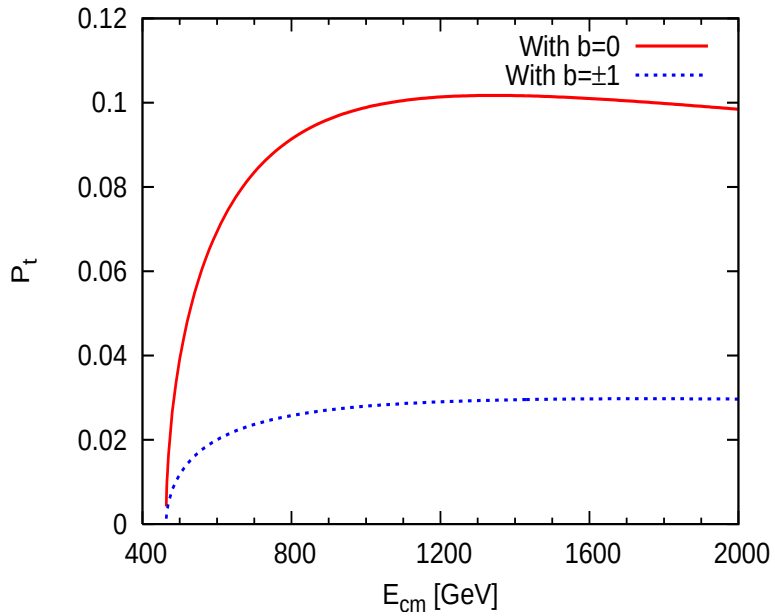

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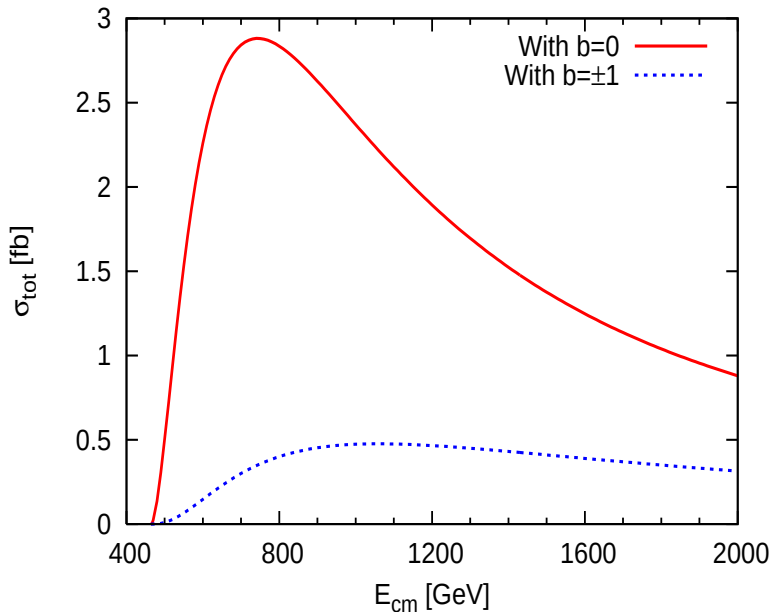

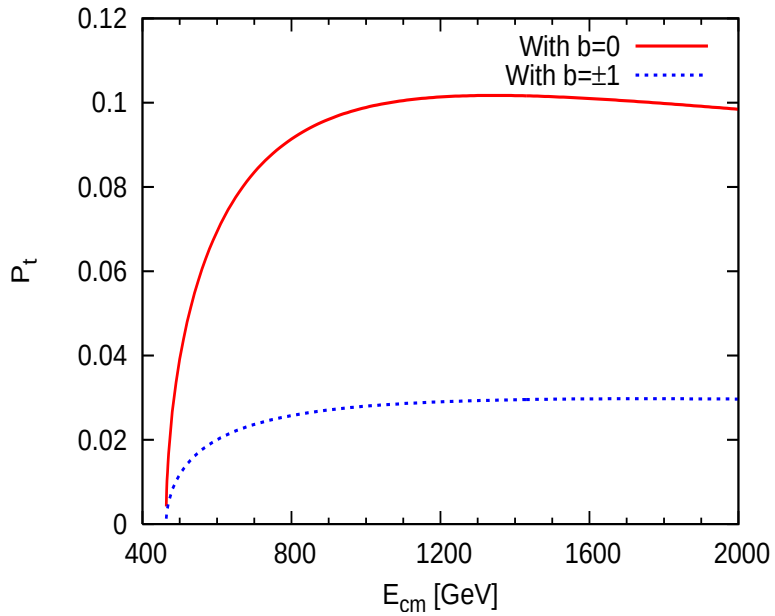

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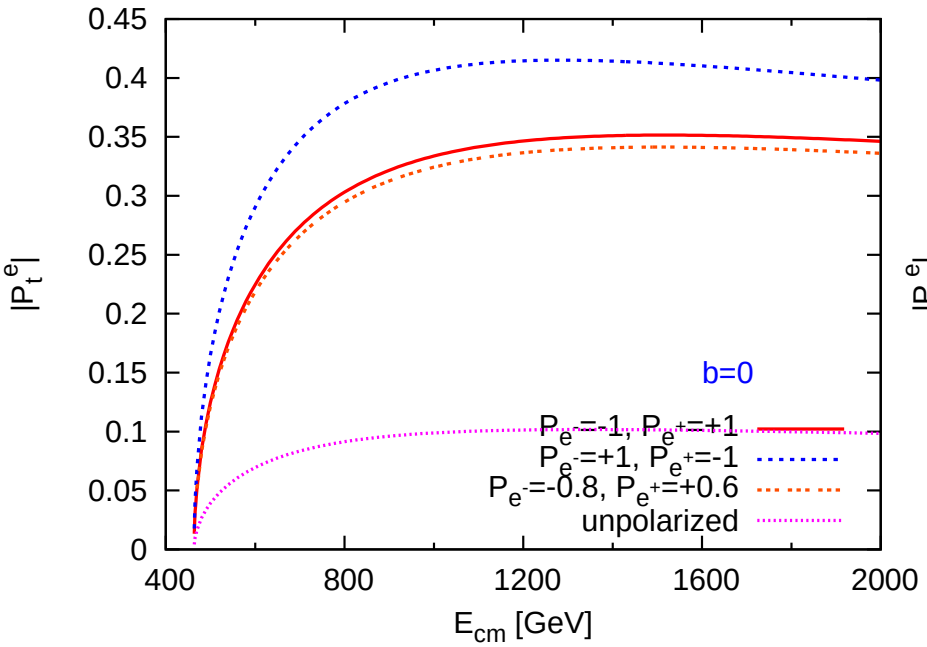
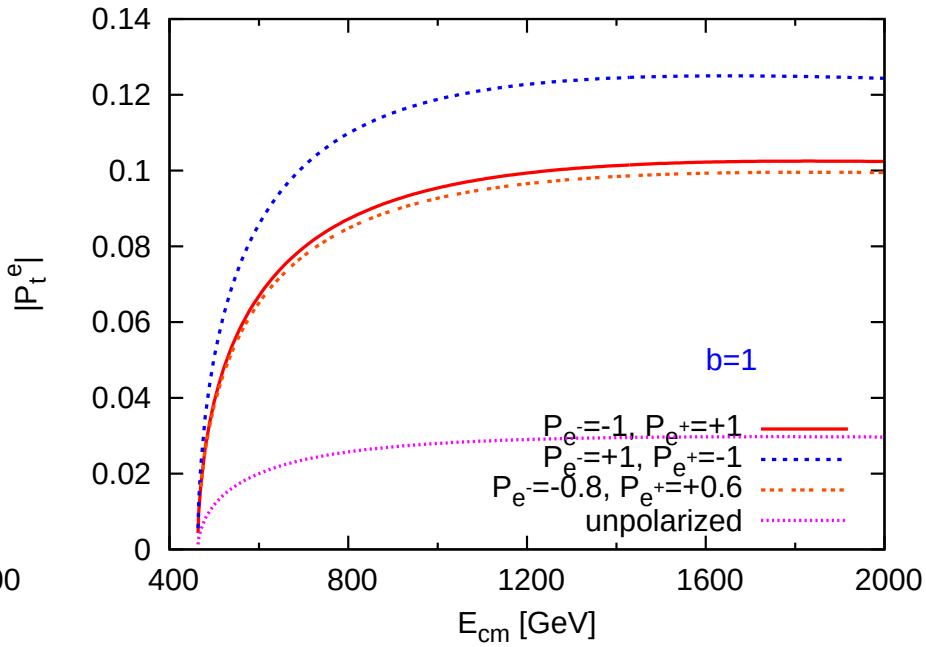
- However, in practice, the cross section values receive higher order radiative corrections from various sectors while the polarization asymmetry may be insensitive to these corrections, these being universal and would be indep. of polarisation of the t.
- Hence, the polarization asymmetry can be a very useful observable to probe b. (Recall that the top polarization can be measured accurately as it decays before hadronization can take place.)

Variation with $E_{\rm cm}$ for $b = 0$ and $b = 1$ (contd.)





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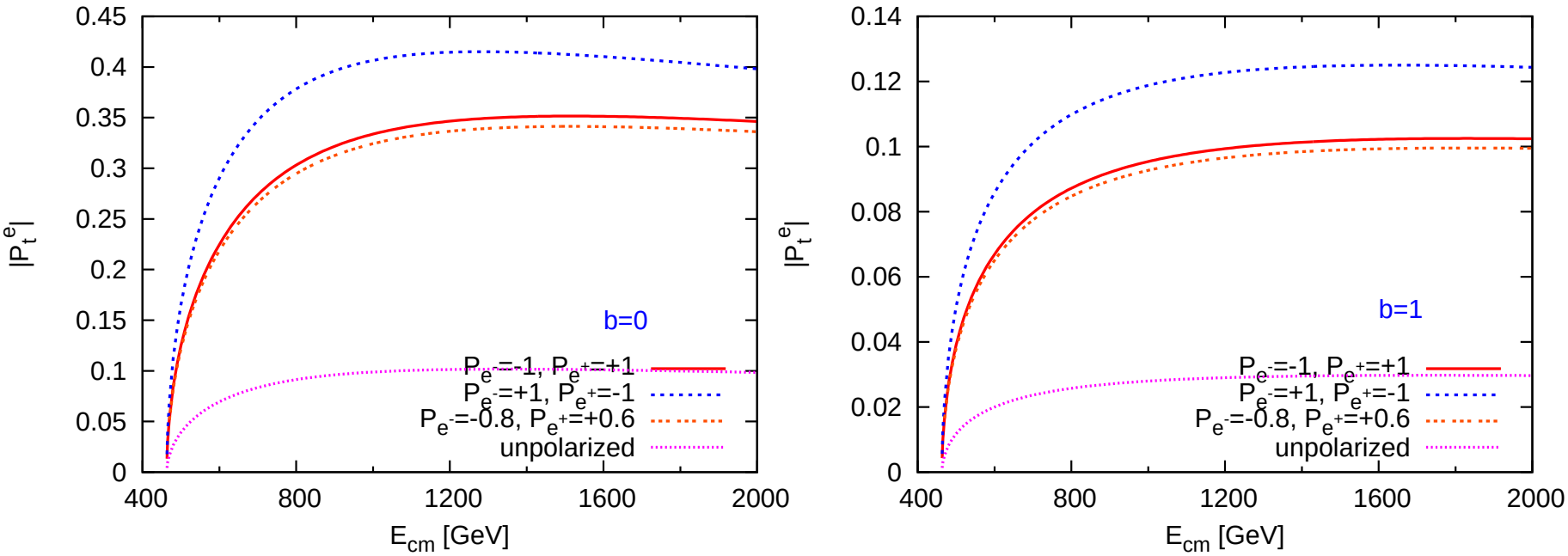

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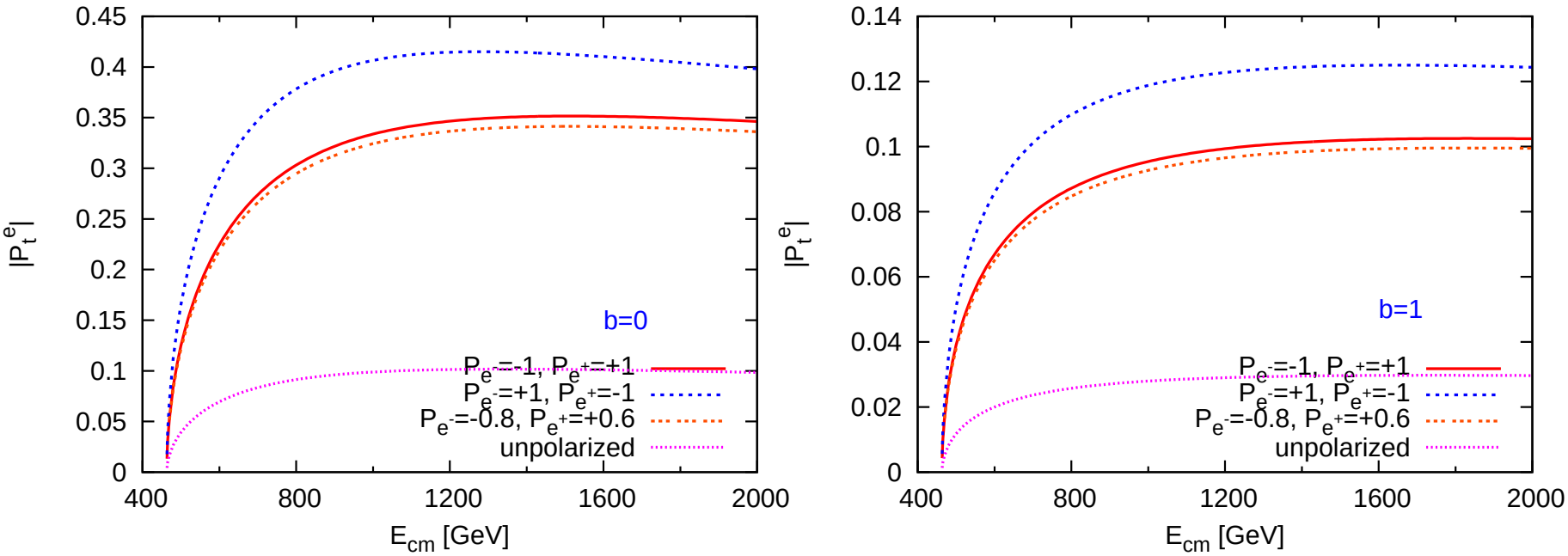

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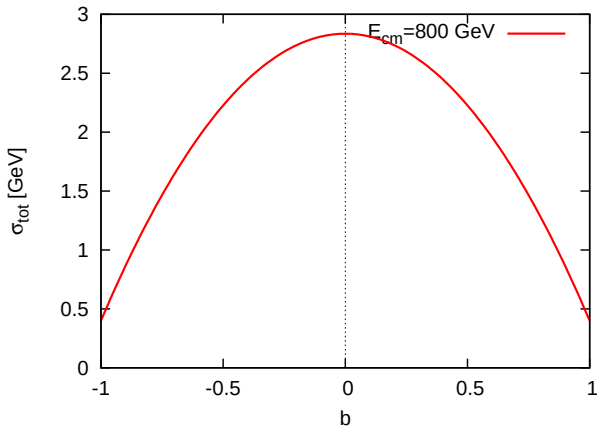
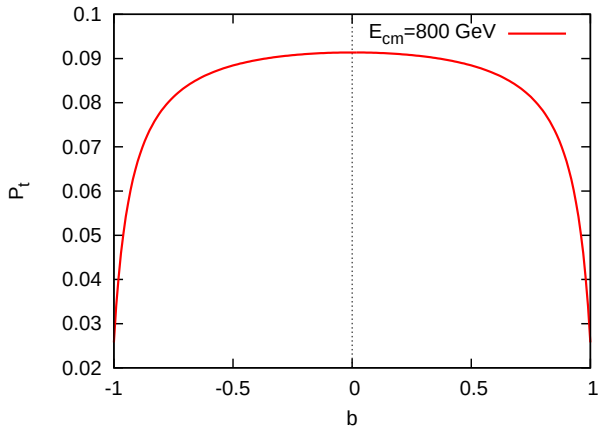
- We choose the realistic values $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$ for our sensitivity analysis.

Variation of σ and P_t with b





- Quadratic variation of σ (integrated over the whole phase space) with b:

$$\sigma_{\text{tot}} = [x_t - y_t b^2]\text{fb}\,, \quad \text{andhence,} \quad P_t = \frac{x_{lr} - y_{lr} b^2}{x_t - y_t b^2}$$

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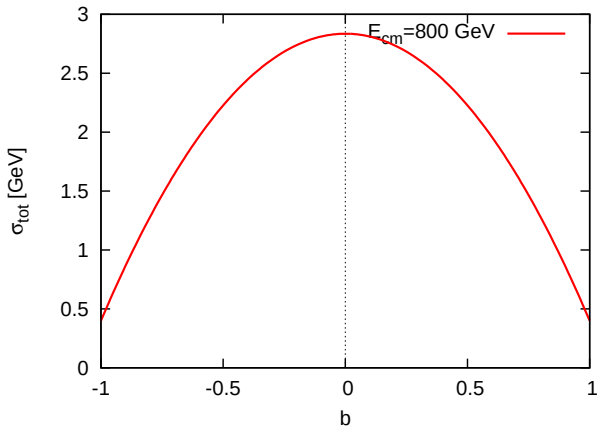

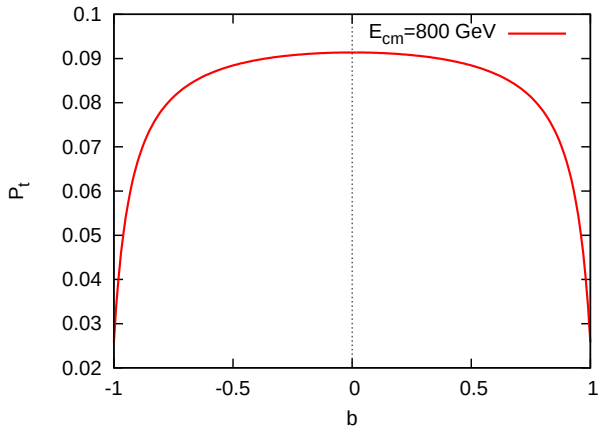

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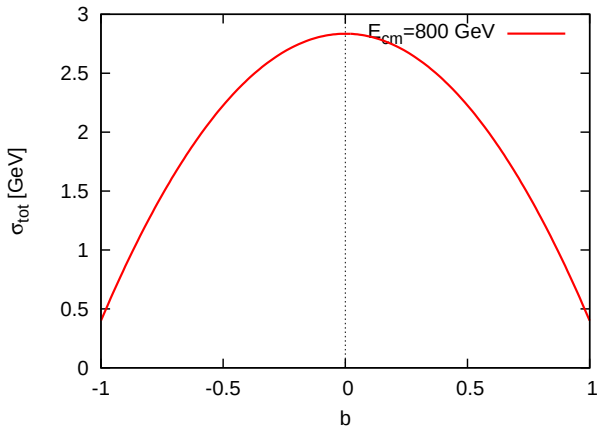

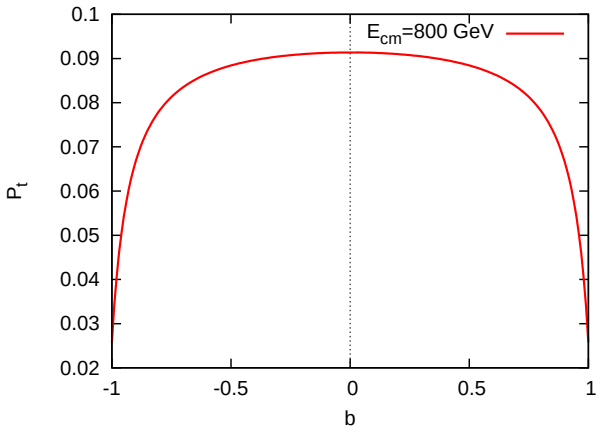

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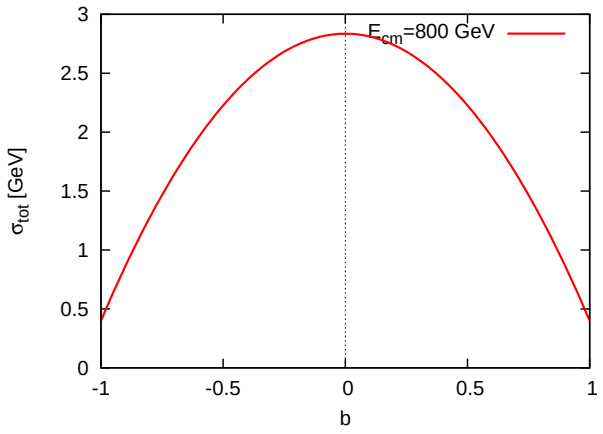

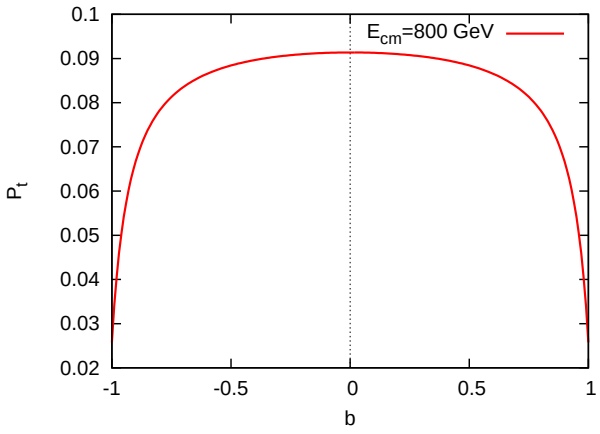

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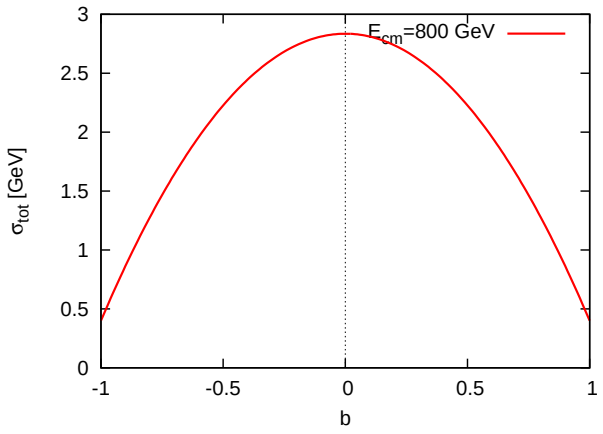

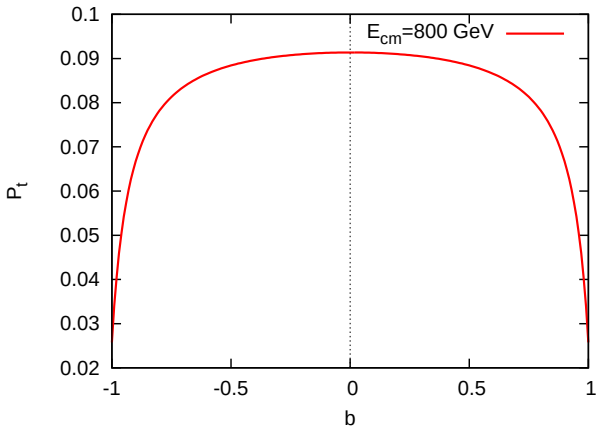

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- We have to construct CP-odd observables for which the ab term is non-zero, in order to probe the CP-mixed state of the Higgs boson.
- Up-down asymmetry of the $\bar{t}$ production w.r.t. the e^- - t plane is an example of such a CP-odd observable.

Up-Down Asymmetry

- The up-down asymmetry of the $\bar{t}$ production w.r.t. the e^- - t plane ($\phi_4' = 0$) is given by

$$A_\phi = \frac{\sigma_{\text{partial}}(0 \leq \phi_4' < \pi) - \sigma_{\text{partial}}(\pi \leq \phi_4' < 2\pi)}{\sigma_{\text{partial}}(0 \leq \phi_4' < \pi) + \sigma_{\text{partial}}(\pi \leq \phi_4' < 2\pi)},$$

$$\text{with } \sin\phi_4' = \frac{\vec{P} \cdot (\vec{p}_3 \times \vec{p'}_4)}{|\vec{P}| \cdot |\vec{p}_3 \times \vec{p'}_4|} \qquad (\vec{P} \equiv \vec{p}_1 - \vec{p}_2)$$

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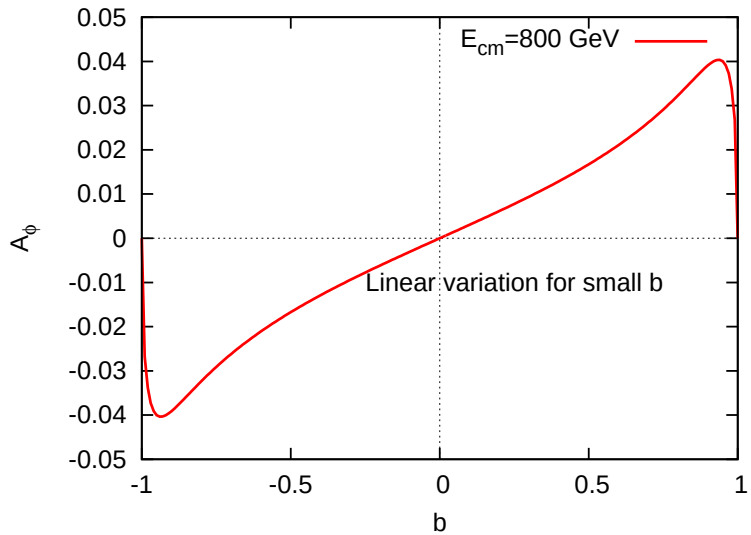

Sensitivity

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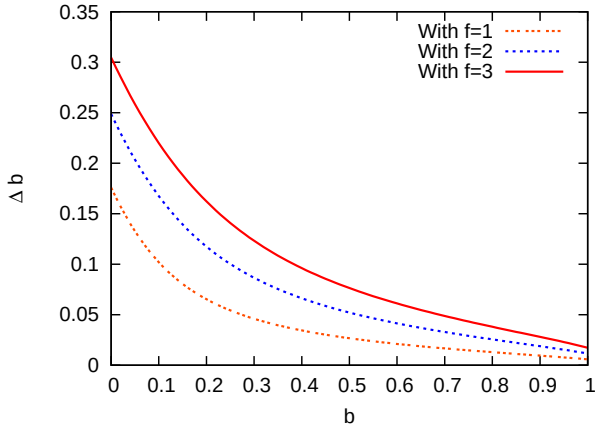
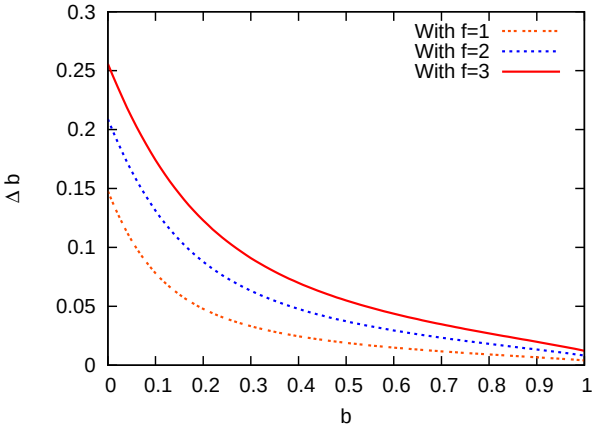
- The sensitivity study for the up-down asymmetry is in progress!

Sensitivity Plots

For cross section measurements



unpolarized



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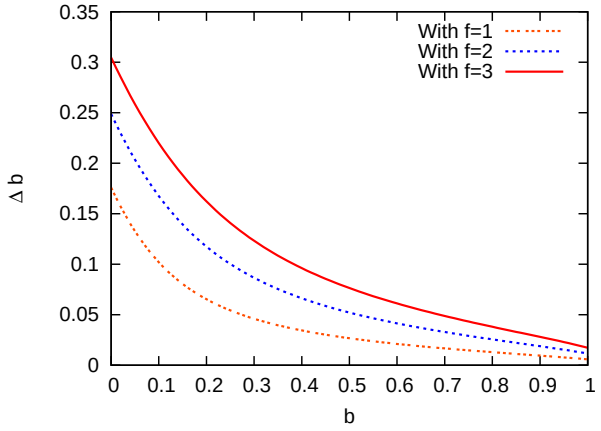

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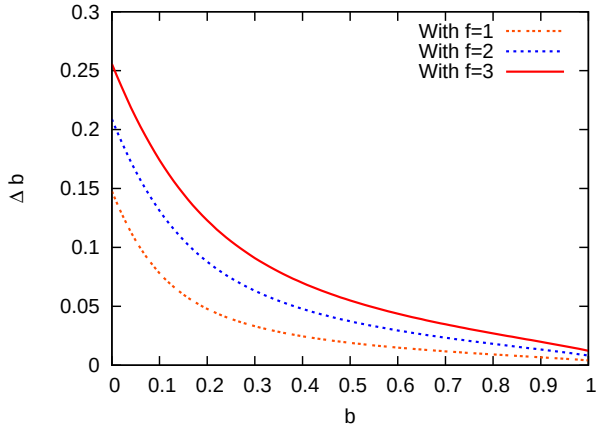

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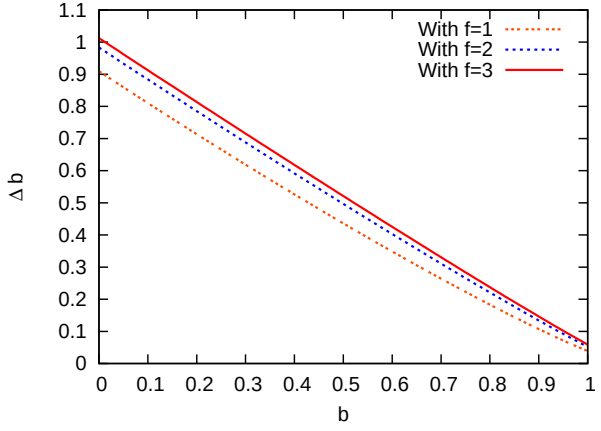

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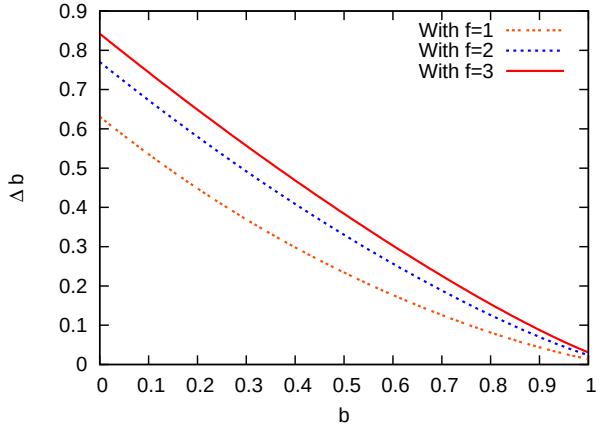

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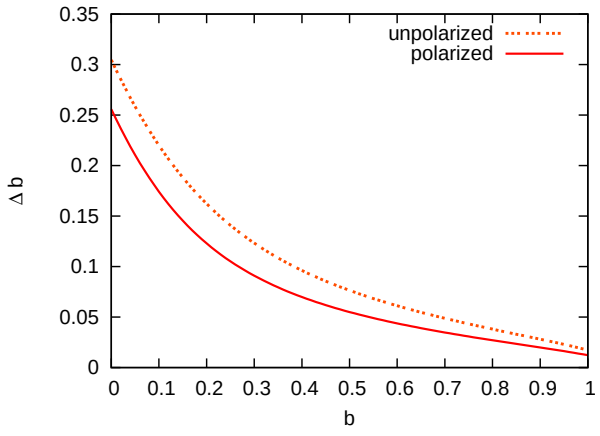

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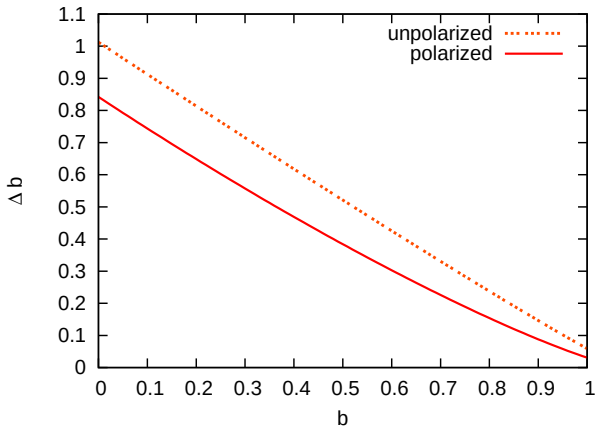

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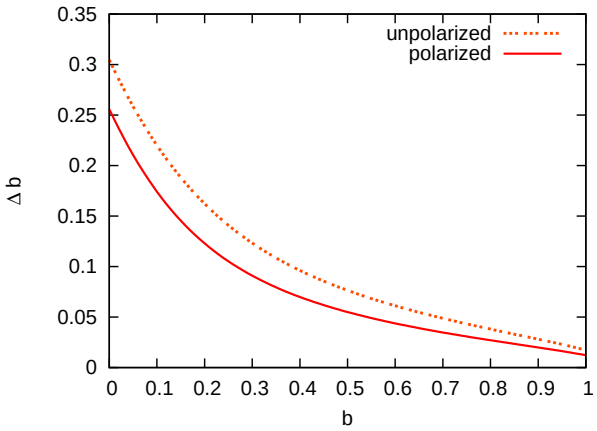

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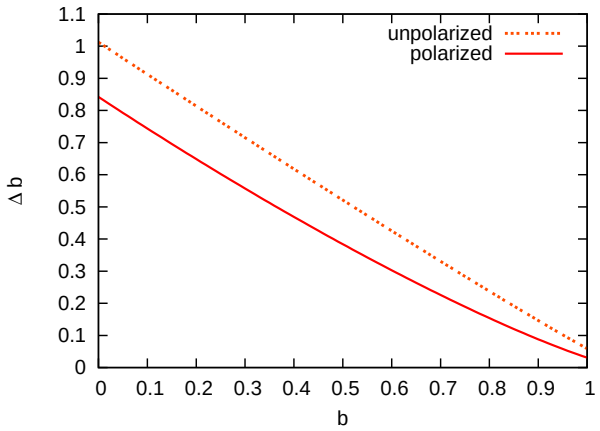

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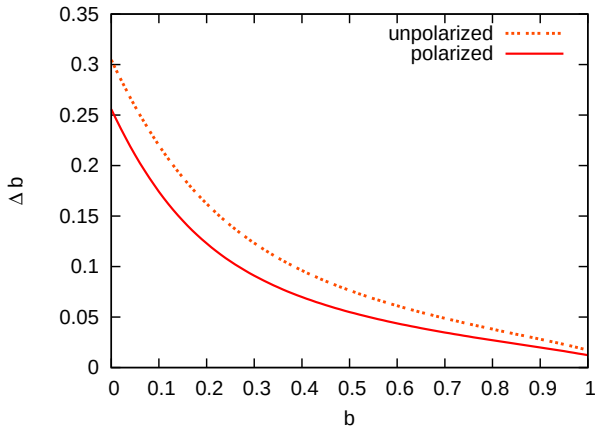

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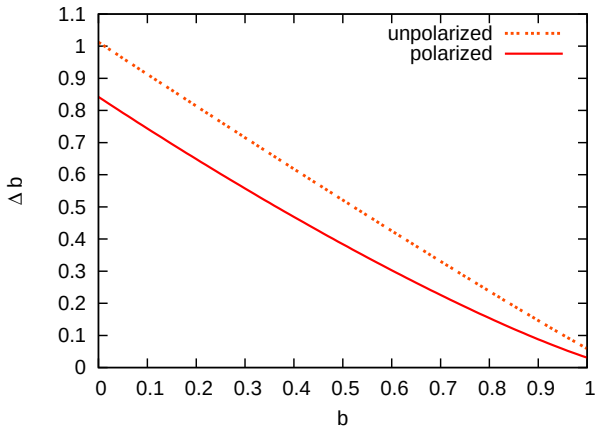

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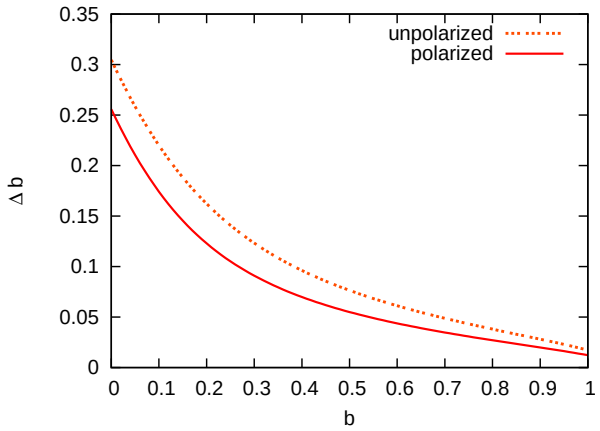

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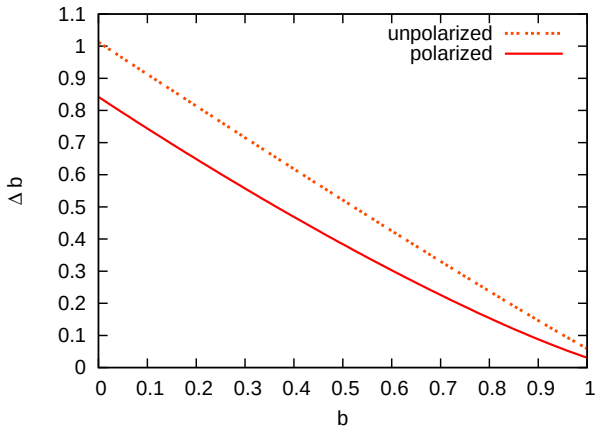

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- This is, anyway, useful in certain cases where other conventional ways can not distinguish a pure CP-odd state from a CP-even state having the same mass.

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- Including the top decay part and calculating the angular distributions of the decay lepton products which are known to be true probes of the non-standard effects in the t-production.
  [Godbole, Rindani, and Singh, *JHEP* **12**, 021 (2006)]

Thank you !