Loops — Summary

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SFB/TR 9

Outline

1. General overview: “Loops for ILC”
2. Contributions in this workshop

- Bhabha scattering
- NLO to 4 particle production
- Sudakov logarithms
- NNLO jets
- 4-loop integrals
- Precision for the MSSM
- Tools
- Other loops

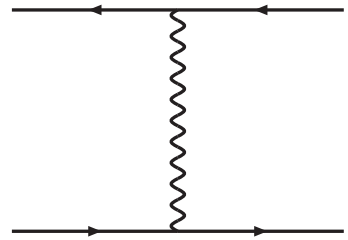
Loops for ILC

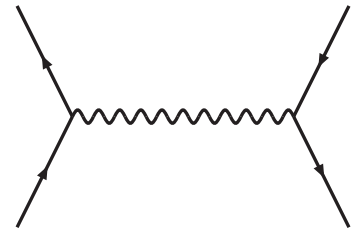
- ILC needs precision ➪ ILC needs loops
- Giga-Z: Z peak + WW threshold
- top threshold
- jets, many particles in final state
- Higgs
- SUSY
- ...

Difficulty	$\sim$	# loops $\oplus$ # legs $\oplus$ # scales

Bhabha scattering

- $e^+e^- \to e^+e^-$





- Luminosity at e^+e^- colliders: $\mathcal{L} \sim \frac{\#(e^+e^-)_{\rm obs}}{\sigma^{\rm theory}_{\rm Bhabha}}$
  - Small-angle scattering: LEP, SLC, ILC (Giga-Z)
  - Large-angle scattering: DAΦNE, KEKB, PEP-II, BEPC, VEPP-2M, . . .
- Tool: MC generators (e.g. BHLUMI [Jadach et al.], BABAYAGA [Balossini et al.], . . .)
- NNLO corrections needed!

NNLO corrections

- 2 approaches:

1) general 2-loop diagrams (s, t and m_e^2)

- (a) reduce to MIs; (b) evaluate MIs;
  (c) limit: $m_e^2/s \to 0$, keep leading term

➪ Fermionic corrections involving m_μ or m_τ

[Actis,Czakon,Gluza,Riemann'07] (Same result as [Becher,Melnikov'07] obtained with approach 2).)

2) Consider $m_e^2 \ll s$ from the very beginning

- expansion-by-region [Smirnov]
- "IR matching": $\boxed{\frac{1}{\epsilon} \quad \leftrightarrow \quad \ln\lambda(\mathrm{IR}), \ln m_e(\mathrm{coll.})}$

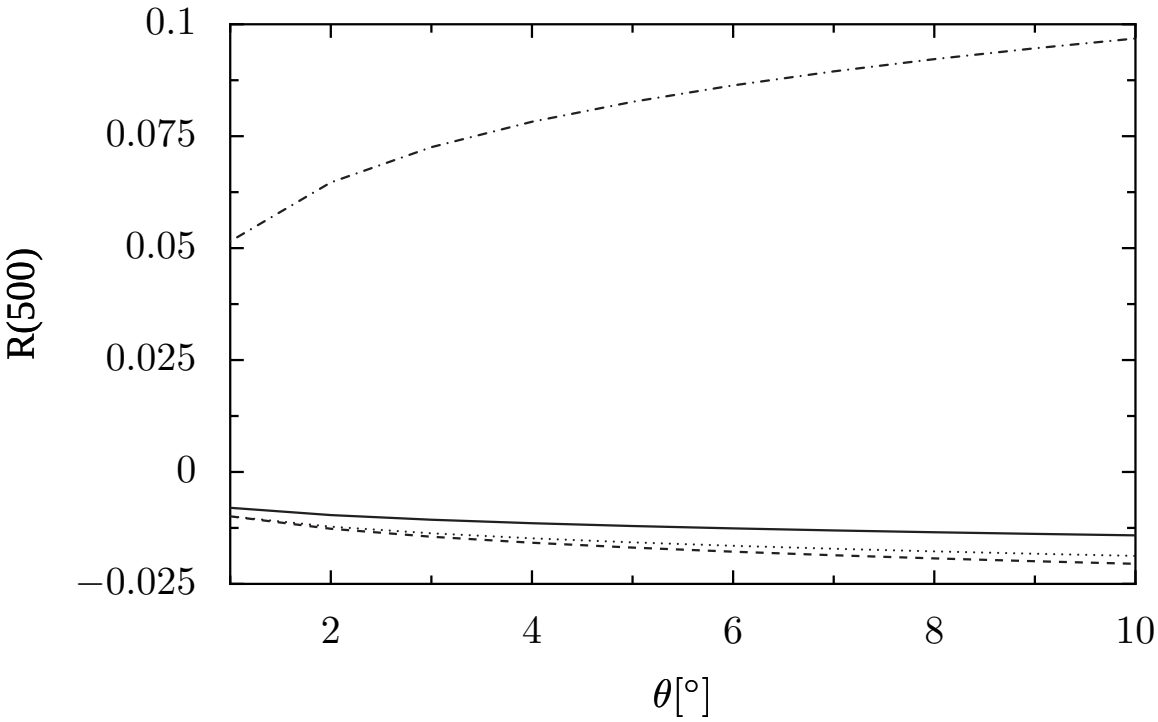
  ➪ photonic corrections [Penin'05]
- More general approach to relate massless and massive amplitudes: ➪ independent check

$$\boxed{\frac{1}{\epsilon} \quad \leftrightarrow \quad \frac{1}{\epsilon}(\mathrm{IR}), \ln m_e(\mathrm{coll.})}$$

[Becher,Melnikov'07]

NNLO corrections

- NNLO QED corrections:
  - fermionic corrections (e): [Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij'04'05]
  - photonic corrections: [Penin'05;Becher,Melnikov'07]
  - fermionic corrections (μ, τ): [Becher,Melnikov'07,Actis et al.'07]



from [Actis,Czakon,Gluza,Riemann'07]

- TODO: "1-loop+γ" $\otimes$ "Born", include in MC, hadr. contr., ...

NNLO corrections

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  - fermionic corrections (e): [Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij'04'05]
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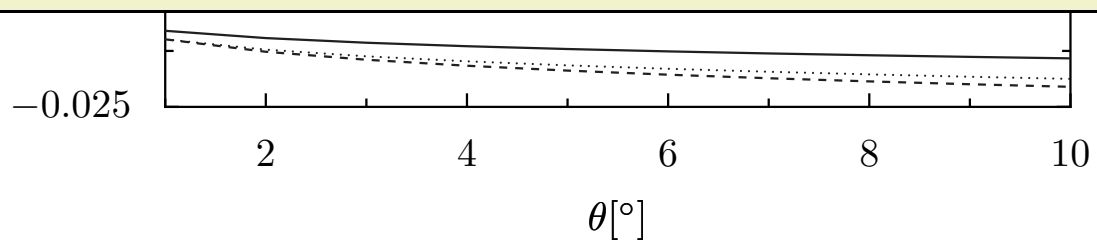
  - fermionic corrections (μ, τ): [Becher,Melnikov'07,Actis et al.'07]

- New method proposed to evaluate IR divergent 1-loop 5-point functions
  Mellin Barnes, numerical evaluation

  [talk by T. Riemann]

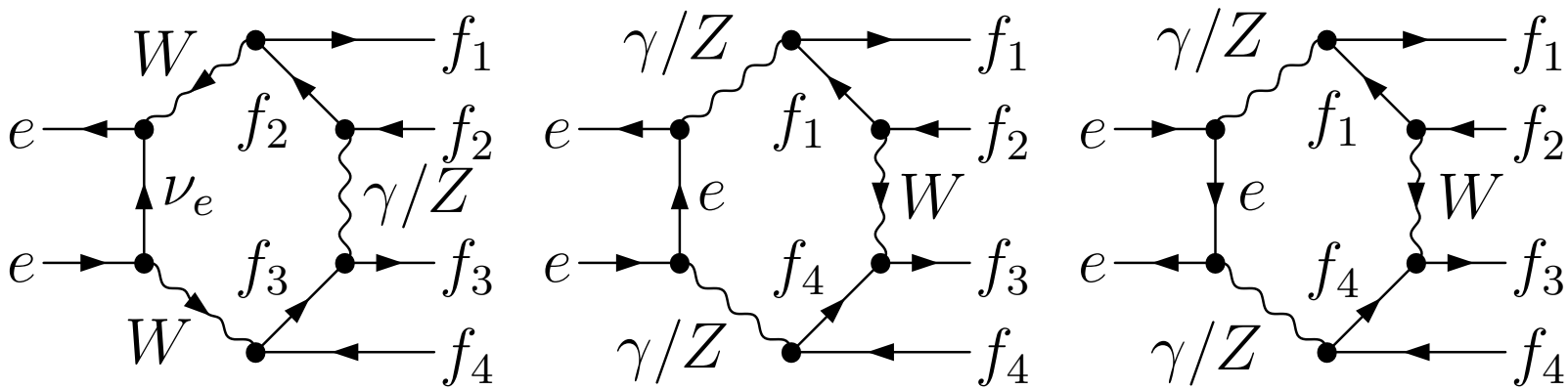


from [Actis,Czakon,Gluza,Riemann'07]

- TODO: "1-loop+γ" $\otimes$ "Born", include in MC, hadr. contr., ...

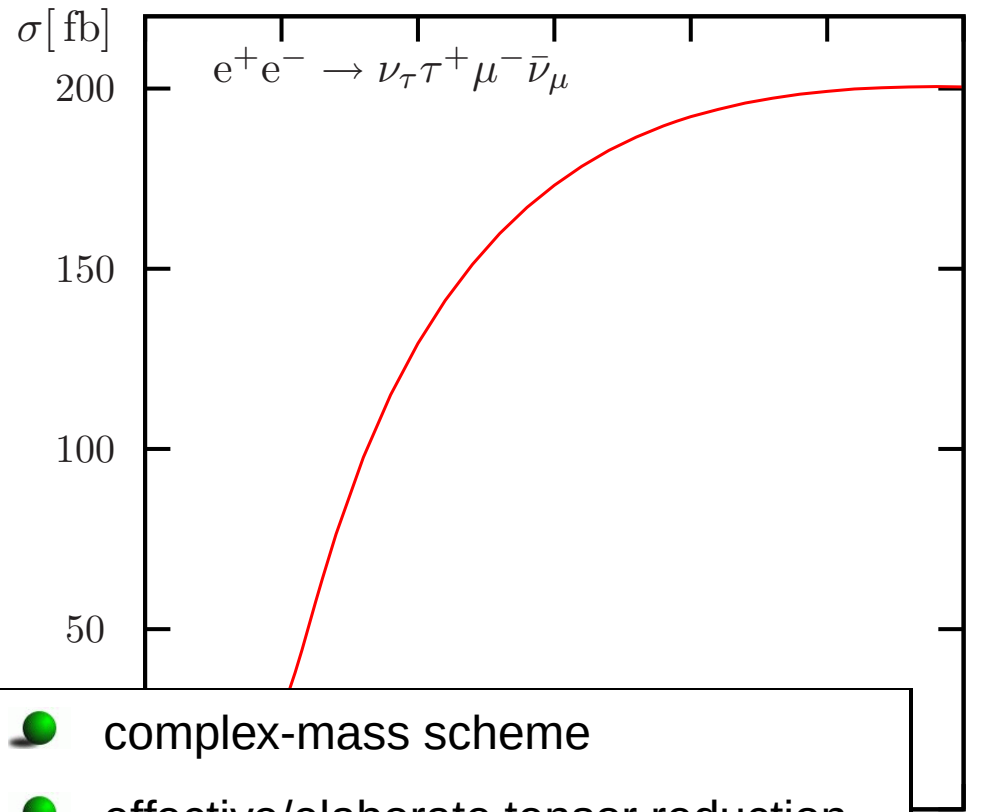
NLO to multi particle production

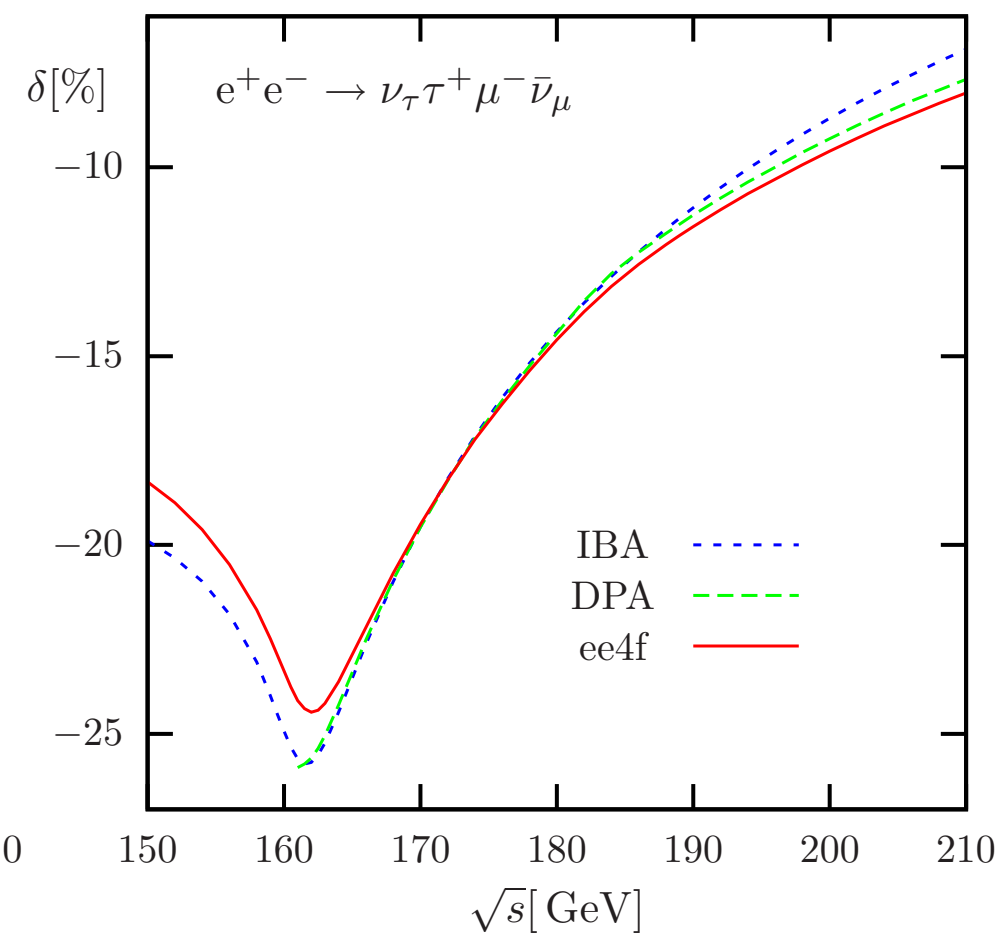
- $e^+e^- \to 4f$ [Denner,Dittmaier,Roth,Wieders'05]
- $e^+e^- \to \nu\bar{\nu}HH$ [GRACE'05]
- $H \to WW/ZZ \to 4f$
  MC event generator PROPHECY4F [Bredenstein,Denner,Dittmaier,Weber'06]



$e^+e^- \to$ 4 fermions

[Denner,Dittmaier,Roth,Wieders'05]





- complex-mass scheme
- effective/elaborate tensor reduction
  [Denner,Dittmaier'05]
- M_W from WW threshold
- TGC alaysis at high energies

LCWS07

W-pair production near threshold with unstable particle effective theory

M_W **measurement from threshold scan at ILC** with error $\lesssim 6$ MeV needs $\sigma(e^+e^- \to 4f)$ with accuracy $\lesssim 0.6\%$ (G.Wilson 01)

Unstable particle effective theory: (Beneke, Chapovsky, Signer, Zanderighi 03)

- Systematic expansion in α and Γ/M
- Hard fluctuations are integrated out, effective theory for non-relativistic Ws, soft and Coulomb-photons.
- Simpler than full NLO $e^+e^- \to 4f$ calculation

W**-pairs near threshold to NLO** (Beneke, Falgari,C.S., Signer, Zanderighi)

- Calculation of total cross section $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}$ completed
- $\sim 0.6\%$ agreement with full $4f$ calculation (Denner et.al. 05):

$\sqrt{s}$ [GeV]	EFT	ee4f [DDRW]	DPA [DDRW]
161	117.41(5)	118.12(8)	115.48(7)
170	399.9(2)	401.8(2)	402.1(2)

- Large remaining $\sim 2\%$ uncertainties from ambiguities in ISR

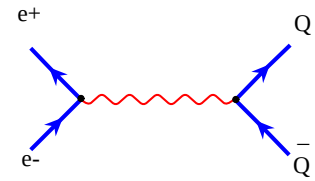
LCWS07: status report from GRACE

[talk by Y. Yasui]

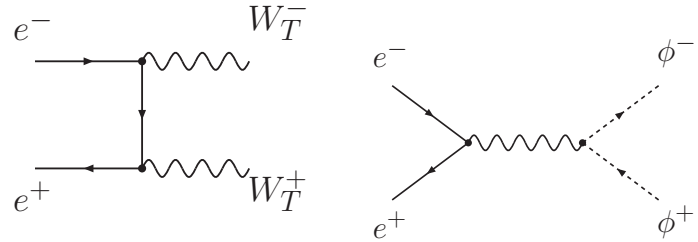
- Projects:
  - GRACE/FORM: full ew $2 \to 3$ to 1 loop
  - GRACE/SUSY: $1 \to 3$ (ew, 1 loop)
- New attempt: numerical integration
  - brute force (extrapolation, sector decomposition, ...)
  - super high precision, HMLIB (octuple or higher precision)

Sudakov logarithms

- New at ILC: virtual W/Z corrections to exclusive reactions



- Four-fermion processes



- WW production

- ➪ large logarithms $\ln s/M_{W/Z}^2$; "$\ln^2$/loop"
  ➪ large contributions: 1-loop: $\approx 30\%$, 2-loop: $\approx 5\%$
- Technique:
  - evolution equations
  - anomalous dimensions
  - ...

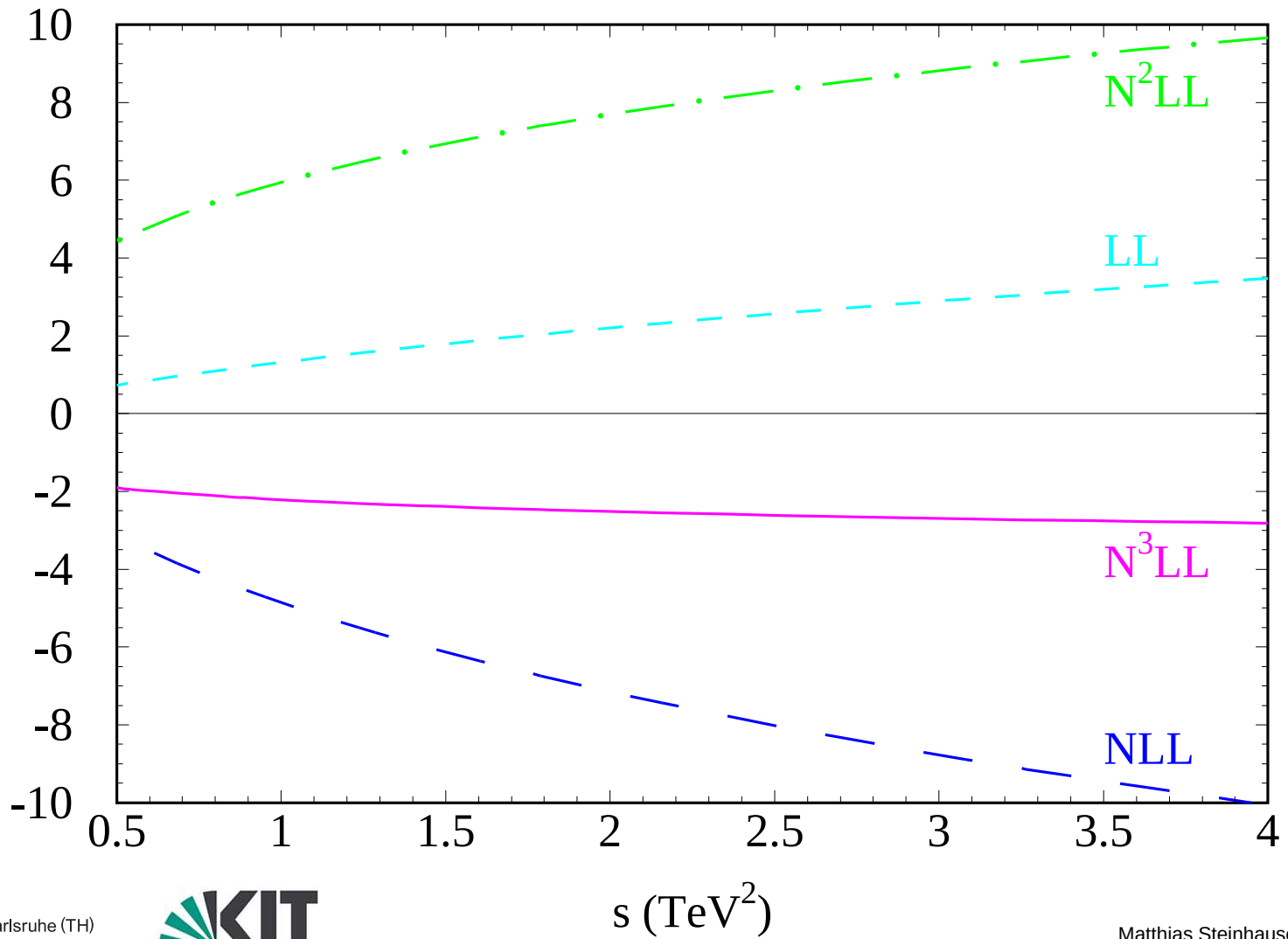
[Mueller'79;Collins'80;Sen'83]

Status (2 loops)

- LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ — All processes
  [Fadin, Lipatov, Martin, Melles '99]
- NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ — Four-fermion processes
  [Kühn, Penin, Smirnov '99]
- NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ — All processes
  [Melles '00]
- NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ — Four-fermion processes
  [Kühn, Moch, Penin, Smirnov '01]
- N^{3}LL: $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ — Four-fermion processes
  [Feucht, Kühn, Penin, Smirnov '05]
- NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ — $e^+e^- \to W^+W^-$
  [Kühn,Metzler,Penin — soon]

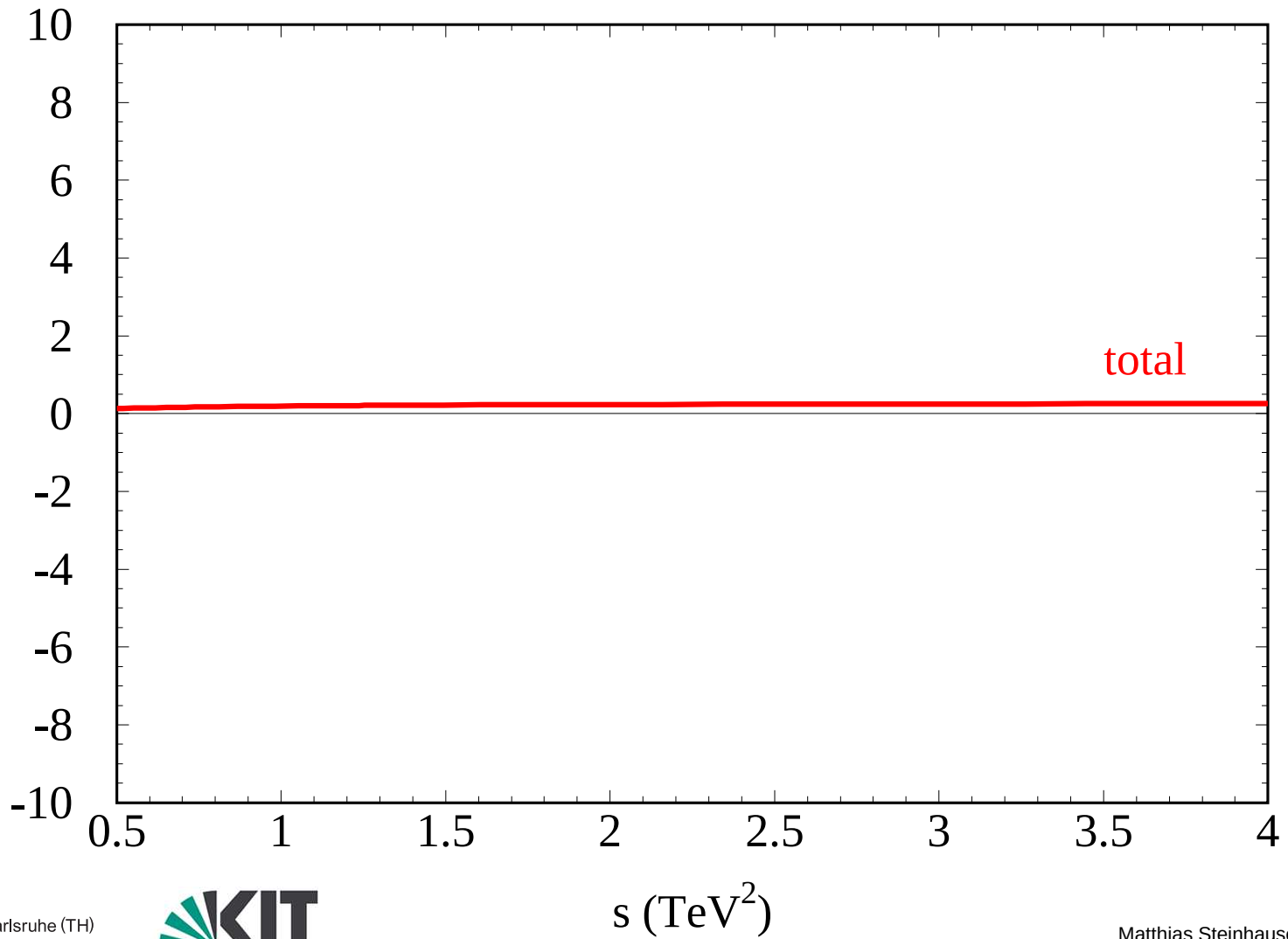
Results — 4 fermions

[Feucht,Kühn,Penin,Smirnov'05]



Results — 4 fermions

[Feucht,Kühn,Penin,Smirnov'05]



Two-loop electroweak NLL corrections

Massless fermionic processes $f_1 f_2 \to f_3 \cdots f_n$

with different $|(p_i + p_j)^2| \gg M_W^2$ and different masses $M_W^2 \sim M_Z^2 \sim m_{\text{top}}^2 \sim M_{\text{Higgs}}^2$:

- complete 2-loop electroweak next-to-leading logarithmic (NLL) corrections in $D = 4 - 2\epsilon$ dimensions $\longrightarrow$ Denner, B.J., Pozzorini, Nucl. Phys. B 761 (2007) 1
- factorizable contributions calculated with 2 independent methods:
  1.) sector decomposition, 2.) expansion by regions & Mellin–Barnes
- non-factorizable contributions shown to vanish due to collinear Ward identities
- result expressed by exponentiated 1-loop terms and β-function coefficients
- universal correction factors, in agreement with existing results

From massless to massive fermions

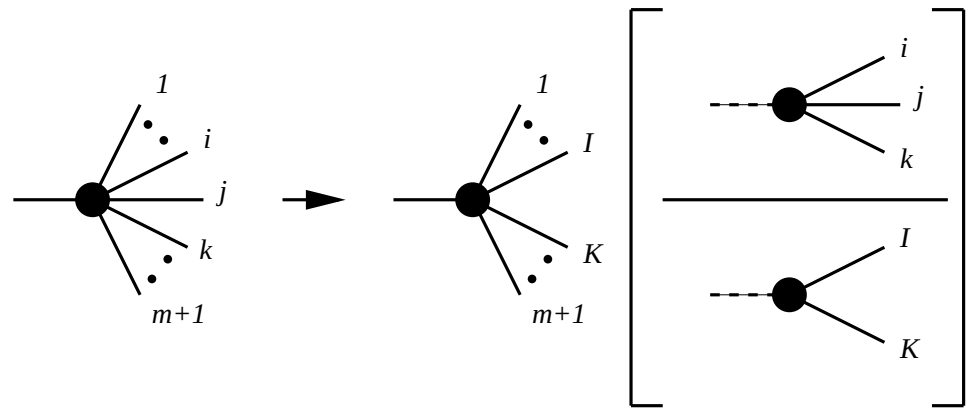
- method also works for massive fermions
- treat fermion mass terms carefully (power singularities $1/m_{\text{top}}^2$, mixing of chiralities)
- contributions to Abelian 2-loop form factor completed, exponentiates 1-loop result
- remaining diagrams will soon be finished ...

$\rightsquigarrow$ **Goal: electroweak NLL corrections for arbitrary processes**

$e^+e^- \to$ 3 jets

- $e^+e^- \to$ 3 jets + event shape ➪ precise α_s
- $\delta\alpha_s$ from jet observables dominated by theory error
- hard vs. soft/colinear (unresoved) gluons; jet algorithm
- real radiation: “double real” and “real gluon + 1-loop”
  2 approaches:
  - subtraction [Weinzierl’03,Kilgore’04,. . . ,Gehrmann et al.’04’05,. . .]
  - direct numerical integration [Binoth,Heinrich’02-..., Anastasiou,Melnikov,Petriello’04]

Antenna subtraction:



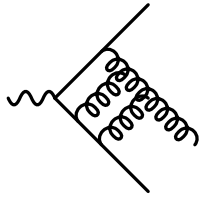
Antenna functions: encapulate singular behaviour due to unresoved emission of partons

Ingredients to NNLO $e^+e^- \to$ 3-jet

[talk by T. Gehrmann]

- Two-loop matrix elements

  $|\mathcal{M}|^2_{\text{2-loop},3 \text{ partons}}$

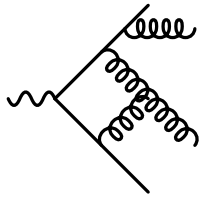
  explicit infrared poles from loop integrals

  L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG;

  S. Moch, P. Uwer, S. Weinzierl

- One-loop matrix elements

  $|\mathcal{M}|^2_{\text{1-loop},4 \text{ partons}}$

  explicit infrared poles from loop integral and

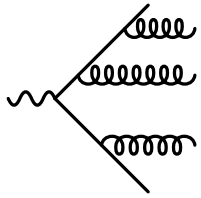
  implicit infrared poles due to single unresolved radiation

  Z. Bern, L. Dixon, D. Kosower, S. Weinzierl;

  J. Campbell, D.J. Miller, E.W.N. Glover

- Tree level matrix elements

  $|\mathcal{M}|^2_{\text{tree},5 \text{ partons}}$

  implicit infrared poles due to double unresolved radiation

  K. Hagiwara, D. Zeppenfeld;

  F.A. Berends, W.T. Giele, H. Kuijf;

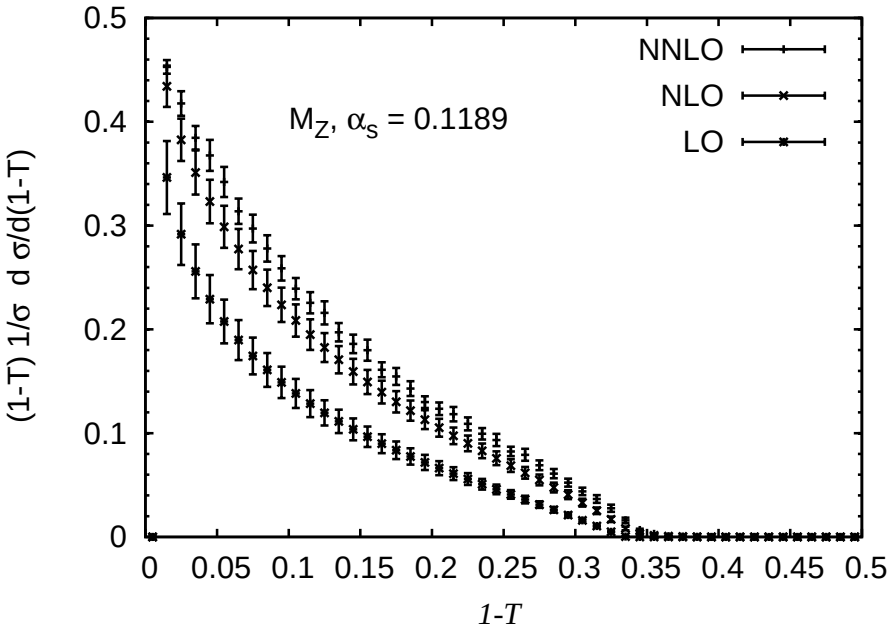
  N. Falck, D. Graudenz, G. Kramer

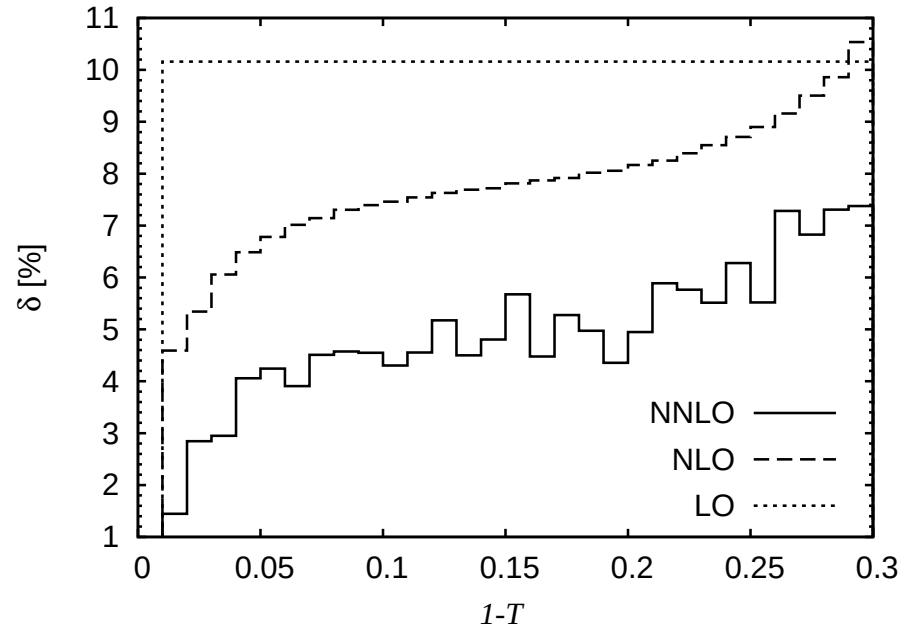
Infrared Poles [illegible] the sum

Results

[talk by T. Gehrmann]

NNLO thrust distribution





- varied $\mu = [M_Z/2; 2\,M_Z]$
- NNLO on the edge of NLO theory uncertainty
- renormalisation scale dependence decreases considerably
- started comparison with LEP data $\longrightarrow \alpha_s$

4-loop integrals

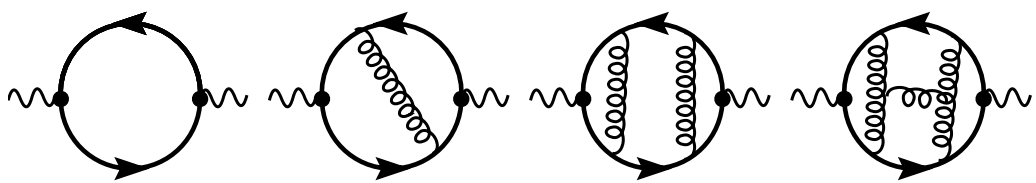
- vacuum integrals ("bubbles"):
  - ρ parameter (➪ Giga-Z)
  - photon polarisation function (➪ m_c, m_b)
  - decoupling of α_s (➪ precise running)
- massless 2-point integrals:
  - $R(s)$ (➪ Giga-Z)
  - τ decay (➪ m_s, α_s)
- Physical problem ➪ Millions of different integrals
  Strategy:
  1) Reduction to master integrals (MIs)
  2) Compute MIs

Bubbles

1) Reduction with Laporta-algorithm [Laporta,Remiddi'96,Laporta'01]
2) MIs: differential equations, difference equations, asymptotic expansion, . . . [Schröder,Vuorinen'05; Chetyrkin,Faisst,Sturm,Tentyukov'06]

- Problem: Laporta-algorithm is very expensive
  e.g.: only 1st derivative of photon polarization function could be computed
- Results:
  - 4-loop ρ parameter

    [Schröder,Steinhauser'05; Chetyrkin,Faisst,Kühn,Maierhöfer,Sturm'06; Boughezal,Czakon'06]

  - 

    Chetyrkin,Kühn,Sturm'06; Boughezal,Czakon,Schutzmeier'06]

Bubbles

1) Reduction with Laporta-algorithm [Laporta,Remiddi'96,Laporta'01]

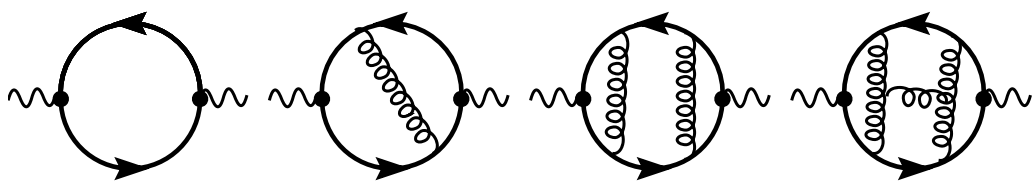
2) MIs: differential equations, difference equations

- Combine 4-loop result with new data on R, Γ_{ee}, ...
  ⇨
  $m_c(m_c) = 1.286(13)$ GeV
  $m_b(m_b) = 4.164(25)$ GeV

  [talk by M. Steinhauser]

- 4 loop ρ parameter

[Schröder,Steinhauser'05; Chetyrkin,Faisst,Kühn,Maierhöfer,Sturm'06; Boughezal,Czakon'06]



Chetyrkin,Kühn,Sturm'06; Boughezal,Czakon,Schutzmeier'06]

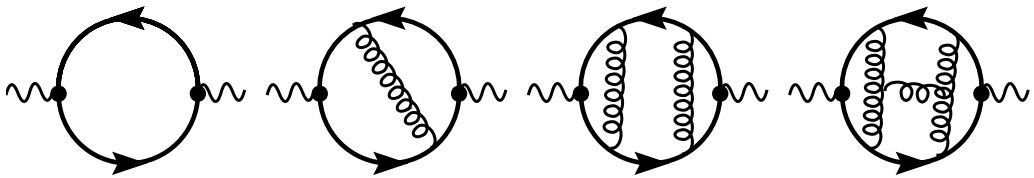
Four-loop QCD corrections to the ρ parameter

K.G. Chetyrkin, M. Faisst, J.H. Kühn, P. Maierhöfer, C. Sturm

- ρ parameter measures relative strength of the charged and neutral current
- It enters the relation for the theory prediction of M_W depending on other standard model parameters
- Four-loop contribution $\mathcal{O}(G_F m_t^2 \alpha_s^3)$ from top- and bottom-quarks to the ρ-parameter in pQCD computed
- Methods:
  - All loop-integrals reduced to smaller set of master integrals
  - Computation of master integrals:
    Method of ε-finite basis + difference equations
    $\rightsquigarrow$ New analytical + numerical results for master integrals
- The four-loop result induces a shift of $\sim$2 MeV to M_W
  $\Longrightarrow$ theoretical uncertainty well below the anticipated accuracy of M_W at ILC ($\delta M_W^{exp.} \sim$6 MeV)

Massless 2-point integrals

- Baikov's method: $I = \sum$ "coef" $\cdot\, MI$ [Baikov'96,...]
  (nice) integral representation for "coef"



  compute for $d \to \infty$ ➪ reconstruct "coef"
- Results:
  - $\Gamma(H \to gg) \sim 1 + 0.65 + 0.20 + 0.02$

    ($M_t = 175$ GeV, $M_H = 120$ GeV) [Baikov,Chetyrkin'06]
  - $R(s)$ to $\mathcal{O}(\alpha_s^4)$:

    $$R(s) = 3[1 + \frac{\alpha_s}{\pi} + \ldots + \left(\frac{\alpha_s}{\pi}\right)^4 (0.022 n_f^3 - 0.80 n_f^2 + \ldots)]$$

    [Baikov,Chetyrkin,Kühn,'02,...]

Precision for the MSSM

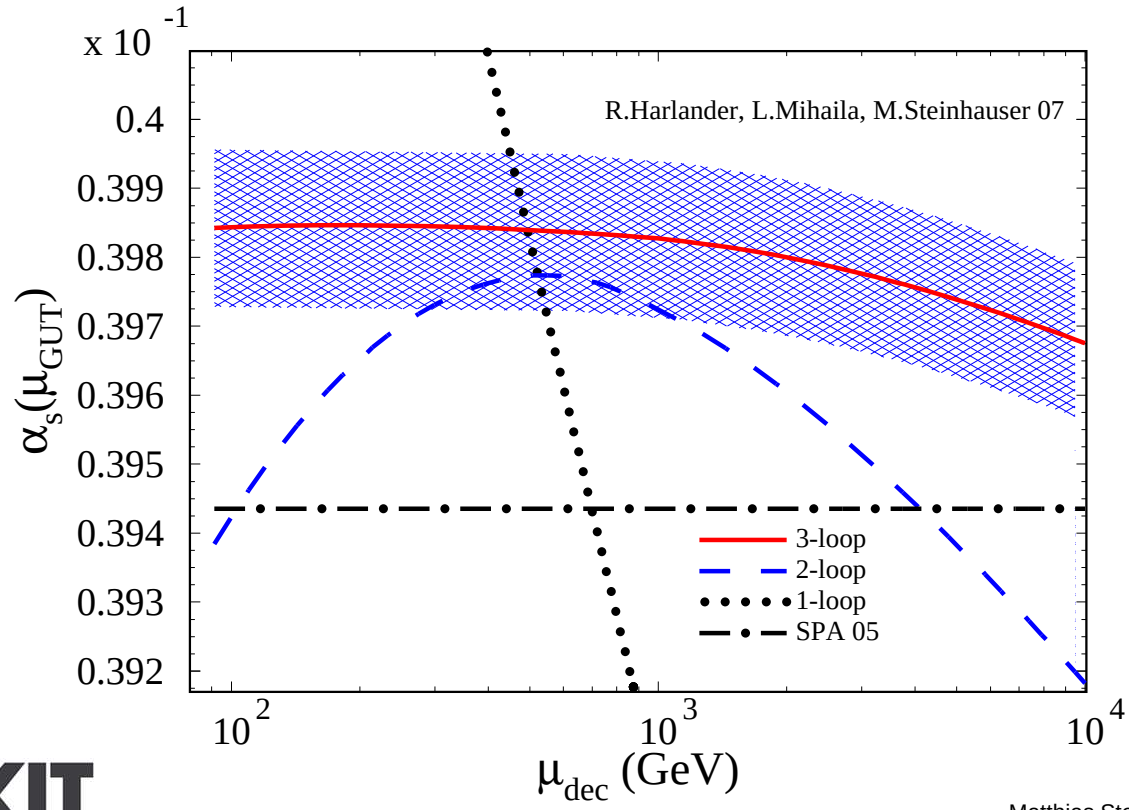
- #loops > 1
- many (more) scales
- Dimensional Regularization breaks SUSY ($\rightarrow$ restoring CTs ??)
- alternative: Regularization by Dimensional Reduction (dim. of vector fields 4-dimensional, as the fermions; integrals are still $d = 4 - 2\epsilon$ dimensional)
- price:
  - add'l field: "ε scalar" ➪ more FRs, more diagrams, ...
  - matching to SM (i.e. to $\alpha_s(M_Z)$ with "Dimensional Regularization") ➪ evanescent couplings, ...
- Does Dimensional Reduction work beyond 1 loop?

[Hollik,Stöckinger'06; Stöckinger'06]

Examples

- typical situation: $\alpha_s^{(5),\overline{\mathrm{MS}}}(M_Z) \longrightarrow \alpha_s^{(\mathrm{full}),\overline{\mathrm{DR}}}(\mu_{\mathrm{GUT}})$

1. $\mu = M_Z \longrightarrow \mu = \mu_{\mathrm{dec}}$
2. match SM and MSSM
3. $\mu = \mu_{\mathrm{dec}} \longrightarrow \mu = \mu_{\mathrm{GUT}}$



Examples

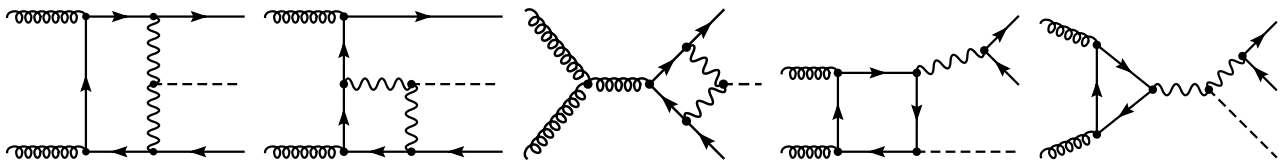
- typical situation: $\alpha_s^{(5),\overline{\mathrm{MS}}}(M_Z) \longrightarrow \alpha_s^{(\mathrm{full}),\overline{\mathrm{DR}}}(\mu_{\mathrm{GUT}})$
- δM_H – 2 loops [Heinemeyer,Hollik,Weiglein,. . .]
- δM_H – 3-loop logarithms [Martin'07]
- $(g-2)_\mu$: 2-loop SUSY [Heinemeyer,Stöckinger,Weiglein'04]
- many 1 loop MSSM [...]
- . . .

Tools/Developments/Projects

- `FeynArts`, `FormCalc`, `LoopTools` [Hahn et al.]
- `MB` [Czakon'06]
- `AMBRE` [Gluza,Kajda,Riemann'07]
- `FeynHiggs` [Heinemeyer,Hahn,...]
- `exp/q2e` [Harlander,Seidensticker,Steinhauser]
- "Torino approach": fully numerical evaluation of 2-loop Feynman integrals
  Applications: $\sin\Theta_{\rm eff}$, $\Gamma(H\to\gamma\gamma)$ (ew corrections)
  [Actis,Passarini,Sturm,Uccirato,...]
- SPA [Aguilar-Saavedra et al.'06]
- ...

Other loops

- 2-loop corrections to $H \to b\bar{b}$:
  $\frac{\Delta\Gamma}{\Gamma_{\rm Born}} \sim (G_F m_t^2)^2 \approx 0.05\%$ [talk by M. Butenschön]
- $\overline{\rm MS}$ – on-shell relation for quark masses:
  full m_c-dependence for m_b [talk by D. Seidel]
- $gg \to q\bar{q}h$: LO = 1 loop



[talk by M. Weber]

- New prescription/method to extract (well-defined) m_t in continuum; e^+e^- (now), pp (to be done)
  based on tower of effective theories ("jet functions", "soft function", ...) [talk by A. Hoang]