

Higgs Triplets, Decoupling and Precision Measurements

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(with M.-C. Chen and S. Dawson)

Outline

- Some motivation
- Renormalization of the SM and different schemes
- Extensions to models beyond the SM
(in particular models with $\Delta\rho \neq 1$ at tree-level)
- Case study: SM plus Triplet Higgs
- One-loop corrections to W boson mass
- Pros and cons of different renormalization schemes
- Decoupling vs. non-decoupling?
- Take Home Message: Correct renormalization procedure is complicated... and it matters!

Motivation

- Pre-LHC Game Plan:
 - Write down your “model of the week”
 - Assume new physics contributes primarily to gauge boson two-point functions
 - Calculate contribution of new (heavy) particles to EW observables (such as Peskin-Takeuchi S, T and U)
 - Extract limits on model parameters (masses, couplings, etc.)
- HOWEVER: this approach must be modified for models which generate corrections to the ρ parameter at tree-level.

Some Examples

- SU(5) GUTs (Georgi and Glashow, PRL32 (1974), 438)
- Little Higgs (without T parity)
- U(1) Extensions of SM
(Mixing of Z and Z' breaks custodial symmetry)
- In general, for models with multiple Higgses in different multiplets:

$$\rho_0 = \frac{\sum_i v_i^2 [I_i(I_i + 1) - I_{3i}^2]}{2 \sum_i v_i^2 I_{3i}^2}$$

where I = isospin and I_3 = 3rd component of neutral component of the Higgs multiplet.

- For example, for the minimal (Standard) model, $I = 1/2$ and $I_3 = -1/2$ and $\rho_0 = 1$
- However, if we add an SU(2) triplet to the mix ($I = 1$ and $I_3 = 0$):

$$\rho_0 = 1 + 4 \frac{v_{trip}^2}{v_{doub}^2}$$

SM Renormalization Schemes

- In the SM gauge sector (after SSB), there are 3 fundamental parameters (g , g' and Higgs vev, v)
- In order to determine all of the SM parameters need (at least) three (well-measured) input observables

- Pick your scheme:

- “On-shell Scheme” (α , M_W and M_Z): $s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$.

$$\rho_0 \equiv \frac{M_{W0}^2}{M_{Z0}^2 c_{\theta_0}^2} = 1$$

- “ M_Z Scheme” (α , G_F and M_Z): $\sin(2\theta_Z) \equiv \sqrt{\frac{4\pi\alpha(M_Z)}{\sqrt{2}G_\mu M_Z^2}}$; $M_W = M_Z \cos\theta_{\text{eff}}$

- “Effective Mixing angle scheme” (α , G_F and $\sin^2\theta_{\text{eff}}$): $M_Z = M_W/\cos\theta_{\text{eff}}$

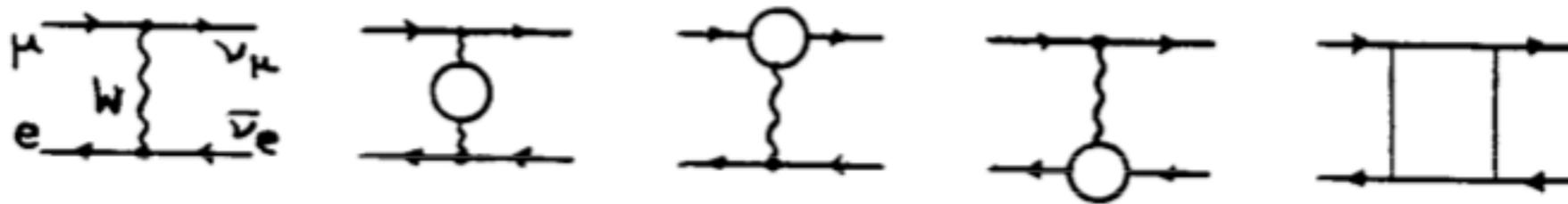
- All schemes identical at tree-level

Muon Decay in the SM

- At tree-level, muon decay (or G_F ... or G_μ) related to input parameters

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8 \sin^2 \theta_W M_W^2}$$

- At one-loop:



$$\begin{aligned} \frac{G_\mu}{\sqrt{2}} &= \frac{e^2}{8s_W^2 M_W^2} \left[1 + \frac{\hat{\Sigma}^{WW}(0)}{M_W^2} + \delta_{VB} \right] \\ &\equiv \frac{e^2}{8s_W^2 M_W^2} [1 + \Delta r] . \end{aligned} \quad (136)$$

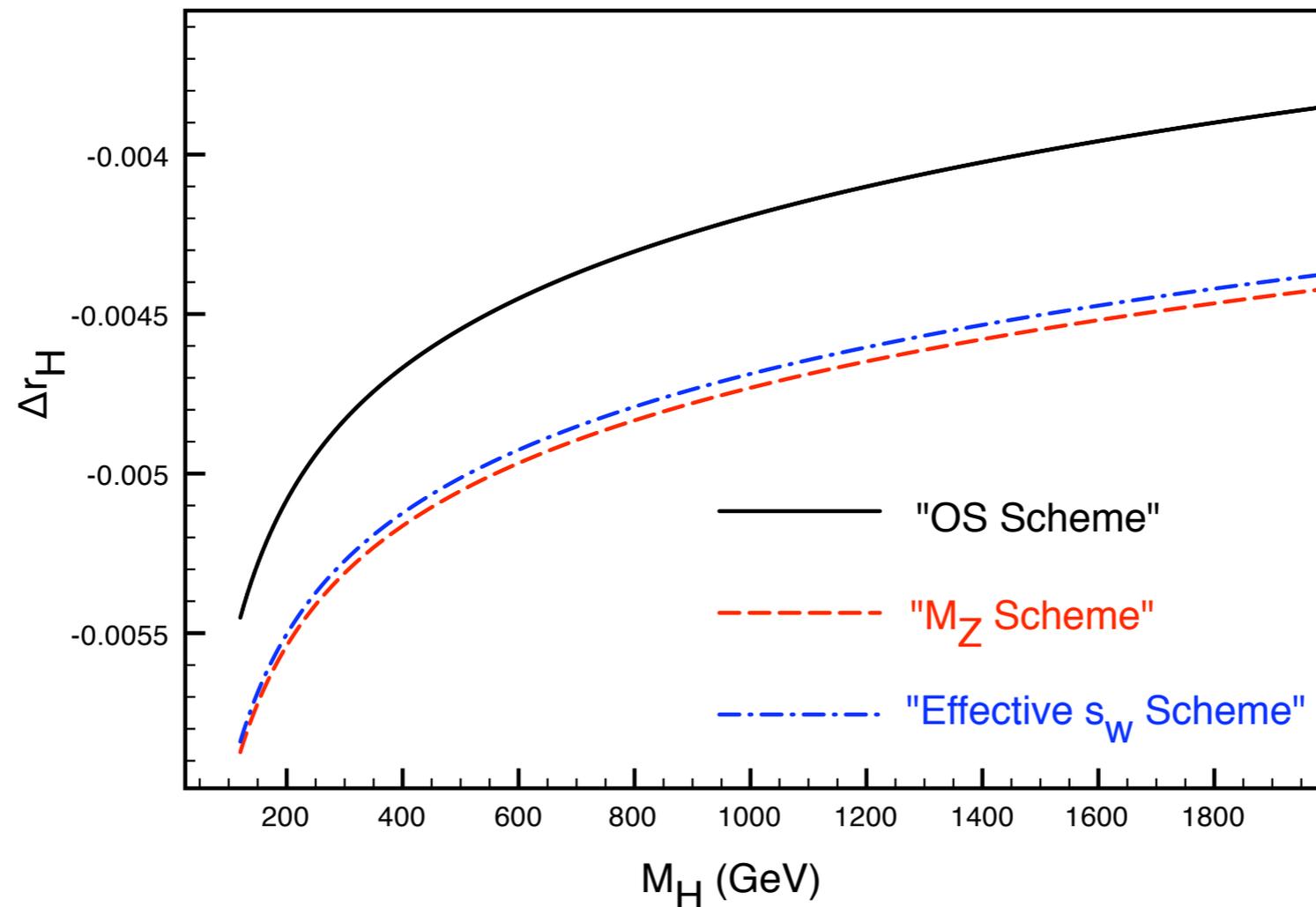
- where:

$$\Delta r_{SM} = -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta \alpha}{\alpha} - \frac{\delta s_\theta^2}{s_\theta^2} \quad (+ \delta_{VB})$$

- The quantity Δr is a physical parameter

Δr_{SM} in Different Renormalization Schemes

- Compute leading SM Higgs mass dependence



- Strong scheme dependence... however, with higher-order corrections, schemes agree!
- Beyond the SM conclusions typically drawn from one-loop results

Renormalization for Models with $\rho_{\text{tree}} \neq 1$

- Can't use relations like: $M_W = M_Z \cos\theta_{\text{eff}}$
- In other words, it seems we need one additional input parameter
- Choices for renormalization scheme:
 - Use four low-energy inputs (e.g., α , G_F , $\sin^2\theta_{\text{eff}}$ and M_Z):
$$\lambda = f(\alpha, G_F, \sin^2\theta_{\text{eff}} \text{ and } M_Z)$$

(Pro: eliminate one parameter; Con: eliminate one parameter)
 - Use only three SM inputs (e.g., α , G_F , and M_Z):
(Pro: full parameter space; Con: loss of predictability?)
 - Use three low-energy inputs plus one “high-energy” input (e.g., measured couplings/masses of new particles)
(Con: no “high-energy” inputs!)

Case Study: SM + Triplet Higgs

The Model

- Simplest extension of SM with $\rho_{\text{tree}} \neq 1$:
SM with a real Higgs doublet *plus* a real isospin ($Y = 0$) triplet

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h^0 + i\chi^0) \end{pmatrix}, \quad \Phi = \begin{pmatrix} \eta^+ \\ v' + \eta^0 \\ -\eta^- \end{pmatrix}$$

- Coupled to gauge fields via usual covariant derivative(s):

$$L = |D_\mu H|^2 + \frac{1}{2} |D_\mu \Phi|^2$$

where:

$$D_\mu H = \left(\partial_\mu + i\frac{g}{2}\tau^a W^a + i\frac{g'}{2}Y B_\mu \right) H \quad D_\mu \Phi = \left(\partial_\mu + i g t_a W^a \right) \Phi$$

- Gauge boson masses: $M_W^2 = \frac{g^2}{4}(v^2 + 4v'^2)$ and $M_Z^2 = \frac{g^2}{4c_\theta^2}v^2$

- ρ parameter @ tree-level:
$$\rho = \frac{M_W^2}{M_Z^2 c_\theta^2} = 1 + 4\frac{v'^2}{v^2}.$$

PDG: $v' < 12$ GeV
(neglecting scalar loops)

More on the Model

- Most general scalar potential:

$$V = \mu_1^2 |H|^2 + \frac{\mu_2^2}{2} |\Phi|^2 + \lambda_1 |H|^4 + \frac{\lambda_2}{4} |\Phi|^4 \\ + \frac{\lambda_3}{2} |H|^2 |\Phi|^2 + \lambda_4 H^\dagger \sigma^a H \Phi_a,$$

- Note: λ_4 has dimensions of mass \rightarrow non-decoupling!
(Chivukula et al., PRD77, (2008))

- After SSB:

$$\begin{pmatrix} H^0 \\ K^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \eta^0 \end{pmatrix} \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} \phi^+ \\ \eta^+ \end{pmatrix}$$

where: $\tan \delta = 2 v'/v$

- Minimize the potential:

$$0 = v \left(\mu_1^2 + \lambda_1 v^2 + \frac{\lambda_3}{2} v'^2 - \lambda_4 v' \right) \\ 0 = v' \left(\mu_2^2 + \lambda_2 v'^2 + \lambda_3 \frac{v^2}{2} \right) - \lambda_4 \frac{v^2}{2}.$$

...and finally

- Trade original parameters for $M_{H^+}, M_{H^0}, M_{K^0}, \gamma, \delta, v$.

$$\lambda_1 = \frac{1}{2v^2} \left(c_\gamma^2 M_{H^0}^2 + s_\gamma^2 M_{K^0}^2 \right)$$

$$\lambda_2 = \frac{2}{v^2} \cot^2 \delta \left[s_\gamma^2 M_{H^0}^2 + c_\gamma^2 M_{K^0}^2 - c_\delta^2 M_{H^+}^2 \right]$$

$$\lambda_3 = \frac{1}{v^2 \tan \delta} \left[(M_{H^0}^2 - M_{K^0}^2) \sin(2\gamma) + M_{H^+}^2 \sin(2\delta) \right]$$

$$\lambda_4 = c_\delta s_\delta \frac{M_{H^+}^2}{v}$$

$$\mu_1^2 = -\frac{M_{H^0}^2}{2} \left(c_\gamma^2 + \frac{s_\gamma c_\gamma}{2} \tan \delta \right) - \frac{M_{K^0}^2}{2} \left(s_\gamma^2 - \frac{s_\gamma c_\gamma}{2} \tan \delta \right) + \frac{M_{H^+}^2}{4} s_\delta^2$$

$$\mu_2^2 = \frac{c_\delta^2}{2} M_{H^+}^2 - \frac{M_{H^0}^2}{2} \left(s_\gamma^2 + \sin(2\gamma) \cot \delta \right) - \frac{M_{K^0}^2}{2} \left(c_\gamma^2 - \sin(2\gamma) \cot \delta \right).$$

- Note: in the $v' \rightarrow 0$ limit...

- $\sin \delta = \sin \gamma = 0$

- $\lambda_4 = 0$

- $M_{H^+} = M_{K^0}$ (from λ_2 relation)

Custodial Symmetry
Restored!



Renormalization and EW Observables in the Triplet Model

Renormalization of the Triplet Model

- EW observable of choice: the W boson mass and compare SM vs. TM
- At tree-level, the W mass is related to the input parameters:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}s_\theta^2 c_\theta^2 M_Z^2 \rho} = \frac{\pi\alpha}{\sqrt{2}s_\theta^2 M_W^2} \quad \rho = \frac{M_W^2}{c_\theta^2 M_Z^2} = 1$$

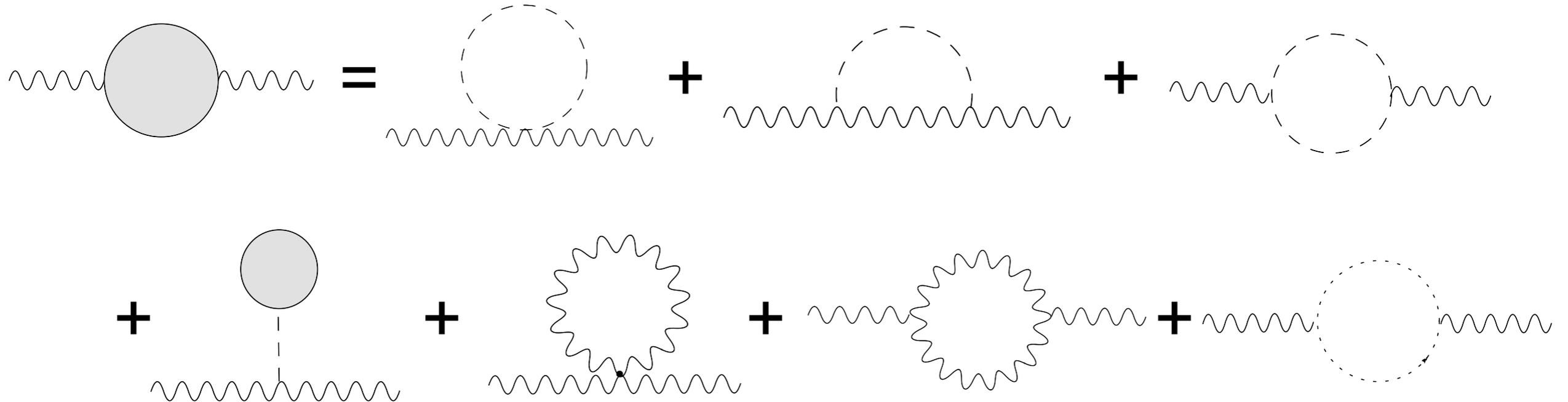
- When $\rho \neq 1$, more inputs are required (?)
- At one-loop level, corrections encoded in Δr :

$$M_W^2 = \frac{\alpha\pi}{\sqrt{2}s_\theta^{eff2} G_\mu} (1 + \Delta r)$$

- And Δr is a function of the one-loop corrected self-energies:

$$\begin{aligned} \Delta r &= -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta\alpha}{\alpha} - \frac{\delta s_\theta^2}{s_\theta^2} \\ &= \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi'_{\gamma\gamma}(0) + 2\frac{s_{\theta,eff}}{c_{\theta,eff}} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} - \frac{\delta s_{\theta,eff}^2}{s_{\theta,eff}^2} \end{aligned}$$

The Loops



- Scalar loops: contributions from H^0 , K^0 and H^\pm (for arbitrary γ and δ)
- SM gauge boson contributions included since different values of M_W and/or M_Z used in “SM” and “TM” calculations of Δr (see below)
- Vertex/box contributions (not shown) also included in order to ensure finite result (“pinch” contributions are a subset of full vertex/box pieces)

Scheme #1

- Input 4 low-energy parameters: (α , G_F , $\sin^2\theta_{eff}$ and M_Z)

$$G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$$

$$M_Z = 91.1876(21) \text{ GeV}$$

$$\alpha^{-1} = 137.035999679(94) \quad .$$

$$\sin^2 \theta_{eff} = .2324 \pm .0012 \quad .$$

From identifying $\sin\theta$ with effective mixing angle measured at Z pole

- CT for $\sin^2\theta_{eff}$:

$$\frac{\delta s_{\theta,eff}^2}{s_{\theta,eff}^2} = \left(\frac{c_{\theta,eff}}{s_{\theta,eff}} \right) \frac{\Pi_{\gamma Z,SM}(M_Z^2)}{M_Z^2} + \mathcal{O}(m_e^2)$$

- Compare results for TM to SM in the “Effective mixing angle scheme” (in order to check decoupling):
 - $M_W(\text{tree})$ in both SM and TM: $M_W(\text{tree}) = 80.159 \text{ GeV}$
 - However, $M_Z(\text{tree})$ in SM different: $M_Z(\text{tree}) = 91.329 \text{ GeV}$
- Note: tadpoles cancel!

Scheme #1 (cont.)

- With the additional input parameter, we can eliminate one of the TM parameters, e.g.:

$$\rho = \frac{M_W^2}{M_Z^2 c_\theta^2} = 1 + 4 \frac{v'^2}{v^2}.$$

- This sets v' and the mixing angle δ :

$$v' = 6.848 \text{ GeV} \longrightarrow \sin\delta = 0.056$$

- Model is over-constrained... i.e., lose ability to scan full parameter space
- In the following, we consider the difference between the TM prediction and the SM...

Testing Decoupling

- Besides renormalization scheme dependence, also interested in (non)decoupling behavior of M_W :

$$M_W^2 = \frac{\alpha\pi}{\sqrt{2}s_\theta^{\text{eff}2}G_\mu}(1 + \Delta r)$$

- First, calculate Δr in TM (using input value of M_Z):

$$\Delta r_{\text{TM}} = \Delta r_{\text{SM}} + \Delta r_1 + f(\sin\delta, \sin\gamma)$$

- Next, calculate Δr in SM (using M_Z calculated from inputs):

$$\Delta r_{\text{eff.}} = \Delta r_{\text{SM}}$$

- Note: difference of two Δr_{SM} quantities $\neq 0$ (because of different M_Z 's)
- Finally, plot the difference:

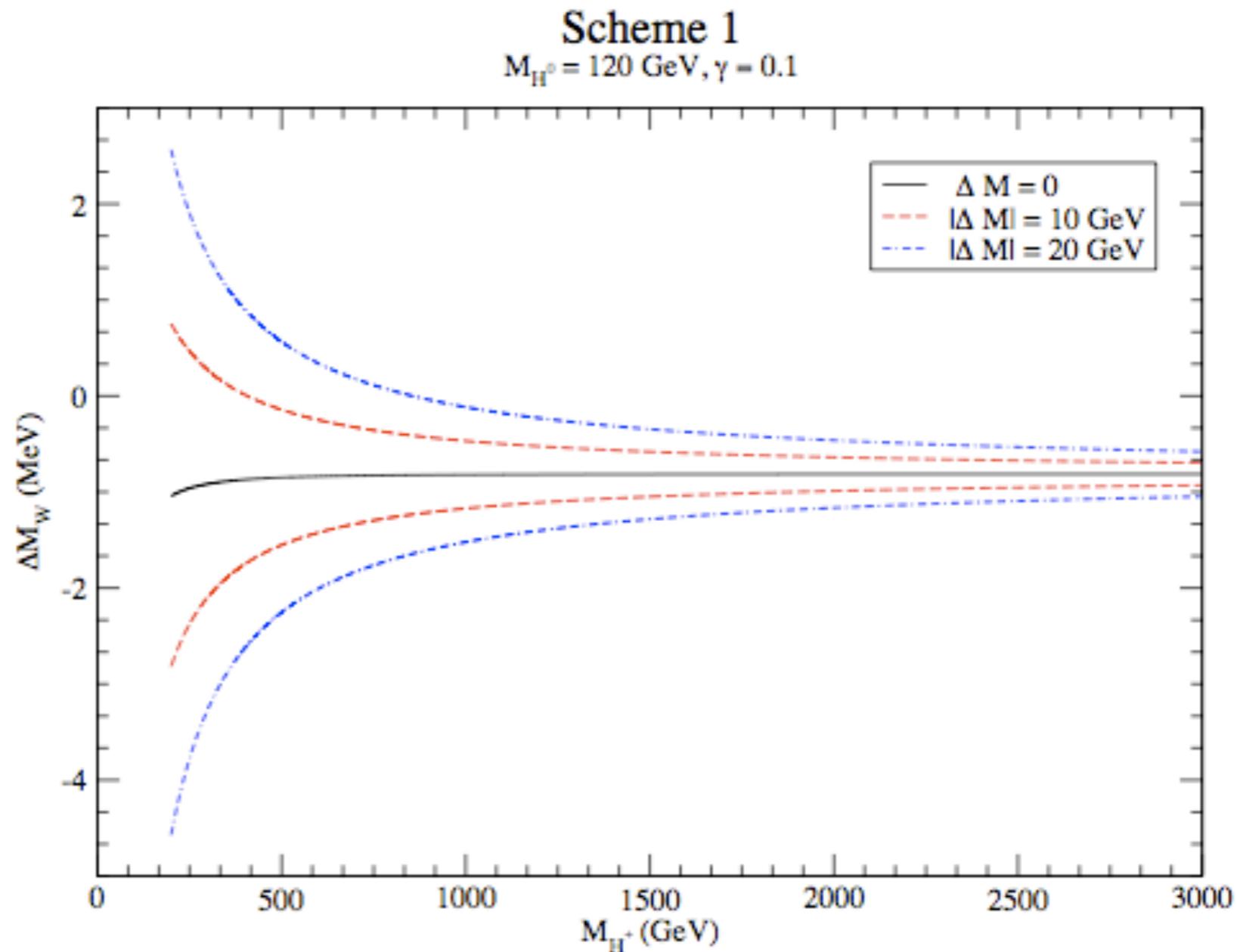
$$\Delta M_W = M_W(\Delta r_{\text{eff.}}) - M_W(\Delta r_{\text{TM}})$$

“Decoupling”
 $\Delta M_W = 0$

Scheme 1 Results

- Consider small mass splittings (perturbativity)
- For $M_{K^0} = M_{H^\pm}$:
 - $v' = \sin\delta = \sin\gamma = 0$
 - Value of ΔM_W due to different M_Z 's used in individual pieces
- For larger splittings, sizable effects at low M_{H^\pm}
- For small values of mixings/mass-splittings:

$$\Delta_{r,1} \rightarrow \frac{\alpha}{24\pi \sin^2 \theta_{eff}} \left\{ \frac{M_{K^0}^2 - M_{H^\pm}^2}{M_{H^\pm}^2} \right\} + \dots$$



Scheme #2

- Input only three low-energy observables (α , G_F , and M_Z) plus one “running” parameter (v')
- Naturally connects with SM “ M_Z Scheme”
- Now, $\sin^2\theta$ and M_W are calculated quantities:

$$\begin{array}{ccc} \text{SM} & & \text{TM} \\ \sin(2\theta_Z) \equiv \sqrt{\frac{4\pi\alpha(M_Z)}{\sqrt{2}G_\mu M_Z^2}} & & \sin(2\theta_Z) = \sqrt{\frac{4\pi\alpha}{M_Z^2} \left[\frac{1}{\sqrt{2}G_\mu} - 4v'^2 \right]} \\ & \searrow & \swarrow \\ & M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu} \cdot \frac{1}{\sin^2\theta_W} & \end{array}$$

- Calculate corrections to M_W in the same manner as Scheme #1
- Claim: “more natural approach to SM limit”
(Chankowski et al., hep-ph/0605302)

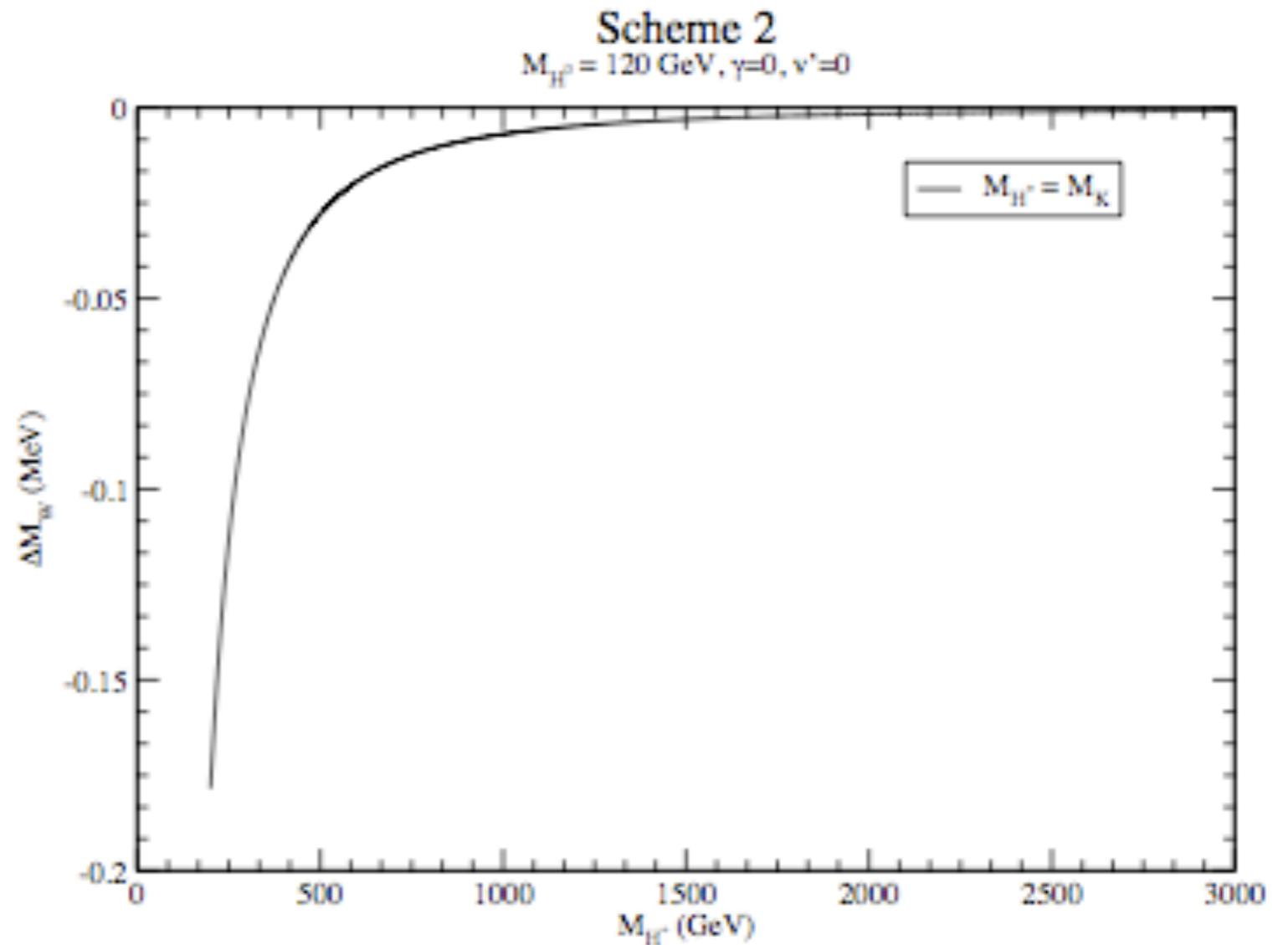
Scheme #2 Results: $v' = 0$

- For $v' = 0$: only solution to minimization conditions...

$$\gamma = 0 \text{ and } M_{K^0} = M_{H^\pm}$$

- No large effects from TM scalar sector

- Decoupling of TM scalar sector is apparent

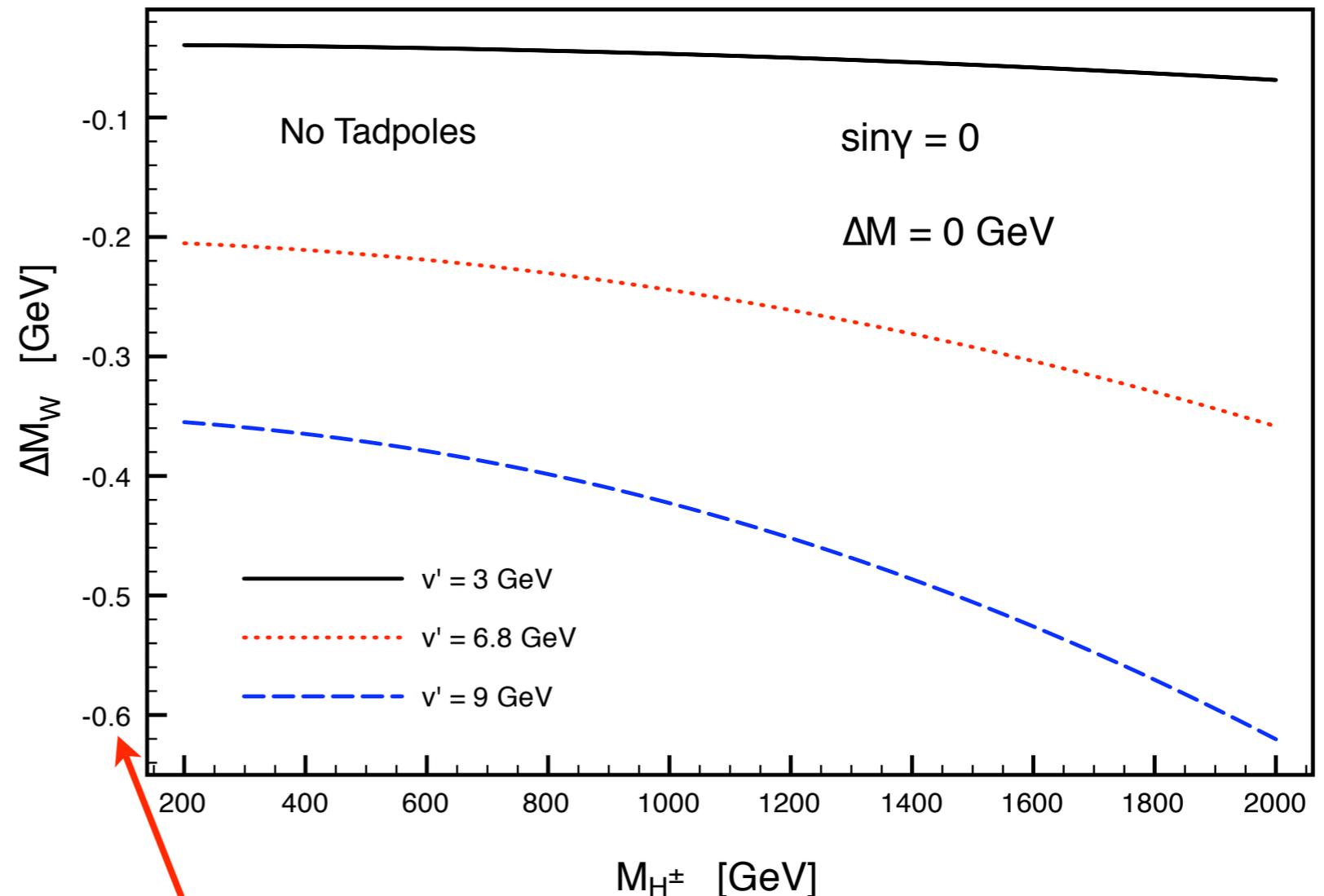


Scheme #2 Results: $v' \neq 0$

- As soon as $v' \neq 0$, then $\lambda_4 \neq 0$
- Since λ_4 has dimensions, we shouldn't expect decoupling
- Large non-decoupling effects from TM scalar sector:

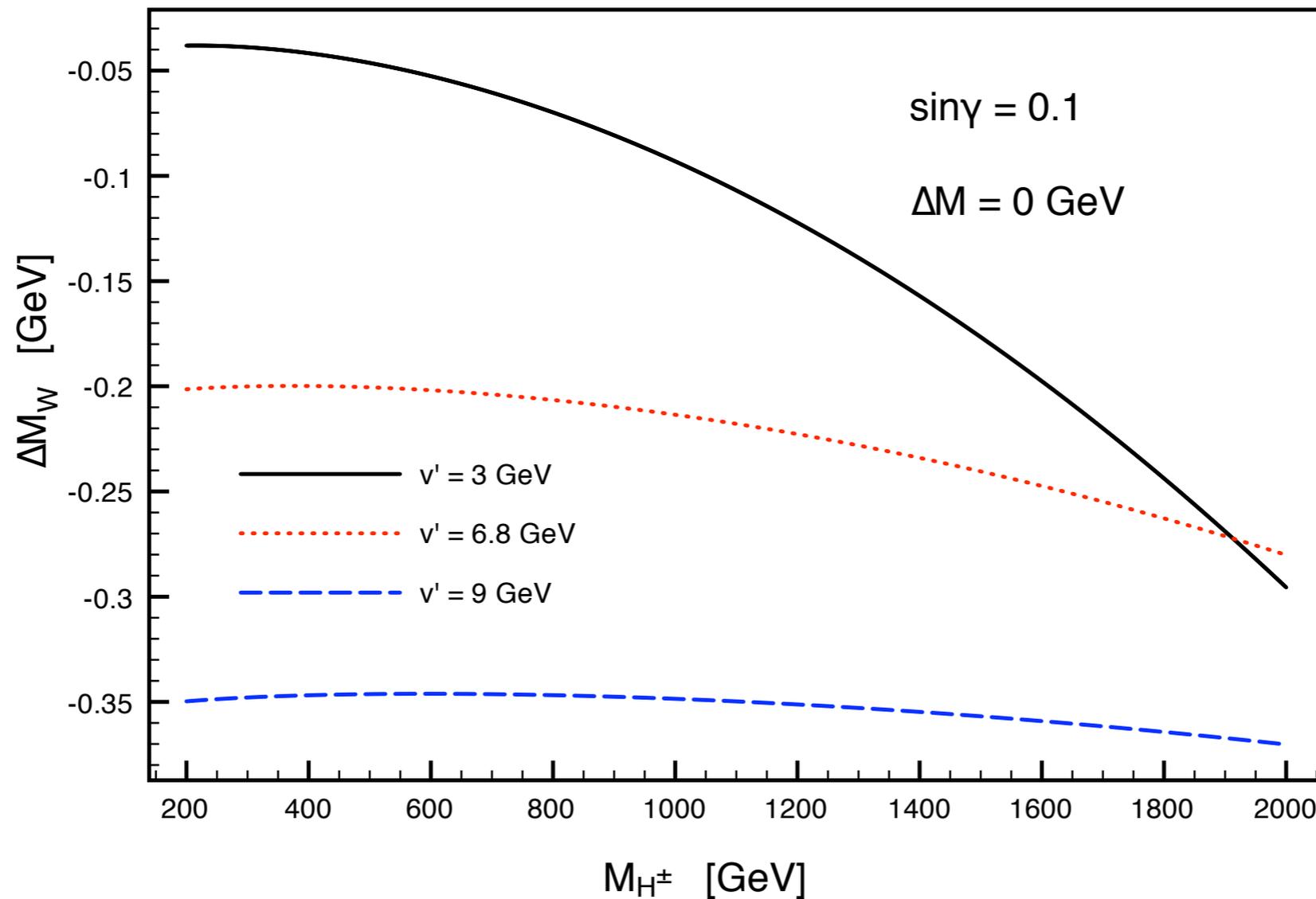
$$\Delta r_1 \approx (v'/v)^2$$

(See Chivukula et al.,
PRD77, 035001 (2008))



Note difference in scale
from Scheme #1!

Scheme #2 Results: $v' \neq 0$



- Large corrections from non-cancellation of M^2 terms:

$$\frac{\delta \hat{s}_Z^2}{\hat{s}_Z^2} \sim \frac{\hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2} \left\{ -\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{1}{1 - 4\sqrt{2}v'^2 G_\mu} \frac{\Pi_{WW}(0)}{M_W^2} \right\}$$

Scheme #2 Results: Attack of the Tadpoles

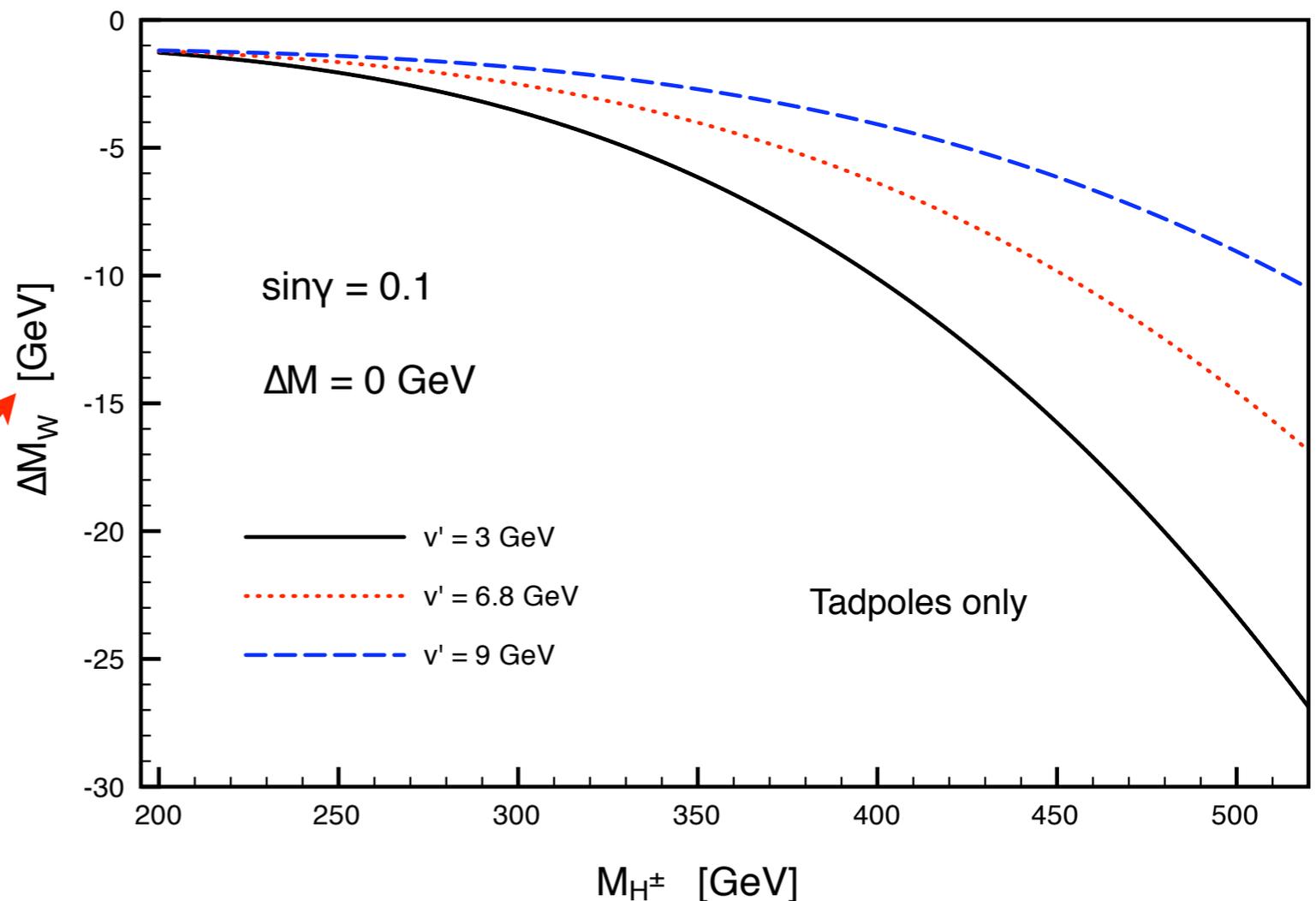
- In SM (and in Scheme #1 for TM), tadpoles cancel
- Not so in Scheme #2 for non-zero v'

$$\Delta r_{\text{triplet}}(\text{Scheme 2})^{\text{tadpole}} = \frac{\hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2} \left\{ -\frac{\Pi_{ZZ}^{\text{tadpole}}}{M_Z^2} + \frac{1}{1 - 4\sqrt{2}v'^2 G_\mu} \frac{\Pi_{WW}^{\text{tadpole}}}{M_W^2} \right\}$$

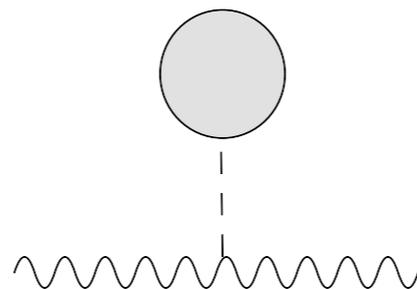
- Tadpole contributions grow as:

$$\Delta r_{\text{tadpoles}} \sim (M_{H^\pm})^2$$

- **Note ridiculous scale!**



Those Darn Tadpoles



- Even for $v'(\text{tree}) = 0$, tadpoles generate an effective v' (Chankowski et al., hep-ph/0605302)
- No physical motivation for definition of v' in simplest Triplet Model (GUTs may have natural way to define v')
- What we're missing is a renormalization condition for v' to cancel tadpole contributions ("Scheme #3"?)
- However, even in "Scheme #3":
 - Fine-tuning?
 - Non-tadpole contributions still large in this scheme!

Conclusions

- Models with $\Delta\rho \neq 1$ at tree-level require four input parameters for a correct renormalization procedure
- Important to compare BSM results with appropriate SM scheme
- Considered two schemes for the Triplet Model
 - Four low-energy input scheme: non-decoupling effects due to different values of M_Z (due to $\Delta\rho \neq 1$)
 - Three low-energy inputs and one running parameter: contributions to Δr much larger than previous scheme
- In both cases, effects of scalar loops critical
- Beware of the tadpoles!
- Correct renormalization procedure is complicated... and it matters!