Top Mass from Jet Distributions

Sonny Mantry
University of Wisconsin at Madison

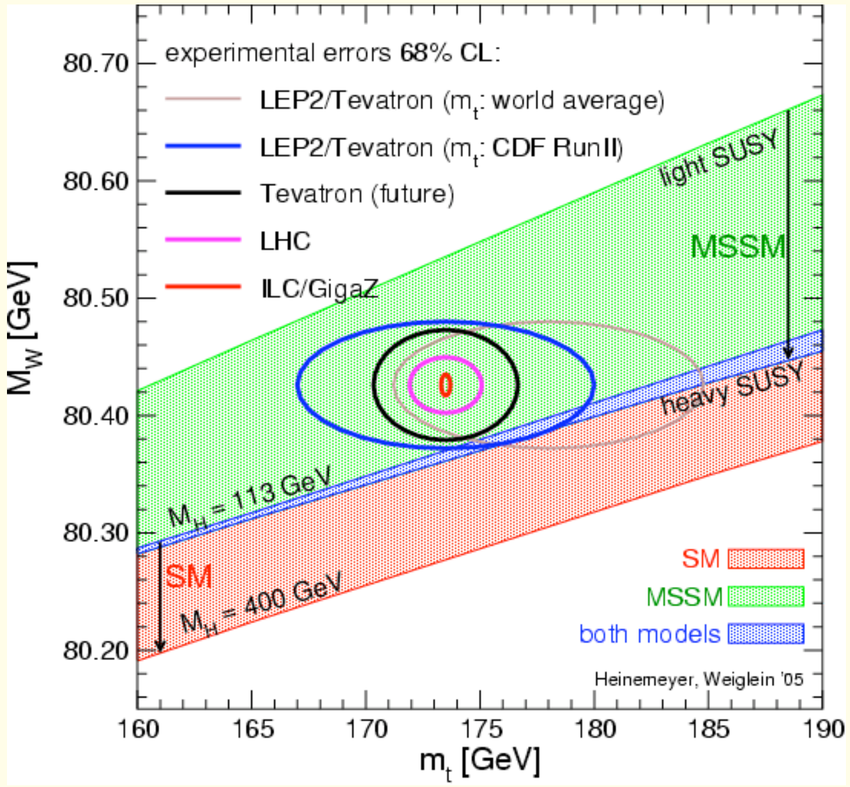
International Linear Collider Workshop
University of Illinois at Chicago, Nov. 17th, 2008

Motivation

- Top quark couples strongly to the Higgs sector and a good probe of new physics.
- The top mass is the dominant source of theoretical uncertainty in EWPOs.
- Typically the uncertainty in the extracted Higgs mass will be limited by the uncertainty in the top mass.

$$\delta m_t \sim \delta m_h.$$

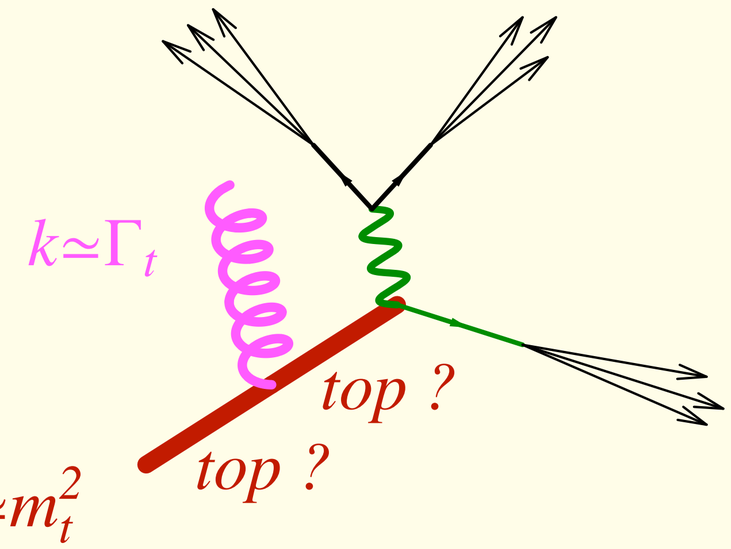
- Good reasons to measure the top mass with high precision.

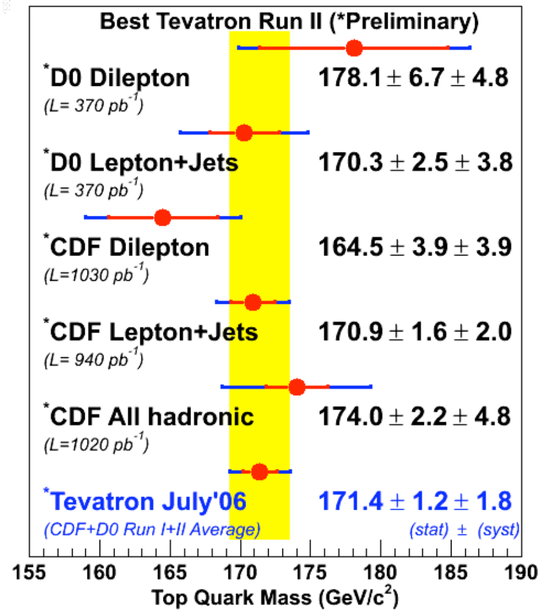


Predictions for the W and top mass in the SM & MSSM

What are we Measuring?

- What is the top mass?
  - Top is a colored parton. Cannot define physical on-shell mass.
  - Top mass is a parameter of the Lagrangian.
  - Top mass parameter is scheme dependent.





- Which top mass?
  - Which mass are the experimentalists measuring?
  - Pole mass? : $\delta m \sim \Lambda_{\rm QCD}$ renormalon ambiguity, poor perturbative behavior.
  - For better precision we need a short distance top mass.
  - How can we extract a short distance mass? Which mass?

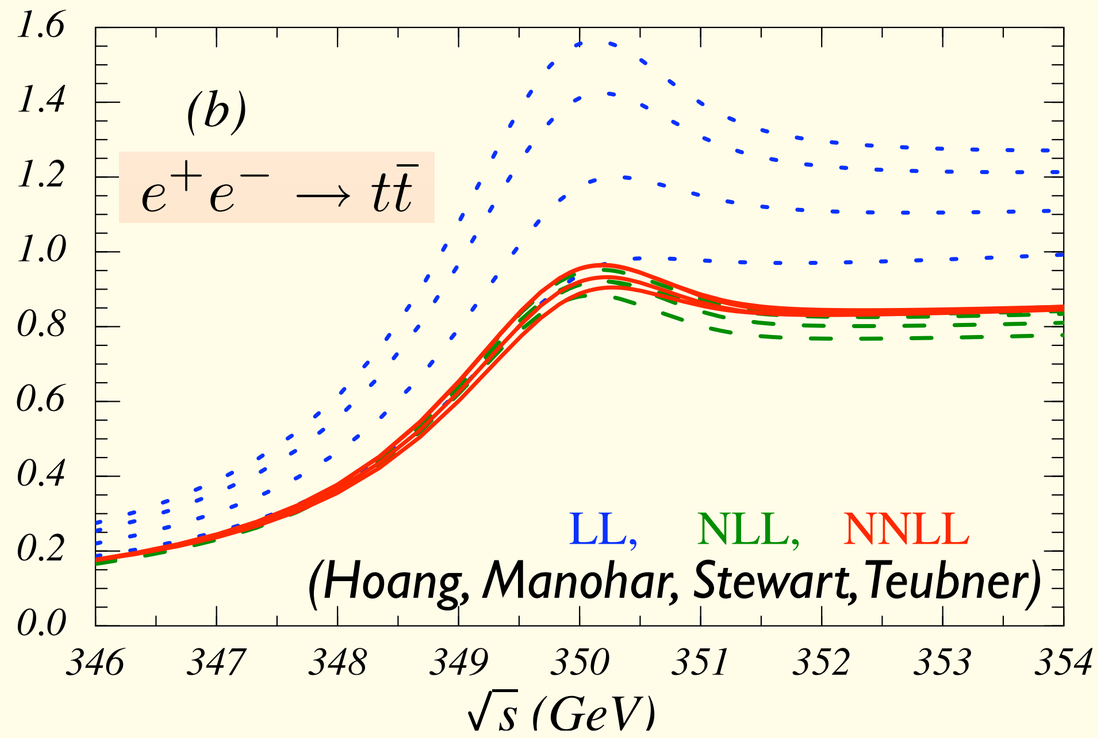
Current Top Mass Mesurement:

$$M_{\rm t} = 170.9 \pm 1.1({\rm stat}) \pm 1.5({\rm syst})\ {\rm GeV}/c^2 \quad \text{(CDF/D0)}$$

Threshold Scan

(Fadin & Khoze; Peskin & Strassler; Hoang, Manohar, Stewart, Teubner,...)

- **Top pair production in the threshold region**
  - Shape of total cross-section sensitive to top mass.
  - Top width provides IR cutoff.
  - Non-perturbative effects are small.

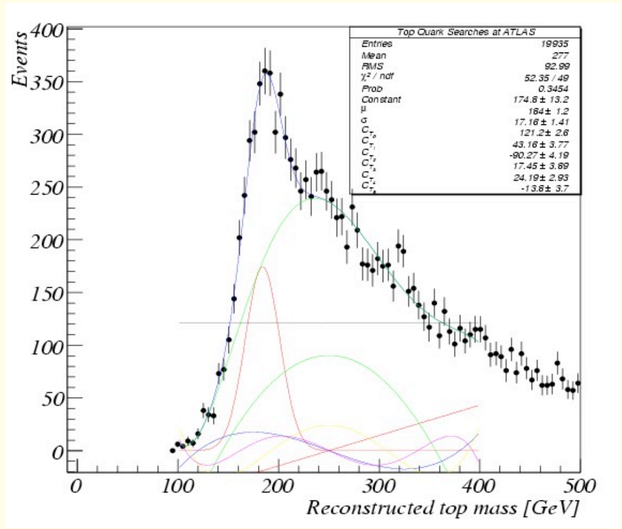


- **Physics well understood**
  - NRQCD is the appropriate EFT.
  - Well defined relation to short distance mass. eg. 1S mass
  - NNLL results known.
  - Theoretical uncertainty:

$$\delta m_t^{th} \sim 100 MeV$$

Jet Reconstruction

- Jet reconstruction methods not so well understood
  - Suitable observables with a well defined relation to a short distance mass ★
  - Summation of large logarithms ★
  - Final state radiation ★
  - Initial state radiation
  - PDFs
  - Jet Energy scale ★
  - Beam remnants
  - ...



★ Issues common to the ILC & LHC

- We study high energy top pair production at the linear collider

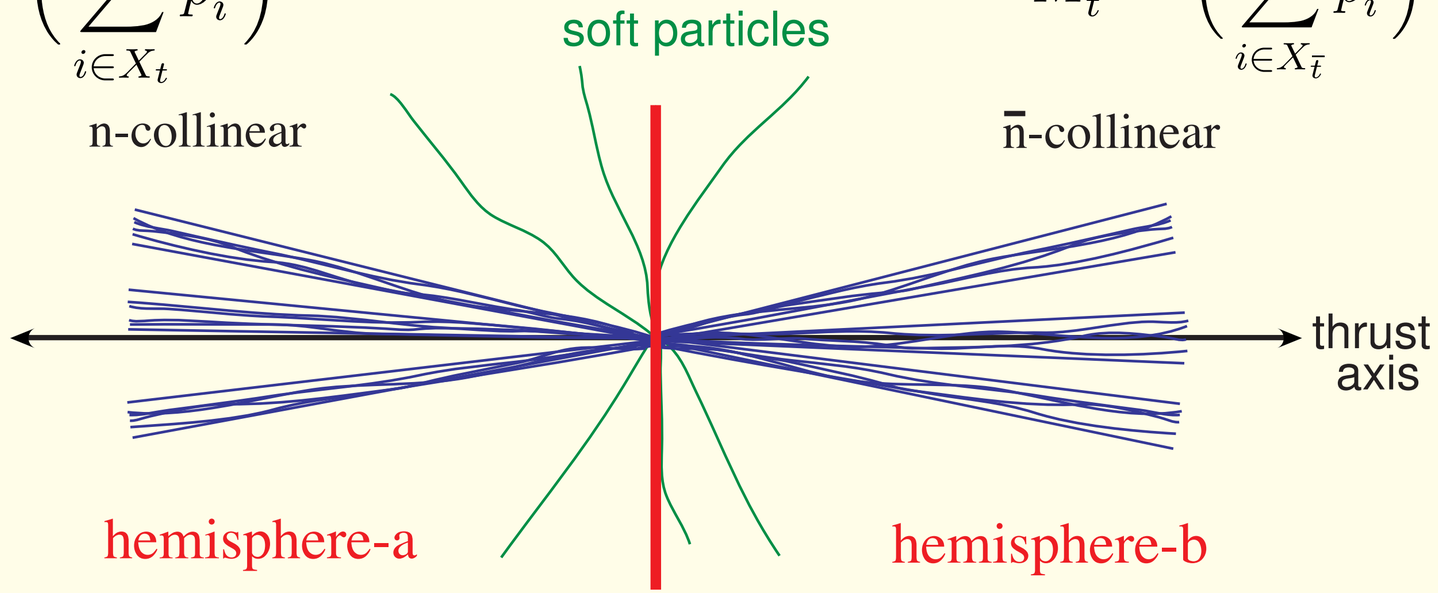
$$e^+e^- \to t\bar{t}X\,, \quad Q \gg m \gg \Gamma > \Lambda_{QCD}$$

Linear Collider Observable

- **Hemisphere invariant mass distribution of top jets:**

$$\left(\frac{d\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}}$$

$$M_t^2 = \Big(\sum_{i\in X_t} p_i^\mu\Big)^2 \qquad M_{\bar{t}}^2 = \Big(\sum_{i\in X_{\bar{t}}} p_i^\mu\Big)^2$$



- **Peak Region**

$$\hat{s}_{t,\bar{t}} \equiv \frac{s_{t,\bar{t}}}{m} \equiv \frac{M_{t,\bar{t}}^2 - m^2}{m} \sim \Gamma \ll m$$

Relevant Energy Scales

- Center of mass energy $Q \sim 1\text{TeV}$
- Top quark mass $m \sim 174\text{GeV}$
- Top quark width $\Gamma \sim 2\text{GeV}$
- Confinement Scale $\Lambda \sim 500 MeV$

Disparate energy scales ⟶ Effective Field Theory!

Effective Field Theories

Kinematics for Top Jets: I

- **High Energy Condition: Top quark pairs are produced with a center of mass energy much larger than the top mass**

$$Q \gg m$$

- **In this limit one can treat top quarks as collinear degrees of freedom in the Soft Collinear Effective Theory (SCET)** *(Bauer, Fleming, Luke, Pirjol, Stewart)***.**

Kinematics for Top Jets: II

- **Invariant Mass Condition: We characterize on shell production by the requirement:**

$$M^2_{t,\bar{t}} - m^2 \lesssim m\Gamma$$

- **This condition looks like the invariant mass constraint on a heavy quark in Heavy Quark Effective Theory (HQET)** *(Isgur, Wise,...)***.**

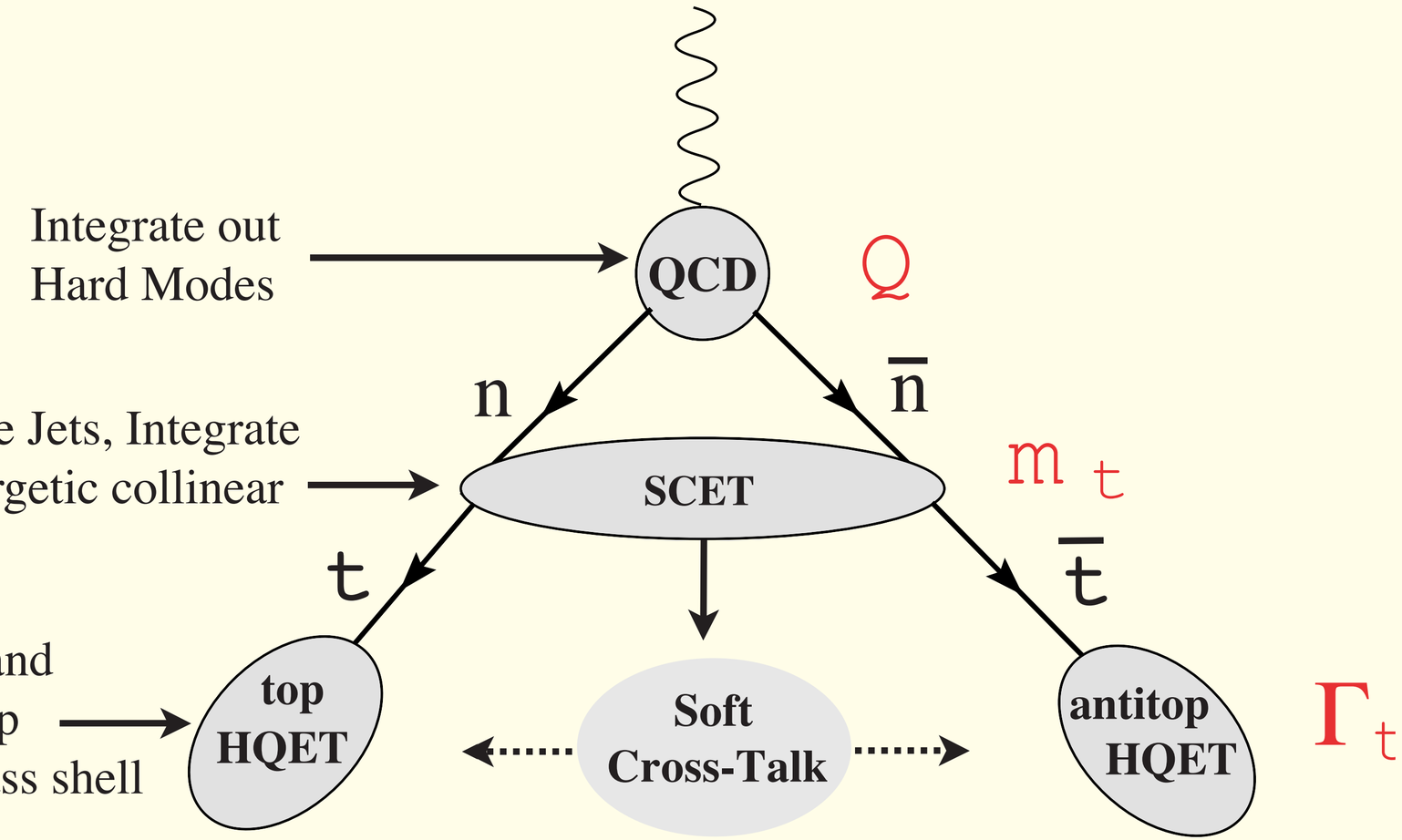
- **HQET has been generalized to unstable particles***(Beneke, Chapovsky, Signer, Zanderighi)***.**

Group Photo of Effective Field Theories

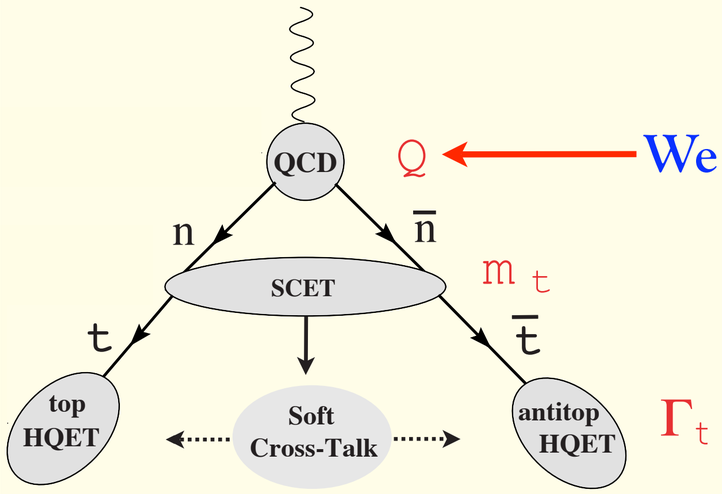
QCD Q

SCET m

HQET Γ



The QCD Cross-Section



- The cross-section in QCD has the general form :

$$\sigma = \sum_X^{res.} (2\pi)^4 \delta^4(p_e + p_{\bar{e}} - p_X) \sum_{ij} L_{\mu\nu}^{(ij)} \langle 0|J_i^\mu(0)|X\rangle\langle X|J_j^{\dagger\nu}(0)|0\rangle$$

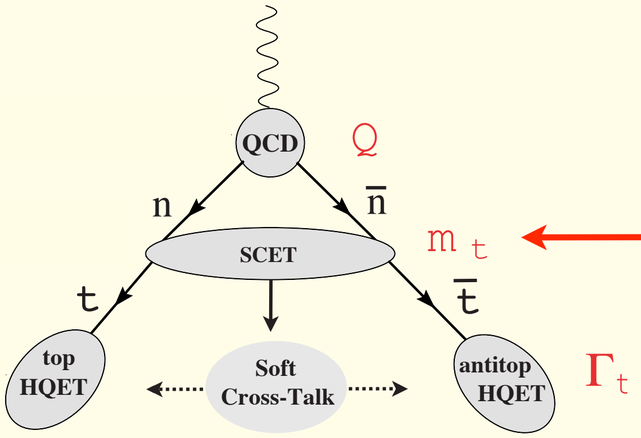
- The sum over final states X is restricted to contain a top jet and an anti-top jet with invariant masses close to the top mass.

- The top quark currents are produced by photon and Z exchange:

$$J_i^\mu(x) = \bar{\psi}(x)\Gamma_i^\mu\psi(x)\,, \quad \Gamma_\gamma^\mu = \gamma^\mu,\ \Gamma_Z^\mu = g^V\gamma^\mu + g^A\gamma^\mu\gamma_5$$

SCET Cross-section

(Fleming, Hoang, Mantry, Stewart)



- **The SCET cross section takes the form:**

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- \; J_n(s_t - Q\ell^+, \mu) J_{\bar{n}}(s_{\bar{t}} - Q\ell^-, \mu) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

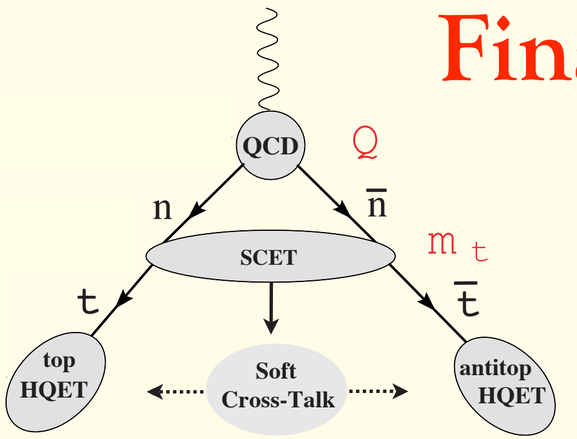
Hard Wilson Coefficient | **Top Jet Function** | **Anti-Top Jet Function** | **Soft Cross Talk Function**

- **The same soft function appears in massless dijets***(Korchemsky & Sterman; Bauer, Lee, Manohar, Wise; Becher & Schwartz)*.

Extraction of strong coupling from LEP data

Final Form of Factorized Cross-Section

(Fleming, Hoang, Mantry, Stewart)



Γ_t ⟵ We are here.

- The HQET cross section takes the form:

Hard Production modes integrated out

"Hard" collinear gluons integrated out

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

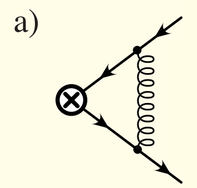
$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$

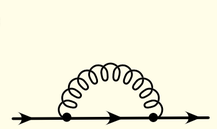
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Matching QCD onto SCET at One Loop

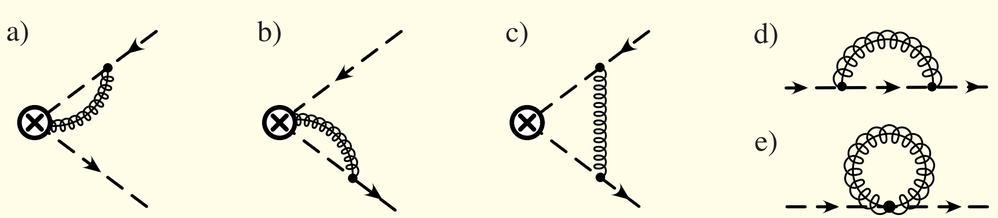
$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

QCD





SCET



- **Matching QCD production current onto SCET at the hard scale:**

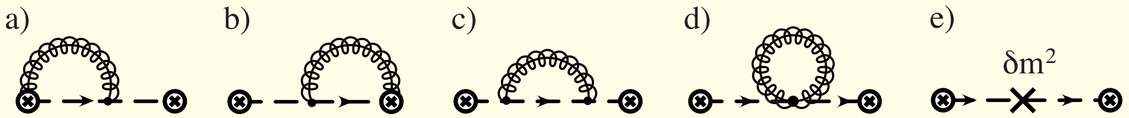
$$C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\ln\frac{-Q^2}{\mu^2} - \ln^2\frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

$$H_Q(Q,\mu_Q) = |C(Q,\mu_Q)|^2 = 1 + \frac{\alpha_s C_F}{4\pi}\left[-2\ln^2\left(\frac{Q^2}{\mu_Q^2}\right) + 6\ln\left(\frac{Q^2}{\mu_Q^2}\right) - 16 + \frac{7\pi^2}{3}\right]$$

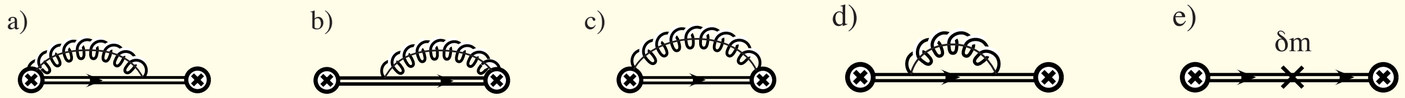
Matching SCET onto BHQET at One Loop

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

SCET



BHQET



- **Matching SCET jet functions onto bHQET jet functions:**

$$J_n(m\hat s, \Gamma, \mu_m) = T_+(m,\mu_m)\, B_+(\hat s, \Gamma, \mu_m)$$
$$J_{\bar n}(m\hat s, \Gamma, \mu_m) = T_-(m,\mu_m)\, B_-(\hat s, \Gamma, \mu_m)$$

$$T_\pm(\mu, m) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right).$$

$$H_m\left(m, \mu_m\right) = T_+(m,\mu_m) T_-(m,\mu_m) = 1 + \frac{\alpha_s C_F}{2\pi}\left(\ln^2\frac{\mu_m^2}{m^2} + \ln\frac{\mu_m^2}{m^2} + 4 + \frac{\pi^2}{6}\right)$$

Who Wants to Run in SCET?

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; J_n(s_t - Q\ell^+, \mu)\, J_{\bar{n}}(s_{\bar{t}} - Q\ell^-, \mu)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Run the Wilson Coefficient:

Run the jet and soft functions:

Top Down

$$\mu \frac{d}{d\mu} H_Q(Q,\mu) = \gamma_{H_Q}(Q,\mu)\, H_Q(Q,\mu)$$

Bottom Up

$$\mu \frac{d}{d\mu} J_{n,\bar{n}}(s,\mu) = \int ds'\, \gamma_{J_{n,\bar{n}}}(s-s')\, J_{n,\bar{n}}(s',\mu)$$

$$\mu \frac{d}{d\mu} S(\ell^+,\ell^-,\mu) = \int d\ell'^+ d\ell'^-\, \gamma_S(\ell^+ - \ell'^+, \ell^- - \ell'^-) S(\ell'^+, \ell'^-, \mu)$$

- **Scale independence of the cross-section requires the equivalence of top down and bottom up running. This provides a check on the consistency of the jet invariant mass definition.**

Who Wants to Run in HQET?

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$

Run the Wilson Coefficient

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Run the Jet & Soft functions

Top Down

$$\mu \frac{d}{d\mu} H_m\left(m, \frac{Q}{m}, \mu\right) = \gamma_{H_m}\left(\frac{Q}{m}, \mu\right) H_m\left(m, \frac{Q}{m}, \mu\right)$$

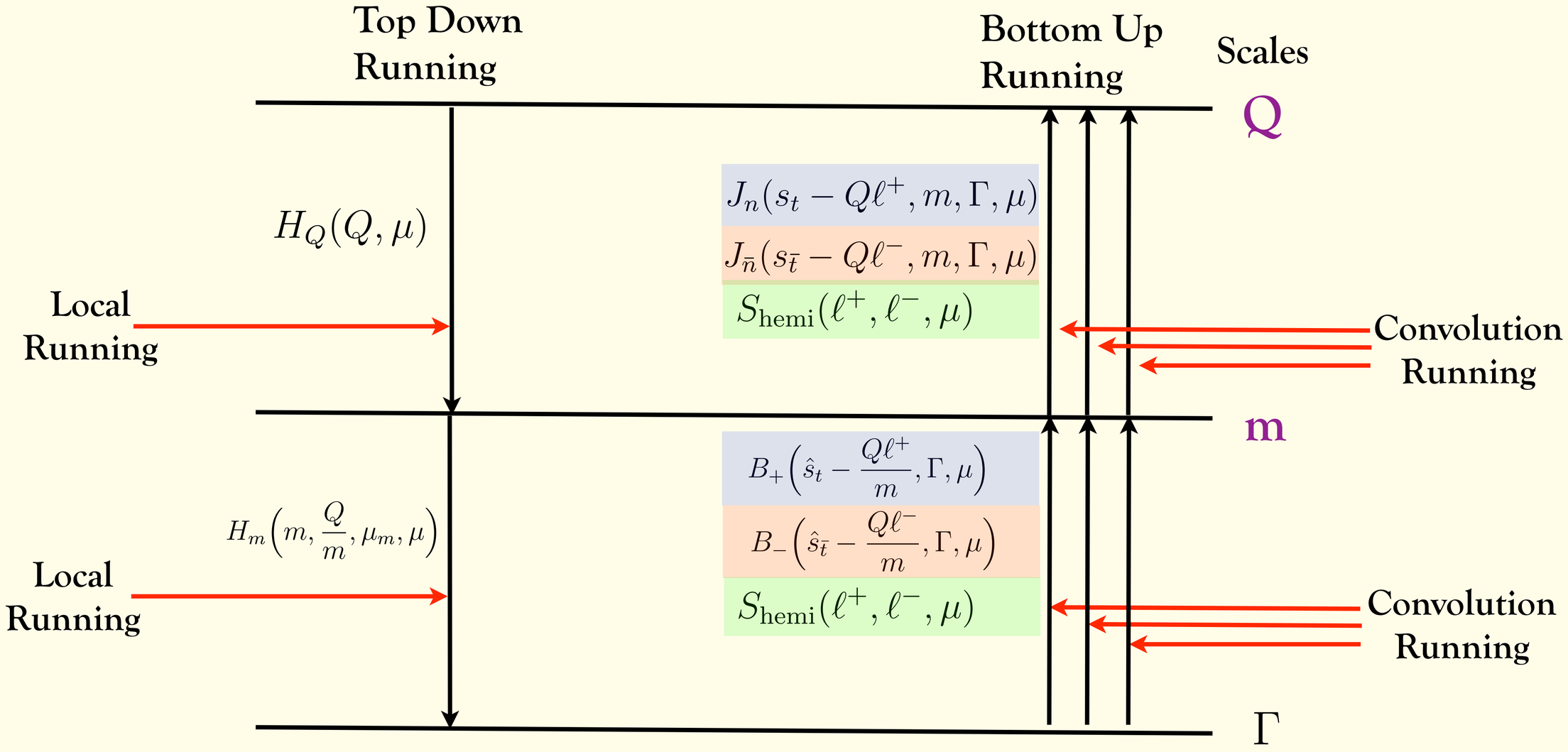
Bottom Up

$$\mu \frac{d}{d\mu} B_\pm(\hat{s}, \mu) = \int d\hat{s}'\, \gamma_{B_\pm}(\hat{s} - \hat{s}')\, B_\pm(\hat{s}', \mu)$$

$$\mu \frac{d}{d\mu} S_{hemi}(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^-\, \gamma_S(\ell^+ - \ell'^+, \ell^- - \ell'^-) S_{hemi}(\ell'^+, \ell'^-, \mu)$$

- **Scale independence of the cross-section requires the equivalence of top down and bottom up running. This provides a check on the consistency of the jet invariant mass definition.**

Equivalence of Top-Down vs. Bottom Up



- Running between the different scales mostly affects only the normalization!

Short Distance Mass for Jets

Connecting the Observable to a Short Distance Mass Scheme

- **Top mass sensitivity comes from the bHQET jet functions**

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

- **We have an analytic formula for the double differential jet invariant mass distribution in terms of the pole mass.**

- **We can now switch to a short distance mass scheme in bHQET.**

$$m_{\text{pole}} = m + \delta m$$

Switching Mass Schemes in bHQET

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Top HQET

$$\mathcal{L}_+ = \bar{h}_{v_+}\Big(iv_+ \cdot D_+ - \delta m + \frac{i}{2}\Gamma\Big)h_{v_+},$$

Anti-Top HQET

$$\mathcal{L}_- = \bar{h}_{v_-}\Big(iv_- \cdot D_- - \delta m + \frac{i}{2}\Gamma\Big)h_{v_-}$$

Top mass scheme

- **Power counting in bHQET requires**

$$\delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma$$

- **Note that this power counting breaks down in the $\overline{\text{MS}}$ scheme:**

$$\delta \overline{m} \sim \alpha_s \overline{m} \gg \Gamma$$

- **We need a short distance mass that respects the power counting of bHQET.**

Top Resonance Mass Schemes

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

- **Top resonance mass schemes are compatible with measurements relying on an underlying Breit-Wigner which incorporates the top width:**

$$\delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma$$

- **Some mass schemes in this context are**
  - Peak Mass
  - Moment Mass
  - Position Mass *(Jain, Scimemi, Stewart)*

$$\left.\frac{d}{d\hat{s}}\, B(\hat{s}, \delta m^{\rm peak}, \Gamma_t, \mu)\right|_{\hat{s}=0} = 0$$

$$\int_{-\infty}^{R} d\hat{s}\ \hat{s}\ B(\hat{s}, \delta m^{\rm mom}, \mu) = 0$$

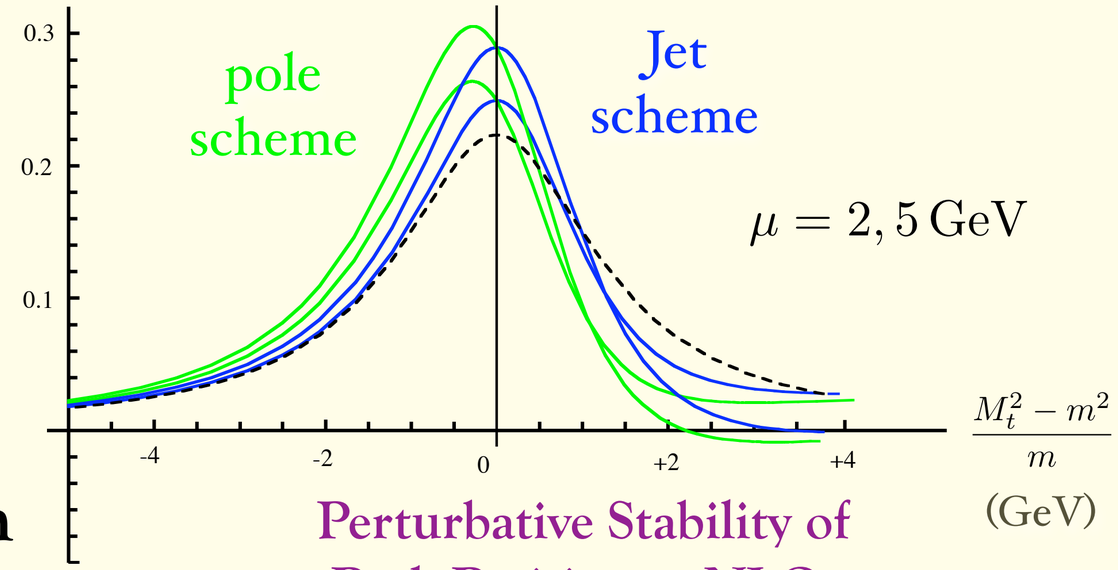
$$\delta m_J = \frac{-i}{2\,\tilde{B}(y,\mu)}\, \frac{d}{dy}\, \tilde{B}(y,\mu)\Big|_{y=-ie^{-\gamma_E}/R}$$

- **These top resonance mass schemes can be related to the more familiar mass schemes.**

Peak Mass

- Define peak mass as:

$$\left.\frac{dB_+(\hat{s},\mu,\delta m_J)}{d\hat{s}}\right|_{\hat{s}=0} = 0$$



Perturbative Stability of Peak Position at NLO.

- In the jet mass scheme the NLO jet function is modified as:

$$\tilde{B}_\pm(\hat{s},\mu) \;=\; B_\pm(\hat{s},\mu) + \frac{1}{\pi m_J}\,\frac{(4\,\hat{s}\,\Gamma)\,\delta m_J}{(\hat{s}^2+\Gamma^2)^2}$$

- At NLO the jet mass is related to the pole mass scheme as follows:

$$m_J(\mu) = m_{\rm pole} - \Gamma\frac{\alpha_s(\mu)}{3}\left[\ln\left(\frac{\mu}{\Gamma}\right) + \frac{3}{2}\right]$$

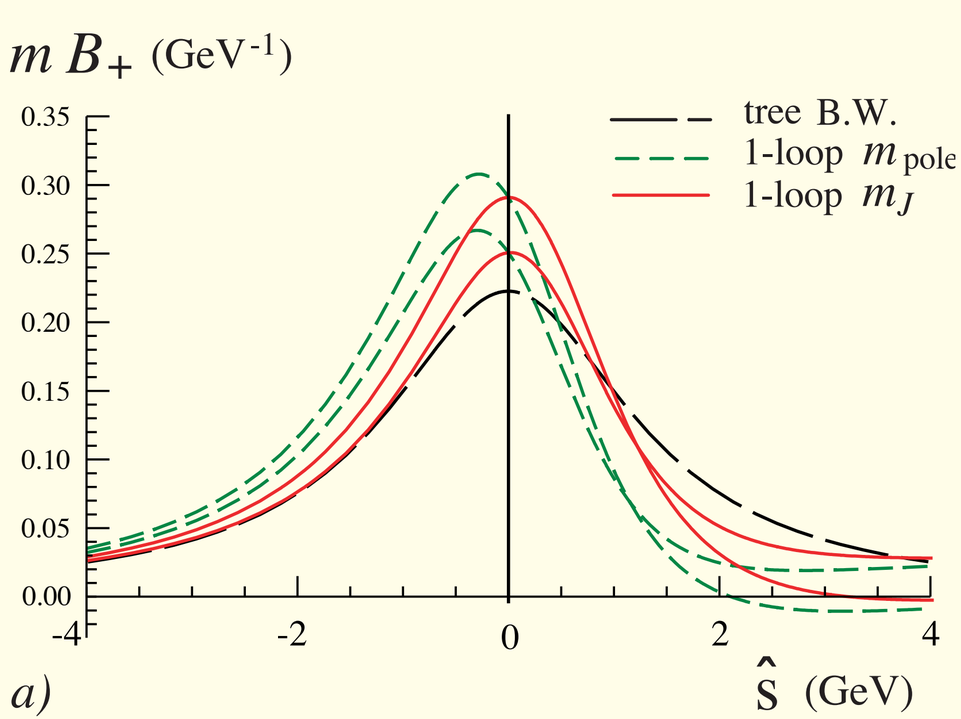
Moment Mass

- **Define the moment mass scheme as:**

$$0 = \int_{-\infty}^{L_m} d\hat{s}\ \hat{s}\ B_+^{\Gamma=0}(\hat{s}, \delta m_J)\,, \qquad L_m \sim\ \Gamma_t .$$

- **At NLO we get:**

$$\delta m_J = L_m\ \frac{\alpha_s(\mu) C_F}{\pi}\ \left[\ln\left(\frac{\mu}{L_m}\right) + \frac{3}{2}\right].$$



IR Group Flow of the Top Mass

(Hoang, Jain, Scimemi, Stewart)

- **Mass schemes can be parameterized by 'R'.**

$$m_{\rm pole} = m(R,\mu) + \delta m(R,\mu)\,,$$

$$\delta m(R,\mu) = R\sum_{n=1}^{\infty}\sum_{k=0}^{n} a_{nk}\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n \ln^k\left(\frac{\mu}{R}\right)$$

- **MSbar and top resonance schemes satisfy**

$$\overline{\rm MS}: R' = \bar{m}(\mu) \gg \Gamma_t, \qquad \text{Top Resonance scheme}: R \sim \Gamma_t \ll m$$

- **Conversion between such schemes can introduce large logs of**

$$\log\frac{R'}{R}$$

- **These logs are summed by an IR group flow(more details in 0803.4214, HJSS).**

$$R\frac{d}{dR}m(R) = -R\,\gamma_R[\alpha_s(R)]$$

Non-Perturbative Effects

Soft Function

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

- **Soft function is non-perturbative and encodes the cross-talk between top jets.**

One can model these effects as

$$S(\ell^+, \ell^-, \mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^-\; S_{\text{part}}(\ell^+ - \tilde{\ell}^+, \ell^- - \tilde{\ell}^-, \mu)\, S_{\text{mod}}(\tilde{\ell}^+, \tilde{\ell}^-)$$

Perturbative Partonic soft function

Model soft function

- **The 'model' soft function is taken to be of the form**

Gap parameter

$$S_{\text{mod}}(\ell^+, \ell^-, \Delta) = f_{\text{exp}}\left(\ell^+ - \Delta, \ell^- - \Delta\right),$$

$$f_{\text{exp}}(\ell^+, \ell^-) = \theta(\ell^+)\theta(\ell^-) \frac{\mathcal{N}(a,b)}{\Lambda^2} \left(\frac{\ell^+\ell^-}{\Lambda^2}\right)^{a-1} \exp\left(\frac{-(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2}\right)$$

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

- **The partonic soft function has a renormalon** *(Gardi; Hoang,Stewart)*

$$S(\ell^+, \ell^-, \mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^-\; S_{\rm part}(\ell^+ - \tilde{\ell}^+ - \delta, \ell^- - \tilde{\ell}^- - \delta, \mu)\, f_{\rm exp}(\tilde{\ell}^+ - \bar{\Delta}, \tilde{\ell}^- - \bar{\Delta})$$

Renormalon present

- **The renormalon corresponds to an ambiguity in the gap parameter:**

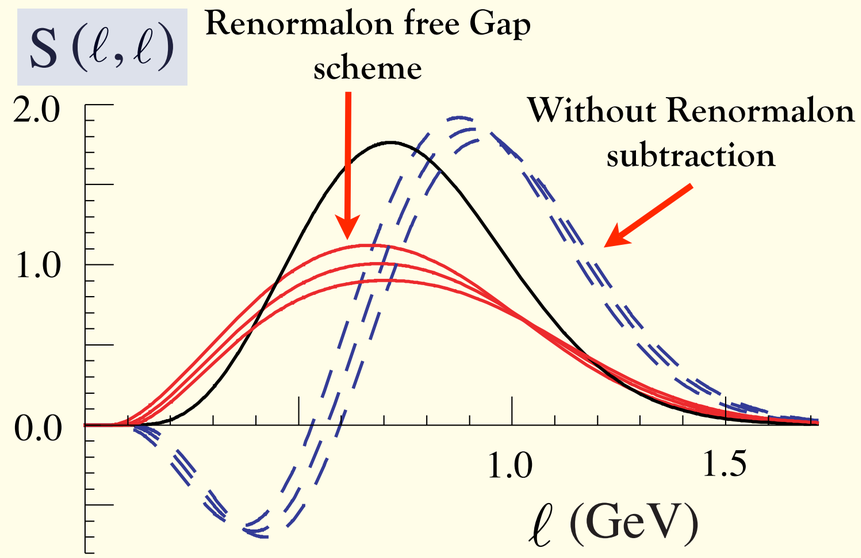
$$\Delta = \bar{\Delta}(\mu) + \delta(\mu)$$

Renormalon free Gap scheme

- **Moment gap scheme:**

$$0 = \int_{-\infty}^{L_\Delta} d\ell^+ \int_{-\infty}^{L_\Delta} d\ell^-\; \ell^+\; S_{\rm part}(\ell^+ - \delta, \ell^- - \delta, \mu)$$

$$\delta_1 = -2L_\Delta \frac{C_F \alpha_s(\mu)}{\pi}\left[\ln\left(\frac{\mu}{L_\Delta}\right) + 1\right]$$

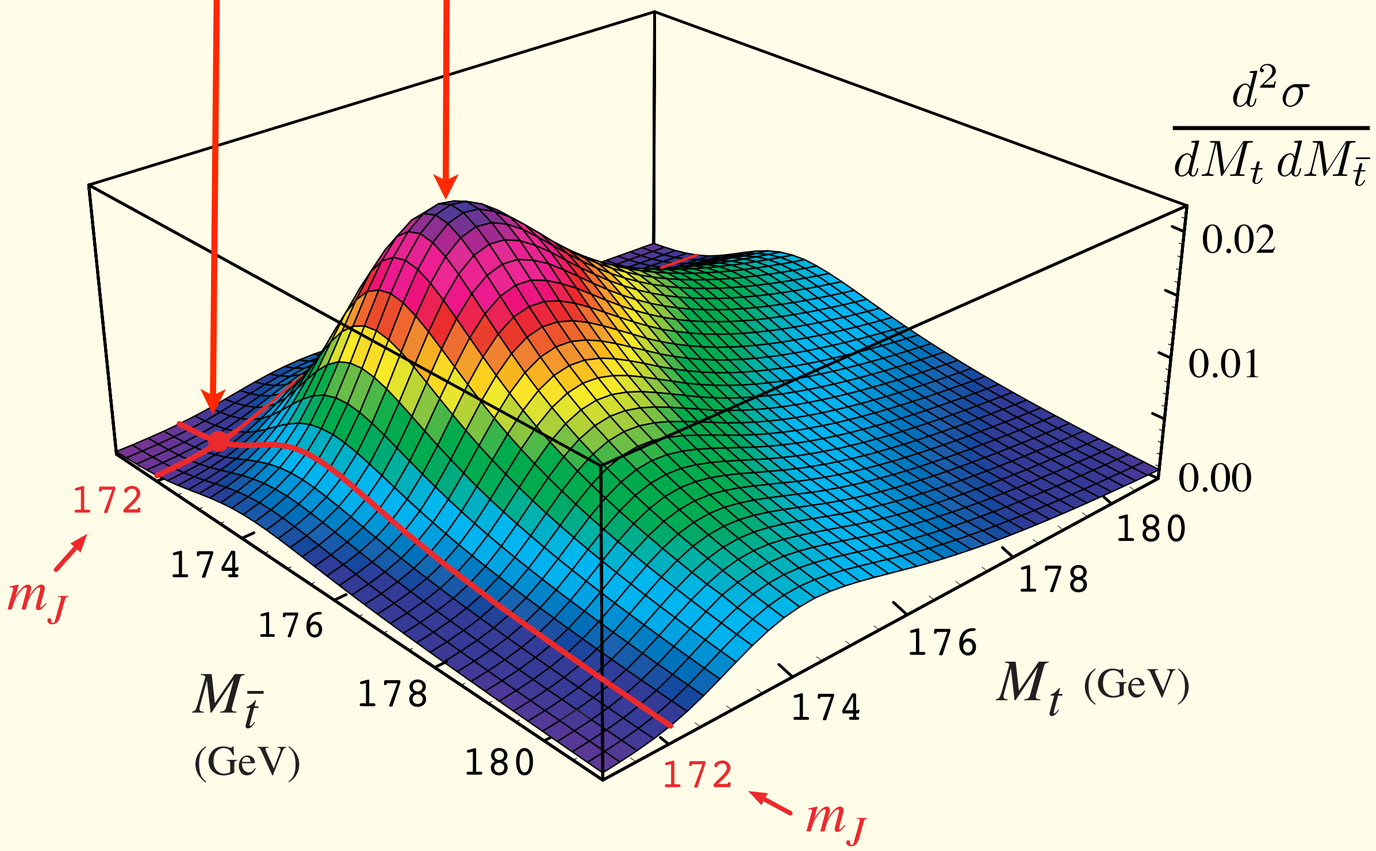


Extraction of the Short Distance Top Mass

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Extract top mass →

$$m = M_{\text{peak}} - \Gamma(\alpha_s + \alpha_s^2 + \cdots) - \frac{Q\Lambda_{QCD}}{m}$$



- **Final result with NLL resummation.**

(Fleming, Hoang, Mantry, Stewart)

Conclusions

- An analytic framework in effective field theory now exists for high energy pair production of tops at a linear collider:

  - Factorization for high energy top pair production at a linear collider.
  - Large logarithms summed using RG equations in effective field theory; NLL resummation
  - Short distance mass schemes suitable for reconstruction from jets; Top resonance schemes
  - Measured peak position can be related to the short distance mass.