Spin tracking update

Positron source meeting July 22 2008

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- Depolarization along linac negligible, spin tracking along BDS still to do
- Sources of depolarization at the IP – new simulation results for CLIC
- CAIN with full polarizations for pair processes
- Theoretical calculations of depolarization processes
  - Operator Method – Baier, Katkov et al
  - Volkov Solution method – 1964(Nikishov-Ritus) approximations
- Updates on Sokolov-Ternov and T-BMT calculations

Depolarization at the IP

There is depolarization (spin flip) due to the QED process of Beamsstrahlung, given by the Sokolov-Ternov equation

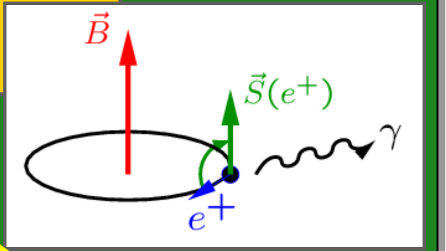
$$dW = -i\frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^\infty K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx$$
$$\text{where}\quad z = \frac{2}{3\upsilon\omega\epsilon_i}\,\frac{\omega_f}{\epsilon_i-\omega_f}$$

The fermion spin can also precess in the bunch fields. Equation of motion of the spin given by the T-BMT equation

$$\frac{d\vec{S}}{dt} = -\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T+(a+1)\vec{B}_L-\gamma\left(a+\frac{1}{\gamma+1}\right)\frac{1}{c^2}\vec{v}\times\vec{E}\right]\times\vec{S}$$

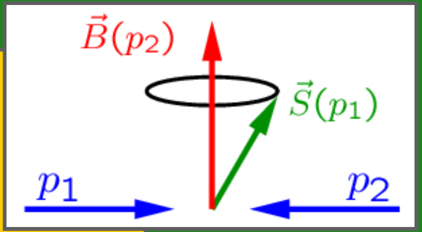
At the IP, the anomalous magnetic moment subject to radiative corrections in the presence of the bunch field

Stochastic spin diffusion from photon emission: Sokolov-Ternov effect, etc.



Parameter set	Depolarization ΔP_{tw}		
	T-BMT	S-T	total
Nominal	0.08%	0.02%	0.10%
low Q	0.04%	0.02%	0.06%
large Y	0.17%	0.02%	0.19%
low P	0.15%	0.09%	0.24%
TESLA	0.11%	0.03%	0.14%

Classical spin precession in inhomogeneous external fields: T-BMT equation.



Depol sims with CLIC parameters (I Bailey) change in polarization vector magnitude

	CLIC-G	ILC nom	ILC (80/30%)
T-BMT	0.10%	0.17%	0.14%
Beamstr.	3.40%	0.05%	0.03%
incoherent	0.06%	0.00%	0.00%
coherent	1.30%	0.00%	0.00%
total	**4.80%**	**0.22%**	**0.17%**

CAIN incoherent pair processes: Breit-Wheeler cross-section with polarizations

- Breit-Wheeler cross-section, CAIN original:

$$\sigma_{orig} \propto 2\left(1-h+\frac{2\epsilon^2-1}{2\epsilon^4}\right)\sinh^{-1}p+\frac{p}{\epsilon}\left(3h-1-\frac{1}{\epsilon^2}\right)$$

where p = electron momentum
ϵ = electron energy
$h = \xi_2 \xi'_2$

full treatment due to Baier & Grozin hep-ph/0209361

$$\frac{d\sigma}{d\cos\theta\, d\phi} = \frac{\alpha^2}{4s^2 x^2 y^2} \sum_{ii'jj'} F^{ii'}_{jj'}\ \xi_j \xi'_{j'}\ \zeta_i \zeta'_{i'}$$

F are functions of scalar products of 4-momenta

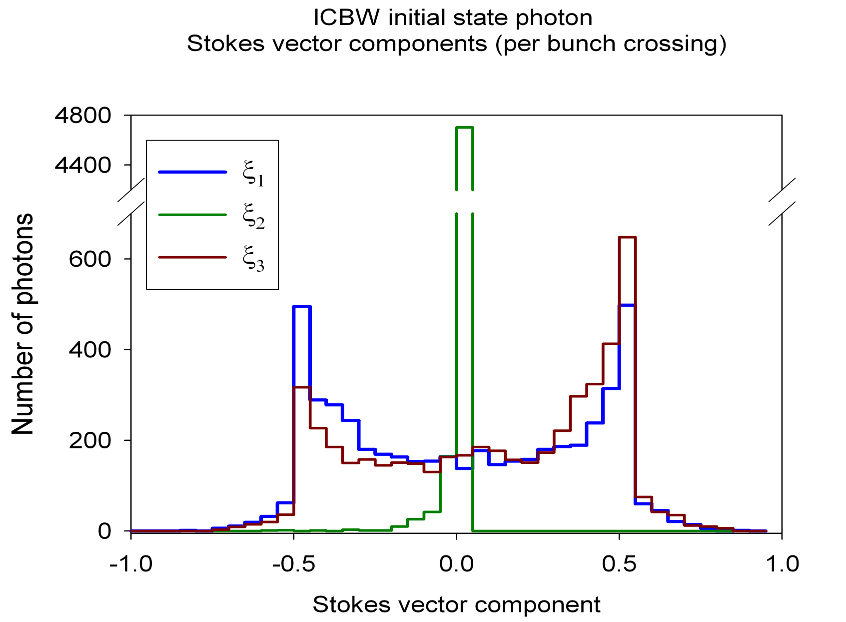
$$\sigma_{new} \propto 2\left(1-h+\frac{2}{\epsilon^2}\left(ha+\xi_1\xi'_1\right)-\frac{ha}{\epsilon^4}\right)\sinh^{-1}p+\frac{p}{\epsilon}\left(3h-1-\xi_1\xi'_1-\xi_3\xi'_3-\frac{ha}{\epsilon^2}\right)$$

where $ha = 1+\xi_3+\xi'_3+\xi_3\xi'_3$

Full expression has similar structure to original CAIN form, so can utilise existing monte-carlo methods

Final pair polarizations $\zeta^{(f)}$

$$\zeta_i^{(f)} = \frac{1}{F} \sum_{ii'jj'} F^{i0}_{jj'} \, \xi_j \xi'_{j'} \quad where \quad F = \sum_{jj'} F^{00}_{jj'} \, \xi_j \xi'_{j'}$$



Beamstrahlung photons have almost no circular polarization component – due to beam field having constant crossed field vectors

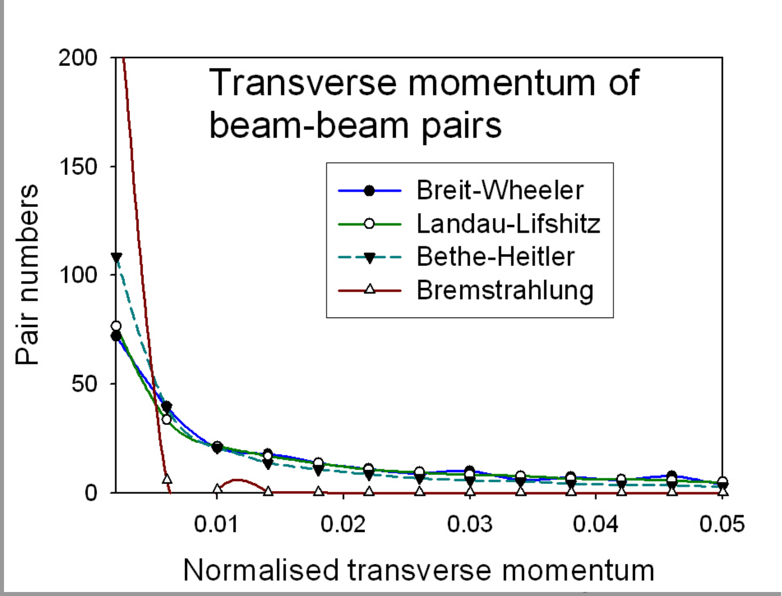
1st two components of the Breit-Wheeler pair polarization depends heavily on the photon circular polarization component, therefore ~0

Original beam polarization contained in the 3rd component of the produced pair

Final e- $(\zeta_1,\zeta_2,\zeta_3)$=(-0.0024,-0.0024, 0.9883)

Final e+ $(\zeta_1,\zeta_2,\zeta_3)$=(0.0023,0.0079, 0.987)

- Pairs have sufficient transverse momentum to distinguish them from outgoing beam, so…
- Could imagine a study to see how sensitive the final pair polarization is to the initial beam polarization, but…
- Breit-Wheeler pairs overlap with L-L and B-H pairs (derived using Equivalent Photon Approximation)



ST - two formulations

Derivation of Sokolov-Ternov eq^n in the literature usually done with the **'operator method'** of Baier Katkov et al

The energy levels of an ultrarelativistic electron in a magnetic field are very close together, so assume motion is classical

$$\frac{\hbar\omega_0}{\epsilon_p} = \frac{B}{B_c}\left(\frac{m}{\epsilon_p}\right)^2$$

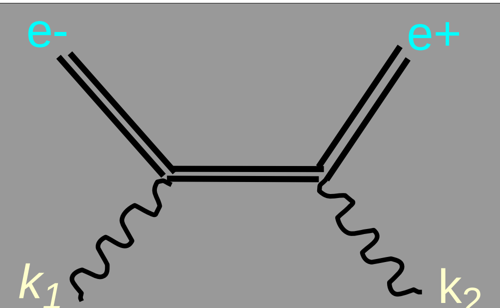
- Operators of the dynamical variables of the electron commute
- Retain commutator between electron and photon variables

Or it can be done without kinematic approximat^{ns} at Lagrangian level using the Bound Interaction Picture

$$L_{BD} = \bar{\psi}_V(x)(i\gamma^\mu\partial_\mu - e\gamma^\nu A^e_\nu) - m\psi_V(x)$$

Make the external field implicit in a bound Dirac Lagrangian L_{BD}. Neglect the interaction between photons and the external field A^e.

- The interaction Lagrangian expresses interaction between the free Maxwell and the Bound Dirac fields
- Requires solutions of the Bound Dirac field operators



fermion solutions represented by double straight lines

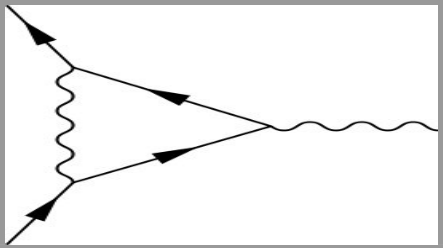
T-BMT Spin tracking: Anomalous magnetic moment in a strong field

Needed in T-BMT equation to calculate the rate of depolarization due to beam-beam effect

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T + (a+1)\vec{B}_L - \gamma\left(a+\frac{1}{\gamma+1}\right)\frac{\beta}{c}\vec{e}_v \times \vec{E}\right]$$

Main contrib^{n} from vertex diagram

$$a = \frac{\alpha}{2\pi} + O(\alpha^2)$$



when fermion is embedded in a strong external field characterised by $\Upsilon = \nu^2 \frac{(k.p)}{m^2}$ the anomalous magnetic moment develops a dependence on Υ and is given by (Baier-Katk)

$$a(\Upsilon) = -\frac{\alpha}{\pi\Upsilon}\int_0^\infty \frac{x}{(1+x)^3}\, dx \int_0^\infty \sin\left[\frac{x}{\Upsilon}\left(t+\frac{1}{3}t^3\right)\right] dt$$

However…we can envisage

- recalculating the vertex diagram in B/P with Volkov solutions replacing all fermion lines
- Making mass correction (including self-energies)