Top effective operators: why ILC?

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The question:

What could ILC possibly offer to study top couplings beyond LHC capabilities?

This question is non-trivial because LHC is an excellent top factory.

This discussion is mainly based on papers

- *"A minimal set of top anomalous couplings"*, NPB '09
- *"A minimal set of top-Higgs anomalous couplings"*, NPB '09
- *"Effective operators in top physics"*, Proc. ICHEP '10
- *"Effective four-fermion operators in top physics: a roadmap"*, hep-ph '10, NPB

Our framework

Preferred framework: gauge-invariant effective operators

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

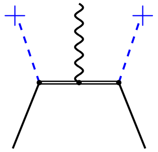
where

$\mathcal{L}_4 = \mathcal{L}_{\text{SM}}$ ➞ SM Lagrangian

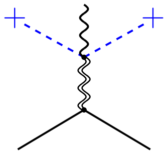
$\mathcal{L}_6 = \sum_x \frac{\alpha_x}{\Lambda^2} O_x$ ➞ O_x gauge-invariant building blocks

Parameterise effects of new physics at scale $\Lambda > v$

New physics contributions to top trilinear couplings

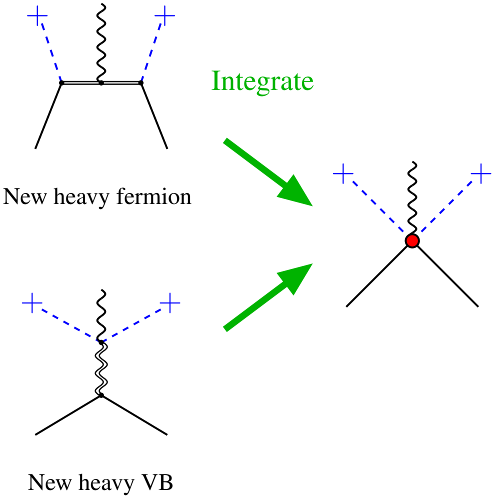


New heavy fermion

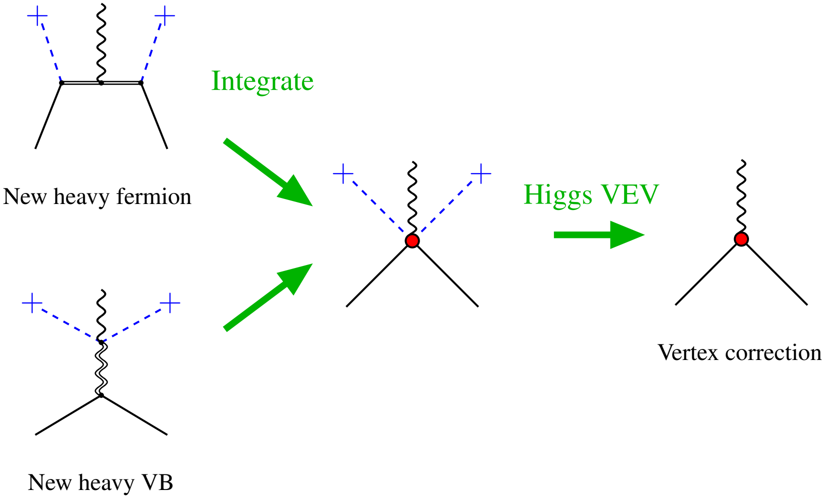


New heavy VB

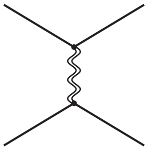
New physics contributions to top trilinear couplings



New physics contributions to top trilinear couplings

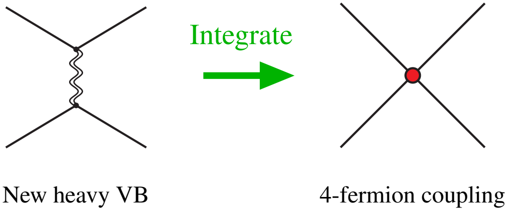


New physics contributions to top 4f operators



New heavy VB

New physics contributions to top 4f operators



Vertex corrections from dim 6 operators:

① Gauge interactions: only γ^μ and $\sigma^{\mu\nu} q_\nu$ terms

② Higgs: only scalar and pseudo-scalar terms

☞ This is general for any two-fermion vertices, not only the top quark!

So simple after eliminating many redundant operators
[JAAS NPB '09]

Example: *Wtb* vertex from dim 6 operators

$$\mathcal{L}_{Wtb} = -\frac{g}{\sqrt{2}} \bar{b}\, \gamma^{\mu} \left(V_L P_L + V_R P_R \right) t\, W_{\mu}^{-} - \frac{g}{\sqrt{2}} \bar{b}\, \frac{i \sigma^{\mu\nu} q_{\nu}}{M_W} \left(g_L P_L + g_R P_R \right) t\, W_{\mu}^{-} + \text{h.c.}$$

$$q = p_t - p_b = p_W$$

Anomalous couplings $\sim \frac{v^2}{\Lambda^2}$ ➞ an expansion seems reasonable

… but which one?

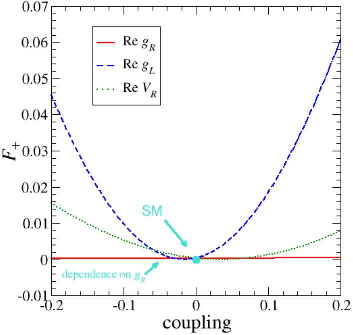
Linear new physics effects $\sim 1/\Lambda^2$ (interference with SM)
quadratic ones $\sim 1/\Lambda^4$

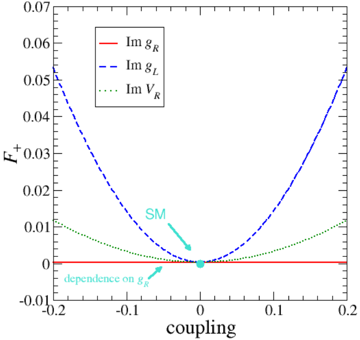
The $1/\Lambda^2$ approximation $(m_b = 0)$

NP structure	$1/\Lambda^2$	$1/\Lambda^4$
$\bar{b}_L \gamma^\mu t_L$	$\delta V_L = C_{\phi q}^{(3,3+3)} \frac{v^2}{\Lambda^2}$	$(\delta V_L)^2 + \text{dim } 8\ \bar{L}L$
$\bar{b}_R \gamma^\mu t_R$	✘	$(\delta V_R)^2 = \frac{1}{4}(C_{\phi\phi}^{33*})^2 \frac{v^4}{\Lambda^4}$
$\bar{b}_R \sigma^{\mu\nu} t_L$	✘	$(\delta g_L)^2 = 2(C_{dW}^{33*})^2 \frac{v^4}{\Lambda^4}$
$\bar{b}_L \sigma^{\mu\nu} t_R$	$\delta g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$	$(\delta g_R)^2 + \text{dim } 8\ \bar{L}R$

keep only $1/\Lambda^2$ in observables ☞ is it sensible?

Example: W helicity fraction F_+ , $F_+ \sim 0$ in the SM





to order $1/\Lambda^2$ ➡ $F_+ = F_+^{\mathrm{SM}}$

approximation not sensible to explore NP effects

In the $1/\Lambda^2$ approximation, many observables do not receive contributions from new physics

Another example: FCNC

FCNC absent in the SM ☞ BSM it is order $1/\Lambda^4$

Then, one must go beyond the $1/\Lambda^2$ approximation to have BSM phenomenology

Killing $1/\Lambda^4$ kills all smoking guns of new physics!

It makes sense to consider the lowest non-zero order for each type of contribution

☞ the leading order approximation

★ justified by phenomenology → different NP structures give different effects

★ consistent within a $1/\Lambda$ expansion

The leading order approximation

NP structure	$1/\Lambda^2$	$1/\Lambda^4$
$\bar{b}_L \gamma^\mu t_L$	$\delta V_L = C_{\phi q}^{(3,3+3)} \frac{v^2}{\Lambda^2}$	$(\delta V_L)^2 + \text{dim 8 } \bar{L}L$
$\bar{b}_R \gamma^\mu t_R$	✗	$(\delta V_R)^2 = \frac{1}{4} (C_{\phi\phi}^{33*})^2 \frac{v^4}{\Lambda^4}$
$\bar{b}_R \sigma^{\mu\nu} t_L$	✗	$(\delta g_L)^2 = 2 (C_{dW}^{33*})^2 \frac{v^4}{\Lambda^4}$
$\bar{b}_L \sigma^{\mu\nu} t_R$	$\delta g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$	$(\delta g_R)^2 + \text{dim 8 } \bar{L}R$

… and with $m_b \neq 0$, $V_R \frac{m_b}{m_t} \sim V_R^2$, $g_L \frac{m_b}{m_t} \sim g_L^2$ of the same order

The leading order approximation

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$\bar{b}_L \sigma^{\mu\nu} t_R$	$\delta g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$	$(\delta g_R)^2 + \text{dim } 8\ \bar{L}R$

… and with $m_b \neq 0$, $V_R \frac{m_b}{m_t} \sim V_R^2$, $g_L \frac{m_b}{m_t} \sim g_L^2$ of the same order

Summary (I)

1. Gauge-invariant effective operators provide a convenient way to parameterise new physics contributions to top interactions
2. In this framework, the general form of NP corrections from dim 6 operators is rather simple after redundant operators are removed
3. A consistent expansion can be made to address $1/\Lambda^2$ corrections to SM processes and new $1/\Lambda^4$ processes absent in the SM
4. There are gauge relations among effective operator contributions to top interactions

   (Nevertheless, measuring all vertices is always useful to overconstrain effective operator coefficients)

ILC vs LHC

The question:

What could ILC possibly offer to study top couplings beyond LHC capabilities?

The answer:

Has gauge symmetry any implication on LHC / ILC complementarity for top couplings from effective operators?

Answer both at the same time

ILC vs LHC

The question:

What could ILC possibly offer to study top couplings beyond LHC capabilities?

~~The answer~~ one more question:

Has gauge symmetry any implication on LHC / ILC complementarity for top couplings from effective operators?

Answer both at the same time

Wtb, Ztt vertices

Consider left-handed γ^μ contributions (RH unrelated)

$$\mathcal{L}_{Wt_Lb_L} = -\frac{g}{\sqrt{2}}\,\bar{b}_L\,\gamma^\mu V_L t_L\,W_\mu^- \qquad V_L = V_{tb} + C_{\phi q}^{(3,3+3)}\,\frac{v^2}{\Lambda^2}$$

$$\mathcal{L}_{Zt_Lt_L} = -\frac{g}{2c_W}\,\bar{t}_L\,\gamma^\mu X_{tt}^L t_L\,Z_\mu \qquad X_{tt}^L = 1 + \left[C_{\phi q}^{(3,3+3)} - C_{\phi q}^{(1,3+3)}\right]\frac{v^2}{\Lambda^2}$$

$$\mathcal{L}_{Zb_Lb_L} = -\frac{g}{2c_W}\,\bar{b}_L\,\gamma^\mu X_{bb}^L b_L\,Z_\mu \qquad X_{bb}^L = -1 + \left[C_{\phi q}^{(3,3+3)} + C_{\phi q}^{(1,3+3)}\right]\frac{v^2}{\Lambda^2}$$

We know from LEP that $\delta X_{bb}^L \simeq 0$ $\rightarrow$ $\boxed{\delta X_{tt}^L = 2\delta V_L\ !}$

LHC: V_L measured with 5% accuracy [ATLAS CSC book]

ILC: X_{tt}^L measured with 2% accuracy [Abe et al. '01]

$\rightarrow$ ILC probes $C_{\phi q}^{(3,3+3)}$ with $5\times$ better precision!

Wtb, Ztt and γtt vertices

$\sigma^{\mu\nu} q_\nu$ contributions

$$\mathcal{L}_{Wt_Rb_L} = -\frac{g}{\sqrt{2}} \bar{b}_R \frac{i\sigma^{\mu\nu} q_\nu}{M_W} g_R t_R \, W_\mu^- \qquad g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$$

$$\mathcal{L}_{Ztt} = -\frac{g}{2c_W} \bar{t} \frac{i\sigma^{\mu\nu} q_\nu}{M_Z} (d_V^Z + i d_A^Z \gamma_5) t \, Z_\mu \qquad d_V^Z = \sqrt{2} \, \mathrm{Re} \left[c_W C_{uW}^{33} - s_W C_{uB\phi}^{33} \right] \frac{v^2}{\Lambda^2}$$

$$d_A^Z = \sqrt{2} \, \mathrm{Im} \left[c_W C_{uW}^{33} - s_W C_{uB\phi}^{33} \right] \frac{v^2}{\Lambda^2}$$

$$\mathcal{L}_{\gamma tt} = -e \bar{t} \frac{i\sigma^{\mu\nu} q_\nu}{m_t} (d_V^\gamma + i d_A^\gamma \gamma_5) t \, A_\mu \qquad d_V^\gamma = \frac{\sqrt{2}}{e} \, \mathrm{Re} \left[s_W C_{uW}^{33} + c_W C_{uB\phi}^{33} \right] \frac{v m_t}{\Lambda^2}$$

$$d_A^\gamma = \frac{\sqrt{2}}{e} \, \mathrm{Im} \left[s_W C_{uW}^{33} + c_W C_{uB\phi}^{33} \right] \frac{v m_t}{\Lambda^2}$$

☞ determine both C_{uW}^{33} and $C_{uB\phi}^{33}$

Wtb, Ztt and γtt vertices

For example:

determine Re C^{33}_{uW} from W helicity fractions
Im C^{33}_{uW} from normal W polarisation in t decays

LHC sensitivity:

$$\mathrm{Re}\, g_R \sim 0.02 \quad \rightarrow \quad \mathrm{Re}\, C^{33}_{uW} \sim 0.23 \quad (\Lambda = 1\ \mathrm{TeV})$$

$$\mathrm{Im}\, g_R \sim 0.04 \quad \rightarrow \quad \mathrm{Im}\, C^{33}_{uW} \sim 0.45 \quad (\Lambda = 1\ \mathrm{TeV})$$

☞ $C^{33}_{uB\phi}$ from $t\bar{t}$ production at ILC with excellent precision (both Z, γ exchange contribute)

Note that γ^μ and $\sigma^{\mu\nu} q_\nu$ couplings have different energy dependence, use measurements at two CM energies

We can also overconstrain C^{33}_{uW}, $C^{33}_{uB\phi}$ using $t\bar{t}\gamma$ production at LHC

Four-fermion operators

General comments

Gauge-invariant 4f operators often give several 4f terms related by gauge symmetry

We are interested here in operators with two leptons (➞ ILC)

probed in:

- top decays ☞ effect $\sim m_t^2/\Lambda^2$
- top FCNC decays ☞ effect $\sim m_t^4/\Lambda^4$
- $e^+e^- \to t\bar{t}$ ☞ effect $\sim s/\Lambda^2$
- $e^+e^- \to t\bar{u}_k$ ☞ effect $\sim s^2/\Lambda^4$

Four-fermion operators

Operators with two 3rd generation quarks + two leptons

	$t\bar{b}e_i\bar{\nu}_j$	$t\bar{t}e_i\bar{e}_j$	$t\bar{t}\nu_i\bar{\nu}_j$	#		$t\bar{b}e_i\bar{\nu}_j$	$t\bar{t}e_i\bar{e}_j$	$t\bar{t}\nu_i\bar{\nu}_j$	#
$O_{\ell q}^{ji33}$	–	✔	✔	6	O_{qe}^{3ij3}	–	✔	–	6
$O_{\ell q'}^{j33i}$	✔	–	✔	6	O_{qde}^{ji33}	✔	–	–	9
O_{eu}^{ji33}	–	✔	–	6	$O_{\ell q\epsilon}^{ji33}$	✔	✔	–	9
$O_{\ell u}^{j33i}$	–	✔	✔	6	$O_{q\ell\epsilon}^{3ij3}$	✔	✔	–	9

ILC benefits

- operators only tested at ILC: $O_{\ell q}$, O_{eu}, $O_{\ell u}$, O_{qe}
- better precision for other ones: $O_{\ell qe}$, $O_{q\ell e}$

- $t\bar{b}e_i\bar{\nu}_j$: probed in $t \to b e_i^+ \nu_j$
- $t\bar{t}e_i\bar{e}_j$: probed in $e^+e^- \to t\bar{t}$ (with $i,j=1$)
- $t\bar{t}\nu_i\bar{\nu}_j$: ?

At this point it is good to remember that

1. Effective operators parameterise NP corrections
2. Any specific NP gives certain operators (not all of them)

   Example: heavy W_R does not give $LLLL$ 4f operators
3. Missing to probe some operators means that we may miss the ones actually produced !

   This is also a motivation to keep operators giving $1/\Lambda^4$ corrections

Four-fermion operators

Operators with one 3rd gen. + one light quark + two leptons

	$t\bar{d}_k e_i \bar{\nu}_j$	$t\bar{u}_k e_i \bar{e}_j$	$t\bar{u}_k \nu_i \bar{\nu}_j$	#		$t\bar{d}_k e_i \bar{\nu}_j$	$t\bar{u}_k e_i \bar{e}_j$	$t\bar{u}_k \nu_i \bar{\nu}_j$	#
$O_{\ell q}^{jik3}$	–	✔	✔	18	O_{qde}^{jik3}	✔	–	–	18
$O_{\ell q'}^{j3ki}$	✔	–	✔	18	$O_{\ell q\epsilon}^{jik3}$	✔	✔	–	18
O_{eu}^{jik3}	–	✔	–	18	$O_{\ell q\epsilon}^{ij3k}$	–	✔	–	18
$O_{\ell u}^{j3ki}$	–	✔	✔	18	$O_{q\ell\epsilon}^{kij3}$	✔	✔	–	18
O_{qe}^{kij3}	–	✔	–	18	$O_{q\ell\epsilon}^{3jik}$	–	✔	–	18

ILC benefits

- better precision for operators giving $t\bar{u}_k e\bar{e}$ terms

- $t\bar{d}_k e_i \bar{\nu}_j$: probed in $t \to d_k e_i^+ \nu_j$
- $t\bar{u}_k e_i \bar{e}_j$: probed in $t \to u_k e_i^+ e_j^-$ and $e^+e^- \to t\bar{u}_k$ $(i,j=1)$
- $t\bar{u}_k \nu_i \bar{\nu}_j$: probed in $t \to u_k \bar{\nu}_i \nu_j$

Summary (II)

The answer: benefits from ILC

1. Much effort is devoted to measure the Wt_Lb_L interaction in single top production at LHC

   ☞ we have shown that measuring Zt_Lt_L at ILC is equivalent but with $5\times$ better sensitivity to NP

   (provided that we know from LEP that Zb_Lb_L is close to SM)

2. LHC + ILC measurements can determine all contributions to Ztt, γtt vertices with good precision

3. Many four-fermion operators can be tested at ILC but not at LHC (some at both: ILC precision better)

ADDITIONAL SLIDES

Operators involving top trilinear interactions

$$O_{\phi q}^{(3,i+j)} = \frac{i}{2} \left[\phi^\dagger (\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I)\phi\right] (\bar{q}_{Li}\gamma^\mu \tau^I q_{Lj})$$

$$O_{\phi q}^{(1,i+j)} = \frac{i}{2} (\phi^\dagger \overleftrightarrow{D^\mu} \phi)(\bar{q}_{Li}\gamma^\mu q_{Lj})$$

$$O_{\phi\phi}^{ij} = i(\tilde{\phi}^\dagger D_\mu \phi)(\bar{u}_{Ri}\gamma^\mu d_{Rj})$$

$$O_{\phi u}^{i+j} = \frac{i}{2} (\phi^\dagger \overleftrightarrow{D^\mu} \phi)(\bar{u}_{Ri}\gamma^\mu u_{Rj})$$

$$O_{uW}^{ij} = (\bar{q}_{Li}\sigma^{\mu\nu}\tau^I u_{Rj})\tilde{\phi}\, W_{\mu\nu}^I$$

$$O_{dW}^{ij} = (\bar{q}_{Li}\sigma^{\mu\nu}\tau^I d_{Rj})\phi\, W_{\mu\nu}^I$$

$$O_{uB\phi}^{ij} = (\bar{q}_{Li}\sigma^{\mu\nu} u_{Rj})\tilde{\phi}\, B_{\mu\nu}$$

$$O_{uG\phi}^{ij} = (\bar{q}_{Li}\lambda^a\sigma^{\mu\nu} u_{Rj})\tilde{\phi}\, G_{\mu\nu}^a$$

$$O_{u\phi}^{ij} = (\phi^\dagger\phi)(\bar{q}_{Li} u_{Rj}\tilde{\phi})$$

Four-fermion operators (I)

$$O_{qq}^{ijkl} = \frac{1}{2}(\bar{q}_{Li}\gamma^\mu q_{Lj})(\bar{q}_{Lk}\gamma^\mu q_{Ll}) \qquad O_{qq'}^{ijkl} = \frac{1}{2}(\bar{q}_{Lia}\gamma^\mu q_{Ljb})(\bar{q}_{Lkb}\gamma^\mu q_{Lla})$$

$$O_{\ell q}^{ijkl} = (\bar{\ell}_{Li}\gamma^\mu \ell_{Lj})(\bar{q}_{Lk}\gamma^\mu q_{Ll}) \qquad O_{\ell q'}^{ijkl} = (\bar{\ell}_{Li}\gamma^\mu q_{Lj})(\bar{q}_{Lk}\gamma^\mu \ell_{Ll})$$

$$O_{\ell\ell}^{ijkl} = \frac{1}{2}(\bar{\ell}_{Li}\gamma^\mu \ell_{Lj})(\bar{\ell}_{Lk}\gamma^\mu \ell_{Ll})$$

$$O_{uu}^{ijkl} = \frac{1}{2}(\bar{u}_{Ri}\gamma^\mu u_{Rj})(\bar{u}_{Rk}\gamma^\mu u_{Rl}) \qquad O_{dd}^{ijkl} = \frac{1}{2}(\bar{d}_{Ri}\gamma^\mu d_{Rj})(\bar{d}_{Rk}\gamma^\mu d_{Rl})$$

$$O_{ud}^{ijkl} = (\bar{u}_{Ri}\gamma^\mu u_{Rj})(\bar{d}_{Rk}\gamma^\mu d_{Rl}) \qquad O_{ud'}^{ijkl} = (\bar{u}_{Ria}\gamma^\mu u_{Rjb})(\bar{d}_{Rkb}\gamma^\mu d_{Rla})$$

$$O_{eu}^{ijkl} = (\bar{e}_{Ri}\gamma^\mu e_{Rj})(\bar{u}_{Rk}\gamma^\mu u_{Rl}) \qquad O_{ed}^{ijkl} = (\bar{e}_{Ri}\gamma^\mu e_{Rj})(\bar{d}_{Rk}\gamma^\mu d_{Rl})$$

$$O_{ee}^{ijkl} = \frac{1}{2}(\bar{e}_{Ri}\gamma^\mu e_{Rj})(\bar{e}_{Rk}\gamma^\mu e_{Rl})$$

Four-fermion operators (II)

$$O_{qu}^{ijkl} = (\bar{q}_{Li} u_{Rj})(\bar{u}_{Rk} q_{Ll}) \qquad O_{qu'}^{ijkl} = (\bar{q}_{Lia} u_{Rjb})(\bar{u}_{Rkb} q_{Lla})$$

$$O_{qd}^{ijkl} = (\bar{q}_{Li} d_{Rj})(\bar{d}_{Rk} q_{Ll}) \qquad O_{qd'}^{ijkl} = (\bar{q}_{Lia} d_{Rjb})(\bar{d}_{Rkb} q_{Lla})$$

$$O_{\ell u}^{ijkl} = (\bar{\ell}_{Li} u_{Rj})(\bar{u}_{Rk} \ell_{Ll}) \qquad O_{\ell d}^{ijkl} = (\bar{\ell}_{Li} d_{Rj})(\bar{d}_{Rk} \ell_{Ll})$$

$$O_{qe}^{ijkl} = (\bar{q}_{Li} e_{Rj})(\bar{e}_{Rk} q_{Ll}) \qquad O_{qde}^{ijkl} = (\bar{\ell}_{Li} e_{Rj})(\bar{d}_{Rk} q_{Ll})$$

$$O_{\ell e}^{ijkl} = (\bar{\ell}_{Li} e_{Rj})(\bar{e}_{Rk} \ell_{Ll})$$

$$O_{qq\epsilon}^{ijkl} = (\bar{q}_{Li} u_{Rj}) \left[(\bar{q}_{Lk} \epsilon)^T d_{Rl} \right] \qquad O_{qq\epsilon'}^{ijkl} = (\bar{q}_{Lia} u_{Rjb}) \left[(\bar{q}_{Lkb} \epsilon)^T d_{Rla} \right]$$

$$O_{\ell q\epsilon}^{ijkl} = (\bar{\ell}_{Li} e_{Rj}) \left[(\bar{q}_{Lk} \epsilon)^T u_{Rl} \right] \qquad O_{q\ell\epsilon}^{ijkl} = (\bar{q}_{Li} e_{Rj}) \left[(\bar{\ell}_{Lk} \epsilon)^T u_{Rl} \right]$$