Open Theoretical Problems for Physics at Future Lepton Colliders

M. E. Peskin
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The organizers asked me to speak on this topic: What do we need now from theorists for the future of the linear collider program ?

There is an obvious answer:

Invent the correct model of reality at the TeV energy scale, and explore detailed aspects of this model that cannot be accessed in the LHC experiments.

This is very ambitious to accomplish a priori:

- ~~4th generation~~
- ~~MUSGRA/cMSSM~~
- most minimal SSM
- Higgs on the IR brane in warped geometry
- ...

I will restrict myself here to three problems that I feel have very high priority.

1. How do we model complex final states in e+e- annihilation with high precision ?

2. What is the correct naturalness criterion that predicts the mass scale of the TeV particle spectrum ?

3. What is the role of precision top quark measurements in uncovering the physics of the TeV scale ?

I omit one topic -- the Higgs boson.

The LHC results so far follow exactly the expectation if Nature contains a Standard Model-like Higgs boson in the mass range

114 GeV - 140 GeV

preferred by the precision electroweak analysis.

It is not yet time to worry about this. Instead, we should prepare for the discovery of the Higgs boson before the end of 2012 -- and even in 2011 if the mass is near 140 GeV.

1. How do we model complex final states in e+e- annihilation with high precision ?

In e+e- annihilation, the entire event is generated by hard processes from a color singlet initial state. This allows detailed theoretical predictions not only for rates but also for event shapes, some at the per mil level.

The analysis of complete events is crucial to the physics. Many issues involve correlation between beam polarization and the polarization of final state particles. Final state polarization are measured from lepton or jet angular distributions.

Multiquark Standard Model processes become increasingly important in LC physics at high energy:

For the measurement of the top-Higgs Yukawa coupling, we study $e^+e^- \to t\bar{t}h^0$. The fully-hadronic mode contains 8 primary jets.

In SUSY, the basic reaction $e^+e^- \to \tilde{\chi}^+\tilde{\chi}^-$ gives 4 jets. However, reactions with heavier states that decay to $\tilde{\chi}$ add jets at each stage of the decay.

What are the issues for a more exact description ?

For the LEP 2 experiments:

Generate $2 \rightarrow 2$ processes with the full $\mathcal{O}(\alpha)$ electroweak radiative corrections.

Generate $2 \rightarrow 4$ processes at tree level.

KORALW implemented this, generating the $2 \rightarrow 4$ processes with electroweak diagrams only.

For the ILC LOIs:

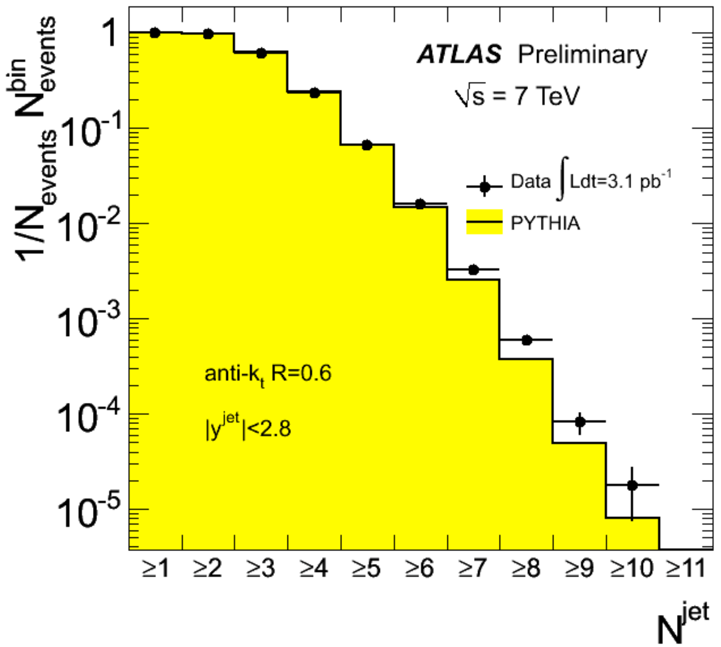
Generate events with similar tools, with the complete set of electroweak diagrams for $2 \rightarrow 6$ processes generated at the tree level by WHIZARD.

These algorithms omit explicit generation of $q\bar{q}$ pairs by QCD. Instead, final quarks initiate parton showers realized by PYTHIA.

This is a reasonable first approximation:

In e+e-, the most important sources of 4, 6, ... quark events are processes with intermediate vector particles (γ - initiated reactions, return to the Z, etc.)

PYTHIA is exact for the first $q\bar{q} \to q\bar{q}g$ splitting, so the use of its parton shower is an approximation only beginning at the 4-quark level. And, in this scheme, there is no double-counting between QCD and EW.



PYTHIA is qualitatively (~20%) correct for complex multiquark final states.

Still, for precision estimates, we should do better.

A. Improvement of the parton shower with matching to higher order QCD matrix elements

This is now the standard method for estimation of SM cross sections for $pp \to W + \text{jets}$ and similar hadron collider processes (implemented in ALPGEN, MadGraph, etc.).

Inclusion of NLO QCD corrections in all stages of the shower is still an open problem. However, there are now serious efforts to build algorithms that at NLO-accurate: MENLOPS (SHERPA), Geneva.

B. Inclusion of Sudakov corrections on the electroweak side

Beginning with Beenacker, many authors have noted that electroweak corrections at high energy behave as

$$-\frac{\alpha}{4\pi s_w^2}\log^2\frac{s}{m_W^2}$$

and give ~ - 10% corrections at TeV energies.

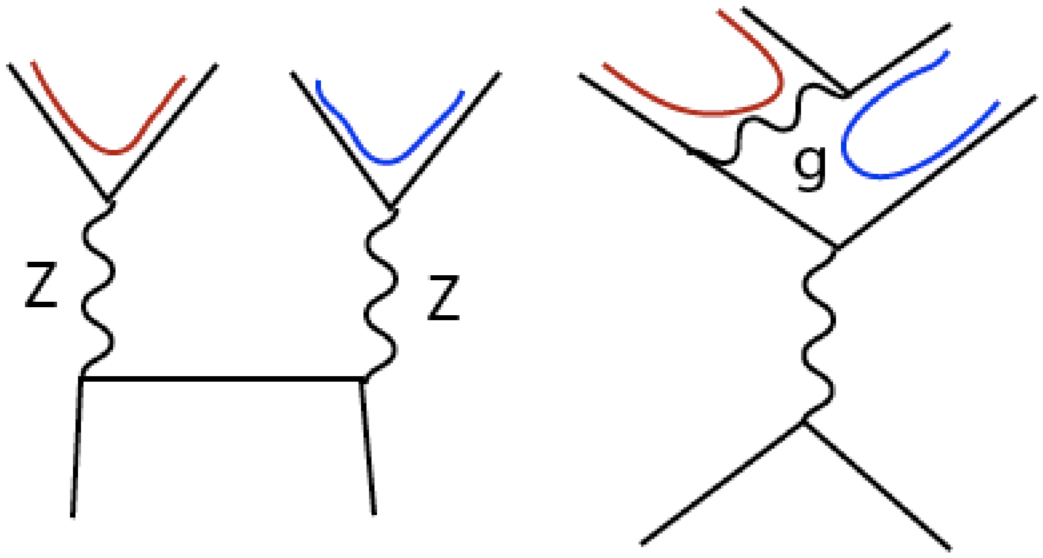
Denner and Pozzorini gave a complete understanding of the leading and next-to-leading terms. The leading terms come from Sudakov form factors from the full SU(2)XU(1) gauge theory. (Pozzorini's thesis: hep-ph/0201077)

In multiparticle processes, every 2-particle channel has similar large radiative corrections.

To take these into account, we need an electroweak analogue of the QCD parton shower.

C. Inclusion of EW/QCD interference

The current scheme does not include interference terms between electroweak and QCD diagrams. This is correct in the first approximation. EW and QCD diagrams do not interfere, because they contain different color flows:



However, the accuracy of this approximation is only $\mathcal{O}(1/N_c^2)$.

To do precision LC physics, we need a description of SM multifermion events that includes all three of these improvements. This would be a matched parton shower incorporating both EW and QCD branchings, made accurate to next-to-leading order.

This is a challenging problem, but QCD theorists are now working on even harder problems for LHC. We will be able to take advantage of what we learn from the LHC to provide excellent tools for the LC.

2. What is the correct naturalness criterion that predicts the mass scale of the TeV particle spectrum ?

In this section, I assume that the TeV-scale solution is SUSY.

For fundamental purposes (e.g. connection to string theory and quantum gravity), we need SUSY only at 10^{18} GeV.

For the connection to grand unification, we need SUSY only at 10 TeV.

The reason that we need SUSY below 1 TeV is to naturally generate the Higgs potential that gives

$$\langle \varphi \rangle = \frac{1}{\sqrt{2}} (246 \text{ GeV})$$

What constraints does this last requirement put on SUSY masses ?

In "The Case for a 500 GeV Linear Collider" (2000), one finds the relation:

$$m_W^2 = -1.3\mu^2 + 0.3m^2(\tilde{g}) + \cdots$$

and the implication is drawn that the $\tilde{\chi}^+$ should be below 250 GeV. The analysis makes use of unification relations between the SUSY-breaking mass terms for top and Higgs and for chargino and gluino.

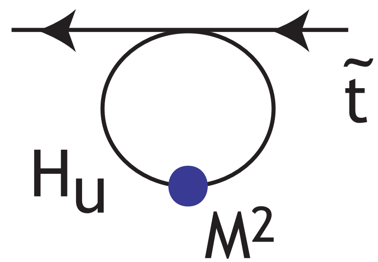
A more precise formula for the W,Z masses in SUSY is:

$$m_Z^2 = 2\,\frac{M_{Hd}^2 - \tan^2\beta M_{Hu}^2}{\tan^2\beta - 1} - 2\mu^2$$

To avoid large unnatural cancellations in this equation, μ must still be small. The top squark mass is constrained indirectly, since top squark loops renormalize M_{Hu}^2. The gluino mass enters more indirectly, through its effect on the top squark mass.

The 1st and 2nd generation squarks enter hardly at all.

The third generation squarks play a crucial role, because it is the diagram



that actually makes the Higgs boson mass parameter negative.

Thus, naturalness dictates that at least one colored superparticle is light, comparable to the Higgs mass scale.

This turns out to be a particle with a small production cross section in pp: about 100 fb for 7 TeV, $m(\tilde{t}) \sim 400\ \mathrm{GeV}$.

In 1996, Cohen, Kaplan, and Nelson proposed the

more minimal supersymmetric model

with only 3rd-generation sfermions, gauginos, Higgsino light.
There are many variations on this theme:

Focus Point Region — Feng Matchev Moroi

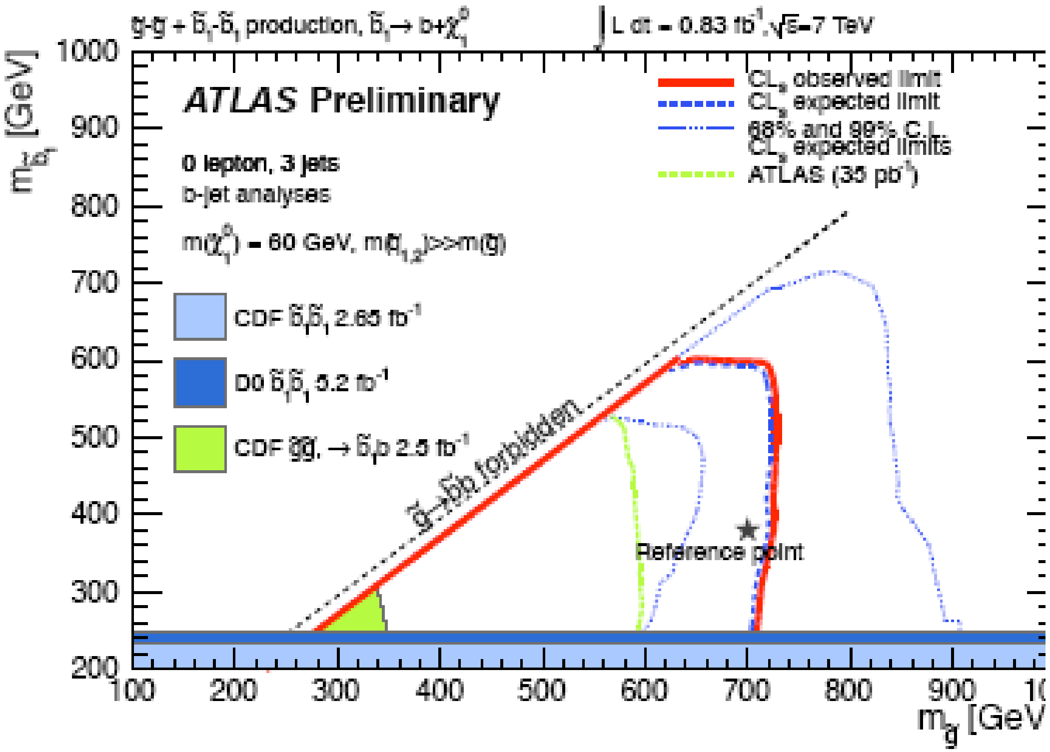
Golden Region — Perelstein Spethmann
(only Higginos and stops below 1 TeV)

Hidden SUSY — Baer, Barger, Huang
(only Higgsinos below 1 TeV)

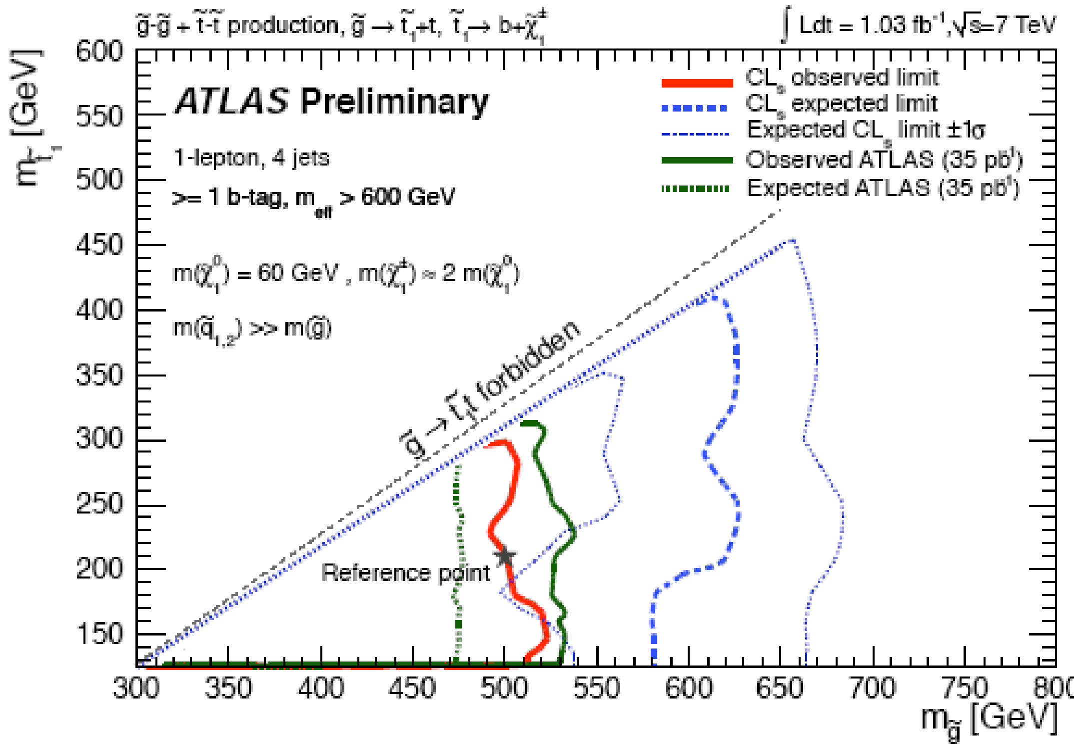
These give "natural" models of the Higgs potential and are barely constrained by the current LHC SUSY limits.

Maxim Perelstein reviewed this story on Monday.

$\tilde{g} \to \tilde{b} + b$



$\tilde{g} \to \tilde{t} + t$



Only one of the standard LC benchmark SUSY points falls into this class of models. This is the point LCC2 used in ILC-dark matter studies:

$$m(\tilde{\chi}^+) = 159 \qquad m(\tilde{g}) = 850 \qquad m(\tilde{t}) = 1976$$

This model has a correct dark matter relic density.

It would be interesting to explore this subset of SUSY parameter space more fully. A useful parameter set is

$$m_{1/2}, m^2_{3L}, \mu, \tan\beta, m_A$$

with all SUSY particles whose masses involve other SUSY-breaking parameters so large that their effects in LC physics can be ignored.

See also the benchmark proposals in AbdusSalam et al, arXiv:1109.3859. But, IMHO, simplified parameter sets are better.

3. What is the role of precision top quark measurements in uncovering the physics of the TeV scale ?

In this section, I assume that the TeV scale solution is NOT SUSY.

From "The Case for a 500 GeV Linear Collider" (2000):

"In the past few years, there has been a theoretical preference for supersymmetry and other weakly-coupled models of electroweak symmetry breaking. If supersymmetric particles are not discovered at the LHC, this situation will change dramatically. In that case, anomalous W and t coupling measurements at an e+e− collider will be among the most central issues in high-energy physics."

In many scenarios of Higgs compositeness, the negative mass term in the Higgs potential is driven by interactions with the top quark. We can explore for this physics by precision measurements on top quark pair production.

This process is governed by 8 form factors, which may be functions of s:

$$\frac{e^2}{s} \cdot \sum_{I,J=L,R} \qquad \bar{t}_I \gamma^\mu t_I \cdot \bar{e}_J \gamma_\mu e_J \cdot \mathcal{F}_{1IJ}(s)$$

$$+ \quad \bar{t}_I \frac{\sigma^{\mu\nu} q_\nu}{2m_t} t_{\overline{I}} \cdot \bar{e}_J \gamma_\mu e_J \cdot \mathcal{F}_{2IJ}(s)$$

The combinations $\mathcal{F}_{2LJ} - \mathcal{F}_{2RJ}$ violate CP and can be used to probe for new CP violation involving top and Higgs.

Already in the Standard Model, the $\mathcal{F}_{1IJ}$ form factors are nontrivially dependent on t and beam polarization due to photon-Z interference:

$$
\begin{aligned}
\mathcal{F}_{1LL}|_{SM} &= \frac{2}{3} + \frac{(\frac{1}{2} - s_w^2)(\frac{1}{2} - 2s_w^2/3)}{c_w^2 s_w^2}\frac{s}{s - m_Z^2} \\
\mathcal{F}_{1LR}|_{SM} &= \frac{2}{3} - \frac{(s_w^2)(\frac{1}{2} - 2s_w^2/3)}{c_w^2 s_w^2}\frac{s}{s - m_Z^2} \\
\mathcal{F}_{1RL}|_{SM} &= \frac{2}{3} - \frac{(\frac{1}{2} - s_w^2)(2s_w^2/3)}{c_w^2 s_w^2}\frac{s}{s - m_Z^2} \\
\mathcal{F}_{1RR}|_{SM} &= \frac{2}{3} + \frac{(s_w^2)(2s_w^2/3)}{c_w^2 s_w^2}\frac{s}{s - m_Z^2}
\end{aligned}
$$

with strong constructive interference for LL, RR and strong destructive interference for LR, RL.

Each form factor has a different characteristic angular dependence. Ignoring CP violation for the moment, we can completely determine the 6 form factors at a given s by fitting the angular distribution for each beam polarization to

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to t\bar{t})$$
$$= A(1+\cos\theta)^2 + B(1-\cos\theta)^2 + C\sin^2\theta$$

Each form factor can realistically be measured to percent accuracy in the LC program.

Some of the physics contained in these form factors is:

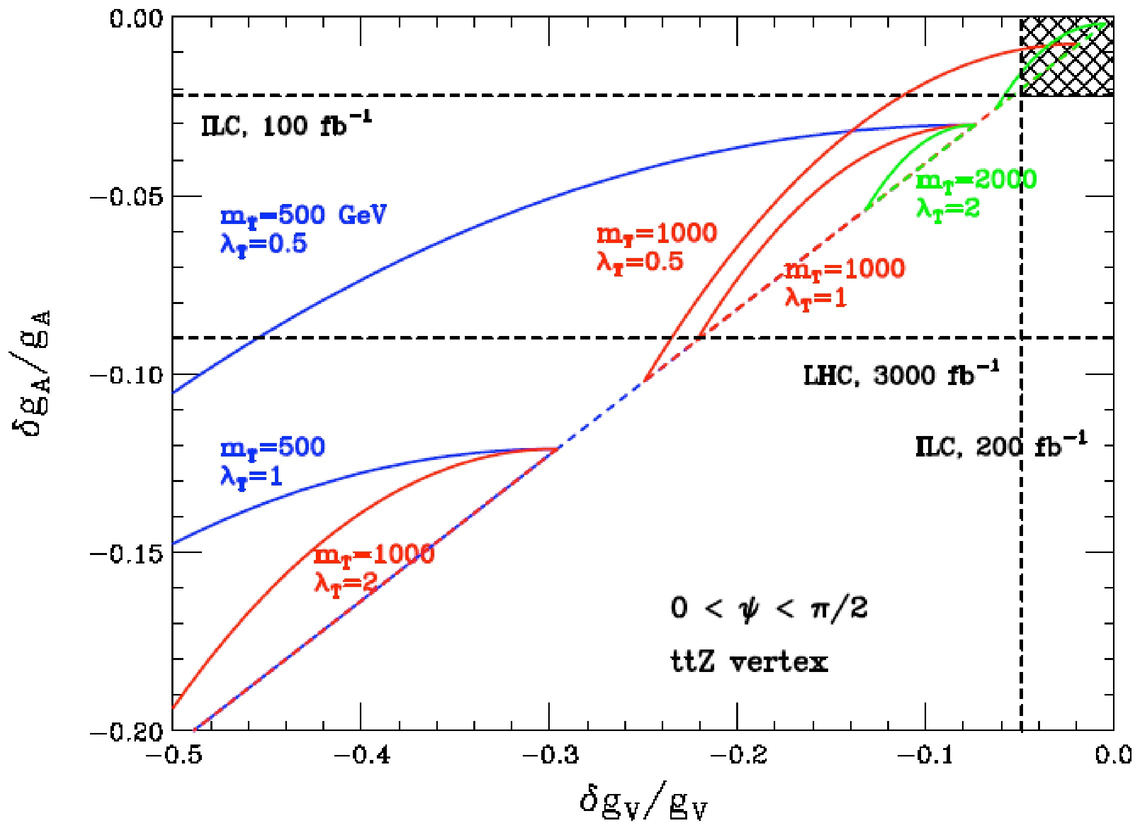
The L,R-handed Z boson couplings to top are not fixed a priori outside of the Standard Model. These couplings can get large corrections in models of Higgs and top compositeness.

In Little Higgs and Randall-Sundrum models, one expects that, specifically, the coupling

$$g_R \, \bar{t}_R \gamma^\mu t_R \cdot Z_\mu$$

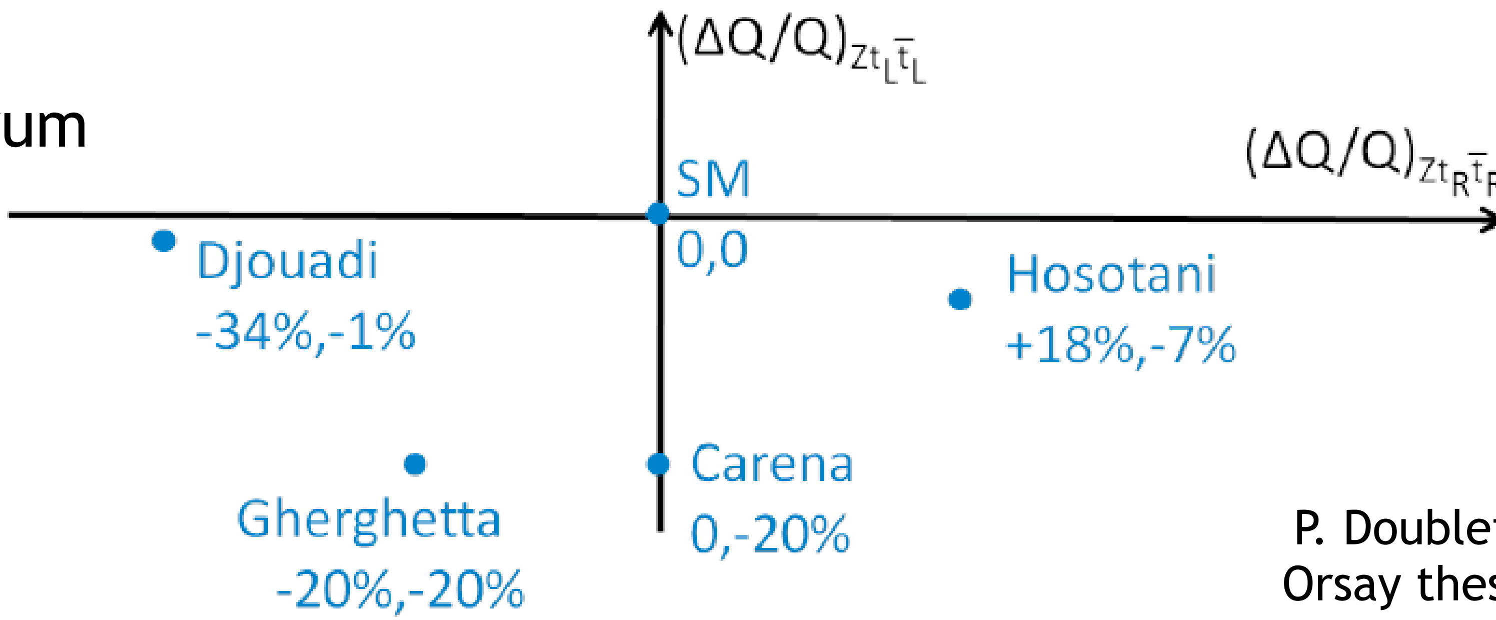
will have a large deviation from the Standard Model value. This effect is not subtle at e+e- colliders, but it is extremely difficult to access at the LHC.

Little Higgs models



Berger, Petriello, Perelstein

Randall-Sundrum models



P. Doublet, Orsay thesis

The L,R handed couplings of the photon to top are fixed at s = 0.

However,

In models of top compositeness, we expect form factor enhancements due to $t\bar{t}$ resonances.

$$\frac{m_R^2}{m_R^2 - s}$$

For a resonance at 3.5 TeV, this is a 2% enhancement of differential cross sections. The relevant $t\bar{t}$ resonances are color-singlet; LHC is mainly sensitive to color-octet.

Non-resonant contributions, from very heavy boson exchanges or contact interactions, also show up as s-dependence of

$$\mathcal{F}_1(s)$$

On Monday, Juan Aguilar-Saavedra showed a range of models with contact interactions and similar effects that might explain the large $t\bar{t}$ forward-backward asymmetry observed at the Tevatron.

In this talk, I have pointed to some aspects of Standard Model physics that need to be understood better to do precision measurements at Linear Colliders, especially at high energies. Much work is needed, but the theory community will get there. The LHC experience will be very helpful.

I have also pointed to two directions in Beyond the Standard Model physics that are very much alive at the current state of the LHC data. We could still have charginos in the range of the ILC at 500 GeV and stops not far above. At ILC, we will certainly have a precision top program, with measurements that might become increasingly central to the problem of EWSB.

There are many more options for new physics at the 1 TeV scale. We theorists also have to keep searching, guided by constraints and, eventually, discoveries from LHC.