Electroweak corrections for top@ILC 2004 - 2014 - 2024

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Thanks to Wolfgang Hollik for re-establishing my interest

Workshop on Top physics at the LC
Session on electroweak couplings and higher order corrections
LPNHE, Paris, France, 5-6 March, 2014,
http://agenda.linearcollider.org/conferenceDisplay.py?ovw=True&confId=6296

Top quarks before their discovery (1995)

I learned the calculation of complete EWRC (electroweak radiative corrections) from two papers:
Passarino, Veltman [1] and 't Hooft, Veltman [2]. This was about 1980.

Top quarks: They were predicted by Kobayashi and Maskawa in 1973 (Nobel prize in 2008).
For some time we expected the top to be not so heavy, e.g. $m_{top} \approx 30$ GeV at the time of TRISTAN planning (1986).

The top virtual effects were studied, also by our group (since 1982):
ZFITTER, `http://zfitter.com`: $\Delta r, \Gamma_Z, \Gamma_W, 2f$-production. See for a summary Akhundov et al., 2013 [3]

Among the very first papers on EWRC to top pair production is that by Fujimoto and Shimizu in 1988 [4], with the title "Radiative corrections to $e^+e^- \to \bar{t}t$ in electroweak theory".
One also might mention the PhD 1989 thesis of Wim Beenakker with a lot of detailed formulae.

Top quarks are there since 1995

The top quark was discovered in 1995 by CDF and D0 at Fermilab with a mass $m_{top} = 173.1$ GeV.
We ignore here the subtleties of on-shell mass and running mass definitions.

Due to the short lifetime, one has to investigate in practice not top pair production, but something like
$e^+e^- \to 6f$
plus radiative corrections plus potentially exotics.

One of the early studies was initiated by Karol Kolodziej with his fast MC generator KARLOMAT (factorizing top production, Kolodziej, Staron, Lorca, TR 2005 [5]).
He used our EWRC from the topfit package [6]:
J. Fleischer, A. Leike, T. Riemann, A. Werthenbach
Electroweak one loop corrections for e^+e^- annihilation into $t\bar{t}$ including hard bremsstrahlung

topfit in 2002-2004: EWRC for $e^+e^- \to t\bar{t}(\gamma)$

The top quark is distinguished by its heavy mass, and we do not know so much about it until now.

In fact, for the proposed Linear Collider (LC) the top quark will be one of the most interesting objects to be studied.
Because we do know a bit about the properties already, top physics has to be precision physics.

My group became involved in dedicated EWRC predictions for top pair production when we all hoped for TESLA.
The TESLA (500 GeV) TDR dates in 1992,
`http://flash.desy.de/tesla/tesla_documentation/`.
References: [7, 8, 6, 9, 10] and Section 7.2 in [11].

Other earlier references are by our Japanese collegues from KEK et al. [12] and our Dutch/German collegues around Wolfgang Hollik [13, 14] and our Russian collegues at JINR Dubna [15, 16].
We arranged for dedicated numerical comparisons. With our study, detailed comparisons of the diverse results were undertaken for the first time [8, 7].

topfit in 2014

Technicalities:
Our Fortran program is topfit.

I successfully reactivated in 2013 the code and can reproduce the old numbers (and the figures and latex files).

We used the package `DIANA` [17, 18] for automatic generation of the diagrams and `FORM` [19] for the further symbolic calculations, and for the numerics the Fortran packages `FF` [20] and `LoopTools` [21].
This I did not activate now.

But:
Is the old stuff useful now for anything and/or anybody?
I do not know a final answer.
It depends.

Today

What are the challenges of today?

What is ”today“?

Today is the time from now until start of accelerator.

Theory demands sometimes long investments.

Frits Berends in 1989:
Give me one million Swiss franks and 10 Russians, and I will calculate the two-loop Bhabha scattering.
He did not.

We know on that topic: The time was estimated more or less OK, but the resources were quite different from what he assumed.

Today, 2.

And this is, in my eyes, the justification of meetings on top physics for an LC@2024 in 2014. Besides the need of holding the physics motivations at the desk.

Building blocks for calculating EWRC for top production:

- Born cross section - with options for exotic contributions
- QED corrections - with some experimental cuts, to some high order in perturbation theory plus some exponentiations - with MC finally
- EWRC - is one-loop sufficient? - if not, we might run into trouble.
- QCD corrections due to the final state
- Mixed EWRC and QCD RC - if higher orders play a role

What might be recyclable:

- There are six degrees of freedom for the "naked" $e^+e^- \to \bar{t}t$ transition, because the electrons are practically massless, and the top quarks are heavy. So, one has 6 form factors to be calculated. Once knownm, they may be integrated into any MC frame. This is what topfit cotnributed to KARLOMAT
- Just numerical comparisons in order to ensure the accuracy of the numerics for the precision level needed. Delivering so-called etalons. Whether they will be needed then, in 2024+X, from us guys in 2014 - no idea.
-

One of the most recent studies is by our Japanese collegues, Khiem et al. [22] (2012).

Hard bremsstrahlung kinematics [Fleischer, TR et al. EPJC 31 (2003) [6]]

The phase space of the process under consideration may be characterized by the following four independent kinematical variables:

- $s' := (p_1 + p_2)^2$ as invariant mass squared of the top pair,
- $\cos\theta$ as cosine of the scattering angle of $\bar{t}$ with respect to the e^- beam axis in the c.m.s. .
- $\cos\theta_\gamma$ as cosine of the polar angle between the three-momenta of $\bar{t}$ and the photon in the c.m.s., which is related to V_2 via an algebraic relation.
- φ_γ as azimuthal angle of the photon in the rest frame of (t, γ) (z-axis defined by $\vec{p}_2$ of $\bar{t}$ in the c.m.s.).

This transforms into the variables

- $r = s'/s$, with $s' := (p_1 + p_2)^2$ as invariant mass squared of the top pair,
- $\cos\theta$ as cosine of the scattering angle of $\bar{t}$ with respect to the e^- beam axis in the c.m.s. .
- $x = 2p_\gamma p_{\bar{t}}/s$, related to $\cos\theta_\gamma$ as cosine of the polar angle between the three-momenta of $\bar{t}$ and the photon in the c.m.s.
- analytical integration over: φ_γ as azimuthal angle of the photon in the rest frame of (t, γ)
  (z-axis defined by $\vec{p}_2$ of $\bar{t}$ in the c.m.s.).

The acollinearity angle ξ is related to x, r, so that one may impose a cut.

topfit performs a three-fold numerical integration over the final-state phase space.
With simple cuts.

There is an option with so-called flux functions, and much faster due to less dimensional integrations.
This allows at most for an s'-cut, but it is an approximation (no correct treatment of final fermion masses) and is finally not recommended.

Kinematics [7]

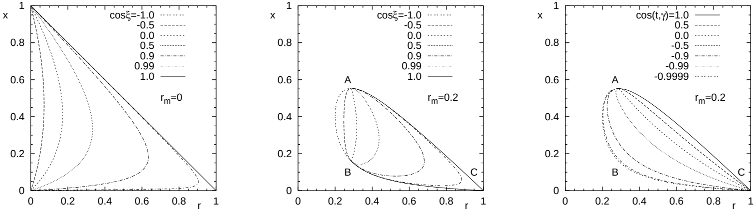

Figure : No cut, acollinearity cut, other cut.
Middle graph - The kinematic region of r and x for different values of the acollinearity angle ξ for (a) $r_m = 4m_t^2/s = 0$ and (b) $r_m = 0.2$.

topfit and Grace, 2002 [Fleischer, Fujimoto, Ishikawa, TR et al. [7]]

$\cos\theta$	$\omega/\sqrt{s}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{tot}}$
−0.9	T : 0.1	0.108839194075	+0.098664253	+0.11408410	0.13144
	T : 0.00001	0.108839194075	−0.017474702	−0.002054858	0.13229
	G : 0.00001	0.108839194076		−0.002054859	0.13206(12)
−0.5	T : 0.1	0.142275069392	+0.12850790	+0.14308121	0.15973
	T : 0.00001	0.142275069392	−0.029702340	−0.015129038	0.16029
	G : 0.00001	0.142275069393		−0.015129039	0.16013(13)
+0.0	T : 0.1	0.225470464033	+0.20239167	+0.21718801	0.23638
	T : 0.00001	0.225470464033	−0.058010508	−0.043214169	0.23476
	G : 0.00001	0.225470464033		−0.043214168	0.23513(14)
+0.5	T : 0.1	0.354666470332	+0.31511723	+0.32933727	0.35651
	T : 0.00001	0.354666470332	−0.109721291	−0.095501257	0.35062
	G : 0.00001	0.354666470332		−0.095501252	0.35104(17)
+0.9	T : 0.1	0.491143715767	+0.43071437	+0.44290816	0.48796
	T : 0.00001	0.491143715767	−0.179672655	−0.16747886	0.47768
	G : 0.00001	0.491143715767		−0.16747886	0.47709(21)

Various differential cross sections. The upper and lower numbers correspond to the `topfit` (T) and GRACE (G) approach, respectively, $\sqrt{s}$ = 500 GeV.

topfit and Grace, 2002 [Fleischer, Fujimoto, Ishikawa, TR et al. [7]]

$\sqrt{s}$	$\sigma^0_{\rm tot}$	$A^0_{\rm FB}$	$\sigma_{\rm SM,tot}$	$\sigma_{\rm SM,FB}$	$\sigma_{\rm tot}$	$A_{\rm FB}$
500	T : 0.5122744	0.4146039	−0.1198972	−0.0855551	0.526337	0.362929
	G : 0.5122751	0.4146042	−0.1198973		0.526371	0.363140
1000	T : 0.1559185	0.5641706	−0.0683693	−0.0522582	0.171916	0.488869
	G : 0.1559187	0.5641710	−0.0683695		0.171931	0.488872

Total cross sections (in pbarn) and forward-backward asymmetries.

$\sigma^0_{\rm tot}$ – Born
$\sigma_{\rm SM,tot}$ – elastic (with soft photons)
$\sigma_{\rm tot}$ – also hard photons, $\omega/\sqrt{s} = 0.00001$.

Table $\sqrt{s} = 500$ GeV [Hahn, Hollik, TR et al., [10]]

$e^+e^- \to t\bar{t}$ $\sqrt{s} = 500$ GeV				
$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$ /pb	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$ /pb	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+w+QED+soft}}$ /pb	Program
−0.9	0.10883 91940 76039	0.12425 90371 32943	0.11408 40955 77861	TOPFIT
−0.9	0.10883 91940 76039	0.12425 90371 33664	0.11408 40955 78964	FA/FC
−0.5	0.14227 50693 93371	0.15684 83718 76069	0.14308 12051 65511	TOPFIT
−0.5	0.14227 50693 93371	0.15684 83718 76250	0.14308 12051 65581	FA/FC
0.0	0.22547 04640 33559	0.24026 68040 30724	0.21718 80097 67412	TOPFIT
0.0	0.22547 04640 33559	0.24026 68040 30032	0.21718 80097 66323	FA/FC
0.5	0.35466 64703 33217	0.36888 65069 94389	0.32933 72739 51692	TOPFIT
0.5	0.35466 64703 33217	0.36888 65069 92599	0.32933 72739 49095	FA/FC
0.9	0.49114 37157 67761	0.50333 75116 05520	0.44290 81673 51494	TOPFIT
0.9	0.49114 37157 67761	0.50333 75116 02681	0.44290 81673 46094	FA/FC

Table : Differential cross-sections for selected scattering angles for τ-production at $\sqrt{s} = 500$ GeV. The three columns contain the Born cross-section, Born including only the weak $\mathcal{O}(\alpha)$ corrections, and Born including the weak and photonic $\mathcal{O}(\alpha)$ corrections. For each angle, the first row represents the TOPFIT result of the Zeuthen group while the second stands for the *FeynArts*/*FormCalc* calculation of the Munich group.

Table $\sqrt{s} = 1000$ GeV [Hahn, Hollik, TR et al., [10]]

$e^+e^- \to t\bar{t} \qquad \sqrt{s} = 1$ TeV			
$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$ /pb	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$ /pb	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+w+QED+soft}}$ /p
-0.9	$0.22785\,42327\,32090 \cdot 10^{-1}$	$0.25521\,28532\,98051 \cdot 10^{-1}$	0.23101 70508 05040
-0.9	$0.22785\,42327\,32090 \cdot 10^{-1}$	$0.25521\,28533\,00748 \cdot 10^{-1}$	0.23101 70508 07714
-0.5	$0.29782\,13110\,31861 \cdot 10^{-1}$	$0.31863\,48943\,59857 \cdot 10^{-1}$	0.28823 01902 00931
-0.5	$0.29782\,13110\,31861 \cdot 10^{-1}$	$0.31863\,48943\,60711 \cdot 10^{-1}$	0.28823 01902 01653
0.0	$0.61180\,06742\,25039 \cdot 10^{-1}$	$0.61591\,61295\,77963 \cdot 10^{-1}$	0.54950 88904 88739
0.0	$0.61180\,06742\,25038 \cdot 10^{-1}$	$0.61591\,61295\,75474 \cdot 10^{-1}$	0.54950 88904 85894
0.5	0.11774 69498 88318	0.11404 76860 51226	0.99417 00898 39905
0.5	0.11774 69498 88318	0.11404 76860 50527	0.99417 00898 32292
0.9	0.18112 20970 86446	0.17134 61927 22790	0.14426 23325 41248
0.9	0.18112 20970 86446	0.17134 61927 21645	0.14426 23325 40061

Table : The same as Tab. 1, but at $\sqrt{s} = 1$ TeV.

Top pair production [Hahn, Hollik, TR et al., [10]]

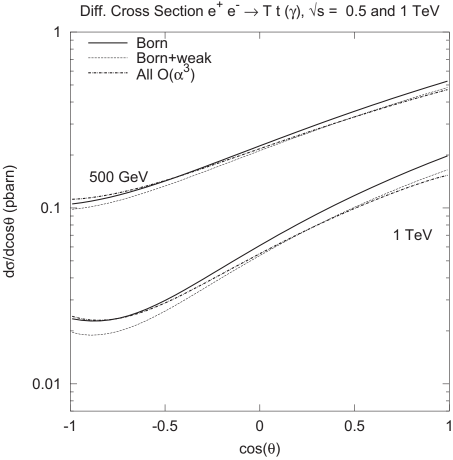

Figure : a figure

Figure CLIC 2004 [TR, Wertenbach, in [11]]

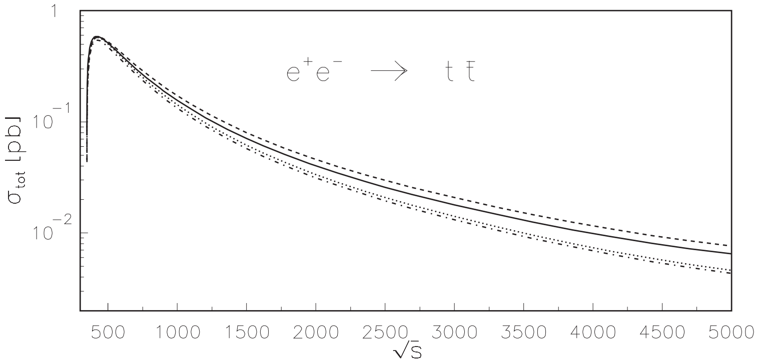

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Figure CLIC 2004 [TR, Wertenbach, in [11]]

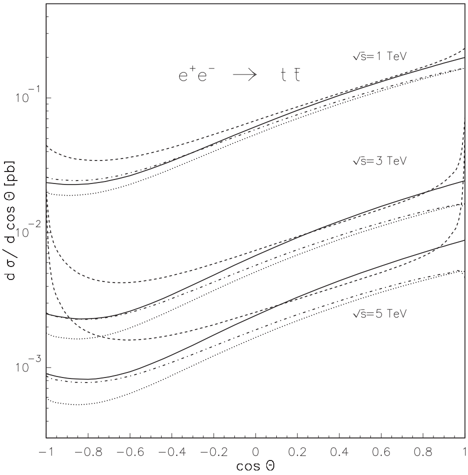

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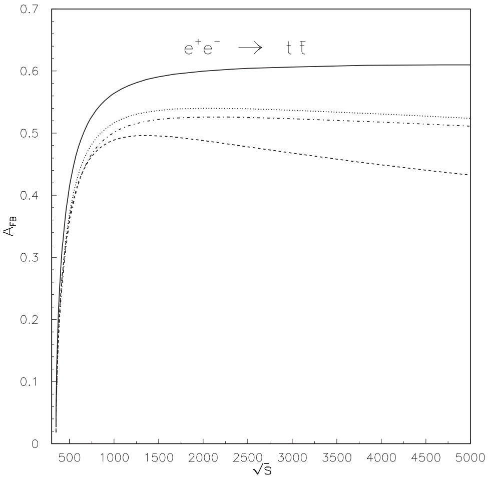
Figure CLIC 2004 [TR, Wertenbach, in [11]]



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The six electroweak form factors

Topfit is as flexible as ZFITTER is, concerning the inclusion of different corrections.

Difference of approach in topfit compared to ZFITTER:
- we made topfit on purpose exactly of order one-loop.
- final state mass is taken into account properly.

In lowest-order perturbation theory, the process $e^+e^- \to t\bar{t}$ is described by four matrix elements

$$\mathcal{M}_1^{ij} = \left[\bar{v}(p_4)\,\gamma^\mu\,\mathbf{G}^i\,u(p_1)\right] \times \left[\bar{u}(-p_2)\,\gamma_\mu\,\mathbf{G}^j\,v(-p_3)\right], \quad i,j = 1,5, \tag{1}$$

with $\mathbf{G}^1 = 1$ and $\mathbf{G}^5 = \gamma_5$, the Born amplitude can be written in a compact form:

$$\mathcal{M}_B = \mathcal{M}_\gamma + \mathcal{M}_Z = \sum\nolimits_{i,j=1,5} F_1^{ij,B}\,\mathcal{M}_1^{ij}, \tag{2}$$

Six f.f.

with

$$F_1^{11,B} = v_e\, v_t \frac{e^2}{s - M_Z^2 + iM_Z\Gamma_Z} + Q_e\, Q_t \frac{e^2}{s} \equiv F_1^{11,B,Z} + F_1^{11,B,\gamma}, \quad (3)$$

$$F_1^{15,B} = -v_e\, a_t \frac{e^2}{s - M_Z^2 + iM_Z\Gamma_Z}, \quad (4)$$

$$F_1^{51,B} = -v_t\, a_e \frac{e^2}{s - M_Z^2 + iM_Z\Gamma_Z}, \quad (5)$$

$$F_1^{55,B} = a_e\, a_t \frac{e^2}{s - M_Z^2 + iM_Z\Gamma_Z}. \quad (6)$$

Six f.f. at one loop

At one-loop level three further basic matrix-element structures are found (in the limit of vanishing electron mass):

$$\mathcal{M}_{\rm 1loop} = \sum_{a=1}^{4} \sum\nolimits_{i,j=1,5} F_a^{ij,\rm 1loop}\, \mathcal{M}_a^{ij}, \tag{7}$$

with

$$\begin{aligned} \mathcal{M}_1^{ij} &= \gamma^\mu \, \mathbf{G}^i \otimes \gamma_\mu \, \mathbf{G}^j, \qquad & \mathcal{M}_2^{ij} = \not{p}_2 \, \mathbf{G}^i \otimes \not{p}_4 \, \mathbf{G}^j, \\ \mathcal{M}_3^{ij} &= \not{p}_2 \, \mathbf{G}^i \otimes \mathbf{G}^j, \qquad & \mathcal{M}_4^{ij} = \gamma^\mu \, \mathbf{G}^i \otimes \gamma_\mu \not{p}_4 \, \mathbf{G}^j, \end{aligned} \tag{8}$$

and correspondingly there are sixteen scalar form factors F_a^{ij} in total. The interferences of these matrix elements with the Born amplitude have to be calculated.

Six f.f.

Only six of the interferences are independent, i.e., $\mathcal{M}_1^{ij}$, ${\mathcal{M}_3}^{11}$ and ${\mathcal{M}_3}^{51}$.

In order to express the results compactly for possible later implementation into a full Monte Carlo program, the virtual corrections are expressed in terms of the six independent, modified, dimensionless form factors $\widehat{F}_1^{ij}, \widehat{F}_3^{11}, \widehat{F}_3^{51}$:

Six f.f., cont'd

$$\widehat{F}_1^{11} = \left[F_1^{11} + \frac{1}{4}(u-t)\,F_2^{11} - \frac{1}{4}(u+t+2m_t^2)\,F_2^{55} + m_t\,(F_4^{55} - F_4^{11})\right], \quad (9)$$

$$\widehat{F}_1^{15} = \left[F_1^{15} - \frac{1}{4}(u+t-2m_t^2)\,F_2^{51} + \frac{1}{4}(u-t)\,F_2^{15}\right], \quad (10)$$

$$\widehat{F}_1^{51} = \left[F_1^{51} + \frac{1}{4}(u-t)\,F_2^{51} - \frac{1}{4}(u+t+2m_t^2)\,F_2^{15} + m_t\,(F_4^{15} - F_4^{51})\right], \quad (11)$$

$$\widehat{F}_1^{55} = \left[F_1^{55} - \frac{1}{4}(u+t-2m_t^2)\,F_2^{11} + \frac{1}{4}(u-t)\,F_2^{55}\right], \quad (12)$$

$$\widehat{F}_3^{11} = \left[F_3^{11} - F_4^{11} + F_4^{55} - m_t\,F_2^{55}\right], \quad (13)$$

$$\widehat{F}_3^{51} = \left[F_3^{51} + F_4^{15} - F_4^{51} - m_t\,F_2^{15}\right]. \quad (14)$$

Six f.f. I

The resulting cross-section formula is:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} c_t \, \beta \, 2\Re e\Big[(u^2+t^2+2m_t^2 s)\left(\bar{F}_1^{11}\bar{F}_1^{11,B*}+\bar{F}_1^{51}\bar{F}_1^{51,B*}\right)$$
$$+(u^2+t^2-2m_t^2 s)\left(\bar{F}_1^{15}\bar{F}_1^{15,B*}+\bar{F}_1^{55}\bar{F}_1^{55,B*}\right)$$
$$+(u^2-t^2)\left(\bar{F}_1^{55}\bar{F}_1^{11,B*}+\bar{F}_1^{15}\bar{F}_1^{15,B*}+\bar{F}_1^{51}\bar{F}_1^{51,B*}+\bar{F}_1^{11}\bar{F}_1^{55,B*}\right)$$
$$+2m_t(tu-m_t^4)\left(\bar{F}_3^{11}\bar{F}_1^{11,B*}+\bar{F}_3^{51}\bar{F}_1^{51,B*}\right)\Big], \quad (15)$$

Six f.f.

where the dimensionless form factors are

$$\bar{F}_1^{ij,B*} \;=\; \frac{s}{e^2}\, F_1^{ij,B*}, \qquad \bar{F}_a^{ij} = \frac{s}{e^2}\left[\frac{1}{2}\delta_{a,1} F_1^{ij,B} + \frac{1}{16\pi^2}\,\widehat{F}_a^{ij,\mathrm{1loop}}\right], \tag{16}$$

and $c_t \;=\; 3$, $\alpha = e^2/4\pi$. The $\bar{F}_a^{ij}$ are defined so that double counting for the Born contributions $F_1^{ij,B}$ is avoided. The factor $1/(16\pi^2)$ is conventional.

The six form factors have been used to include into the Monte Carlo package of Karol Kolodziej the weak corrections properly and efficient.

See [5]: K. Kolodziej, TR et al., ”Factorizable electroweak $o(\alpha)$ corrections for top quark pair production and decay at a linear e^+e^- collider“, EPJ C46 (2006).

Sometimes unexpected applications happen

Two weeks ago I got an email from a Belle researcher (TF):

ich arbeite an der Messung von Fermionpaar-Asymetrien (zunaechst nur Muonpaare) bei Belle ($\sqrt{(}s) = 10.58 GeV$). Ich hoffe Du hast Zeit und Lust mir bei einigen Fragen/Problemen zu helfen. Ich moechte gerne ZFITTER verwenden ...

After a while it became clear to me that this is an ideal application not of ZFITTER, but of topfit.

Summary

What will be the final accuracy from the experimental side?

Which observables are most interesting?

Do we need for the EWRC any, or even complete 2-loop contributions?

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