Prospects for Precision Momentum Scale Calibration

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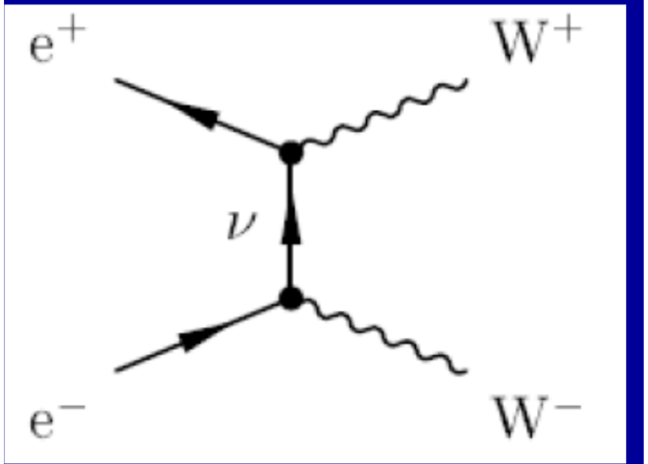
May 13th 2014

Motivation and Context

- Physics at a linear collider can benefit greatly from a precise knowledge of the center-of-mass energy.
  - Examples: m_t, m_W, m_H, $m_{Z,}$ m(chargino)
- The $\sqrt{s_P}$ method based on di-muon momenta promises much better statistical precision than other methods.
  - See my talk at the Hamburg LC2013 workshop last year
  - Needs a precision knowledge of the tracker momentum scale
- Here, I discuss prospects for a precision understanding of the tracker momentum scale with an emphasis on studies with J/psi's.
- Precision = 10 ppm or better

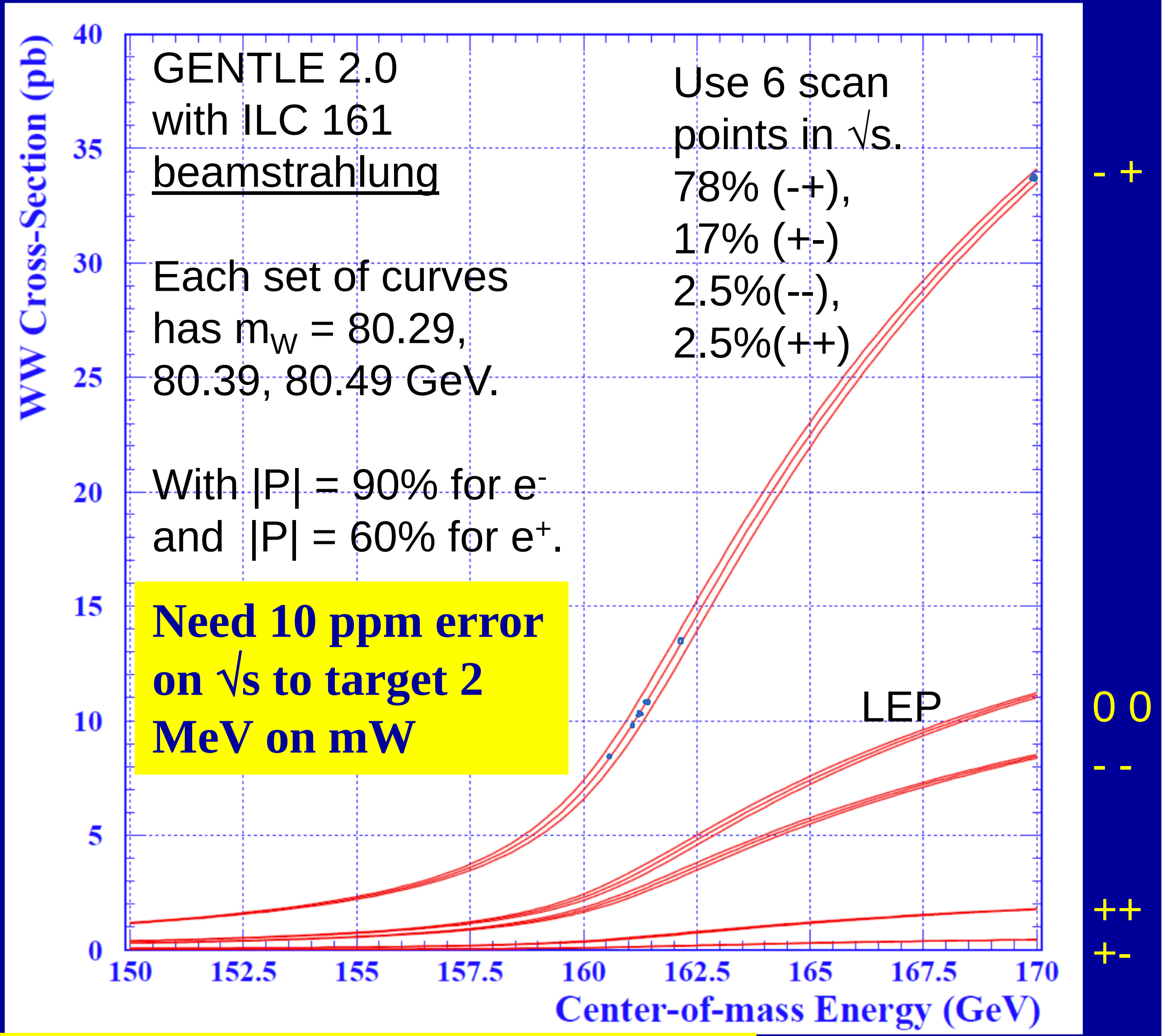
Polarized Threshold Scan



Use (-+) helicity combination of e^- and e^+ to enhance WW.

Use (+-) helicity to suppress WW and measure background.

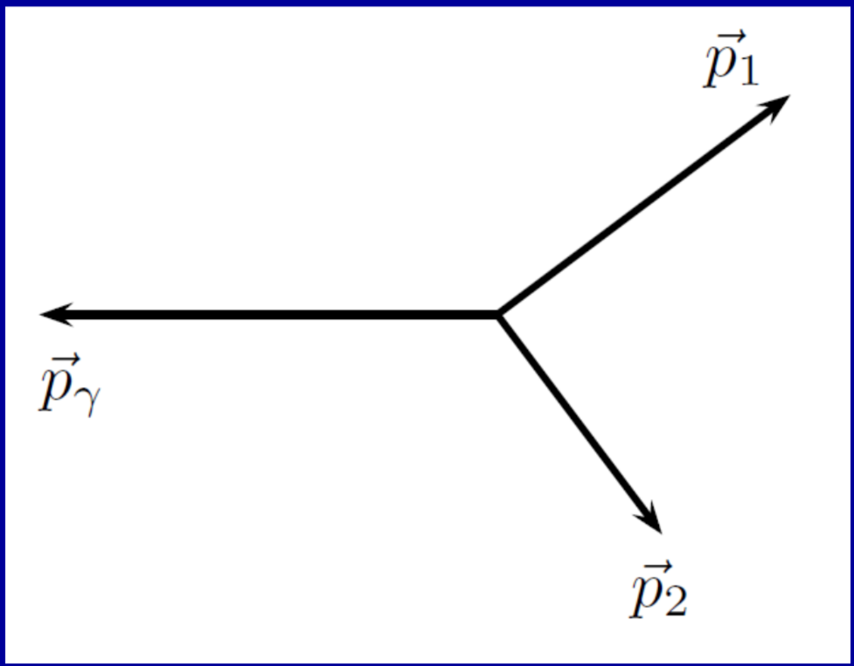
Use (--) and (++) to control polarization (also use 150 pb qq events)



Experimentally very robust. Fit for eff, pol, bkg, lumi

Method P
Use muon momenta. Measure $E_1 + E_2 + |\mathbf{p}_{12}|$.

Proposed and studied initially by T. Barklow



Under the assumption of a massless photonic system balancing the measured di-muon, the momentum (and energy) of this photonic system is given simply by the momentum of the di-muon system.

So the center-of-mass energy can be estimated from the sum of the energies of the two muons and the inferred photonic energy.

$$(\sqrt{s})_P = E_1 + E_2 + |\, \mathbf{p_1} + \mathbf{p}_2 \,|$$

In the specific case, where the photonic system has zero p_T, the expression is particularly straightforward. It is well approximated by

$$\sqrt{s}_P = p_T \left(\frac{1+\cos\theta_1}{\sin\theta_1} + \frac{1+\cos\theta_2}{\sin\theta_2} \right)$$

where p_T is the p_T of each muon. Assuming excellent resolution on angles, the resolution on $(\sqrt{s})_P$ is determined by the θ dependent p_T resolution.

Method can also use non-radiative return events with $m_{12} \gg m_Z$

Summary Table

ECMP errors based on estimates from weighted averages from various error bins up to 2.0%. Assumes (80,30) polarized beams, equal fractions of +- and -+.

Preliminary

(Statistical errors only …)

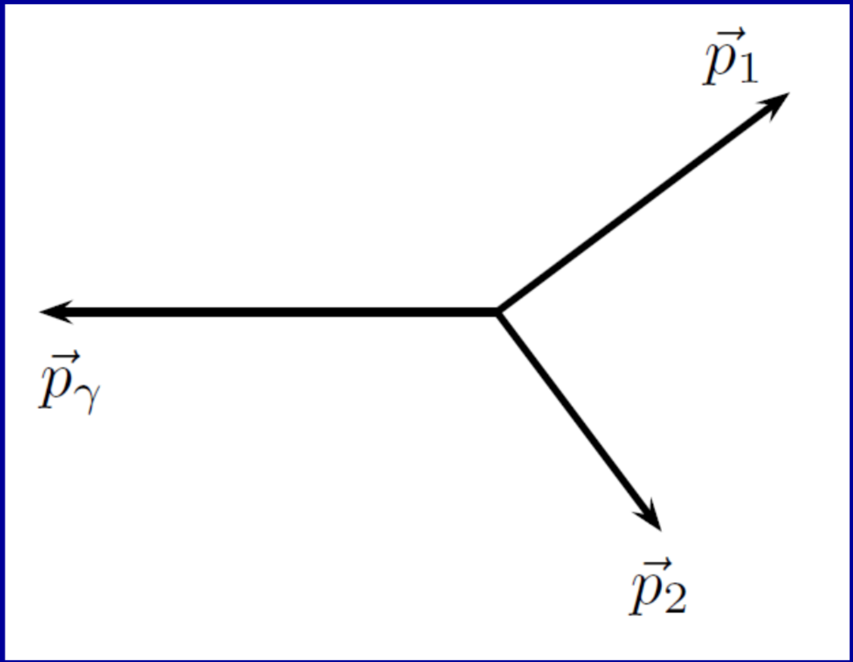
ECM (GeV)	L (fb^{-1})	$\Delta(\sqrt{s})/\sqrt{s}$ Angles (ppm)	$\Delta(\sqrt{s})/\sqrt{s}$ Momenta (ppm)	Ratio
161	161	-	4.3	
250	250	64	4.0	16
350	350	65	5.7	11.3
500	500	70	10.2	6.9
1000	1000	93	26	3.6

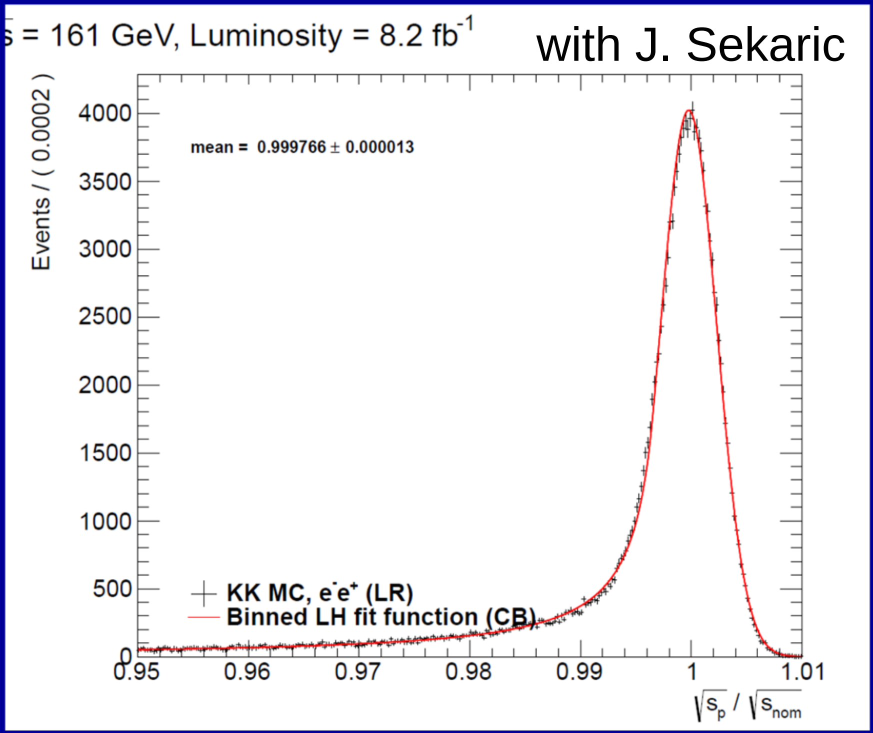
< 10 ppm for 150 – 500 GeV CoM energy

161 GeV estimate using KKMC.

"New" In-Situ Beam Energy Method

$e^+ e^- \rightarrow \mu^+ \mu^- (\gamma)$





Use muon momenta.
Measure $E_1 + E_2 + |\mathbf{p}_{12}|$ as an estimator of $\sqrt{s}$

ILC detector momentum resolution (0.15%), gives beam energy to better than 5 ppm statistical. Momentum scale to 10 ppm => 0.8 MeV beam energy error projected on m_W. (J/psi)

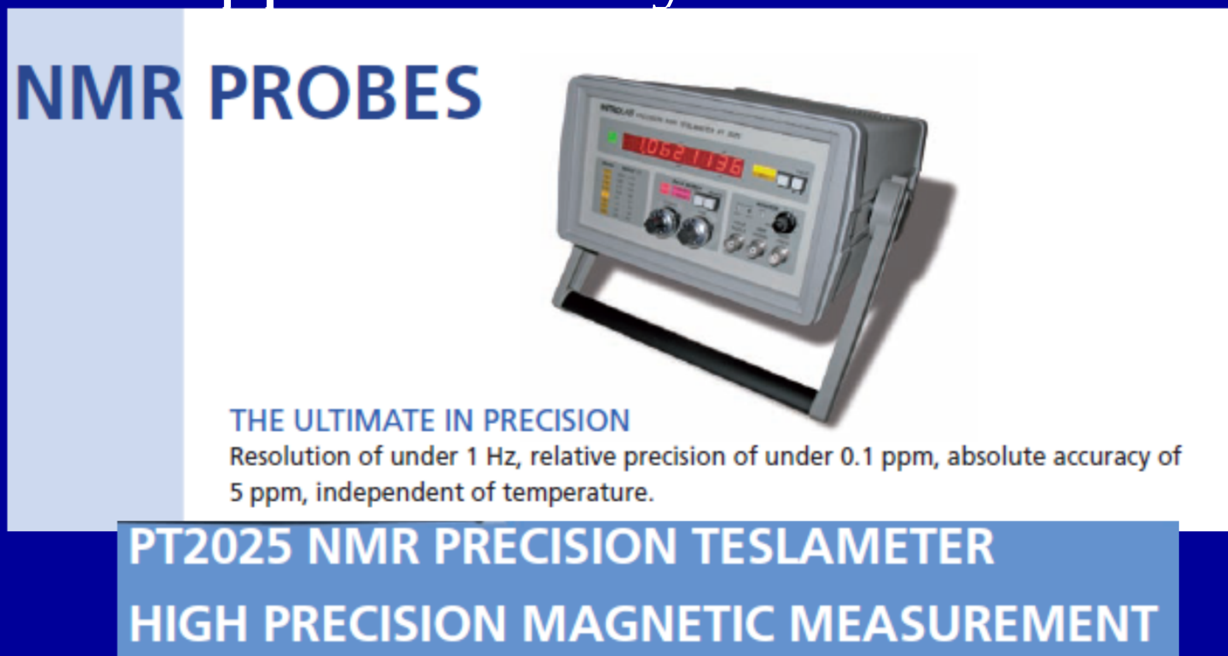
Beam Energy Uncertainty should be controlled for $\sqrt{s} <= 500$ GeV

Momentum measurement basics

- In uniform field – helical trajectory
- $p_T = q\ B\ R$
- p_T (GeV/c) = 0.2997925 B (T) R (m)
  - Errors in momentum scale likely from
    - Knowledge of absolute value of B
    - Alignment errors.
    - Field inhomogeneities.

NMR ?

- Commercial NMR probes can achieve of order ppm accuracy.



- In practice such measurements have never been fully exploited in collider detector environments.

Candidate Decay Modes for Momentum-Scale Calibration

Particle	$n_{Z\,\mathrm{had}}$	Decay	BR (%)	$n_{Z\,\mathrm{had}} \cdot$ BR	Γ/M	PDG $(\Delta M/M)$
J/ψ	0.0052	$\mu^-\mu^+$	5.93	0.00031	3.0×10^{-5}	3.6×10^{-6}
K^0_S	1.02	$\pi^-\pi^+$	69.2	0.71	1.5×10^{-14}	4.8×10^{-5}
Λ	0.39	π^-p	63.9	0.25	2.2×10^{-15}	5.4×10^{-6}
D^0	0.45	$K^-\pi^+$	3.88	0.0175	8.6×10^{-13}	7.0×10^{-5}

Table 1: Candidate standard candles for momentum scale calibration and abundances in Z decay.

Momentum Scale Study

- Studies done with ILD fast-simulation SGV
  - "covariance matrix machine"
  - Using ILD model in SGV
- Plus – various vertex fitters (see later).
- Main J/psi study done with PYTHIA Z decays.
- Now also have some single-particle studies where I am able to specify the decay-point.
  - Current approach and/or SGV does not yet work appropriately for large d0/R. (needed for K0, Λ)

Mass Sensitivity to Momentum-Scale Shift

20 GeV parent momentum.

Dependence of mass on CM decay angle of negative particle.

J/ψ has largest sensitivity (and largest Q-value)

-100 ppm shift in p +100 ppm shift in p

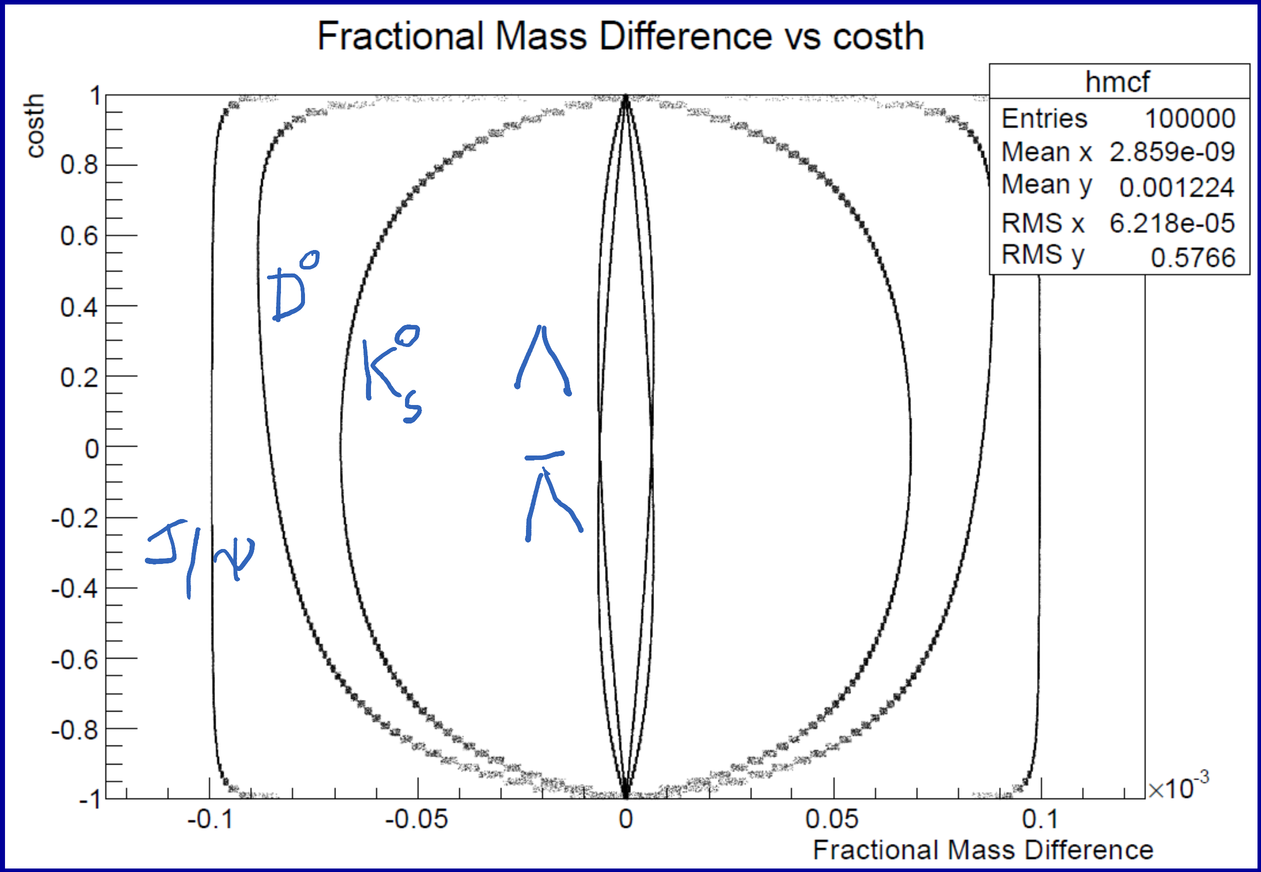


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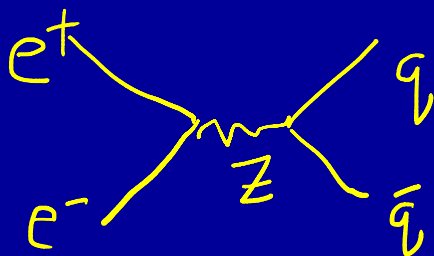
Table 1: Candidate standard candles for momentum scale calibration and abundances in Z decay.

Particle	Decay	Sensitivity	σ_M/M	Stat. Error (10^7 Z)	Stat. Error (10^9 Z)	PDG limit
J/ψ	$\mu^-\mu^+$	0.99	1.2×10^{-3}	22 ppm	2.2 ppm	3.6 ppm
K^0_S	$\pi^-\pi^+$	0.55	2.3×10^{-3}	1.6 ppm	0.16 ppm	87 ppm
Λ	π^-p	0.044	3.8×10^{-4}	5.5 ppm	0.55 ppm	123 ppm
D^0	$K^-\pi^+$	0.77	1.2×10^{-3}	3.7 ppm	0.37 ppm	91 ppm

Table 2: Estimated momentum scale statistical errors assuming 100% acceptance.

J/ψ Based Momentum Scale Calibration



$\sigma_{had} = 30\,nb$ at $\sqrt{s} \simeq m_Z$

$f_{b\bar{b}} \equiv R_b = 22\%$

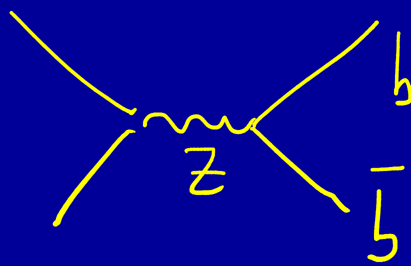
Most $Z \to J/\psi\, X$ believed to be from $B_{hadron} \to J/\psi\, X$

$B(Z_{had} \to J/\psi\, X) \cdot B(J/\psi \to \mu^+\mu^-) \simeq 3.0 \times 10^{-4}$

$\Rightarrow$ Expect 300,000 events with 10^9 hadronic Z's

J/psi's from Z

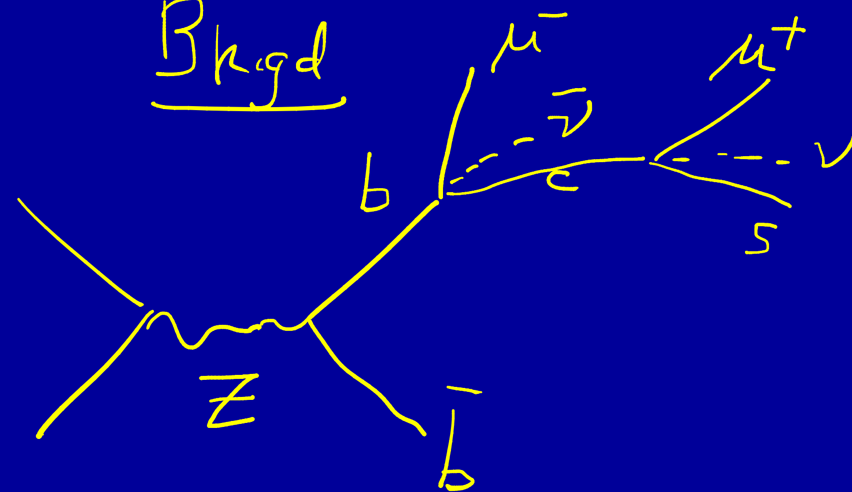
Signal



$b \rightarrow J/\psi\, X$

$\hookrightarrow \mu^+\mu^-$

Bkgd

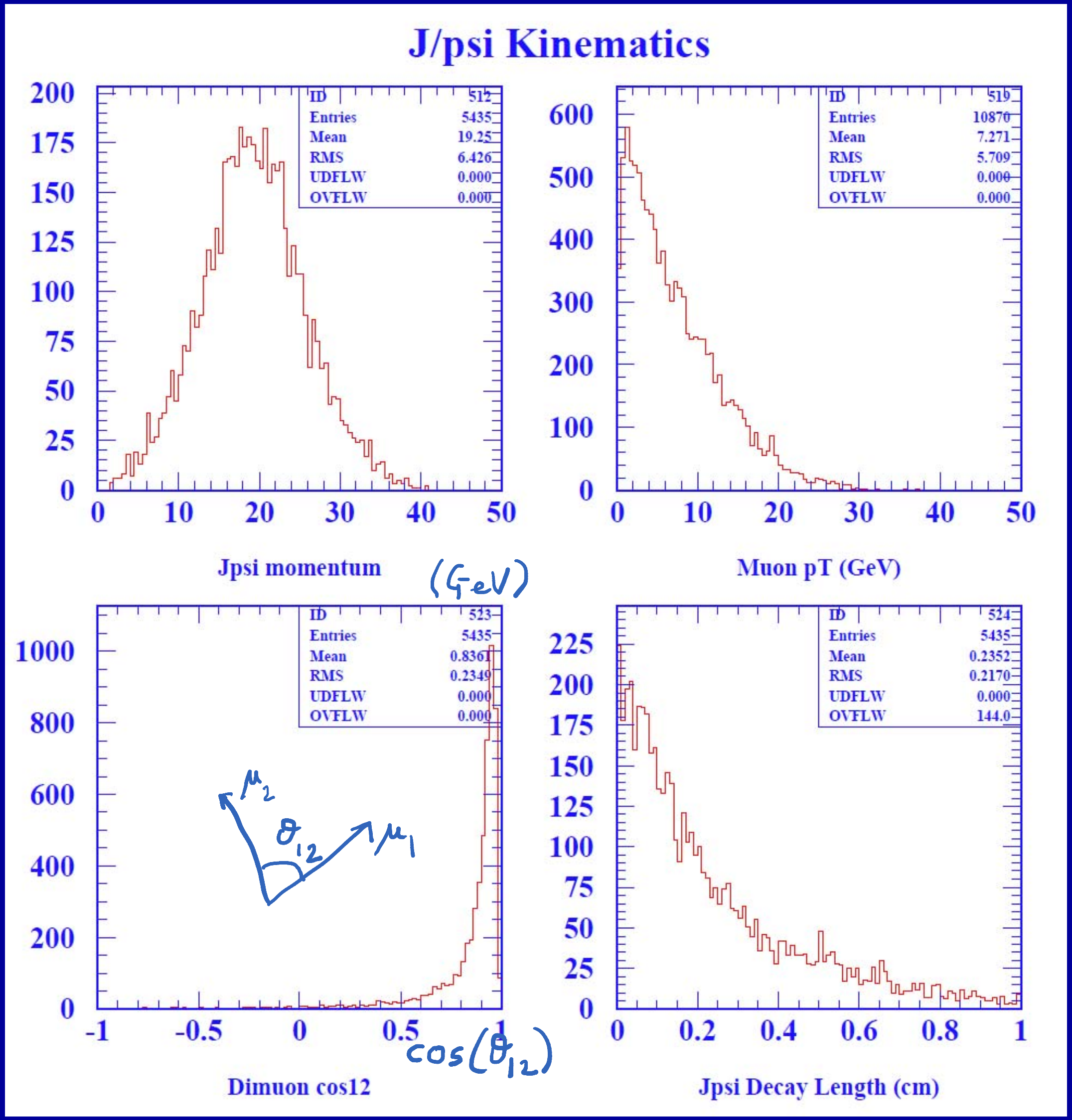


semi-leptonic cascades

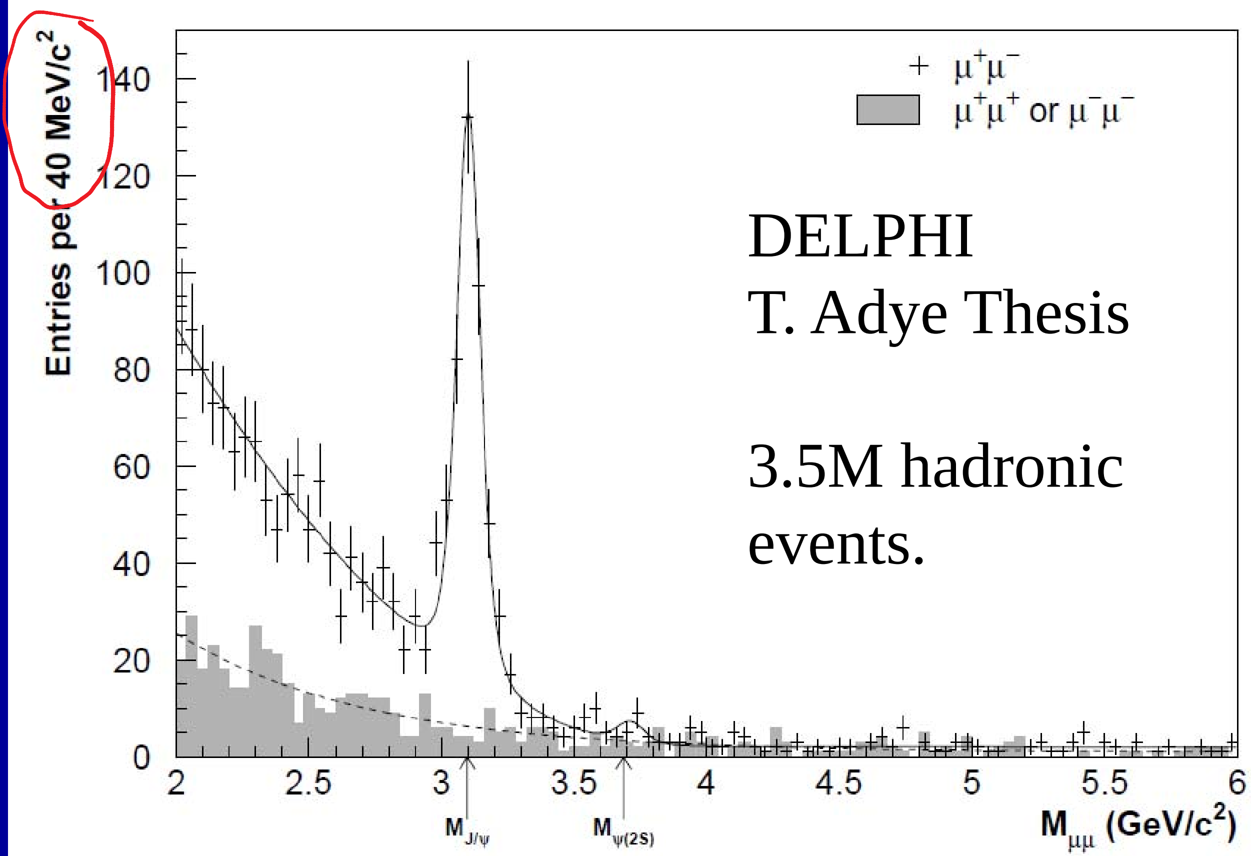
$b \rightarrow c\, l\, \nu$

$\hookrightarrow s\, l\, \nu$

J/psi Kinematics from Z→bb



Example LEP data



Opposite-sign fit (1941 candidates; 495 in $2.95 \le M_{\mu\mu} < 3.25$ GeV/c^2 window)

J/ψ fraction in window	$f_{J/\psi} =$	(73.3 ± 2.1)	%
hemiparabola fraction	$P_N =$	(69.9 ± 1.6)	%
total $\psi(2S)$s	$N_{\psi(2S)} =$	16.7 ± 6.6	
J/ψ mass	$M_{J/\psi} =$	(3102.3 ± 3.4)	MeV/c^2

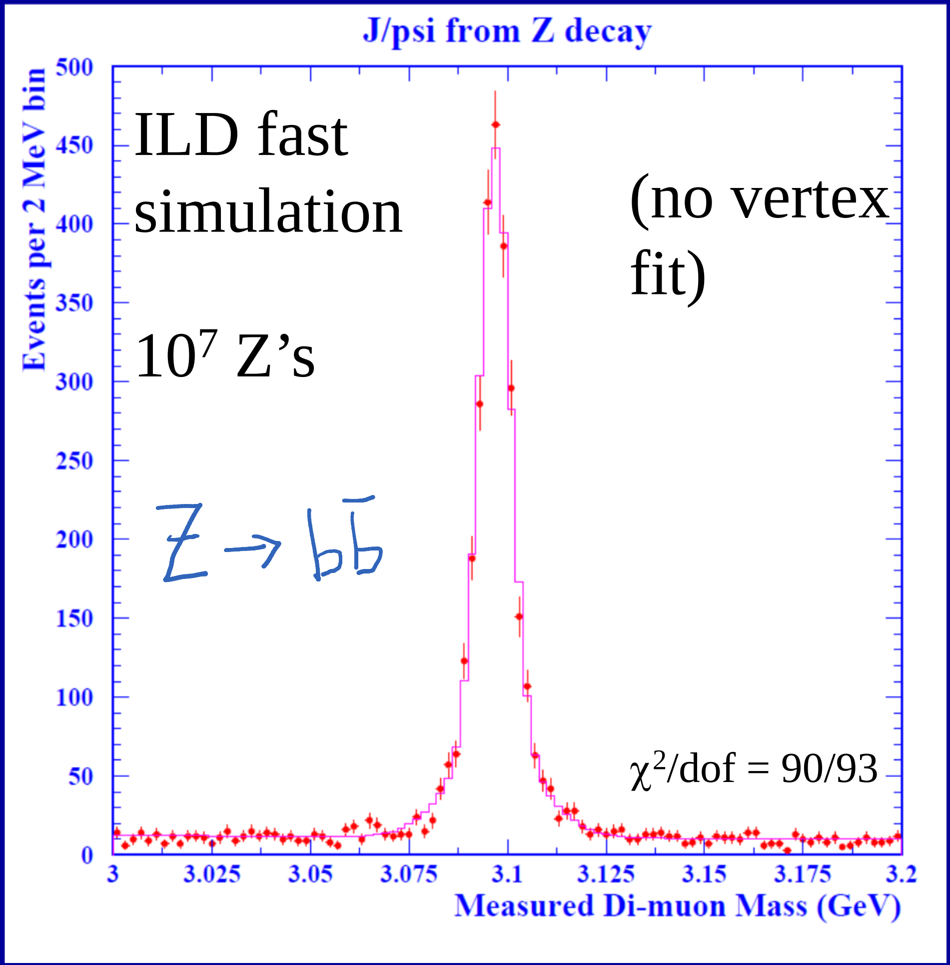
Momentum Scale with J/psi

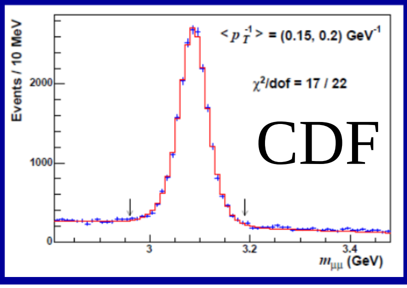
With 10^9 Z's expect statistical error on mass scale of < 3.4 ppm given ILD momentum resolution.

Most of the J/psi's are from B decays.

J/psi mass is known to 3.6 ppm.

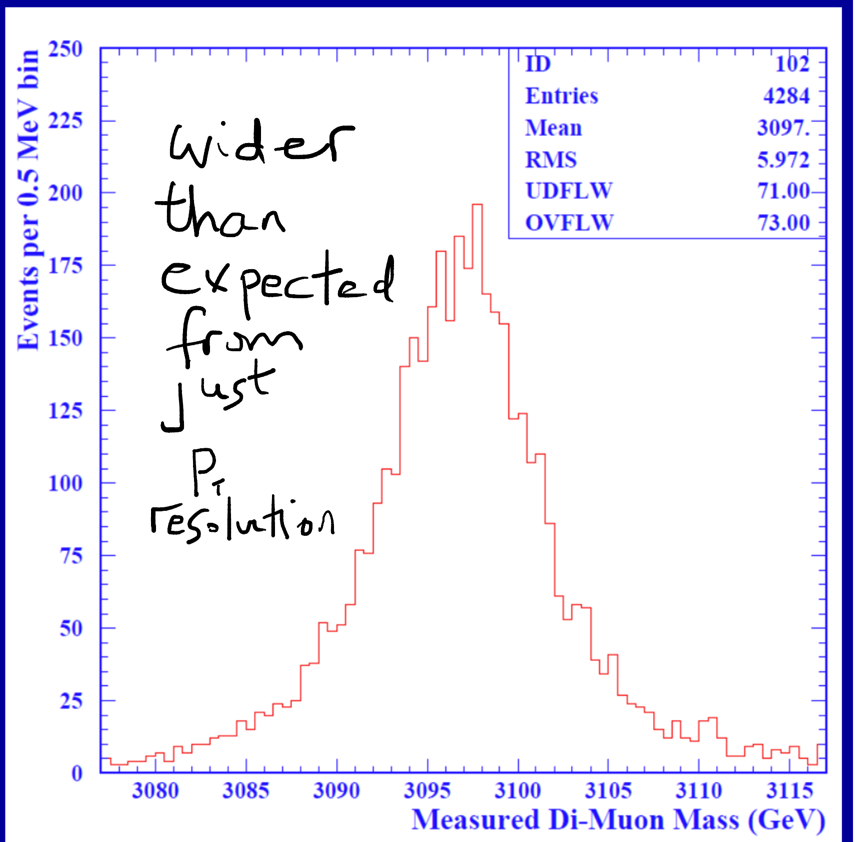
Can envisage also improving on the measurement of the Z mass (23 ppm error)

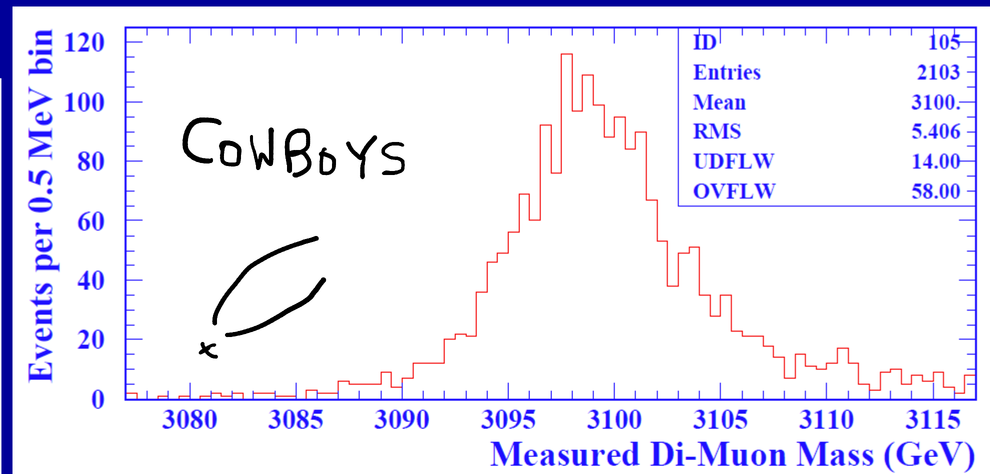


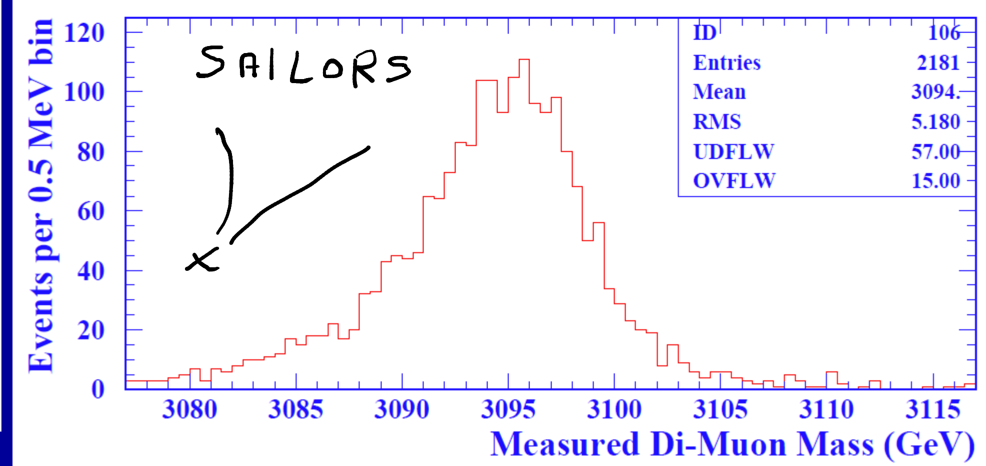


Double-Gaussian + Linear Fit

Is the mass resolution as expected?







=> Need to calculate mass using the track parameters at the di-muon vertex.

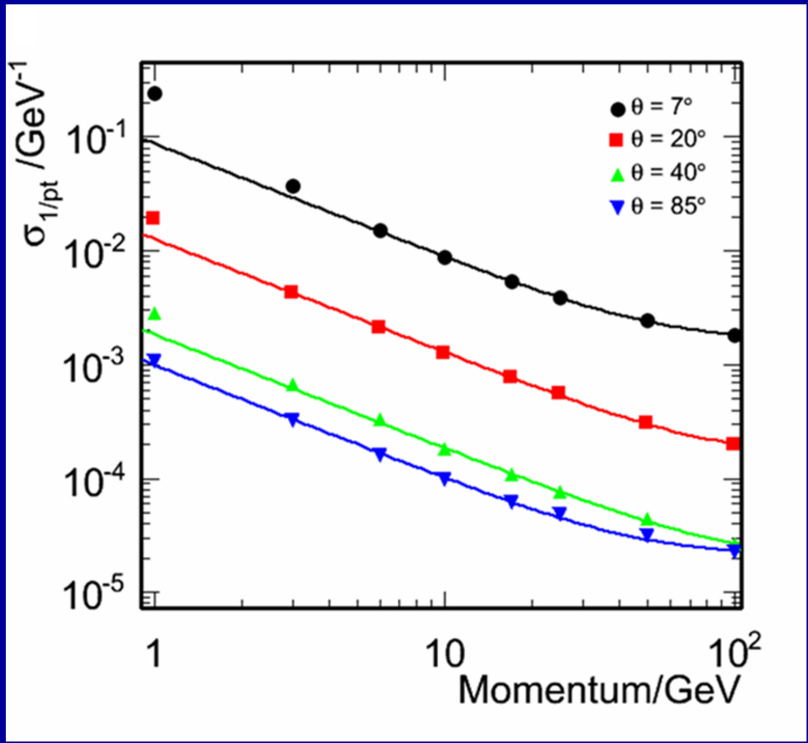
Momentum Resolution

$P_T(\mathrm{GeV}/c) = 0.3\, z\, B(T) R(m)$

Define track curvature

$\kappa \equiv \frac{1}{R} \sim \frac{1}{P_T}$

$(\Delta\kappa)^2 = (\Delta\kappa_{res})^2 + (\Delta\kappa_{MS})^2$



$\sigma_{1/p_T} = a \oplus b/(p_T \sin\theta)$

$a = 2 \times 10^{-5}\,\mathrm{GeV}^{-1}$ and $b = 1 \times 10^{-3}$

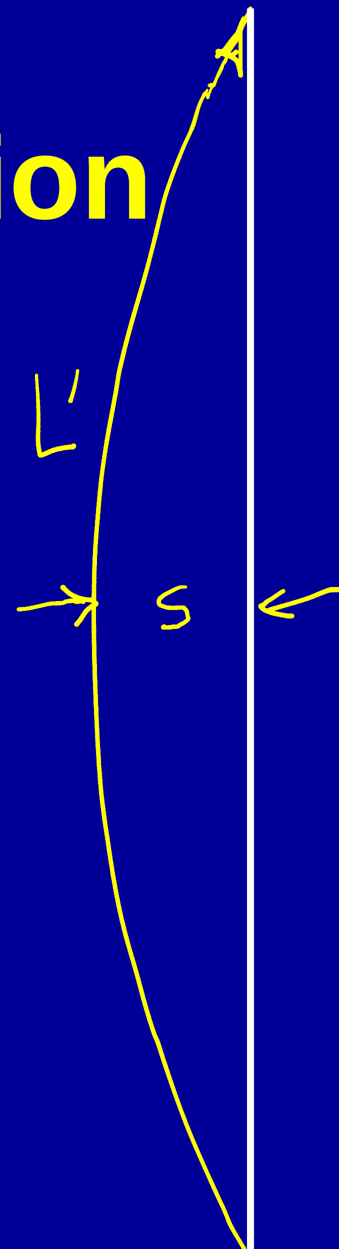
See PDG

Momentum Resolution

$$\Delta K_{res} = \sqrt{\frac{720}{N+4}}\,\frac{\epsilon}{L'^2}$$

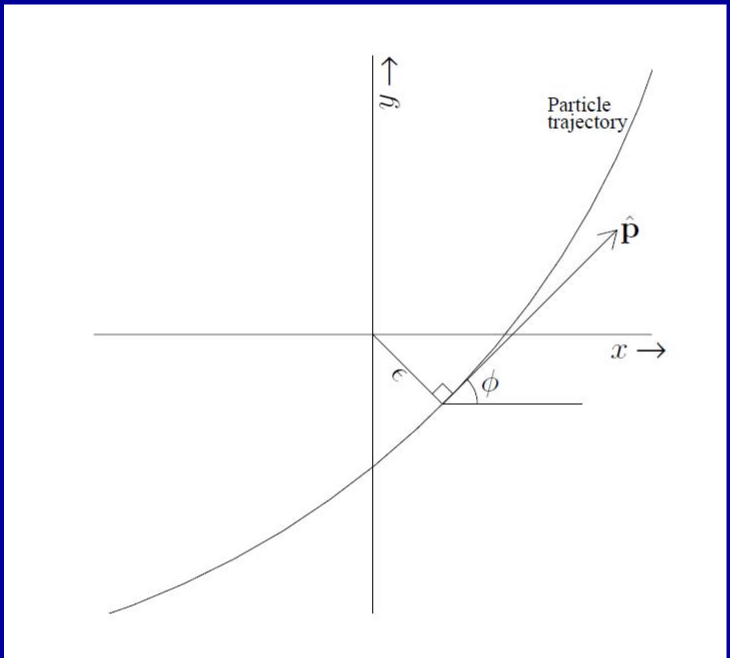
$720 \rightarrow 320 \rightarrow 256 \rightarrow 136$

$$\Delta K_{MS} \approx \frac{0.016}{L\,p_T \sin\theta}\sqrt{\frac{L}{X_0}}$$



Resolution depends on number of points (N), track-lengths (L and L'), point-resolution (ε) and material thickness.

Track/Helix Parameterization



Track parameters

$$\vec{q} = \begin{pmatrix} \epsilon \\ z \\ \theta \\ \phi \\ \kappa \end{pmatrix} \qquad \begin{matrix} d_0 \\ z_0 \\ \tan\lambda \\ \phi_0 \\ \rho \quad \frac{1}{R} \end{matrix}$$

Note: often impt.
Sign conventions.

Vertex Fit

Idea.

$\vec{q}_1, V_1$

$\vec{q}_2, V_2$

10 measurements

V_1, V_2 independent.

Adjust $\vec{q}_1, \vec{q}_2$ subject to the constraint that they originate from a common point in 3-d.

Fit parameters (9)

$\vec{r}_v = (x_v, y_v, z_v)$

$\vec{p}_1 \quad (K_1, \theta_1, \phi_1)$

$\vec{p}_2 \quad (K_2, \theta_2, \phi_2)$

$\Rightarrow \chi^2_{fit}, \ \vec{q}\,'_{12} = (\vec{r}_v, \vec{p}_1, \vec{p}_2), \ V_{12} \qquad$ (1 dof)

Vertex Fitters

A Method for Finding the Least Squares Estimate of the Intersection Point of Two Helices in Space

R. J. Royston and J. Gregory
Argonne National Laboratory, Argonne, Illinois

When the helical trajectories of two charged particles moving away from a common point in a magnetic field are reconstructed from measurements on the tracks, the reconstructed tracks are perturbed by measurement and other errors and do not, in general, intersect. A method is given for adjusting the reconstructed tracks in a least squares manner so that they do intersect.

280 Communications of the ACM

Volume 9 / Number 4 / April, 1966

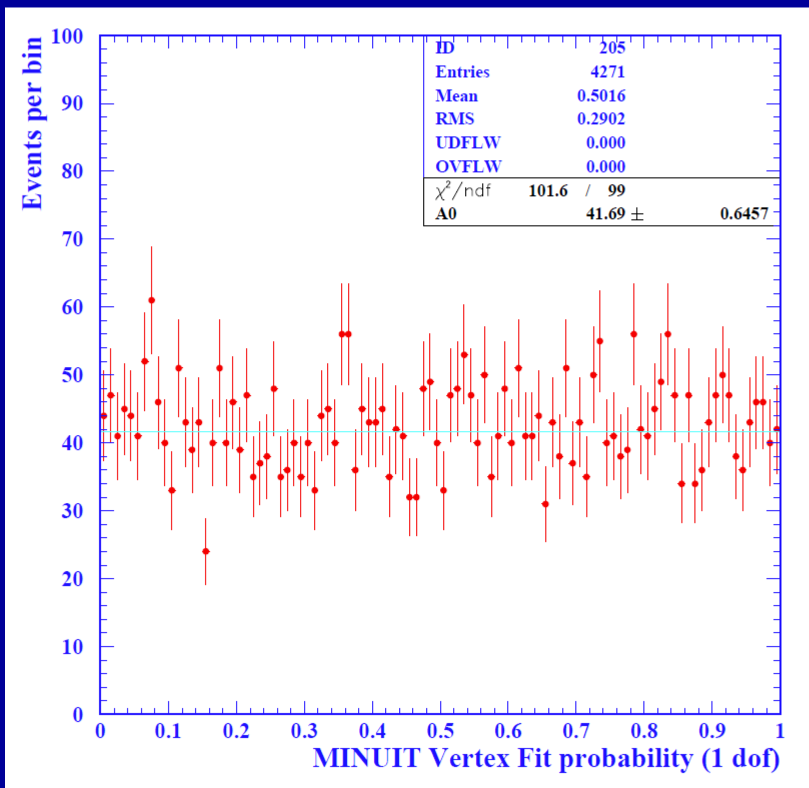
In the 48 years since 1966, Moore's law implies a factor of 2^{24} increase in CPU power. Essentially what can now be done in 1s used to take 1 year.

All vertex fitters seem to have "fast" in their title.

I investigated the OPAL and DELPHI vertex fitters, but after finding a few bugs and features, decided to revert to MINUIT.

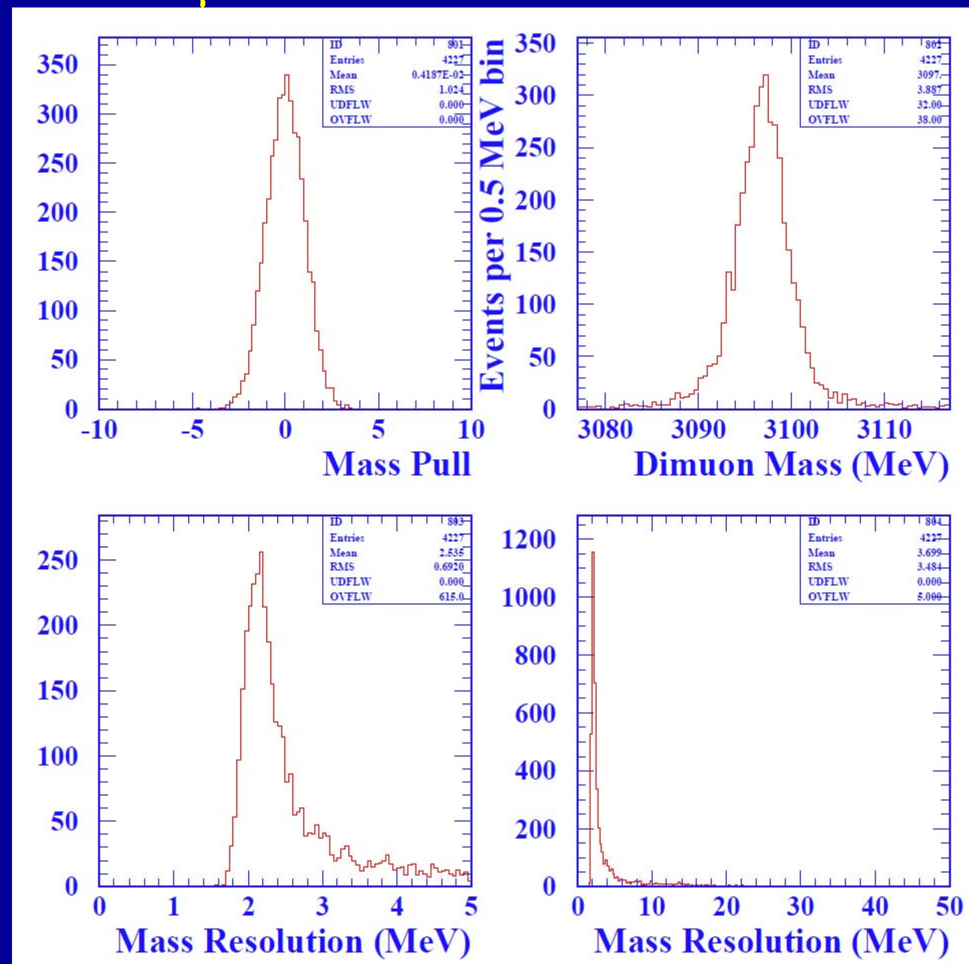
J/Psi (from Z) Vertex Fit Results

Implemented in MINUIT by me.
(tried OPAL and DELPHI fitters – but some issues)



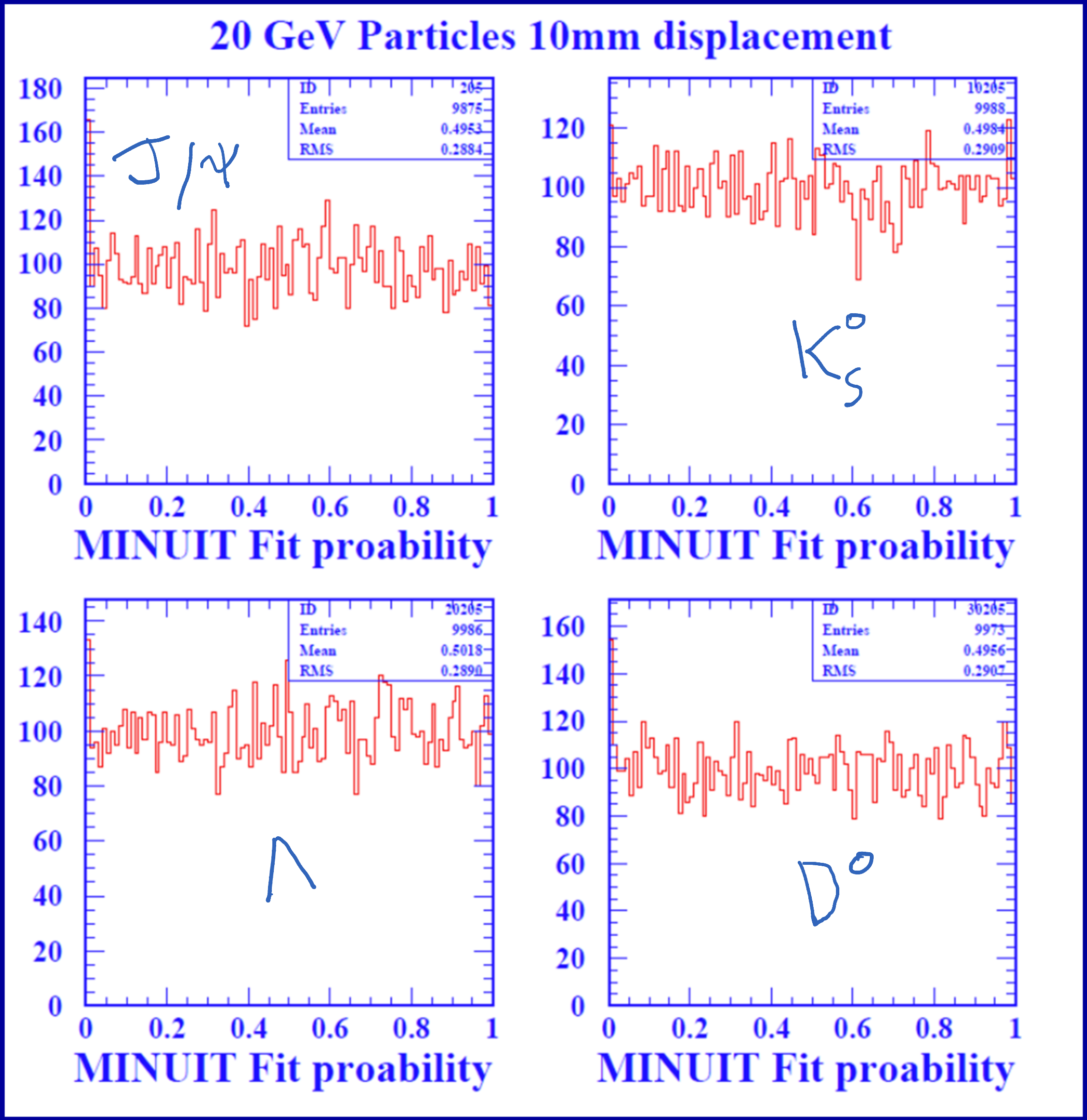
Mass errors calculated from V_{12}, cross-checked with mass-dependent fit parameterization

With $P_{fit} > 1\%$ cut



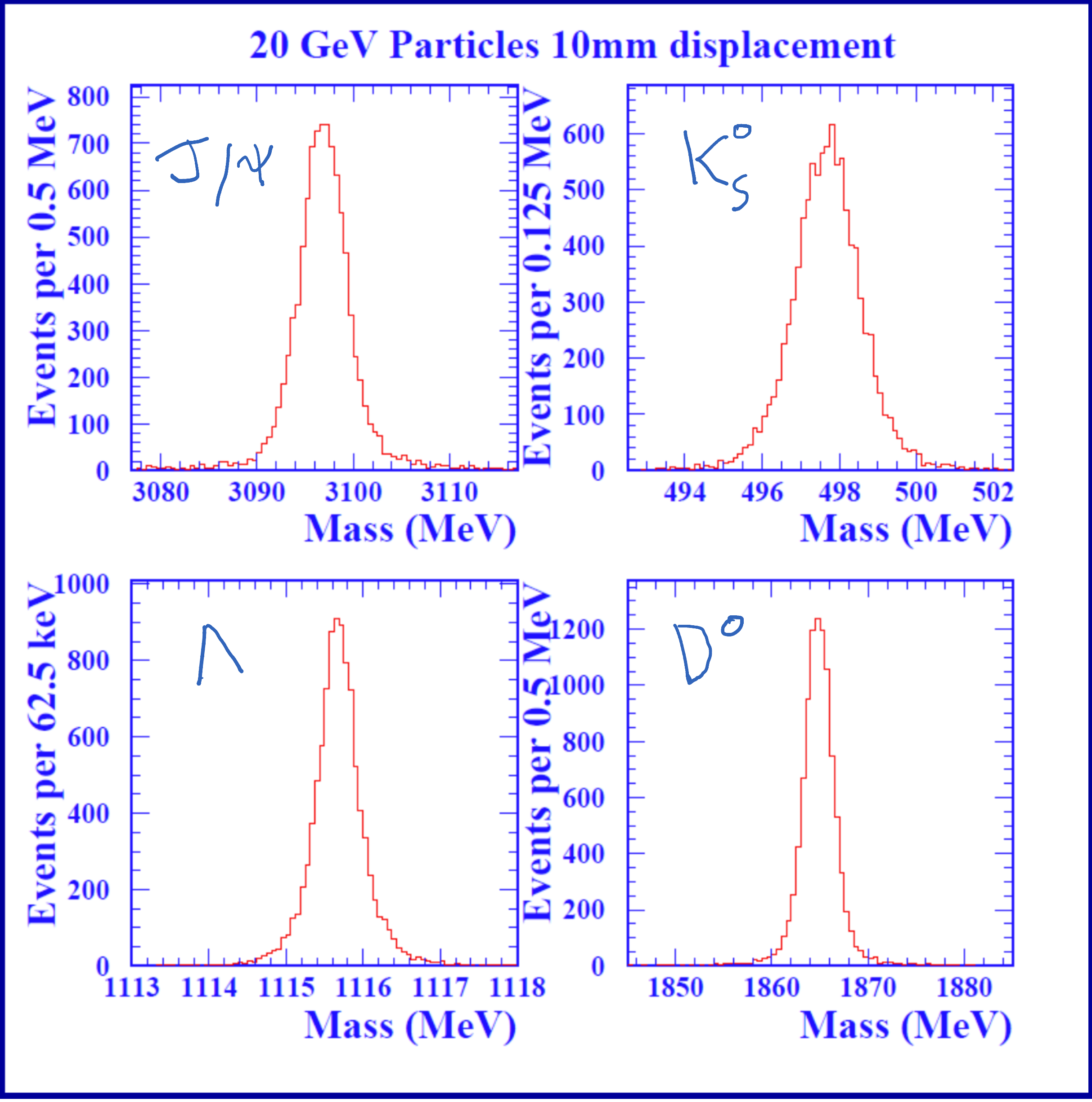
$$\text{pull} \equiv \frac{m_{fit} - m_{gen}}{\Delta m_{fit}}$$

Single particle studies

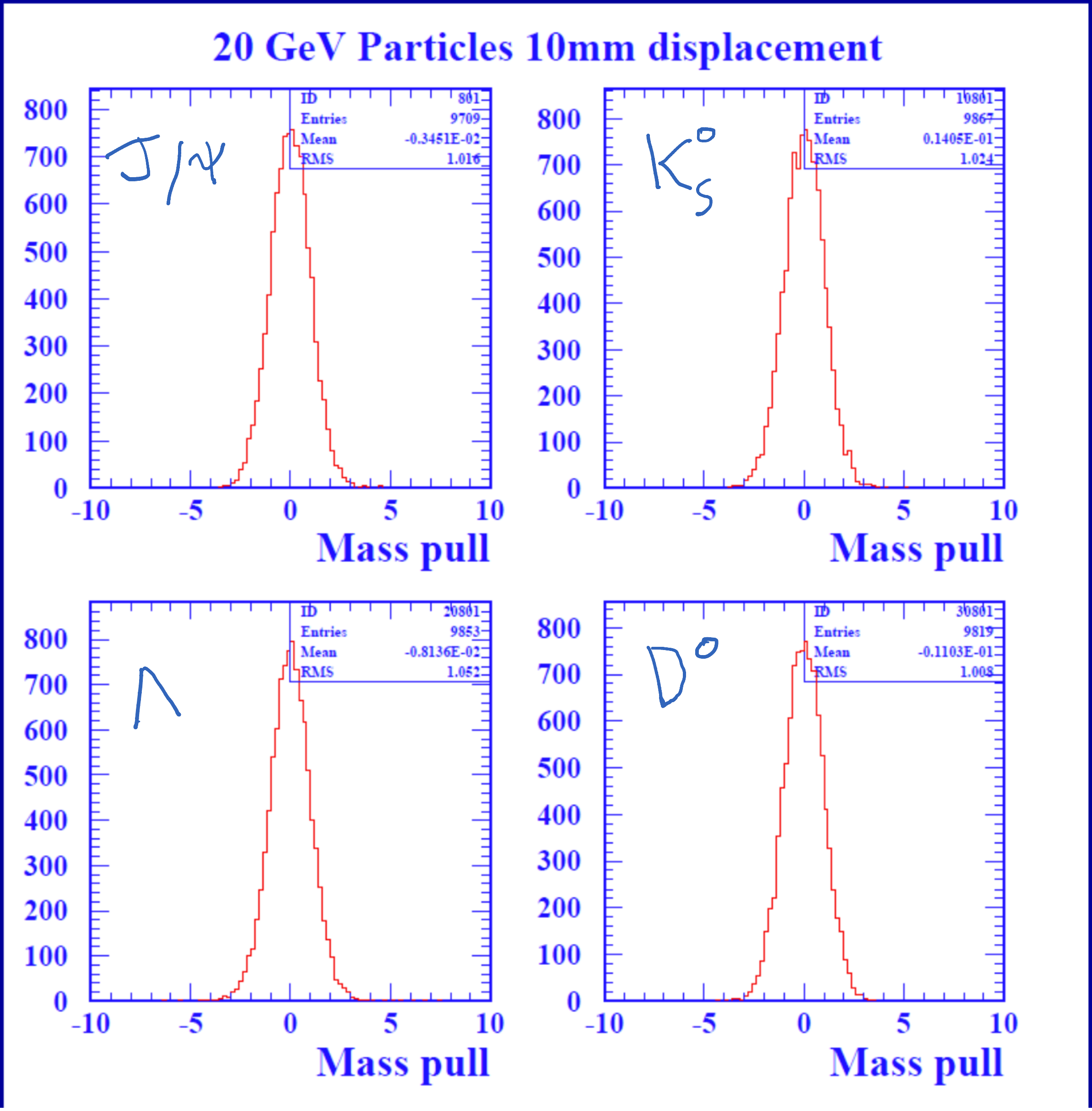


Mass Plots

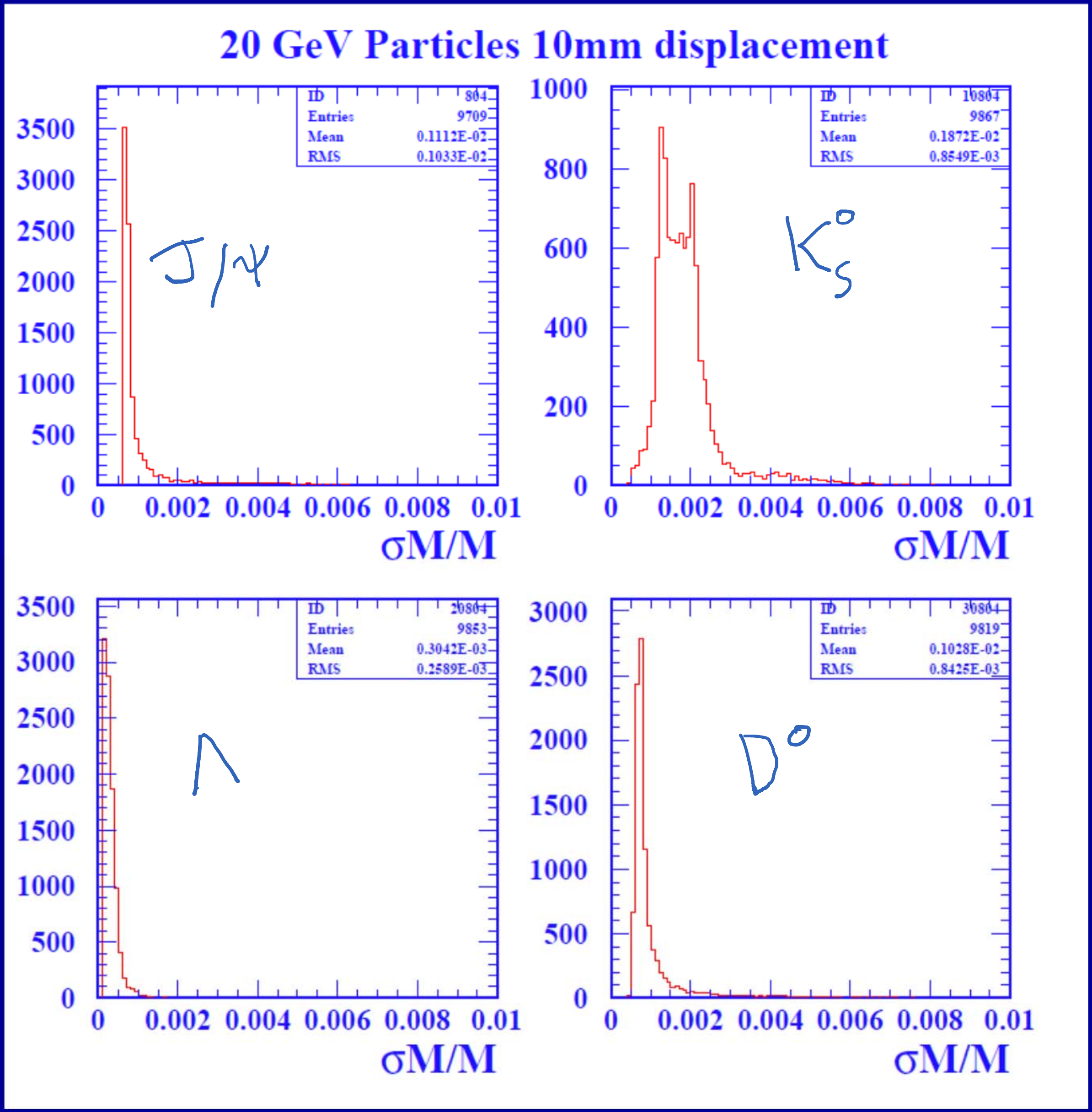
After fit ($P_{fit}>1\%$)



Mass Pulls


20 GeV Particles 10mm displacement
J/ψ
ID 801
Entries 9709
Mean -0.3451E-02
RMS 1.016
K^0_S
ID 10801
Entries 9867
Mean 0.1405E-01
RMS 1.024
Λ
ID 20801
Entries 9853
Mean -0.8136E-02
RMS 1.052
D^0
ID 30801
Entries 9819
Mean -0.1103E-01
RMS 1.008
Mass pull

Mass Resolution



Bottom-line with Z events

- Without vertex fit and using simple mass fit, expect **statistical** error on J/psi mass of 3.4 ppm from 10^9 hadronic Z's.
- With vertex fit => 2.0 ppm
- With vertex fit and per-event errors => 1.7 ppm.
- (Note background currently neglected. (S:B) in ± 10 MeV range is about 135:1 wrt semi-leptonic dimuons background from Z->bb, and can be reduced further if required)
- Neglected issues likely of some eventual importance :
  - J/psi FSR, Energy loss.
  - Backgrounds from hadrons misID'd as muons
  - Alignment, field homogeneity etc ..

Prospects at higher energies

J/psi: • b bbar cross-section comparison

$\sqrt{s}$ (GeV)	91	161	250	350	500	1000
$\sigma_{b\bar{b}}$ (pb)	6600	25	9.9	4.9	2.5	0.7

Table 3: Unpolarized $b\bar{b}$ cross-sections

- Other modes: HX, ttbar
- (prompt) J/psi production from gamma-gamma collisions (DELPHI: 45 pb @ LEP2)
- Best may be to use J/psi at Z to establish momentum scale, improve absolute measurements of particle masses (eg. D^0)
  - Use D^0 for more modest precision at high energy (example top mass application)

Improving on the Z Mass and Width etc?

- With the prospect of controlling √s at the few ppm level, ILC can also target much improved Z line-shape parameters too.
- The "Giga-Z" studies were quite conservative in their assumptions on beam energy control and this is the dominant systematic in many of the observables.

Summary

- m_W can potentially be measured to 2 MeV at ILC from a polarized threshold scan.
- Needs beam energy controlled to 10 ppm
  - Di-muon momentum-based method has sufficient statistics ($\sqrt{s}$=161 GeV)
  - Associated systematics from momentum scale can be controlled with good statistics using J/psi’s collected at $\sqrt{s}$=91 GeV
    - Statistics from J/psi in situ at $\sqrt{s}$=161 GeV is an issue. Sizable prompt cross-section from two-photon production (45 pb) in addition to b’s.