W mass measurement at future e^+e^- colliders

The three most promising approaches to measuring the W mass at an e^+e^- collider are:

- Polarized threshold scan of the W^+W^- cross-section as discussed in [1].
- Kinematically-constrained reconstruction of W^+W^- using constraints from four-momentum conservation and optionally mass-equality as was done at LEP2.
- Direct measurement of the hadronic mass. This can be applied particularly to single-W events decaying hadronically or to the hadronic system in semi-leptonic W^+W^- events.

The three different methods are summarized in the following tables. There is one reasonably complete study related to a polarized threshold scan at ILC [1] which has been updated for this Snowmass workshop. There is also a new much more precise method for determining the beam energy in situ using di-muon events at ILC which has been developed in more depth during this workshop and was presented at [2]. This gives the potential to reduce the beam energy uncertainty on the W mass to 0.8 MeV (limited by stand-alone momentum scale uncertainties estimated at 10 ppm). This previously important systematic for the threshold method - and dominant systematic for the kinematically-constrained reconstruction method appears to be no longer such a critical issue. The reported tables should be taken as reasonable indications of the potential performance. W mass measurements were statistics limited for these methods at LEP2. It is clear that large improvements in the systematics are feasible at future machines like ILC. Exactly how much better can be done is something that can not be predicted with absolute certainty, given the orders of magnitude of improvement. In practice it is something that typically can only be pinned down once a detector is operating. In general the experience has been that predictions tend to err significantly on being too conservative.

ΔM_W [MeV]	LEP2	ILC	ILC
$\sqrt{s}$ [GeV]	161	161	161
$\mathcal{L}$ [fb^{-1}]	0.040	100	480
$P(e^-)$ [%]	0	90	90
$P(e^+)$ [%]	0	60	60
statistics	200	2.4	1.1
systematics	70	3.0	1.6
experimental total	210	3.9	1.9
beam energy	13	0.8	0.8
theory	-	(1.0)	(1.0)
total	210	4.0	2.1

Table 1: Current and preliminary anticipated uncertainties in the measurement of M_W at e^+e^- colliders close to WW threshold.

Table 5 has projected results for running close to WW threshold. ILC can collide highly longitudinally polarized electrons and highly longitudinally polarized positrons - this is particularly advantageous for a threshold scan. In the tables it is assumed that if ILC undertakes a dedicated scan near threshold that this would be done once the highest polarization levels considered achievable have been achieved. The estimated uncertainties assume that the beam energy scale can be established from collision data at the level of 1 part in 10^5 leading to a corresponding experimental uncertainty on m_{W} of 0.8 MeV. This has been shown to be statistically feasible using di-muon events provided that the momentum scale is determined to the same precision. This appears feasible using J/ψ events in Z decays. The ILC numbers are based on a detailed and updated study with realistic assumptions on detection efficiency, polarization determination, backgrounds, efficiency and normalization errors using a 6-point scan with four different beam helicity combinations. The ILC numbers include the (small) effects from beamstrahlung on the cross-section and take advantage of the 150 pb cross-section of multi-hadron production for determinining the beam polarizations from the data. In addition, the table includes an indicative estimate of the anticipated theoretical uncertainty associated with interpreting cross-section measurements near threshold in terms of m_{W} of 1.0 MeV. A detailed assessment of the anticipated theoretical shape and normalization uncertainties on the cross-section behavior with center-of-mass energy and including the effects of realistic experimental acceptance for all the four-fermion final states would in principle be needed to report a firm theoretical error estimate. In the table for the ILC, the systematics are essentially currently included in the overall error as the multi-parameter fit adjusts the systematics as nuisance parameters constrained within a priori uncertainties taken as 0.1% for relative efficiency and absolute integrated luminosity. The beam polarizations and backgrounds are fitted simultaneously from the data. In the context of the polarized scan this measurement is essentially *statistics* dominated.

ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	172-209	250	350	500
$\mathcal{L}$ [fb^{-1}]	3.0	500	350	1000
$P(e^-)$ [%]	0	80	80	80
$P(e^+)$ [%]	0	30	30	30
beam energy	9	0.8	1.1	1.6
luminosity spectrum	N/A	1.0	1.4	2.0
hadronization	13	1.3	1.3	1.3
radiative corrections	8	1.2	1.5	1.8
detector effects	10	1.0	1.0	1.0
other systematics	3	0.3	0.3	0.3
total systematics	21	2.4	2.9	3.5
statistical	30	1.5	2.1	1.8
total	36	2.8	3.6	3.9

Table 2: Current and preliminary estimated experimental uncertainties in the measurement of M_W at e^+e^- colliders from kinematic reconstruction in the $q\bar{q}\ell\nu_\ell$ channel with $\ell = e, \mu$.

Table 2 has projected results for kinematic reconstruction using the semi-leptonic channels as was used at LEP2. Details of this method are in the recently submitted LEP2 legacy paper [3] and the systematics discussed there are used as the basis for this discussion. At

LEP2 the fully hadronic channel was also used. It is not expected to be competitive at the sub-10 MeV level because of final-state interaction effects and so is neglected for these projections. There have not been dedicated studies on the semi-leptonic channel for ILC, but the measurements at LEP2 can be used to estimate/bracket some of the primary uncertainties. The beam energy uncertainty is taken again as a 10^{-5} uncertainty at 250 GeV leading to an error of 0.8 MeV. At higher energies this uncertainty is scaled linearly with center-of-mass energy reflecting in part less statistics for in-situ checks. Systematic errors associated with knowledge of the luminosity spectrum, dL/dx_1dx_2, are estimated to be at the 1 MeV level at 250 GeV and will increase with center-of-mass energy. The table assumes a linear dependence. Two of the primary systematics associated with the W mass measurement at LEP2, namely from hadronization and detector effects will be controlled much better with the modern ILC detectors and a more than one hundred times larger data-set. As an example let us consider the error associated with the muon energy scale. In the LEP2 analysis by the OPAL experiment, a 0.3% uncertainty on the muon energy scale was assessed leading to an error on the W mass of 7 MeV. For the anticipated errors on the muon energy scale at ILC (10 ppm), the corresponding systematic error is reduced to being completely negligible (naively 0.02 MeV). The hadronization errors which dominated the LEP2 systematic uncertainty were a result of several effects. The much larger statistics envisaged at ILC will allow the kaon and proton fractions in W decays to be measured at least ten times better and the particle-flow based jet reconstruction should make it more feasible to use identified particles in reconstructing jets. Given the improvements in the detector and statistics, improvements in the leading experimental systematics by a factor of 10 can be envisaged. The radiative corrections systematic can presumably be improved with further work. The growing importance of ISR at higher center-of-mass energies suggests that this systematic will degrade as the center-of-mass energy increases. The effective statistical error is not completely straightforward to estimate as it includes effects from ISR and beamstrahlung which often degrade the validity of the kinematic constraints both of which are substantially larger at higher center-of-mass energy. It has been shown that these effects can be ameliorated in the fully hadronic channel [4] by allowing for such photon radiation. It is expected that similar methods will be useful to improve the effective resolution in the semi-leptonic channel too although this is not as highly constrained given the unobserved neutrino. This method is likely to be *systematics* dominated.

Table 3 has projected results from the direct measurement of the hadronic mass. This measurement depends primarily on how well the hadronic mass scale can be determined. It essentially does not depend at all on measurements of the beam energy or luminosity spectrum and so is very complementary to the previous two methods. In the particle-flow approach it is in principle possible to cast this as primarily a “bottom-up” problem of determining the tracker momentum scale, the electro-magnetic calorimeter scale and the calorimeter energy scale for neutral hadrons and it is these components that affect the jet energy scale. Over the course of the envisaged ILC program it is anticipated that the samples of Z’s decaying to hadrons where the Z mass is currently known to 2.1 MeV should make it feasible to target a 3.0 MeV error originating from the jet energy scale. The hadronization error is anticipated to be dominated by knowledge of the K_L^0 and neutron fractions. The pile-up entry refers to primarily $\gamma\gamma \rightarrow$ hadrons events coincident with W events. The contribution of such events to the measured hadronic mass can be mitigated

ΔM_W [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [fb^{-1}]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

Table 3: Preliminary estimated experimental uncertainties in the measurement of M_W at e^+e^- colliders from direct reconstruction of the hadronic mass in single-W and WW events where one W decays hadronically. Does not include WW with $q\bar{q}\ell\nu_\ell$ where $\ell = e, \mu$.

and is not expected to be a dominant systematic error - but it will be more problematic at higher center-of-mass energies. The statistical error depends on the jet energy resolution and the consequent hadronic mass resolution. The hadronic mass resolution for a particular event varies substantially depending primarily on the fractions of energy in charged particles, photons and neutral hadrons in the event. The effective hadronic mass resolution is therefore a strong function of the analysis method. A full convolution fit with more advanced reconstruction techniques like π^0 mass-constrained fitting offers the potential to improve the W mass statistical error by a factor of 2.2 over that naively estimated from the observed average jet energy resolution in full simulation studies. In the estimates above, we have been conservative and have assumed that the actual improvement factor of a realistic and mature analysis is 1.4. This method is likely to be *systematics* dominated.

ILC Polarized Threshold Scan Details

The study of [1] has been updated and improved. The updates include the use of beam parameters consistent with the ILC TDR design and experimental performance appropriate to the envisaged ILC detectors. The previous study was done in 1999 and had very conservatively assumed experimental characteristics similar to the LEP detectors. The improvements include a re-optimization of the fraction of the luminosity associated with each beam helicity combination which results from the better detector performance.

The updated assumptions on the experimental event selection and the associated systematics are given in Table 4. These correspond to a factor of two reduction in the event selection inefficiency and a factor of two reduction in the non-WW backgrounds compared to that essentially achieved with the LEP detectors. Further improvement beyond these expected performance numbers is not out of the question.

The re-optimized running strategy devotes 78% of the integrated luminosity to the “signal” helicity combination (L-, R+), 17% to the “background” helicity combination (R-, L+) and 5% equally shared amongst the polarization constraining like-sign helicity combinations

Channel	Efficiency (%)	Unpolarized σ_{bkgd} (fb)	Eff. syst. (%)	Bkgd syst. (%)
ll	87.5	10	0.1	free
lh	87.5	40	0.1	free
hh	83.5	200	0.1	free

Table 4: Experimental assumptions for the WW event selection near threshold using a polarized scan

of (L- L+) and (R- R+). The optimization was done assuming 90% electron beam polarization and 60% positron beam polarization. The center-of-mass energies used in the 6-point scan are (160.6, 161.2, 161.4, 161.6, 162.2, 170.0) GeV with integrated luminosities in the ratios of 1:5:5:5:1:6 respectively. The current scan is optimized for measuring m_{W}. There is room for further optimization and alternative strategies. Alternative scans better suited to measuring Γ_W can also be envisaged.

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$\sqrt{s}$ [GeV]	161	161	161
$\mathcal{L}$ [fb^{-1}]	0.040	100	480
$P(e^-)$ [%]	0	90	90
$P(e^+)$ [%]	0	60	60
statistics	200	2.4	1.1
background		2.0	0.9
efficiency		1.2	0.9
luminosity		1.8	1.2
polarization		0.9	0.4
systematics	70	3.0	1.6
experimental total	210	3.9	1.9
beam energy	13	0.8	0.8
theory	-	(1.0)	(1.0)
total	210	4.0	2.1

Table 5: Current and preliminary anticipated uncertainties in the measurement of M_W at e^+e^- colliders close to WW threshold. The total does not include the theory guesstimate.

Systematics are fitted for so the most relevant number is the experimental total. The systematic effects on m_{W} are evaluated by fixing all the nuisance parameters or all but one. Correlations amongst the systematic variations on the cross-section determinations mean that the total systematic is not the quadrature sum of the estimated individual systematic errors.

References

[1] "Precision measurement of the W mass with a polarized threshold scan at a linear collider." G. W. Wilson. In *2nd ECFA/DESY Study 1998-2001* 1498-1505 LC-PHSM-

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[2] "Investigating In-Situ Center-of-Mass Energy Determination with Di-Muon Events", G. W. Wilson, ECFA LC2013 Workshop, Hamburg, May 2013.

[3] S. Schael *et al.* [ALEPH and DELPHI and L3 and OPAL and LEP Electroweak Working Group Collaborations], "Electroweak Measurements in Electron-Positron Collisions at W-Boson-Pair Energies at LEP," arXiv:1302.3415 [hep-ex].

[4] M. Beckmann, B. List and J. List, "Treatment of Photon Radiation in Kinematic Fits at Future e^+e^- Colliders," Nucl. Instrum. Meth. A **624**, 184 (2010) [arXiv:1006.0436 [hep-ex]].