Model Dependence Error in Hadronic Recoil ZH Cross Section and Why Recoil ZH Cross Section Measurements May Not Matter If We Measure σXBR(H→BSM) Well

Tim Barklow (SLAC)
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Mark Thomson's analysis of $\sigma(ZH)$ with $Z \to q\bar{q}$ uses two measurements to obtain the cross section:

$$\sigma(ZH) = \sigma(ZH)\bullet BR(visible) + \sigma(ZH)\bullet BR(invisible)$$

$\sigma(ZH)\bullet BR(visible)$

Final(?) Results

Process	σ/fb	ε_{presel}	$\varepsilon_{\mathcal{L}>0.70}$	$N_{\mathcal{L}>0.70}$
$q\bar{q}$	25180	0.5 %	<0.1 %	6211
$q\bar{q}l\nu$	5914	6.4 %	0.1 %	3895
$q\bar{q}q\bar{q}$	5847	4.2 %	0.4 %	10818
$q\bar{q}ll$	1704	1.2 %	0.1 %	1218
$q\bar{q}\nu\bar{\nu}$	325	0.6 %	<0.1 %	35
$H\nu_e\bar{\nu}_e$		- %		
HZ	93.4	44.0 %	20.3 %	9493
$H \to$ invis.		0.6 %	<0.1 %	–
$H \to q\bar{q}/gg$		43.5 %	20.6 %	6211
$H \to WW^*$		44.7 %	19.5 %	2240
$H \to ZZ^*$		40.0 %	18.1 %	254
$H \to \tau^+\tau^-$		47.6 %	21.4 %	738
$H \to \gamma\gamma$		42.8 %	22.1 %	32
$H \to Z\gamma$		41.8 %	17.6 %	17
$H \to \mu^+\mu^-$		39.5 %	20.6 %	3

★ For optimal cut
- signal ~9.5k events
- background ~ 19k events

15 % improvement c.f. LCWS analysis

Efficiencies same to ~10 % !!!

⇨ almost model independent

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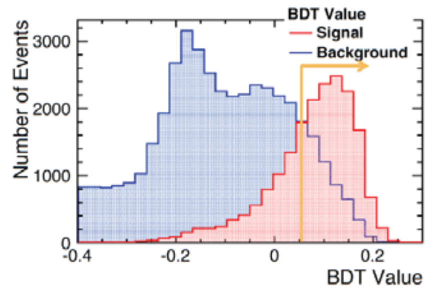
$\sigma(ZH)\bullet BR(invisible)$

BDT Selection

★ Preliminary results (7 variable BDT selection)



Signal

Channel	Efficiency
Z H → qq invis.	20.7 %

Backgrounds

Channel	Efficiency	Events
qqlν	<0.1 %	900
qqll	<0.1 %	4
qqνν	1.5 %	2414

★ Assuming no invisible decays (1 sigma stat. error):

⇨ $\Delta\sigma_{\text{invis}} = \pm 0.57\ \%$

(CLIC beam spectrum, 500 fb^{-1} @ 350 GeV, no polarisation)

In order to use this cross section measurement in our Higgs analyses we have to quantify the penalty associated with the fact that $\sigma(ZH) \bullet BR(visible)$ is "almost model independent". By how much must we blow up $\Delta\sigma(ZH) \bullet BR(visible)$ to account for the fact that the efficiencies differ by 10% or more?

Model Indepedence

★ **Combining visible + invisible analysis: wanted M.I.**

- **i.e. efficiency independent of Higgs decay mode**

Decay mode	$\varepsilon^{\rm vis}_{\mathscr{L}>0.70}$	$\varepsilon^{\rm invis}_{\rm BDT>0.08}$	$\varepsilon^{\rm vis}+\varepsilon^{\rm invis}$
$H \to$ invis.	<0.1 %	20.7 %	20.7 %
$H \to q\bar{q}/gg$	20.6 %	<0.1 %	20.6 %
$H \to WW^*$	19.5 %	<0.1 %	19.8 %
$H \to ZZ^*$	18.1 %	0.9 %	19.0 %
$H \to \tau^+\tau^-$	21.4 %	0.1 %	21.5 %
$H \to \gamma\gamma$	22.1 %	<0.1 %	22.1 %
$H \to Z\gamma$	17.6 %	<0.1 %	17.1 %
$H \to \mu^+\mu^-$	20.6 %	<0.1 %	20.6 %
$H \to WW^* \to q\bar{q}q\bar{q}$	19.3 %	<0.1 %	19.3 %
$H \to WW^* \to q\bar{q}l\nu$	19.6 %	<0.1 %	19.6 %
$H \to WW^* \to q\bar{q}\tau\nu$	19.9 %	<0.1 %	19.9 %
$H \to WW^* \to l\nu l\nu$	22.0 %	0.3 %	22.3 %
$H \to WW^* \to l\nu\tau\nu$	16.7 %	0.3 %	17.0 %
$H \to WW^* \to \tau\nu\tau\nu$	12.2 %	1.3 %	13.6 %

Very similar efficiencies (first eight decay modes)

Look at wide range of WW topologies (WW* decay modes)

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I propose an approach where we use all of our σ•*BR* measurements for visible Higgs decays to obtain an estimate of the average signal efficiency for σ(*ZH*)•*BR*(*visible*). It is then straightforward to propagate the σ•*BR* errors to the error on σ(*ZH*)•*BR*(*visible*), This means that one must take into account the correlation between the σ•*BR* measurements and our σ(*ZH*) measurement from hadronic Z decays when we fit for couplings and total width. It also means that we must develop σ•*BR* analyses for all possible visible Higgs decays -- but this we have to do anyway to extract the best errors on Higgs couplings and total width.

Let

$$\Psi \equiv \sigma(ZH) \bullet BR(visible)$$

$\Omega =$ Number of signal + background events in $\sigma(ZH) \bullet BR(visible)$ analysis

$\mathrm{B} =$ Predicted number of background events in $\sigma(ZH) \bullet BR(visible)$ analysis

Ξ = Average efficiency for signal events to pass $\sigma(ZH) \bullet BR(visible)$ analysis

$L =$ luminosity

$$\Psi = \frac{\Omega - \mathrm{B}}{L\,\Xi} = \frac{1}{\Xi}\sum_i \psi_i \xi_i = \sum_i \psi_i \quad \text{where}$$

$\psi_i = \sigma(ZH) \bullet BR_i$

$\xi_i =$ efficiency for events from Higgs decay i to pass $\sigma(ZH) \bullet BR(visible)$ analysis

$$\Xi = \frac{\sum_i \psi_i \xi_i}{\sum_i \psi_i}$$

$$\psi_i = \frac{\omega_i - \beta_i}{L\,\eta_i}$$

$\omega_i =$ Number of signal + background events in $\sigma(ZH)\bullet BR_i$ analysis

$\beta_i =$ Predicted number of background events in $\sigma(ZH)\bullet BR_i$ analysis

η_i = efficiency for Higgs decay i to pass $\sigma\bullet BR_i$ analysis

K_i = number of signal + background events common to had Z recoil and $\sigma\bullet BR_i$ analyses

E = number of signal + background events unique to had Z recoil analysis

ε_i = number of signal + background events events unique to $\sigma\bullet BR_i$ analysis

$$\Omega = \mathrm{E} + \sum_i \mathrm{K}_i \qquad S \equiv \Omega\text{-}\mathrm{B} \qquad \mathrm{T} \equiv \frac{\sqrt{S + \mathrm{B}}}{S}$$

$$\omega_i = \mathrm{K}_i + \varepsilon_i \qquad s_i \equiv \omega_i - \beta_i \qquad \tau_i \equiv \frac{\sqrt{s_i + \beta_i}}{s_i}$$

$$\lambda_i \equiv \frac{\mathrm{K}_i}{\omega_i} \qquad N \equiv L\,\sigma_{ZH} \qquad r_i \equiv BR_i \qquad \delta_i \equiv \xi_i - \Xi$$

$$(\Delta\Psi)^2 = \left(\frac{\partial\Psi}{\partial\Omega}\right)^2 V_{\Omega\Omega} + \left(\frac{\partial\Psi}{\partial\Xi}\right)^2 V_{\Xi\Xi} + 2\frac{\partial\Psi}{\partial\Omega}\frac{\partial\Psi}{\partial\Xi} V_{\Omega\Xi}$$

$$\frac{\partial\Psi}{\partial\Omega} = \frac{1}{L\,\Xi} = \frac{\Psi}{\Omega}\left(1 - \frac{\mathrm{B}}{\Omega}\right)^{-1} \qquad \frac{\partial\Psi}{\partial\Xi} = -\frac{\Omega - \mathrm{B}}{L\,\Xi^2} = -\frac{\Psi}{\Xi}$$

$$V_{\Omega\Omega} = \mathrm{E} + \sum_i \mathrm{K}_i = \Omega$$

$$V_{\Xi\Xi} = \frac{1}{L^2\Psi^2}\sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2}(\varepsilon_i + \mathrm{K}_i)$$

$$V_{\Omega\Xi} = \frac{1}{L\,\Psi}\sum_i \frac{\xi_i - \Xi}{\eta_i}\mathrm{K}_i$$

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = \frac{1}{\Omega^2}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2} V_{\Omega\Omega} + \frac{1}{\Xi^2} V_{\Xi\Xi} - \frac{2}{\Omega\Xi}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1} V_{\Omega\Xi}$$

$$= \frac{1}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2} + \frac{1}{L^2\Xi^2\Psi^2}\sum_i \frac{(\xi_i-\Xi)^2}{(\eta_i)^2}(\varepsilon_i + \mathrm{K}_i) - \frac{2}{L\,\Omega\Xi\Psi}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1}\sum_i \frac{\xi_i-\Xi}{\eta_i}\mathrm{K}_i$$

$$= \frac{1}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2} + \frac{1}{L^2\Xi^2\Psi^2}\sum_i \frac{(\xi_i-\Xi)^2}{(\eta_i)^2}(L\eta_i\psi_i + \beta_i) - \frac{2}{L\,\Omega\Xi\Psi}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1}\sum_i \frac{\xi_i-\Xi}{\eta_i}\lambda_i(L\eta_i\psi_i + \beta_i)$$

$$= \frac{1}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2}\left[1+\frac{L}{\Omega}\sum_i \frac{(\xi_i-\Xi)^2}{\eta_i}\psi_i\left(1+\frac{\beta_i}{s_i}\right) - \frac{2L}{\Omega}\sum_i(\xi_i-\Xi)\psi_i\lambda_i\left(1+\frac{\beta_i}{s_i}\right)\right]$$

$$= \frac{S+\mathrm{B}}{S^2}\left\{1+\frac{L}{\Omega}\sum_i(\xi_i-\Xi)\psi_i\left(\frac{s_i+\beta_i}{s_i^2}\right)\left[(\xi_i-\Xi)L\psi_i - 2\lambda_i s_i\right]\right\}$$

$$= \mathrm{T}^2\left\{1+\frac{N^2}{\Omega}\sum_i r_i^2\tau_i^2\left[\delta_i^2 - 2\lambda_i\eta_i\delta_i\right]\right\}$$

This is our result for the error on $\sigma(ZH)\bullet BR(visible)$ given the approach outlined on page 4

What if we don't do a hadronic Z recoil measurement and instead only use $\sigma(ZH)\bullet BR_i$ to calculate $\sigma(ZH)\bullet BR(visible) = \sum_i \sigma(ZH)\bullet BR_i$?

$$\Psi' = \sum_i \psi_i \qquad \psi_i = \frac{\omega_i - \beta_i}{L\,\xi_i}$$

$$(\Delta\Psi')^2 = \sum_i \left(\frac{\partial\Psi'}{\partial\omega_i}\right)^2 \omega_i \ , \qquad \frac{\partial\Psi'}{\partial\omega_i} = \frac{1}{L\eta'_i}$$

$$(\Delta\Psi')^2 = \frac{1}{L^2}\sum_i = \frac{1}{L^2}\sum_i \frac{s_i + \beta_i}{\xi_i^{\,2}}$$

$$\left(\frac{\Delta\Psi'}{\Psi'}\right)^2 = \left(\sum_i \frac{\omega_i - \beta_i}{L\,\xi_i}\right)^{-2} \frac{1}{L^2}\sum_i \frac{s_i + \beta_i}{\xi_i^{\,2}}$$

$$= \frac{S + \mathrm{B}}{S^2}\frac{L}{\Omega}\Xi^2 \sum_i \frac{\psi_i}{\xi_i}\left(1 + \frac{\beta_i}{s_i}\right)$$

Compare this now with our formula for $\left(\frac{\Delta\Psi}{\Psi}\right)^2$ for $\lambda_i = 1$:

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = \frac{S + \mathrm{B}}{S^2}\left\{1 + \frac{1}{\Omega}\sum_i \omega_i \left[\left(1 - \frac{\Xi}{\xi_i}\right)^2 - 2\left(1 - \frac{\Xi}{\xi_i}\right)\right]\right\}$$

$$= \frac{S + \mathrm{B}}{S^2}\left\{1 + \frac{1}{\Omega}\sum_i \omega_i \left[1 - \frac{2\Xi}{\xi_i} + \left(\frac{\Xi}{\xi_i}\right)^2 - 2 + 2\frac{\Xi}{\xi_i}\right]\right\} = \left(\frac{\Delta\Psi'}{\Psi'}\right)^2$$

But the question posed on the last page is more than just an exercise to check the formula for $\left(\frac{\Delta\Psi}{\Psi}\right)^2$. Why DON'T we calculate $\sigma(ZH)\bullet BR(visible)$ using $\sum_i \sigma(ZH)\bullet BR_i$ if we are confident in our measurement of $\sigma(ZH)\bullet BR(H \to visible\, BSM)$?

In fact this is what Michael Peskin does implicitly in his fits when he uses the constraint $\sum_i BR_i = 1$. If this constraint implicitly includes an hadronic recoil ZH cross section measurement, then the importance of the leptonic recoil ZH cross section measurement should diminish when this constraint is imposed.

I asked Michael to redo his Snowmass 2013 fits with the ILC Higgs White paper recoil cross section errors blown up by various amounts, with and without the constraint $\sum_i BR_i = 1$. Here are the results, where "sigma error" refers to the ILC Higgs White Paper $\Delta\sigma$(ZH):

varying the error in e+e- -> Z h with a bound on BRinv and BRother

error estimates for Higgs couplings and Higgs total width

	W	Z	b	g	gam	tau	c	Gtot
sigma error x 1	4.666	0.7935	4.635	6.009	17.66	5.122	6.296	8.877
sigma error x 2	4.683	0.9348	4.681	6.014	17.67	5.16	6.318	8.893
sigma error x 4	4.69	0.9837	4.699	6.016	17.67	5.174	6.326	8.899
sigma error x 8	4.692	0.9972	4.704	6.016	17.67	5.178	6.328	8.9

varying the error in e+e- -> Z h with no bound on BRother

error estimates for Higgs couplings and Higgs total width

	W	Z	b	g	gam	tau	c	Gtot
sigma error x 1	4.803	1.3	5.244	6.315	17.79	5.66	6.697	11.44
sigma error x 2	5.305	2.6	5.707	6.705	17.94	6.092	7.066	14.56
sigma error x 4	6.958	5.2	7.27	8.077	18.49	7.576	8.379	23.16
sigma error x 8	11.38	10.4	11.57	12.1	20.57	11.77	12.3	42.83

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ILC 500

varying the error in e+e- -> Z h with a bound on BRinv and BRother

error estimates for Higgs couplings and Higgs total width

	W	Z	b	g	gam	tau	c	Gtot
sigma error x 1	0.5337	0.5475	1.013	2.038	8.382	1.951	2.563	2.103
sigma error x 2	0.554	0.6251	1.013	2.04	8.382	1.951	2.563	2.121
sigma error x 4	0.5609	0.6503	1.013	2.041	8.382	1.951	2.563	2.128
sigma error x 8	0.5628	0.6571	1.013	2.042	8.382	1.951	2.563	2.129

varying the error in e+e- -> Z h with no bound on BRother

error estimates for Higgs couplings and Higgs total width

	W	Z	b	g	gam	tau	c	Gtot
sigma error x 1	1.136	0.9824	1.583	2.293	8.461	2.307	2.808	4.87
sigma error x 2	2.046	1.965	2.324	2.855	8.63	2.867	3.284	8.369
sigma error x 4	3.971	3.93	4.121	4.442	9.277	4.45	4.729	15.98
sigma error x 8	7.88	7.859	7.957	8.128	11.51	8.132	8.288	31.57

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ILC 500up

varying the error in e+e- -> Z h with a bound on BRinv and BRother

error estimates for Higgs couplings and Higgs total width

	W	Z	b	g	gam	tau	c	Gtot
sigma error x 1	0.2412	0.2425	0.4714	0.9583	3.96	0.8895	1.196	0.9328
sigma error x 2	0.2485	0.2762	0.4722	0.9591	3.96	0.8902	1.196	0.9365
sigma error x 4	0.251	0.287	0.4725	0.9593	3.96	0.8904	1.196	0.9379
sigma error x 8	0.2517	0.29	0.4726	0.9594	3.96	0.8905	1.196	0.9382

varying the error in e+e- -> Z h with no bound on BRother

error estimates for Higgs couplings and Higgs total width

	W	Z	b	g	gam	tau	c	Gtot
sigma error x 1	0.5137	0.4386	0.7319	1.072	3.995	1.056	1.308	2.21
sigma error x 2	0.9171	0.8772	1.055	1.314	4.067	1.301	1.513	3.757
sigma error x 4	1.775	1.754	1.85	2.008	4.341	2	2.144	7.145
sigma error x 8	3.519	3.509	3.557	3.643	5.299	3.638	3.719	14.1

It is clear from these results that $\sigma(ZH){\bullet}BR(visible)$ is being calculated implictly using $\sigma(ZH){\bullet}BR(visible) = \sum_i \sigma(ZH){\bullet}BR_i$ when the constraint $\sum_i BR_i = 1$ is imposed, and that the importance of the leptonic recoil ZH cross section measurement is diminished in this case.

It appears that we have no need for a separate direct hadronic ZH recoil cross section measurement once we are confident that we have $\sigma(ZH){\bullet}BR(H \rightarrow visible\, BSM)$ under control. In fact, it appears that we don't even need the classic leptonic ZH recoil cross section measurement in this case!

Caveats :

Michaels's results assume that the true $BR(H \to \text{ BSM}) = 0$
and that $BR(H \to \text{ visible BSM}) < 0.9\%$ at 95% CL can be achieved.
This has yet to be demonstrated, but seems plausible given the
results for $BR(H \to \text{ invisible BSM})$.

Also, we have to check how these conclusions are
altered if the true $BR(H \to \text{ BSM}) = 1\%$, or 10%.