ゲージ一重項スカラー場による拡張模型と階層性問題

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Based on arXiv:1608.00087

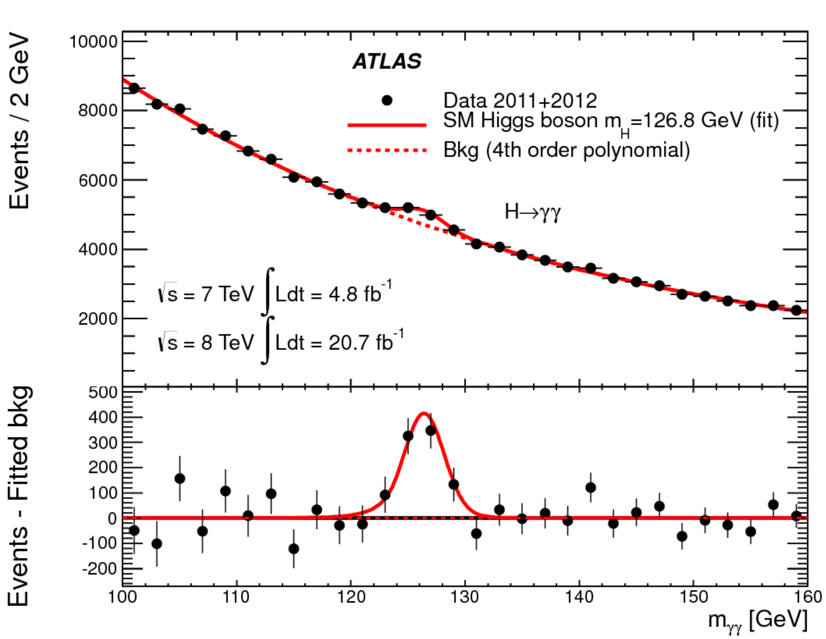
Outline

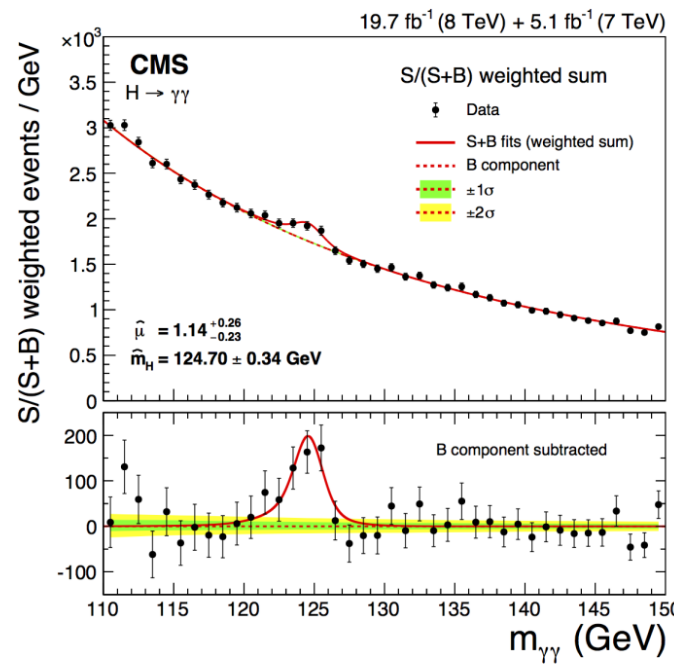
- Introduction
  - Vacuum stability
  - Hierarchy problem
- Our study
  - Singlet scalar extension
- Summary

Introduction
-vacuum stability-

Higgs mass

LHC discovered SM-like Higgs boson in 2012.





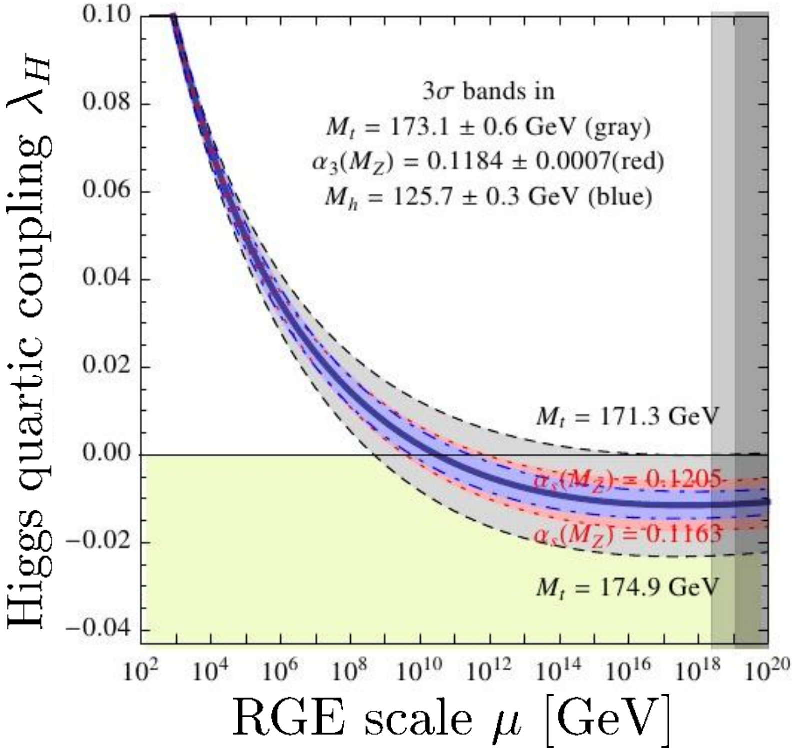
Recent combined analyses give Higgs mass as

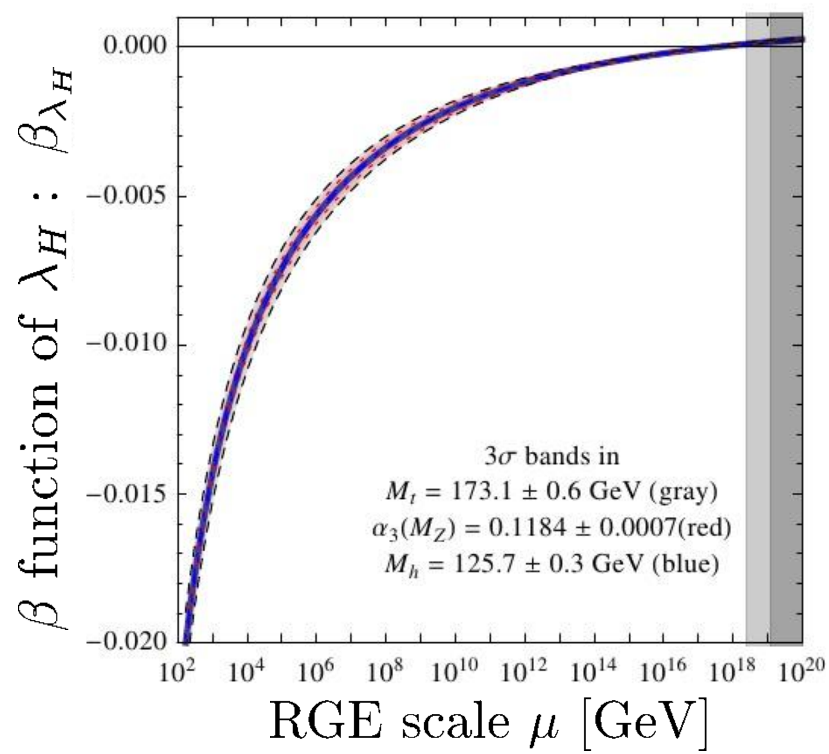
$$M_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

ATLAS and CMS collaborations [arXiv:1503.07589[hep-ex]]

Higgs critical behavior

M_h=125GeV suggests critical behavior in SM





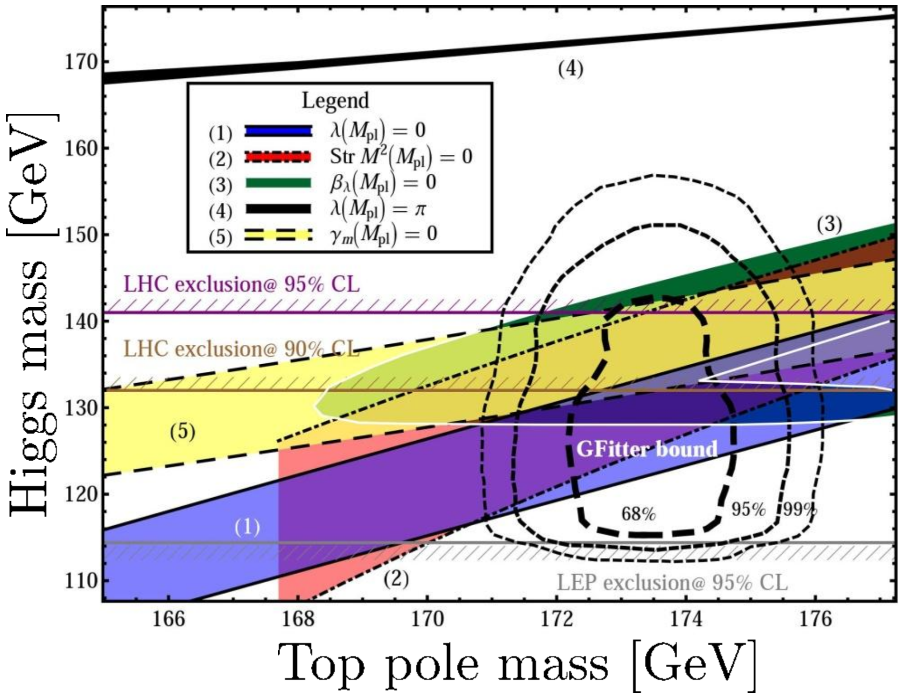
$\lambda_H(M_{\rm Pl}) \simeq 0$ $\qquad$ $\beta_{\lambda_H}(M_{\rm Pl}) \simeq 0$

[Buttazzo, et al., arXiv:1307.3536]

Higgs critical behavior

- Multiple-point principle (MPP): $\lambda_H(M_{\rm Pl}) = \beta_{\lambda_H}(M_{\rm Pl}) = 0$
  - $M_h = 121.8 \pm 11\,\text{GeV}$ for $M_t = 173.1 \pm 4.6\,\text{GeV}$

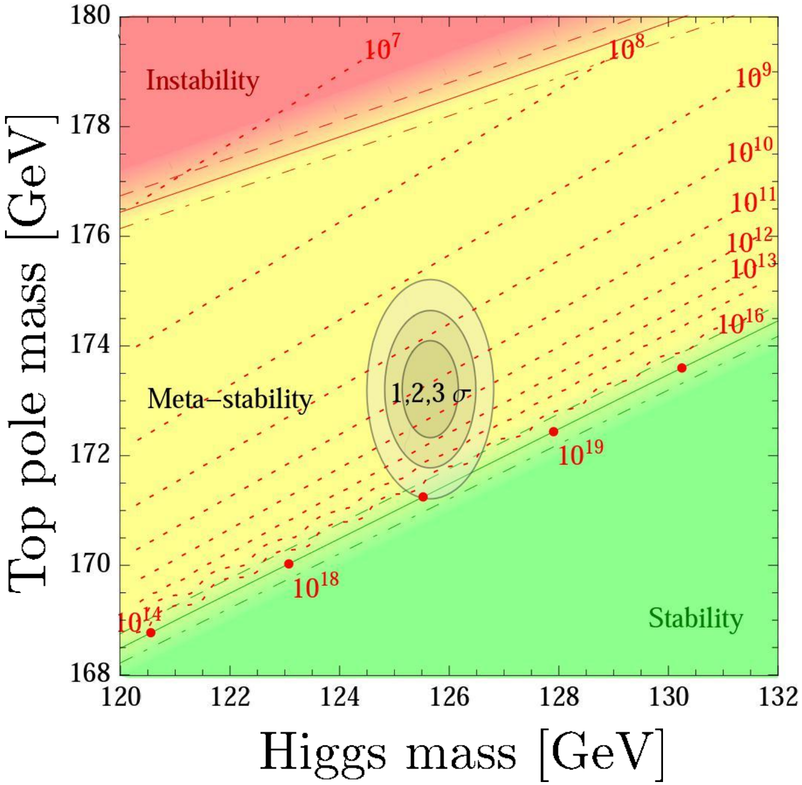
  ['95,' 01 Froggatt, Nielsen]
- Asymptotic safety of gravity
  - $M_h \simeq 126.5\,\text{GeV}$ for $M_t = 171.3\,\text{GeV}$ ['10 Shaposhnikov, Wetterich]
- Planck scale boundary conditions ['12 Holthausen, Lim, Lindner]



Vacuum stability

The EW vacuum is meta-stable in SM.

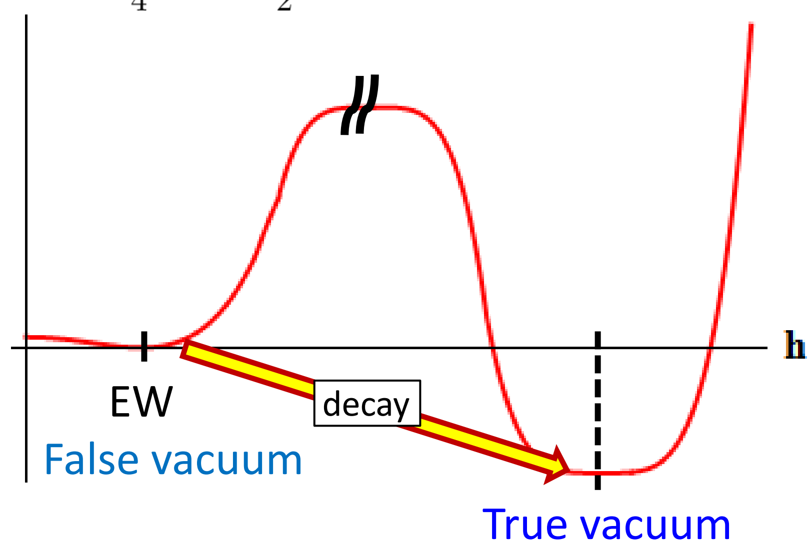
phenomenologically safe



[Buttazzo, et al., arXiv:1307.3536]

Higgs potential

$$V_{\text{eff}}(\mathbf{h}) = \frac{1}{4}\lambda_H h^4 + \frac{1}{2} m_H^2 h^2$$

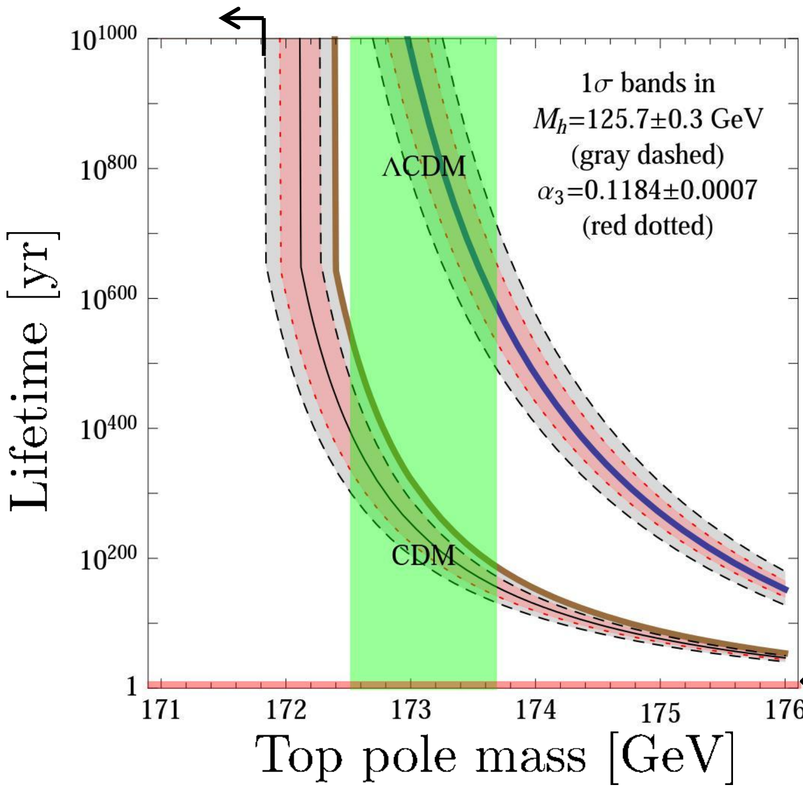


Our Universe may decay!!

Lifetime of EW vacuum

Lifetime is much longer than age of our Universe.

$\lambda_H(\Lambda_B) > 0$ (Vacuum is stable)



Age of our Universe
(T_U = 13.8 Gyr)

[Buttazzo, et al., arXiv:1307.3536]

Lifetime

$$\tau = \begin{cases} \left(\frac{55}{3\pi}\right)^{1/4} \frac{e^{S(\Lambda_B)/4}}{\Lambda_B} \approx \frac{T_U}{p_0^{1/4}} & \text{for CDM} \\ \frac{3H^3 e^{S(\Lambda_B)}}{4\pi\Lambda_B^4} \approx \frac{0.02\,T_U}{p_0} & \text{for } \Lambda\text{CDM} \end{cases}$$

$$\begin{cases} \Lambda_B \text{ satisfys } \beta_{\lambda_H}(\Lambda_B) = 0 \\ S(\Lambda_B) = \frac{8\pi^2}{3|\lambda_H(\Lambda_B)|} \\ p_0 = 0.15\,\frac{\Lambda_B^4}{H_0^4}\,e^{-S(\Lambda_B)} \end{cases}$$

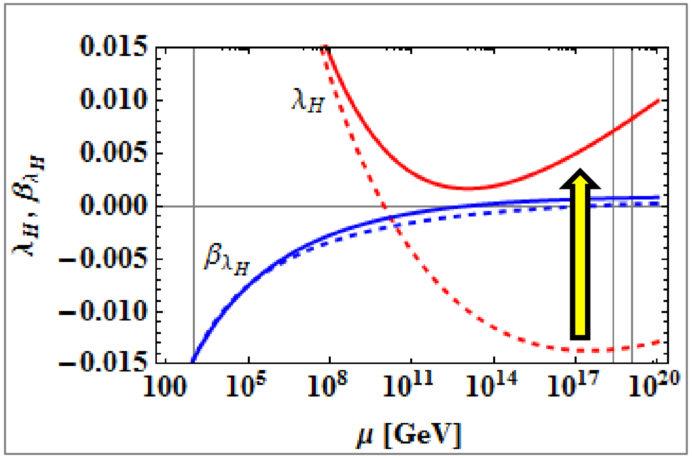
Toward stable vacuum

To stabilize vacuum, λ_H must be positive at any energy scale.

$\rightarrow$ Practically, β_{λ_H} should be larger than SM case.

$$\beta_{\lambda_H} = \frac{1}{(4\pi)^2}\left[\lambda_H\left(24\lambda_H + 12y_t^2 - 3g_Y^2 - 9g_2^2\right) - 6y_t^4 + \frac{3}{8}g_Y^4 + \frac{9}{8}g_2^4 + \frac{3}{4}g_Y^2 g_2^2\right]$$

Ex.)



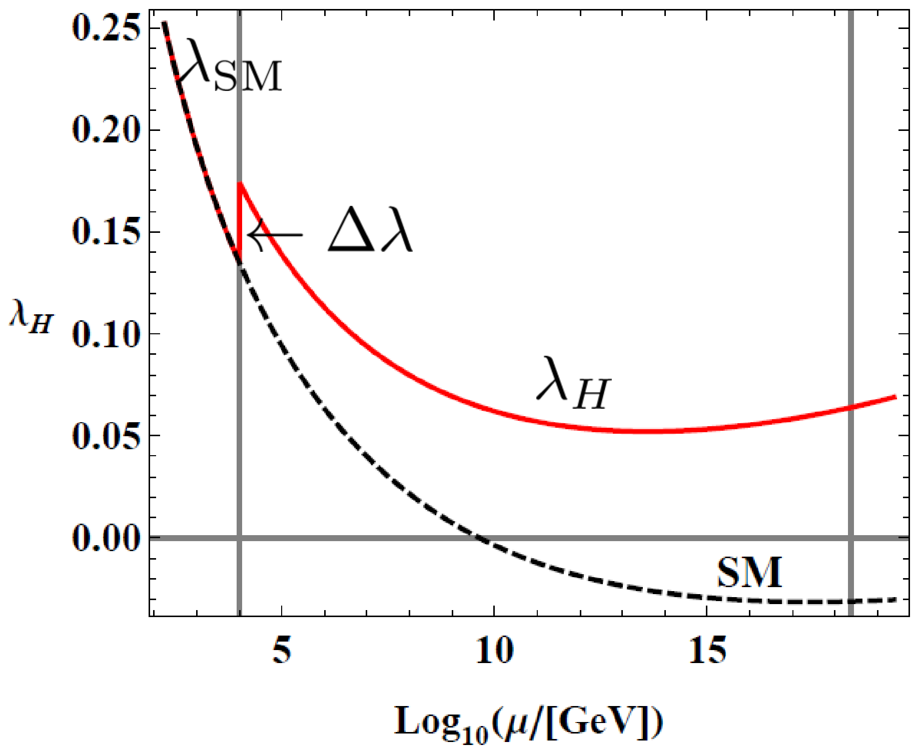
[Simple solutions]

- Singlet scalar extension
  - Scalar mixing contributes $+\lambda_{\rm mix}^2$
- Gauge sector extension
  - Additional gauge contributes $+g'^4$
- Change of running gauge couplings
  - Larger gauge couplings lead smaller y_t

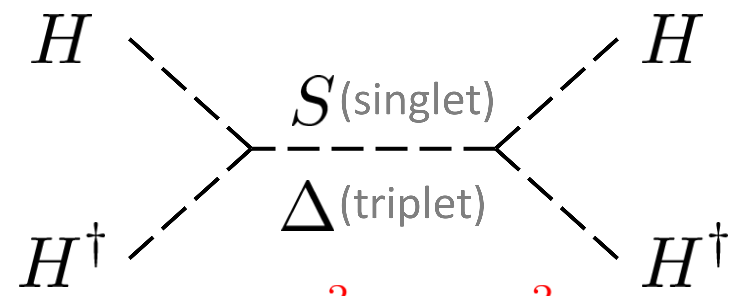
Toward stable vacuum

- Tree-level threshold correction

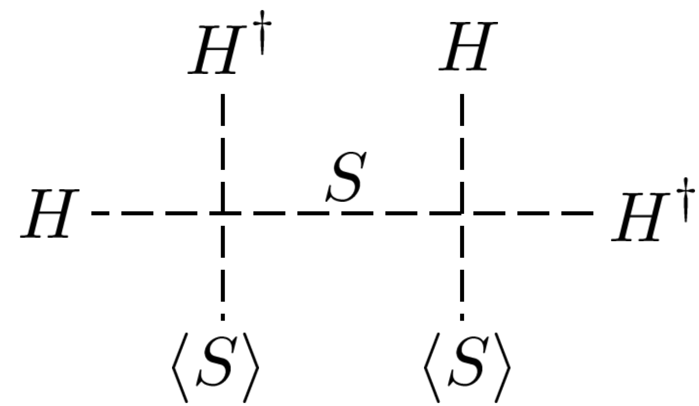
['12 J. Elias-Miro, J. R. Espinosa, G. F. Giudice, H. M. Lee and A. Strumia]



$$\lambda_{\rm SM} = \lambda_H - \Delta\lambda$$



$$\Delta\lambda = \frac{\mu_{HS}^2}{m_S^2} \text{ or } \frac{\mu_{H\Delta}^2}{m_\Delta^2}$$



$$\Delta\lambda = \frac{\lambda_{HS}^2}{\lambda_S}$$

Introduction
-hierarchy problem-

Mass correction

Quantum correction for scalar field is given by

$$m_H^2(\Lambda,\mu) = \bar{m}_H^2(\Lambda) + \sum_{A=S,V,F} (-1)^{2J_A}(2J_A+1)\frac{g_A}{16\pi^2}\left[\Lambda^2 - \bar{m}_A^2(\Lambda)\ln\frac{\Lambda^2}{\mu^2}\right]$$

Λ: cutoff scale, μ: renormalization scale, $\bar{m}_A$: bare mass, $(\Lambda > \mu > m_A)$

J_A: spin, g_A: coupling betweein S and A ($g_F = y_F^2$ for fermion)

['14 A.Kobakhidze, K.L.McDonald]

- There are two types of corrections
  - Quadratic divergence($\propto \Lambda^2$)
  - Logarithmic divergence($\propto \ln \Lambda^2/\mu^2$)

$\Lambda^2 \gg m_H^2$ (may) cause hierarchy problem!!

fine-tuning between $\bar{m}_H^2$ and δm_H^2

Veltman condition

Quadratic div. disappears if couplings satisfy

$$\sum(-1)^{2J_A}(2J_A+1)g_A=0$$

In the SM, it corresponds to $M_h^2+2M_W^2+M_Z^2-4M_t^2=0$

Veltman condition ['81 M.J.G.Veltman]

Veltman condition at the EW scale suggests

$$M_h^2=4\times 173^2-2\times 80^2-91^2\ [\mathrm{GeV}^2]\simeq(314\,\mathrm{GeV})^2$$

It conflicts with 125GeV Higgs

- Veltman condition might be realized at the UV scale with beyond-SM fields
  - Bare Higgs mass vanishes around the Planck scale ['12 Y.Hamada, H.Kawai, K.Oda]
- But, Veltman condition is not motivated by any symmetry $\rightarrow$ It is not different from fine-tuning
  (Even if it is satisfied by some physics, we cannot distinguish it from fine-tuning)

Supersymmetry

Exact supersymmetry $\left(\sum(-1)^{2J_A}(2J_A+1)=0\right)$ requires

$\triangleright$ $\sum(-1)^{2J_A}(2J_A+1)g_A=0$ (cancellation of quadratic div.)

$\triangleright$ $\sum(-1)^{2J_A}(2J_A+1)g_A m_A^2=0$ (cancellation of logarithmic div.)

(These equations are guaranteed by non-renormalization theorem)

- The hierarchy problem can be solved by SUSY, but it should be broken above the EW scale
  - We need soft SUSY-breaking terms, which lead

$$\sum(-1)^{2J_A}(2J_A+1)g_A m_A^2 \sim M_{\rm SUSY}^2$$

Then, Higgs mass is $M_h^2 \sim \dfrac{M_{\rm SUSY}^2}{16\pi^2}$

⇨ $M_{\rm SUSY}$ should be a few TeV

But, such a low scale SUSY particle is excluded…

Hierarchy problem

In fact, quadratic divergence is always subtracted.

['72 G.'t Hooft, M.J.G.Veltman], ['72 C.G.Bollini, J.J.Giambiagi] (Dimensional regularization)
['11 K.Fujikawa] (Subtraction scheme) ['12 H.Aoki, S.Iso] (Wilsonian renormalization)

⇨ We need consider only logarithmic divergence

Log. div. is interpreted by β-function of Higgs mass parameter.

- RGE of the Higgs mass parameter in the SM

$$\frac{dm_H^2}{d\ln\mu} = \frac{1}{16\pi^2} m_H^2 \left[12\lambda_H + 6y_t^4 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right]$$

⇨ $m_H^2(\mu) \sim m_H^2(\Lambda)$ Order of magnitude does not change

- RGE of the Higgs mass parameter in some extended SM

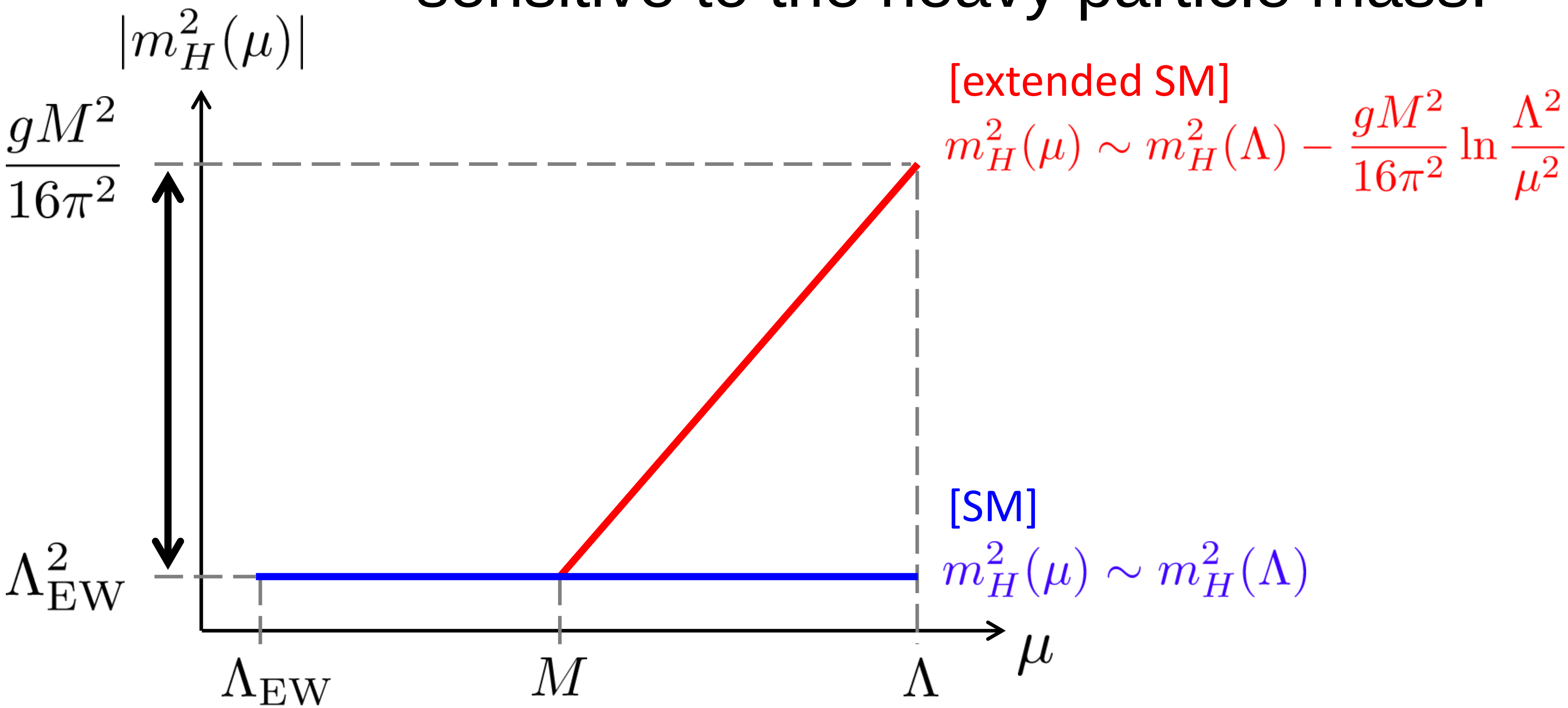
$$\frac{dm_H^2}{d\ln\mu} = \frac{1}{16\pi^2} m_H^2 \left[12\lambda_H + 6y_t^4 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right] + \frac{1}{16\pi^2} gM^2$$

⇨ $m_H^2(\mu) \sim m_H^2(\Lambda) - \frac{gM^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}$ Large contribution arises

$(\text{for } \mu > M)$

Hierarchy problem

Renormalization condition at the UV scale is sensitive to the heavy particle mass.



Hierarchy problem arises only when $M \gg \Lambda_{\rm EW}$.

Our study

Setup

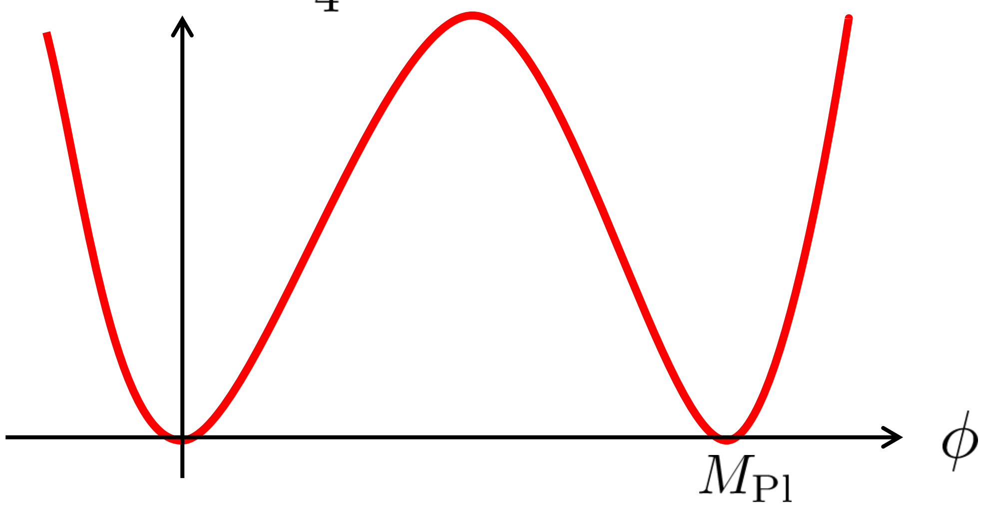
We consider a scalar singlet extension of the SM with the MPP conditions at the Planck scale.

$$\text{All } \lambda(M_{\text{Pl}}) = \beta_\lambda(M_{\text{Pl}}) = 0$$

- The multiple-point principle (MPP) means that there are two degenerate vacua in the scalar potential

['95 C. D. Froggatt and H. B. Nielsen]

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$



Within the SM,
the MPP conditions suggest
M_h = 129GeV for M_t = 173GeV.

inconsistent with M_h = 125GeV

Potential analysis

Potential

$$V(H,S) = \frac{\lambda_H}{2}(H^\dagger H)^2 + m_H^2 H^\dagger H + \frac{\lambda_S}{8}S^4 + \frac{\mu_S}{3}S^3 + \frac{m_S^2}{2}S^2 + \frac{\lambda_{HS}}{2}S^2 H^\dagger H + \mu_{HS} S H^\dagger H \quad (m_S^2 > 0,\ \mu_{HS} > 0)$$

$$H = \left(0,\ (v_H + h)/\sqrt{2}\right)^T,\ S = v_S + s,$$

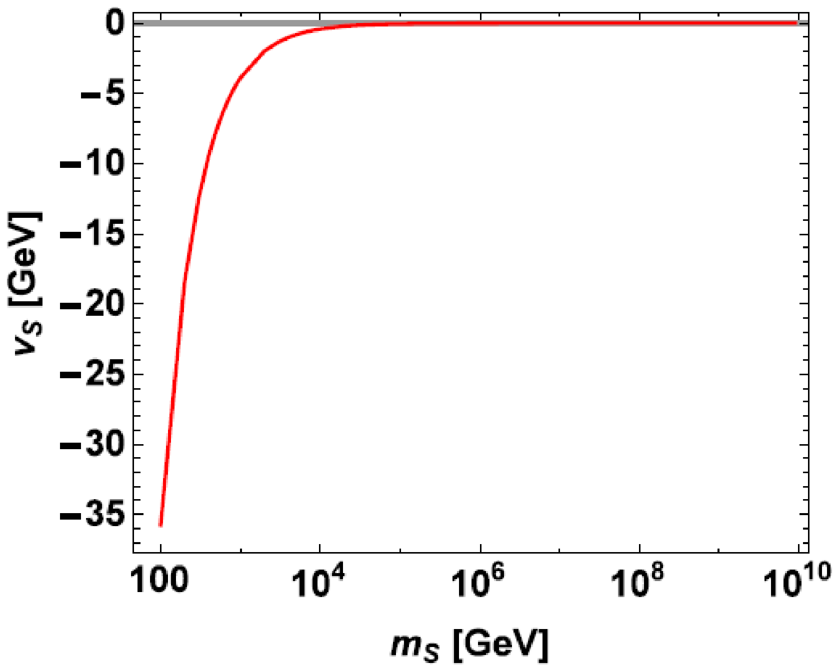
Stationary conditions

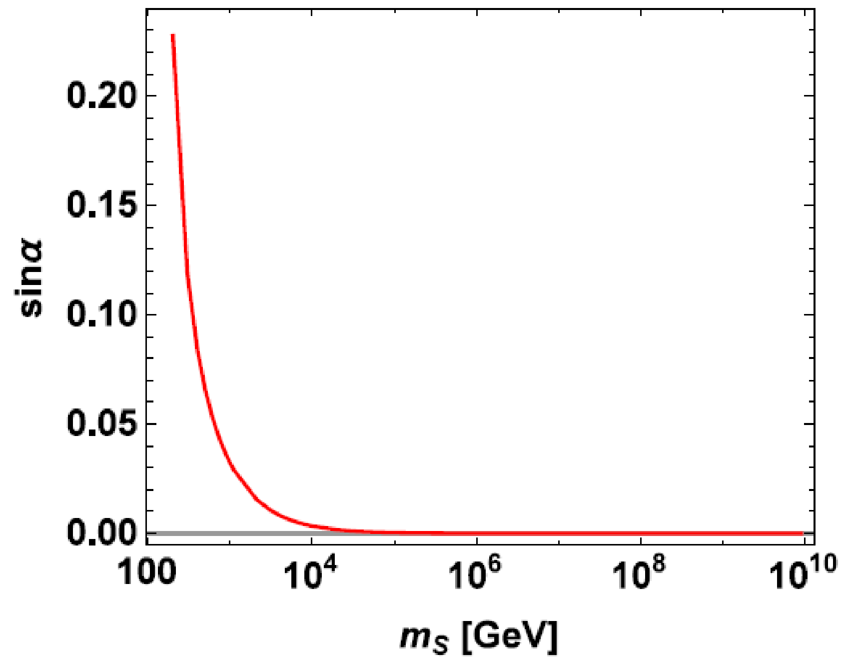
$$\begin{cases} \left.\dfrac{\partial V}{\partial h}\right|_{h\to 0,\, s\to 0} = \dfrac{v_H}{2}\left(\lambda_H v_H^2 + 2m_H^2 + \lambda_{HS} v_S^2 + 2\mu_{HS} v_S\right) = 0 \\ \left.\dfrac{\partial V}{\partial s}\right|_{h\to 0,\, s\to 0} = \dfrac{1}{2}\left[v_S\left(\lambda_S v_S^2 + 2\mu_S v_S + 2m_S^2 + \lambda_{HS} v_H^2\right) + \mu_{HS} v_H^2\right] = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} v_H^2 = -\dfrac{2}{\lambda_H}\left(m_H^2 + \mu_{HS} v_S\right) \\ v_S = -\dfrac{\mu_{HS} v_H^2}{2m_S^2} \end{cases}$$

The MPP conditions induce
$\lambda_S = \lambda_{HS} = 0,\ \mu_S = 0$

Scalar mixing



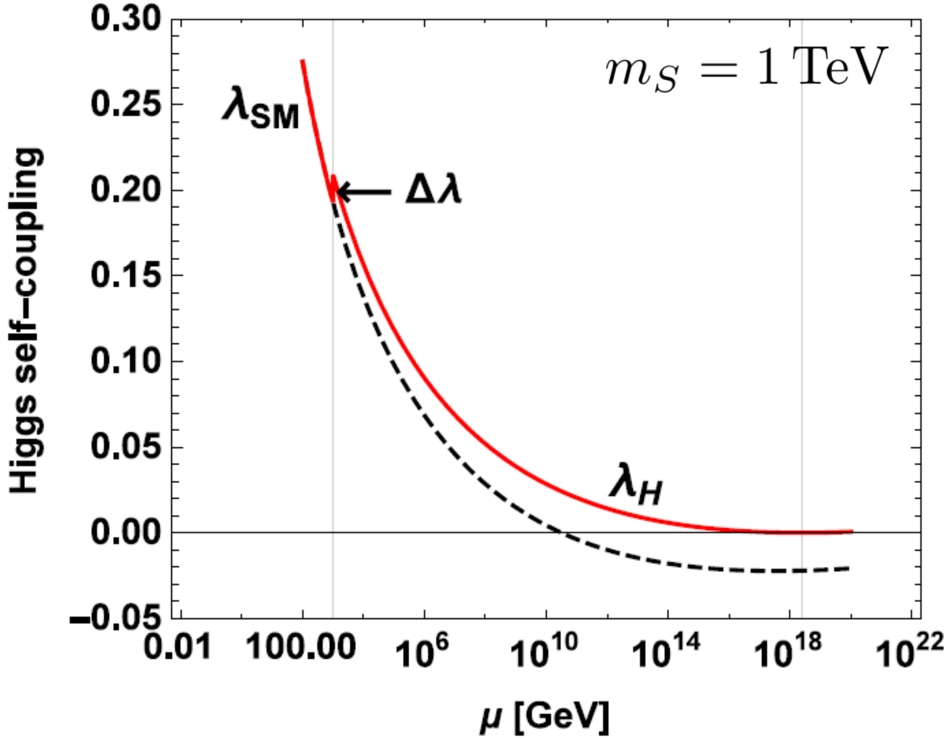


$$\tan 2\alpha = \frac{2\mu_{HS} v_H}{m_S^2 - \lambda_H v_H^2} \quad \rightarrow \quad \sin\alpha \approx \frac{\mu_{HS} v_H}{m_s^2} = 2\frac{-v_S}{v_H} \quad \text{for } m_S^2 \gg |m_H^2|$$

LHC Run 1 result: sinα ≤ 0.36 [ATLAS-CONF-2015-044]

More precise experiments are needed to test our model.

Renormalization group evolution



Conditions

$$\begin{cases} \lambda_{\rm SM} = \lambda_H - \dfrac{\mu_{HS}^2}{m_S^2} \text{ at } \mu = m_S \\ \lambda_H(M_{\rm Pl}) = 0 \\ \beta_{\lambda_H}(M_{\rm Pl}) = 0 \end{cases}$$

The EW vacuum can be stable because of $\lambda \geq 0$.

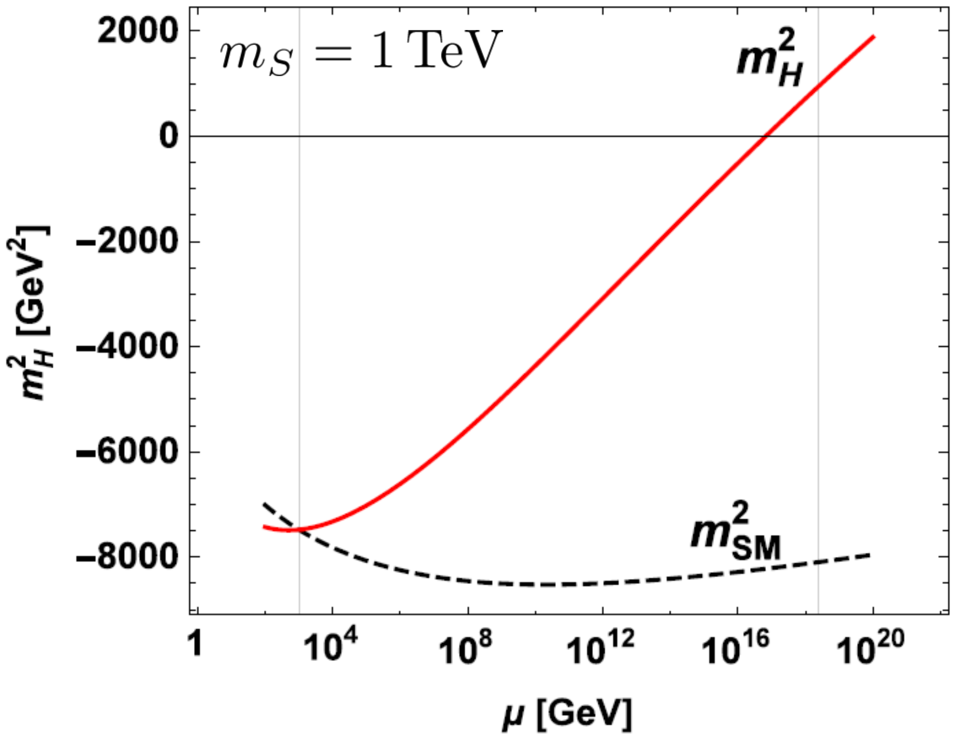
To realize $\beta_{\lambda_H}(M_{\rm Pl}) = 0$, for $\alpha_s(M_Z) = 0.1179$, 0.1185 and 0.1191

$M_t = 172.322\,\mathrm{GeV}$, $172.687\,\mathrm{GeV}$ and $173.052\,\mathrm{GeV}$, respectively.

our model predictions

$M_t = 172.99 \pm 0.91\,\mathrm{GeV}$ (ATLAS), $M_t = 172.44 \pm 0.48\,\mathrm{GeV}$ (CMS)

Renormalization group evolution



Beta function of the Higgs mass

$$\beta_{m_H^2} = \beta_{m_H^2}^{\rm SM} + \frac{1}{16\pi^2}(\lambda_{HS} m_S^2 + 2\mu_{HS}^2)$$

$m_S \gg M_h$ ⇩ $\lambda_{HS} = 0$

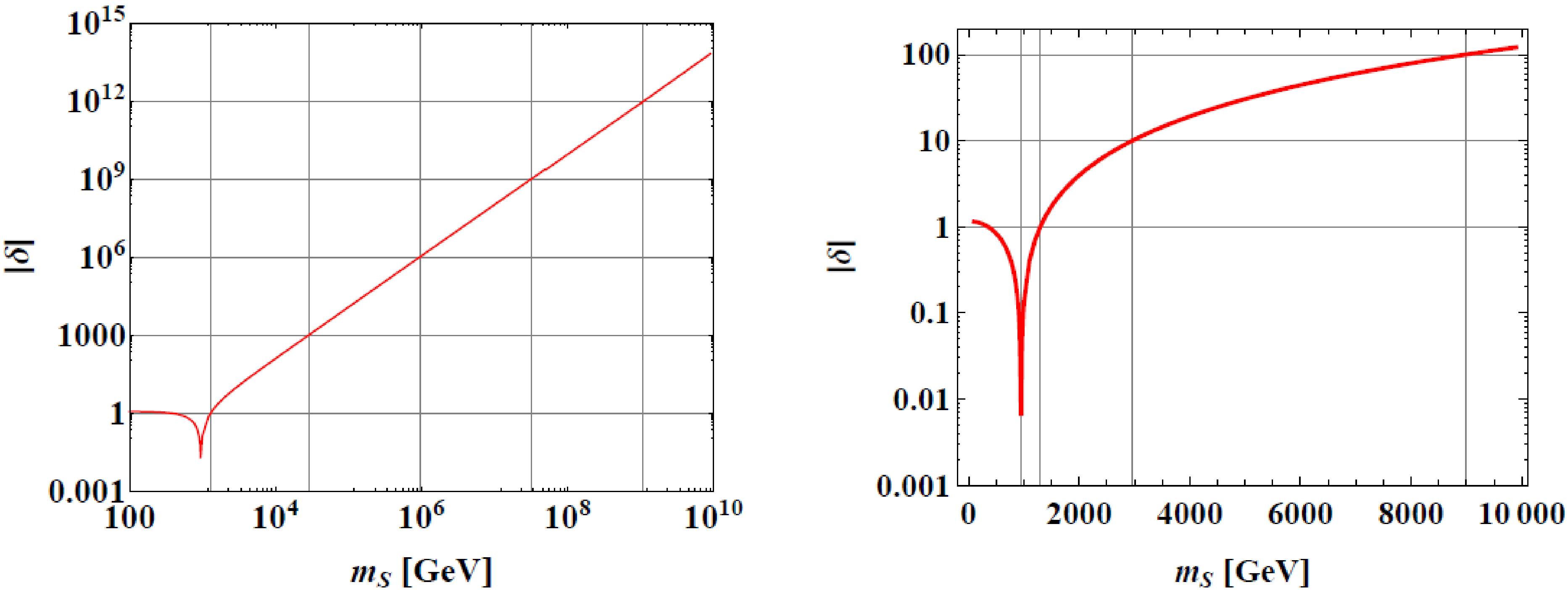
$$m_H^2(\mu) \approx m_H^2(M_{\rm Pl}) - \frac{\mu_{HS}^2}{16\pi^2} \ln\left(\frac{M_{\rm Pl}}{\mu}\right)^2$$

The EW symmetry is radiatively broken for $m_S \gtrsim 950$GeV.

($m_H^2(M_{Pl}) > 0$ for $m_S \gtrsim 950$GeV.)

But, large m_s causes hierarchy problem!!

Higgs mass correction



We define the fine-tuning level as $\delta \equiv \frac{m_H^2(M_{\rm Pl})}{|m_H^2(v_H)|} = \frac{2m_H^2(M_{\rm Pl})}{M_h^2}$

δ = 1, 10 and 100 correspond to $m_S \simeq$ 1.3TeV, 3.0TeV and 9.0TeV.

For the naturalness, the singlet scalar should exist at O(1)TeV.

Summary

- Standard model
  - The EW vacuum is meta-stable
  - The EW symmetry breaking occurs by hand
- The extended SM causes hierarchy problem
  - Solution: only introduce TeV scale particle
- Singlet scalar extension with the MPP conditions
  - The EW vacuum can be stable
  - The EW symmetry can be radiatively broken
  - The Singlet scalar should be O(1) TeV for naturalness
  - In order to test our model, high sensitivities for the Higgs and/or the top quark are needed

Thank you !!

BacKup

Bardeen's argument

From the Bardeen's argument ['95 W.A.Bardeen]

"We have argued that the Standard Model does not, by itself, have a fine tuning problem due to the approximate ***scale invariance*** *of the perturbative expansion."*

In the SM

- Only Higgs mass parameter is dimensionful
- Its β-function (anomalous dimension) is small

The SM satisfies $m_H^2 \simeq 0 \ (|m_H^2| \ll M_{\rm Pl}^2)$ and $\beta_{m_H^2} = \mathcal{O}(1)$, and this approximate scale invariance stabilize the EW scale.

Scale invariance

Derivative of scale (dilatation) current is given by trace of the energy-momentum tensor:

$$\partial^\mu J_\mu = \Theta^\mu{}_\mu = \underline{(\text{soft terms})} + \beta_{\lambda_i} \mathcal{O}_i$$

Anomalous term

$$\Cup$$

$$(m_H^2 + \delta m_H^2) H^\dagger H \quad (\delta m_H^2 \propto m_H^2)$$

Even if $m_H = 0$, scale invariance is broken by anomalous term, but it does not generate quadratic divergence.

(Coleman-Weinberg mechanism, strong dynamics like QCD, Landau pole, etc.)

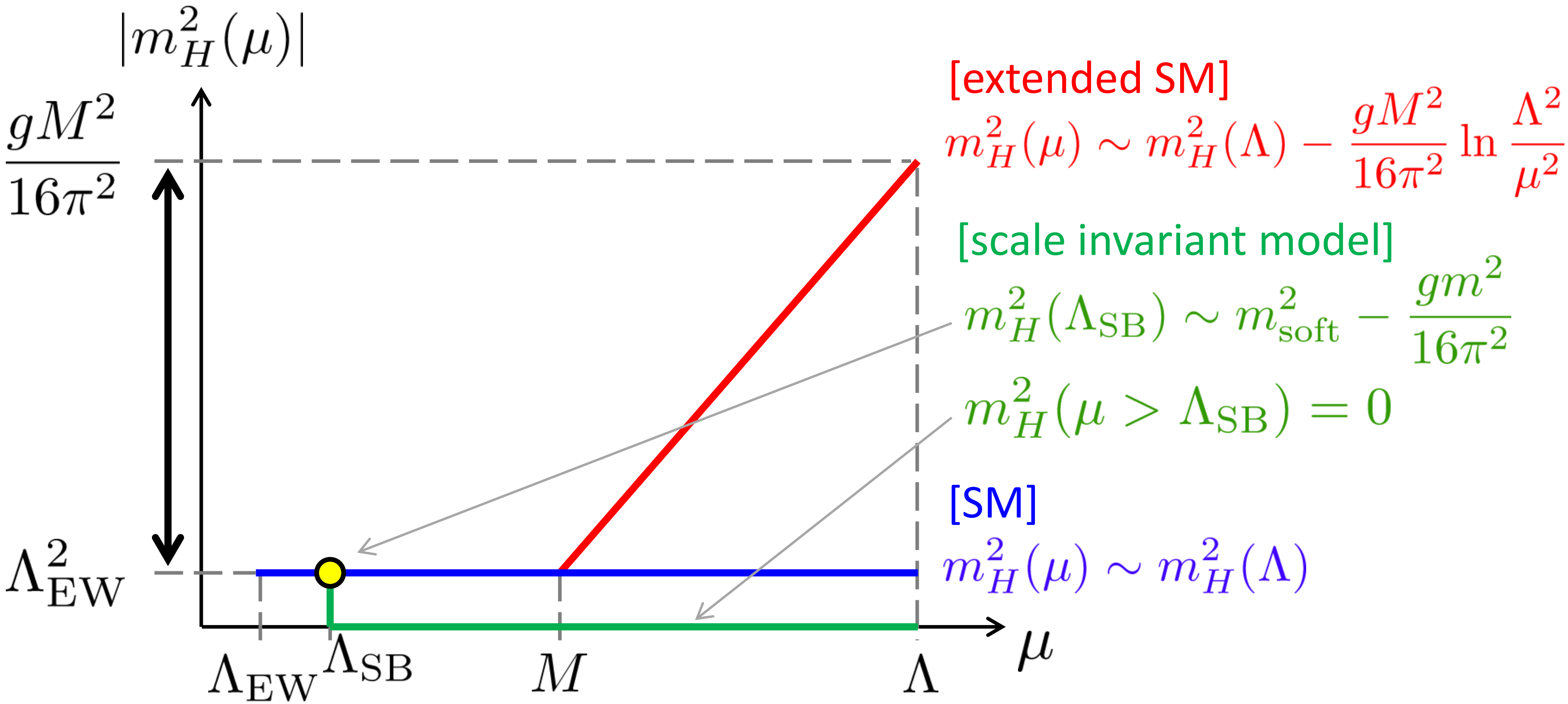
Thus, $m_H \to 0$ is called *"classically"* scale invariance.

In the classically scale invariant model, dimensional transmutation (or generation of nonzero m_H) should occur at energy scale not so far from the EW scale.

Hierarchy problem

Dimensional transmutation leads Higgs mass term.

(denoted by m_{soft}^2)



Hiererchy problem can be solved if $\underbrace{\frac{gm^2}{16\pi^2}}_{\text{radiative correction } (m \propto \Lambda_{\mathrm{SB}})} < m_{\mathrm{soft}}^2 \sim \Lambda_{\mathrm{EW}}^2$

Right-handed neutrino extension

We can introduce right-handed neutrinos without breaking the MPP scenario.

$$-\mathcal{L}_N = Y_\nu^\dagger \overline{L}\tilde{H}N + Y_N S\overline{N}N + \frac{1}{2}M_N\overline{N^c}N + \text{h.c.}$$

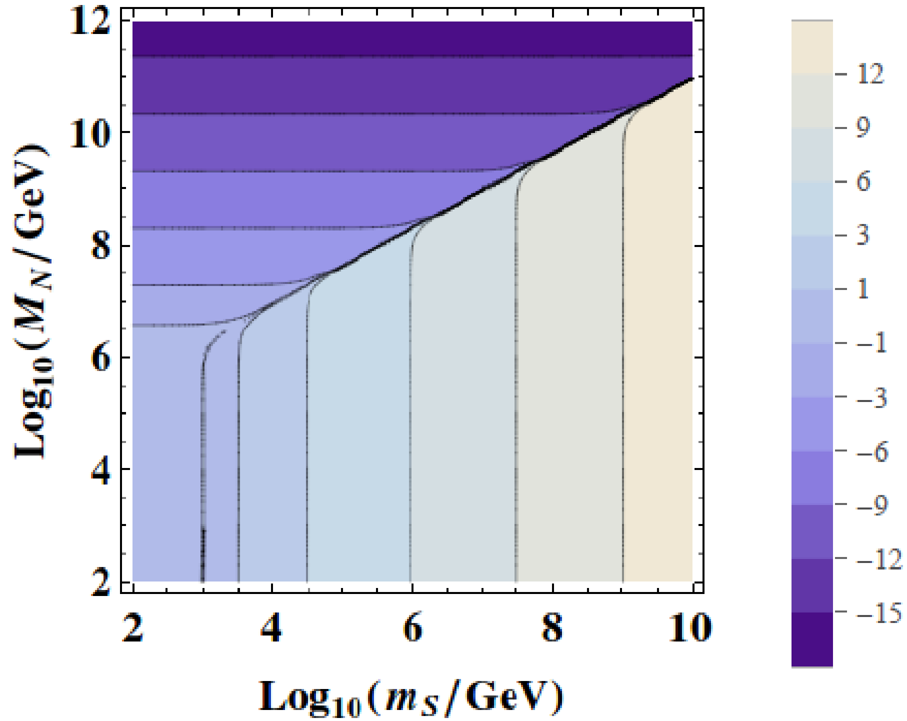
$$(\beta_{\lambda_S}(M_{\text{Pl}}) = 0 \to Y_N = 0)$$

The Higgs mass is given by

$$m_H^2(v_H) \approx m_H^2(M_{\text{Pl}}) - \frac{\mu_{HS}^2}{16\pi^2}\ln\left(\frac{M_{\text{Pl}}}{m_S}\right)^2 + \frac{4\,\text{tr}[Y_\nu^\dagger M_N^2 Y_\nu]}{16\pi^2}\ln\left(\frac{M_{\text{Pl}}}{M_N}\right)^2$$

Because these correction terms have the opposite sign, accidental cancellation of the corrections can occur.

Right-handed neutrino extension



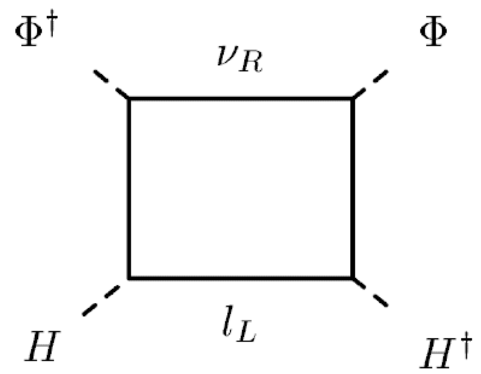
Accidental cancellation: $\mathrm{Log}_{10}\left(\frac{M_N}{\mathrm{GeV}}\right) \approx 4 + \frac{2}{3}\mathrm{Log}_{10}\left(\frac{m_S}{\mathrm{GeV}}\right)$

Even if mS and MN are much larger than the EW scale,
sum of the Higgs mass corrections can vanish.

Higgs mass correction

Δm_H^2 should be lower than Higgs mass: $\Delta m_H^2 \lesssim (125\mathrm{GeV})^2$

- Neutrino (one-loop)



$$\Delta m_H^2 \sim \frac{Y_\nu^2 Y_M^2 v_\Phi^2}{16\pi^2} \sim \frac{m_\nu M_N^3}{16\pi^2 v_H^2}$$

$$m_\nu \sim Y_\nu^2 v_H^2 / M_N \sim 0.1\mathrm{eV}$$

$$\Rightarrow M_N \lesssim 10^7\mathrm{GeV} \Rightarrow v_\Phi \lesssim \left(\frac{0.1}{Y_N}\right) \times 10^8\ \mathrm{GeV}$$

Beta functions

The one-loop beta functions for the SM are given by

$$\beta_{g_Y} = \frac{g_Y^3}{16\pi^2}\frac{41}{6}, \qquad \beta_{g_2} = \frac{g_2^3}{16\pi^2}\left(-\frac{19}{6}\right), \qquad \beta_{g_3} = \frac{g_3^3}{16\pi^2}\left(-7\right),$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + \frac{9}{2}y_t^2\right),$$

$$\beta_{\lambda_{\rm SM}} = \frac{1}{16\pi^2}\left[\lambda_{\rm SM}\left(12\lambda_{\rm SM} - 9g_2^2 - 3g_Y^2 + 12y_t^2\right) + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_Y^2 + \frac{3}{4}g_Y^4 - 12y_t^4\right],$$

$$\beta_{m^2_{\rm SM}} = \frac{m^2_{\rm SM}}{16\pi^2}\left(6\lambda_{\rm SM} - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2\right)$$

Beta functions

For a real singlet scalar extension of the SM, the one-loop beta functions of the gauge and the top Yukawa couplings do not change.

The beta functions of the other couplings are given by

$$\beta_{\lambda_H} = \frac{1}{16\pi^2}\left[\lambda_H\left(12\lambda_H - 9g_2^2 - 3g_Y^2 + 12y_t^2\right) + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_Y^2 + \frac{3}{4}g_Y^4 - 12y_t^4 + \lambda_{HS}^2\right],$$

$$\beta_{\lambda_S} = \frac{1}{16\pi^2}\left(9\lambda_S^2 + 4\lambda_{HS}^2\right),$$

$$\beta_{\lambda_{HS}} = \frac{\lambda_{HS}}{16\pi^2}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS} + 3\lambda_S\right),$$

$$\beta_{\mu_S} = \frac{1}{16\pi^2}\left(9\lambda_S\mu_S + 6\lambda_{HS}\mu_{HS}\right),$$

$$\beta_{\mu_{HS}} = \frac{1}{16\pi^2}\left[\mu_{HS}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS}\right) + \lambda_{HS}\mu_S\right],$$

$$\beta_{m_H^2} = \frac{1}{16\pi^2}\left[m_H^2\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2\right) + \lambda_{HS}m_S^2 + 2\mu_{HS}^2\right],$$

$$\beta_{m_S^2} = \frac{1}{16\pi^2}\left(3\lambda_S m_S^2 + 4\mu_S^2 + 4\lambda_{HS}m_H^2 + 4\mu_{HS}^2\right).$$

Beta functions

In addition to the real singlet scalar, we introduce right-handed neutrinos.

The one-loop beta functions of the gauge couplings do not change.

The beta functions of the other couplings are given by

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + \frac{9}{2}y_t^2 + \mathrm{Tr}(Y_\nu^\dagger Y_\nu)\right),$$

$$\beta_{Y_\nu} = \frac{1}{16\pi^2}\left[Y_\nu\left(-\frac{9}{4}g_2^2 - \frac{3}{4}g_Y^2 + 3y_t^2 + \mathrm{Tr}(Y_\nu^\dagger Y_\nu) + \frac{3}{2}Y_\nu^\dagger Y_\nu\right) + 2Y_N^2 Y_\nu\right],$$

$$\beta_{Y_N} = \frac{1}{16\pi^2}\left[Y_N\left(4\mathrm{Tr}(Y_N^2) + 12Y_N^2 + (Y_\nu Y_\nu^\dagger)^T\right) + Y_\nu Y_\nu^\dagger Y_N\right],$$

$$\beta_{\lambda_H} = \frac{1}{16\pi^2}\left[\lambda_H\left(12\lambda_H - 9g_2^2 - 3g_Y^2 + 12y_t^2 + 4\mathrm{Tr}(Y_\nu^\dagger Y_\nu)\right) + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_Y^2 + \frac{3}{4}g_Y^4 \right.$$
$$\left. - 12y_t^4 + \lambda_{HS}^2 - 4\mathrm{Tr}(Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu)\right],$$

$$\beta_{\lambda_S} = \frac{1}{16\pi^2}\left[\lambda_S\left(9\lambda_S + 16\mathrm{Tr}(Y_N^2)\right) + 4\lambda_{HS}^2 - 128\mathrm{Tr}(Y_N^4)\right],$$

$$\beta_{\lambda_{HS}} = \frac{1}{16\pi^2}\left[\lambda_{HS}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS} + 3\lambda_S + 2\mathrm{Tr}(Y_\nu^\dagger Y_\nu) + 8\mathrm{Tr}(Y_N^2)\right) \right.$$
$$\left. - 32\mathrm{Tr}(Y_N^2 Y_\nu^\dagger Y_\nu)\right],$$

Beta functions

In addition to the real singlet scalar, we introduce right-handed neutrinos.

The one-loop beta functions of the gauge couplings do not change.

The beta functions of the other couplings are given by

$$\beta_{\mu_S} = \frac{1}{16\pi^2}\left[\mu_S\left(9\lambda_S + 12\mathrm{Tr}(Y_N^2)\right) + 6\lambda_{HS}\mu_{HS} - 96\mathrm{Tr}(M_N Y_N^3)\right],$$

$$\beta_{\mu_{HS}} = \frac{1}{16\pi^2}\left[\mu_{HS}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS} + 2\mathrm{Tr}(Y_\nu^\dagger Y_\nu) + 4\mathrm{Tr}(Y_N^2)\right)\right.$$
$$\left. + \lambda_{HS}\mu_S - 16\mathrm{Tr}(M_N Y_N Y_\nu^\dagger Y_\nu)\right],$$

$$\beta_{M_N} = \frac{1}{16\pi^2}\left[M_N(Y_\nu Y_\nu^\dagger)^T + \left(Y_\nu Y_\nu^\dagger\right)M_N + 4\mathrm{Tr}(M_N Y_N)Y_N + 12 M_N Y_N^2\right],$$

$$\beta_{m_H^2} = \frac{1}{16\pi^2}\left[m_H^2\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 2\mathrm{Tr}(Y_\nu^\dagger Y_\nu)\right) + \lambda_{HS}m_S^2 + 2\mu_{HS}^2\right.$$
$$\left. - 4\mathrm{Tr}(Y_\nu^\dagger M_N^2 Y_\nu)\right],$$

$$\beta_{m_S^2} = \frac{1}{16\pi^2}\left[m_S^2\left(3\lambda_S + 8\mathrm{Tr}(Y_N^2)\right) + 4\mu_S^2 + 4\lambda_{HS}m_H^2 + 4\mu_{HS}^2 - 48\mathrm{Tr}(M_N^2 Y_N^2)\right],$$

Boundary conditions

To solve the RGEs,

[D.Buttazzo, et al., arXiv:1307.3536]

We take the following boundary conditions:

$$g_Y(M_t) = 0.35761 + 0.00011\left(\frac{M_t}{\mathrm{GeV}} - 173.10\right),$$

$$g_2(M_t) = 0.64822 + 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.10\right),$$

$$g_3(M_t) = 1.1666 - 0.00046\left(\frac{M_t}{\mathrm{GeV}} - 173.10\right) + 0.00314\left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right),$$

$$y_t(M_t) = 0.93558 + 0.00550\left(\frac{M_t}{\mathrm{GeV}} - 173.10\right) - 0.00042\left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right),$$

$$\alpha_3(M_Z) = 0.1184 \pm 0.0007.$$