Polarization Measurement from Collision Data

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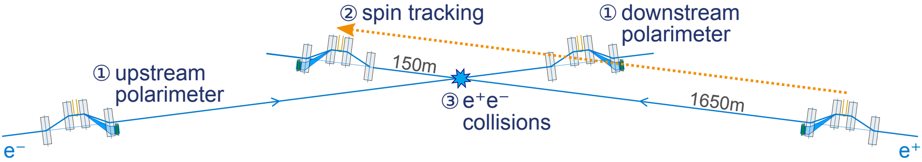
30.11.2016

Determination of the Polarization from Collision Data



Permille-level precision of the luminosity-weighted average polarization at the IP

- Previous Work:
  - Using the information from W-pair production (**Ivan Marchesini**)
  - Using the information from single W, γ, Z events (**Graham W. Wilson**)
- Current Work:
  - Combining all relevant processes, including all uncertainties and their correlations
  - Compensating for a non-perfect helicity reversal
  - Including constraints from the polarimeter measurement

The New Unified Approach

χ^2-Method:

- Combining all suitable processes

$$\chi^2 = \sum_{\text{process}} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)^T \Xi^{-1} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)$$

- Considering all statistical and systematical uncertainties

$$\vec{\sigma} := \begin{pmatrix}\sigma_{-+} & \sigma_{+-} & \sigma_{--} & \sigma_{++}\end{pmatrix}^T$$

$$\Xi := \Xi_N + \Xi_B + \Xi_\varepsilon + \Xi_{\mathcal{L}};$$

- Using a full covariance Matrix (Ξ)

$$\left(\Xi_\varepsilon\right)_{ij} = \text{corr}\left(\vec{\sigma}_i^\varepsilon,\ \vec{\sigma}_j^\varepsilon\right) \frac{\partial \vec{\sigma}_i}{\partial \varepsilon_i} \frac{\partial \vec{\sigma}_j}{\partial \varepsilon_j} \Delta\varepsilon_i \Delta\varepsilon_j$$

Compensation non-perfect helicity reversal: Using 4 independent parameters

$$\underbrace{P_{e^-}^- = -80\%,}_{\text{"left"-handed } e^-\text{-beam}} \quad \underbrace{P_{e^-}^+ = 80\%,}_{\text{"right"-handed } e^-\text{-beam}} \quad \underbrace{P_{e^+}^- = -30\%,}_{\text{"left"-handed } e^+\text{-beam}} \quad \underbrace{P_{e^+}^+ = 30\%,}_{\text{"right"-handed } e^+\text{-beam}}$$

Alternative parametrization: Average Polarization and deviation

$$P_{e^\pm}^- = -\left|P_{e^\pm}\right| + \tfrac{1}{2}\delta_{e^\pm}$$
$$P_{e^\pm}^+ = \left|P_{e^\pm}\right| + \tfrac{1}{2}\delta_{e^\pm}$$

Achievable Limit of the Statistical Precision

Currently implemented processes:

Process	Channel
single $W^{\pm}$	$e\nu l\nu,\ e\nu q\bar{q}$
WW	$q\bar{q}q\bar{q},\ q\bar{q}l\nu,\ l\nu l\nu$
ZZ	$q\bar{q}q\bar{q},\ q\bar{q}ll,\ llll$
$ZZWW$Mix	$q\bar{q}q\bar{q},\ l\nu l\nu$
Z	$q\bar{q},\ ll$

Statistical precision H-20:

[%]	500	350	250	500	250
$P^-_{e^-}$	0.2	0.3	0.1	0.08	0.09
$P^+_{e^-}$	0.05	0.06	0.03	0.02	0.02
$P^-_{e^+}$	0.1	0.1	0.06	0.04	0.04
$P^+_{e^+}$	0.2	0.3	0.1	0.08	0.08

- **Consider best case using σ_{tot}:**
  - Assumption of a perfect 4π detector
  - No background
  - No systematic uncertainties
- **Comparison to previous analyses:**
  - Both previous anlyses
    - Using fiducial cuts
    - Statistical uncertainties only
  - Single boson: $\mathcal{L} = 2\text{ab}^{-1}$
    No background estimation

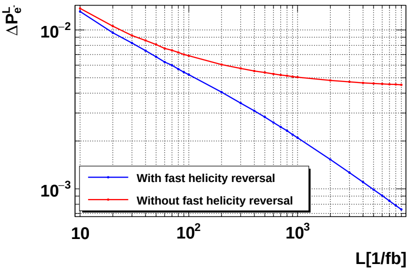
    $P_{e^-}: 0.085\%\quad \delta_{e^-}: 0.12\%$

    $P_{e^+}: 0.22\%\quad \delta_{e^+}: 0.32\%$
  - W-pairs (angular fit): $\mathcal{L} = 500\text{fb}^{-1}$
    Full background estimation

    $P_{e^-}: 0.08\%\quad P_{e^+}: 0.34\%$

Systematic Uncertainties and their Correlations

Systematic quantity		related to:
Integrated luminosity	$\mathcal{L}$	accelerator
Selection efficiency	ε	detector
Background estimate	B	theory

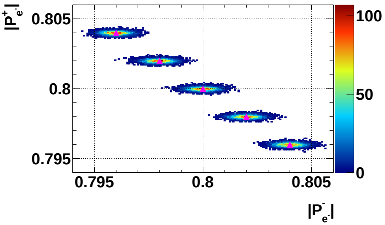


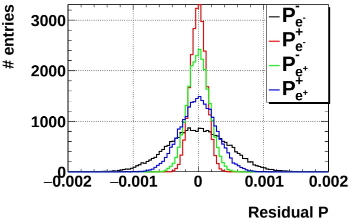
Remark:
A non-perfect helicity reversal has close to no influence on the precision due to compensation of the unified approach

- **Uncertainties influenced by**
  - Detector calibration and alignment
  - Machine performance
  - B assumed constant and small
  - ⇒ $\Delta\mathcal{L}$, $\Delta\varepsilon$ are time dependent
- **Correlations:**
  - Data sets taken concurrently
  - Generate correlations
  - ⇒ Lead to cancellation of systematic uncertainties

⇒ **Fast helicity reversal**
  - Fast switch between $\sigma_{\pm\pm}$ measurements e.g. train-by-train
  - ⇒ Faster than changes in calibrations, alignments, etc.

Testing for a Non-Perfect Helicity Reversal



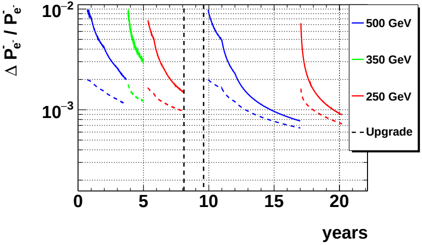


- **Variation in the absolute polarization**
  - Toy Measurement for 5 different polarization discrepancies for both beams
  - Nominal initial polarizations: $|P_{e-}| = 80\%$, $|P_{e+}| = 30\%$
  - Statistical uncertainties only
- χ^2**-Fit:**
  - Correct determination of the 4 polarization values
  - No noticeable changes in the uncertainties

⇒ **Non-Perfect Helicity Reversal:**

⇒ No noticeable impact on polarization precision using total cross sections

⇒ In addition: consider polarimeter information



[%]	500	350	250	500	250
Without Constraint					
$P_{e^-}^-$	0.2	0.3	0.1	0.08	0.09
With Constraint					
$P_{e^-}^-$	0.1	0.1	0.1	0.07	0.07

Simplified approach: (as a first step)

- Neglect spin transport
- Using $\Delta P/P = 0.25\%$:
- Gaussian distribution
  - Mean: $|P_{e-}| = 80\%, |P_{e+}| = 30\%$
  - Width: ΔP

Implementation:

$$\chi^2 += \sum_P \left[\frac{\left(P^{\pm}_{e\pm} - \mathcal{P}^{\pm}_{e\pm}\right)^2}{\Delta\mathcal{P}^2} \right]$$

- $P^{\pm}_{e\pm}$: 4 fitted parameters
- $\mathcal{P}^{\pm}_{e\pm}$: Polarimeter measurement
- $\Delta\mathcal{P}$: Polarimeter uncertainty

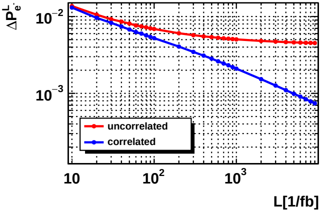
Conclusion

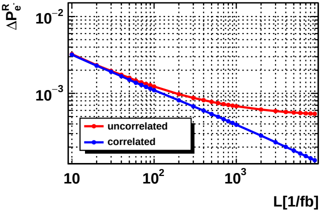
- **New unified approach combing all suitable cross sections and the polarimeter measurement**
  - ⇒ Statistical precision of a permille-level is achievable
  - ⇒ Impact of time-dependent systematic uncertainties can be reduced due to a fast helicity reversal
  - ⇒ Non-perfect helicity reversal has no impact on the statistical precision

Outlook

- **Considering systematic quantities for each process**
  - → Determination of the selection efficiency ε and background estimation B
  - → Determination of the systematic uncertainties: ΔB, $\Delta\varepsilon$, $\Delta\mathcal{L}$
  - → Determination of realistic correlation factors
- **Implementing differential cross sections within the χ^2-method**
  - → Using the angular information of a process to further improve the precision
  - → Consider a well separation from possible BSM effects

Backup Slides


$\Delta P^L_{e^-}$
10^{-2}
10^{-3}
uncorrelated
correlated
10
10^2
10^3
L[1/fb]


$\Delta P^R_{e^-}$
10^{-2}
10^{-3}
uncorrelated
correlated
10
10^2
10^3
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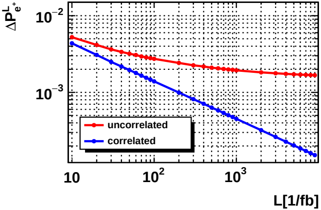
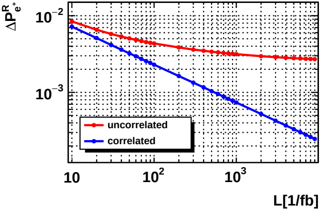
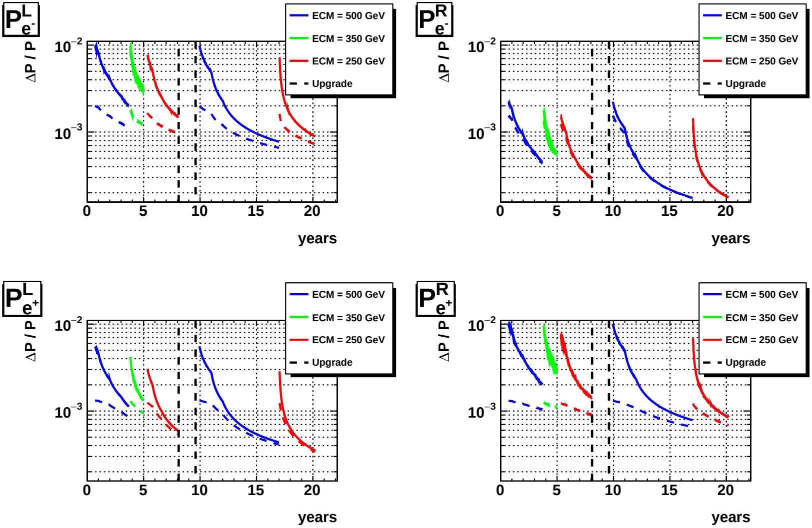

$\Delta P^L_{e^+}$
10^{-2}
10^{-3}
uncorrelated
correlated
10
10^2
10^3
L[1/fb]

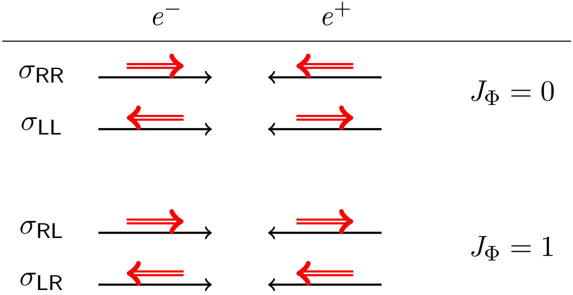

$\Delta P^R_{e^+}$
10^{-2}
10^{-3}
uncorrelated
correlated
10
10^2
10^3
L[1/fb]



$P^L_{e^-}$
$P^R_{e^-}$
$P^L_{e^+}$
$P^R_{e^+}$
ΔP / P
years
ECM = 500 GeV
ECM = 350 GeV
ECM = 250 GeV
Upgrade

Polarization at a $e^- e^+$ Collider

- Helicity is the projection of the spin vector on the direction of motion
- In case of massless particles, helicity is equal to chirality
- If $E_{\text{kin}} \gg E_0 \quad \longrightarrow \quad m_e \approx 0$



- For a bunch of particles the polarization is defined as:

$$P := \frac{N_R - N_L}{N_R + N_L}$$

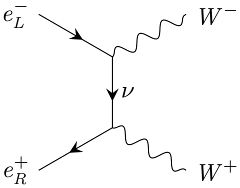
Collision Data

Improvement by Constraints from Polarimeter Measurement

Concept

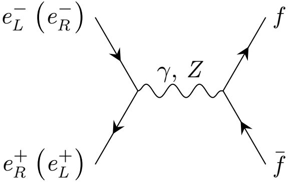
Example Processes:

W-pair production:



$\sigma_{\mathsf{LL}} = \sigma_{\mathsf{RR}} = \sigma_{\mathsf{RL}} = 0$

s-channel spin-1 particle:



$\sigma_{\mathsf{LL}} = \sigma_{\mathsf{RR}} = 0$

- Calculation of the P from polarized σ measurement of well known SM-process
  - $\rightarrow$ Using the information of their chiral structure
- Requirement to consider a process:
  - Theoretical very well known
    - $\rightarrow$ Reduction of theoretical uncertainties
  - High absolute cross section (high rate)
    - $\rightarrow$ Minimizing the statistical error
  - Large left-right-asymmetry
    - $\rightarrow$ Minimizing the influence of systematic uncertainties
  - Well separable from possible BSM-effects
- Feature of the ILC:
  Using 4 different polarization configuration
  ($\rightarrow$ signs of the polarizations)

$\Rightarrow$ Task: Providing the absolute scale calibration

Special Case: The Modified Blondel Scheme (MBS)

- Constraints for the Modified Blondel Scheme:
  - Process must fulfill: $\sigma_{\text{LL}} \equiv \sigma_{\text{RR}} \equiv 0$
  - Perfect helicity reversal: $+|P| \longleftrightarrow -|P| \quad \Rightarrow \quad |P| \equiv$ const.
- Unique solution:

  4 possible cross section measurements: $\sigma_{-+},\ \sigma_{+-},\ \sigma_{--},\ \sigma_{++}$

  Maximal 4 unknown quantities: $\sigma_{\text{LR}},\ \sigma_{\text{RL}},\ |P_{e^-}|,\ |P_{e^+}|$
- Solve for $|P_{e^{\mp}}|$:

$$\sigma_{\pm\pm} = \frac{(1\pm|P_{e^-}|)}{2}\frac{(1\mp|P_{e^+}|)}{2}\cdot\sigma_{RL} + \frac{(1\mp|P_{e^-}|)}{2}\frac{(1\pm|P_{e^+}|)}{2}\cdot\sigma_{LR}$$

- Modified Blondel-Scheme:

$$|P_{e^{\mp}}| = \sqrt{\frac{(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++})(\pm\sigma_{-+}\mp\sigma_{+-}+\sigma_{--}-\sigma_{++})}{(\sigma_{-+}+\sigma_{+-}+\sigma_{--}+\sigma_{++})(\pm\sigma_{-+}\mp\sigma_{+-}-\sigma_{--}+\sigma_{++})}}$$

- Uncertainties are calculated via analytic error propagation

The Unified Approach: χ^2-Method

- Desire for a more general approach:
  - Consider any process with a polarization dependence + using several processes at once
  - Compensate non-perfect helicity reversal: $+\left|P^R\right| \longleftrightarrow -\left|P^L\right|$
- Consider a χ^2-Method: Using all 4 chiral cross sections

$$\chi^2 = \sum_{\text{process}} \left\{ \sum_{\pm\pm} \left[\frac{\left(\sigma^{\text{data}} - \sigma^{\text{theory}}\right)^2}{\Delta\sigma^2} \right] \right\}$$

- Compensate non-perfect helicity reversal: 4 free parameters

$$\underbrace{P_L^- = -80\%,}_{\text{left-handed } e^-\text{-beam}} \quad \underbrace{P_R^- = 80\%,}_{\text{right-handed } e^-\text{-beam}} \quad \underbrace{P_L^+ = -30\%,}_{\text{left-handed } e^+\text{-beam}} \quad \underbrace{P_R^+ = 30\%,}_{\text{right-handed } e^+\text{-beam}}$$

- Error determination via toy experiments

Polarized Cross Section

- Theoretical polarized cross section:

$$\sigma\left(P_{e^-},P_{e^+}\right)=\frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\mathsf{LL}}+\frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\mathsf{RR}}$$
$$+\frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\mathsf{LR}}+\frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\mathsf{RL}}$$

- Measured polarized cross section:

$$\sigma\left(P_{e^-},P_{e^+}\right)=\frac{N}{\varepsilon\cdot\mathcal{L}}=\frac{D-\langle B\rangle}{\varepsilon\cdot\mathcal{L}};$$

Statistic quantity: selected data D, number of events N

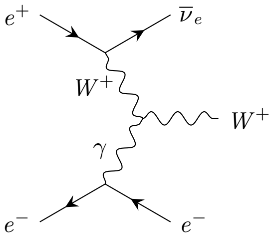
Systematic quantity: background B, selection efficiency ε, integrated luminosity $\mathcal{L}$

- Cross section of the 4 polarization configurations

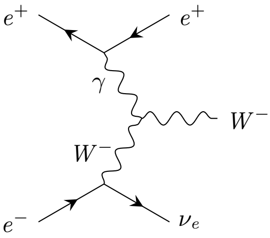
$$\sigma_{--}:=\sigma\left(-|P_{e^-}|,-|P_{e^+}|\right)\qquad\sigma_{++}:=\sigma\left(+|P_{e^-}|,+|P_{e^+}|\right)$$
$$\sigma_{-+}:=\sigma\left(-|P_{e^-}|,+|P_{e^+}|\right)\qquad\sigma_{+-}:=\sigma\left(+|P_{e^-}|,-|P_{e^+}|\right)$$

Previous Single $W^{\pm}$, Z, γ Study: Leading Diagrams

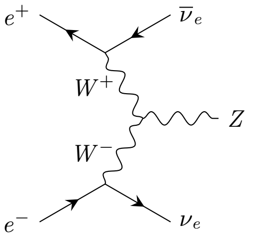
Single W^+



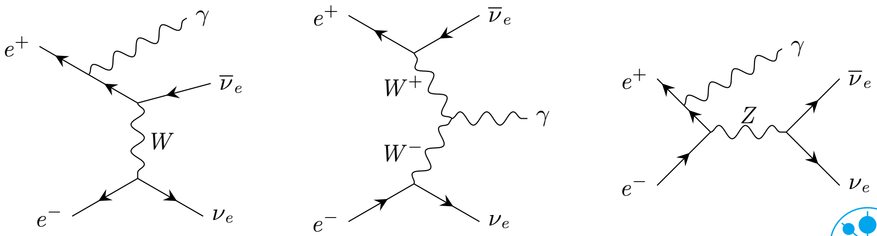
Single W^-



Single Z



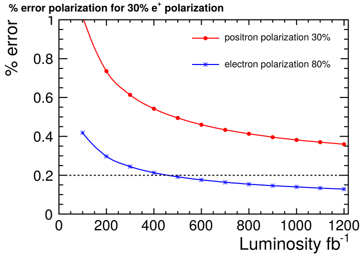
Single γ



Comparison to the Previous W-Pair Study

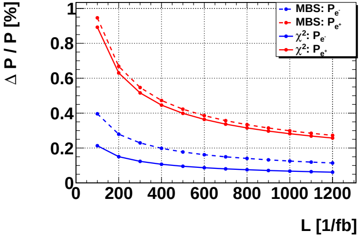
Study by Ivan Marchesini:

- Using $e^- e^+ \rightarrow W^+ W^- \rightarrow q\bar{q}l\nu$
- Statistical uncertainties only
- Consider equal absolute polarizations (MBS)
- Including full background study



Adjustment of the current study:

- Limited to $e^- e^+ \rightarrow W^+ W^- \rightarrow q\bar{q}l\nu$
- Forced equal absolute polarizations $\left(\left|P^L\right| \equiv \left|P^R\right|\right)$
- Including same background estimation and selection efficiency



Comparison:

⇒ χ^2-method yields better precision under same conditions than the MBS

Comparison to Previous Single $W^{\pm}$, γ, Z Study

Study by Graham W. Wilson

- Using 4 Processes simultaneously:

$$e^- e^+ \to \nu\bar{\nu}\gamma; \qquad e^- e^+ \to \nu\bar{\nu}Z$$
$$e^- e^+ \to e^+ \nu W^- \to e^+ \nu \mu^- \bar{\nu}$$
$$e^- e^+ \to e^- \bar{\nu} W^+ \to e^- \bar{\nu} \mu^+ \nu$$

- Consider equal absolute polarizations
  2 Parameters: P_{e^-}, P_{e^+}

- Consider deviations: 4 Parameters

$$P^L_{e^\pm} = -|P_{e^\pm}| + \tfrac{1}{2}\delta_\pm$$
$$P^R_{e^\pm} = \quad |P_{e^\pm}| + \tfrac{1}{2}\delta_\pm$$

parameters		$\Delta P/P,\ \mathcal{L} = 2\text{ab}^{-1}$	
#	P	**Previous**	**Current**
2	P_{e^-}	0.07%	0.051%
	P_{e^+}	0.22%	0.21%
4	P_{e^-}	0.085%	0.088%
	δ_{e^-}	0.12%	0.19%
	P_{e^+}	0.22%	0.23%
	δ_{e^+}	0.32%	0.56%

$\mathcal{L}$ equally distributed between $\sigma_{\pm\pm}$

Statistical precision only

Comparison to Current analysis

- Differences:

Previous: Constraint on δ: $\Delta\delta < 10^{-3}$

Current: direct fit of $P^{L,R}_{e^\pm}$

- Very similar precision even without additional constraint on δ

Combining W-Pair + Single W, Z, γ

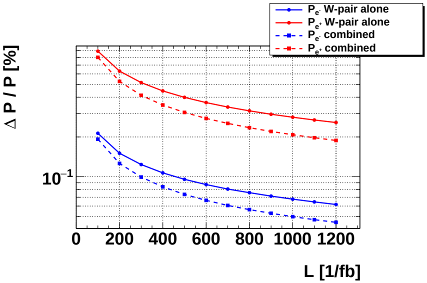
Combined vs. W-Pairs alone

- W-Pair yields only enough information for 2 parameter fit P_{e-}, P_{e+}
- Large improvement
  $\rightarrow$ due to additional processes
- Combined: fit of 4 parameters is possible P^L_{e-}, P^R_{e-}, P^L_{e+}, P^R_{e+}

$\Rightarrow$ Compensation for a non-perfect helicity reversal

Combined vs. Single Boson

- The 4 processes Single $W^{\pm}$, Single Z, Single γ yields a large analysis power
- Combined precision dominated by single boson processes



$\Delta P/P$, $\mathcal{L} = 2\text{ab}^{-1}$

	single W, Z, γ	Combined
P_{e-}	0.088%	0.079%
δ_{e-}	0.19%	0.18%
P_{e+}	0.23%	0.16%
δ_{e+}	0.56%	0.51%

Consider Correlated Uncertainty

Implementing correlated uncertainty:

$$\chi^2 = \sum_{\text{process}} \sum_{i \in \pm\pm} \frac{\left(\sigma_i^{\text{data}} - \sigma_i^{\text{theory}}\right)^2}{\Delta\sigma_i^2} \longrightarrow \sum_{\text{process}} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)^T \Xi^{-1} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)$$

$$\vec{\sigma} := \begin{pmatrix} \sigma_{-+} & \sigma_{+-} & \sigma_{--} & \sigma_{++} \end{pmatrix}^T$$

$$\Xi := \Xi_N + \Xi_B + \Xi_\varepsilon + \Xi_\mathcal{L}; \qquad \text{e.g. } \left(\Xi_\varepsilon\right)_{ij} = \text{corr}\left(\vec{\sigma}_i^\varepsilon,\ \vec{\sigma}_j^\varepsilon\right) \frac{\partial\vec{\sigma}_i}{\partial\varepsilon_i} \frac{\partial\vec{\sigma}_j}{\partial\varepsilon_j} \Delta\varepsilon_i \Delta\varepsilon_j$$

Occurrence of correlated uncertainties:

- Fast switch between $\sigma_{\pm\pm}$
- Faster than change in e.g $\delta\mathcal{L}$

→ $\Delta\sigma_{\pm\pm}\,(\Delta\mathcal{L})$ becomes correlated

⇒ $\text{corr}\left(\vec{\sigma}_i^\mathcal{L},\ \vec{\sigma}_j^\mathcal{L}\right) \neq 0 \quad \forall i \neq j$

Consider disadvantageous situation:

- $\varepsilon = 0.6$
- $\Delta\varepsilon/\varepsilon = 0.01$
- $\Delta\mathcal{L}/\mathcal{L} = 0.001$

→ Studying the impact of correlations

Outlook

- **Open issues**
  - Implementing fiducial cuts for all processes → correct description of all systematics
  - Including a complete background analyses
- **Further Improvement**
  - Consider also differential cross sections
  - Study the possibility to use fiducial and differential cross sections simultaneously

Collision Data

Improvement by Constraints from Polarimeter Measurement

Consider Polarimeter Information

Simplified approach: (as a first step)

- Assume polarimeter measure directly at IP (neglect spin transport)
- Use nominal polarimeter uncertainty $\Delta P/P = 0.25\%$:
- Toy polarimeter measurement:

  Gaus-smeared

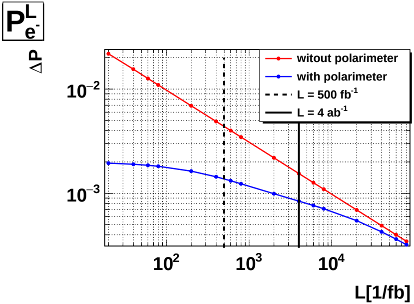
  - Mean: $P_{e^-} = 80\%,\ P_{e^+} = 30\%$
  - Width: ΔP

Implementation

$$\chi^2 += \sum_P \left[\frac{\left(P_{e^\pm}^{L,R} - \mathcal{P}_{e^\pm}^{L,R} \right)^2}{\Delta \mathcal{P}^2} \right]$$

- $P_{e^\pm}^{L,R}$: 4 fitted Parameter
- $\mathcal{P}_{e^\pm}^{L,R}$: Polarimeter measurement
- $\Delta \mathcal{P}$: Polarimeter uncertainty

Impact of the Polarimeter Constraint



For idealized situation:

- Better polarization precision, especially for lower integrated luminosities
- More robust against large Poisson fluctuations in the cross section measurement

Next step: add more realism

- Spin tracking including misalignments in the BDS
- Include impact of collision effect
- Use upstream and downstream polarimeter separately