

Top Physics at ILC



Jürgen R. Reuter, DESY





Top quark: **special while heaviest SM particle?**
or: **the only standard particle of the SM?**

SM / top candles can be used as
Telescopes for [indirect] BSM searches



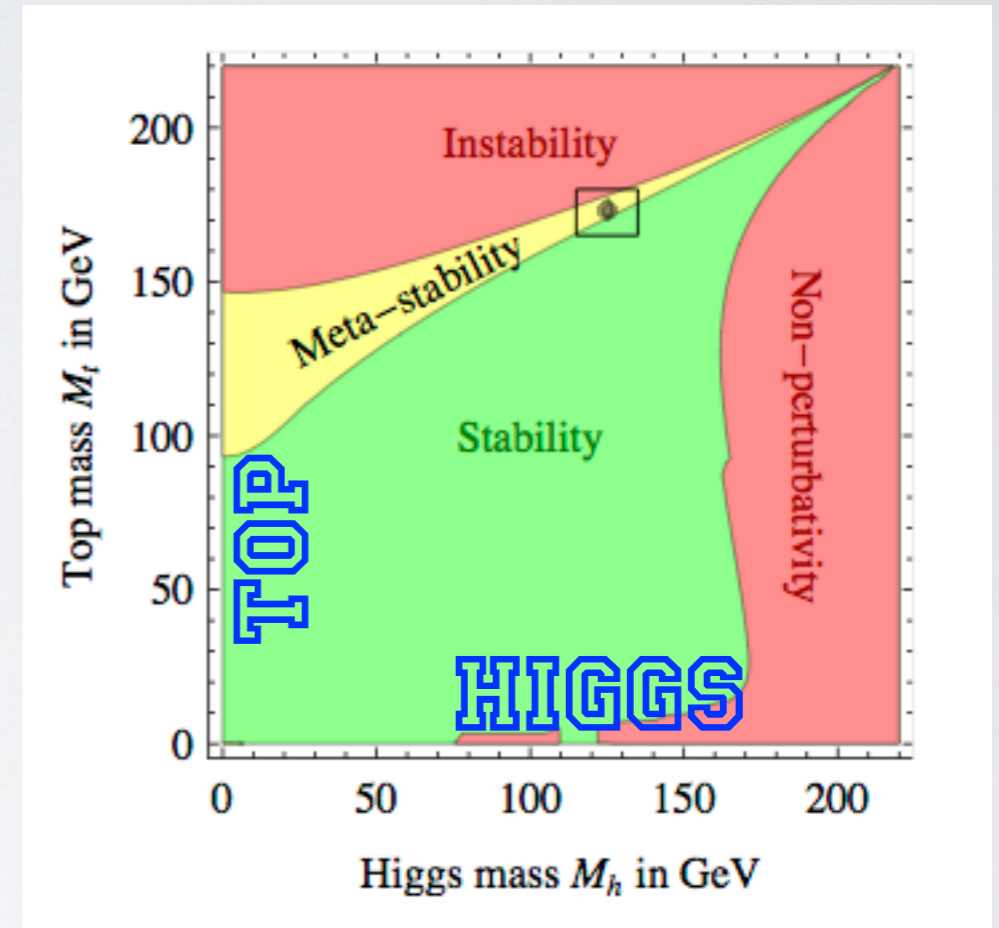
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Paradigmatic Standard Candle Telescopes



Electroweak vacuum & excitations:



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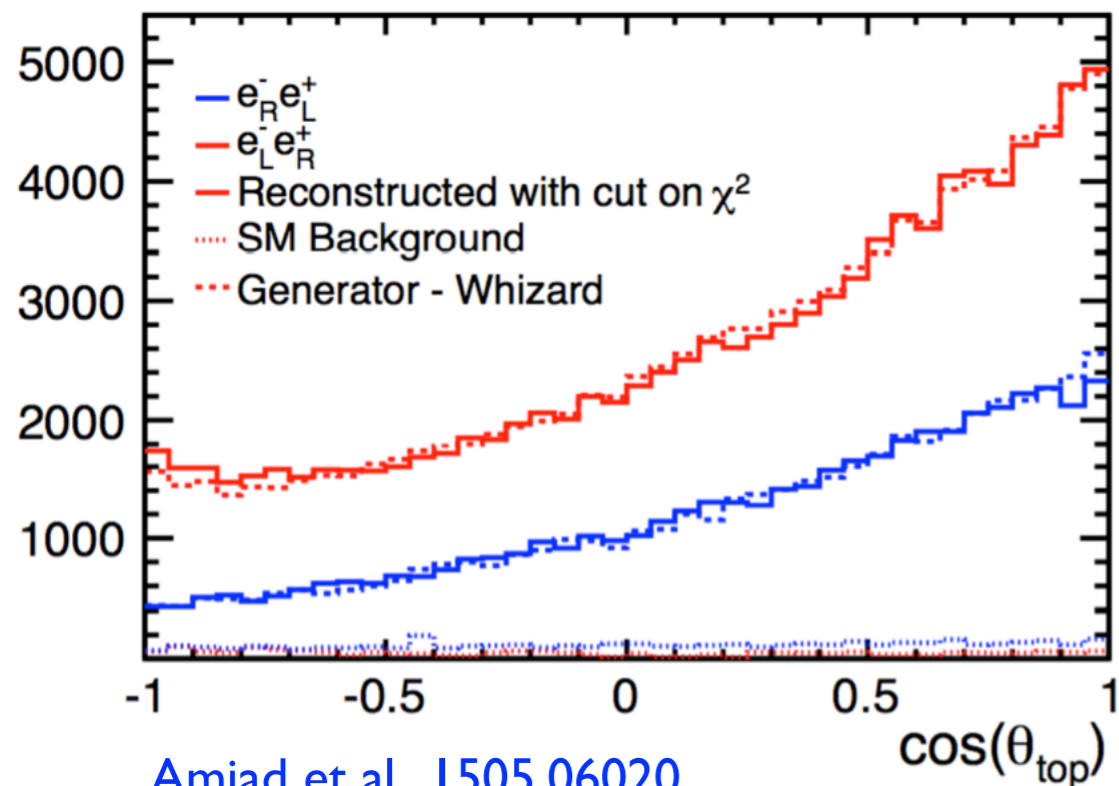
(note: plot under assumptions of **NO additional BSM**)

Telescope of Anomalous top couplings

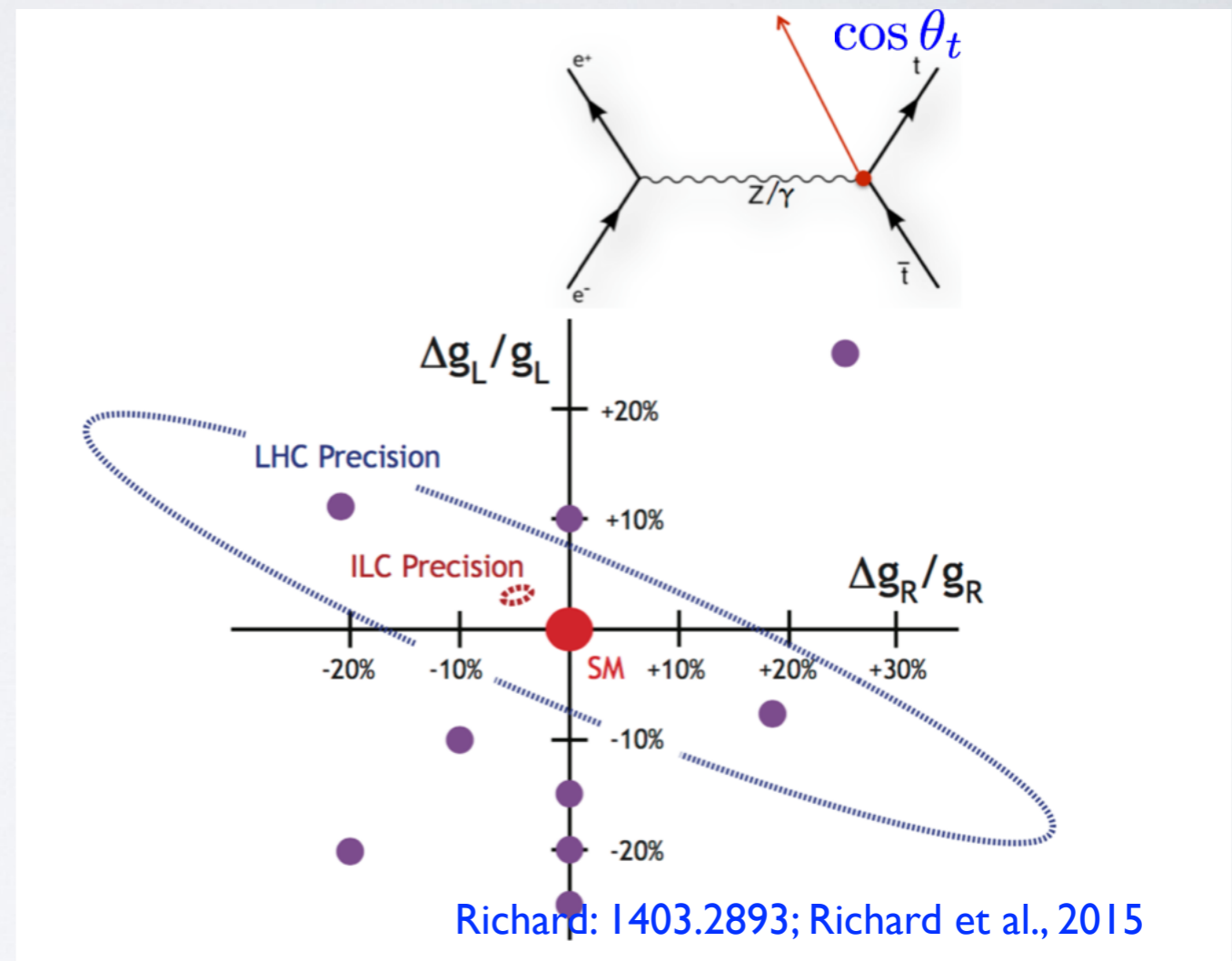
Anomalous Top couplings as BSM probes

$$\Gamma_{\mu}^{ttX}(k^2, q, \bar{q}) = ie \left\{ \gamma_{\mu} \left(\tilde{F}_{1V}^X(k^2) + \gamma_5 \tilde{F}_{1A}^X(k^2) \right) + \frac{(q - \bar{q})_{\mu}}{2m_t} \left(\tilde{F}_{2V}^X(k^2) + \gamma_5 \tilde{F}_{2A}^X(k^2) \right) \right\}$$

- Strong handle on BSM (e.g. compositeness)
- Excellent top reconstruction in e^+e^-
- Study of CP properties possible (!)



Amjad et al., 1505.06020



- Paradigm processes at lepton colliders: **precision determination of m_t and Y_t**
- Major background for EW measurements (VVV and VBS); any [most] BSM searches
- Investigate processes of increasing complexity: $2 \rightarrow 2$ to $2 \rightarrow 4$ to $2 \rightarrow 6$

$e^+e^- \rightarrow$	$n_{\text{loop diags}}$	Max. prop.	n_{hel}
$t\bar{t}$	2	3	16
$W^+W^-b\bar{b}$	157	5	144
$b\bar{b}\bar{\nu}_e e^- \nu_\mu \mu^+$	830	5	16
$t\bar{t}H$	17	4	16
$bW^+\bar{b}W^-H$	1548	6	144
$b\bar{b}\bar{\nu}_e e^- \nu_\mu \mu^+ H$	7436	6	16

On-Shell process: $e^+e^- \rightarrow t\bar{t}$

- NLO QCD [[Jersak/Laermann/Zerwas, 1982](#)]
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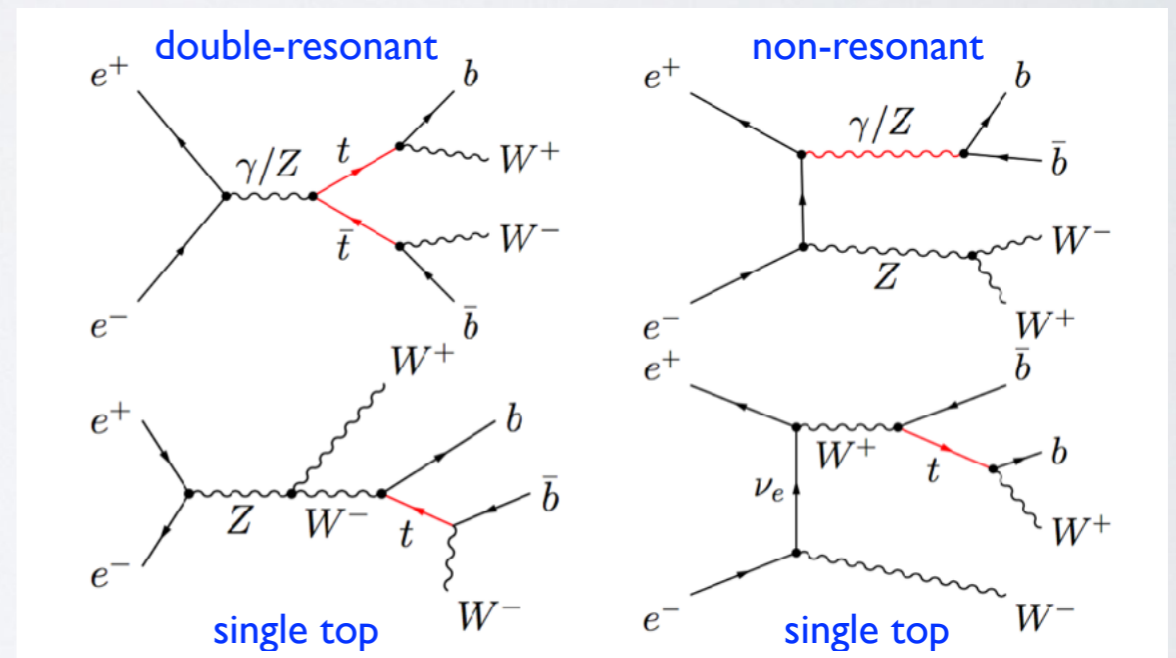
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ILC: tt continuum production (on- & off-shell)

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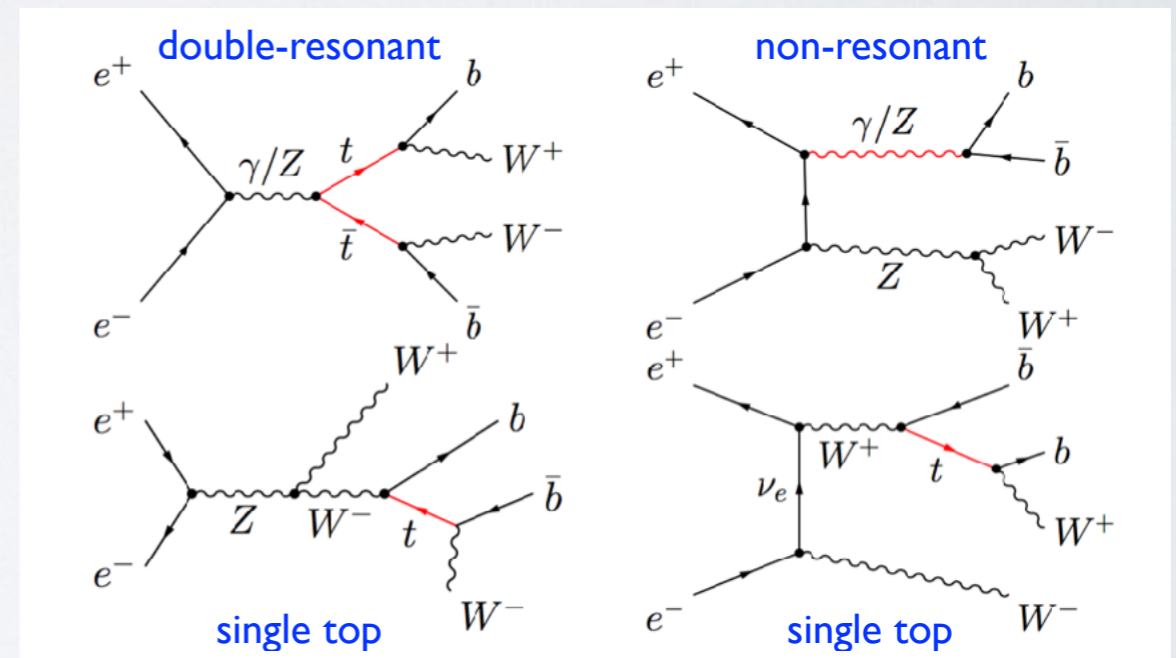
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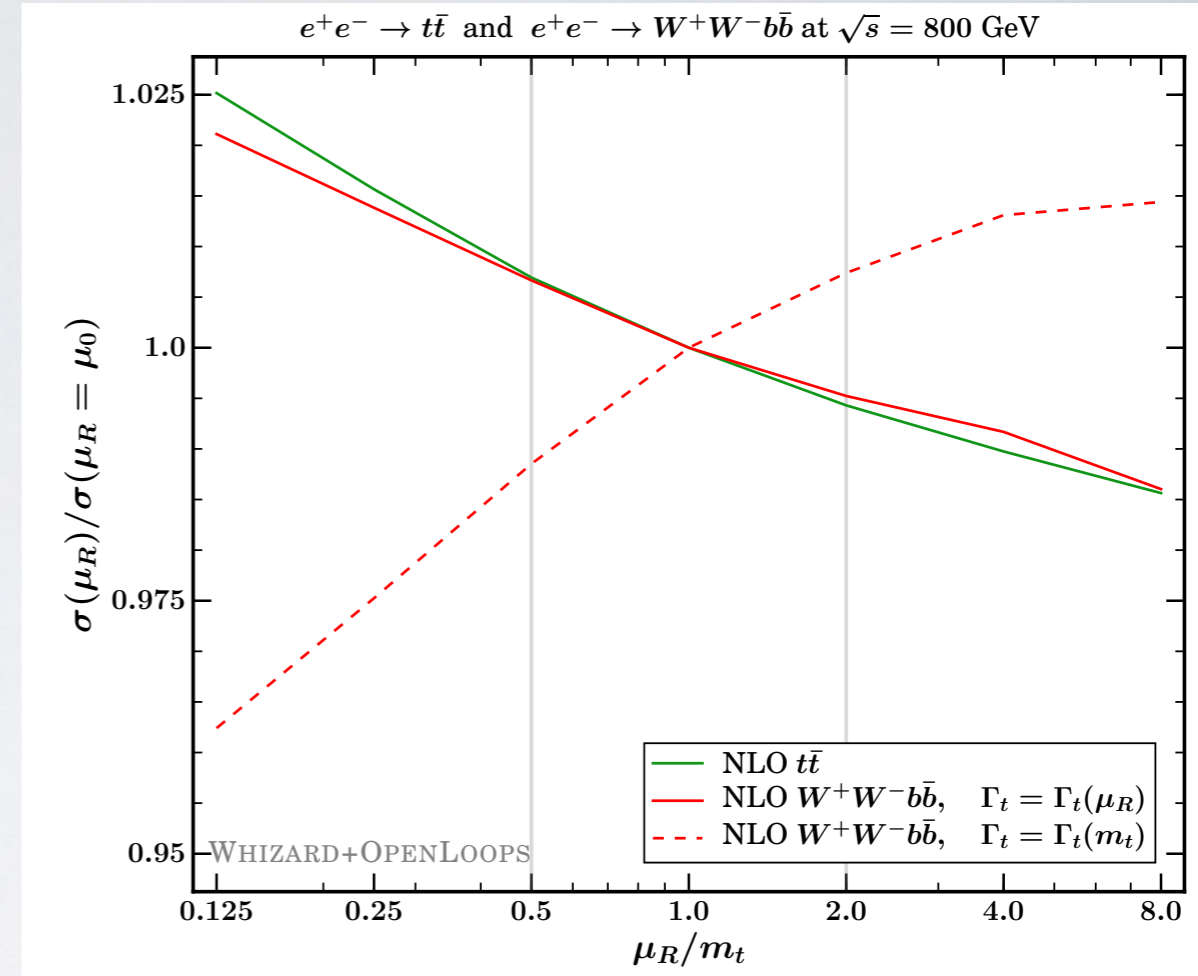
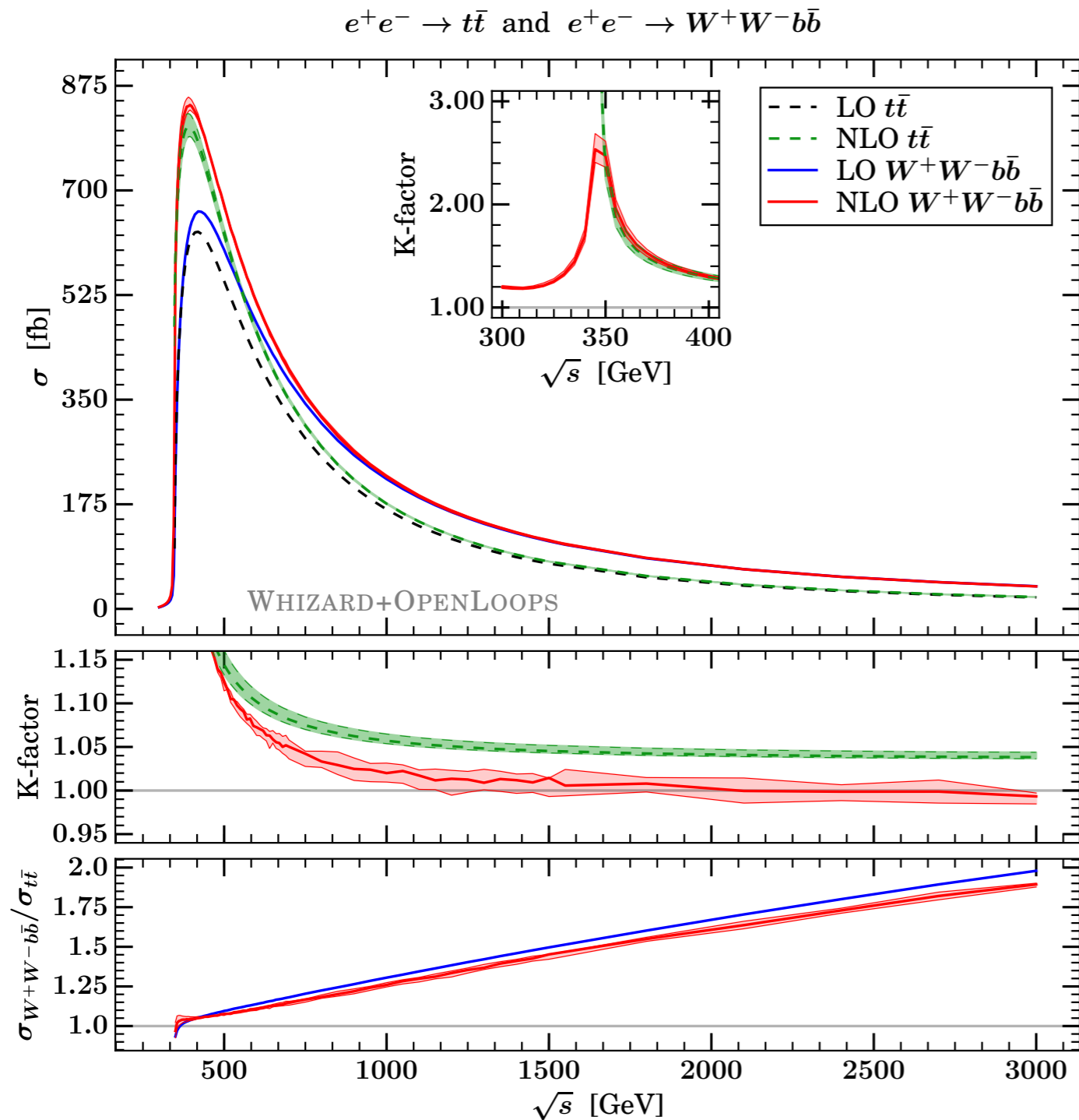


Top width: $t \rightarrow W^+b$

- NLO QCD [Jezabek/Kühn, 1989]



NLO QCD Results for off-shell $e^+e^- \rightarrow t\bar{t}$

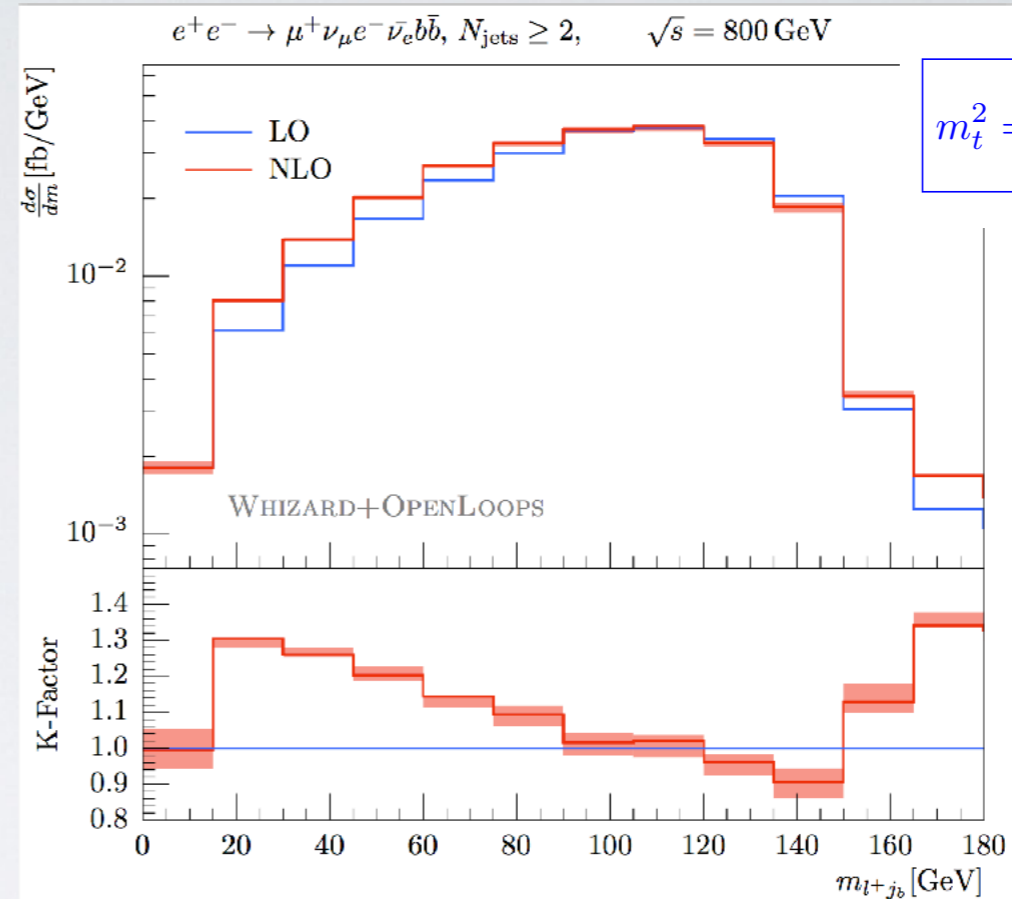
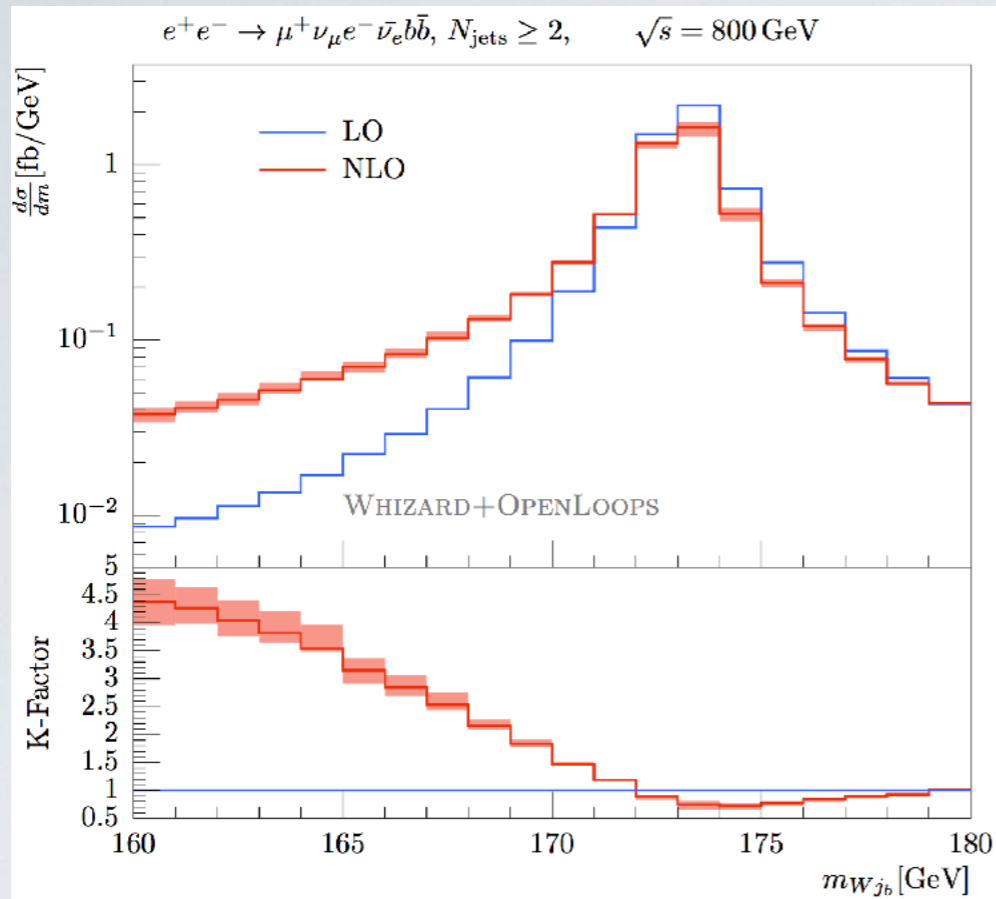


\sqrt{s} [GeV]	$e^+e^- \rightarrow t\bar{t}$			$e^+e^- \rightarrow W^+W^-b\bar{b}$		
	σ^{LO} [fb]	σ^{NLO} [fb]	K-factor	σ^{LO} [fb]	σ^{NLO} [fb]	K-factor
500	548.4	$627.4^{+1.4\%}_{-0.9\%}$	1.14	600.7	$675.1^{+0.4\%}_{-0.8\%}$	1.12
800	253.1	$270.9^{+0.8\%}_{-0.4\%}$	1.07	310.2	$320.7^{+1.1\%}_{-0.7\%}$	1.03
1000	166.4	$175.9^{+0.7\%}_{-0.3\%}$	1.06	217.2	$221.6^{+1.1\%}_{-1.0\%}$	1.02
1400	86.62	$90.66^{+0.6\%}_{-0.2\%}$	1.05	126.4	$127.9^{+0.7\%}_{-1.5\%}$	1.01
3000	19.14	$19.87^{+0.5\%}_{-0.2\%}$	1.04	37.89	$37.63^{+0.4\%}_{-0.9\%}$	0.993

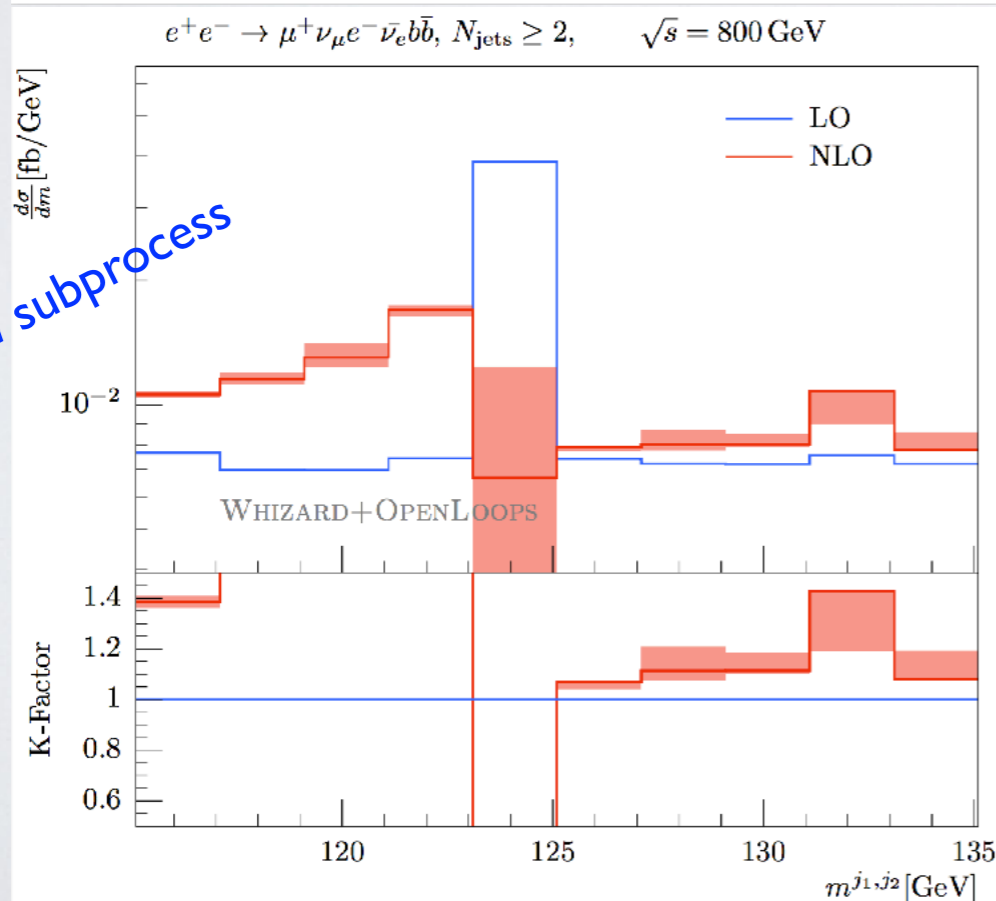
Chokouf/Kilian/Lindert/Pozzorini/JRR/Weiss, 2016



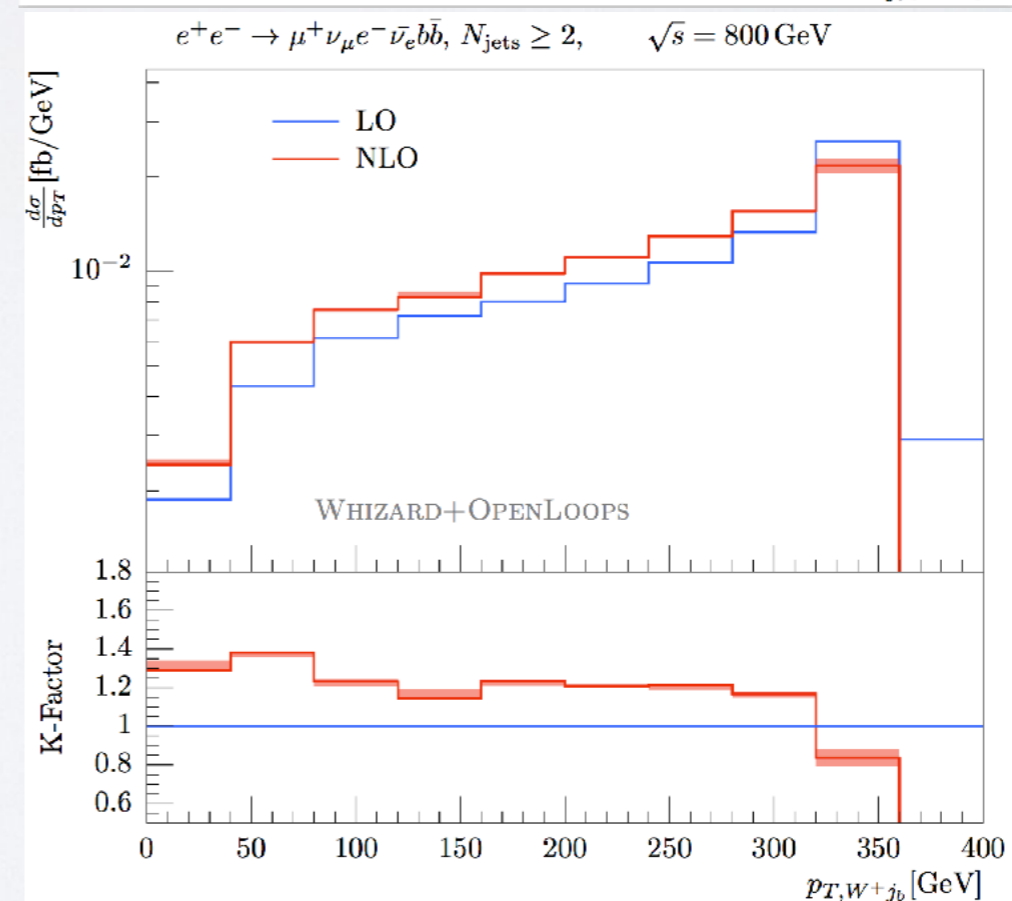
Differential Results for off-shell $e^+e^- \rightarrow tt$



$$m_t^2 = m_W^2 + \frac{2\langle m_{ljb}^2 \rangle}{1 - \langle \cos \theta_{ljb} \rangle}$$



WW \bar{h} subprocess



Top-Forward Backward Asymmetry

$$A_{FB} = \frac{\sigma(\cos \theta_t > 0) - \sigma(\cos \theta_t < 0)}{\sigma(\cos \theta_t > 0) + \sigma(\cos \theta_t < 0)}$$

Gluon emission symmetric in $\theta \Rightarrow$
NLO QCD corrections small

A_{FB} of the top quark

$e^+e^- \rightarrow$	A_{FB}^{LO}	A_{FB}^{NLO}	$A_{FB}^{\text{NLO}} / A_{FB}^{\text{LO}}$
$t\bar{t}$	-0.535	-0.539	1.013
A_{FB} $W^+W^-b\bar{b}$	-0.428	-0.426	0.995
A_{FB} $\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
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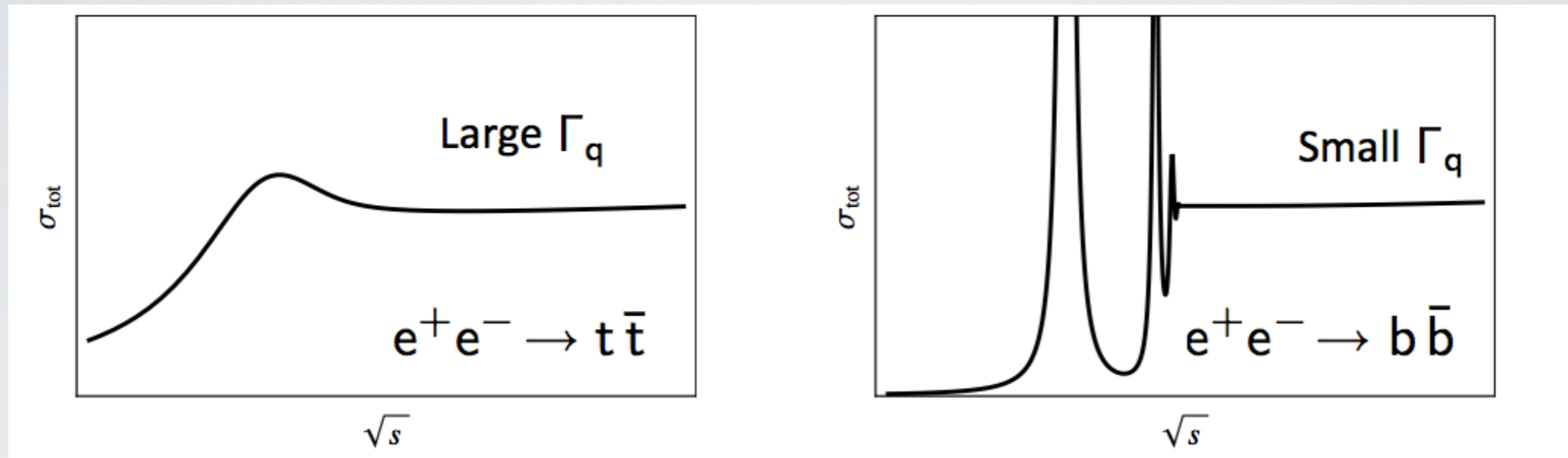
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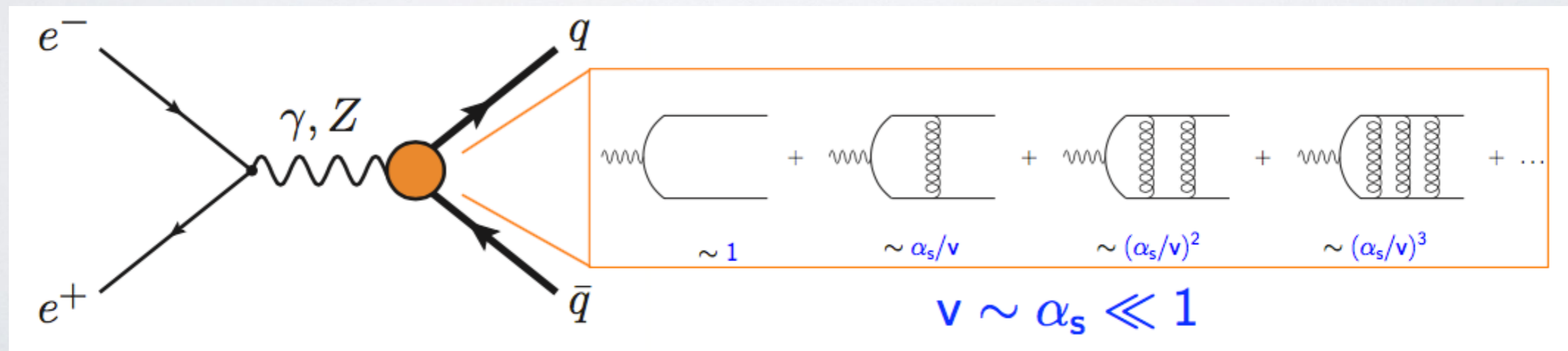
Top Threshold at ILC350

ILC top threshold scan best-known method to measure top quark mass, $\Delta M \sim 30\text{-}50 \text{ MeV}$

Heavy quark production at lepton colliders, qualitatively:



Threshold region: top velocity $v \sim \alpha_s \ll 1$



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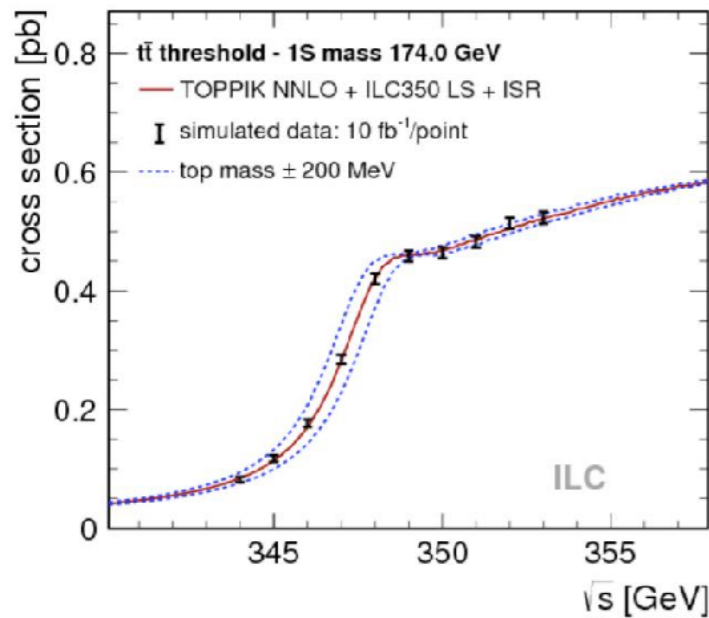
- Default parameters:

$$M^{1S} = 172 \text{ GeV}, \quad \Gamma_t = 1.54 \text{ GeV},$$

$$\alpha_s(M_Z) = 0.118$$

$$M^{1S} = M_t^{pole} \left(1 - \Delta_{(Coul.)}^{LL/NLL} \right)$$

Marquard et al.



error source	Δm_t^{PS} [MeV]
stat. error (200 fb ⁻¹)	13
theory (NNNLO scale variations, PS scheme)	40
parametric (α_s , current WA)	35
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 – 50
total (stat. + syst.)	40 – 75

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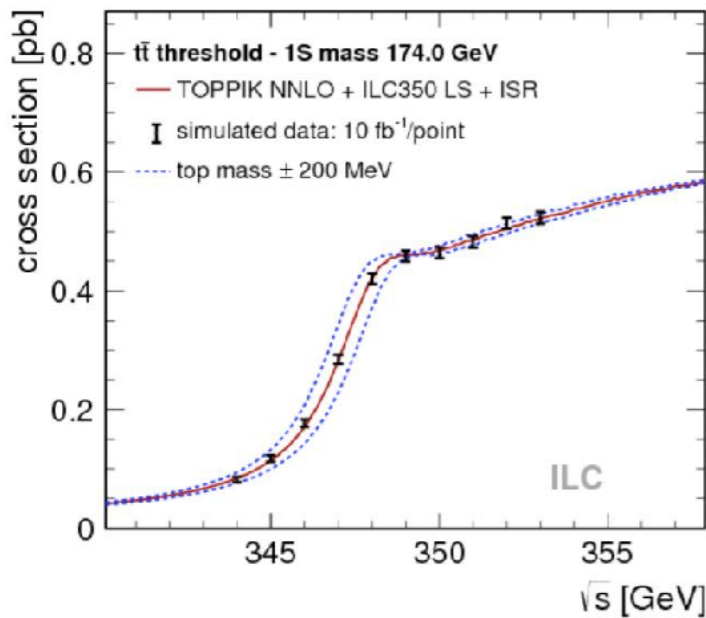
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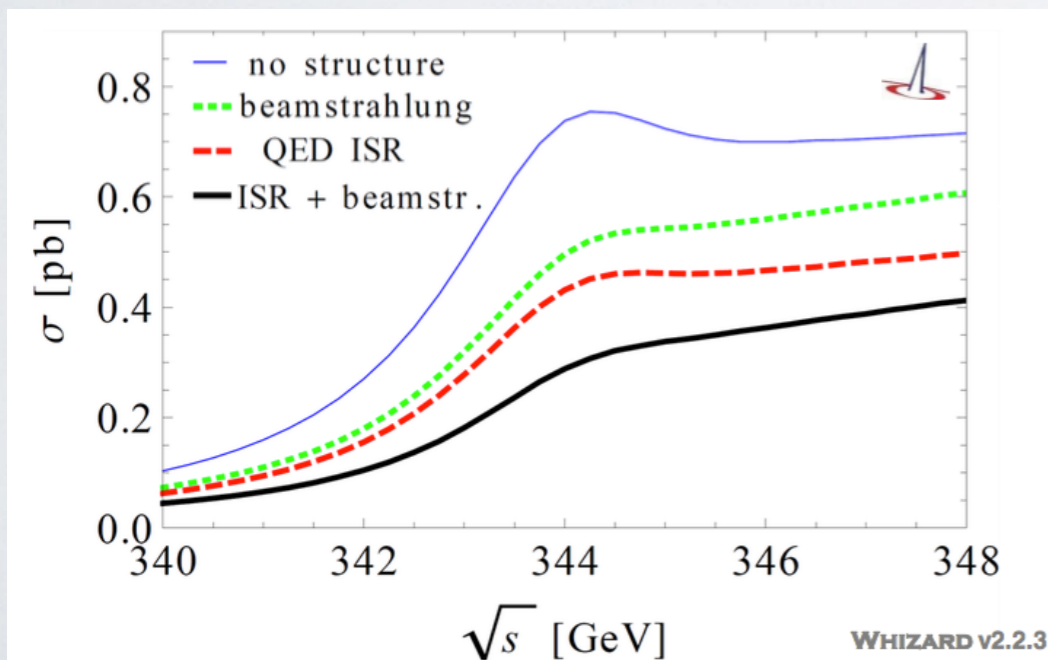
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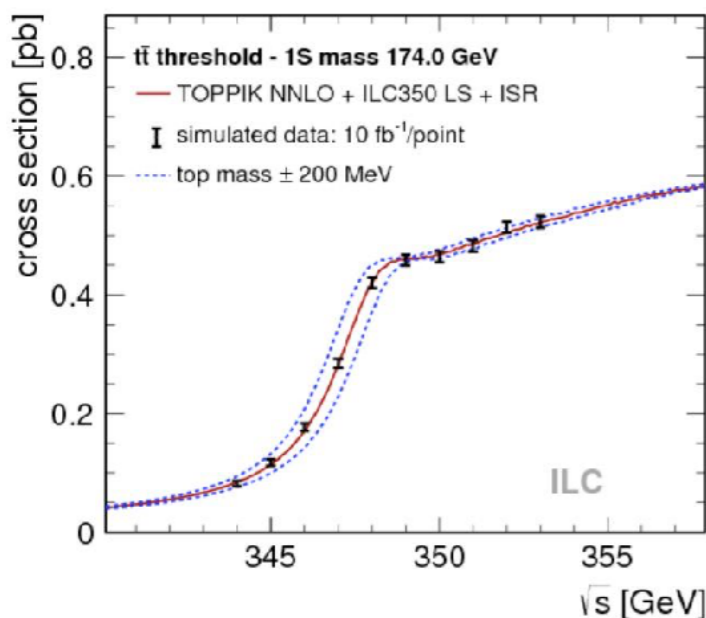
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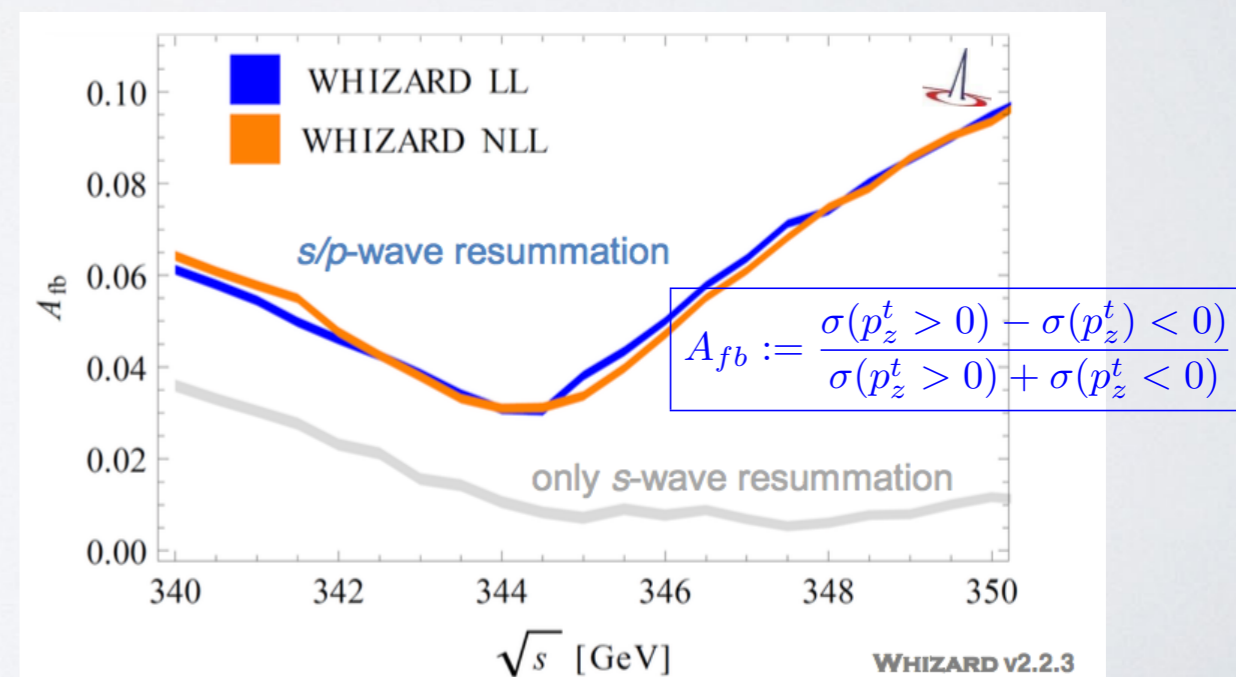
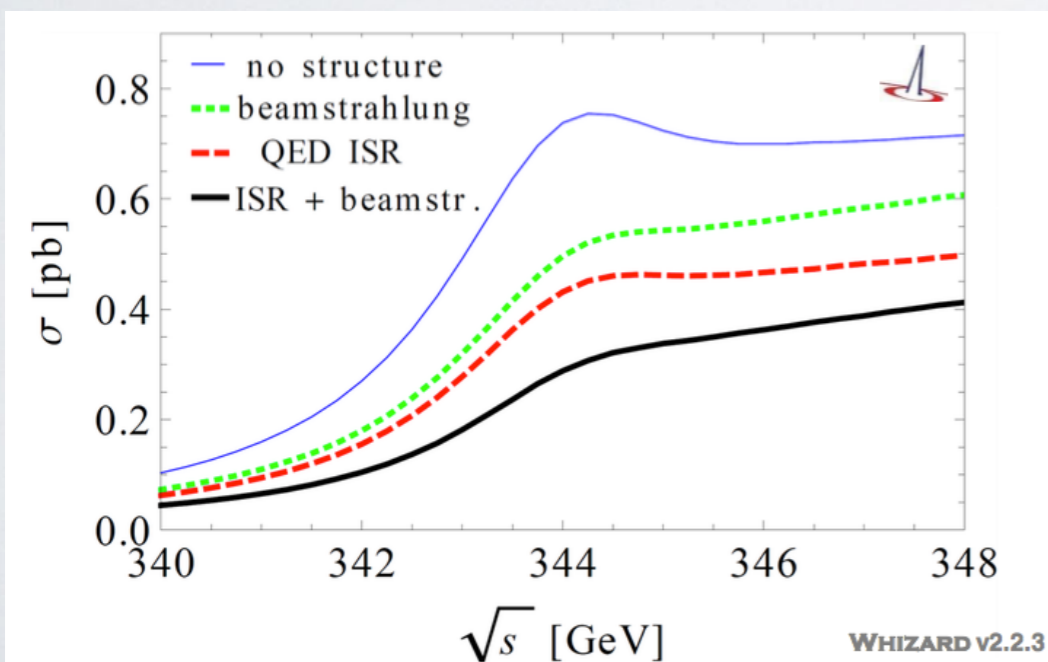
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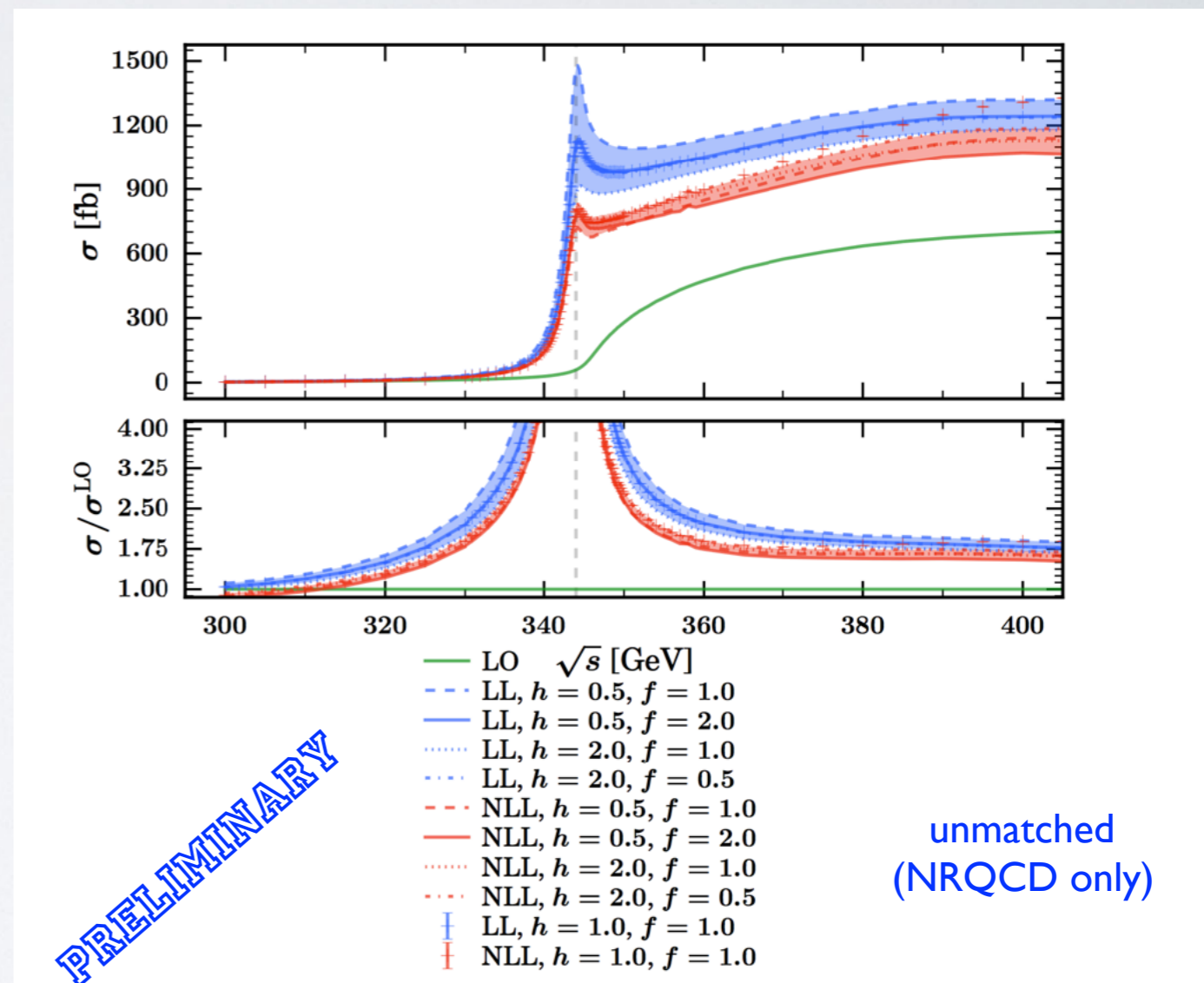
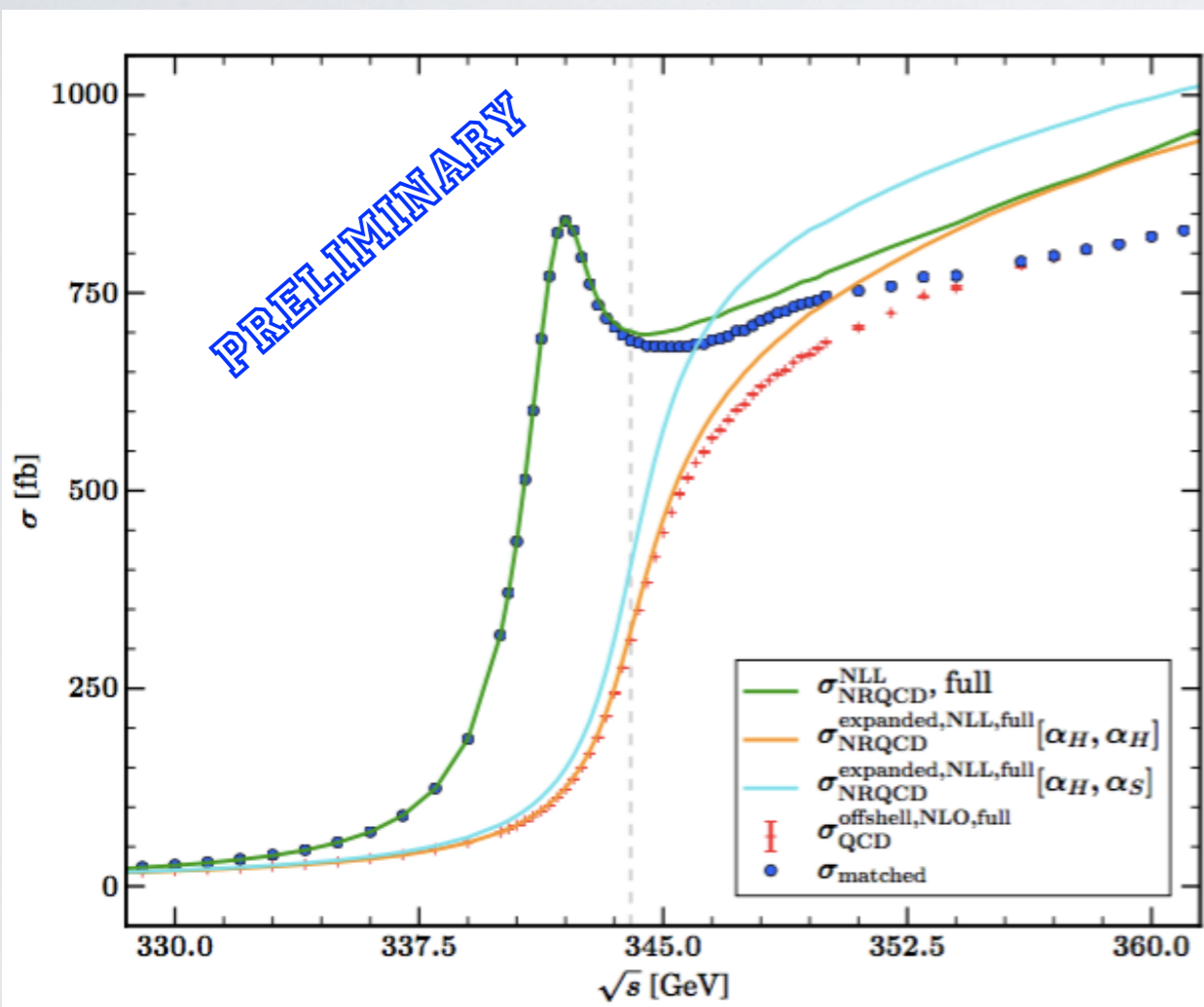
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Matching to continuum at LO and NLO

- Transition region between relativistic and resummation effects
- CLIC benchmark energies:
0.38 TeV, 1.4 TeV, 3.0 TeV

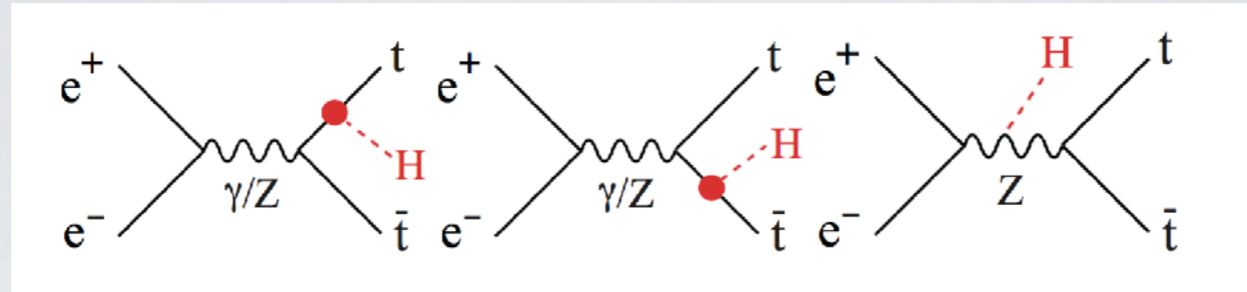
- Measurement not fully inclusive
- Important for determination of experimental efficiencies



Bach/Chokouf /Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss work in progress

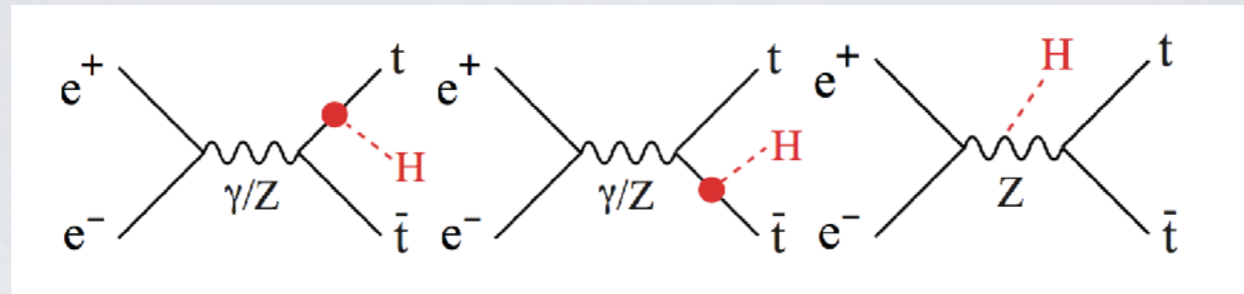


Top pair + Higgs production

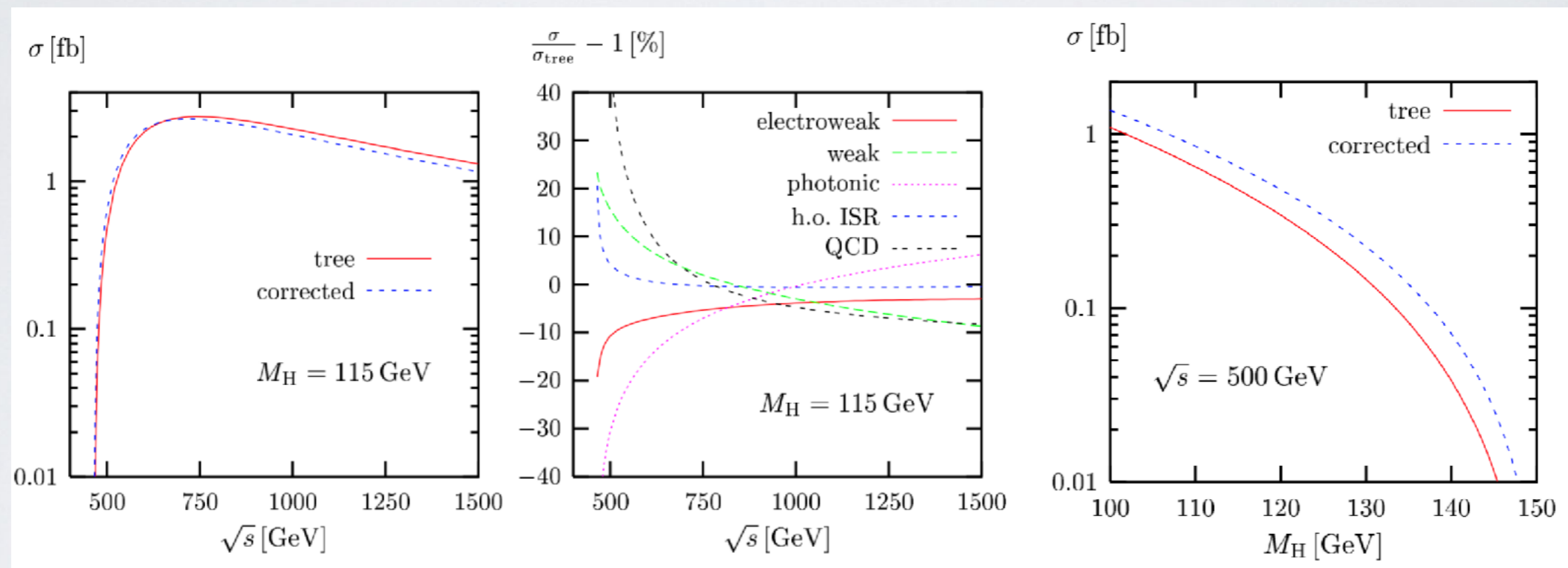


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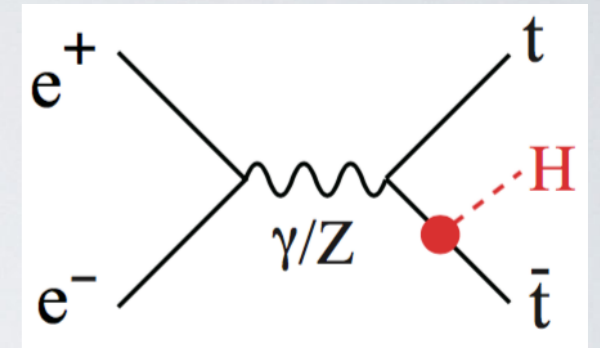
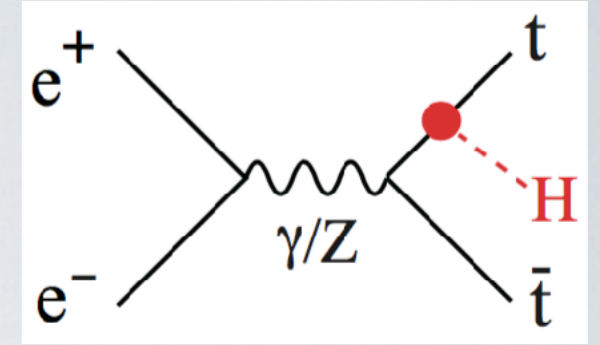
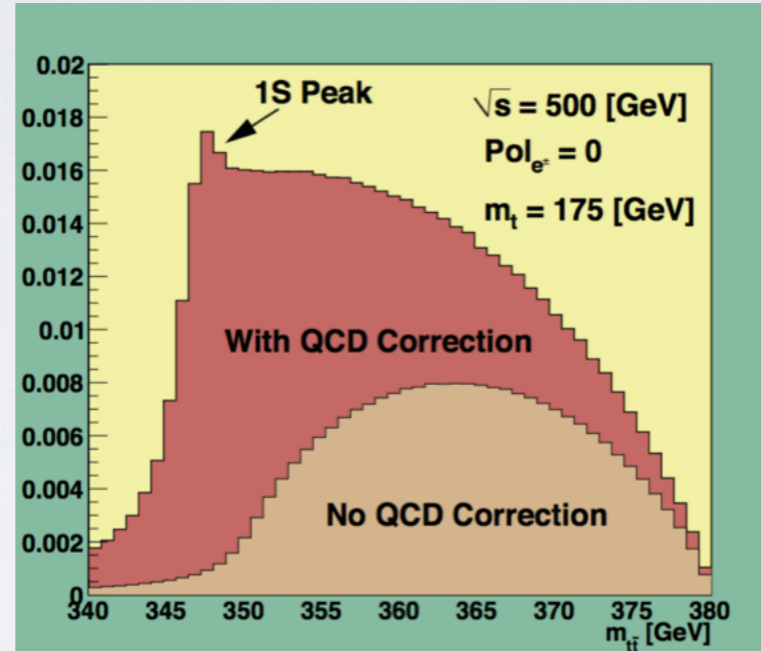
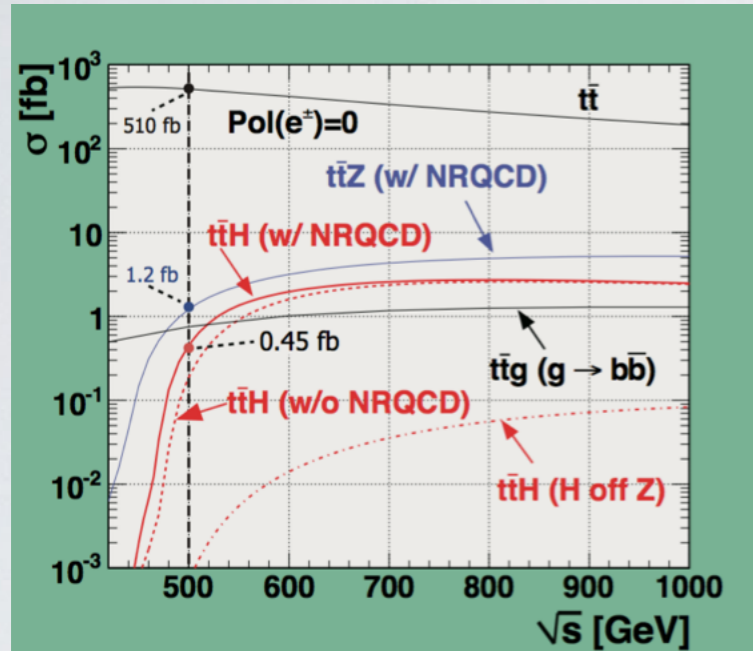
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- Photonic and gluonic corrections largely cancel
- **Large effects at threshold (ISR, but more so QCD → more soon)**
- Depends crucially on Higgs mass: **sweet spot for $M(h) = 125$ GeV is for $E \sim 800$ GeV**

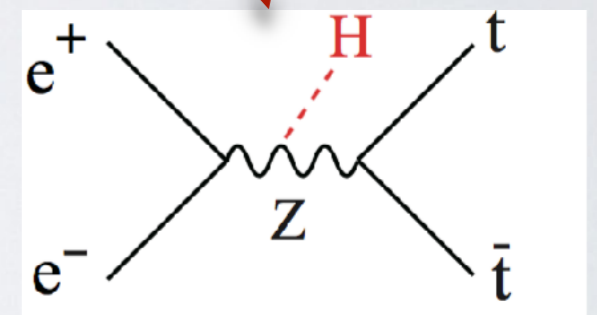
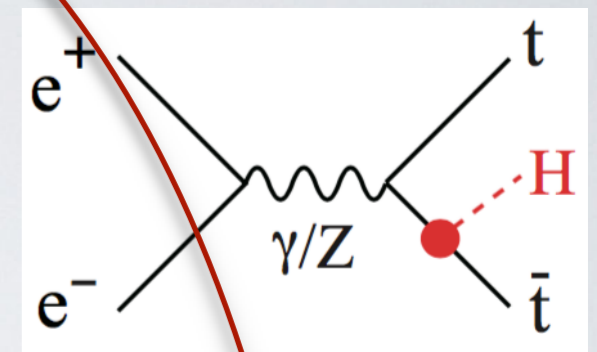
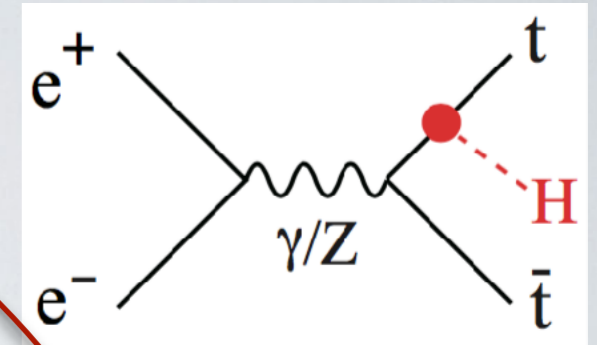
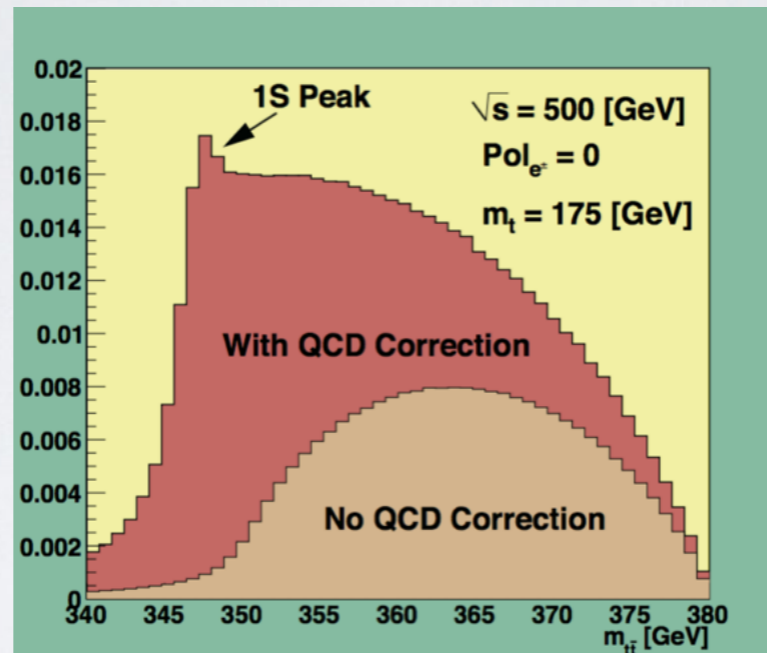
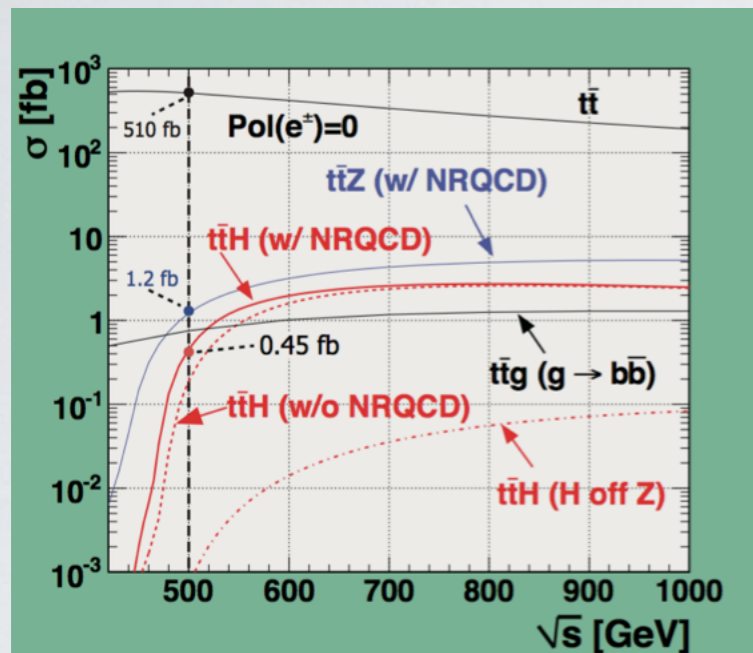
The top Yukawa coupling

- Measurement possible for $\sqrt{s} \gtrsim 500$ GeV [$t\bar{t}h$ associated production]
- 8% Higgsstrahlung contamination, $Z^* \rightarrow t\bar{t}$
- **Cross section rises by factor of 4 between 500 and 550 GeV (!)**
- Slight enhancement at threshold due to QCD Coulomb gluons



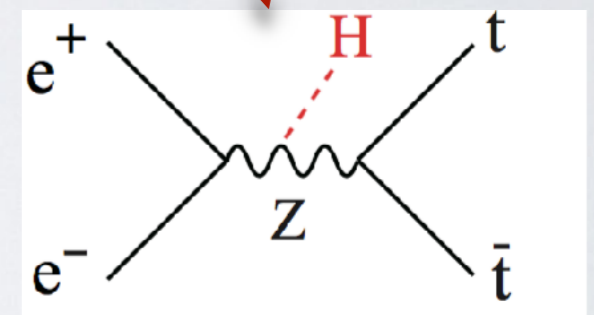
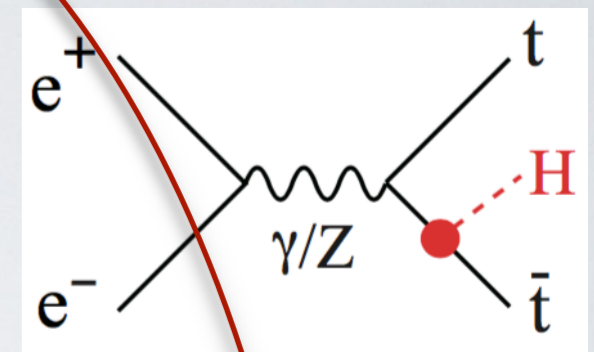
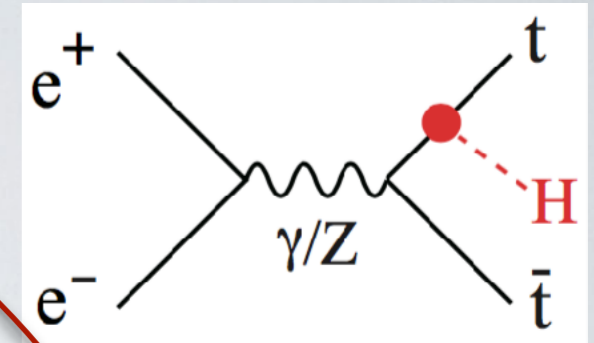
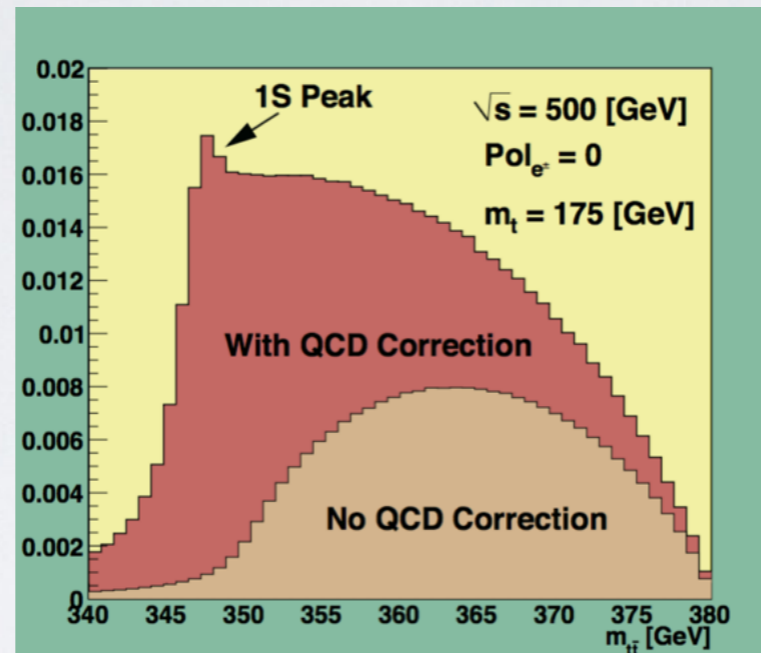
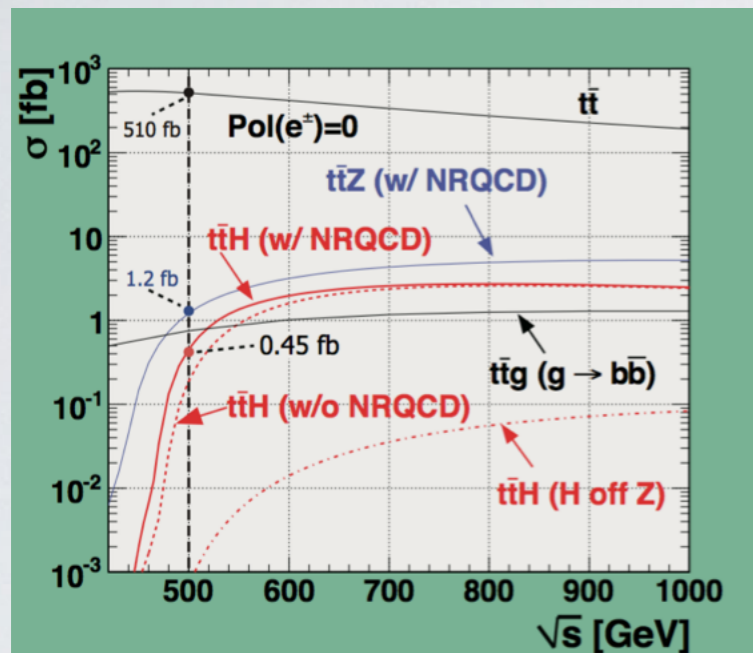
The top Yukawa coupling

- Measurement possible for $\sqrt{s} \gtrsim 500$ GeV [$t\bar{t}h$ associated production]
- 8% Higgsstrahlung contamination, $Z^* \rightarrow t\bar{t}$
- **Cross section rises by factor of 4 between 500 and 550 GeV (!)**
- Slight enhancement at threshold due to QCD Coulomb gluons



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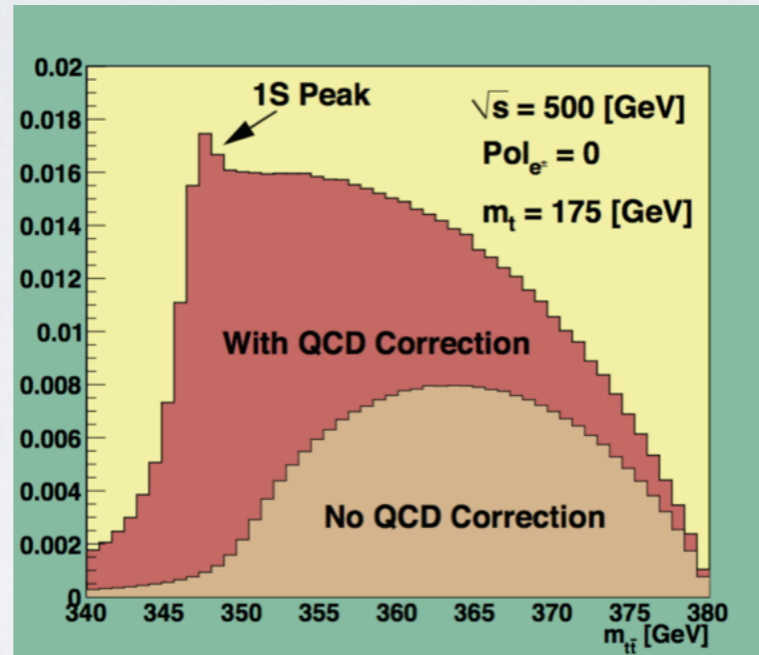
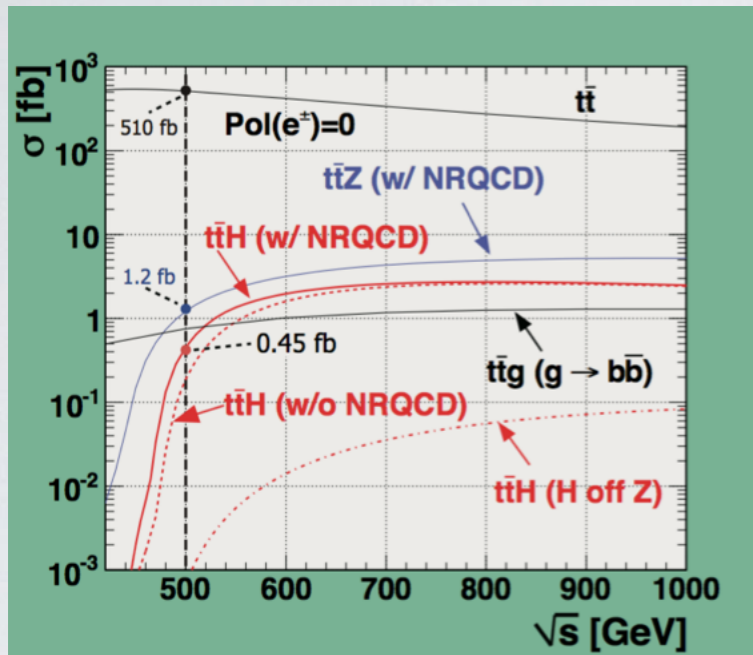
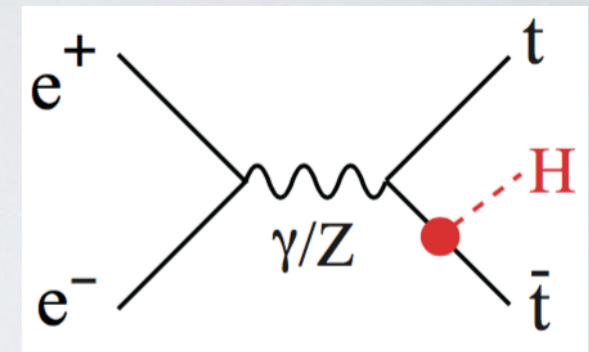
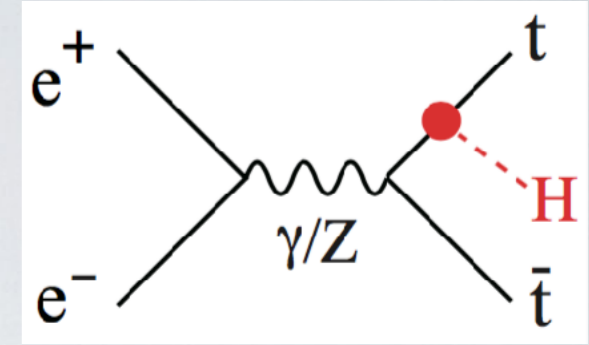


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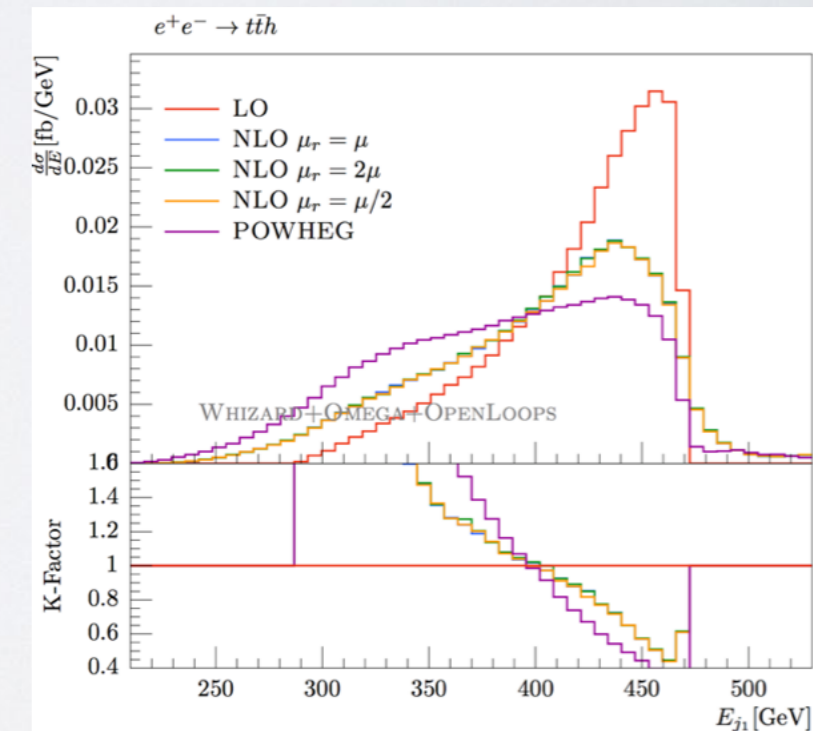
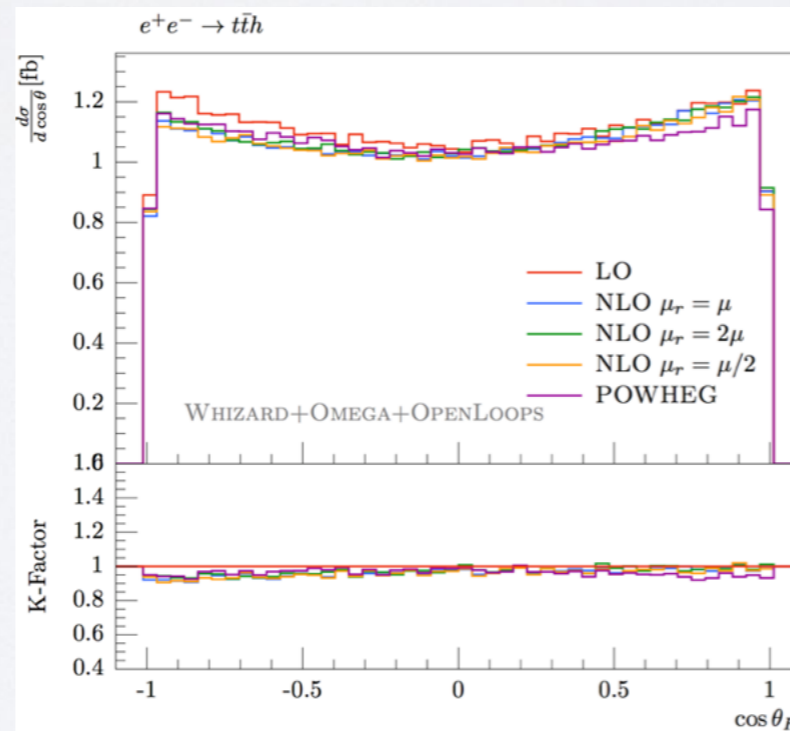


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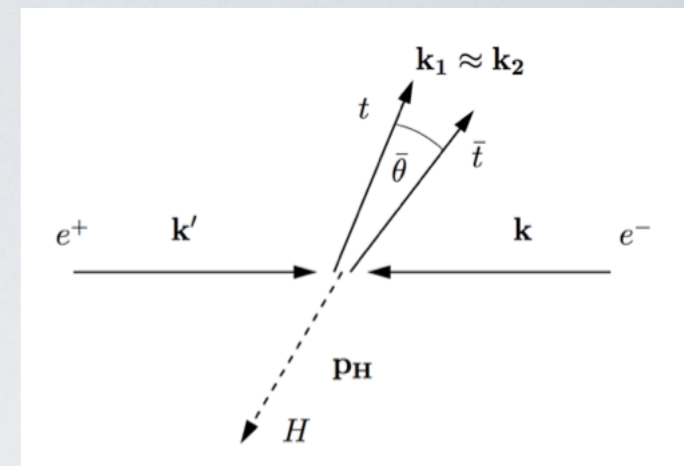
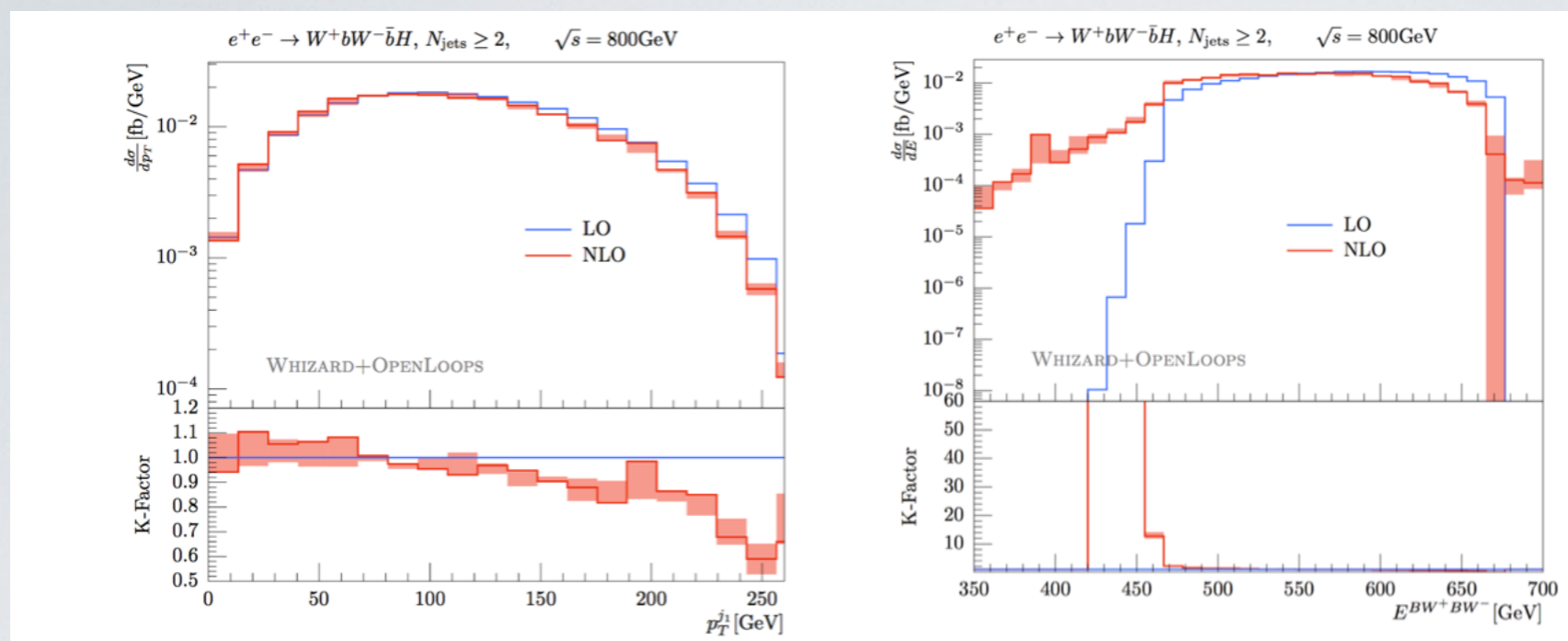
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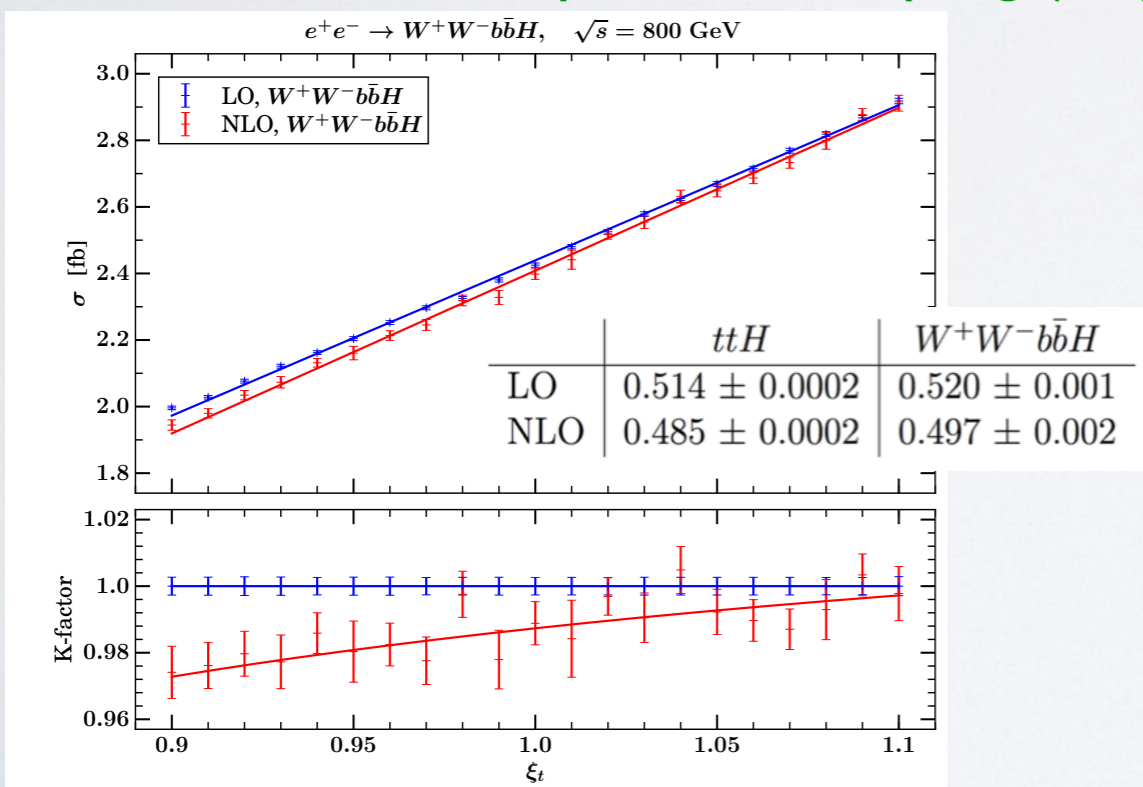


Differential Results for off-shell ttH



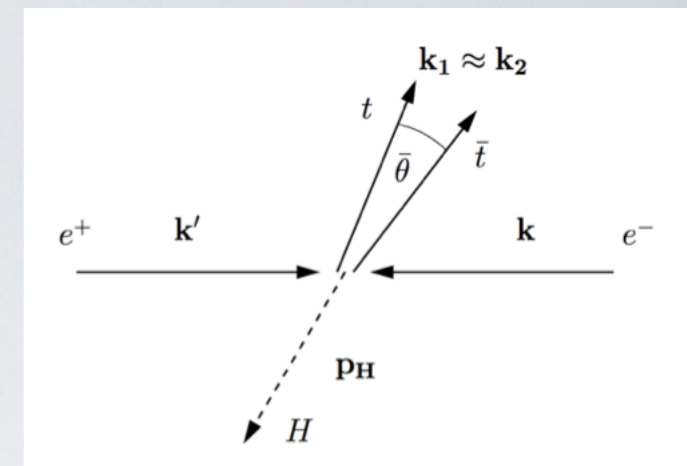
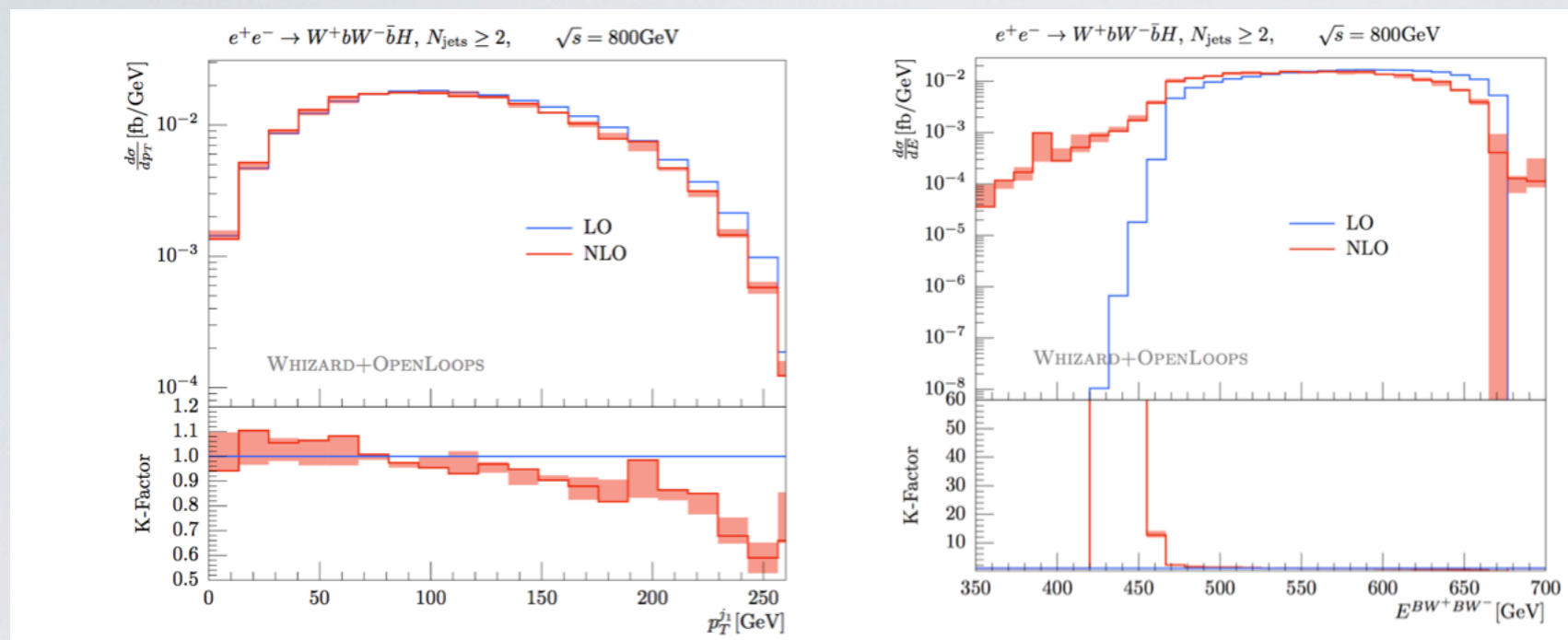
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Determination of top Yukawa coupling (ttH)



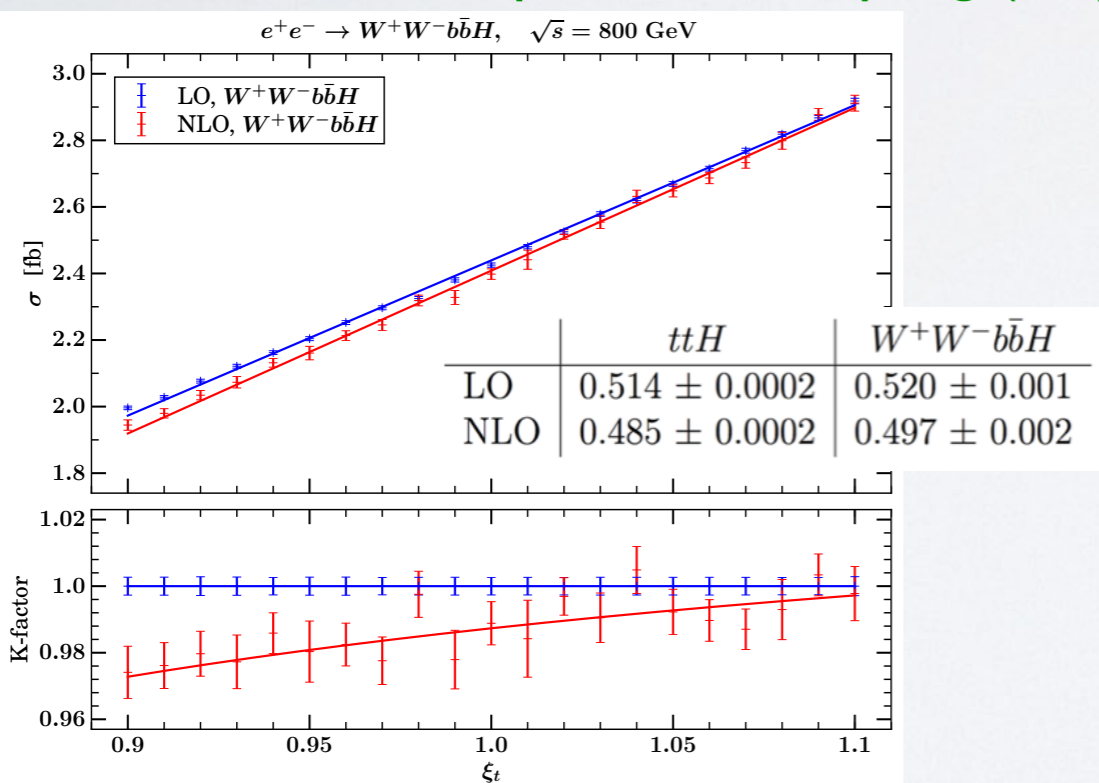
Chokouf /Kilian/Lindert/Pozzorini/JRR/Weiss, 2016

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Determination of top Yukawa coupling (ttH)



Polarized Results (tt)

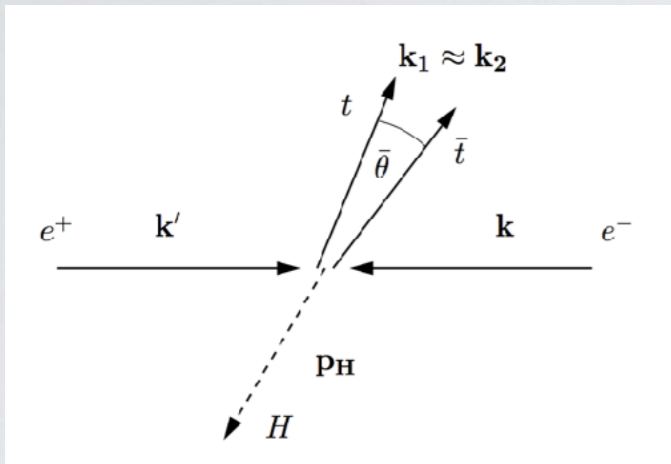
- ILC will always run polarized
- Polarized I-loop amplitudes beyond BLHA

$P(e^-)$	$P(e^+)$	$\sqrt{s} = 800 \text{ GeV}$			$\sqrt{s} = 1500 \text{ GeV}$		
		$\sigma^{\text{LO}}[\text{fb}]$	$\sigma^{\text{NLO}}[\text{fb}]$	K-factor	$\sigma^{\text{LO}}[\text{fb}]$	$\sigma^{\text{NLO}}[\text{fb}]$	K-factor
0%	0%	253.7	272.8	1.075	75.8	79.4	1.049
-80%	0%	176.5	190.0	1.077	98.3	103.1	1.049
+80%	0%	176.5	190.0	1.077	53.2	55.9	1.049
-80%	30%	420.8	452.2	1.074	124.9	131.0	1.048
-80%	60%	510.7	548.7	1.074	151.6	158.9	1.048
80%	-30%	208.4	224.5	1.077	63.0	66.1	1.049
80%	-60%	240.3	258.9	1.077	72.7	76.3	1.049

Chokouf/Kilian/Lindert/Pozzorini/JRR/Weiss, 2016

Threshold enhancement @ 500 GeV for $t\bar{t}h$

- ▶ Close to threshold: Coulomb enhancement of non-relativistic top pair
- ▶ Energy distribution of Higgs enhanced in top pair recoil [Farrell/Hoang, 2005-2006]



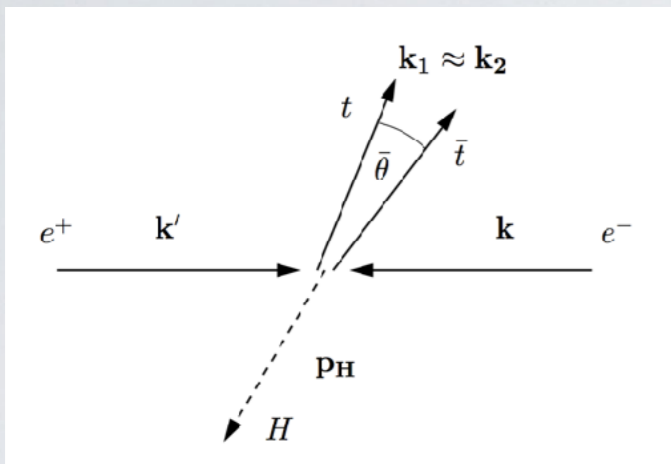
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Enhancement most pronounced in Higgs energy distribution

$$\lim_{E_h \rightarrow E_h^{\max}} \frac{d\sigma(e^+e^- \rightarrow t\bar{t}h)}{E_h} \longrightarrow \text{vNRQCD resummation}$$

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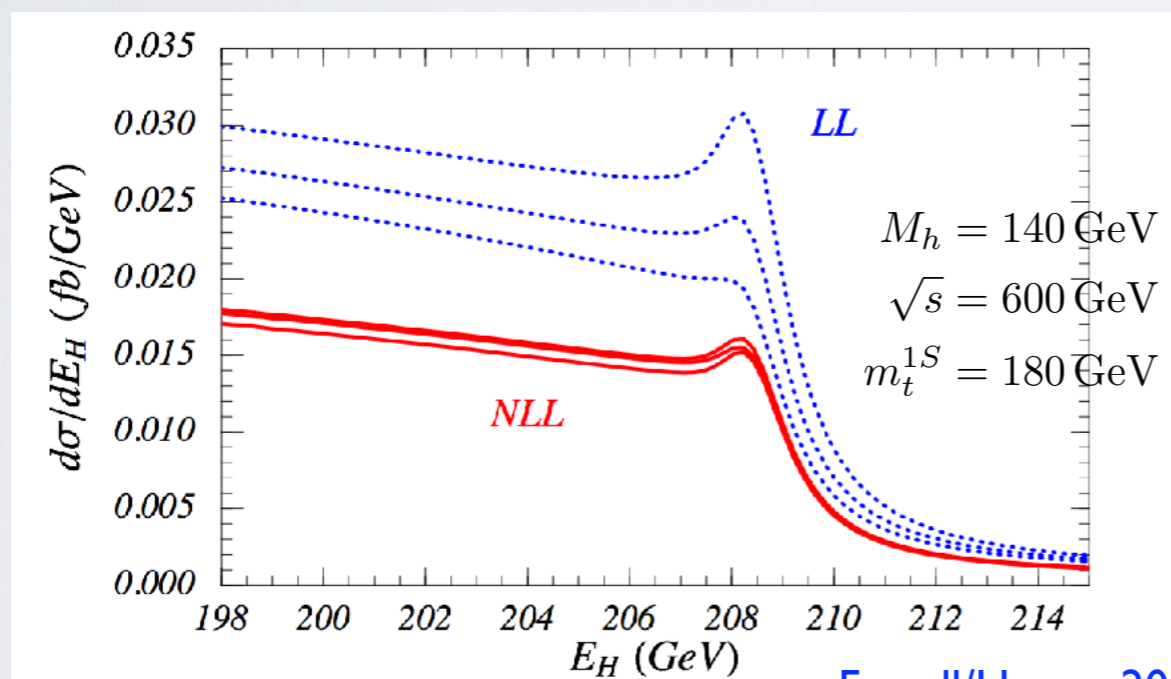
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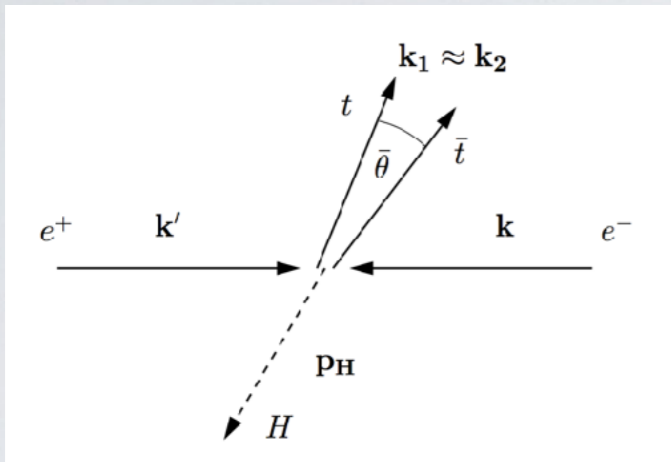
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Farrell/Hoang, 2005

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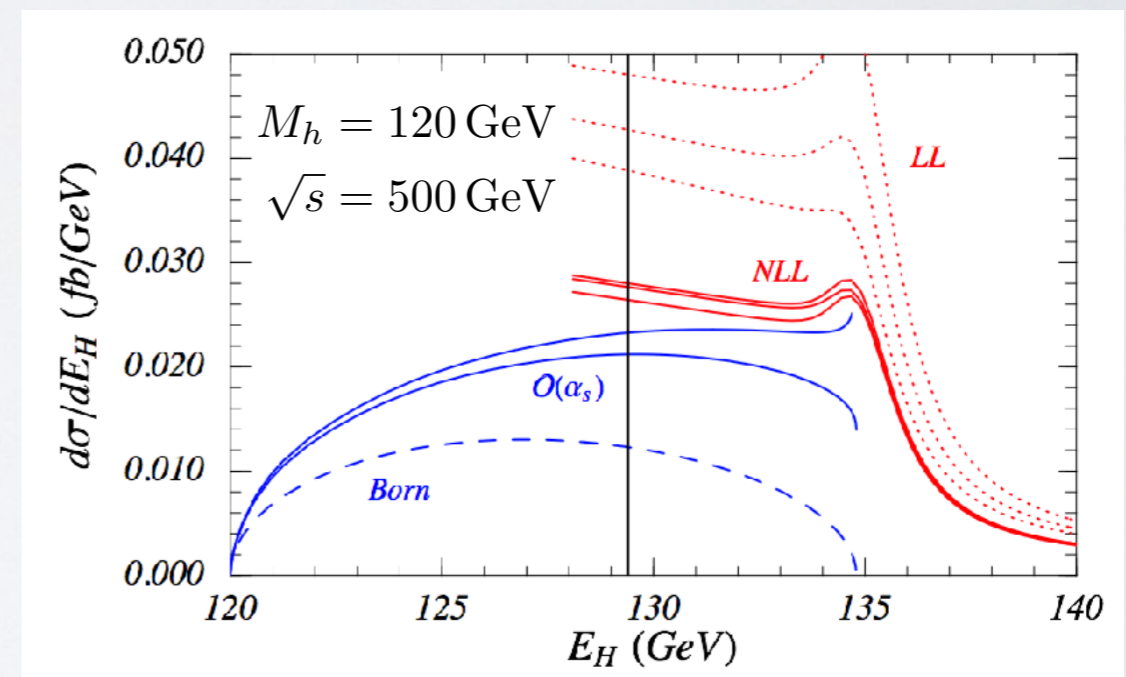
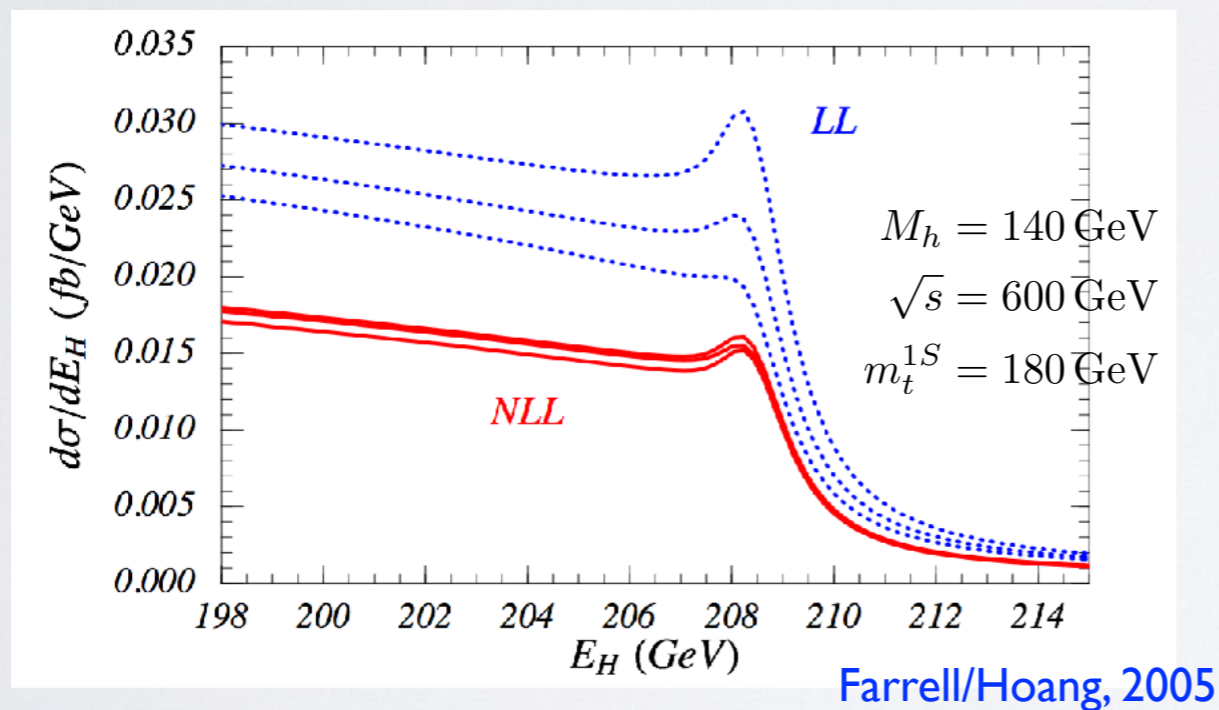
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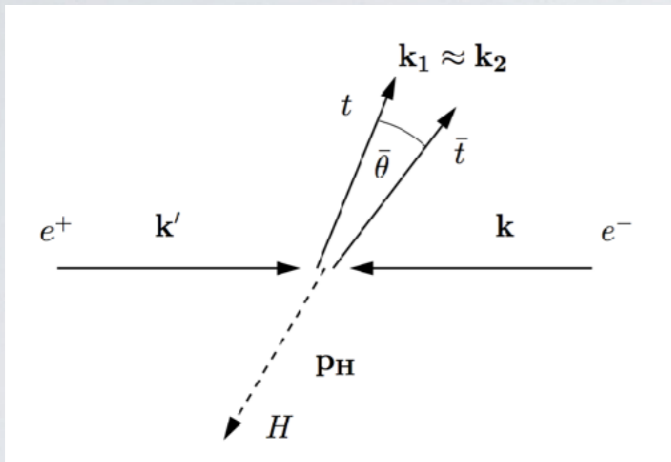
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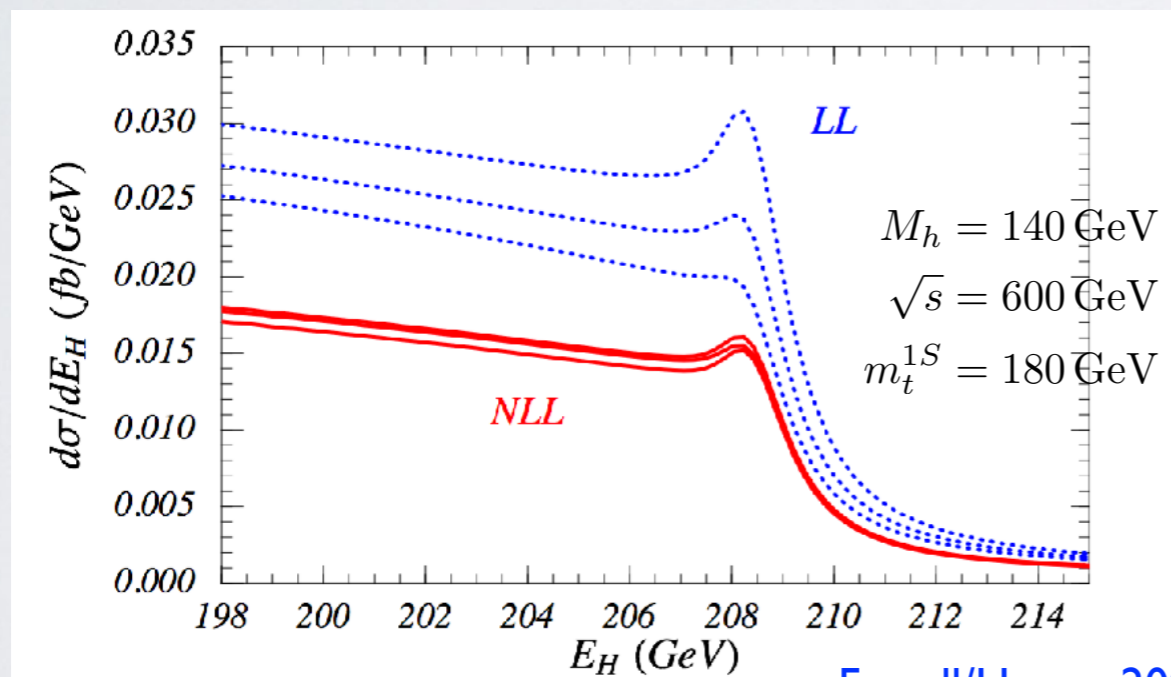
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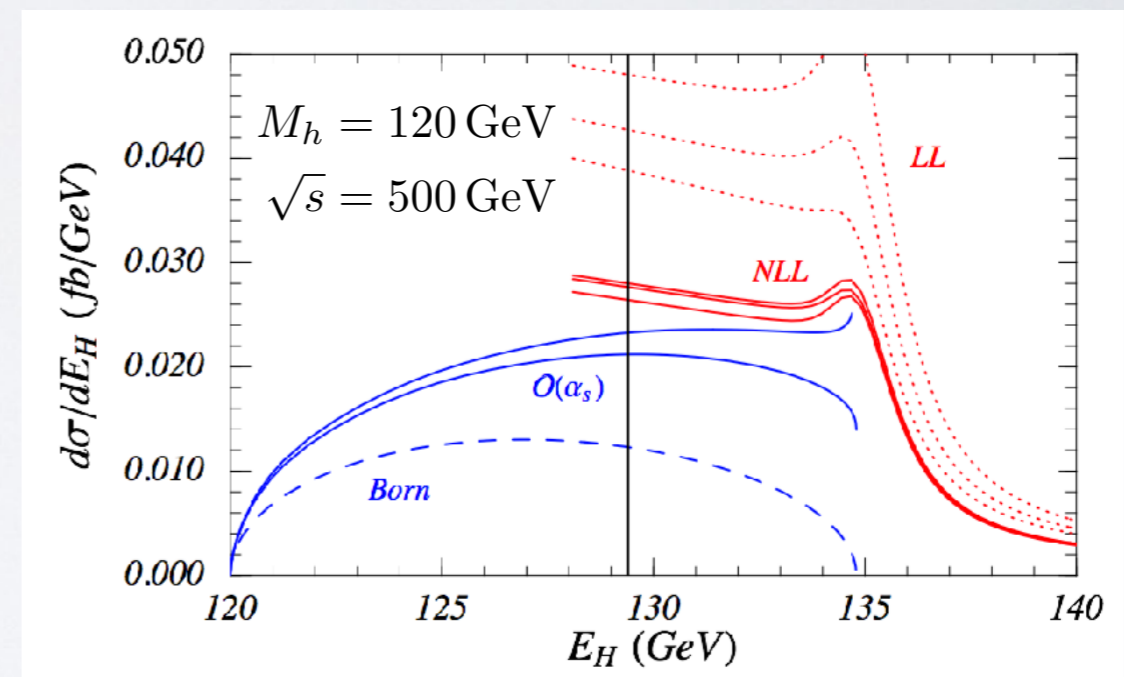
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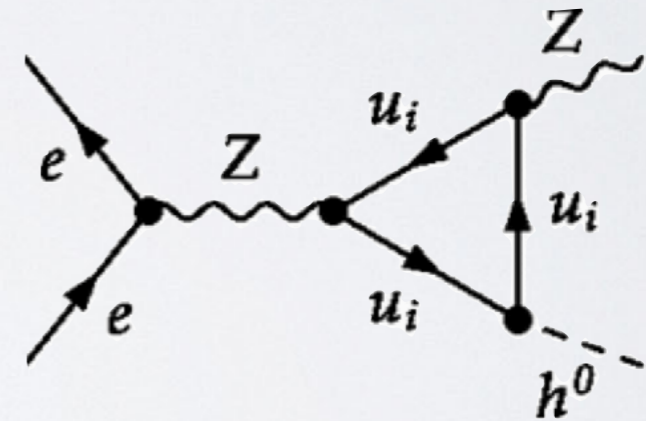
Farrell/Hoang, 2005



- ▶ Goal: properly matched NLL threshold/NLO cont. [Chokoufe/Hoang/JRR/Stahlhofen/Weiss, in prep.]

Top Physics for 5.99 Billion \$?

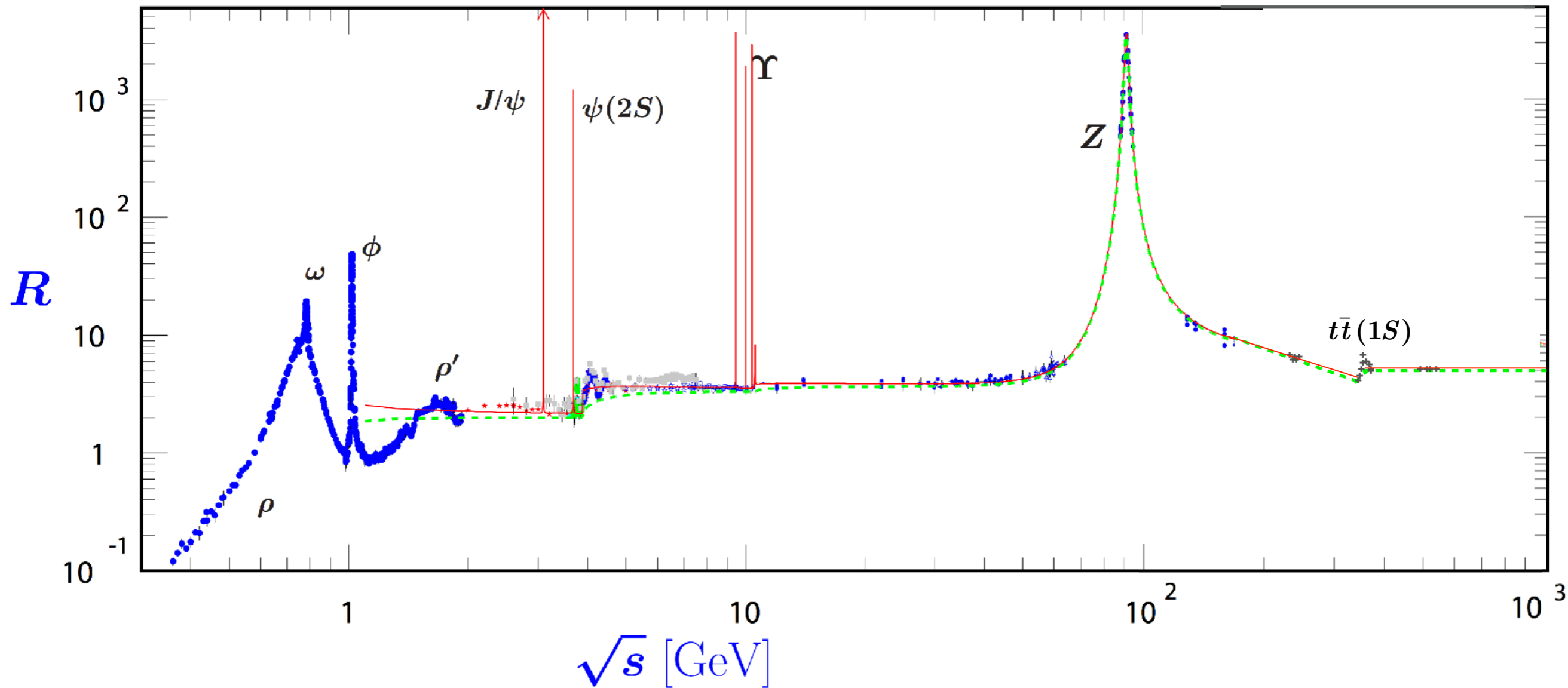
- Staged approach: start at 250 GeV without dedicated top programme
- Off-shell top measurements from $WbWb$ continuum
- Equiv. to single top: $tWb \longrightarrow WbWb$
- No recent systematic study: interplay of QCD and EW corrections
- Top mass determination from Higgs recoil measurement (radiative corr.)



Conclusions & Outlook

- Precision measurements at ILC elucidate top [-Higgs] sector
- Determination of fundamental parameters: m_t, Y_t, g_t^L, g_t^R
- BSM reach in top sector supersedes LHC searches in most models
- For almost all measurements: theory uncertainties dominate
- NLO QCD corrections well under control (10% at max, 1-2% at 3 TeV)
- Differential results as NLO fixed-order histograms (at the moment)
- Need for combined & **matched** NLO QCD+EW corrections in the future
- Work on this has started, also for the inclusion in event generators
- Top threshold in e^+e^- : measure top quark with ca. 50 MeV precision
- Assessment of theory uncertainties (NNLL + N(NN)LO matching)

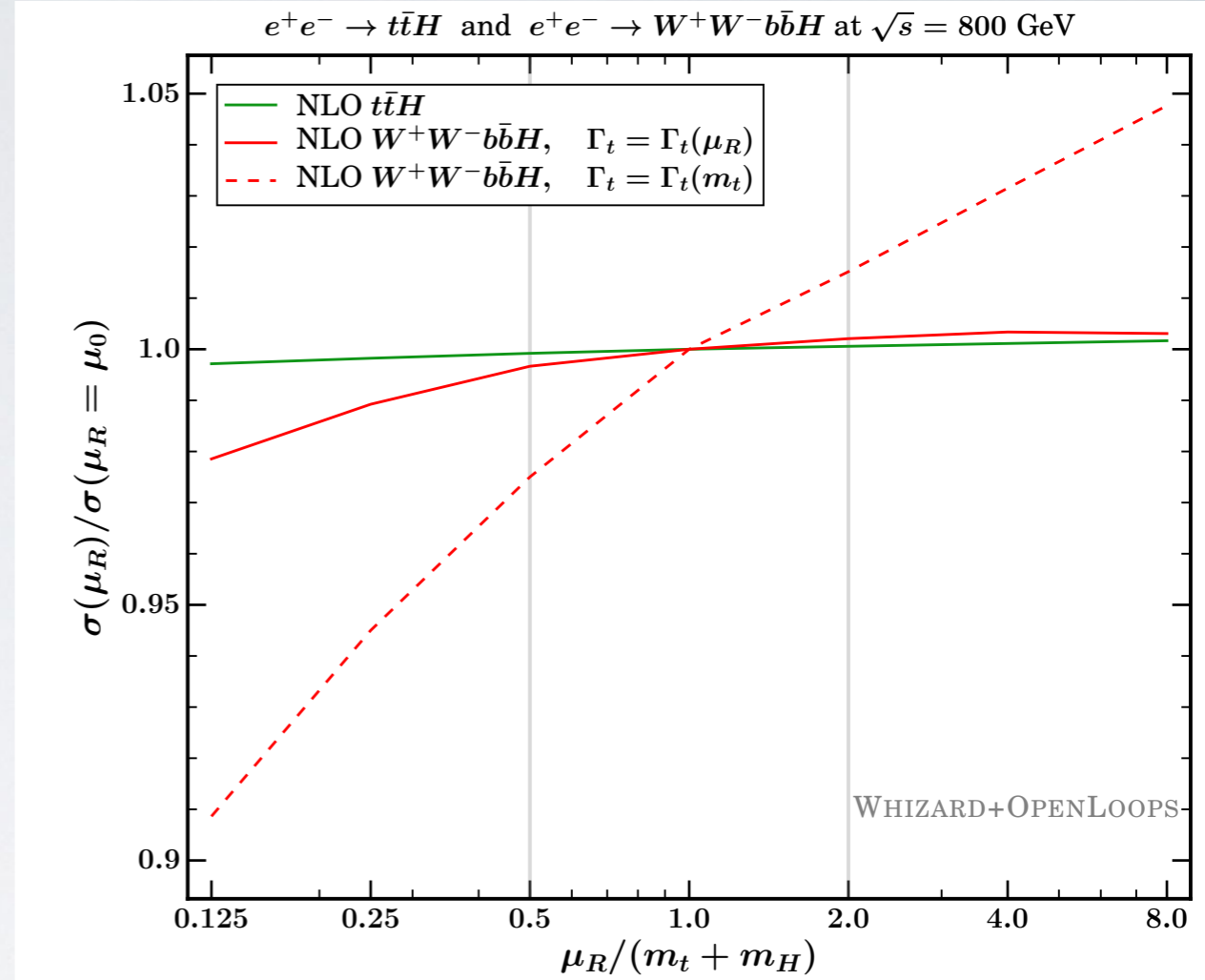
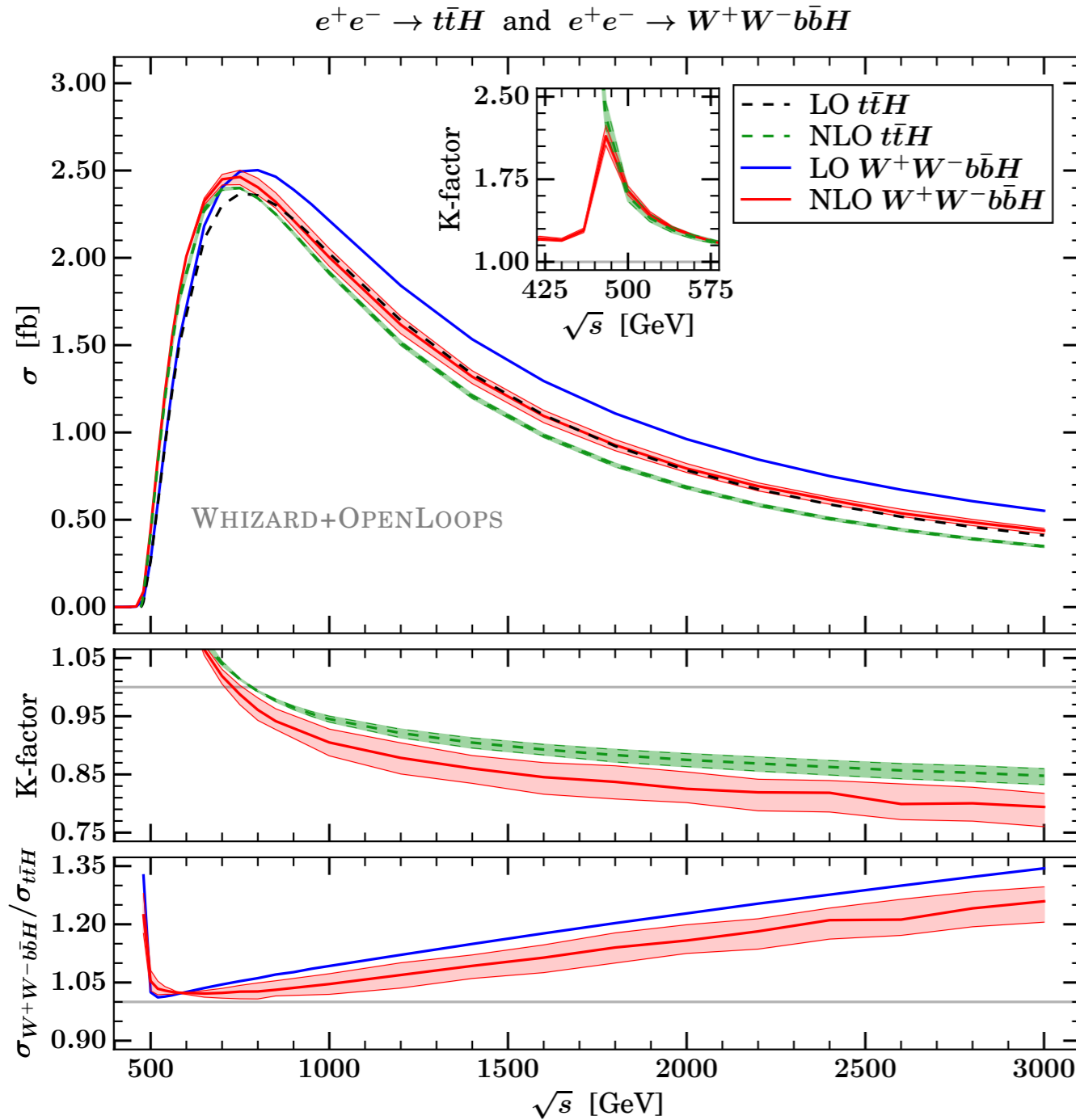
Looking forward to future data



BACKUP SLIDES



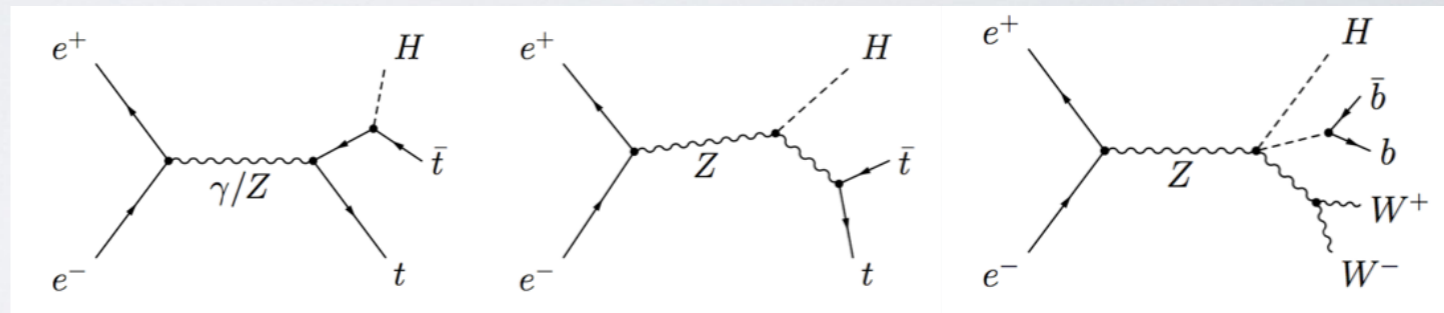
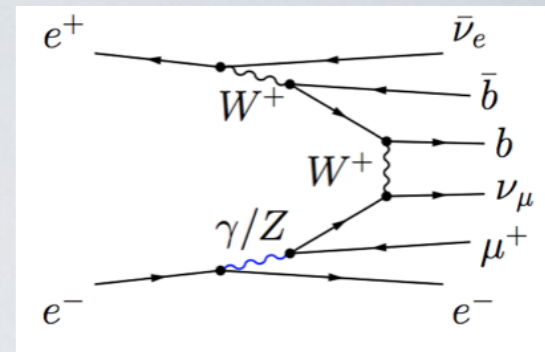
NLO QCD Results for off-shell $e^+e^- \rightarrow ttH$



\sqrt{s} [GeV]	$e^+e^- \rightarrow t\bar{t}H$			$e^+e^- \rightarrow W^+W^-b\bar{b}H$		
	σ^{LO} [fb]	σ^{NLO} [fb]	K-factor	σ^{LO} [fb]	σ^{NLO} [fb]	K-factor
500	0.26	$0.42^{+3.6\%}_{-3.1\%}$	1.60	0.27	$0.44^{+2.6\%}_{-2.4\%}$	1.63
800	2.36	$2.34^{+0.1\%}_{-0.1\%}$	0.99	2.50	$2.40^{+2.1\%}_{-1.9\%}$	0.96
1000	2.02	$1.91^{+0.5\%}_{-0.5\%}$	0.95	2.21	$2.00^{+2.5\%}_{-2.5\%}$	0.90
1400	1.33	$1.21^{+0.9\%}_{-1.0\%}$	0.90	1.53	$1.32^{+2.6\%}_{-3.0\%}$	0.86
3000	0.41	$0.35^{+1.4\%}_{-1.8\%}$	0.84	0.55	$0.44^{+2.9\%}_{-4.3\%}$	0.79

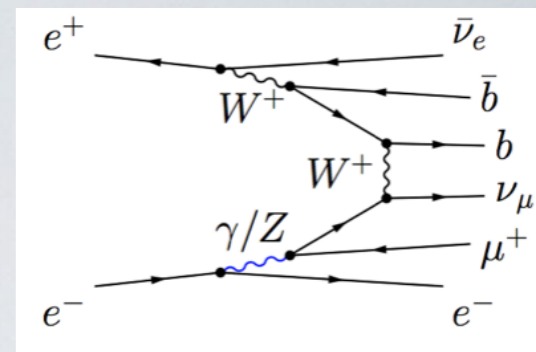
ILC: tt and ttH (on- & off-shell)

- Using massive b quarks: no cuts necessary for $e^+e^- \rightarrow W^+W^-bb$
- Full process $e^+e^- \rightarrow \mu^+\nu_\mu e^- \bar{\nu}_e bb$ exhibits Coulomb singularity:
- ttH production: 8% contamination from Higgsstrahlung
- Contribution from quartic SM vertices

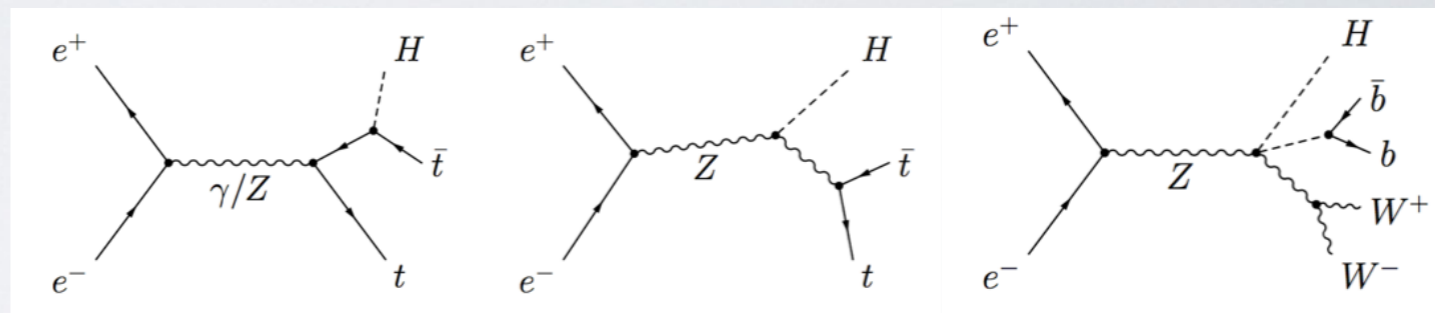


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INPUT PARAMETERS:



$$m_Z = 91.1876 \text{ GeV},$$

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$$m_H = 125 \text{ GeV}$$

$$\Gamma_H = 0.000431 \text{ GeV}$$

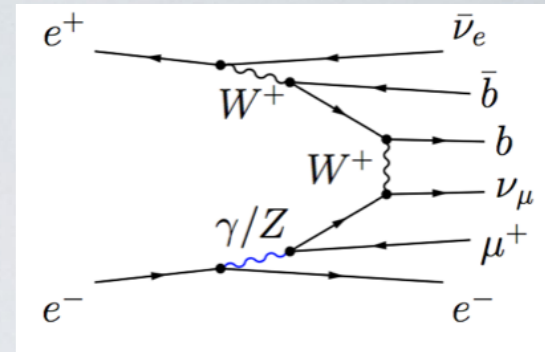
complex mass scheme:

$$\mu_i^2 = M_i^2 - i\Gamma_i M_i \quad \text{for } i = W, Z, t, H$$

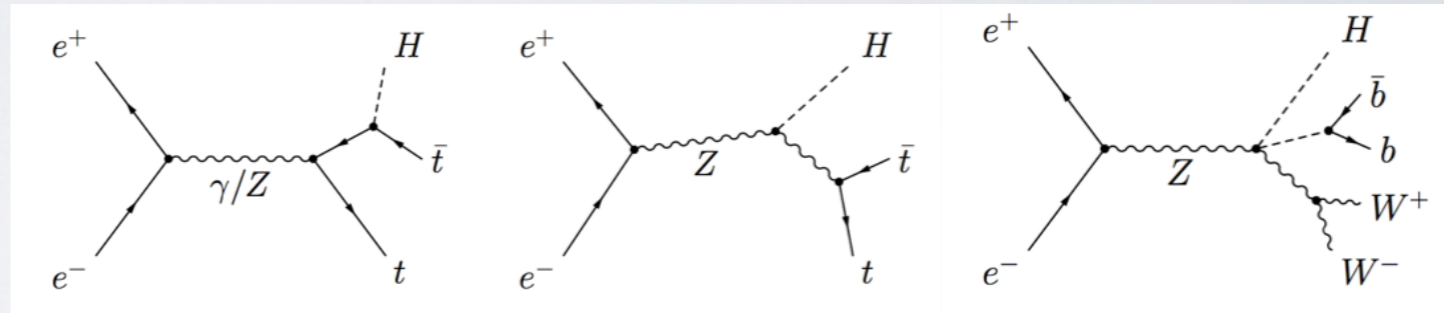
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- Typical pentagons/hexagons:

