# ADVANCED DIFFERENTIAL METHODS FOR THE MEASUREMENT OF KINEMATIC EDGES

And their application to the lepton colliders SUSY measurement



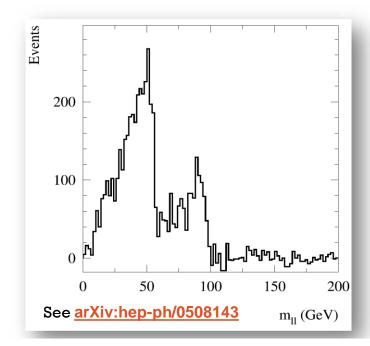
## KINEWATIC EDGES

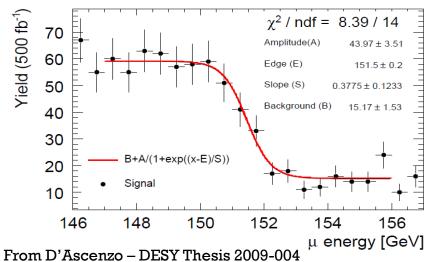
### Model independent signature

- Position depends only on kinematics (special relativity)
- Specific interaction irrelevant

### BSM searches

• Many searches rely on the identification and measurement of kinematic edges







## KINEWATIC EDGES WEASUREMENT

### Fitting an arbitrary function

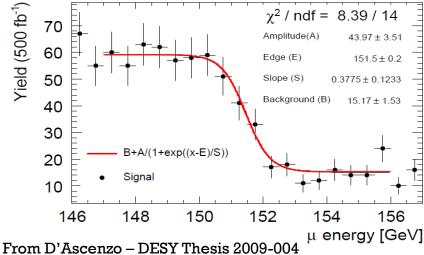
- Typically no physical reason behind the choice
- The function depends on the BG shape
- The function should depend on the "distortion sources"

### End point detection

- Model independent
- Conceptually slightly different
- Important cross-check

### What is an edge?

 Locus of the local maxima of the first derivative and 0 of the second derivative









## DIFFERENTIAL EDGE MEASUREMENT

### Derivative regularization

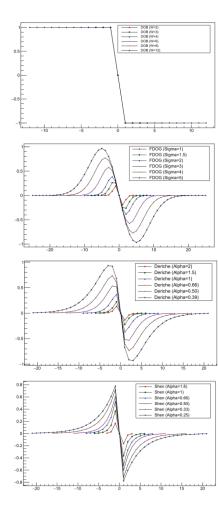
- The differential operator is not robust against noise
- Need some kind of preliminary smoothing
- Edge detection operator most used in image processing

### Implementation

- Regularization and differential operator merged (filter)
- Convolution of the filter with the target function
- Measurement of the response peak position

### Implemented filters

- Difference of boxes: Moving average
- First derivative of a Gaussian: Gaussian smoothing
- Deriche: Continuous exponential smoothing
- Shen: Exponential smoothing with discontinuity at the origin





## LOCALIZATION BENCHMARK PARAMETER

### Definition

- The accuracy in the determination of the edge position in a noisy environment
- Measured with a toy MC by randomly fluctuating the template distribution within its errors

#### Bias

- The average distance between the true and measured position
- If different than 0 defines the calibration to apply to obtain the "true position"

#### Error

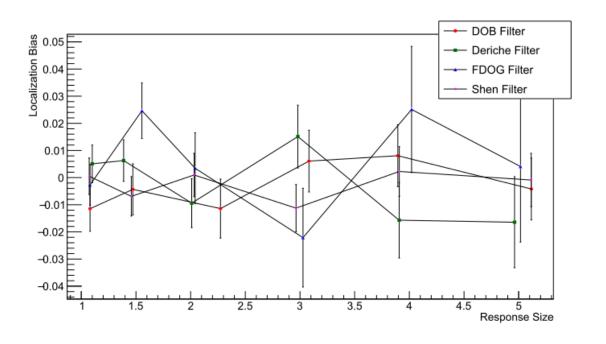
- RMS of the measured position
- · Defines the measurement uncertainty

### Depends on

- Peak interpolation algorithm
- Signal shape
- Filter size
- Signal to noise ratio



## IDEAL STEP

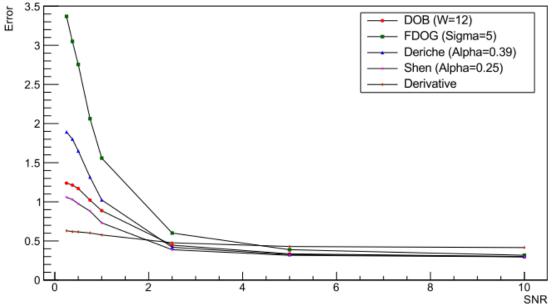


### Bias compatible with 0

• Cross-check for the quality of the implementation

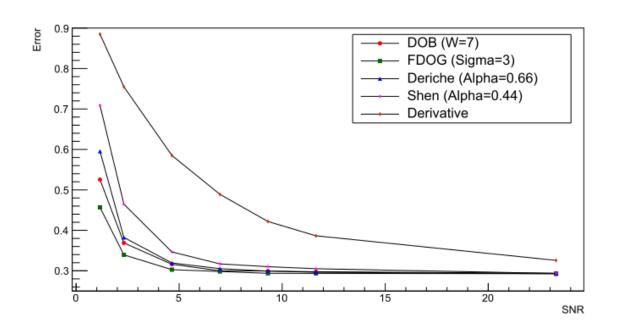
### Asymptotic decrease

• Similar localization at high Signal-to-Noise ratio





## SMOOTH STEP

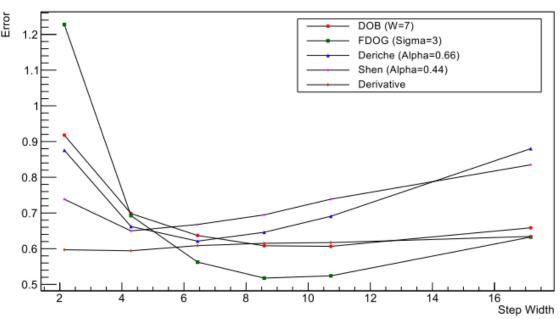


### Asymptotic behavior

- Similar to the ideal case
- The common derivative now is much worse

### Size optimization

- Error is minimized at fix width/size ratios
- Optimization depends on the filter type
- FDOG is typically the best at its optimal size.







# SELECTRON-NEUTRALINO MASS MEASUREMENT

## ILC SELECTRON PRODUCTION

### Production processes

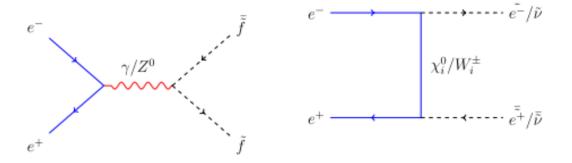
- Both s- and t- channel processes
- Constructive interference
- T-channel dominance

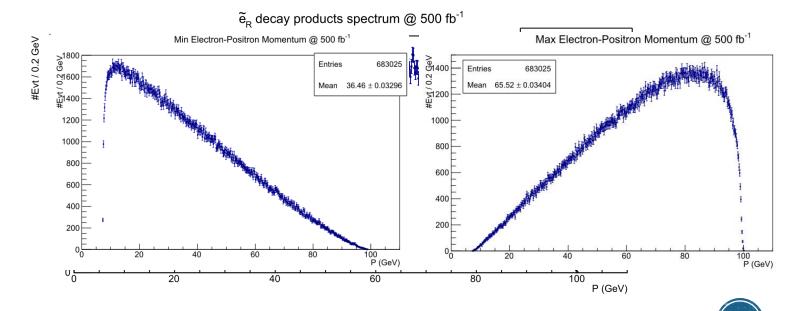
### Kinematic edges

- Momentum edges depending on the SUSY masses
- 2 edges for 2 masses
- Separate measurement

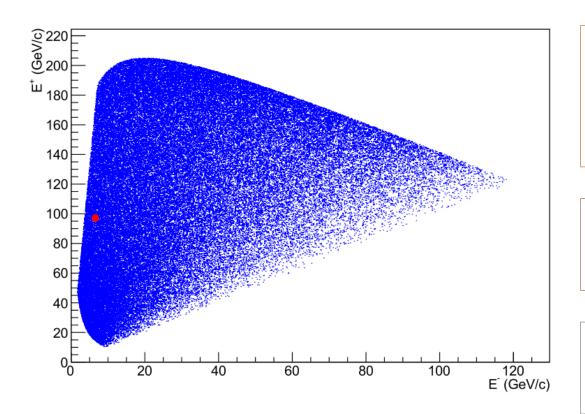
### **Experimental distortions**

- Beamstrahlung
- ISR
- Detector reconstruction
- Selection





## ANALYSIS LIMITS



### LEP limits

- Neutralino mass > 46 GeV
- Selectron mass > 97.5 GeV
- Mass difference > 10 GeV

### **ILC** parameters

- 500 GeV
- 500 fb<sup>-1</sup>
- 80% right handed electrons
- 30% left handed positrons

### Edge limits

Max edge: 10-205 GeVMin edge: 2.5-120 GeV

### STC4 SUSY Benchmark

Selectron mass: 126.235 GeV
Neutralino mass: 95.586 GeV



## HIGH EDGE SELECTION

## Optimized to reduce distortions of the high edge

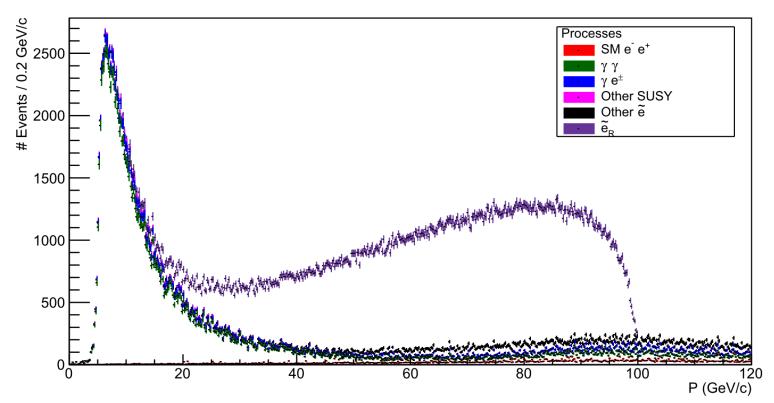
- Low energy peak irrelevant
- Would work for almost all the available mass range
- About 82% acceptance of the signal sample

### Edge position measurement

• Localization error: 42 MeV

Localization bias: -605 MeV

### Momentum of the stiffest jet





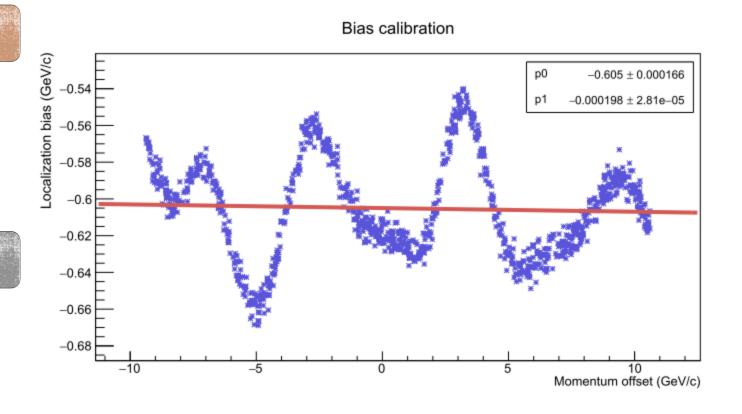
## HIGH EDGE CALIBRATION

### Technique

- Apply a random offset to the signal sample
- Adding the signal to a fixed background
- Using the new histogram as a template for the full edge measurement procedure

### Result

- Flat calibration curve
- Fluctuation due to the background uncertainties
- Bias error (RMS of the results): 26 MeV





## LOW EDGE SELECTION

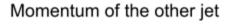
### Stricter analysis

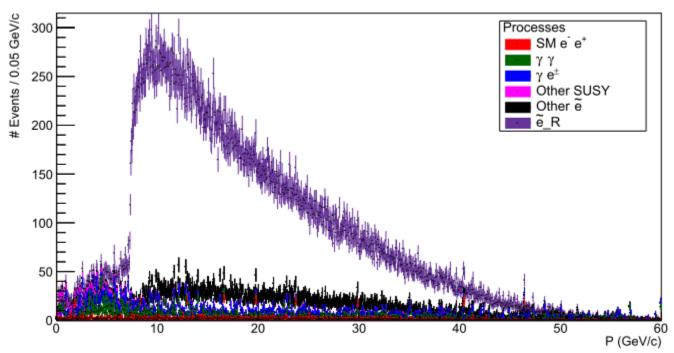
- Removal of the low energy background
- Additional collinearity and coplanarity cuts
- 22% acceptance but 4.26 total signal purity

### Edge position measurement

• Localization error: 14 MeV

Localization bias: 94 MeV







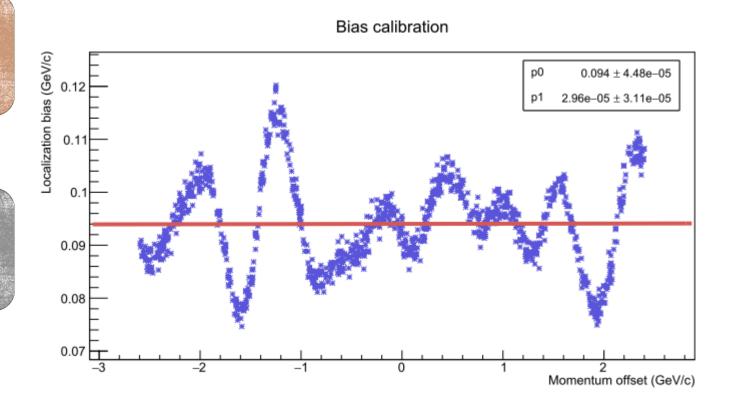
## LOW EDGE CALIBRATION

## Same technique as before

• Smaller range, proportional to the binning chosen

### Results

- Flat calibration curve
- Bias error: 8.4 MeV





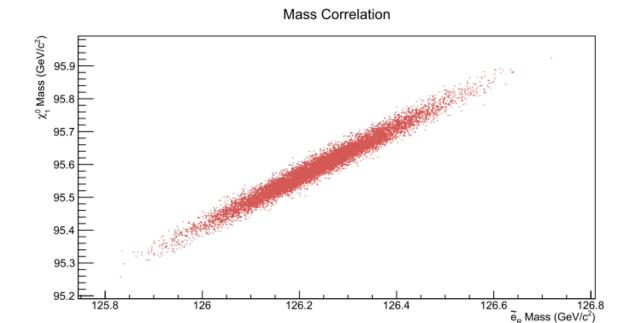
## MASS WEASUREMENT

### Procedure

- Use the final selection histograms as templates
- Generate a new pair for each data point and measure the edge position
- Subtract the bias, variable according to the calculated error
- Extract mass errors and correlations
- Consistent with analytic error propagation assuming edge positions as independent variables

### Final results

- Measured selectron mass: 126.242 +- 0.119 GeV
- True selectron mass: 126.235 GeV
- Neutralino mass: 95.594 +- 0.091 GeV
- True neutralino mass: 95.586 GeV
- Covariance: 0.0106 GeV^2
- Correlation coefficient: 0.749





## CONCLUSIONS

## CONCLUSIVE SUMMARY

Differential operator is the most direct way to measure the position of an edgelike structure

• Differential operators must be made robust against noise to be applied to experimental distributions

### Differential edge detection algorithm

- Robust differential operator
- Data-driven optimization procedure

### Selectron-LSP mass measurement

- Applied to the STC4 SUSY benchmark point.
- Cut based analysis

### Final results

- Selectron mass error: 119 GeV (was 210)
- Neutralino mass error: 91 GeV (was 160)



## BACKUP



## OPTIMIZATION PROCEDURE

### MC Toy optimization

- Use the input distribution as a function template.
- Vary the input according to the template n times and measure the edge position to evaluate the localization error

### Fixed binning

- Shen or Deriche for small features (depending on the noise amplitude)
- FDOG for extended edges

### Variable binning (histograms)

- Optimize the binning to extend the edge size keeping the relative noise low enough
- Use the MC Toy optimization to minimize the error

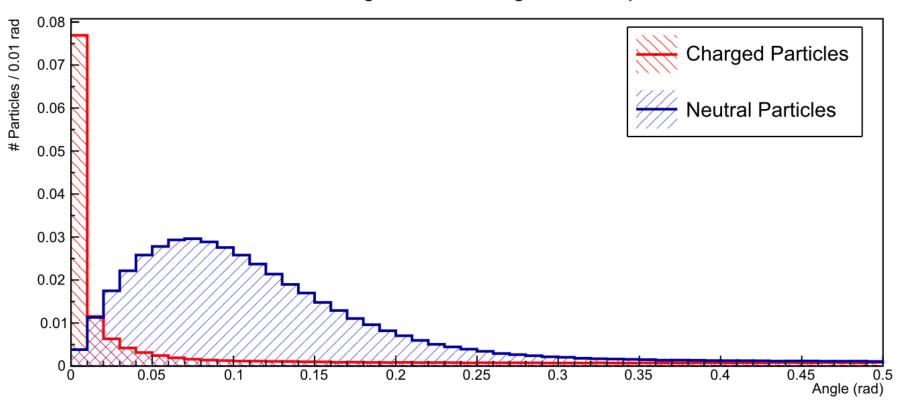




# ELECTRON MOMENTUM RECONSTRUCTION

## ANGULAR DISTANCE - ZOOM IN

Angular distance against seed particle





## JET FINDER COMPARE

