Top electroweak couplings study using di-muonic state at $\sqrt{s} = 500$ GeV, ILC

Full ILD detector simulation

50th General meeting

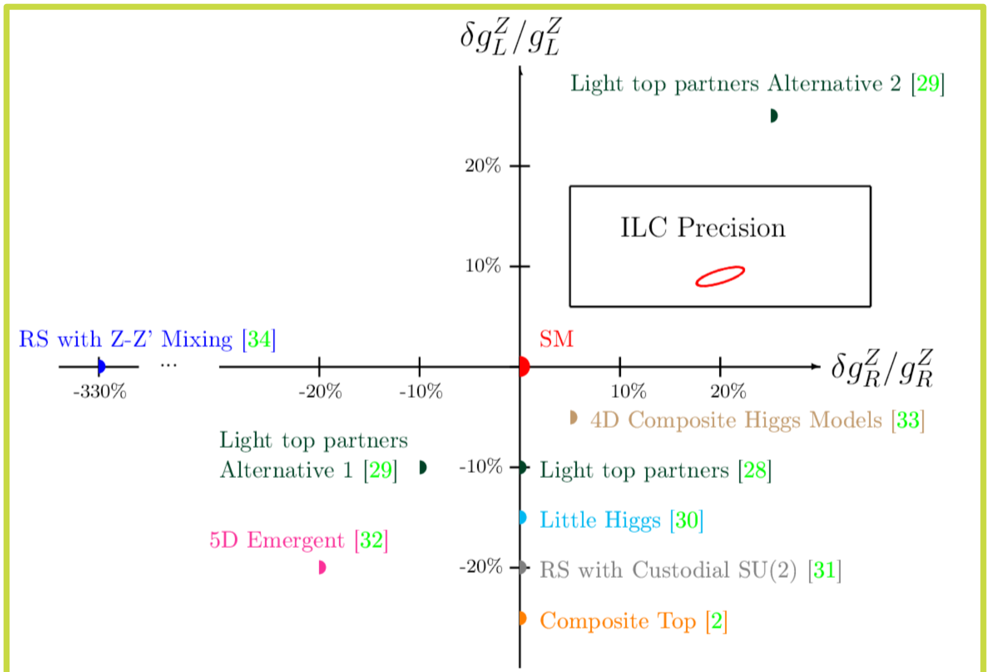
Yo Sato[A]

Emi Kou[B], Francois Le Diberder[B], Hitoshi Yamamoto[A]

Tohoku University[A], LAL[B]

Top electroweak couplings

The top quark mass is comparable with the electroweak symmetry breaking scale. One can speculate that top quark plays a special role for the EWSB, for example such composite models. Therefore top quark electroweak couplings are good probes for New Physics.



Plot shows the predicted deviations from the Standard model of Z^0 couplings to t_L and t_R in composite models

Precision expected at the ILC will allow to distinguish between models.

arXiv:1505.06020 [hep-ph]

Matrix element method

The most efficient method when all the kinematics can be reconstructed.

- Results of previous study show that 10 form factors can be fitted simultaneously at less than a percent precision.

Statistical uncertainties and correlation with the SM LO as normalization

Kheim, E.K. Kurihara, Le Diberder: arXiv:1503:04247

$\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V}$	$\mathcal{R}e\ \delta\tilde{F}^{Z}_{1V}$	$\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A}$	$\mathcal{R}e\ \delta\tilde{F}^{Z}_{1A}$	$\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V}$	$\mathcal{R}e\ \delta\tilde{F}^{Z}_{2V}$	$\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A}$	$\mathcal{R}e\ \delta\tilde{F}^{Z}_{2A}$	$\mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A}$	$\mathcal{I}m\ \delta\tilde{F}^{Z}_{2A}$
0.0037	−0.18	−0.09	+0.14	+0.62	−0.15	0	0	0	0
	0.0063	+.14	−0.06	−0.13	+0.61	0	0	0	0
		0.0053	−0.15	−0.05	+0.09	0	0	0	0
			0.0083	+0.06	−0.04	0	0	0	0
				0.0105	−0.19	0	0	0	0
					0.0169	0	0	0	0
						0.0068	−0.15	0	0
							0.0118	0	0
								0.0069	−0.17
									0.0100

500 GeV&500 fb^{-1} Polarization 50/50 between ±80% and ±30%

Emi Kou (LAL-Orsay)
LFC 15, Trento,
7-11 Sep. 2015

This result is at parton level ignoring the detector effect, ISR and so on.

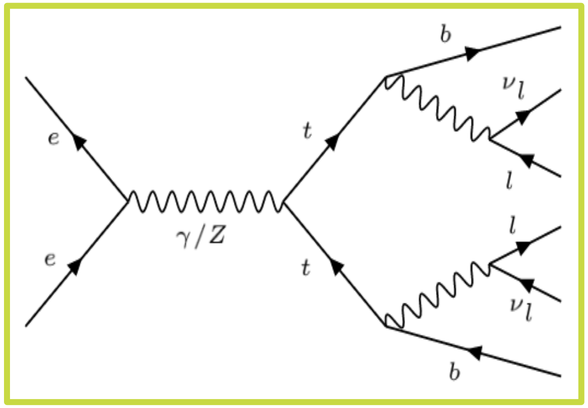
→More realistic study is required !

Setting of this study

Sample

The top pair production di-muonic state;

$t\bar{t} \rightarrow b\bar{b}\mu^{+}\mu^{-}\nu\bar{\nu}$ at $\sqrt{s}$ = 500 GeV



Situation

- ✓ The hadronization of b and $\bar{b}$ quark
- ✓ The detector effects (= full simulation)

} case A.

- ✓ ISR and beamsstrahlung
- ✓ Gluon emission from top quark
- ✓ γγ → hadrons background

} case B. (all five items above)

- × Background events

Reconstruction

Process of the reconstruction

- Isolated leptons tagging
- B-jets reconstruction using the Thrust axis method
- Kinematical reconstruction

Thrust axis method

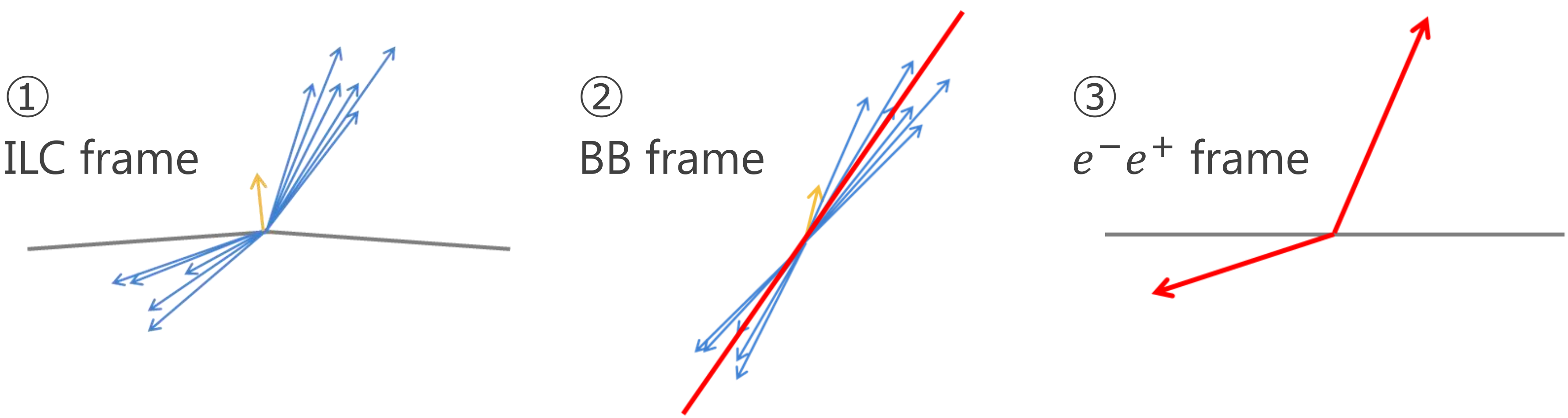
① Collect all hadronized particles in the ILC frame

② Boost them into the BB frame and calculate thrust axis

③ Boost the vectors along the thrust axis into the rest frame of e^-e^+

case A. using same parameters with ②

case B. considering ISR/BS effects using → introduce the k_{e^-}, k_{e^+}



Considering ISR/BS effects

Collinear approximation:
Photons are emitted on the beam directions by ISR/BS

$$\vec{e}^{\,-} = \hat{\eta}_{e^-} E_{e^-} \quad (1)$$
$$\vec{e}^{\,+} = \hat{\eta}_{e^+} E_{e^+} \quad (2)$$

with,

$$\hat{\eta}_{e^-} = (\sin\theta_c, 0, \cos\theta_c) \quad (3)$$
$$\hat{\eta}_{e^+} = (\sin\theta_c, 0, -\cos\theta_c) \quad (4)$$
$$E_{e^\pm} = E = 250 \text{ GeV} \quad (5)$$

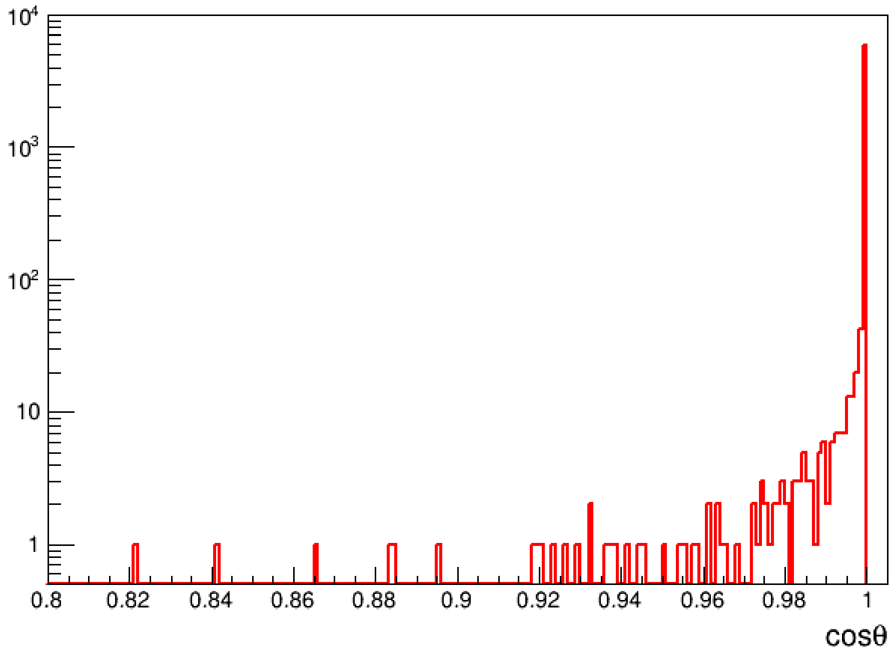
where θ_c is the beam crossing angle, $\theta_c = 7$ mrad.

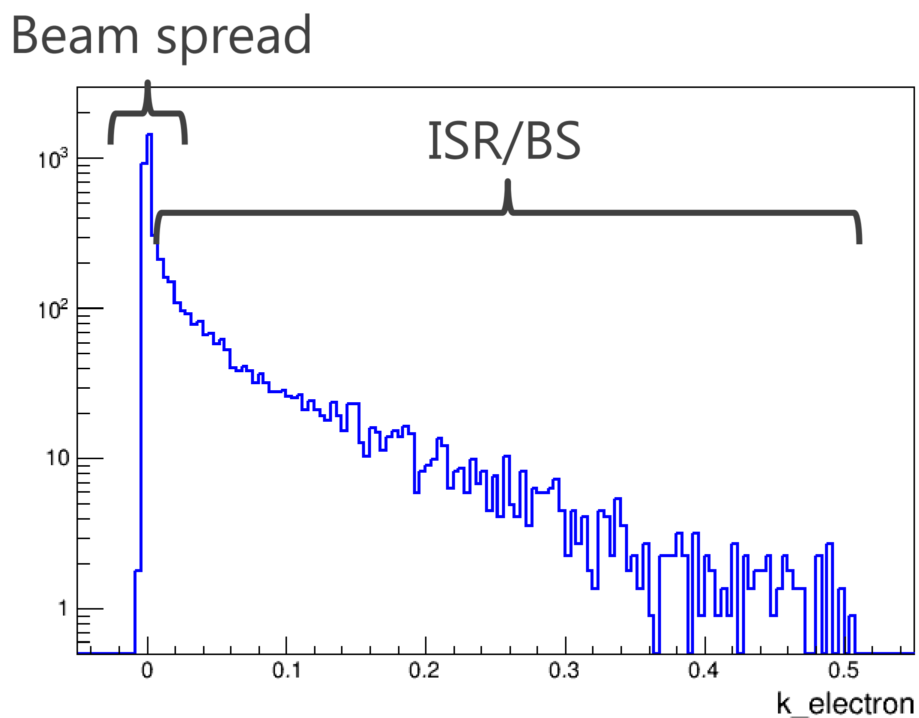
In this approximation, the directions are not changed but only the energies are changed. Then the electron and positron thre-momenta become:

$$(\vec{e}^{\,-})^* = \hat{\eta}_{e^-} E^*_{e^-} = \hat{\eta}_{e^-} E(1 - k_{e^-}) \quad \text{with} \quad k_{e^-} = \frac{E - E^*_{e^-}}{E} \quad (6)$$
$$(\vec{e}^{\,+})^* = \hat{\eta}_{e^+} E^*_{e^+} = \hat{\eta}_{e^+} E(1 - k_{e^+}) \quad \text{with} \quad k_{e^+} = \frac{E - E^*_{e^+}}{E} \quad (7)$$

where $E^*_{e^\pm}$ is the energy of electron or positron just before collision.

Distribution of $\cos\theta$ and k_{e^-} (MC truth)





$\cos\theta$

(θ : angle between e^{-*} and beam direction)

$$\frac{250 - E^*_{e^-}}{250} (= k_{e^-})$$

(Indeed, it is not same as k_{e^-} because $\cos\theta$ has width)

Kinematical reconstruction

Strategy of the kinematical reconstruction

- **There are 8 unknown kinematics in this state**

  (= the momenta of two neutrinos and energy of two b-jets)

- **Impose 8 constraints** (= initial state constraints and $m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$)

  → Solutions are obtained in terms of (θ_t, ϕ_t) → $(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-})$

  But the equation is nonlinear. Furthermore an ambiguity of b-charge remains.

  → **Typically 4 solutions per event.**

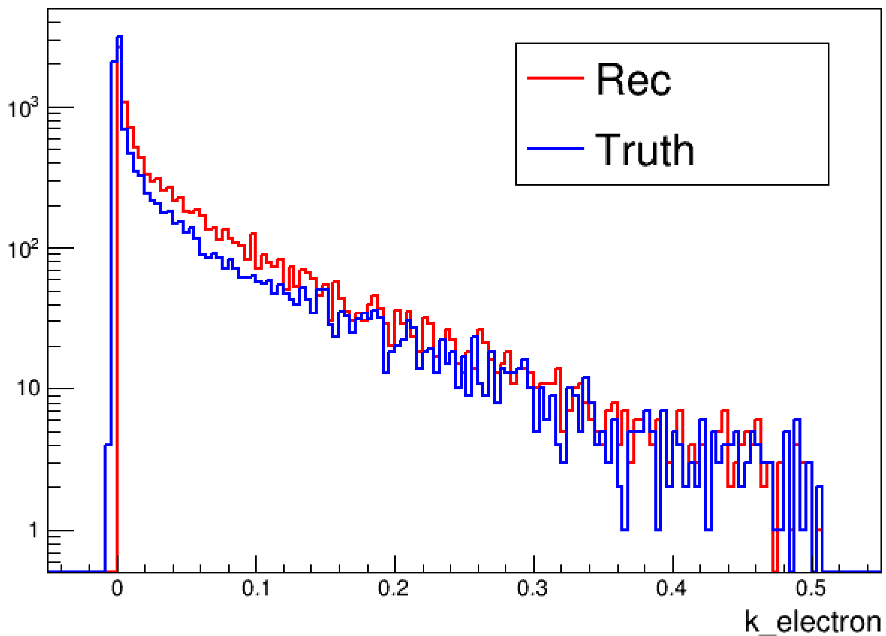
- **Select the optimal solution**

  Compare $E_b^{\text{meas.}}$ (by thrust axis method) and $E_b^{\text{rec.}}$ (by kinematical reconstruction).

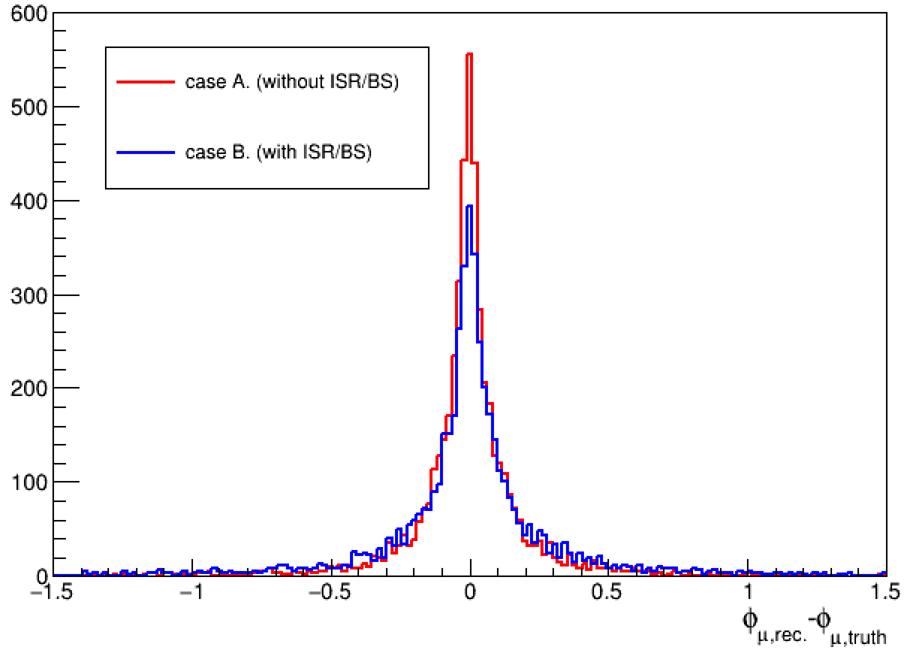
and (k_{e^-}, k_{e^+}) in case B.

Kinematical reconstruction : Results

k_{e^-} (only **case B.**)



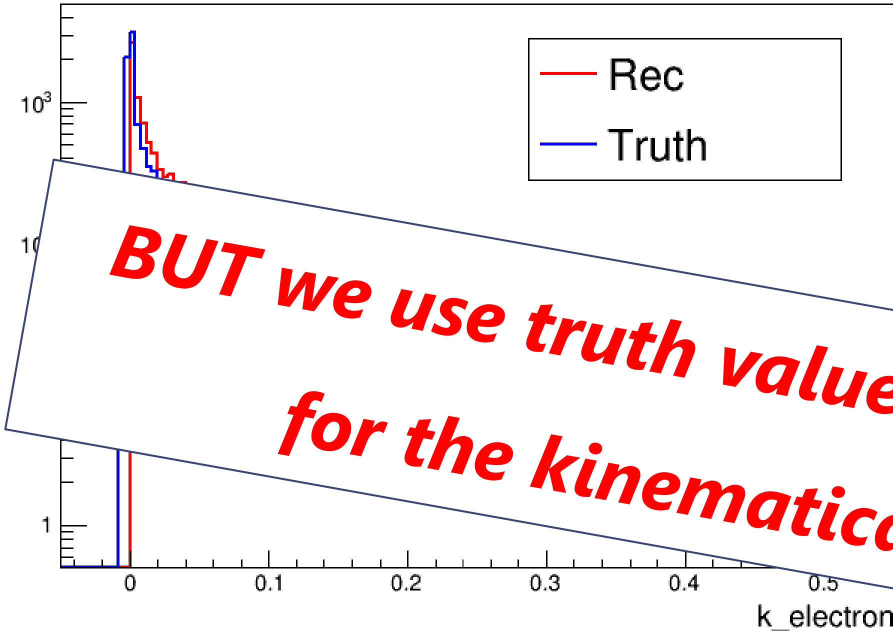
$\phi_{\mu^+,rec.} - \phi_{\mu^+,truth}$



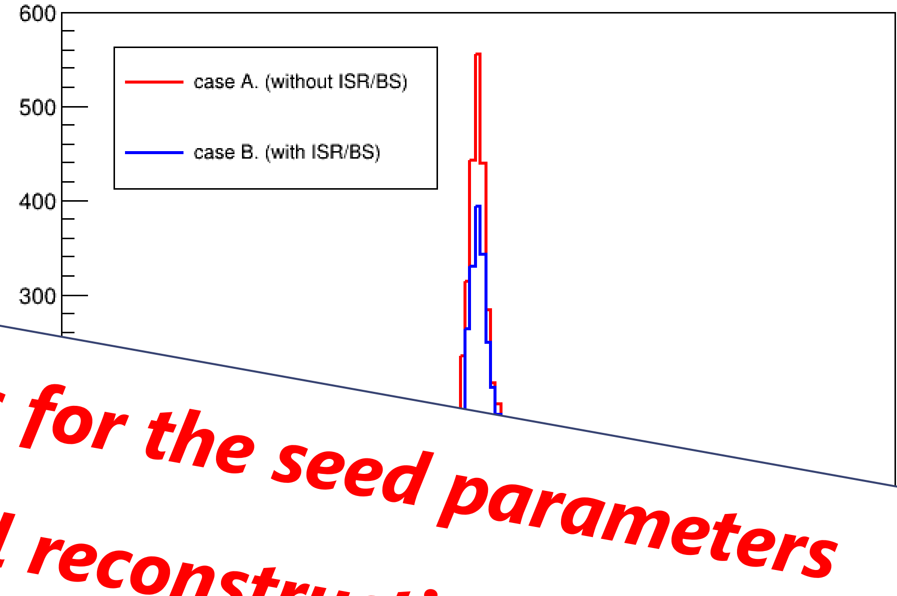
- **case B.** is not that bad comparing with **case A.**
  → Need to check if it is enough for the analysis using MEM

Kinematical reconstruction : Results

k_{e^-} (only **case B.**)



$\phi_{\mu^+,rec.} - \phi_{\mu^+,truth}$



BUT we use truth values for the seed parameters for the kinematical reconstruction !!!

- **case B.** is not that bad comparing with **case A.**

  → Need to check if it is enough for the analysis using MEM

Status of the 10 form factors fit (case A.)

Δ_{F1}	-0.0067 $\pm$ 0.0082
Δ_{F2}	0.035 $\pm$ 0.017
Δ_{F3}	-0.056 $\pm$ 0.012
Δ_{F4}	0.035 $\pm$ 0.018
Δ_{F5}	-0.022 $\pm$ 0.026
Δ_{F6}	0.042 $\pm$ 0.045
Δ_{F7}	-0.0081 $\pm$ 0.015
Δ_{F8}	0.010 $\pm$ 0.032
Δ_{F9}	0.013 $\pm$ 0.024
Δ_{F10}	-0.010 $\pm$ 0.022

(Preliminary)

5000 events, after cut on the $\chi^2_{\text{tot.}}$ to keep ~83% of the events without ISR/BS

Some small biases are observed (eg. Δ_{F3}) at few percent level

→ **No show stopper yet !!!**

→ Use DBD samples for the matrix element method

Summary & Plan

- **Started realistic study using DBD samples**
  - Accuracy of kinematical reconstruction is not that bad comparing the samples without ISR/BS
- **Analysis with Matrix element method**
  - 10 form factors can be fitted at percent precision in **case A.**

  → Apply Matrix element method on DBD samples (**case B.**)

- **Visit Francois at LAL to learn MEM and statistics in March**
- **Borys, master student from Ukraine will join us to work on seeds issue at LAL.**

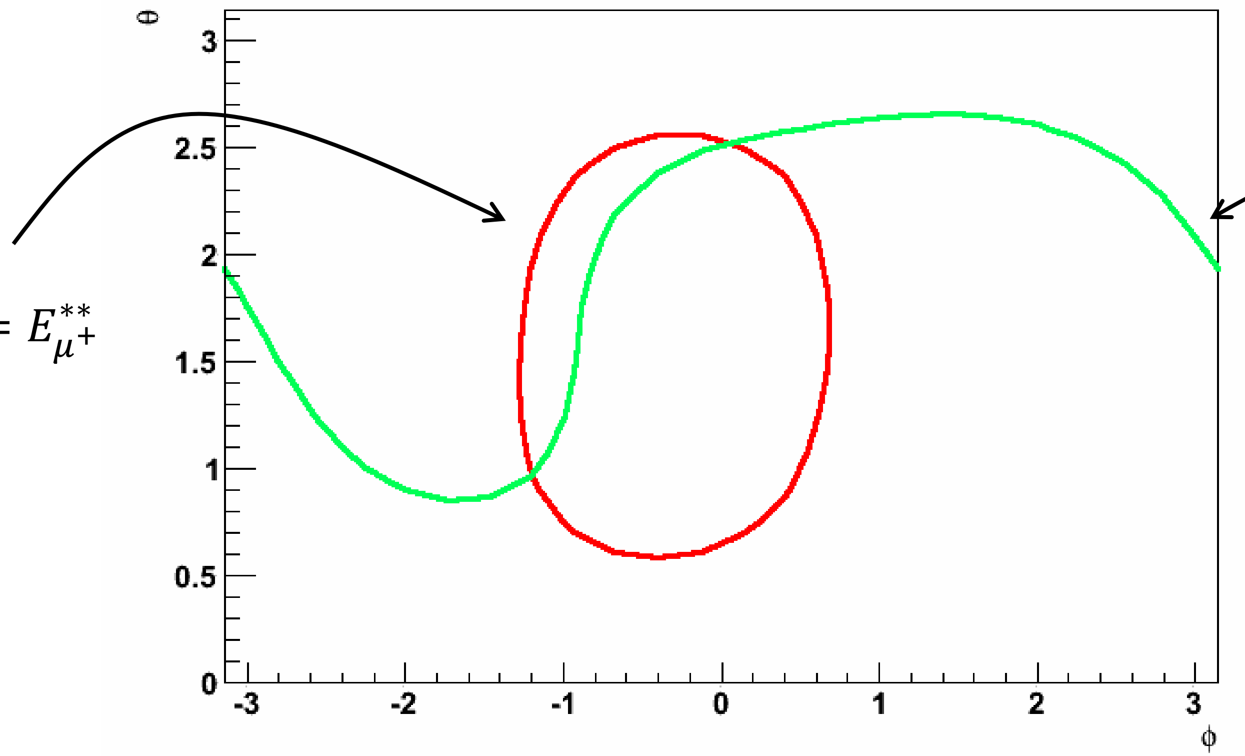
Back up

Kinematical constraints

In the W rest frame, the energy of isolated lepton is equal to $m_W/2$

(with ignoring ISR and bremsstrahlung)

$$\frac{m_{W^+}}{2} = E^{**}_{\mu^+}$$

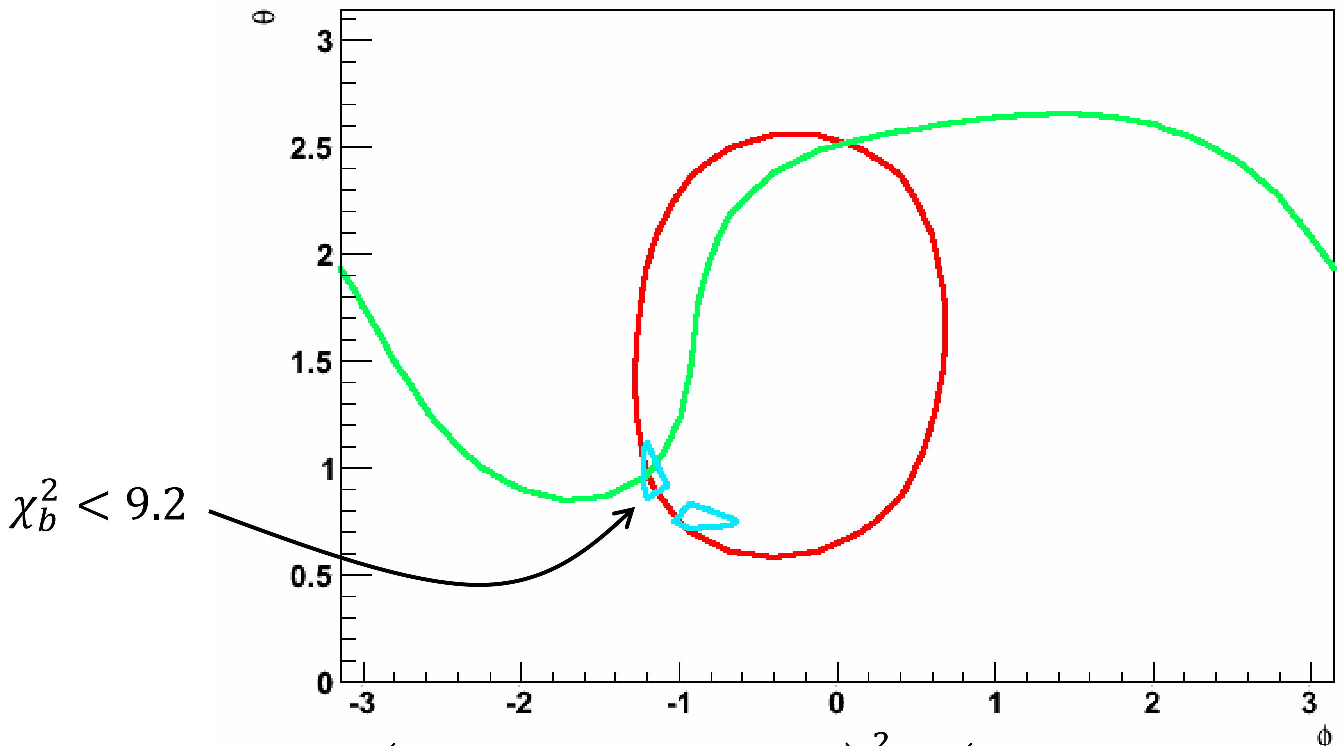


$$\frac{m_{W^-}}{2} = E^{**}_{\mu^-}$$

Measurements of b-quark energies

To select a right solution, we can use the measurement of b-jets energy.

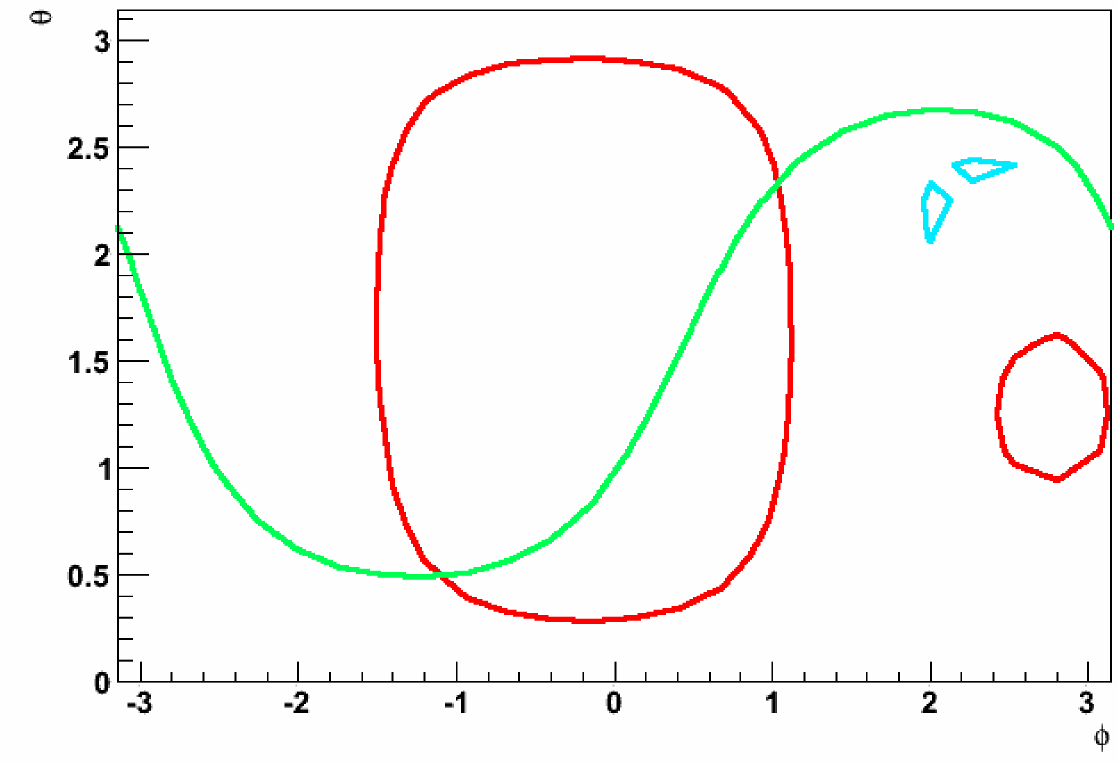
(Because this figure is at parton level, the χ_b^2 doesn't make sense.)



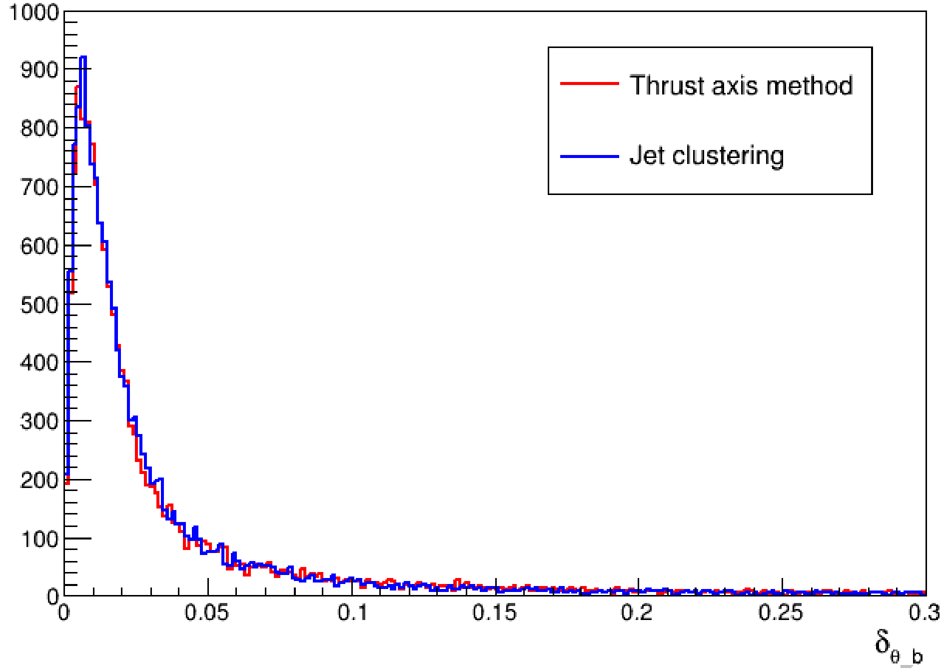
$$\chi_b^2(\phi_t,\theta_t) = \left(\frac{E_{b,mes} - E_{b,reco}(\phi_t,\theta_t)}{\sigma_j\sqrt{E_{b,mes}}}\right)^2 + \left(\frac{E_{\bar{b},mes} - E_{\bar{b},reco}(\phi_t,\theta_t)}{\sigma_j\sqrt{E_{\bar{b},mes}}}\right)^2$$

Miss combination of b-quarks

When we use the anti-b direction for the top reconstruction, the measurements of energy of b-jets excludes this combination.



Comparison Thrust axis method and Jet clustering



The angle between truth direction and reconstructed direction of b quark

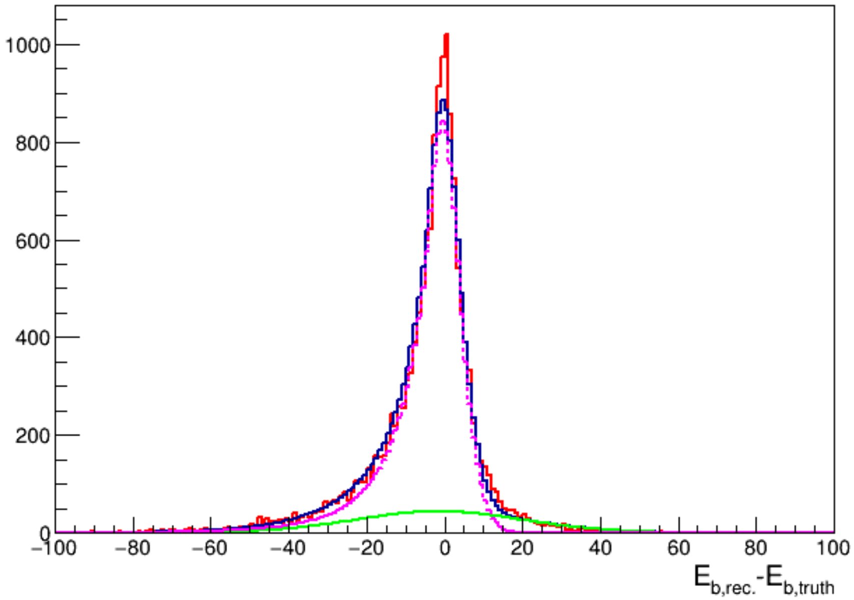
Red : Thrust axis method

Blue : Jet clustering (LCFIPlus)

Two methods produce almost same precision for direction of b-quark.

→We select the thrust axis method for this study so far.

Assessment of energy of b-jet



Deviation of energy of b-jet

(using thrust axis method)

Red : Original

Blue : Fitted

(Blue is a sum of Pink and Green)

To estimate the b-jet energy resolution we use multiple Crystal Ball functions for fitting.

$$CB(x|\alpha, n, \mu, \sigma) = \begin{cases} N \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), & \frac{x-\mu}{\sigma} > -\alpha \\ N \cdot A \cdot \left(B - \frac{x-\mu}{\sigma}\right)^{-n}, & \frac{x-\mu}{\sigma} < -\alpha \end{cases}$$

So far we use $\sigma_{jet} E_b^K$ with two parameters σ_{jet} and K for σ.

χ^2 algorithm : General

1. Define the χ_μ^2 ;

$$\chi_\mu^2 = \chi_{\mu^+}^2 + \chi_{\mu^-}^2, \quad \chi_{\mu^\pm}^2 = \left(\frac{E_{\mu^\pm}^{**}(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}) - m_{W^\pm}/2}{\sigma\left[E_{\mu^\pm}^{**}\right]} \right)^2$$

The energy of $\mu^\pm$ in the $W^\pm$ rest frame, $E_{\mu^\pm}^{**}$, must be equal to $m_{W^\pm}/2$.

2. Define the δ_b^2 ;

$$\delta_b^2 = -2 \log L_b - 2 \log L_{\bar{b}}, \quad L_b = \mathrm{CB}\left(E_b^{\mathrm{meas.}} - E_b^{\mathrm{rec.}}(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-})\right)$$

The likelihood function is obtained from the assessment of b-jets energy.

3. Compound $\chi_{\mathrm{tot.}}^2$; $\chi_{\mathrm{tot.}}^2 = \chi_\mu^2 + \delta_b^2$

One minimizes the $\chi_{\mathrm{tot.}}^2$ to obtain the optimal solution; $(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-})$.

χ^2 algorithm : Optional

The direction of b-jets is obtained by thrust axis method.

→ Add 4 angles $(\theta_b, \phi_b, \theta_{\bar{b}}, \phi_{\bar{b}})$ to the minimization parameters

→ Add constraints of 4 angles $(\theta_b, \phi_b, \theta_{\bar{b}}, \phi_{\bar{b}})$ to $\chi^2_{\mathrm{tot.}}$ as follows;

$$\chi^2_{\mathrm{direction}} = \chi^2_{\theta_b} + \chi^2_{\phi_b} + \chi^2_{\theta_{\bar{b}}} + \chi^2_{\phi_{\bar{b}}}, \qquad \chi^2_{\theta_b} = \left(\frac{\theta_b^{\mathrm{meas.}} - \theta_b}{\sigma[\theta_b^{\mathrm{meas.}}]} \right)^2$$

$(\chi^2_{\phi_b}, \chi^2_{\theta_{\bar{b}}}, \chi^2_{\phi_{\bar{b}}}$ are same as $\chi^2_{\theta_b})$

$$(\chi^2_{\mathrm{tot.}})' = \chi^2_{\mathrm{tot.}} + \chi^2_{\mathrm{direction}}$$

We can use $(\chi^2_{\mathrm{tot.}})'$ instead of $\chi^2_{\mathrm{tot.}}$ to get the optimal solution,

which is written in $(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}, \theta_b, \phi_b, \theta_{\bar{b}}, \phi_{\bar{b}})$

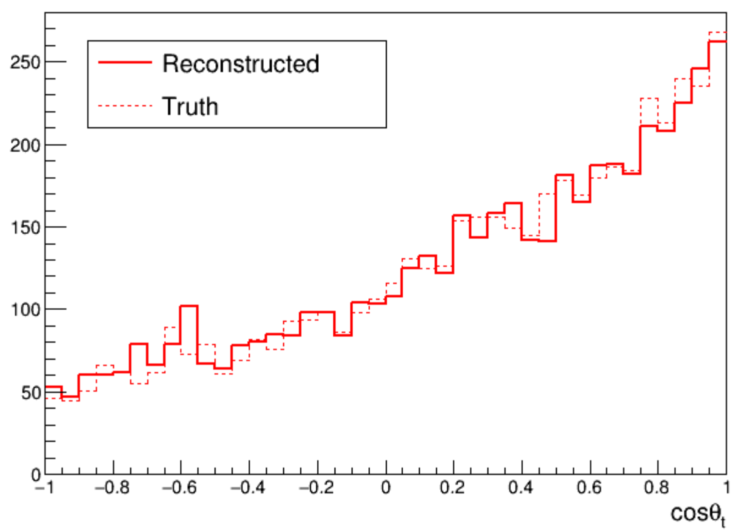
Kinematical reconstruction : Results

Reconstructed particles → 9 helicity angles :

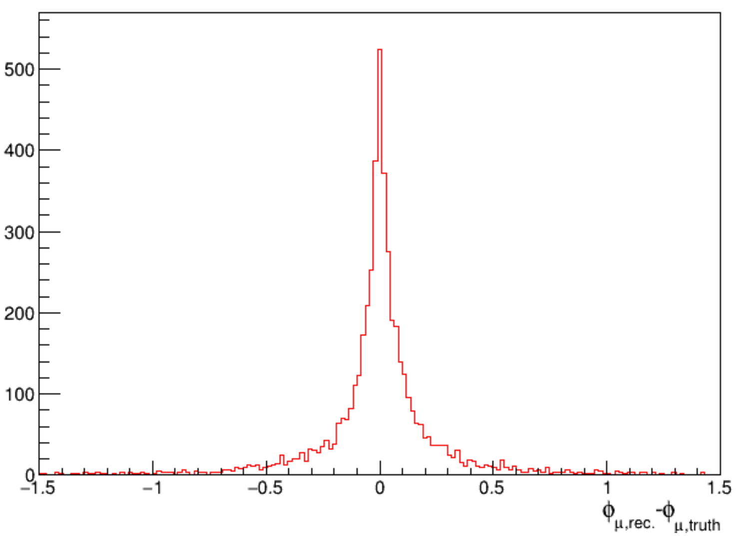
$$\cos\theta_t\,,\cos\theta_{W^+}\,,\phi_{W^+},\cos\theta_{\mu^+}\,,\phi_{\mu^+},\cos\theta_{W^-}\,,\phi_{W^-},\cos\theta_{\mu^-}\,,\phi_{\mu^-}$$

(→ Matrix element squared → Fit Form Factors)

eg.) $\cos\theta_t$



eg.) $\phi_{\mu^+,rec.} - \phi_{\mu^+,truth}$



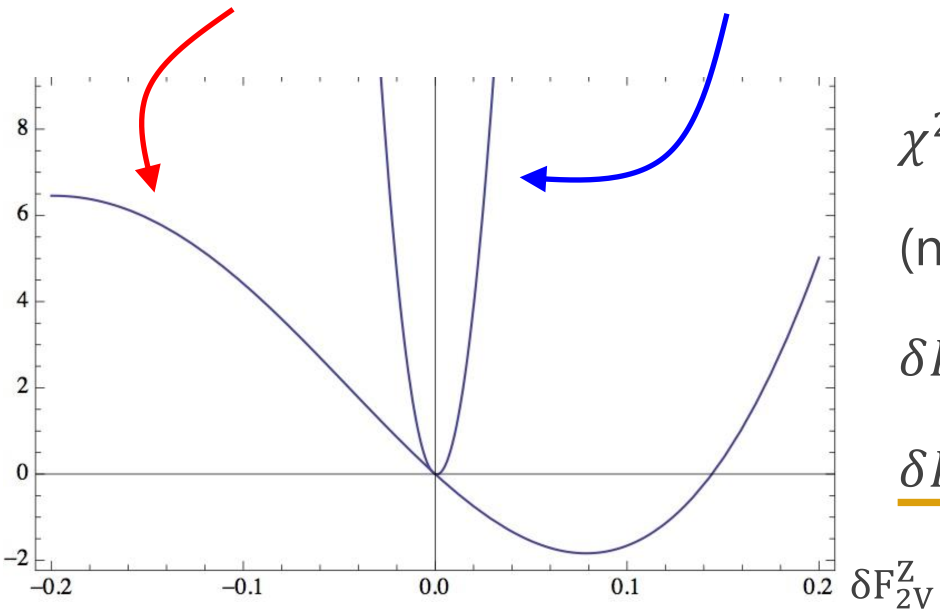
The ratio of wrong assignment of b-quark is only 2.1 % !

(cf. ~16 % for the semi-leptonic analysis, Yo Sato Top@LC 2016)

Analysis with Matrix element method

Illustration of analysis : 5000 unpolarized events

- using only $\cos\theta_t$
- using the complete set of 9 helicity angles



$\chi^2(\delta F^{Z}_{2V})$ function

(normalized such that $\chi^2(0) = 0$)

$\delta F^{Z}_{2V} = 0.080 \pm 0.05$ by $\cos\theta_t$

$\delta F^{Z}_{2V} = 0.001 \pm 0.01$ by the complete set

Number of events not used on purpose to compare the intrinsic power of these two to determine a single form factor, F^{Z}_{2V}.