# Oriented event shapes

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In collaboration with Germán Rodrigo

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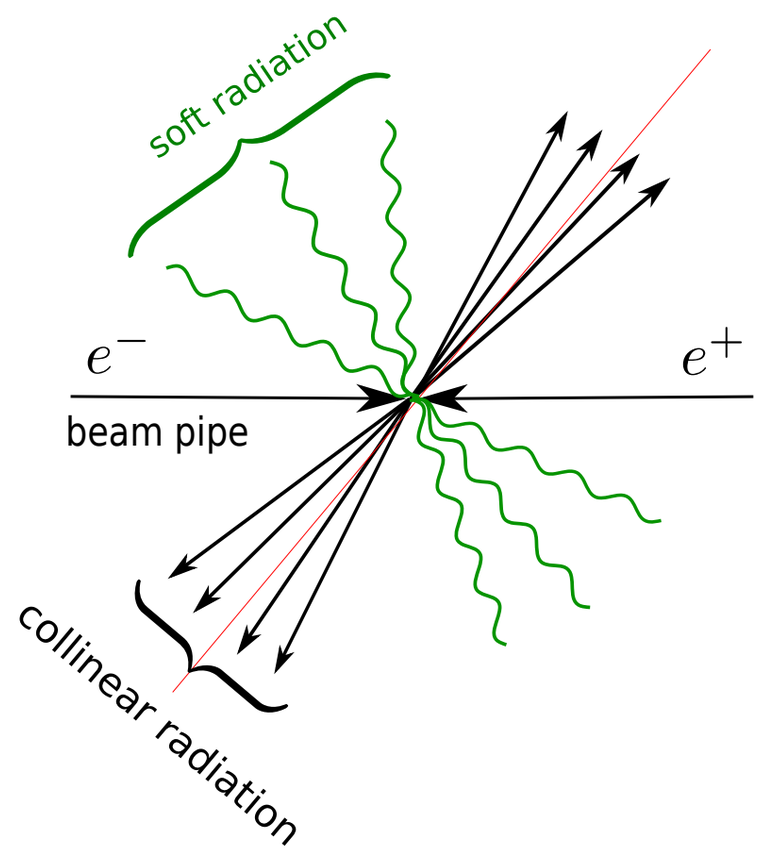
# Outline

- Introduction and Motivation
- Leading power factorization theorem (SCET)
- LO and NLO fixed-order results
- An all-order proof
- NNLO fixed-order results
- (Rough) comparison to data
- Sub-leading power factorization theorem (SCET)
- Outlook

# Introduction and Motivations

# (Un)Oriented event shapes

$e^+ e^- \to \text{jets}$

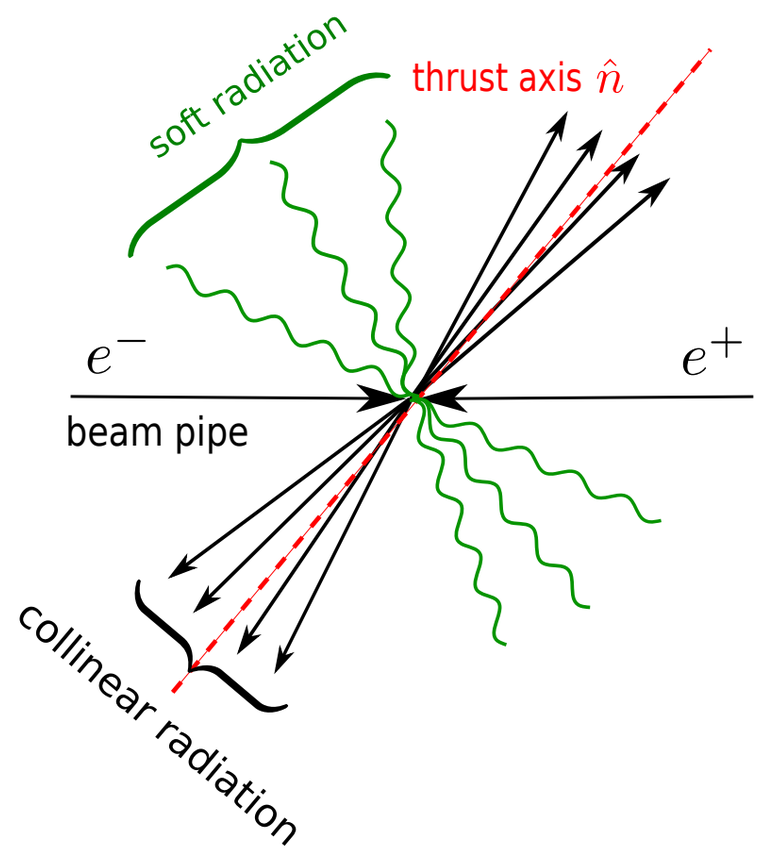



Dijet configuration:
Soft and collinear radiation

# (Un)Oriented event shapes

$e^+ e^- \to \text{jets}$

Collinear radiation
centered around thrust axis




Dijet configuration:
Soft and collinear radiation

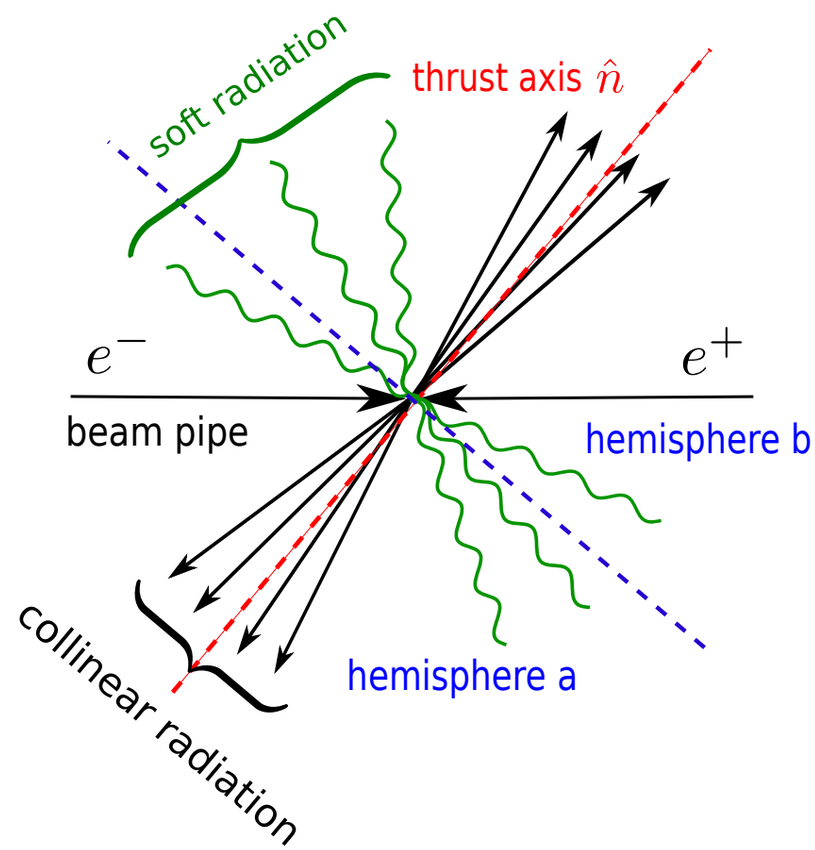
$$\tau_J = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q}$$

One needs to find the axis $\hat{n}$ that minimizes the value of thrust

# (Un)Oriented event shapes

$e^+ e^- \to \text{jets}$

Sum of mass in each hemisphere similar to value of thrust




Dijet configuration:
Soft and collinear radiation

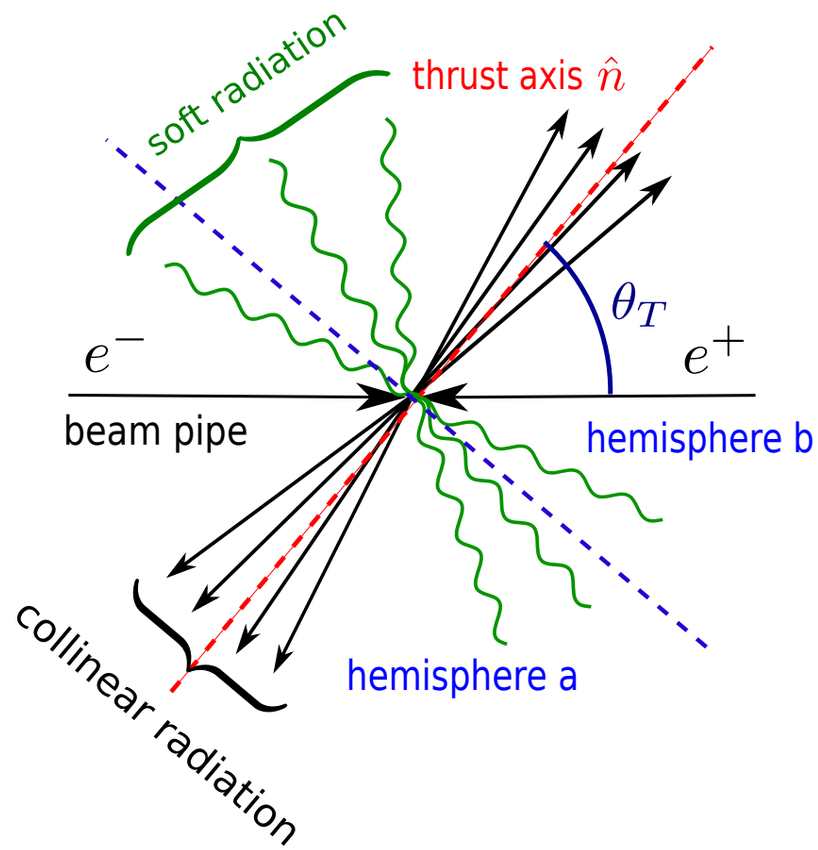
$$\tau_J = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q} \underset{Q\tau_J \ll Q}{\sim} \frac{M_1^2 + M_2^2}{Q^2}$$

One needs to find the axis $\hat{n}$ that minimizes the value of thrust

The plane normal to the thrust axis defines two hemispheres

# (Un)Oriented event shapes $e^+ e^- \to \text{jets}$

If only interested in shape of the event: unoriented event shape

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One needs to find the axis $\hat{n}$ that minimizes the value of thrust

The plane normal to the thrust axis defines two hemispheres

One can measure the angle between beam and $\hat{n}$ to define Oriented Event Shapes

# Thrust

$$e^+ e^- \to \text{jets}$$

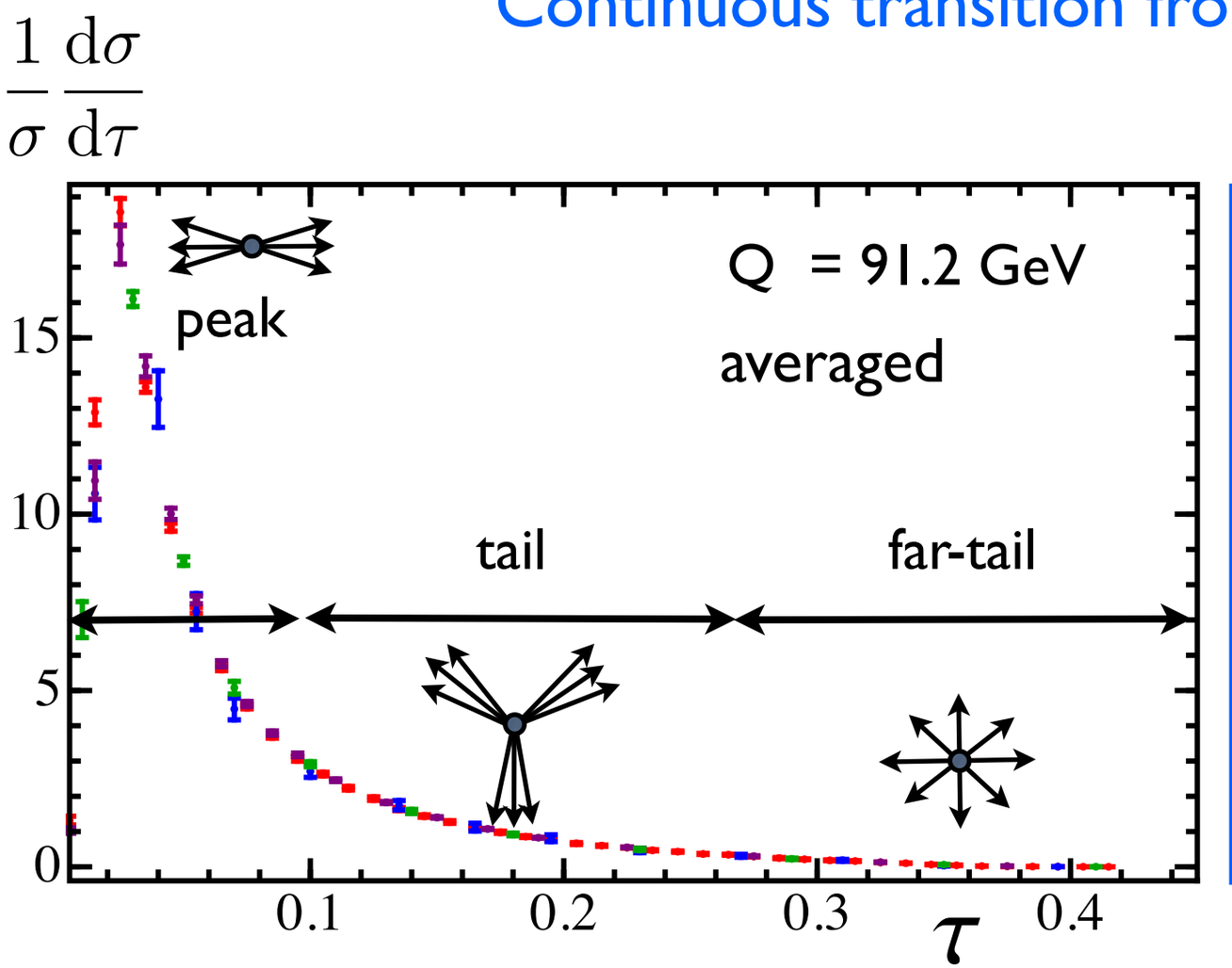
Most widely used event shape

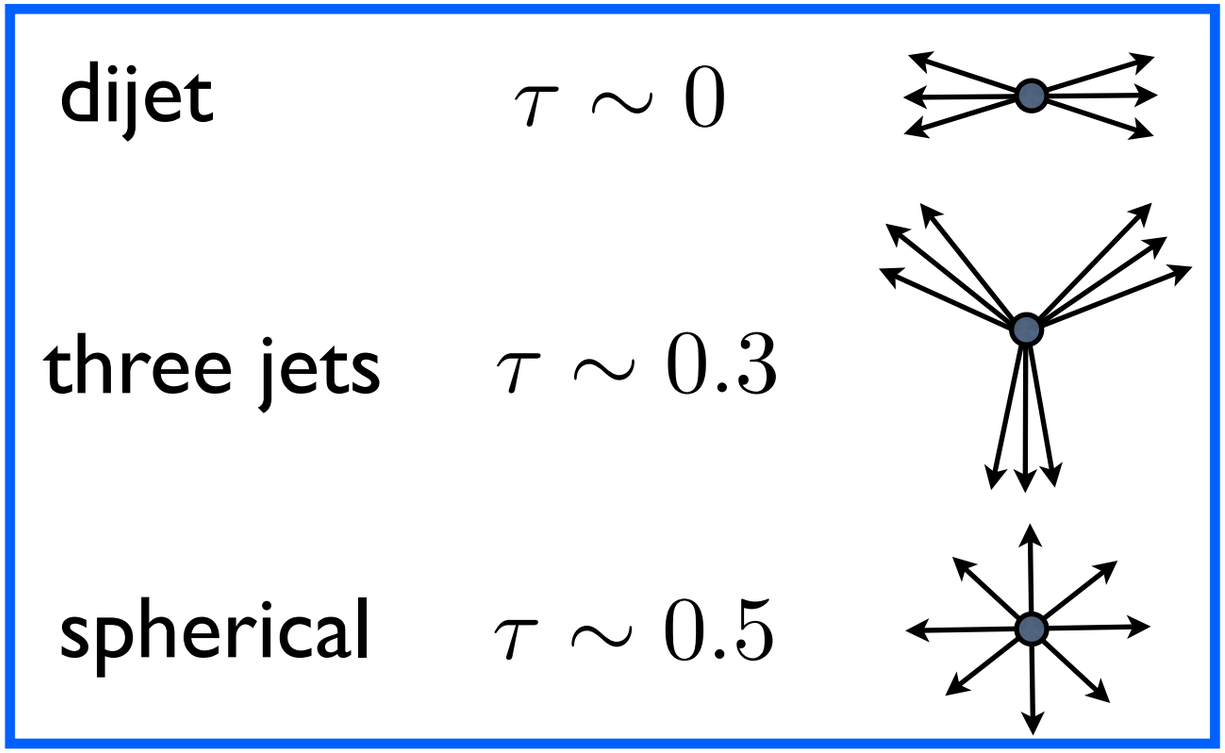
$$\tau_J = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q} \underset{Q\,\tau_J \ll Q}{\sim} \frac{M_1^2 + M_2^2}{Q^2}$$

IR and collinear safe

- Single sum
- Requires minimization

Continuous transition from 2-jet to 3-jet, ... multi-jet events



# Heavy Jet Mass

$$e^+ e^- \to \text{jets}$$

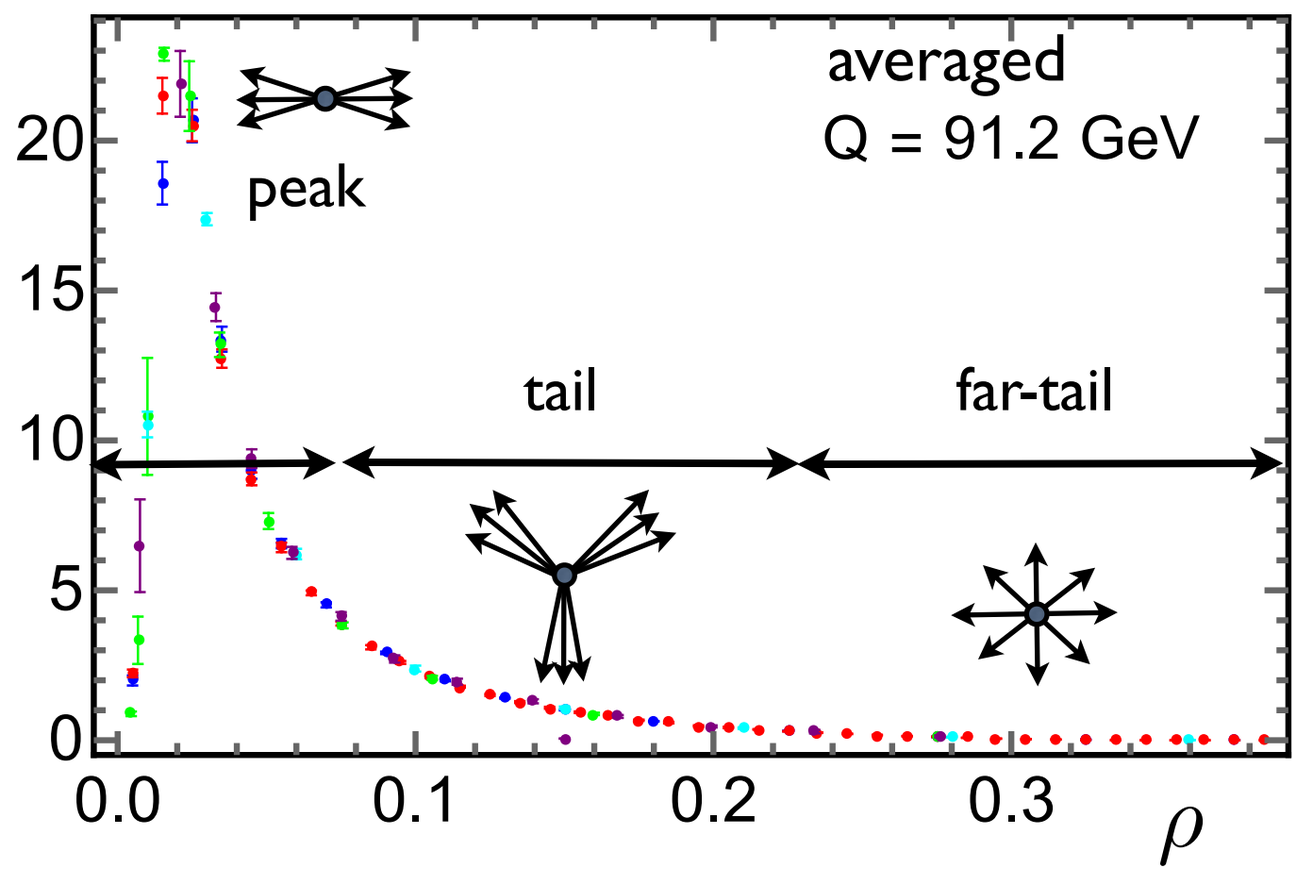
Far tail is hard to describe

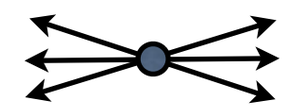
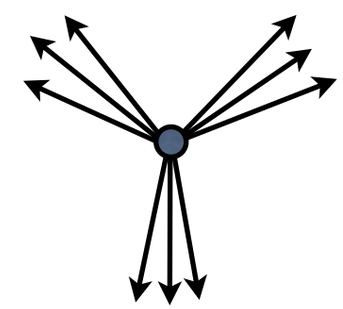
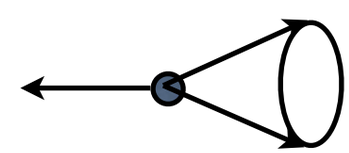
$$\rho = \frac{1}{Q^2} \max(M_1^2, M_2^2)$$

IR and collinear safe

- Uses both energy and momentum
- Much larger and involved hadronization corrections
- Requires knowledge of thrust axis

Continuous transition from 2-jet to 3-jet, ... conical events




| | | |
|---|---|---|
| dijet | $\rho \approx 0$ |  |
| three jets | $\rho \approx 0.3$ |  |
| conical | $\rho \approx 0.5$ |  |

# C-parameter

$$e^+ e^- \to \text{jets}$$

linearized momentum tensor

$$\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$$

with eigenvalues $\lambda_{1,2,3}$

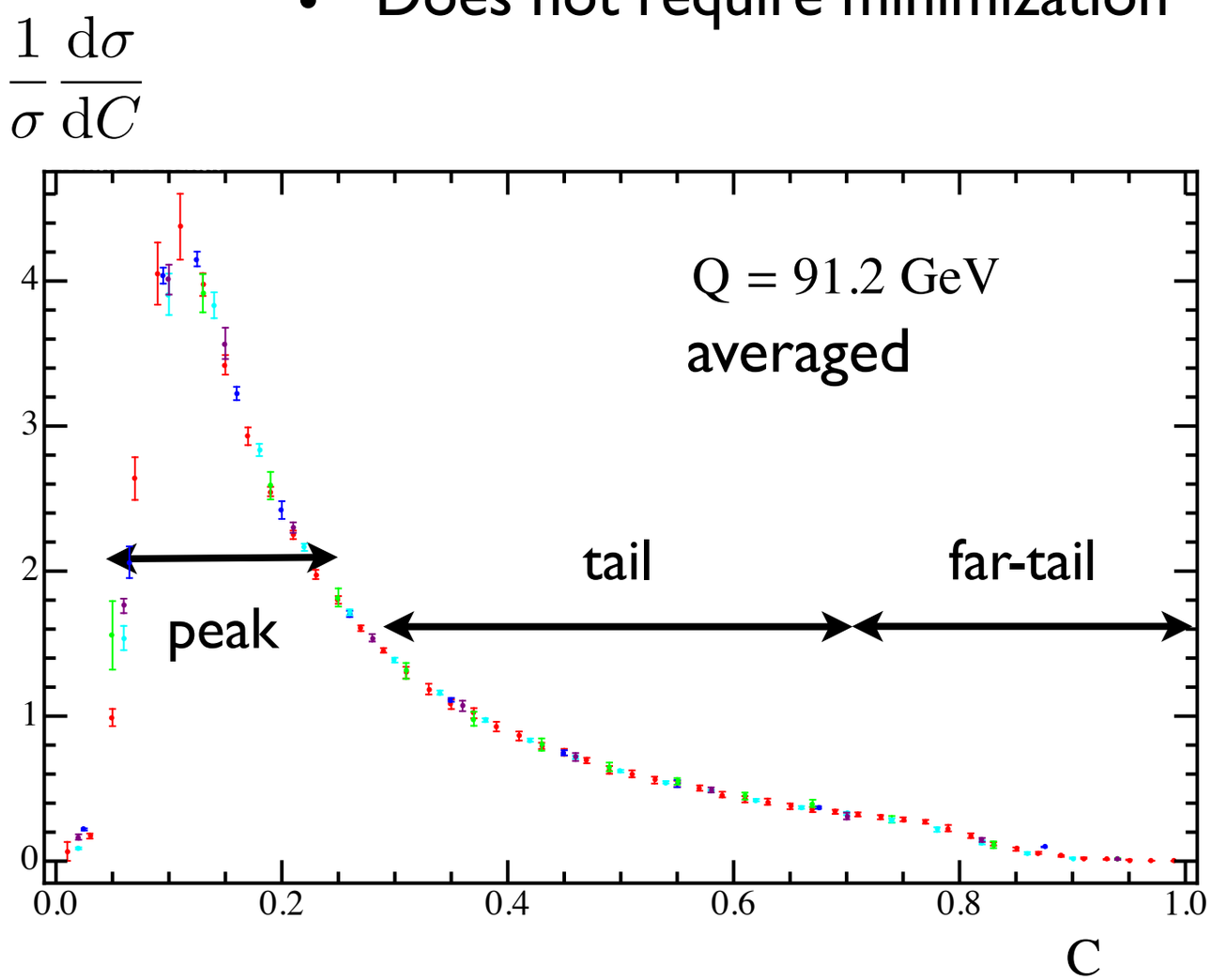
$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

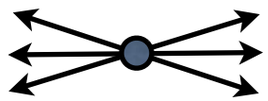
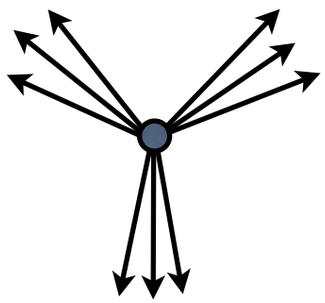
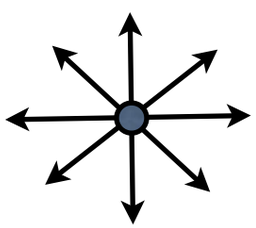
$$C = 3\left(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3\right) = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

IR and collinear safe

- Double sum
- Does not require minimization

Continuous transition from 2-jet to 3-jet, ... multi-jet events




| | | |
|---|---|---|
| dijet | $C \sim 0$ |  |
| three jets | $C \sim 0.75$ |  |
| spherical | $C \sim 1$ |  |

# Why do we care about oriented ES?

- Allows for so far unexplored tests of QCD.
- Can have a large impact in the measurement of $\sin(\theta_W)$ .
- Can have an impact in measurements of $\alpha_s(m_Z)$.
- Additional handle in top quark mass measurements.
- Can help improving quality of Monte Carlo generators.
- Nice simplified laboratory to study power-suppressed factorization theorems and resummation.
- Could be the first case of subleading power corrections for a massive Event shape in SCET/bHQET.
- Data measurements from ALEPH and DELPHI exist, but no analysis of them with high-precision theoretical predictions exist yet.

# Leading power factorization theorem

# LO factorization theorem in SCET

The leading order factorization theorem is derived in a way completely analogous to unoriented event shapes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{1}{2Q^2} \int \mathrm{d}^4x \, e^{iq\cdot x} \sum_{i=V,A} L^i_{\mu\nu} \, \langle\, 0 \,|\, j_i^{\mu\,\dagger}(x) \, \delta(e - \hat{e}) \, j_i^{\nu}(0) \,|\, 0 \,\rangle$$

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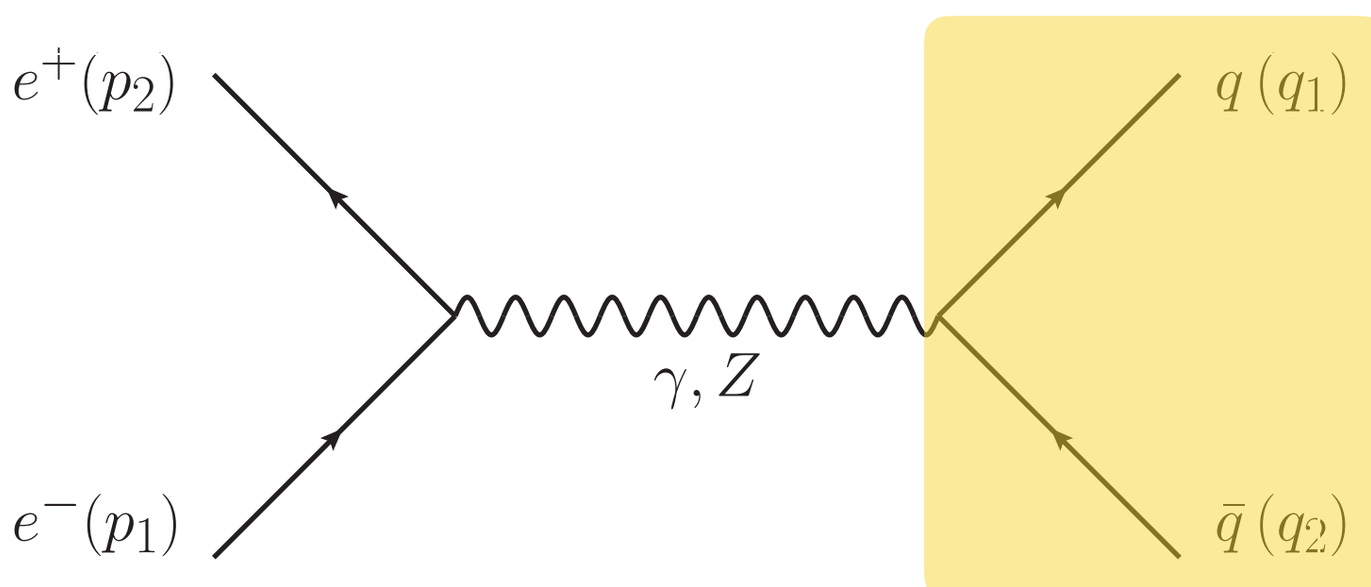

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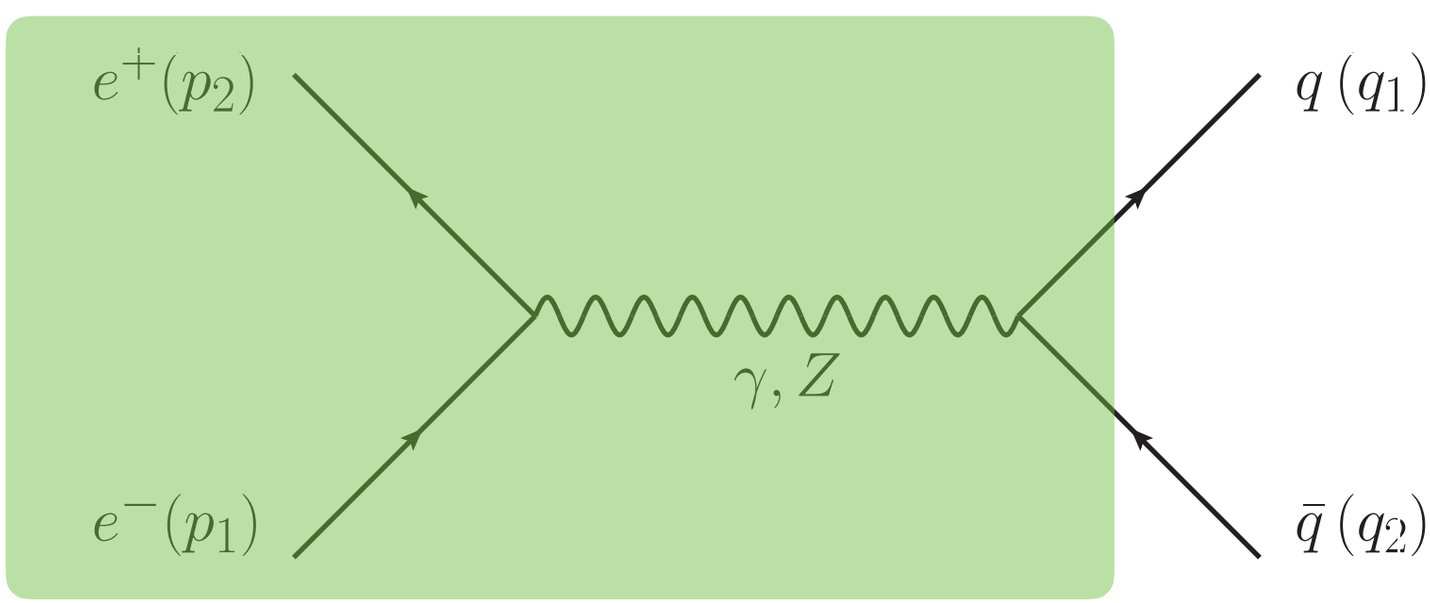
$$L_j^{\mu\nu} = \frac{16\pi^2\alpha^2}{Q^4} L_j(Q^2)\mathcal{V}^{\mu\nu}$$

$$\mathcal{V}^{\mu\nu} = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - \frac{Q^2}{2}\, g^{\mu\nu}$$ oriented

$$\langle \mathcal{V}^{\mu\nu} \rangle = \frac{1}{3}\left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)$$ averaged



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Matching QCD currents into SCET operators

$$j_i^\mu = \sum_{\hat{n}} \sum_{\tilde{p}_1,\tilde{p}_2} C_{n,\bar{n}}(\tilde{p}_1,\tilde{p}_2,\mu)\, \mathcal{O}_{n\bar{n}}(x;\tilde{p}_1,\tilde{p}_2)\,,$$

$$\mathcal{O}_{n\bar{n}}(x;\tilde{p}_1,\tilde{p}_2) = e^{i(\tilde{p}_1-\tilde{p}_2)\cdot x}\, \bar{\chi}_{n,\tilde{p}_1}(x) Y_n(x) \Gamma_i^\mu \overline{Y}_{\bar{n}}(x) \chi_{\bar{n},\tilde{p}_2}$$

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Wilson coefficient

collinear "jet" fields (contains collinear Wilson lines)

soft Wilson lines

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Factorize event shape measurement as well (not always possible!)

$$\delta(e-\hat{e}) = \int \mathrm{d}e_n\, \mathrm{d}e_{\bar{n}}\, \mathrm{d}e_s\, \delta(e_n-\hat{e}_n)\, \delta(e_{\bar{n}}-\hat{e}_{\bar{n}})\, \delta(e_s-\hat{e}_s)\, \delta(e-e_n-e_{\bar{n}}-e_s)\,,$$

# LO factorization theorem in SCET

Next steps: introduce SCET currents into QCD computation

perform x integration

require label momentum conservation

Use fierz identities

Perform analytically the sum over perp momenta labels

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Next one has to contract the Lorentz indices:

$$(n_\mu\bar{n}_\nu + n_\nu\bar{n}_\mu - 2g_{\mu\nu})\mathcal{V}^{\mu\nu} = Q^2(1+\cos^2\theta_T)$$
$$(n_\mu\bar{n}_\nu + n_\nu\bar{n}_\mu - 2g_{\mu\nu})\langle\mathcal{V}^{\mu\nu}\rangle = \frac{2}{3}Q^2$$

Which leads to our final result:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{S}}}{\mathrm{d}\cos\theta_T\,\mathrm{d}e} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{S}}}{\mathrm{d}e}$$

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with $\frac{\mathrm{d}\sigma_\mathrm{S}}{\mathrm{d}e}$ the usual factorization theorem for averaged event shapes:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_\mathrm{S}}{\mathrm{d}e} = H(Q,\mu)\int \mathrm{d}e_n\,\mathrm{d}e_{\bar{n}}\,J_n(e_n,\mu)\,J_{\bar{n}}(e_{\bar{n}},\mu)\,S(e-e_s-e_n,\mu)$$

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hadronization inside

# LO factorization theorem in SCET

These matrix elements are known at 2 or 3 loops
Anomalous dimensions known to 3-loops
Therefore precision is $\mathrm{N^3LL} + \mathcal{O}(\alpha_s^3)$

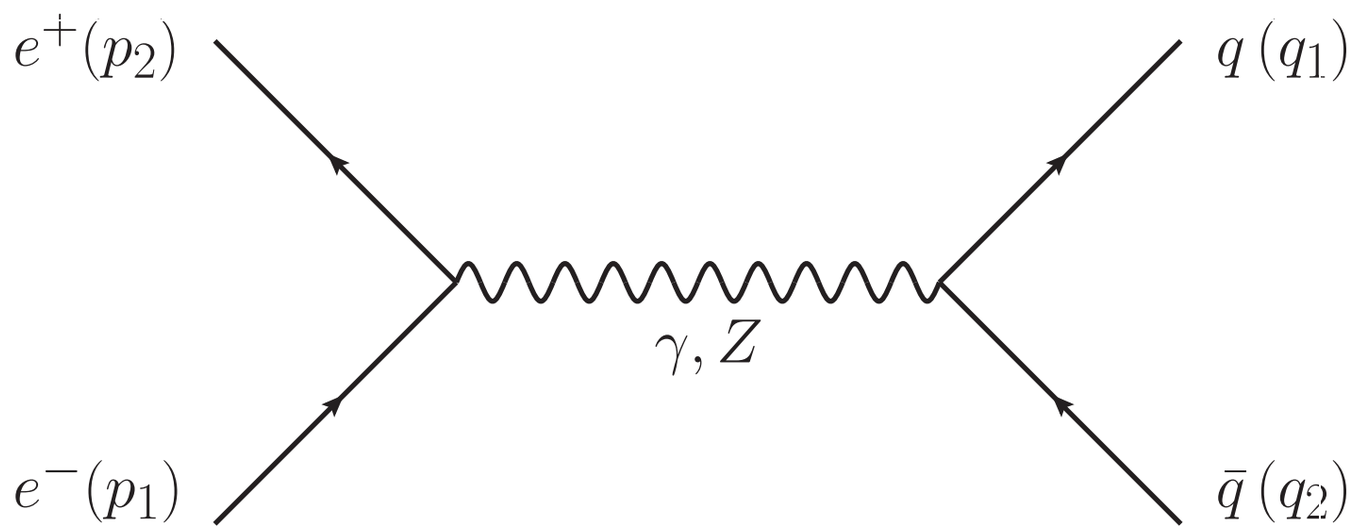
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_\mathrm{S}}{\mathrm{d}e} = H(Q,\mu)\int \mathrm{d}e_n\,\mathrm{d}e_{\bar{n}}\,J_n(e_n,\mu)\,J_{\bar{n}}(e_{\bar{n}},\mu)\,S(e-e_s-e_n,\mu)$$

LO result at fixer-order

# LO and NLO results

$\mathcal{O}(\alpha_s^0)$ result

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^0}{\mathrm{d}\cos\theta_T\,\mathrm{d}e} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\delta(e)\,.$$



# LO and NLO results

$\mathcal{O}(\alpha_s^0)$ result $$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^0}{\mathrm{d}\cos\theta_T\,\mathrm{d}e} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\delta(e)\,.$$

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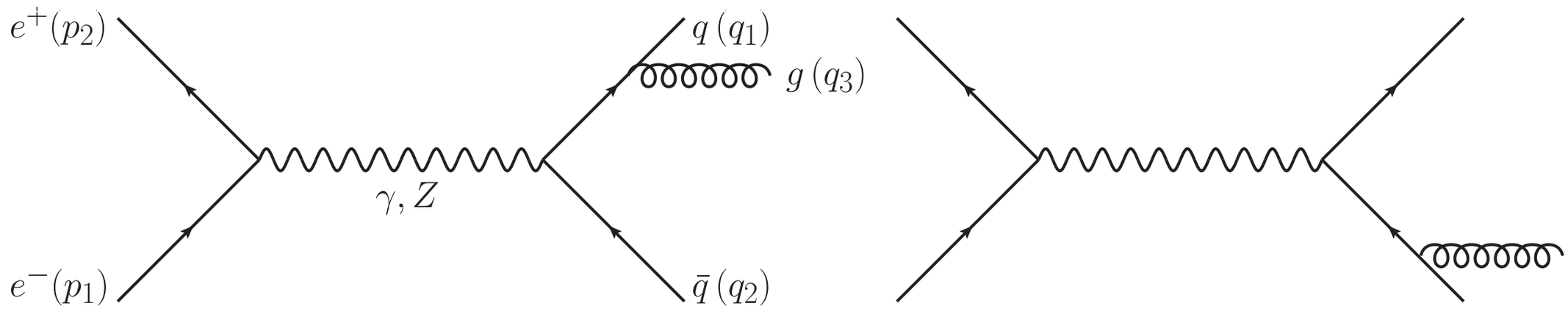

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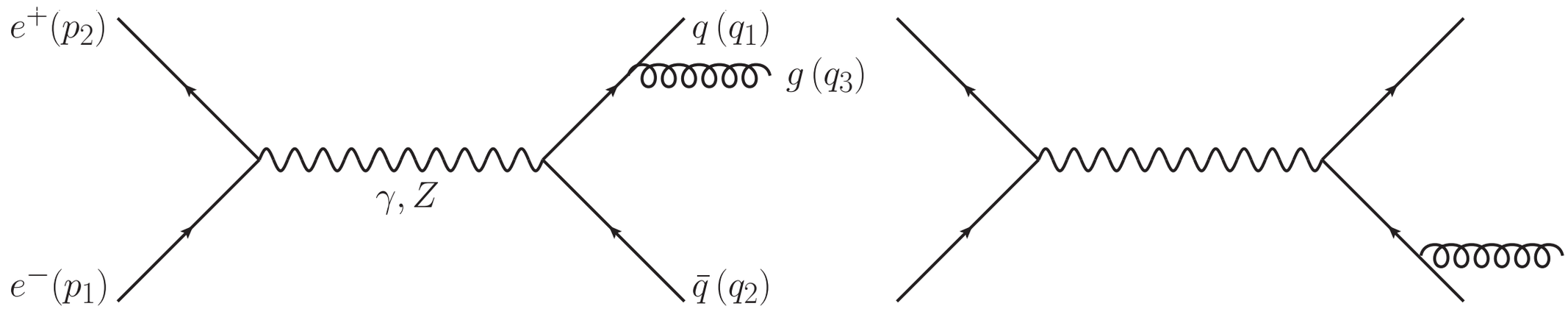


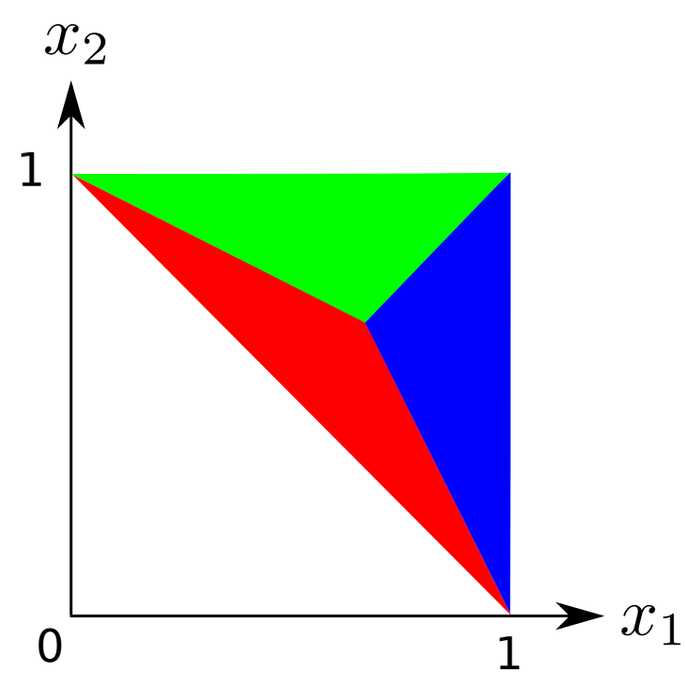

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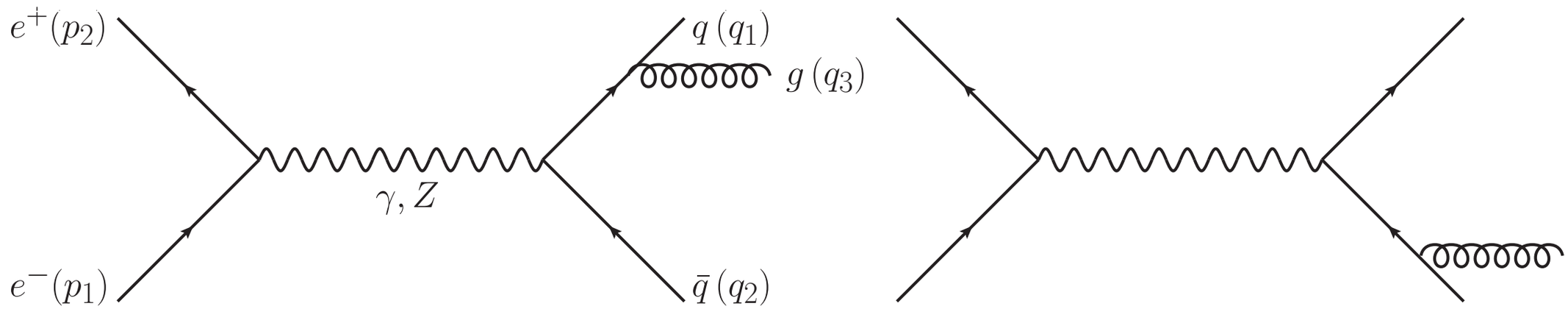


$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}\cos\theta_T} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}x_1\mathrm{d}x_2} + \left(1-3\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}_{\mathrm{ang}}}{\mathrm{d}x_1\mathrm{d}x_2}$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{\alpha_s}{2\pi}C_F\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}_{\mathrm{ang}}}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{3\alpha_s}{8\pi}C_F(1-x_3)\left[\frac{\theta(x_1-x_2)\,\theta(x_1-x_3)}{x_1^2} + \frac{\theta(x_2-x_1)\,\theta(x_2-x_3)}{x_2^2} + \frac{2\,\theta(x_3-x_1)\,\theta(x_3-x_2)}{x_3^2}\right]$$

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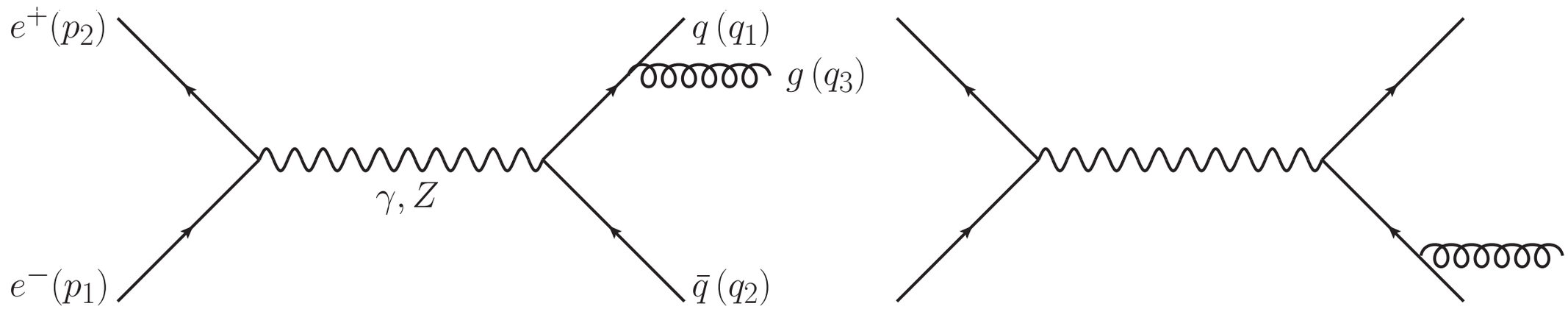


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soft singularities unreachable

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}_{\mathrm{ang}}}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{3\alpha_s}{8\pi}C_F(1-x_3)\left[\frac{\theta(x_1-x_2)\,\theta(x_1-x_3)}{x_1^2} + \frac{\theta(x_2-x_1)\,\theta(x_2-x_3)}{x_2^2} + \frac{2\,\theta(x_3-x_1)\,\theta(x_3-x_2)}{x_3^2}\right]$$

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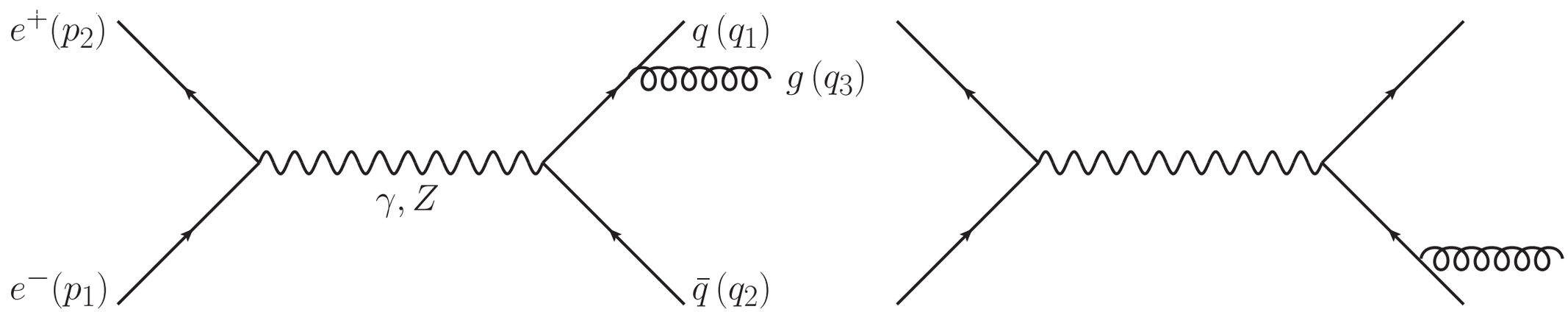


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This angular decomposition remains true to all orders, massive or massless particles, partons or hadrons

*[VM, Rodrigo (2013)]*

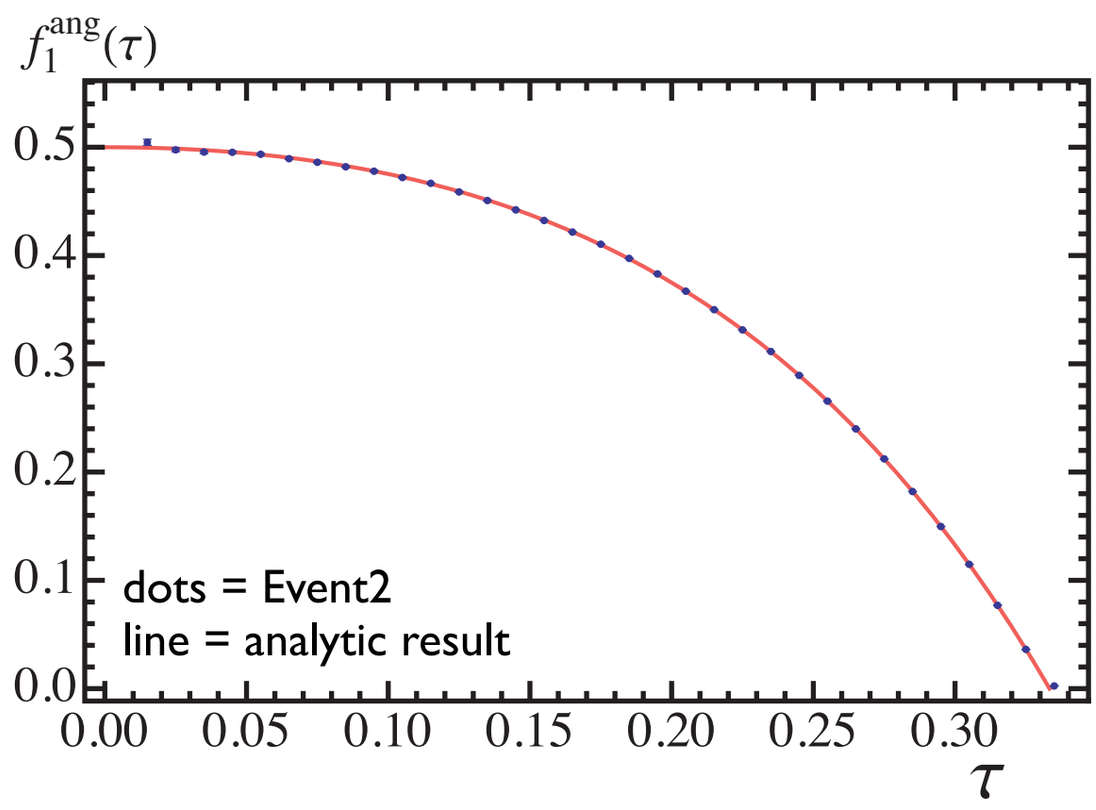
# NLO results

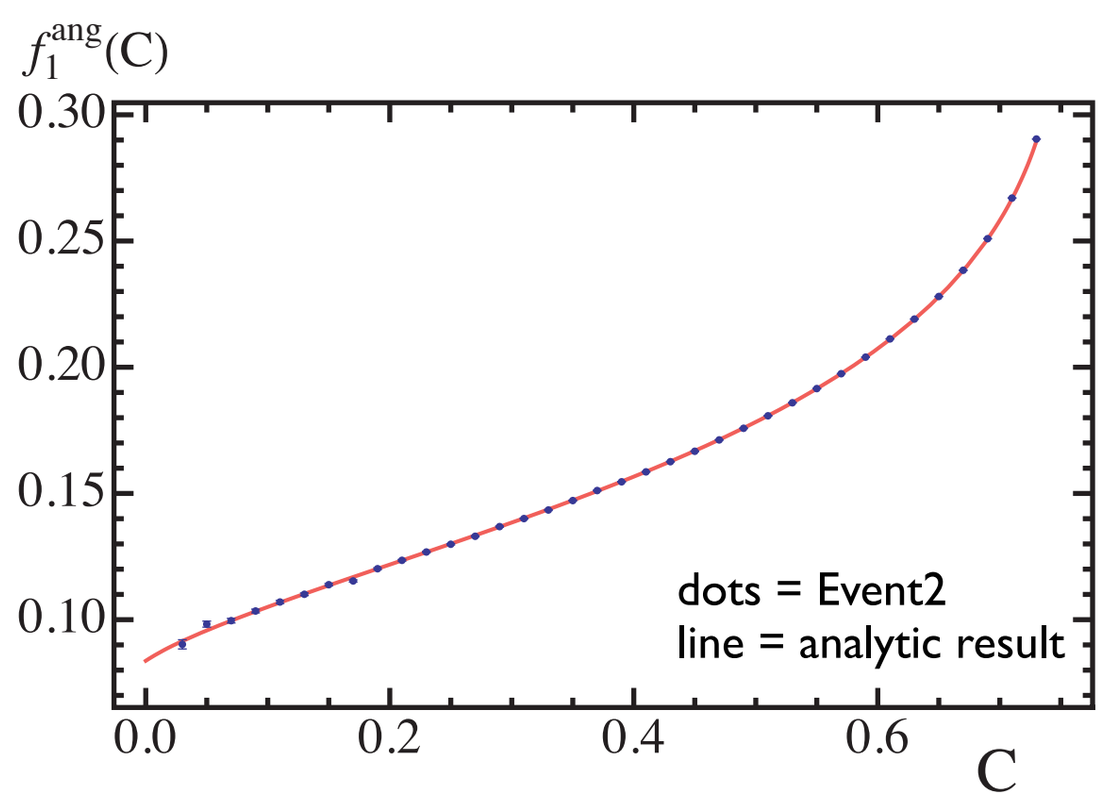
Next one has to project into the desired event shape

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{LO}}_{\mathrm{ang}}}{\mathrm{d}\tau} = \frac{3\,\alpha_s}{8\,\pi}\,C_F\,\frac{(1-3\,\tau)(1+\tau)}{(1-\tau)^2} = \frac{\alpha_s}{\pi}\,f_1^{\mathrm{ang}}(\tau) \qquad \Sigma_1^{\mathrm{ang}}(\tau) = -\,\frac{3}{8}\,C_F\left[\frac{\tau\,(7-3\tau)}{1-\tau} + 8\,\log(1-\tau)\right]$$

Thrust and Heavy Jet Mass and sum of hemisphere masses coincide at this order. Analytic, simple result. Also for cumulant distribution.

For C-parameter very complicated analytic expression in terms of incomplete elliptic functions. Cumulant integrated numerically. *[VM in progress]*



# An all-order proof

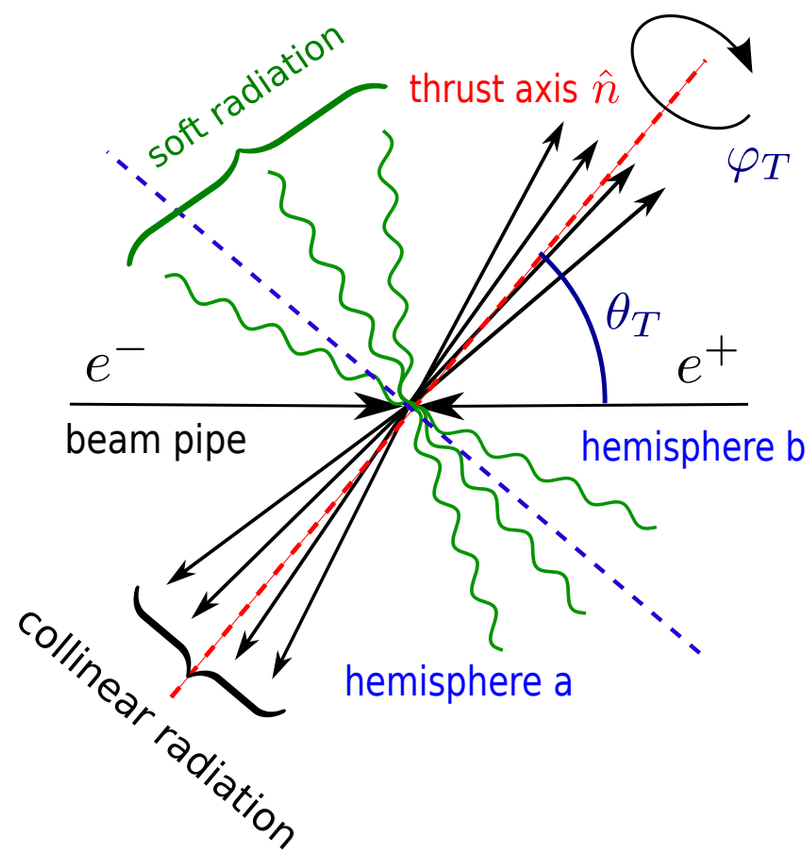
# Key ingredients of the proof

Most general Lorentz decomposition of Hadronic Tensor

Phase space factorization

Thrust remains invariant if particles clustered within the same hemisphere

Terms linear in $\cos\theta_T$ or $\sin\theta_T$ vanish upon average over the polar angle




If parity violation is taken into account and one can distinguish between the two hemispheres, then also $\cos\theta_T$ terms can appear

# Results of the proof

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_T\,\mathrm{d}e} = \frac{3}{8}(1+\cos^2\theta_T)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} + (1-3\cos^2\theta_T)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{ang}}}{\mathrm{d}e}$$

averaged distribution

angular distribution

Each distribution can be expanded in powers of $\alpha_s$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{ang}}}{\mathrm{d}e} = \frac{\alpha_s(Q)}{\pi}f_1^{\mathrm{ang}}(e) + \left(\frac{\alpha_s(Q)}{\pi}\right)^2 f_2^{\mathrm{ang}}(e) + \left(\frac{\alpha_s(Q)}{\pi}\right)^3 f_3^{\mathrm{ang}}(e) + \ldots$$

Finally one can define the total angular hadronic cross section

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_T} = \frac{3}{8}(1+\cos\theta_T^2)R_{\mathrm{had}}(Q) + (1-3\cos\theta_T^2)R_{\mathrm{ang}}(Q)$$

$$R_{\mathrm{ang}} = \frac{\alpha_s(Q)}{\pi}R_1^{\mathrm{ang}} + \left(\frac{\alpha_s(Q)}{\pi}\right)^2 R_2^{\mathrm{ang}} + \left(\frac{\alpha_s(Q)}{\pi}\right)^3 R_3^{\mathrm{ang}} + \ldots$$

whose leading terms can be computed analytically from $\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{ang}}^{\mathrm{LO}}}{\mathrm{d}x_1\mathrm{d}x_2}$

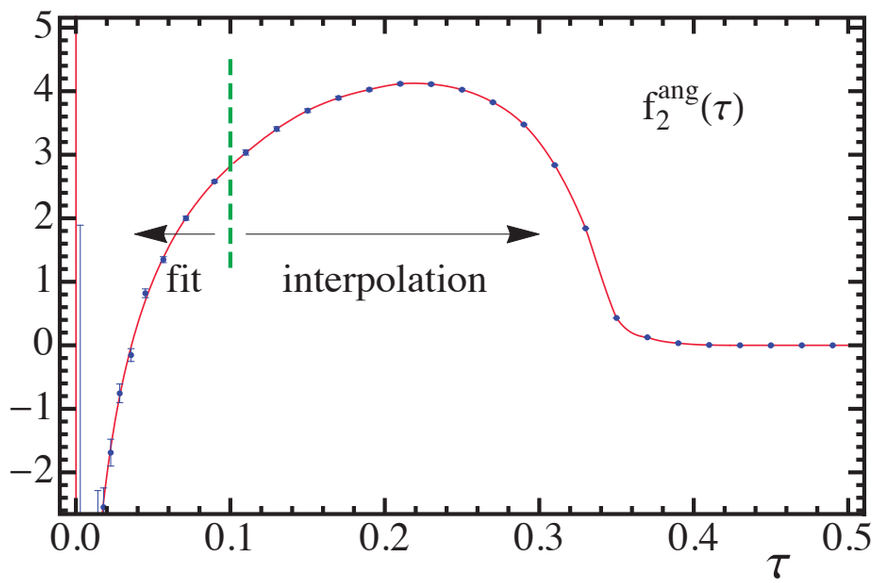
$$R_1^{\mathrm{ang}} = \frac{3\,C_F}{8}\left[8\,\log\left(\frac{3}{2}\right) - 3\right]$$

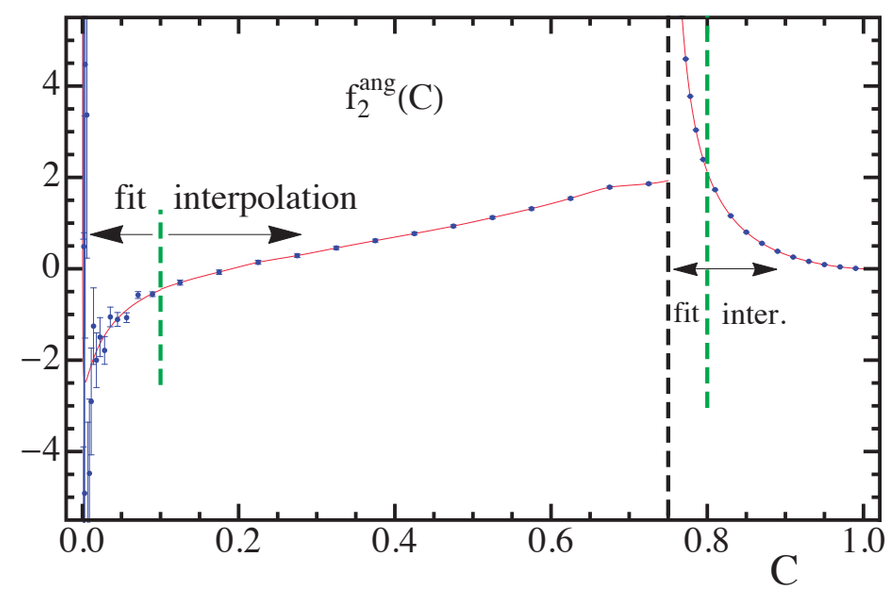
# NLO result at fixer-order

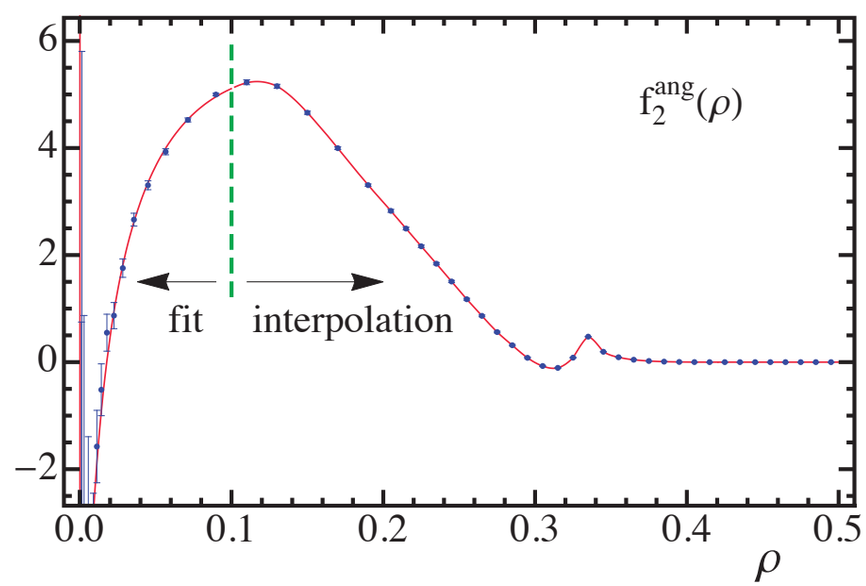
# NLO results

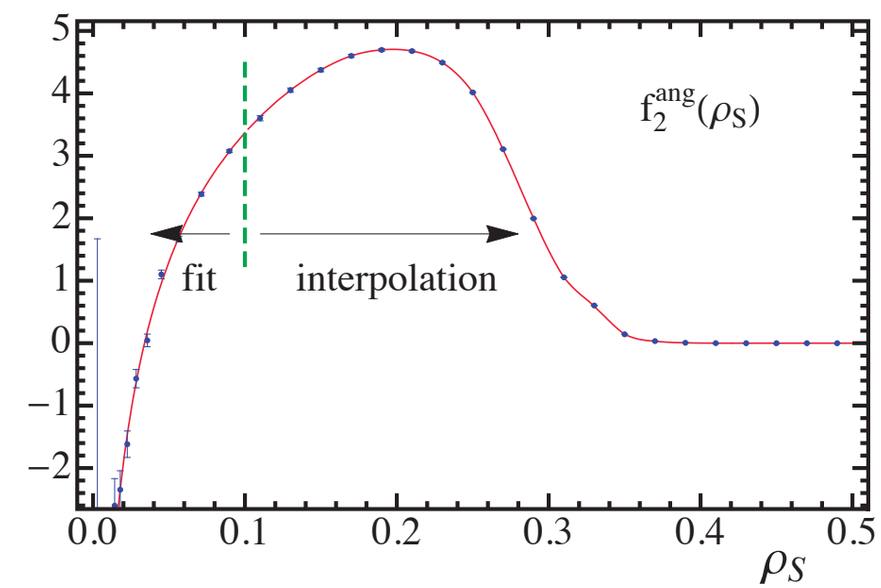
These can be extracted from EVENT2, directly projecting out the angular term

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{ang}}}{\mathrm{d}e} = \frac{3}{8}\int_{-1}^{1}\mathrm{d}\cos\theta_T\,(2-5\cos^2\theta_T)\,\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_T\,\mathrm{d}e}$$



# NLO averaged total cross section

The total angular hadronic cross section has to be computed by extrapolating to $e_c \longrightarrow 0$ the cumulant cross section

$$\Sigma_{\rm ang}(e_c) = \frac{1}{\sigma_0}\int_0^{e_c} {\rm d}e\, \frac{{\rm d}\sigma_{\rm ang}}{{\rm d}e}$$

A similar "trick" can't be used for the averaged total cross section

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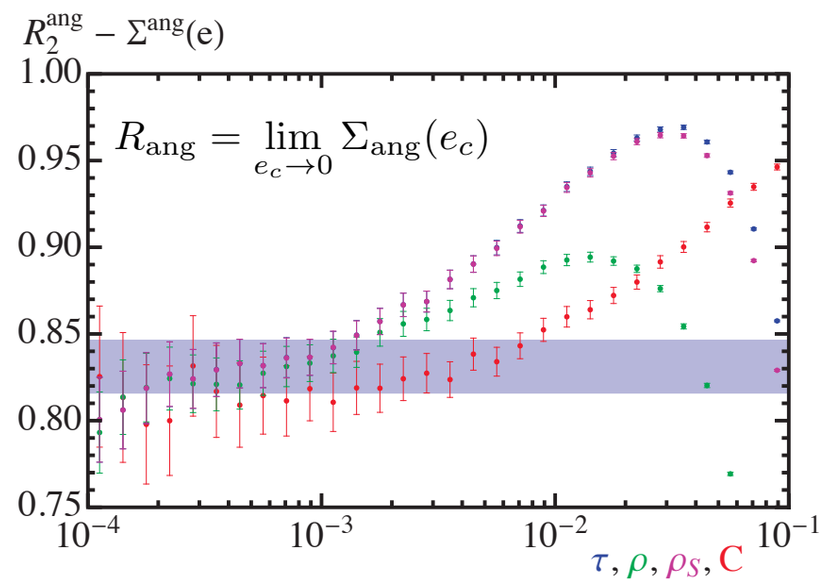
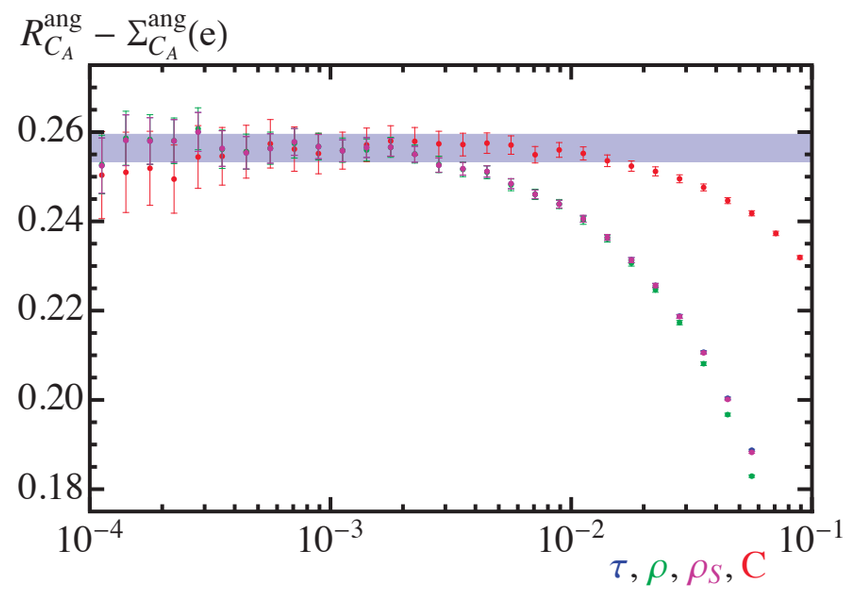
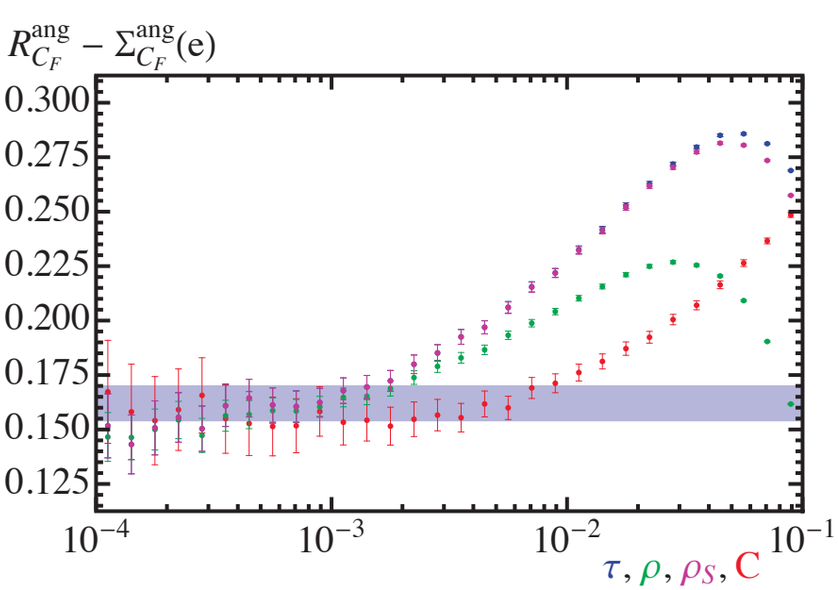
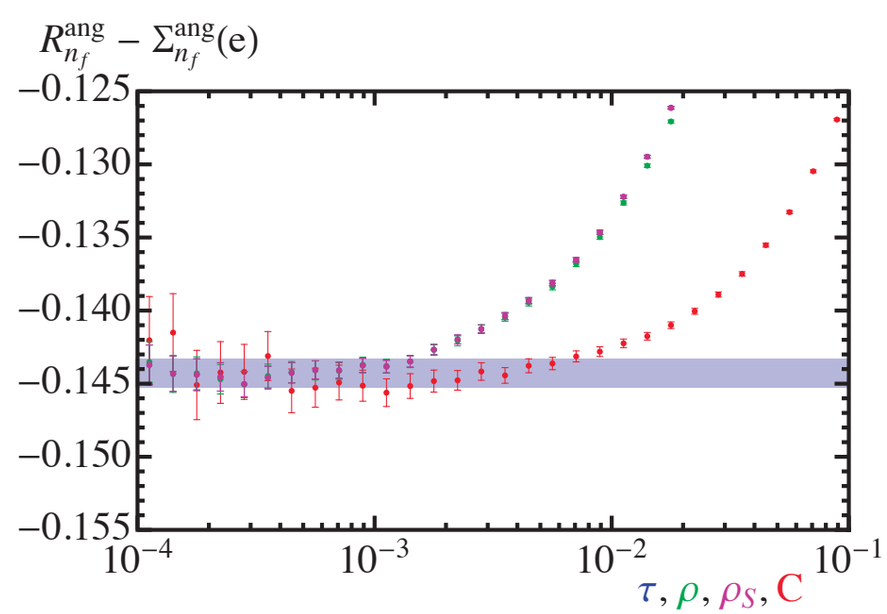



We use four event shapes to perform the extrapolation to 0

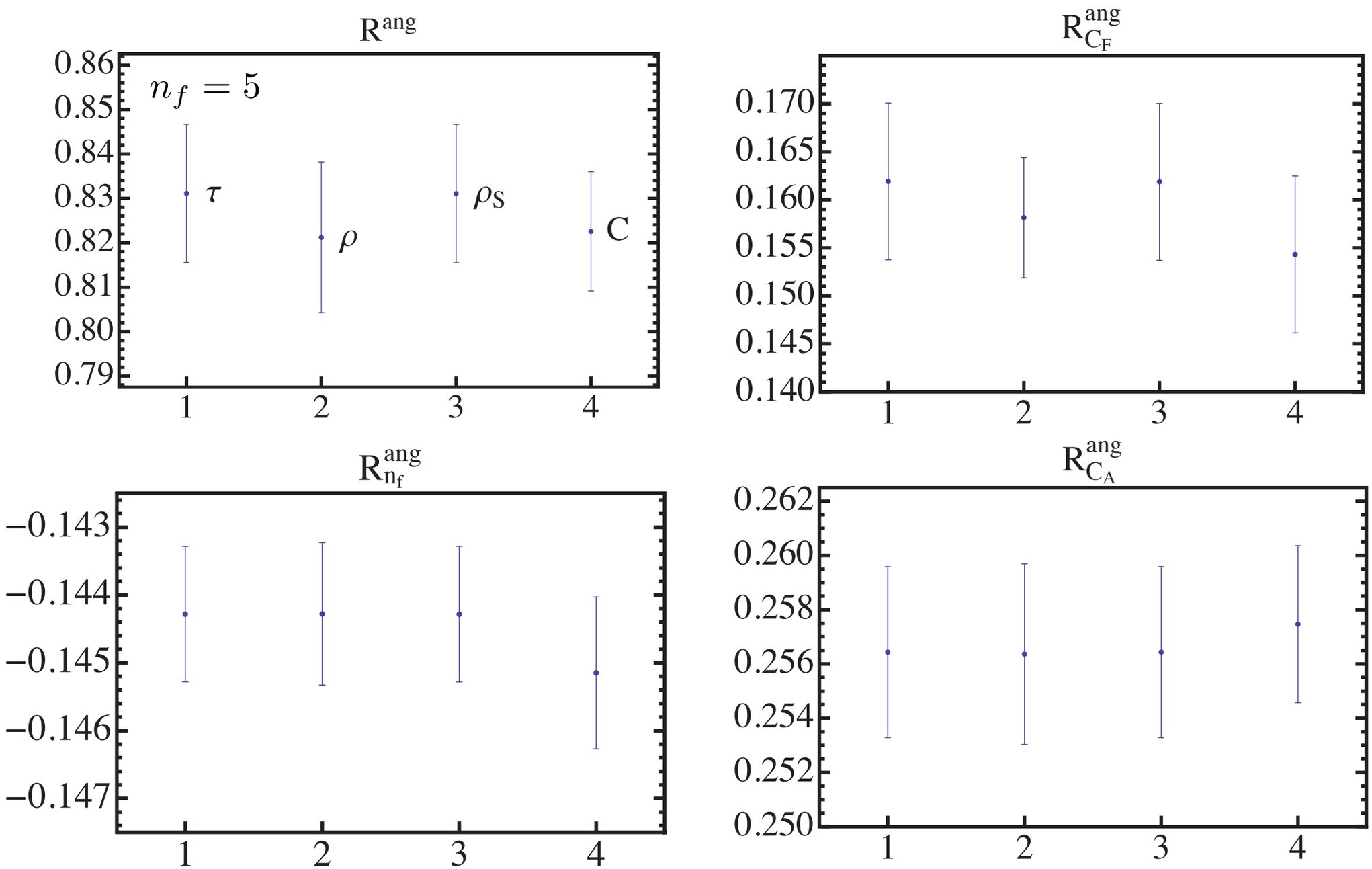



Logarithmic binning close to e = 0 is crucial to get stable results

# NLO averaged total cross section

Nice agreement among the various ES extrapolations for each color structure.



# NLO averaged total cross section

Nice agreement among the various ES extrapolations for each color structure.

Disagreement with previous determination by *[B. Lampe (1992)]*, who performs a linear extrapolation in the resolution parameter.

$$R_2^{\rm ang} = \quad 0.831 \pm \ 0.014 \pm \ 0.006\,,$$
$$R_{C_F}^{\rm ang} = \quad 0.162 \pm \ 0.008 \pm \ 0.002\,,$$
$$R_{C_A}^{\rm ang} = \quad 0.256 \pm \ 0.029 \pm \ 0.001\,,$$
$$R_{n_f}^{\rm ang} = -\,0.1443 \pm 0.0008 \pm 0.0006\,,$$

$$R_2^{\rm ang}|_{\rm Lampe} = -0.044 \pm 0.012$$

We have a few cross checks on our estimate:

Our result is independent of event shape

Our LO Event-2 cross sections agree with analytic computation

Our NLO Event-2 cross sections agree with sub-leading factorization theorem at small e

Same technique and Event2 runs have been used to determine other constants

Further checks are possible and will be performed in the near future

(Rough)
comparison to data

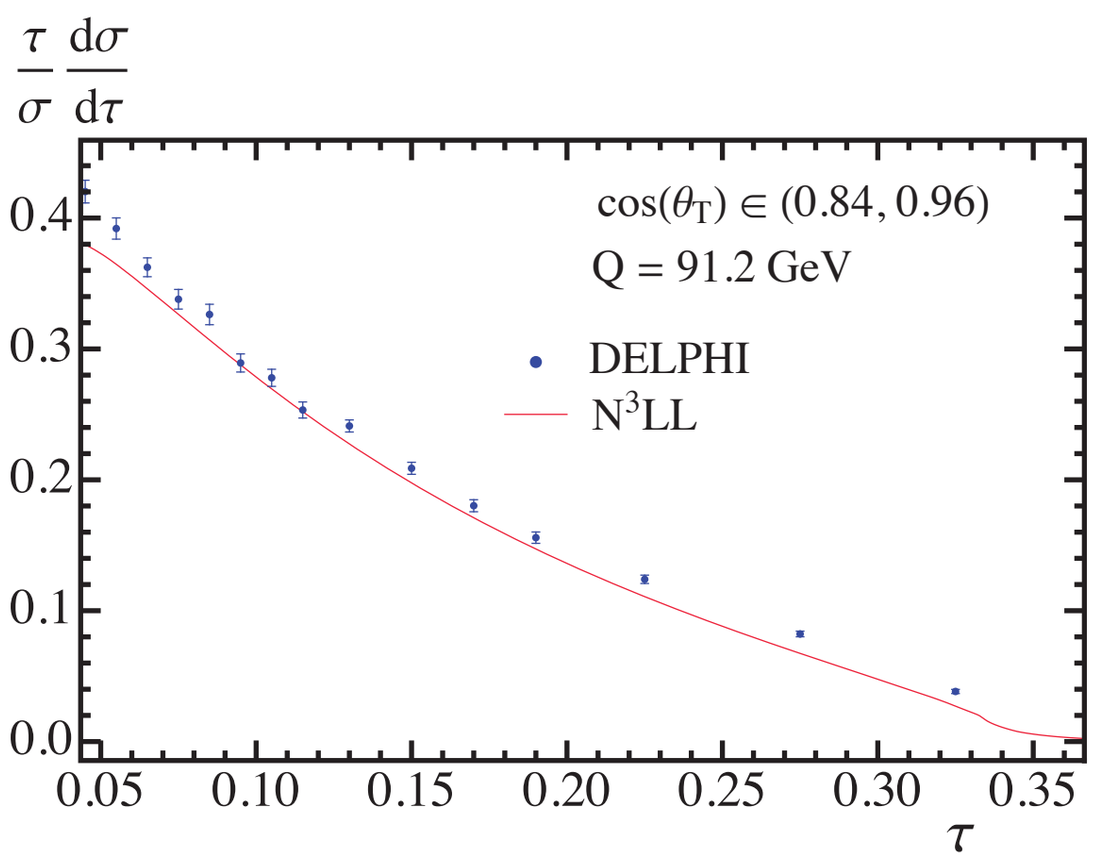
# Comparison to data

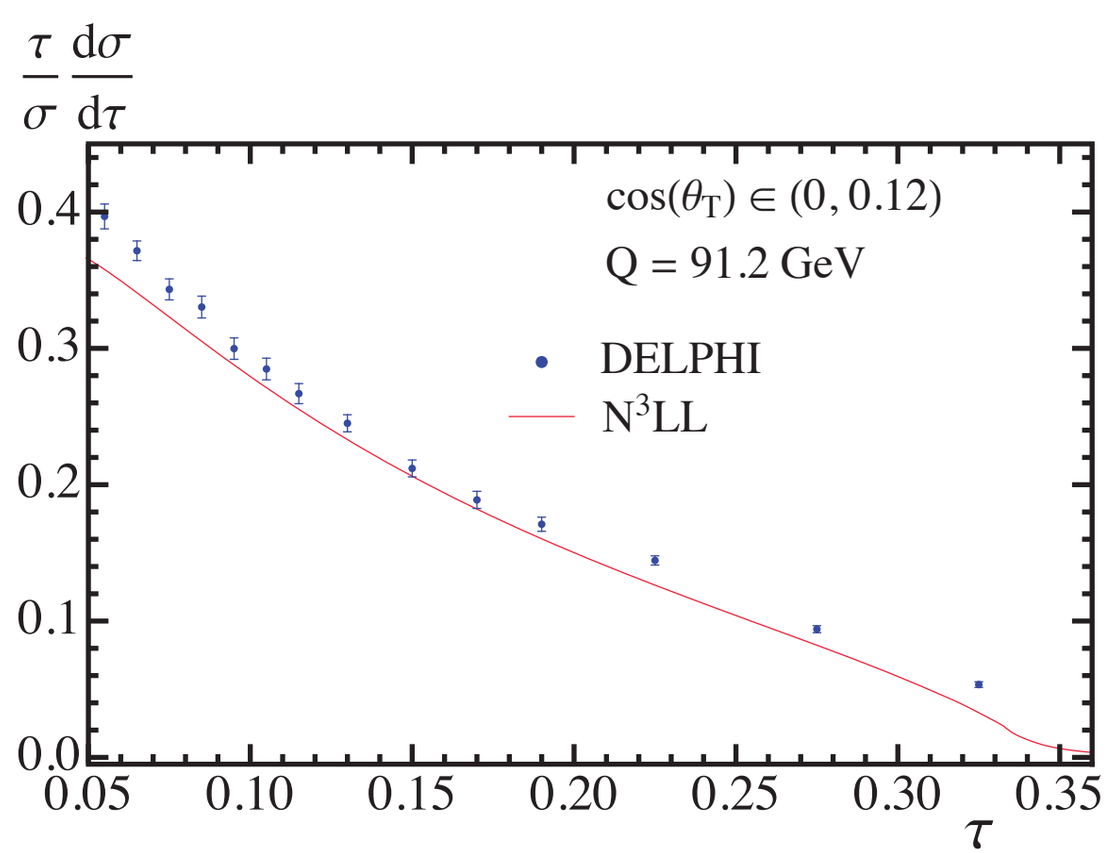
Singular (with N$^3$LL resummation) and non-singular terms (fixed-order) included (3-loop for averaged, 2-loop for angular)

No hadronization power corrections implemented

No estimate of perturbative errors

Uses world average for $\alpha_s$



# Sub-leading power factorization theorem

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*[Hagiwara, Kirilin (2010)]*

We have adapted their factorization theorem from thrust to any event shape

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_s^{\mathrm{ang}}}{\mathrm{d}e} = \frac{8}{3}H_3(Q,\mu)\int \mathrm{d}e_n\,\mathrm{d}e_{\bar{n}}\Sigma_\perp(e_n,\mu)\,J_{\bar{n}}(e_{\bar{n}},\mu)\,S(e-e_s-e_n,\mu)$$

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New matching coefficient

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New matching coefficient

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leading soft function

Leading nonperturbative power corrections are the same as for averaged cross section

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New matching coefficient

leading jet function

leading soft function

New (sub-leading) jet function

$$\Sigma_\perp\left(p^2,\mu^2\right) = \frac{g_s^2}{(p\cdot n)\,Q^2 N_{\mathrm{c}}}\frac{1}{\pi}\,\mathrm{Im}\left[i\int \mathrm{d}^{\mathcal{D}}x e^{-ipx}\left\langle 0\left|\mathrm{T}\left\{\left(\bar{\xi}'_{n_+}\widetilde{\not{A}}_{\perp,n_+}W_{n_+}\right)(x)\,\frac{\not{n}}{2}\left(W^\dagger_{n_+}\widetilde{\not{A}}_{\perp,n_+}\xi'_{n_+}\right)(0)\right\}\right|0\right\rangle\right]$$

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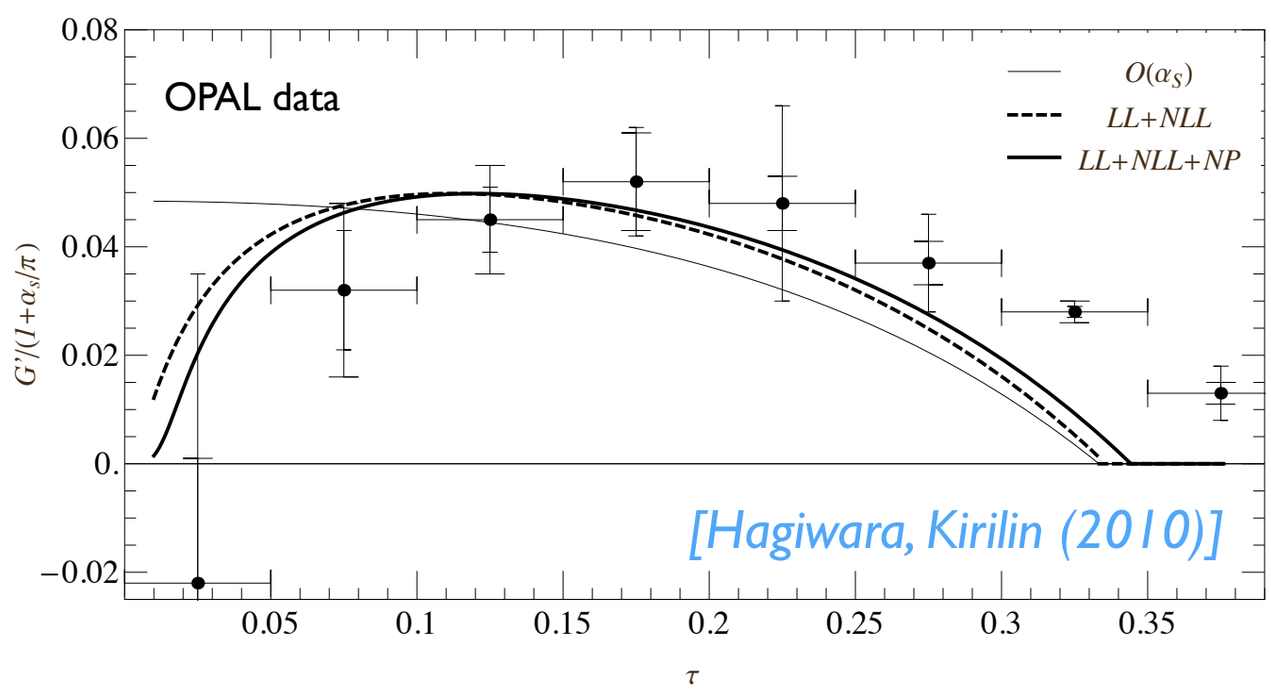
Effects of resummation appear to be important




This formula reproduces LO and NLO results in the dijet limit

# Outlook

# Outlook

Generalize the proof to massive Event shapes both in SCET and bHQET

Compute the LO Fixed-oder and SCET distributions with heavy quarks

Extract the three-loop fixed-order angular term from EERAD3 (possible?)

Compute the missing anomalous dimension to achieve $N^2LL$ precision

Further clarify the disagreement with Lampe

Create a fast computer code that matches the resummed expression with fixed-order results, and includes power corrections in a renormalon-free scheme

Compare to data and fit for $\alpha_s$

Include massive oriented event shapes in the MC mass calibration program.

Manpower is most welcome for most of these tasks!

# Backup slides