Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

Damping Rings – Part 2

FIFTH INTERNATIONAL ACCELERATOR SCHOOL FOR LINEAR COLLIDERS

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Lecture A3, Part 2 – Low Emittance Ring Design

A. Radiation Damping and Equilibrium Emittance

B. Damping Ring Lattices

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Damping Ring Lectures

Lecture A3, Part 1 – Damping Ring Basics
- Introduction to Damping Rings
- General Linear Beam Dynamics

Lecture A3, Part 2 – Low Emittance Ring Design
- Radiation Damping and Equilibrium Emittance
- Damping Ring Lattices

Lecture A3, Part 3 – Damping Ring Technical Systems
- Systems Overview
- Review of Selected Systems for ILC and CLIC
- R&D Challenges

Lecture A3, Part 4 – Beam Dynamics
- Overview of Impedance and Instability Issues
- Review of Selected Collective Effects
- R&D Challenges

Outline of DR Lecture I, Part 2

Re-cap of Yesterday's Lecture

Radiation Damping and Equilibrium Emittance

- Radiation Damping
- Synchrotron Equations of Motion
- Synchrotron Radiation Integrals
- Quantum Excitation and Equilibrium Emittance
- Summary of Beam Parameters and Radiation Integrals

Damping Ring Lattices

- ILC Damping Ring Design Optimization
  - The DCO Lattice
  - Summary of Parameters and Design Choices
  - Moving to a Smaller Circumference Ring
- CLIC Damping Ring Design Optimization
  - The CLIC DR Lattice
  - Summary of Parameters and Design Choices

Review I

There are several points that I hope you were able to take away from yesterday's lecture:

1. The most basic input for the damping ring is the target luminosity at the collision point

$$\boxed{\mathcal{L} = \frac{N^2 f_{coll}}{4\pi\sigma_x\sigma_y}\mathcal{H}_D}$$

Thus we need to provide:

1. High intensity bunches
2. High repetition rate
3. Small transverse beam sizes

The choices for these parameters are largely driven by the main linac technology choice

Review II

2. Interfaces to the upstream (beam acceptance from sources) and downstream (bunch length target from the bunch compressors) systems also impose critical constraints.
3. A short review of the ILC damping ring and CLIC damping ring design parameters provides a glimpse at how varied damping ring solutions can be. Nevertheless, many of the technology challenges are quite closely related.
4. Yesterday's lecture concluded with a walk through the basics of ring physics

Let's continue with that line of development…

Synchrotron Radiation and Radiation Damping

Up to this point, we have treated the transport of a relativistic electron (or positron) around a storage ring as a conservative process. In fact, the bending field results in the particles emitting synchrotron radiation.

The energy lost by an electron beam on each revolution is replaced by radiofrequency (RF) accelerating cavities. Because the synchrotron radiation photons are emitted in a narrow cone (of half-angle $1/\gamma$) around the direction of motion of a relativistic electron while the RF cavities are designed to restore the energy by providing momentum kicks in the $\hat{s}$ direction, this results in a gradual loss of energy in the transverse directions. This effect is known as *radiation damping*.

Synchrotron Radiation

We will only concern ourselves with electron/positron rings. The instantaneous power radiated by a relativistic electron with energy E in a magnetic field resulting in bending radius ρ is:

$$\boxed{P_\gamma = \frac{cC_\gamma E^4}{2\pi\rho^2} = \frac{e^2c^3}{2\pi}C_\gamma E^2 B^2} \quad \text{where} \quad C_\gamma = 8.85\times10^{-5}\, m/(GeV)^3$$

We can integrate this expression over one revolution to obtain the energy loss per turn:

$$\boxed{U_0 = \frac{C_\gamma E^4}{2\pi}\oint\frac{ds}{\rho^2} = \frac{C_\gamma E^4}{2\pi} I_2} \quad \text{where} \quad I_2 \text{ is the 2nd radiation integral}$$

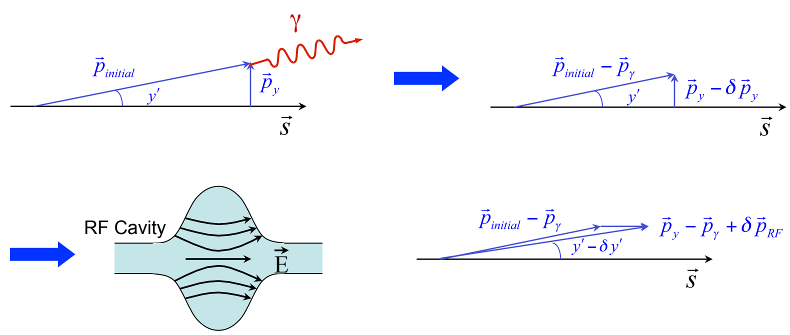
For a lattice with uniform bending radius (iso-magnetic) this yields:

$$U_0[eV] = 8.85\times10^4\,\frac{E^4[GeV]}{\rho[m]}$$

If this energy were not replaced, the particles would lose energy and gradually spiral inward until they would be lost by striking the vacuum chamber wall. The RF cavities replace this lost energy by providing momentum kicks to the beam in the longitudinal direction.

Radiation Damping of Vertical Betatron Motion

We look first at the vertical dimension where, for an ideal machine, we do not need to consider effects of vertical dispersion.



The change in y' after the RF cavity can be written as:

$$\delta y' = -y'\frac{\delta p_{RF}}{p} = -y'\frac{\delta E}{E}$$

Radiation Damping (Vertical)

Recall that an oscillation with amplitude A is described by:

$$A^2 = \gamma y^2 + 2\alpha y y' + \beta y'^2$$

If we assume that the β-function is slowly varying, so that $\alpha = -\beta'/2 \sim 0$, we can write:

$$\delta\left(A^2\right) \approx \underbrace{\delta\left(\gamma y^2\right)}_{=0} + \delta\left(\beta y'^2\right)$$

$$\Rightarrow A\delta A = \beta y'^2 \frac{\delta y'}{y'} = -\beta y'^2 \frac{\delta E}{E}$$

and (using the solution to Hill's equation we obtained previously):

$$y'(s) \approx -\frac{A}{\sqrt{\beta_y(s)}} \sin\left[\psi_y(s) + \phi_0\right]$$

Substituting and averaging then gives:

$$\frac{\delta A}{A} = -\frac{1}{2}\frac{\delta E}{E_0}$$

Radiation Damping (Vertical)

Thus the damping decrement, ie, the fractional decrease in amplitude in one revolution, is:

$$\alpha_y = \frac{\langle \delta A \rangle}{A T_0} = \frac{U_0}{2 E_0 T_0}$$

We can re-write this in exponential decay form as:

$$A(t) = A(0)\exp\left(-\alpha_y t\right)$$

or equivalently, the damping of the vertical emittance is given by:

$$\boxed{\varepsilon(t) = \varepsilon(0)\exp\left(-2\alpha_y t\right)}$$

Radiation Damping (Transverse)

The situation for horizontal radiation damping is somewhat more complicated than the vertical case because of the presence of dispersion generated by the bending magnets. A similar procedure to that followed for the vertical case yields the result:

$$\boxed{\alpha_x = \frac{U_0}{2E_0T_0}\left(1-\mathcal{D}\right)}$$

$$\mathcal{D} = \frac{I_4}{I_2} \quad \text{with} \quad I_2 = \oint \frac{ds}{\rho^2} \quad \text{and} \quad I_4 = \oint \frac{D}{\rho}\left(\frac{1}{\rho^2} + 2k\right) ds$$

It is usual to write the transverse damping decrements as:

$$\boxed{\alpha_i = \frac{U_0}{2E_0T_0} J_i \quad \text{with} \quad J_x = 1 - \mathcal{D} \quad \text{and} \quad J_y = 1}$$

The transverse emittances will damp as:

$$\boxed{\frac{d\varepsilon_i}{dt} = -2\alpha_i \varepsilon_i}$$

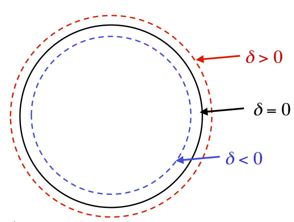
Synchrotron Motion

As particles circulate in a ring, the phase of their passage through the RF accelerating cavities must stay synchronized with respect to the RF frequency in order for their orbits to be stable. This stability is provided by the principle of phase focusing. In the relativistic limit we take:

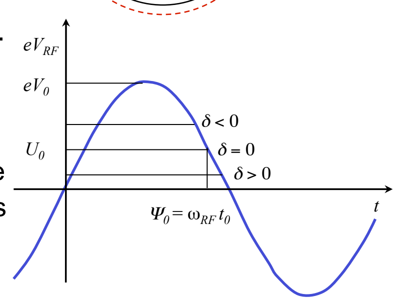
$$\delta = \frac{\Delta p}{p} = \frac{\Delta E}{E}$$

The arrival time for each particle is given by:

$$\frac{\Delta t}{T_0} = \frac{\Delta C}{C} = \alpha_c \delta$$



where α_c is the momentum compaction factor. Thus particles with $\delta>0$ will be delayed and will receive a smaller kick from the RF while particles with $\delta<0$ will arrive early and receive a larger kick as long as the default arrival time in the RF cavity is as shown on the right. This leads to synchrotron oscillations around a stable point.



Synchrotron Equations of Motion

For our description of the longitudinal motion, we will use the variables:

$$\delta = \frac{\Delta E}{E_0} \quad \text{and} \quad \tau = t - t_0$$

where the *0* subscripts are for the synchronous particle.

Thus we can write:
$$\boxed{\frac{d\tau}{dt} = -\alpha_c \delta}$$

and
$$\boxed{\frac{d\delta}{dt} = \frac{eV_{RF}(\tau) - U(E)}{E_0 T_0}}$$

Note that we write the energy loss term as a function of E

where we have assumed that any synchrotron oscillations are far slower than the revolution time (a good assumption in practice) so that using the average energy loss per turn is valid. For small values of τ the RF voltage can be linearized as:

$$\boxed{V_{RF}(\tau) = \frac{U_0}{e} + \tau \left.\frac{dV}{dt}\right|_{t=t_0} = \frac{U_0}{e} + \tau\omega_{RF} V_0 \cos\Psi_s} \quad \text{where } \sin\Psi_s = \frac{U_0}{eV_0}$$

Synchrotron Equation of Motion

We can now write:
$$\boxed{\frac{d^2\delta}{dt^2} + 2\alpha_E \frac{d\delta}{dt} + \omega_s^2 \delta = 0}$$

Synchrotron EOM

where:
$$\alpha_E = \frac{1}{2T_0} \left.\frac{dU(E)}{dE}\right|_{E=E_0}$$

$$\omega_s^2 = \frac{e\alpha_c \omega_{RF} V_0 \cos\Psi_s}{E_0 T_0}$$

The solutions to the synchrotron EOM can be written as:

$$\delta(t) = A_E e^{-\alpha_E t} \cos(\omega_s t - \Psi_s)$$

with
$$\tau(t) = \frac{-\alpha_c A_E}{E_0 \omega_s} e^{-\alpha_E t} \sin(\omega_s t - \Psi_s)$$

which describes the oscillation in energy and time of a particle with respect to the ideal synchronous particle.

Energy Oscillation Damping

There are a couple points to note about the synchrotron EOM.

- First, we note that the synchrotron motion is intrinsically damped towards the motion of the synchronous particle. In the δ-τ plane, an off-energy particle will exponentially spiral towards the origin – the synchronous particle's parameters
- Second, the damping coefficient, α_E, is dependent on the energy of the particle. This happens in two ways. First the power radiated depends on energy. Secondly, the time it takes an electron to complete a revolution around the ring depends on the circumference of the orbit which also depends on the energy. Thus we still have some work to do to understand the rate of damping.

We start by writing the energy lost in one turn as:

$$U = \int_0^T P_\gamma dt$$

Radiation Damping of Synchrotron Motion

We want to convert the integral over time to an integral over s. For a particle that is not on the closed orbit, the path length that it traverses can be written as:

$$d\ell = \left(1 + \frac{x}{\rho}\right) ds \quad \Rightarrow \quad dt = \frac{d\ell}{c} = \frac{1}{c}\left(1 + \frac{x}{\rho}\right) ds$$

where x represents the orbit displacement due to the energy deviation. We can thus write the time differential as:

$$dt = \left(1 + \frac{D\delta}{\rho}\right) ds$$

and the energy loss per turn becomes:

$$U = \frac{1}{c}\oint P_\gamma \left(1 + \frac{D\delta}{\rho}\right) ds$$

Radiation Damping

Evaluating $\left.\frac{dU}{dE}\right|_{E=E_0}$ yields (after a bit of work):

$$\boxed{\alpha_E = \frac{1}{2T_0}\frac{dU}{dE} = \frac{U_0}{2T_0E_0}J_E}$$

where $$J_E = 2 + \mathcal{D} = 2 + \frac{I_4}{I_2}$$

and $$I_2 = \oint \frac{1}{\rho^2}ds \qquad I_4 = \oint \frac{D}{\rho}\left(\frac{1}{\rho^2} + 2k\right)ds \qquad k = \frac{1}{B\rho}\frac{dB_y}{dx}$$

Thus an energy deviation will damp with a time constant

$$\boxed{\tau_E = \frac{2T_0E_0}{J_EU_0}}$$

Summary of Radiation Damping

We can now summarize the radiation damping rates for each of the beam degrees of freedom:

$$\alpha_E = \frac{U_0}{2T_0E_0}J_E \qquad J_E = 2 + \mathcal{D} \qquad \mathcal{D} = 1 + \frac{I_4}{I_2}$$

$$\alpha_x = \frac{U_0}{2T_0E_0}J_x \qquad J_x = 1 - \mathcal{D}$$

$$\alpha_y = \frac{U_0}{2T_0E_0}J_y \qquad J_y = 1$$

and we can immediately write: $$\boxed{J_E + J_x + J_y = 4}$$
Robinson's Theorem

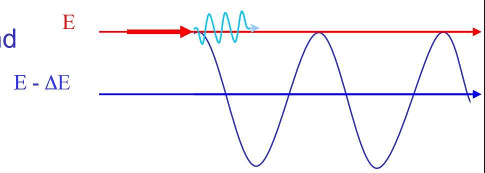
For separated function lattices, $\mathcal{D} \ll 1$ and the longitudinal damping occurs at roughly twice the rate of the damping in the two transverse dimensions.

Radiation damping plays a very special role in electron/positron rings because it provides a direct mechanism to take *hot* injected beams and reduce the equilibrium parameters to a regime useful for high luminosity colliders and high brightness light sources. At the same time, the radiated power plays a dominant role in the design of the technical systems – we will discuss some aspects of this further in tomorrow's lecture.

Equilibrium Beam Properties

Now that we have determined the radiation damping rates, we can explore the equilibrium properties of the beam

- The emission of photons by the beam is a random process around the ring
- Photons are emitted within a cone around the direction of the beam particle with a characteristic angle $1/\gamma$



- This quantized process excites oscillations in each dimension
- In the absence of resonance or collective effects, which also serve to *heat* the beam, the balance between quantum excitation and radiation damping results in the equilibrium beam properties that are characteristic of a given lattice

Quantum Excitation - Longitudinal

We will first look at the impact of quantum excitation in the longitudinal dimension.

For the very short timescales corresponding to photon emission, we can take the equations of motion we previously obtained for synchrotron motion and write:

$$\delta_E^2(t)+\frac{E_0^2\omega_s^2}{\alpha_c^2}\tau^2(t)=A_E^2$$

where A_E is a constant of the motion.

We want to consider the change in A_E due to the emission of individual photons. The emission of an individual photon will not affect the time variable, however, it will cause an instantaneous change in the value of δ_E.

Quantum Excitation - Longitudinal

Thus we can write:

$$\Delta\delta = A_0 \cos\omega_s (t - t_0) - \frac{u}{E_0}\cos\omega_s (t - t_1) = A_1 \cos\omega_s (t - t_1)$$

where u is the energy radiated at time t_1. Thus

$$A_1^2 = A_0^2 + \left(\frac{u}{E_0}\right)^2 - \frac{2A_0 u}{E_0}\cos\omega_s (t_1 - t_0)$$

and $$\Delta A^2 = \left\langle A^2 - A_0^2 \right\rangle = \frac{u^2}{E_0^2}$$

We can thus write the average change in synchrotron amplitude due to photon emission as:

$$\boxed{\frac{d\left\langle A^2 \right\rangle}{dt} = \mathcal{N}\left(\frac{u}{E_0}\right)^2}$$

where $\mathcal{N}$ is the rate of photon emission and u is the photon energy.

Quantum Excitation - Longitudinal

If we now include the radiation damping term, the net change in the synchrotron amplitude can be written as:

$$\frac{d\left\langle A^2 \right\rangle}{dt} = -2\alpha_E \left\langle A^2 \right\rangle + \mathcal{N}\frac{u^2}{E_0^2}$$

The equilibrium properties of a bunch are obtained when the rate of growth from quantum excitation and the rate of damping from radiation damping are equal. For an ensemble of particles where we identify the RMS energy amplitude with the energy spread, we can then write the equilibrium condition as:

$$\sigma_\delta^2 = \left(\frac{\sigma_E}{E_0}\right)^2 = \frac{\left\langle A^2 \right\rangle}{2} = \frac{\left\langle \mathcal{N}\left\langle u^2 \right\rangle \right\rangle_s}{4\alpha_E E_0^2}$$

Photon Emission

$\left\langle \mathcal{N}\left\langle u^2\right\rangle\right\rangle_s$ is the ring-wide average of the photon emission rate, $\mathcal{N}$, times the mean square energy loss associated with each emission. In other words:

$$\mathcal{N} = \int_0^\infty n(u)du \quad \text{and} \quad \mathcal{N}\left\langle u^2\right\rangle = \int_0^\infty u^2 n(u)du$$

where $n(u)$ is the photon emission rate at energy u, and

$$\left\langle \mathcal{N}\left\langle u^2\right\rangle\right\rangle_s = \frac{1}{C}\oint \mathcal{N}\left\langle u^2\right\rangle ds$$

where C is the ring circumference. Derivations of the photon spectrum emitted in a magnetic field are available in many texts and we will simply quote the result:

$$\mathcal{N}\left\langle u^2\right\rangle = 2C_q\gamma^2\frac{E_0P_\gamma}{\rho} \quad \text{where} \quad C_q = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} = 3.84\times10^{-13}m$$

Energy Spread and Bunch Length

Integrating around the ring then yields the beam energy spread:

$$\boxed{\sigma_\delta^2 = \left(\frac{\sigma_E}{E_0}\right)^2 = C_q\gamma^2\frac{I_3}{J_EI_2} \quad \text{where} \quad I_3 = \oint\frac{ds}{|\rho|^3}}$$

Using our solution to the synchrotron equations of motion, the bunch length is related to the energy spread by:

$$\boxed{\sigma_\ell = \frac{c\alpha_c}{\omega_s E_0} \quad \text{where} \quad \omega_s^2 = \frac{e\alpha_c\omega_{RF}V_0\cos\Psi_s}{E_0T_0}}$$

We note that the bunch length scales inversely with the square root of the RF voltage.

Quantum Excitation - Horizontal

In order to evaluate the impact of the radiated photon on the motion of the emitting electron, we recall

$$A^2 = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

The change in closed orbit due to losing a unit of energy, u, is given by:

$$\delta x = -D(s)\frac{u}{E_0}$$

$$\delta x' = -D'(s)\frac{u}{E_0}$$

and we can then write:

$$\boxed{\delta A^2 = \left(\gamma D^2 + 2\alpha DD' + \beta D'^2\right)\frac{u^2}{E_0^2} = \mathcal{H}(s)\frac{u^2}{E_0^2}}$$

where $\mathcal{H}(s)$ is the *curly-H* function.

Horizontal Emittance

We can then write an excitation term for the rms emittance as:

$$\left.\frac{d\varepsilon_x}{dt}\right|_{QE} = \frac{1}{2}\frac{d\langle A^2\rangle}{dt} = \frac{\left\langle \mathcal{N}\mathcal{H}\langle u^2\rangle\right\rangle_s}{2E_0^2}$$

Equating this expression to the damping rate yields (after some calculation) the equilibrium horizontal emittance:

$$\boxed{\varepsilon_x = C_q\frac{\gamma^2\left\langle\dfrac{\mathcal{H}}{\rho^3}\right\rangle}{J_x\left\langle\dfrac{1}{\rho^2}\right\rangle} = C_q\frac{\gamma^2 I_5}{J_x I_2}}$$

where we have defined the next synchrotron radiation integral:

$$\boxed{I_5 = \oint\frac{\mathcal{H}}{\rho^3}ds}$$

Quantum Excitation - Vertical

In the vertical dimension, where we assume the ideal case of no vertical dispersion, the quantum excitation of the emittance is determined by the opening angle of the emitted photons. The resulting perturbation to the vertical motion can be described as:

$$\delta y = 0 \qquad \delta y' = \frac{u}{E_0}\theta_\gamma$$

and we can write: $$\delta\left\langle A^2\right\rangle = \left(\frac{u\theta_\gamma}{E_0}\right)^2 \beta_y$$

Thus, proceeding as we have on the preceding pages, we can write the expression for the equilibrium emittance as:

$$\varepsilon_y = \frac{\left\langle \mathcal{N}\left\langle u^2\right\rangle \beta_y\right\rangle_s \left\langle\theta_\gamma^2\right\rangle}{4E_0^2} = \frac{\left\langle \mathcal{N}\left\langle u^2\right\rangle \beta_y\right\rangle_s}{4\gamma^2 E_0^2}$$

$$\boxed{\varepsilon_y \approx \frac{C_q}{2J_y I_2}\oint \frac{\beta_y}{\rho^3} ds}$$

Vertical Emittance & Emittance Coupling

For typical storage ring parameters, the vertical emittance due to quantum excitation is negligible. Assuming a typical β_y values of a few 10's of meters and bending radius of ~100m, we can estimate $\varepsilon_y \leq 0.1$ pm. The observed sources of vertical emittance are:

- ***emittance coupling*** whose source is ring errors which couple the vertical and horizontal betatron motion
- ***vertical dispersion*** due to vertical misalignment of the quadrupoles and sextupoles and angular errors in the dipoles

The vertical and horizontal emittances in the presence of a collection of such errors around a storage ring is commonly described as:

$$\varepsilon_y = \frac{\kappa}{1+\kappa}\varepsilon_0; \quad \varepsilon_x = \frac{1}{1+\kappa}\varepsilon_0 \quad \text{for } 0 < \kappa < 1$$

ε_0 is the natural emittance.

Radiation Integrals and Equilibrium Quantities

Summary of Radiation Integrals:

$$I_1 = \oint \frac{D(s)}{\rho} ds$$

$$I_2 = \oint \frac{1}{\rho^2} ds$$

$$I_3 = \oint \frac{1}{|\rho|^3} ds$$

$$I_4 = \oint \frac{D(s)}{\rho}\left(\frac{1}{\rho^2} + 2k\right) ds$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho|^3} ds$$

$$\mathcal{H} = \gamma D^2 + 2\alpha D D' + \gamma D'^2$$

Summary of Equilibrium Beam Properties:

$$\alpha_c = \frac{I_1}{C}$$

$$U_0 = \frac{C_\gamma E^4}{2\pi} I_2 \quad \text{where} \quad C_\gamma = 8.85\times 10^{-5} m/(GeV)^3$$

$$\alpha_i = \frac{U_0}{2E_0T_0} J_i, \quad i = x, y, E$$

$$J_x = 1 - \mathcal{D}; \quad J_y = 1; \quad J_E = 2 + \mathcal{D}; \quad \mathcal{D} = \frac{I_4}{I_2}$$

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{C_q \gamma^2 I_3}{J_E I_2} \quad \text{where} \quad C_q = 3.84\times 10^{-13} m$$

$$\sigma_\ell = \frac{c\alpha_c}{\omega_s E_0} \quad \text{where} \quad \omega_s^2 = \frac{e\alpha_c \omega_{RF} V_0 \cos\Psi_s}{E_0 T_0}; \quad \sin\Psi_s = \frac{U_0}{eV_0}$$

$$\varepsilon_x = \frac{C_q \gamma^2 I_5}{J_x I_2}; \quad \varepsilon_y \approx \frac{C_q}{2J_y I_2} \oint \frac{\beta_y}{\rho^3} ds \quad \text{(quantum excitation)}$$

Emittance Scaling in Lattices

The natural emittance of a lattice is given by:

$$\varepsilon_0 = \frac{C_q \gamma^2}{J_x} \frac{I_5}{I_2}$$

The ratio $\frac{I_5}{I_2}$ can be tailored to provide very low emittance. It can be shown that the natural emittance scales approximately as:

$$\varepsilon_0 \approx F \frac{C_q \gamma^2}{J_x} \theta^3$$

where F is a function of the lattice design and θ is the bending angle from the dipoles in each lattice cell. The natural emittance can be made small by having small bending angles in the dipoles of each lattice cell and by optimizing F. The theoretical minimum emittance (TME) lattice has

$$F \approx \frac{1}{12\sqrt{15}}$$

Unfortunately, designing a very low emittance lattice in this way may have serious impact on the cost and/or performance of a low emittance ring.

Achieving Ultra-Low Emittance

The path to low emittance that is pursued in a damping ring, is to provide insertion devices, wigglers, which dominate the radiation damping of the machine. For a sinusoidal wiggler, we can write the energy loss around the ring as:

$$U_0 = \frac{C_\gamma E^4}{2\pi}\left(\oint_{dipoles}\frac{1}{\rho^2}ds + \int_0^{L_{wiggler}}\frac{1}{\rho_{wig}^2}ds\right) = U_{dip} + U_{wig}$$

The overall length of the wiggler section, along with the wiggler period and peak field, can be adjust to make the second term dominate the radiation losses in the ring and hence the damping rate. The expressions

$$\varepsilon_{dip} = C_q\gamma^2\frac{I_{5dip}}{I_{2dip}} \quad \text{and} \quad \varepsilon_{wig} = C_q\gamma^2\frac{I_{5wig}}{I_{2wig}}$$

give the emittance contributions of the dipole and wiggler regions, respectively. We can then write the natural emittance of the ring as:

$$\varepsilon_0 = \frac{\varepsilon_{dip}}{1+F} + \frac{\varepsilon_{wig}F}{1+F} \quad \text{where} \quad F = \frac{U_{wig}}{U_{dip}}$$

Thus, if the wiggler radiation dominates, the emittance contribution due to the dipoles is reduced by a factor of F and the ring emittance is dominated by the intrinsic wiggler emittance. In fact, the wiggler emittance can be quite small by placing the wigglers in zero dispersion regions with small β_x.

Segue

At this point we have reviewed the key physics concepts involved in the design of fixed energy lepton rings.

Now we will take a look at lattice designs for ILC and CLIC damping rings *and how they've been forced to evolve*.

Hopefully you will leave today's lecture with an appreciation of

- The features that a real design must include
- How designs are optimized to
  - Handle interface requests
  - Reduce cost
  - Improve robustness

The ILC Damping Rings Lattice

At the time of the ILC Reference Design Report, the ILC damping rings lattice was based on a variant of the TME (theoretical minimum emittance) lattice. As noted earlier, however, there is some flexibility in the choice of lattice style in a wiggler dominated ring.

Thus, the the RDR damping ring design was superseded by a design that employs a FODO lattice. The FODO-based design offers greater flexibility in setting the momentum compaction of the damping rings and was chosen to be the basis for further ILC DR design work over the last 2 years. Of course, change never stops and present design efforts are focused on a move to a shorter (3.2 km) ring. We will come back to this later.

It should be noted that much of the design work for each of these lattices is associated with the injection/extraction straights, RF and wiggler regions, and other specialty segments of the accelerator.

Technical Design Phase Baseline Parameters

Main Parameters

Table 1: Main parameters of the baseline lattice for the RDR and proposed for the EDR.

	RDR [1]	Proposed for EDR		
		low rf	nominal	high threshold
Beam energy	5 GeV	5 GeV		
Harmonic number	14516	14042		
RF frequency	650 MHz	650 MHz		
RF voltage[1]	22.1 MV	13.2 MV	21.6 MV	25.8 MV
Number of rf cavities	18	8	16	16
Momentum compaction factor[1]	4×10^{-4}	1.1×10^{-4}	1.8×10^{-4}	2.7×10^{-4}
Natural rms bunch length[1]	9 mm	6.6 mm	6 mm	6.6 mm[(2)]
Natural energy spread	0.13%	< 0.13%		
Natural emittance	5 µm	< 8 µm		
Transverse damping times	25 ms	< 25 ms		
Betatron acceptance ($A_x + A_y$)	> 0.01 m	> 0.01 m		
Energy acceptance	± 0.5%	± 0.5%		

[1] These parameters should be variable over some range: see Table 2.
[2] Can be reduced to 6 mm with 30.8 MV total rf voltage (18 cavities): see Table 2.

Max. value for bunch compressors

Updated target (previously 4×10^{-4})

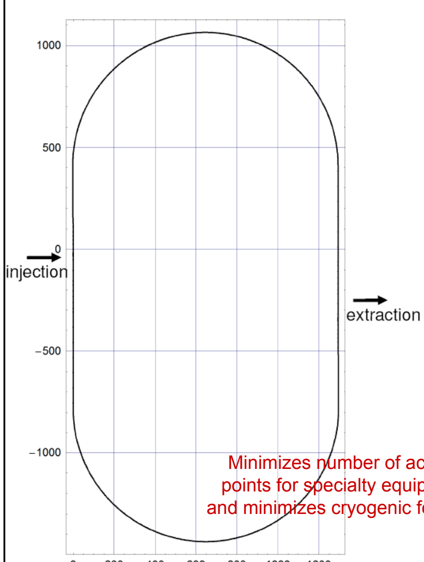
Parameter Comparisons

	OCS8	FODO4			FODO5			DCO		
Beam energy (GeV)	5.00									
Circumference (m)	6476.440									
RF frequency (MHz)	650									
Harmonic number	14042									
Number of straight sections	8	4			8			2		
Arc cell type	TME	FODO			FODO			FODO		
Arc cell length (m)	38.9	29.4			28.4			21.0		
Number of arc cells	128	184			188			192		
Number of dipoles per arc cell	1	2			2			1		
Arc dipole length (m)	6	2			2			2		
Arc dipole field (T)	0.146	0.142			0.139			0.273		
Number of quadrupoles per arc cell	4	2			2			2		
Number of sextupoles per arc cell	4	2			2			2		
Natural rms bunch length (mm)	9.00	9.00			6.00			6.00		
Natural energy spread (10^{-3})	1.28	1.28			1.28			1.27		
Transverse damping time (ms)	25	25			25			21		
Approximate phase advance per cell	90	60	72	90	72	90	108	72	90	100
Momentum compaction factor (10^{-4})	4.0	6.0	4.0	2.0	4.00	2.5	1.7	2.8	1.7	1.3
Normalised natural emittance (m)	5.2	5.4	4.2	3.4	3.9	3.1	2.6	6.5	4.7	4.3
RF voltage (MV)	21.2	31	22	15	45	29	21	32	21	17
RF acceptance (%)	1.46	1.65	1.48	1.21	2.70	2.45	2.17	2.35	1.99	1.72
Synchrotron tune	0.059	0.091	0.061	0.038	0.089	0.056	0.037	0.061	0.038	0.028
Horizontal tune	49.23	40.29	46.28	58.29	50.30	61.30	72.28	64.75	75.20	80.45
Natural horizontal chromaticity	-64	-48	-54	-74	-63	-79	-108	-77	-95	-107
Vertical tune	53.34	41.25	47.24	57.25	51.26	62.24	69.23	61.40	71.40	75.90
Natural vertical chromaticity	-64	-49	-55	-73	-63	-80	-100	-76	-93	-104

March 4, 2008 TILC08 Global Design Effort 35

The DCO Lattice

Wolski, Korostelev



- Arcs consist of a total of 192 FODO cells
- Flexibility in tuning momentum compaction factor, given by phase advance per arc cell:
  - **72° phase advance: α_p=2.8×10^{-4}**
  - **90° phase advance: α_p=1.7×10^{-4}**
  - **100° phase advance: α_p=1.3×10^{-4}**
- No changes in dipole strengths needed for different working points.
- Racetrack structure has two similar straights containing:
  - **injection and extraction in opposite straights**
  - **phase trombones**
  - **circumference chicanes**
  - **rf cavities**
  - **"doglegs" to separate wiggler from rf and other systems**
  - **wiggler**

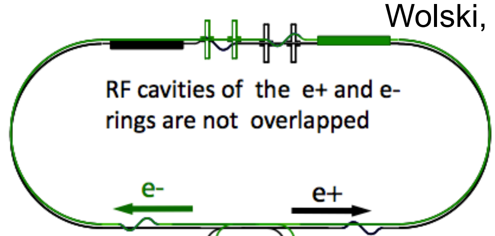
November 1, 2010 A3 Lectures: Damping Rings - Part 2 36

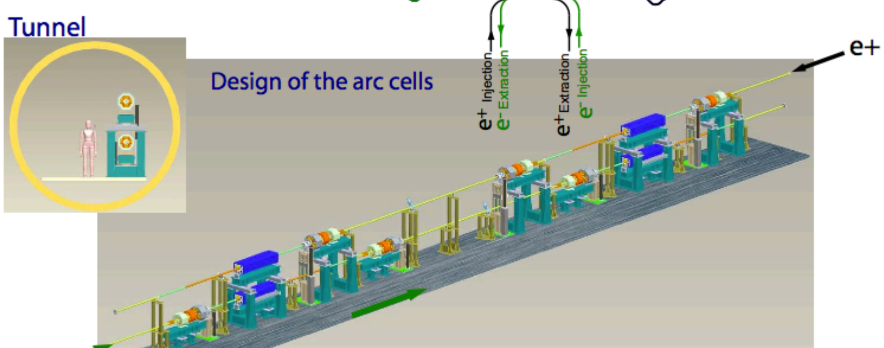
DCO4 Layout

Wolski, Korostelev

e- Damping Ring
e+ Damping Ring

The lattice of e- DR is identical to the lattice of the e+ DR.





Beams circulate in opposite directions in the two rings.

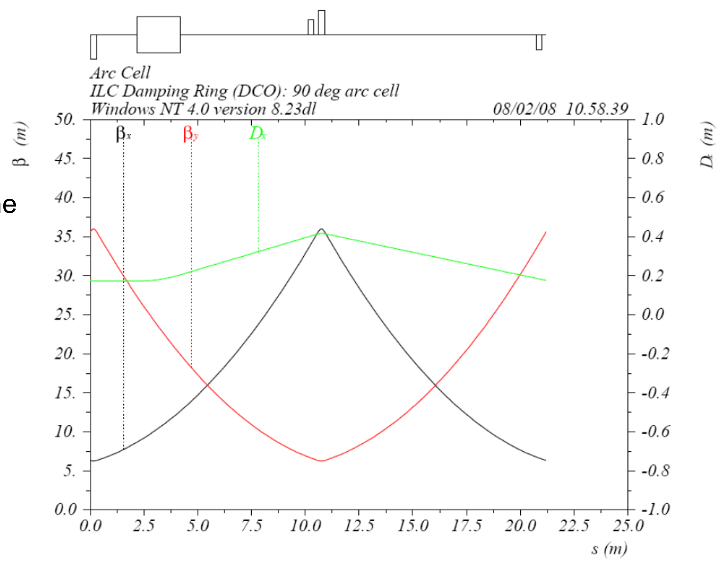
DCO Design Parameters

Beam energy	5 GeV
Circumference	6476.440 m
RF frequency	650 MHz
Harmonic number	14042
Transverse damping time	21.0 ms
Natural rms bunch length	6.00 mm
Natural rms energy spread	1.27×10^{-3}

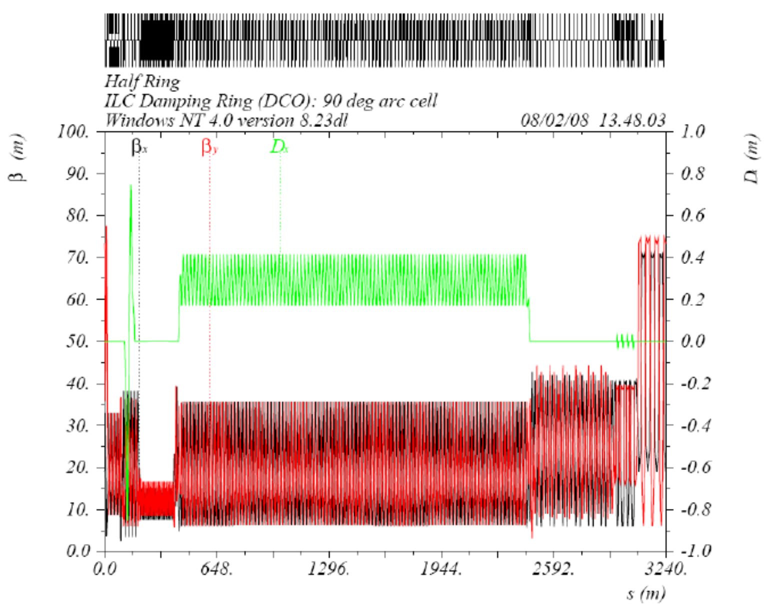
Phase advance per arc cell (approximate)	72°	90°	100°
Momentum compaction factor	2.80×10^{-4}	1.73×10^{-4}	1.29×10^{-4}
Normalised natural emittance	6.53 μm	4.70 μm	4.27 μm
RF voltage	31.6 MV	21.1 MV	17.2 MV
RF acceptance	2.35%	1.99%	1.72%
Synchrotron tune	0.061	0.038	0.028
Horizontal tune	64.750	75.200	80.450
Natural horizontal chromaticity	-76.5	-95.1	-106.9
Vertical tune	61.400	71.400	75.900
Natural vertical chromaticity	-75.6	-93.4	-103.5

Arc Cell

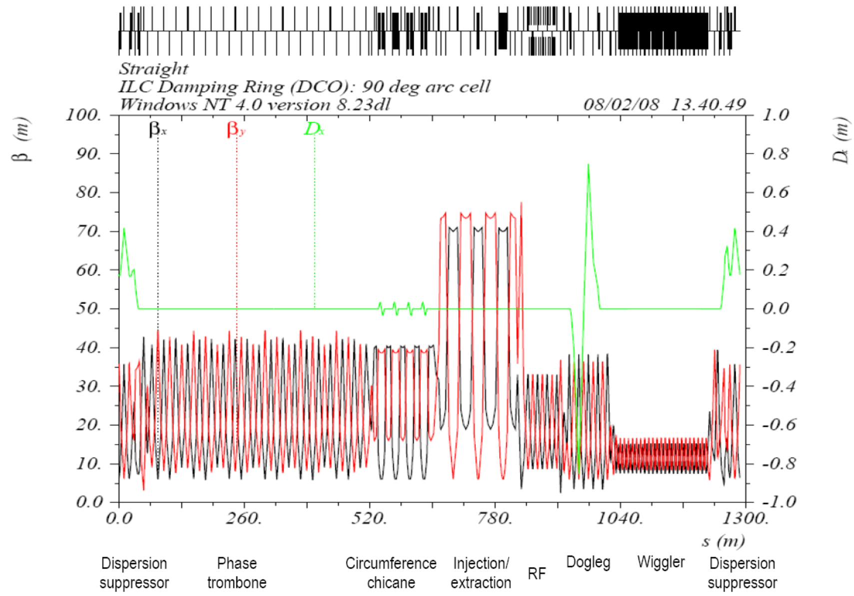
The arc cell design is a slightly non-standard FODO cell which utilizes relatively little dipole in each cell to help control the dispersion in the design.



Half Ring

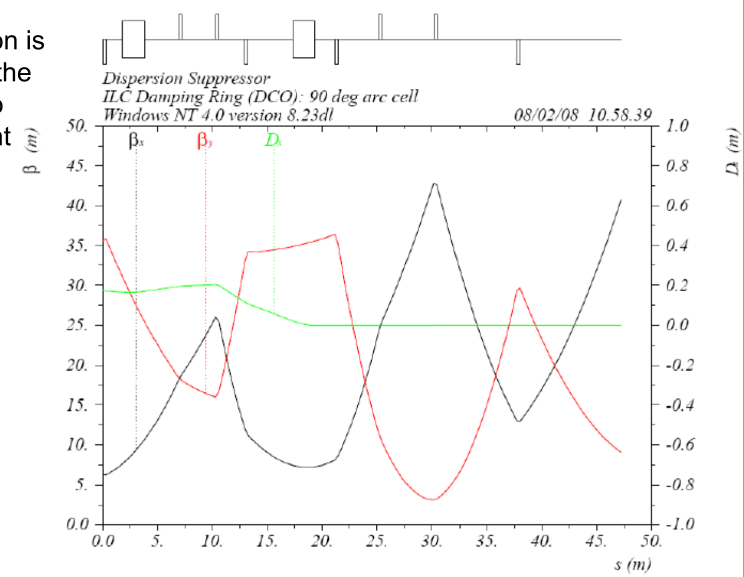


DCO Straight Section



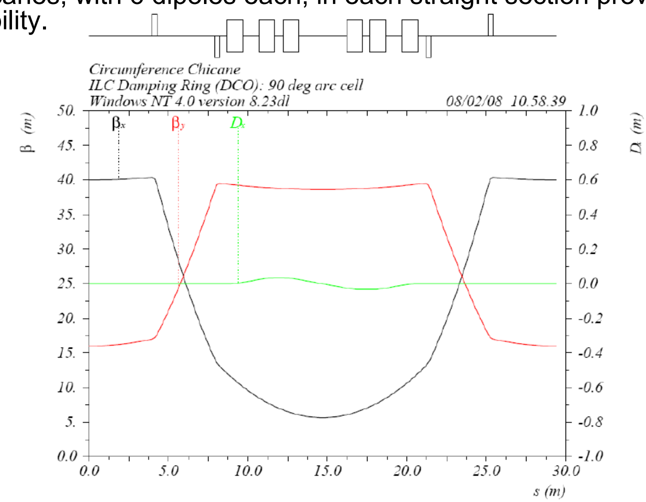
Dispersion Suppressor Section

A dispersion suppressor section is utilized to match the arcs with the zero dispersion straight sections



Chicanes

Because the ring RF frequency must be locked to the main linac RF, an important feature of the DR lattice is the need to adjust the circumference of the ring while maintaining a fixed RF frequency. Estimates of our ability to maintain the circumference suggest that adjustments on the order of ±1 cm are required. A set of 4 chicanes, with 6 dipoles each, in each straight section provide this range of flexibility.



Other Features of the DCO Lattice

Other key features of the DCO lattice include:

- Space in the injection and extraction optics to accommodate up to 33 kicker modules
  - Each module includes a stripline kicker of 30 cm length and 20 mm gap
  - 30 modules with the plates operating at ±7 kV are required for operation
- Space in the straights for up to 24 RF cavities.
  - Assuming 1.7 MV per module, 19 cavities are required to provide a 6 mm bunch length in the high momentum compaction (α_c = 2.8×10^{-4}) configuration
- The dogleg sections provide 2 m transverse shift of the beamline after each wiggler straight
  - The dogleg will allow installation of a photon dump to handle the forward radiation from each wiggler section
  - It will also serve to protect sensitive downstream hardware from the wiggler radiation fan.
  - This arrangement allows the RF and wiggler sections to be quite close and hence minimizes the amount of cryogenic transfer line required.

How Do We Validate the Design?

At this point we have a lattice design that satisfies a great many design details. There are several steps that need to be followed in order to validate the design.

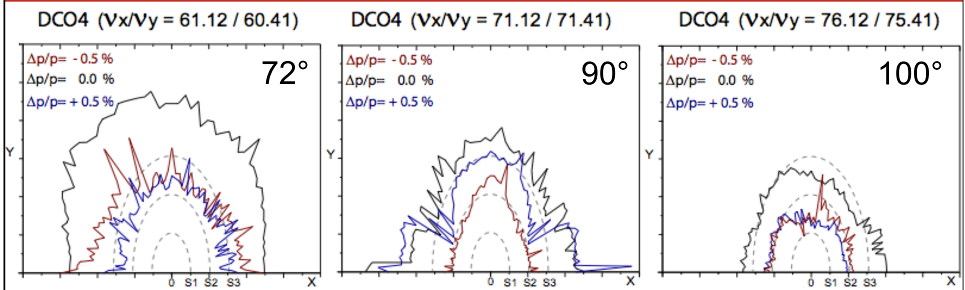
1. Verify that all of the magnet strengths are viable – this is particularly an issue for small rings with strong focusing elements. For the ILC DR, a major question was whether a suitable wiggler could be constructed. Luckily there was a working example, the CESR-c wiggler, which satisfied this criteria.
2. Verify that other components (eg, kickers) can be manufactured – in the case of the ILC DR the design choice was dependent on the development of fast high voltage pulsers to drive the strip-line kickers
3. Verify the basic performance of the ring by tracking particles. There are 2 analyses that you will regularly encounter if you work on rings: Dynamic Aperture and Frequency Map Analysis
4. Finally, a major issue for high intensity rings is the analysis of collective effects that can de-stabilize the beam. This work is typically not complete until the ring is operating and the machine tells you what was missed.

Dynamic Aperture

Non-linear particle dynamics can limit the phase space volume of stable particles. The principal source of the non-linear behavior arises from the sextupoles used to correct the ring chromaticity. Typically a great deal of effort is spent in trying to achieve the "best" sextupole distribution for a given ring. Of course there are other sources of non-linearities (eg, magnet field errrors). Given the extreme performance parameters that we are pursuing, it is preferable to include all such errors in the analysis.

In calculating the dynamic aperture, particles with a range of initial coordinates and energies are tracked through the ring, typically for several hundred to several thousand turns. The particles which are not lost define the stable phase space volume.

DCO4 Dynamic Aperture



These dynamic aperture plots show the maximum initial amplitudes of stable trajectories. It is customary to overlay either the injected or equilibrium beam size on the plot. Significant margin is usually desirable in a design because machine errors (which cannot always be included) will degrade it.

- S1,2,3 lines give the nominal 1,2,3σ contours for the injected beam
- In reality, S1 corresponds to the maximum injected coordinates expected for the positron beam
- Blue and red lines indicate particles with ±0.5% energy deviations
- Solid black line indicates on energy particles

Frequency Map Analysis I

FMA provides another method to characterize the performance of our lattice. It provides valuable information about how the tunes of particles with different initial oscillation amplitudes evolve and can help identify conditions/errors in the machine that need to be closely examined.

In order to understand FMA, we need to first say a few words about the tune plane and resonances.

In any oscillatory system, an excitation at a resonant frequency can result in rapid amplitude growth. For a particle accelerator, once this growth becomes sufficiently large, particles will be lost from the machine. Let's consider the simplest accelerator example:

A machine is operating with an integer number of betatron wavelengths per revolution and an error in a corrector results in an unintended kick to the beam on every revolution. Because the machine is operating at an integer tune, the kicks will be additive over successive turns until the amplitude grows so large that the beam exits the machine.

Frequency Map Analysis II

Tune plane and resonances (cont'd)

More generally, since the x and y motions of the beam behave as coupled resonators, the general condition for resonances that may potentially be excited can be expressed as:

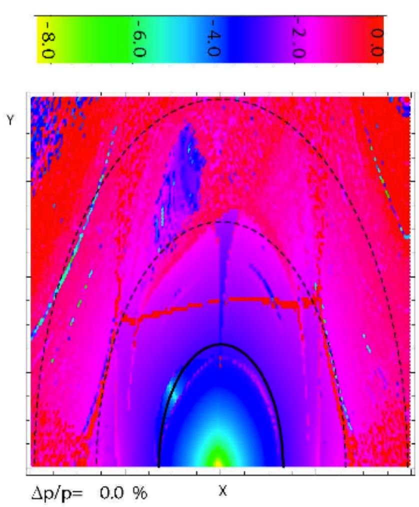
$$mQ_x + nQ_y = p$$

where m, n and p are all integers. The order of the resonance is given by |m| + |n|. Needless to say, the working point for a given lattice is explicitly chosen to avoid such resonances.

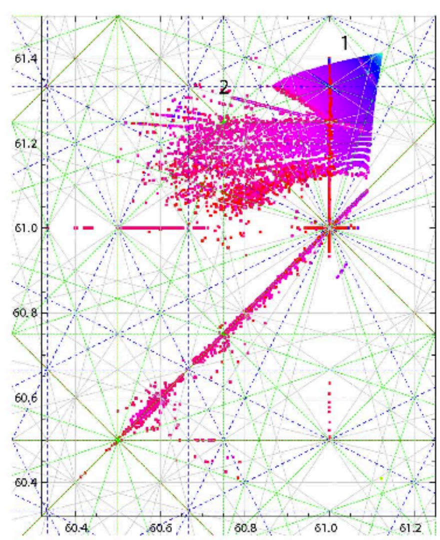
FMA

Although the zero-amplitude working point may be far away from any such resonance conditions, the non-linearities in the machine will result in the tunes of individual particles with non-zero amplitudes exploring a much larger footprint in tune space. FMA shows how particles diffuse in the tune plane and can help identify resonances that may be of concern. To do this operates on turn-by-turn data where it monitors the evolution of the short term tunes. You may note that there is no distinction made about whether the data is simulated or real.

FMA Results for the DCO4 Lattice



$$\Delta\nu = \log_{10}\sqrt{\left(\nu_x^{1024} - \nu_x^{512}\right)^2 + \left(\nu_y^{1024} - \nu_y^{512}\right)^2}$$



1) 3νx + 2νy = 306

2) νx + 4νy = 306

Are We Done?

At this point it appears that we have a lattice that satisfies the ILC needs. Does that mean the design process is complete?

The short answer is ***NO***!

Further design modifications have been requested and are currently undergoing complete evaluation. A change in parameters has opened the possibility of reducing the number of bunches by a factor of 2. This certainly generates great interest because we can, roughly speaking, simply scale down the circumference of the ring by a factor of 2 ⇨ ***cost savings***

Even if that weren't the case, continued technical development and further evaluation of impedance and instability issues is required.

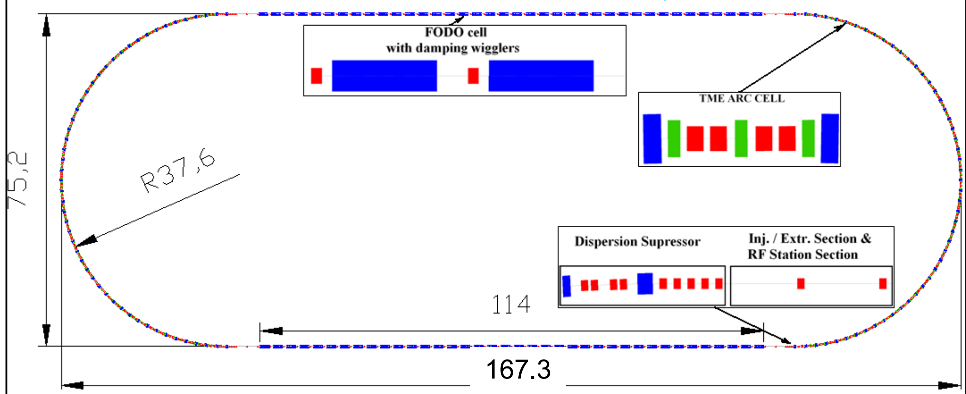
The CLIC Damping Ring

At this point we take a short tour of the present state of design of the CLIC DR. I will acknowledge in advance Y. Papaphilippou and the CLIC DR team for the information presented here

Design Parameters	CLIC	ILC
Energy [GeV]	2.86	5.0
Circumference [m]	420.56	6476
Energy loss/turn [MeV]	4.2	10.3
RF voltage [MV]	4.9	24
Momentum Compaction Factor	$8\text{x}10^{-5}$	$1.3\text{-}2.8\times10^{-4}$
Damping time trans/long [ms]	1.88/0.96	21/11
# dipoles / wigglers	100/52	192/88
Dipole/ wiggler field [T]	1.4/2.5	1.6

CLIC Damping Ring Layout

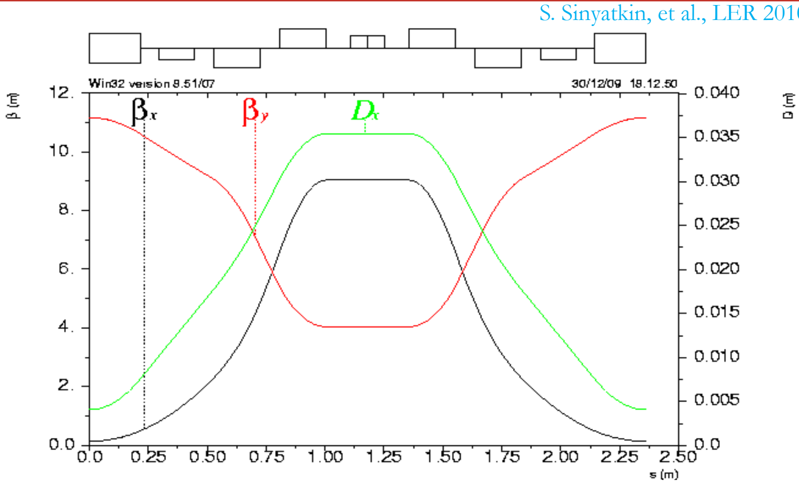
S. Sinyatkin, et al., LER 2010



- Racetrack shape with
  - 96 TME arc cells (4 half cells for dispersion suppression)
  - 26 Damping wiggler FODO cells in the long straight sections (LSS)
  - Space reserved upstream the LSS for injection/extraction elements and RF cavities

Arc Cell

S. Sinyatkin, et al., LER 2010

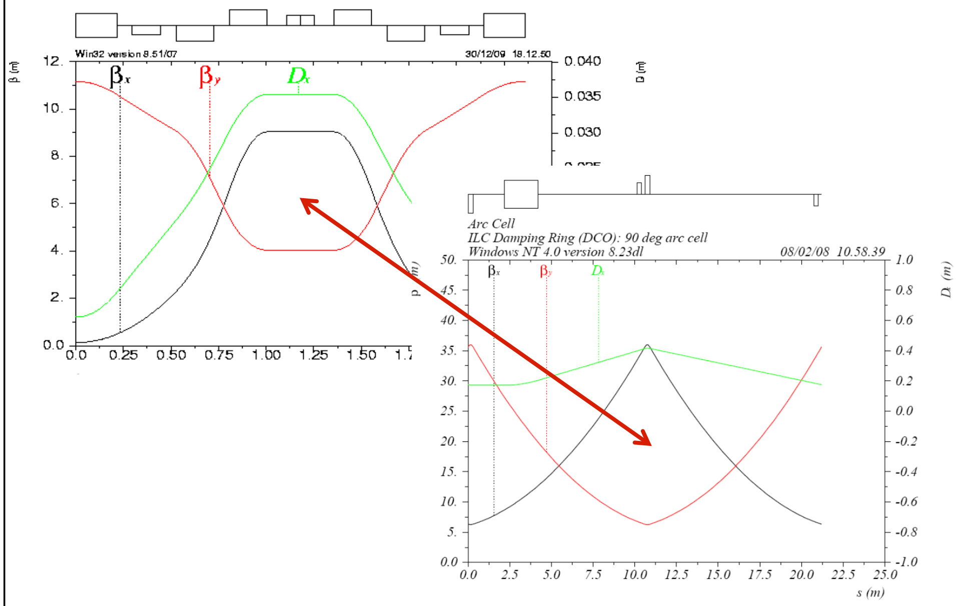


Special Features:

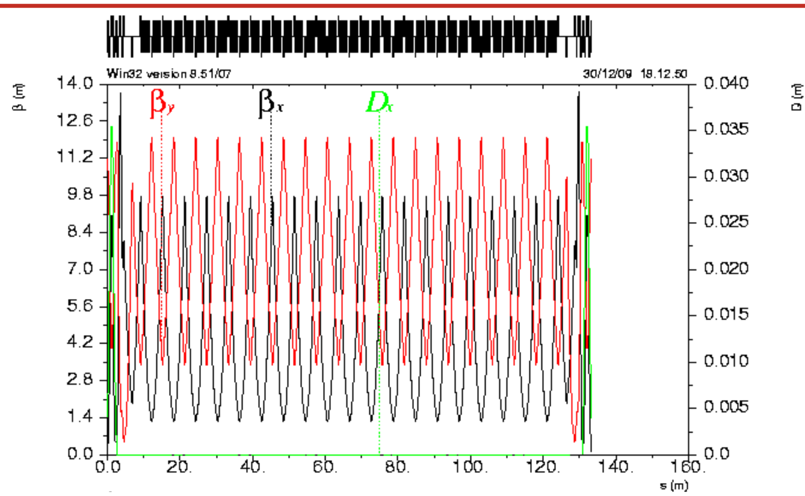
2.36m-long TME cells with bends including small gradient

Detailed control of optics functions throughout the cell enable mitigation of IBS effects

Compare with DCO4



Long Straight Section

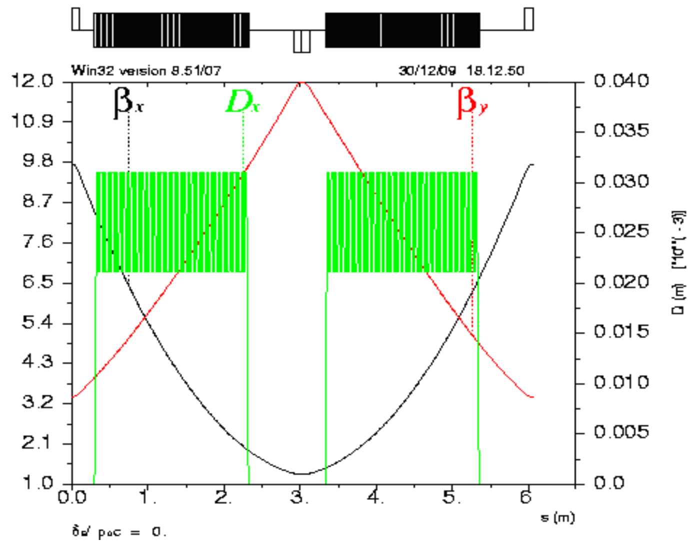


The straight sections are dominated by the 6m FODO cells for the wigglers

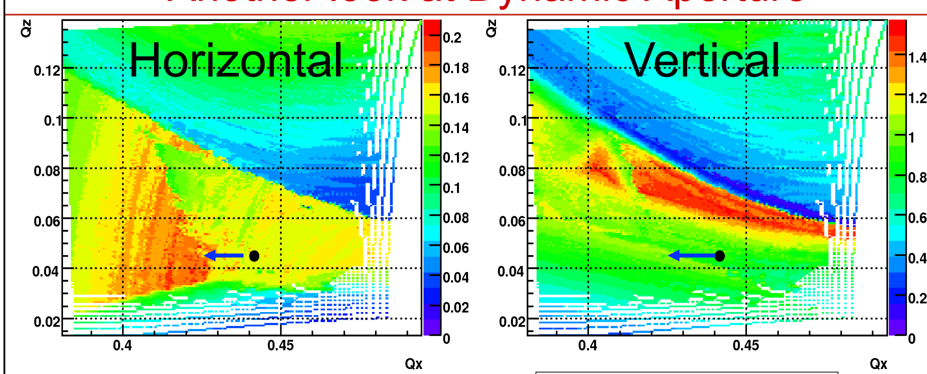
Wiggler Cell

Horizontal phase advance has been optimized for low emittance (we will come back to this later)

Short drift sections are required due to the need for photon stops



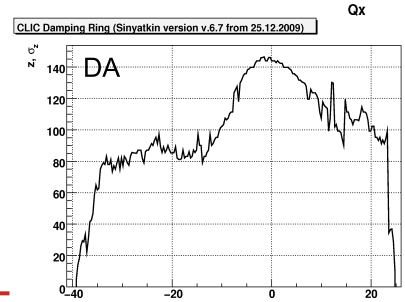
Another look at Dynamic Aperture



Tune scan simulation to determine optimum working point

Dynamic aperture appears very good, but

- Wigglers, misalignments and field errors not yet included



CLIC DR Parameter Summary

At its present stage of development, work has been carried out to ensure that the basic ring components can be built and fit into a physical layout

A significant difference from the ILC design is the use of a combined function (dipole-quad) in the arcs

This has allowed a design with good dynamic aperture and much reduced sensitivity to IBS effects to emerge

But, as with the ILC, much work remains…

Parameters	**Value**
Energy [GeV]	2.86
Circumference [m]	420.56
Coupling	0.0013
Energy loss/turn [MeV]	4.2
RF voltage [MV]	4.9
Natural chromaticity x / y	-168/-60
Momentum compaction factor	8e-5
Damping time x / s [ms]	1.9/ 0.96
Dynamic aperture x / y [σ_{inj}]	30 / 120
Number of dipoles/wigglers	100/52
Cell /dipole length [m]	2.36 / 0.43
Dipole/Wiggler field [T]	1.4/2.5
Bend gradient [1/m^2]	-1.10
Max. Quad. gradient [T/m]	73.4
Max. Sext. strength [kT/m^2]	6.6
Phase advance x / z	0.452/0.056
Bunch population, [10^9]	4.1
IBS growth factor	1.4
Hor./ Ver Norm. Emittance [nm.rad]	400 / 4.5
Bunch length [mm]	1.6
Longitudinal emittance [keVm]	5.5

Summary

During today's lecture, we have reviewed the basics of storage ring physics with particular attention on the effect know as radiation damping which is central to the operation of storage and damping rings. We have also had an overview of the key design elements presently incorporated into the ILC damping ring lattice and the CLIC damping ring lattice. The homework problems will provide an opportunity to become more familiar with some of these issues.

Tomorrow we will look in greater detail at specific systems and specific physics effects which play significant roles in the successful operation of a damping ring.