Damping Rings Lecture 3A Part 2 Homework Problems

1. A key contribution of damping wigglers in the damping ring is to lower the damping time of the beam. For a ring having total wiggler length of L_w and wigglers with peak field B_w, show the dependence of the damping time on the wiggler parameters. You may assume the wiggler field has a simple sinusoidal form in s. In other words: $B=B_w\sin(k_w s)$ where B_w is the peak wiggler field and the period of the wiggler is given by $2\pi/k_s$. Hint: It is often easiest to scale quantities with respect to the beam rigidity…

2. In addition to shortening the damping time, the wigglers also act to increase the energy spread of the beam. In the same level of approximation as in the preceding problem, now calculate the contribution of the wiggler to the energy spread of the beam.

3. Discuss the trade-off between damping time and energy spread in determining the energy of the damping ring. Recall that the ILC DR baseline design employs approximately 200 m of wigglers with a peak field of 1.6 T and that the energy spread that can be tolerated by the downstream systems is <0.15%.

4. The emittance in a wiggler dominated ring can be written as:

$$\varepsilon_0 = \frac{\varepsilon_{dip}}{1+F} + \frac{\varepsilon_{wig} F}{1+F} \quad \text{where} \quad F = \frac{U_{wig}}{U_{dip}}$$

Assuming that F>>1, we have: $\varepsilon_0 \approx \varepsilon_{wig} = C_q \dfrac{\gamma^2}{J_x} \dfrac{I_{5,wig}}{I_{2,wig}}$

(a) For this problem assume that the wiggler field can be written as $B(s)=\cos(ks)$ [this form will make solving the boundary conditions somewhat simpler]. Using the differential equation for the dispersion function in Lecture 1, write an expression for the dispersion function. Match the edge of the wiggler to a zero dispersion region at s=0 (ie, D=D'=0) and solve for the dispersion function.

(b) Now that you have an expression for the dispersion, integrate $I_{5,wig}$ over a half period of the wiggler. You may assume that β is constant for this integration. Hint: What does this imply about the value of the α term in the integral?

(c) Now determine the value of $I_{2,wig}$ and obtain an expression for the emittance. If you assume $\lambda_{wig} << \beta$, can you simplify your result?

5. There is enough information in the lecture notes to pick out suitable parameters for the various designs we have been looking at. Pick one and calculate the wiggler-dominated emittance. Does your answer agree with the quoted numbers?