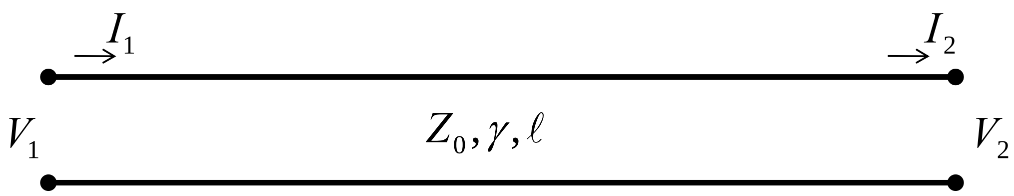
Room temperature RF

Part 1: Introduction



TRANSMISSION LINES

Maxwell's equations in vacuum

Maxwell's equations:

$$\nabla \times \vec{H} - \varepsilon_0 \frac{\partial}{\partial t}\vec{E} = 0 \quad \nabla \cdot \mu_0 \vec{H} = 0$$
$$\nabla \times \vec{E} + \mu_0 \frac{\partial}{\partial t}\vec{H} = 0 \quad \nabla \cdot \varepsilon_0 \vec{E} = 0$$

We know that **homogeneous plane waves** are solutions, let's demonstrate this!

- We chose the Cartesian coordinate z to be in the direction of propagation.
- We chose the Cartesian coordinate y to be in the direction of the electric field vector (called also the polarization). The magnetic field vector will thus be in x-direction.
- With these two condition we can say that the fields are TEM.
- We look at a special solution with sinusoidal time dependence.
- With this knowledge of the expected solution we can write

$$\vec{E} = \vec{u}_y \operatorname{Re}\left\{E_x(z)e^{j\omega t}\right\}$$
$$\vec{H} = \vec{u}_x \operatorname{Re}\left\{H_y(z)e^{j\omega t}\right\}$$

Let's pretend not to know the z-dependence and derive it here to verify the solution. We have equally introduced here the notation of complex amplitudes which are convenient for a simple notation for ω-domain analysis. It allows to replace the time derivative operator by a multiplication:

$$\frac{d}{dt} \equiv j\omega$$

Time domain vs. frequency domain

- The Fourier transform allows to analyze in either ω- or t-domain (and to transform in the respective other) as long as the equations are **linear** (LTI: linear time-invariant).

FT:

$$g(t) \circ\!\!-\!\!\bullet\ G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t)e^{j\omega t}dt$$

IFT:

$$G(\omega) \bullet\!\!-\!\!\circ\ g(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega)e^{-j\omega t}d\omega$$

- One would prefer to use ω-domain:
  - with single frequency operation,
  - with systems that take a long time to reach a steady state oscillation,
  - for beam impedance calculations.
- One would prefer to use t-domain:
  - for transient responses (wide spectrum),
  - for wakefield calculations,
  - whenever things become nonlinear ...
- In the following, I will use primarily the ω-domain (steady state)!

Homogeneous plane wave (1)

Maxwell's equations:

$$\nabla\times\vec{H}-\varepsilon_0\frac{\partial}{\partial t}\vec{E}=0 \quad \nabla\cdot\mu_0\vec{H}=0$$
$$\nabla\times\vec{E}+\mu_0\frac{\partial}{\partial t}\vec{H}=0 \quad \nabla\cdot\varepsilon_0\vec{E}=0$$

With only E_x and H_y:

$$\vec{u}_x\frac{\partial}{\partial z}H_y+\vec{u}_x\,\varepsilon_0\frac{\partial}{\partial t}E_x=0$$
$$\vec{u}_y\frac{\partial}{\partial z}E_x+\vec{u}_y\mu_0\frac{\partial}{\partial t}H_y=0$$

2nd and 4th equations are automatically satisfied.

With $\frac{d}{dt}\equiv j\omega$ we get

$$\frac{d}{dz}H_y=-j\omega\,\varepsilon_0\,E_x$$
$$\frac{d}{dz}E_x=-j\omega\,\mu_0\,H_y$$

Homogeneous plane wave (2)

Let us write
$$\frac{d}{dz}H_y=-j\omega\,\varepsilon_0\,E_x$$
$$\frac{d}{dz}E_x=-j\omega\,\mu_0\,H_y$$
in matrix form:

$$\frac{d}{dz}\begin{pmatrix}E_x\\H_y\end{pmatrix}=\begin{pmatrix}0 & -j\omega\mu_0\\-j\omega\varepsilon_0 & 0\end{pmatrix}\cdot\begin{pmatrix}E_x\\H_y\end{pmatrix}$$

Even though there are simpler methods to solve this, let's introduce **eigenvectors** here!

BEGIN excursion "calculus eigensystem"

Reminder of some calculus

- The equation $\mathbf{A}\cdot\vec{x}=\vec{y}$ with the quadratic (non-singular) matrix $\mathbf{A}$ maps a vector space on itself.
- The eigenvectors $\vec{v}$ of $\mathbf{A}$ are mapped by $\mathbf{A}$ without rotation: $\mathbf{A}\cdot\vec{v}=\vec{v}\,\lambda$. λ is called eigenvalue.
- Only the direction of eigenvectors is given, their length is not determined and can be chosen.
- Writing the n eigenvectors of the n x n matrix as columns of a matrix $\mathbf{V}$, one obtains

$$\mathbf{M}\cdot\begin{bmatrix}\vec{v}_1 & \cdots & \vec{v}_n\end{bmatrix}=\begin{bmatrix}\vec{v}_1 & \cdots & \vec{v}_n\end{bmatrix}\cdot\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \lambda_n\end{bmatrix}$$

Eigensystem of a matrix

- The system of eigenvalues λ_i and eigenvectors $\vec{v}_i$ is the ***eigensystem*** of $\mathbf{A}$.
- Writing $\begin{bmatrix}\vec{v}_1 & \cdots & \vec{v}_n\end{bmatrix}=\mathbf{V}$ and $\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \lambda_n\end{bmatrix}=\mathbf{\Lambda}$,

  the eigensystem satisfies $\mathbf{A}\cdot\mathbf{V}=\mathbf{V}\cdot\mathbf{\Lambda}$.
- This allows to transform $\mathbf{A}$ to a diagonal matrix:

$$\mathbf{V}^{-1}\cdot\mathbf{A}\cdot\mathbf{V}=\mathbf{\Lambda}$$

$$\mathbf{A}=\mathbf{V}\cdot\mathbf{\Lambda}\cdot\mathbf{V}^{-1}$$

Eigensystem, number example:

■ *Mathematica* **example:**

In[1]:= `A = {{4, 4}, {1, 7}}; A // TraditionalForm`

Out[1]//TraditionalForm=

$$\begin{pmatrix} 4 & 4 \\ 1 & 7 \end{pmatrix}$$

In[2]:= `Eigensystem[A]`

Out[2]= `{{8, 3}, {{1, 1}, {-4, 1}}}`

In[3]:= `Λ = DiagonalMatrix[Eigensystem[A]〚1〛]; Λ // TraditionalForm`

Out[3]//TraditionalForm=

$$\begin{pmatrix} 8 & 0 \\ 0 & 3 \end{pmatrix}$$

In[4]:= `V = Eigensystem[A]〚2〛ᵀ; V // TraditionalForm`

Out[4]//TraditionalForm=

$$\begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix}$$

In[5]:= `A.V == V.Λ // FullSimplify`

Out[5]= `True`

$$\begin{bmatrix} 4 & 4 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} 8$$

$$\begin{bmatrix} 4 & 4 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -12 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix} 3$$

END excursion “calculus eigensystem”

Eigensystem for the homogeneous plane wave (1)

Apply this method to $\dfrac{d}{dz}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = \begin{pmatrix} 0 & -j\omega\mu_0 \\ -j\omega\varepsilon_0 & 0 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ H_y \end{pmatrix}$

■ **Eigensystem homogeneous plane wave**

In[1]:= `A = {{0, -ⅉ ω μ}, {-ⅉ ω ϵ, 0}}; A // TraditionalForm`

Out[1]//TraditionalForm=

$$\begin{pmatrix} 0 & -i\mu\omega \\ -i\epsilon\omega & 0 \end{pmatrix}$$

In[2]:= `Λ = DiagonalMatrix[Eigensystem[A]〚1〛];`
`Simplify[Λ, Assumptions → {μ > 0, ϵ > 0}] // TraditionalForm`

Out[2]//TraditionalForm=

$$\begin{pmatrix} -i\sqrt{\epsilon\mu}\,\omega & 0 \\ 0 & i\sqrt{\epsilon\mu}\,\omega \end{pmatrix}$$

In[3]:= `V = Eigensystem[A]〚2〛ᵀ; Simplify[V, Assumptions → {μ > 0, ϵ > 0}] // TraditionalForm`

Out[3]//TraditionalForm=

$$\begin{pmatrix} \sqrt{\frac{\mu}{\epsilon}} & -\sqrt{\frac{\mu}{\epsilon}} \\ 1 & 1 \end{pmatrix}$$

In[4]:= `Simplify[Inverse[V], Assumptions → {μ > 0, ϵ > 0}] // TraditionalForm`

Out[4]//TraditionalForm=

$$\begin{pmatrix} \frac{\sqrt{\frac{\epsilon}{\mu}}}{2} & \frac{1}{2} \\ -\frac{\sqrt{\frac{\epsilon}{\mu}}}{2} & \frac{1}{2} \end{pmatrix}$$

Eigensystem for the homogeneous plane wave (2)

$$\frac{d}{dz}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = \begin{pmatrix} 0 & j\omega\mu_0 \\ j\omega\varepsilon_0 & 0 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ H_y \end{pmatrix}$$

$$\frac{d}{dz}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\mu_0}{\varepsilon_0}} & -\sqrt{\frac{\mu_0}{\varepsilon_0}} \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} -j\omega\sqrt{\mu_0\varepsilon_0} & 0 \\ 0 & j\omega\sqrt{\mu_0\varepsilon_0} \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} \sqrt{\frac{\varepsilon_0}{\mu_0}} & 1 \\ -\sqrt{\frac{\varepsilon_0}{\mu_0}} & 1 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ H_y \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{\frac{\varepsilon_0}{\mu_0}} & 1 \\ -\sqrt{\frac{\varepsilon_0}{\mu_0}} & 1 \end{pmatrix} \cdot \frac{d}{dz}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = \begin{pmatrix} -j\omega\sqrt{\mu_0\varepsilon_0} & 0 \\ 0 & j\omega\sqrt{\mu_0\varepsilon_0} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\frac{\varepsilon_0}{\mu_0}} & 1 \\ -\sqrt{\frac{\varepsilon_0}{\mu_0}} & 1 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ H_y \end{pmatrix}$$

$$\frac{d}{dz}\begin{pmatrix} H_y + \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \\ H_y - \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \end{pmatrix} = \begin{pmatrix} -j\omega\sqrt{\mu_0\varepsilon_0} & 0 \\ 0 & j\omega\sqrt{\mu_0\varepsilon_0} \end{pmatrix} \cdot \begin{pmatrix} H_y + \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \\ H_y - \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \end{pmatrix}$$

Eigensystem for the homogeneous plane wave (3)

- The system $\frac{d}{dz}\begin{pmatrix} H_y + \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \\ H_y - \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \end{pmatrix} = \begin{pmatrix} -j\omega\sqrt{\mu_0\varepsilon_0} & 0 \\ 0 & j\omega\sqrt{\mu_0\varepsilon_0} \end{pmatrix} \cdot \begin{pmatrix} H_y + \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \\ H_y - \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_x \end{pmatrix}$

  now consists of two independent, uncoupled ODE's!
- With abbreviations $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, $k = \omega\sqrt{\mu_0\varepsilon_0}$ and introducing the new variables $a_+ = H_y + E_x / Z_0$ and $a_- = H_y - E_x / Z_0$ (the eigenvectors), this system can conveniently be rewritten as:

$$\frac{d}{dz} a_+ = -j\,k\,a_+$$

$$\frac{d}{dz} a_- = j\,k\,a_-$$

The solutions are now simple:

$$a_+ = a_{+0}\, e^{-jkz}$$

$$a_- = a_{-0}\, e^{+jkz}$$

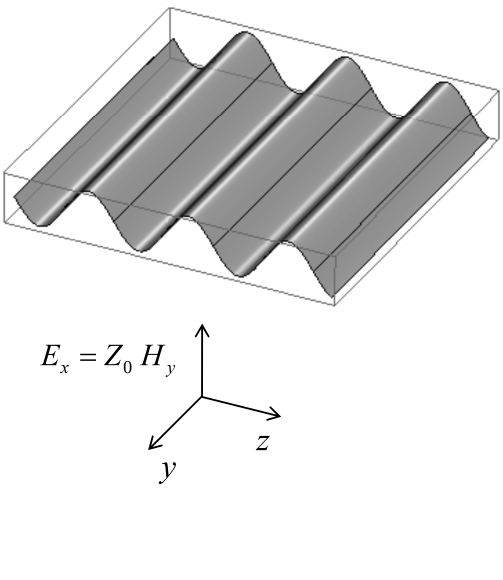
Interpretation of these solutions

$$a_+ = a_{+0}\, e^{-jkz}$$
$$a_- = a_{-0}\, e^{+jkz}$$

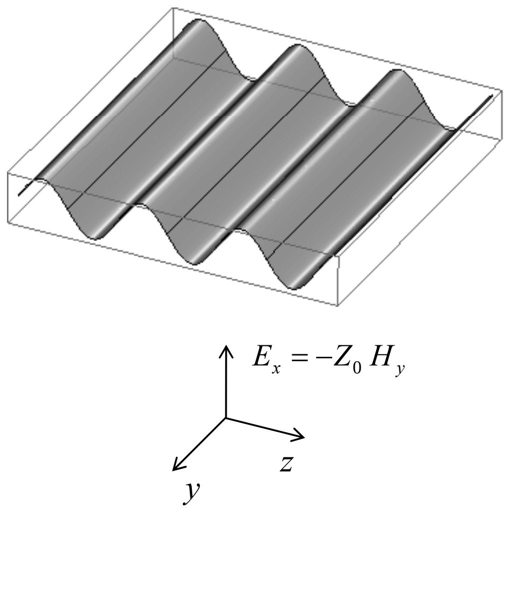
- With the above assumptions on polarization and time dependence:
  - We find independent forward and backward waves with amplitudes a_+ and a_-.
  - These are the eigenvectors of the matrix of the governing differential equation.
  - For the forward wave we find that the transverse electric and magnetic fields components have the constant ratio $Z_0 = \sqrt{\mu_0/\varepsilon_0}$. This ratio is called the wave impedance of free space.
  - The forward wave has z-dependence e^{-jkz}, the backward wave e^{jkz}.
  - The propagation constant is $jk = j\omega\sqrt{\mu_0\varepsilon_0} = \omega/c_0$.
  - The phase velocity is $\omega/k = 1/\sqrt{\mu_0\varepsilon_0} = c_0$, i.e. the speed of light.

Homogeneous plane wave

forward wave:



backward wave:

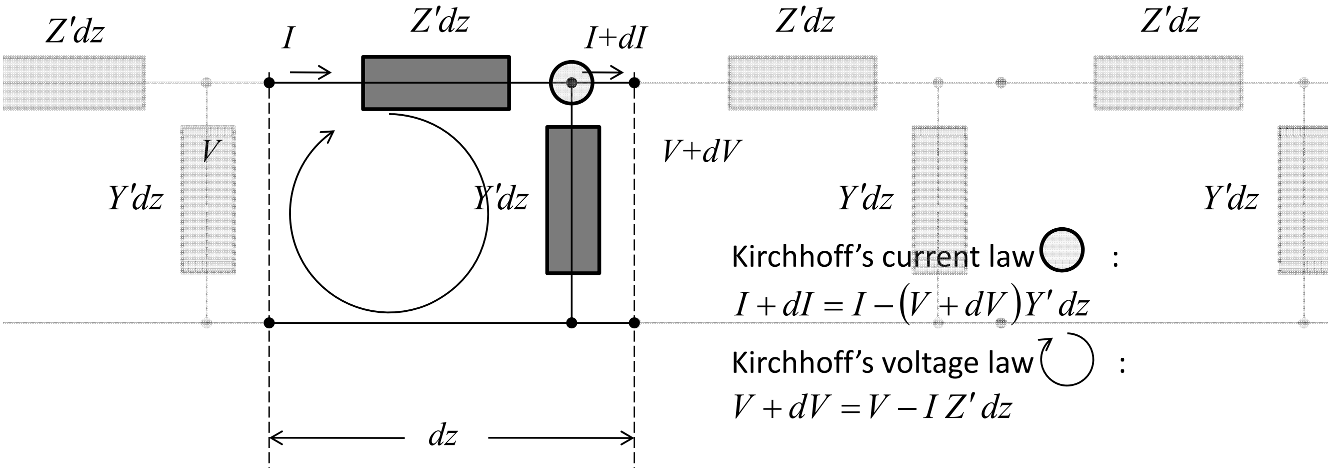


From free space to transmission lines

The above was made more complicated than necessary, why?

- The same form of equation applies to voltages and currents (or electric and magnetic field amplitudes) on transmission lines.
  ... but also:
- This method can be generalized.
- Eigenvectors and eigenvalues are very important throughout waveguides and cavities and they will keep coming back!
- Eigenvalues for waveguides are propagation constants, the corresponding eigenvectors are waveguide modes (German: Eigenwellen)
- Eigenvalues for cavities (resonators) are frequencies, the corresponding eigenvectors are cavity modes (German: Eigenschwingungen).
- In the general case, eigenvalues are complex, but the system of eigenvectors can in general be renormalized such that the eigenvector matrix is unitary.

Differential equation of a transmission line (TL)



Kirchhoff's current law :

$I + dI = I - (V + dV)Y'\,dz$

Kirchhoff's voltage law :

$V + dV = V - I\,Z'\,dz$

Z' is the series impedance per unit length,
Y' is the shunt admittance per unit length.

$$\frac{d}{dz}\begin{pmatrix} V \\ I \end{pmatrix} = \begin{pmatrix} 0 & -Z' \\ -Y' & 0 \end{pmatrix} \cdot \begin{pmatrix} V \\ I \end{pmatrix}$$

Transmission line equations

Now this equation $\frac{d}{dz}\begin{pmatrix} V \\ I \end{pmatrix} = \begin{pmatrix} 0 & -Z' \\ -Y' & 0 \end{pmatrix} \cdot \begin{pmatrix} V \\ I \end{pmatrix}$ has the same form as

$$\frac{d}{dz}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = \begin{pmatrix} 0 & -j\omega\mu_0 \\ -j\omega\varepsilon_0 & 0 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ H_y \end{pmatrix}$$ that we have just solved.

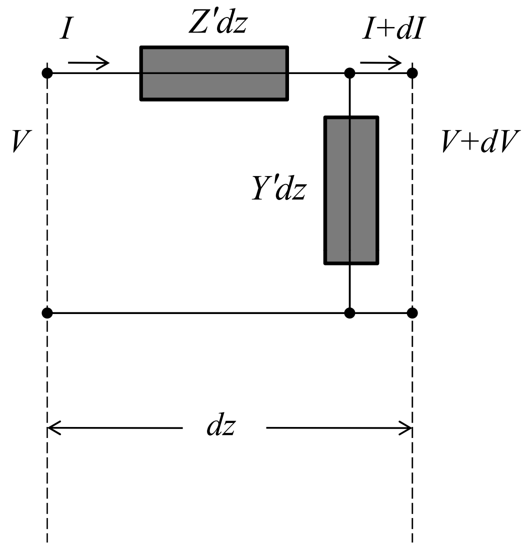
This allows to apply the same method; we get eigenvalues $\{-\gamma, \gamma\}$ where $\gamma = \sqrt{Z'Y'}$.

The eigenvectors are $\begin{matrix} V_+ = V + Z_0\, I \\ V_- = V - Z_0\, I \end{matrix}$ with $Z_0 = \sqrt{Z'/Y'}$.

The transformed equations are $\begin{matrix} \frac{d}{dl} V_+ = -\gamma\, V_+ \\ \frac{d}{dl} V_- = +\gamma\, V_- \end{matrix}$, the solutions are $\begin{matrix} V_+ = V_{+0}\, e^{-\gamma l} \\ V_- = V_{-0}\, e^{+\gamma l} \end{matrix}$.

Characteristics of the TL

- $Z_0 = \sqrt{Z'/Y'}$ is called the characteristic impedance.
- $\gamma = \sqrt{Z'Y'}$ is called propagation constant.



Solutions of the TL equation:

The solution can again be interpreted:

- The solution of $\frac{d}{dl}V_+ = -\gamma\, V_+,\ V_+ = V_{+0}\, e^{-\gamma l}$ describes a forward wave,
  the solution of $\frac{d}{dl}V_- = +\gamma\, V_-,\ V_- = V_{-0}\, e^{+\gamma l}$ describes a backward wave.
- Assuming arbitrary Z' and Y' leads in general to a complex propagation constant $\gamma = \alpha + j\,\beta$.
- The voltage and current on the line according to the solution are $V = \frac{V_+ + V_-}{2}$, $I = \frac{V_+ - V_-}{2\,Z_0}$, i.e. superpositions of forward and backward travelling waves.
- If there is only a forward (backward) wave, the voltage to current ratio is $Z_0(-Z_0)$.

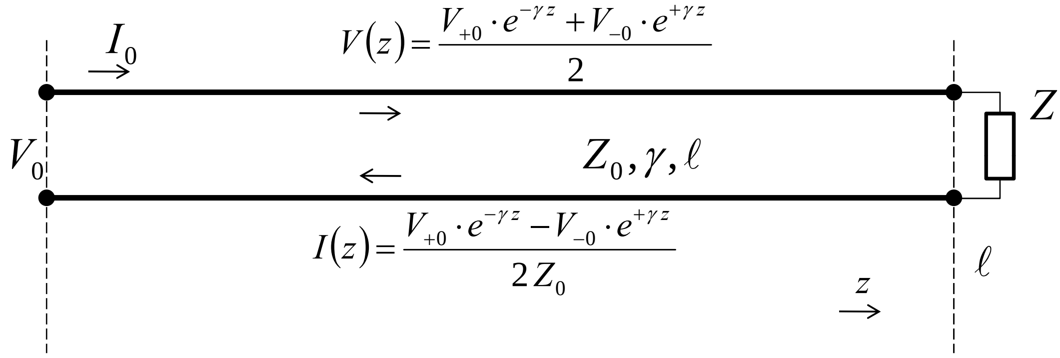
Normalized wave amplitudes

- The power transported on a transmission line is $P = V\,I^*$.
- Written with eigenvectors V_+ and V_- of the forward and backward waves this becomes

$$P = \frac{\left(V_+ + V_-\right)\left(V_+^* - V_-^*\right)}{4Z_0} = \frac{\left|V_+\right|^2 - \left|V_-\right|^2}{4Z_0}$$

- Since the length of the eigenvectors can be chosen, it could be used here to normalize them to directly describe the power flow; e.g. choosing
  $a = \frac{V_+}{2\sqrt{Z_0}}$ and $b = \frac{V_-}{2\sqrt{Z_0}}$ as amplitudes of forward and backward waves results in a simple expression for the power flow: $P = |a|^2 - |b|^2$
- Wave amplitudes are a convenient means to describe reflections and transmissions in a microwave network.
- The S-matrix (see below) describes the relations between wave amplitudes.

Voltage and current along the line



$$V(z) = \frac{V_{+0} \cdot e^{-\gamma z} + V_{-0} \cdot e^{+\gamma z}}{2}$$

$$I(z) = \frac{V_{+0} \cdot e^{-\gamma z} - V_{-0} \cdot e^{+\gamma z}}{2 Z_0}$$

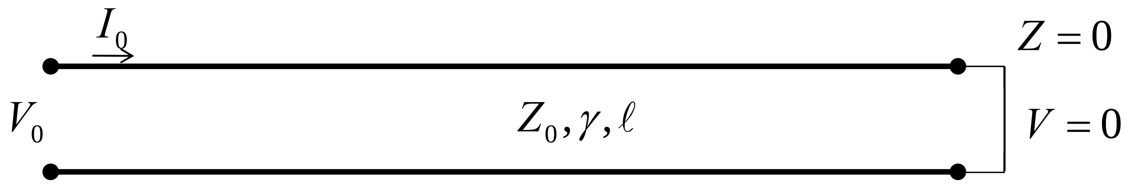
$$V(\ell) = \frac{V_+(\ell) + V_-(\ell)}{2} = Z \cdot I(\ell) = Z \frac{V_+(\ell) - V_-(\ell)}{2 Z_0}$$

This can be solved for $V_-(\ell)/V_+(\ell)$:

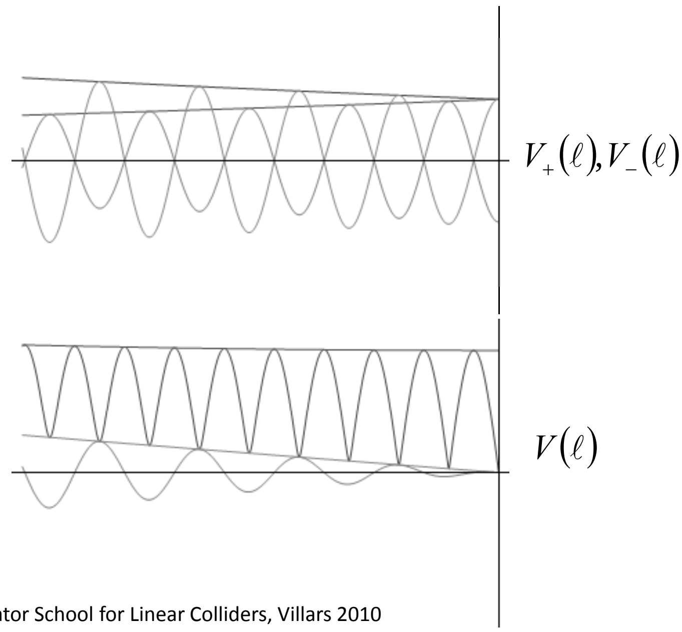
$$\frac{V_-(\ell)}{V_+(\ell)} = \frac{Z - Z_0}{Z + Z_0} \equiv r_{end}$$

r is called the reflection coefficient

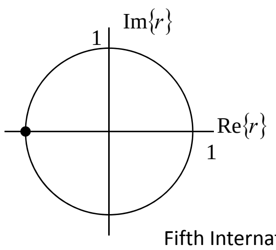
Short circuit



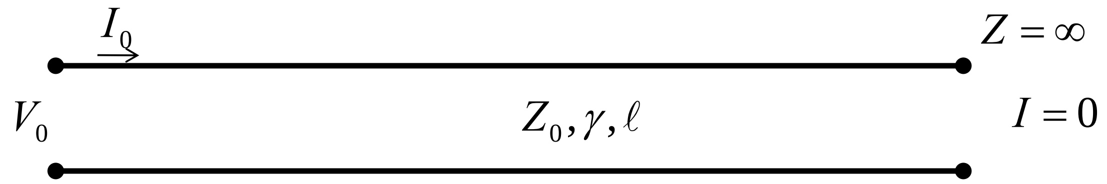
$$V(\ell) = \frac{V_+(\ell) + V_-(\ell)}{2} = 0$$



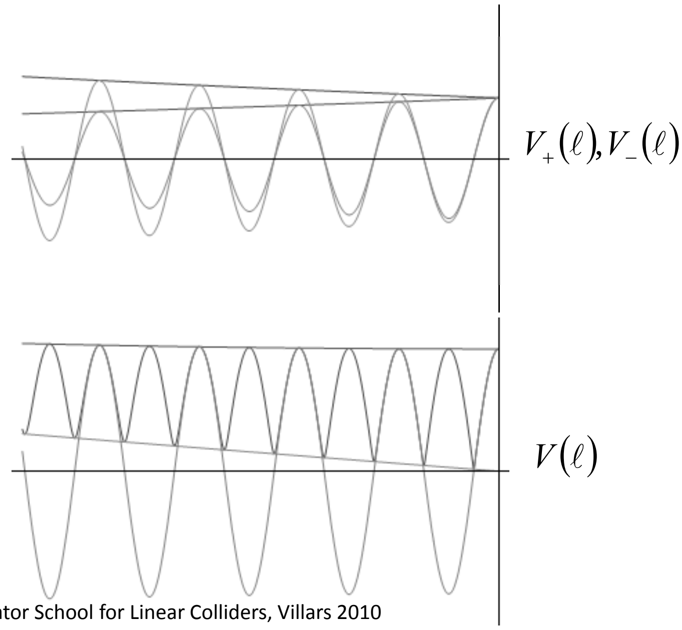
$$r_{short} = \frac{Z - Z_0}{Z + Z_0} = -1$$



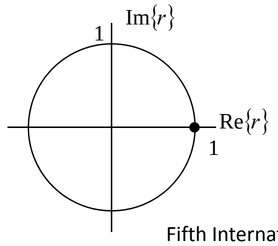
Open circuit



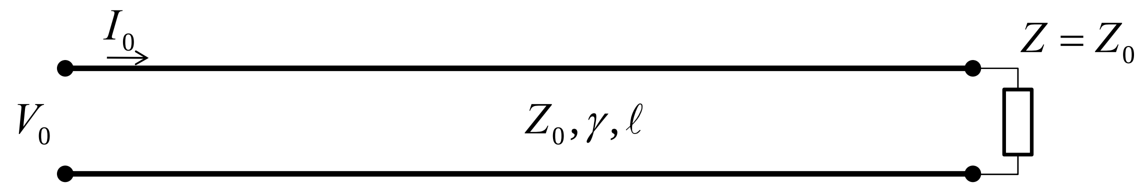
$$I(\ell) = \frac{V_+(\ell) - V_-(\ell)}{2 Z_0} = 0$$



$$r_{open} = \frac{Z - Z_0}{Z + Z_0} = +1$$

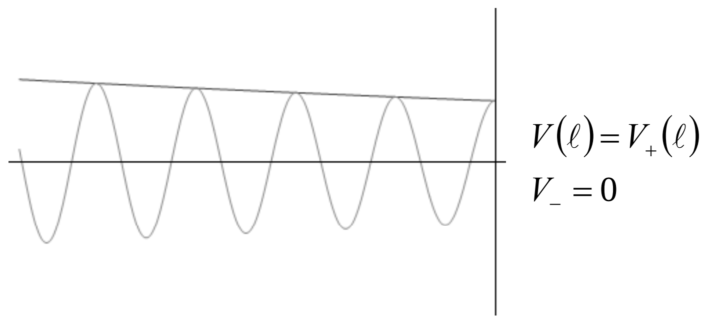


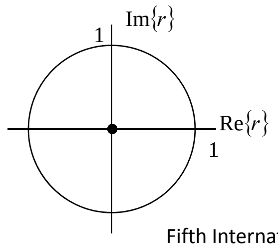
Matched load (termination)



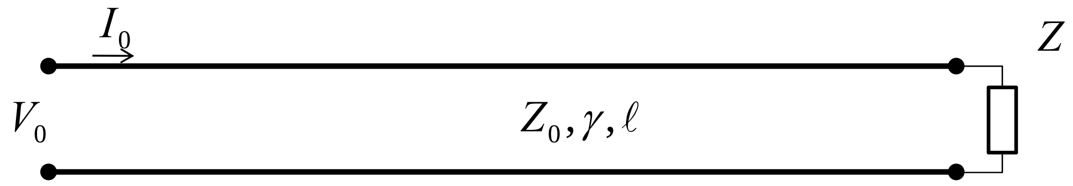
$$V(\ell) = Z_0 \, I(\ell)$$

$$r_{match} = \frac{Z - Z_0}{Z + Z_0} = 0$$

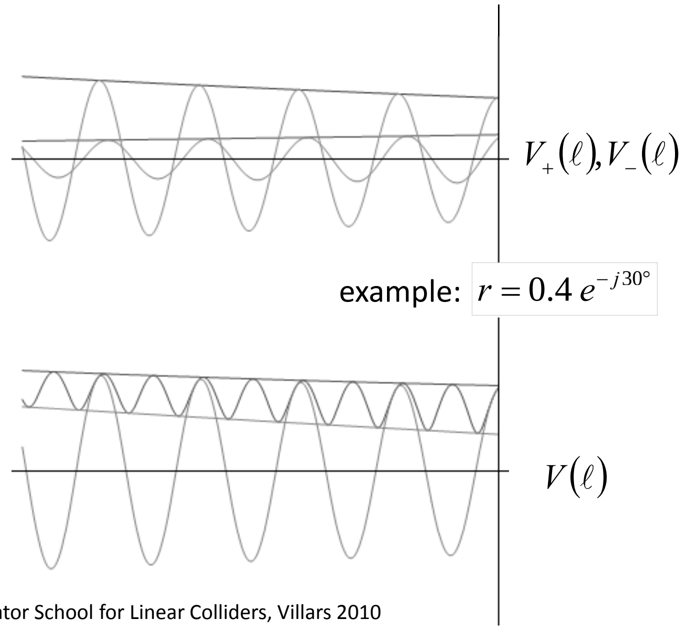




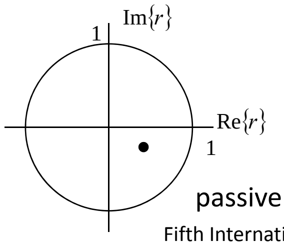
General mismatch



$$r = \frac{Z - Z_0}{Z + Z_0}$$



example: $r = 0.4\, e^{-j30°}$



passive: $|r| \le 1$

BEGIN excursion “S-matrix, T-matrix, periodic structures”

S-matrix

- The S-matrix describes the scattering off a general microwave network (n-port):

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

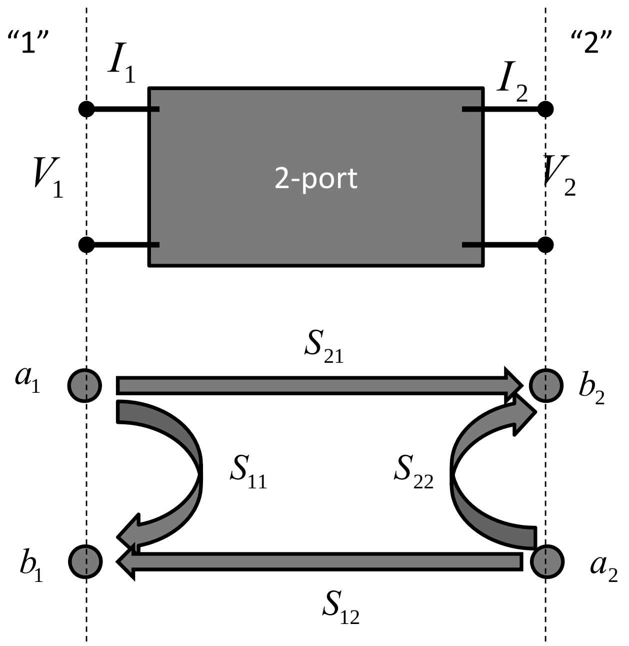
- The elements of the S-matrix (the S-parameters) are the quantities measured by network analysers (NWA).
- Each port may have its own Z_0!
  A transmission line is a simple case (a 2-port), its S-matrix is

$$\mathbf{S}_{TL} = \begin{pmatrix} 0 & e^{+\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{pmatrix}$$

- The termination is a 1-port, its S-matrix is just the reflection coefficient.

$$r = \frac{Z - Z_0}{Z + Z_0}$$

Signal Flow Chart



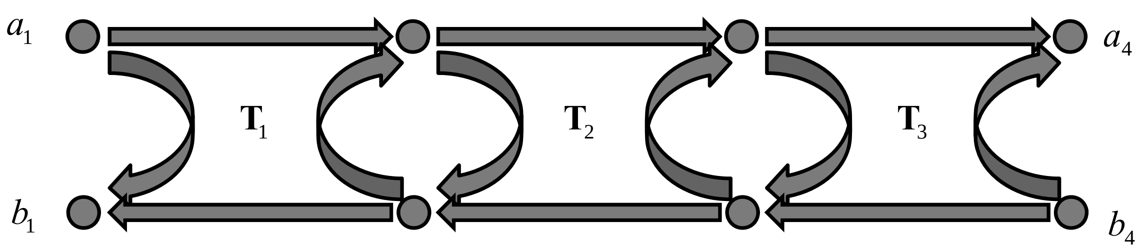
Signal flow chart

When working with S-parameters, it can be useful to work with the "signal flow chart" rather than an equivalent circuit with voltages and currents.

T-matrix

- The T-matrix (Transmission) is more convenient than the S-matrix when cascading 2-ports.
- The T-matrix is defined as $\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \cdot \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$.
- The conversion from S-matrix to T-matrix and inverse is given by

$$\mathbf{T} = \frac{1}{S_{21}} \begin{pmatrix} -\det(\mathbf{S}) & S_{11} \\ -S_{22} & 1 \end{pmatrix} \qquad \mathbf{S} = \frac{1}{T_{22}} \begin{pmatrix} T_{12} & \det(\mathbf{T}) \\ 1 & -T_{21} \end{pmatrix}$$



- The T-matrix for the network consisting of these 3 2-ports is now simply.

$$\mathbf{T} = \mathbf{T}_1 \cdot \mathbf{T}_2 \cdot \mathbf{T}_3$$

Periodic structures

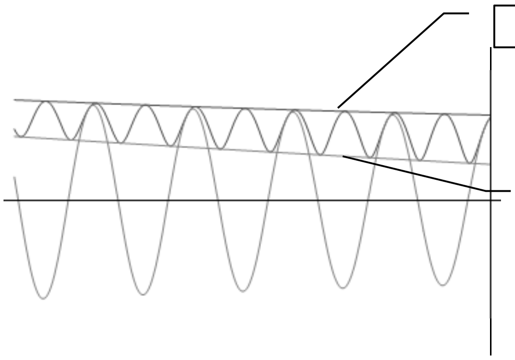
- An example where many 2-ports are cascaded are periodic structures.
- If one cell is described by $\mathbf{T}$, the n-cell periodic structures is described by

$$\mathbf{T}^n$$

- To calculate the power of a matrix, one again needs to calculate eigenvectors (see above!)
- With the eigensystem of $\mathbf{T}$ written again as $\mathbf{T}\cdot\mathbf{V}=\mathbf{V}\cdot\mathbf{\Lambda}$
  one finds $\mathbf{T}^n=\mathbf{V}\cdot\mathbf{\Lambda}\cdot\mathbf{V}^{-1}\cdot\mathbf{V}\cdot\mathbf{\Lambda}\cdot\mathbf{V}^{-1}\cdot\mathbf{V}\cdot\mathbf{\Lambda}\cdot\mathbf{V}^{-1}\cdot\ldots\cdot\mathbf{V}\cdot\mathbf{\Lambda}\cdot\mathbf{V}^{-1}$
  which is very simply $\mathbf{T}^n=\mathbf{V}\cdot\begin{pmatrix}\lambda_1^n & 0\\ 0 & \lambda_2^n\end{pmatrix}\cdot\mathbf{V}^{-1}$.
- The eigenvectors of periodic structures are also known as *Bloch*-waves.

END excursion "S-matrix, T-matrix, periodic structures"

VSWR



Voltage maximum: $V_{\max}\propto\left|e^{-\gamma z}\right|+\left|r\,e^{\gamma z}\right|$

Voltage minimum: $V_{\min}\propto\left|e^{-\gamma z}\right|-\left|r\,e^{\gamma z}\right|$

The VSWR is the "Voltage Standing Wave Ratio":

$$VSWR=\frac{\left|e^{-\gamma z}\right|+\left|r\,e^{\gamma z}\right|}{\left|e^{-\gamma z}\right|-\left|r\,e^{\gamma z}\right|}$$

For a lossless TL: $VSWR=\dfrac{1+|r|}{1-|r|}$ and $|r|=\dfrac{VSWR-1}{VSWR+1}$

Special case: lossless TEM TL

Propagation constant:

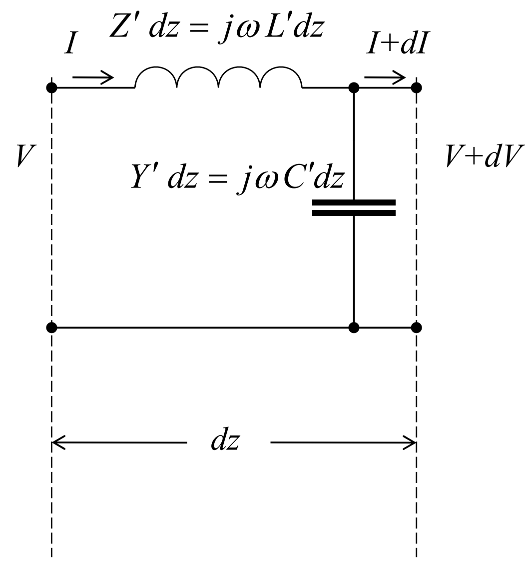
$$\gamma = \sqrt{Z'\,Y'} = j\omega\sqrt{L'C'}$$

$\gamma = j\beta$ is purely imaginary.

Characteristic impedance:

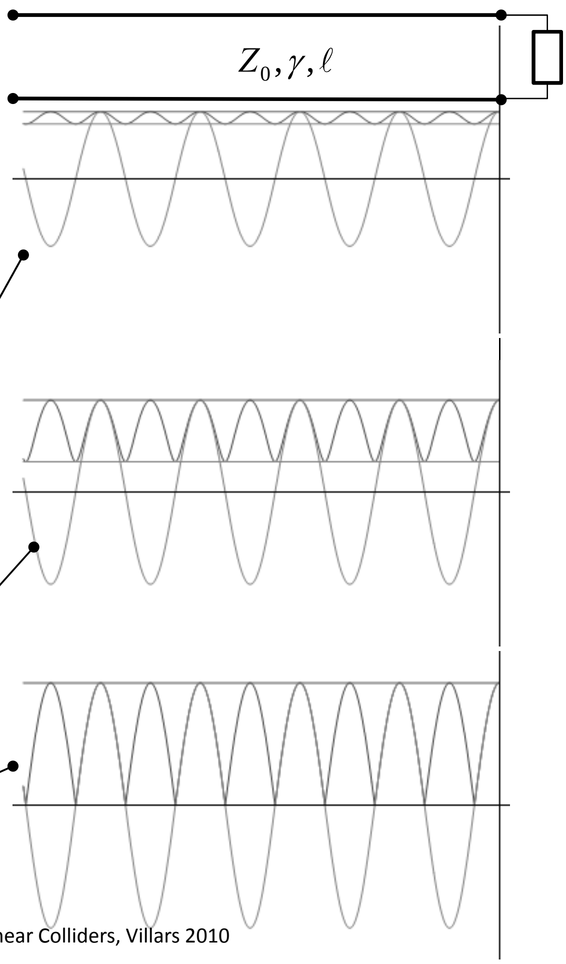
$$Z_0 = \sqrt{Z'/Y'} = \sqrt{\frac{L'}{C'}}$$

Z_0 is purely real.



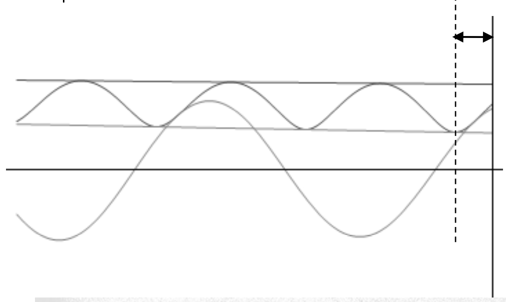
Mismatch examples (lossless case)

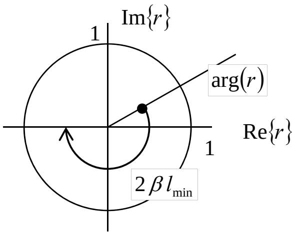
|r|	|r| *in* dB	*VSWR*
0	-∞ dB	1
.01	-40 dB	1.02
.1	-20 dB	1.22
.2	-14 dB	1.5
.5	-6 dB	3
1	0 dB	∞



Measurement line

- Note the envelope of the standing wave pattern on the line!
- This pattern follows the equation $\left| e^{-\gamma z} + r\, e^{\gamma z} \right|$.
- The minima are spaced $\lambda/2$.
- The first minimum occurs when the backward wave subtracts from the forward wave, i.e. when the local r is negative real.

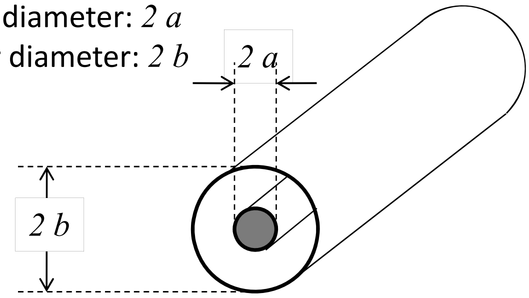




- Before the advent of NWA's, this technique was used to determine the complex reflection coefficient using a probe in a slotted line. (hp ad from 1950)

TEM TL example: Coaxial line

Inner diameter: $2\,a$
Outer diameter: $2\,b$



Fields are TEM: $\vec{E} = \vec{u}_\rho E_\rho$

$\vec{H} = \vec{u}_\varphi H_\varphi$

From Maxwell's equations:

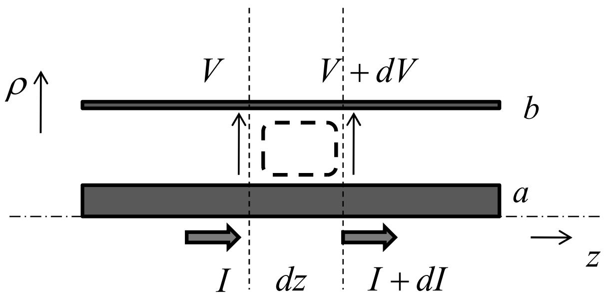
$$H_\varphi = \frac{I}{2\pi\,\rho}$$

$$E_\rho = \frac{1}{\rho} \frac{V}{\ln(b/a)}$$

Define:
I: current through inner conductor,

V: Voltage between inner and outer conductor, $V = \int_a^b E_\rho \, d\rho$.

Characteristics of a coaxial line:



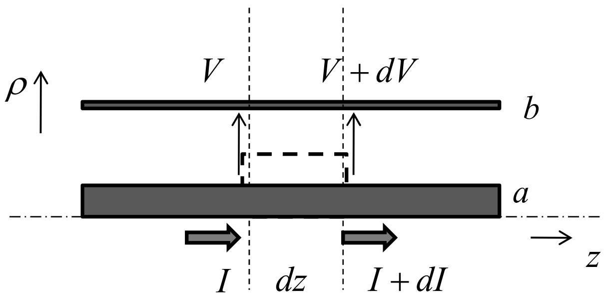
1. Integrate around the dashed line and apply $\oint \vec{E}\cdot d\vec{s} = -j\omega \iint \vec{B}\cdot d\vec{A}$:

$$dV = -j\omega\, dz \int_a^b B_\varphi\, d\rho$$

$$dV = -\frac{j\omega\mu}{2\pi} dz \int_a^b \frac{1}{\rho} d\rho\, I$$

$$dV = -\frac{j\omega\mu}{2\pi} dz \ln\left(\frac{b}{a}\right) I$$

Characteristics of a coaxial line:



2. Integrate on the dashed closed surface around the inner conductor and apply $\oiint \left(\vec{J} + j\omega\varepsilon\vec{E}\right)\cdot d\vec{A} = 0$

$$dI = -j\omega\varepsilon\, dz \int_0^{2\pi} E_\rho\, d\varphi$$

$$dI = -j\omega\varepsilon\, dz \frac{2\pi}{\ln(b/a)} V$$

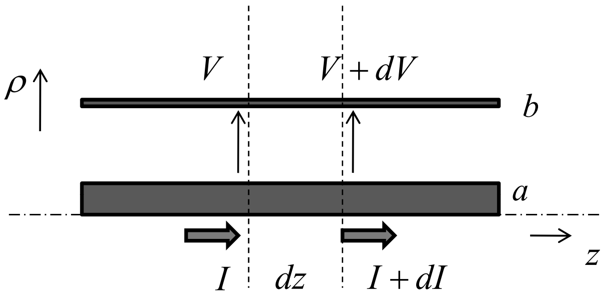
Result:

$$\frac{dV}{dz} = -\frac{j\omega\mu}{2\pi}\ln\left(\frac{b}{a}\right) I$$

$$\frac{dI}{dz} = -j\omega\varepsilon \frac{2\pi}{\ln(b/a)} V$$

compare to $\frac{d}{dz}\begin{pmatrix} V \\ I \end{pmatrix} = \begin{pmatrix} 0 & -Z' \\ -Y' & 0 \end{pmatrix}\cdot\begin{pmatrix} V \\ I \end{pmatrix}$

Characteristics of a coaxial line:



$$\frac{dV}{dz} = -\frac{j\omega\mu}{2\pi}\ln\left(\frac{b}{a}\right) I$$

$$\frac{dI}{dz} = -j\omega\varepsilon\frac{2\pi}{\ln(b/a)} V$$

$$Z' = \frac{j\omega\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$Y' = j\omega\varepsilon\frac{2\pi}{\ln(b/a)}$$

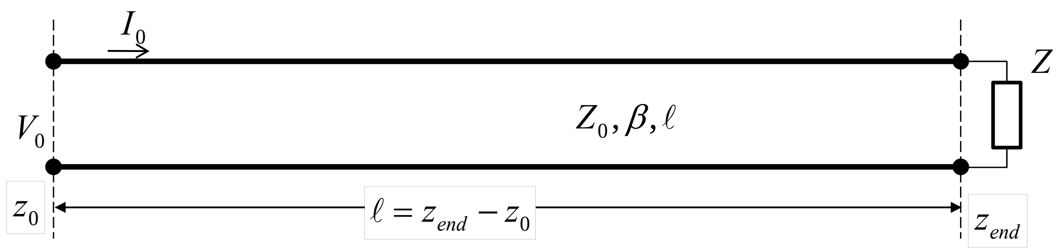
Result:

Propagation constant: $\gamma = j\omega\sqrt{\mu\varepsilon}$

Characteristic impedance $Z_0 = \frac{1}{2\pi}\ln\left(\frac{b}{a}\right)\sqrt{\frac{\mu}{\varepsilon}}$

Note: $\sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 c = 376.73\,\Omega$ $\qquad \frac{\mu_0 c}{2\pi} \approx 60\,\Omega$

Impedance transformation



What is the input impedance V_0 / I_0 ?

Easier: What is the input reflection coefficient r ?

$$V_+ = V_{+0}\, e^{-j\beta(z-z_{end})}$$

$$V_- = V_{-0}\, e^{+j\beta(z-z_{end})}$$

$$\Rightarrow \quad r(l) = r_{end}\, e^{-2j\beta(z-z_{end})}$$

$$\Rightarrow \quad Z_{in} = \frac{V_0}{I_0} = Z_0\frac{1+r(z_0)}{1-r(z_0)}$$

$$\frac{Z_{in}}{Z_0} = \frac{1+r_{end}e^{2j\beta\ell}}{1-r_{end}e^{2j\beta\ell}}$$

Impedance transformation (2)

- Inserting $r_{end} = \dfrac{Z - Z_0}{Z + Z_0}$ into $\dfrac{Z_{in}}{Z_0} = \dfrac{1 + r_{end} e^{2 j \beta \ell}}{1 - r_{end} e^{2 j \beta \ell}}$ results in

$$\frac{Z_{in}}{Z_0} = \frac{Z + j Z_0 \tan(\beta \ell)}{j Z \tan(\beta \ell) + Z_0}$$

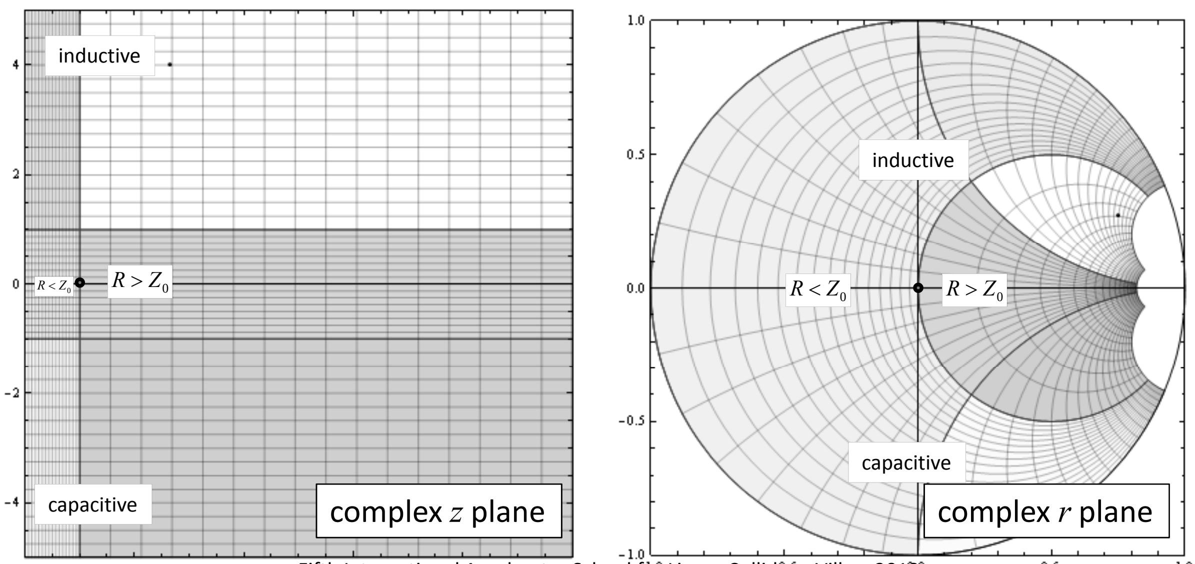
- In the general case with a complex propagation constant:

$$\frac{Z_{in}}{Z_0} = \frac{Z + Z_0 \tanh(\gamma \ell)}{Z \tanh(\gamma \ell) + Z_0}$$

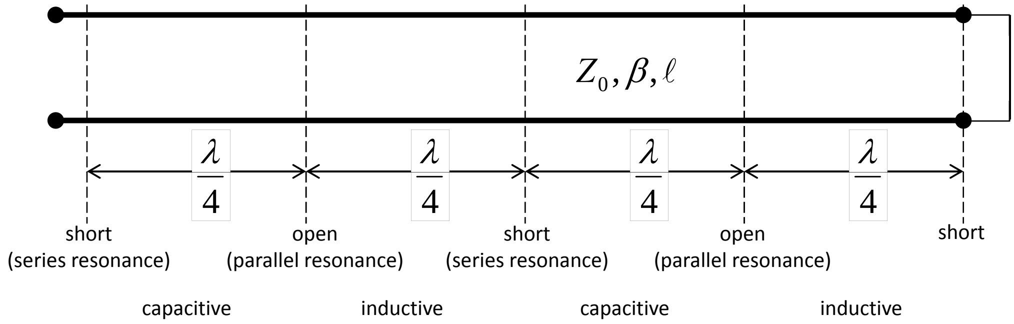
Smith Chart

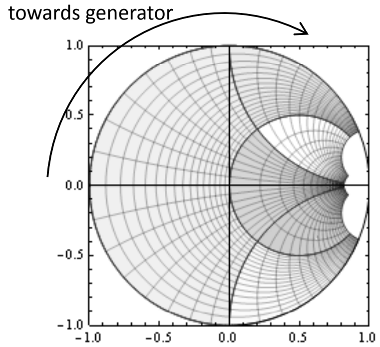
The transformations $z = \dfrac{1 - r}{1 + r}$ and $r = \dfrac{z - 1}{z + 1}$ with complex r and z

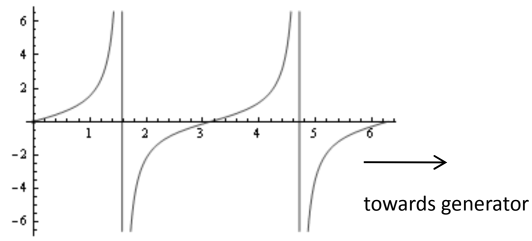
are “conformal maps” that map generalized circles into generalized circles:



Example:



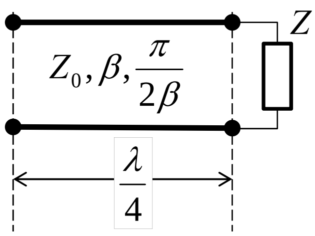




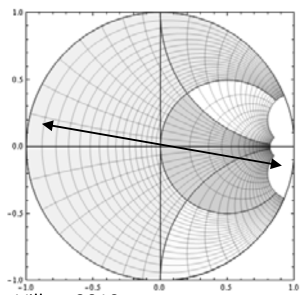
Quarter-wave transformer

Special case: $\ell = \frac{\lambda}{4}$

$\beta L = 90°$



- $\frac{Z_{in}}{Z_0} = \frac{Z + j Z_0 \tan(\beta \ell)}{j Z \tan(\beta \ell) + Z_0}$ now becomes $\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z}$.
- This can be used to transform an impedance Z to an impedance Z_0^2/Z
  (opposite in the Smith chart)
- Example from other fields:
  - Coating on optical lenses to minimize reflection.



Free space as transmission line(1)

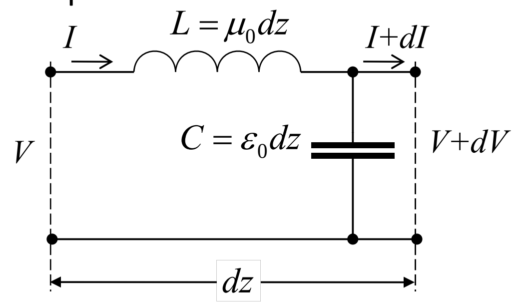
- The similarity between the TL equations

$$\frac{d}{dz}\begin{pmatrix} V \\ I \end{pmatrix} = \begin{pmatrix} 0 & -Z' \\ -Y' & 0 \end{pmatrix} \cdot \begin{pmatrix} V \\ I \end{pmatrix}$$

  and the equations of a homogeneous plane wave

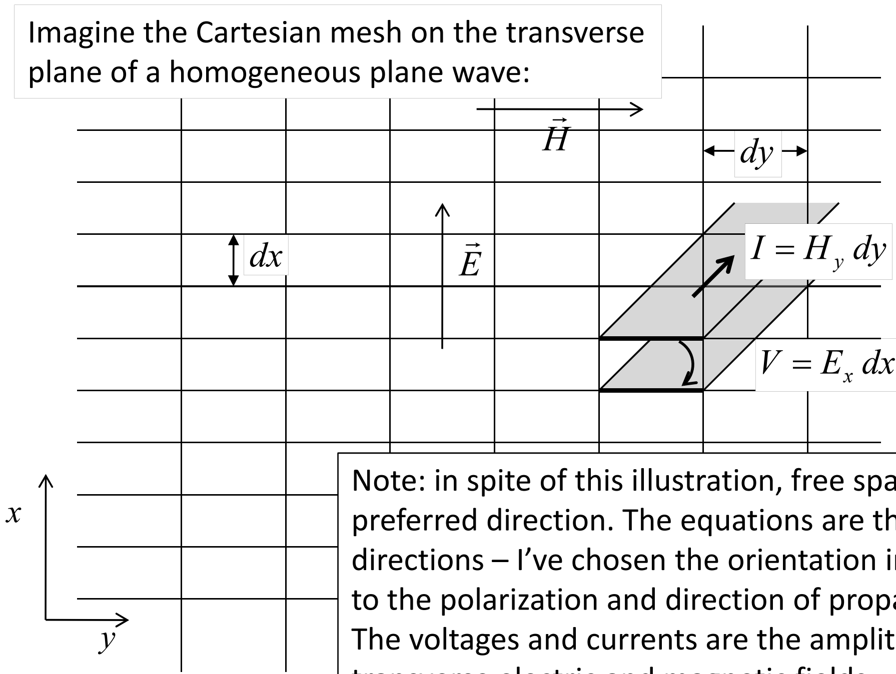
$$\frac{d}{dz}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = \begin{pmatrix} 0 & -j\omega\mu_0 \\ -j\omega\varepsilon_0 & 0 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ H_y \end{pmatrix}$$

  allows us to write the "transmission line of free space" and draw an equivalent circuit:
- The voltages and currents can be understood as local quantities $V = E_x\,dx$ and $I = H_y\,dy$.



Free space as transmission line(2)

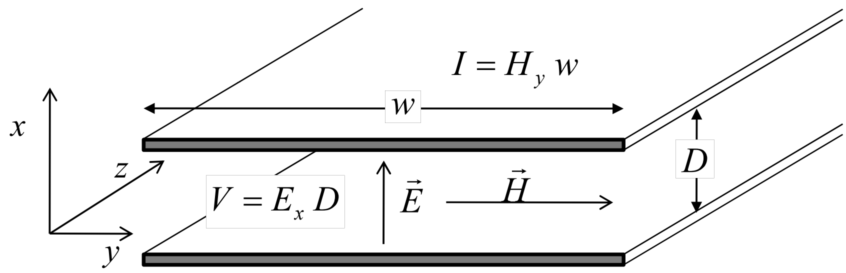
Imagine the Cartesian mesh on the transverse plane of a homogeneous plane wave:

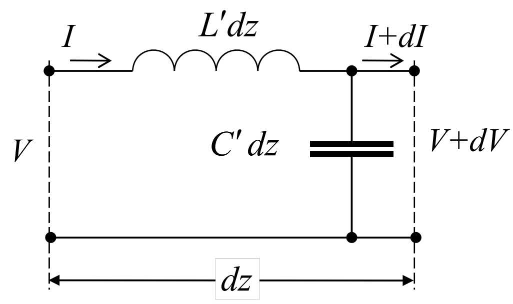


Note: in spite of this illustration, free space has no preferred direction. The equations are the same in all directions – I've chosen the orientation in order to fit to the polarization and direction of propagation.
The voltages and currents are the amplitudes of the transverse electric and magnetic fields.

Parallel plate TL

- This brings us naturally to the parallel plate transmission line:





WAVEGUIDES

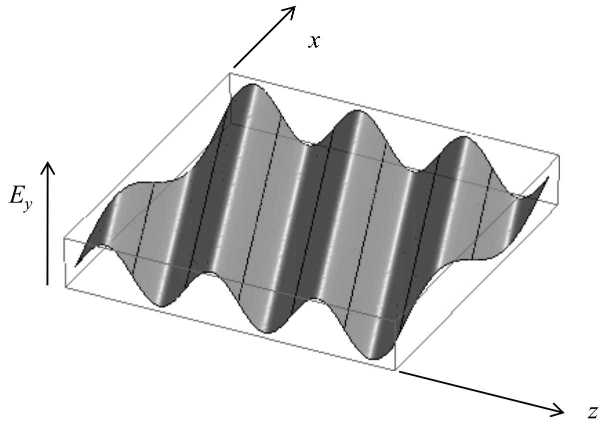
Fifth International Accelerator School for Linear Colliders, Villars 2010

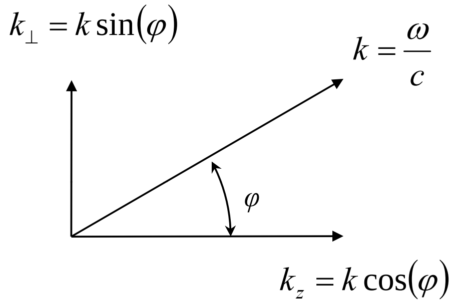
Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$
$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$

$$\vec{k}\cdot\vec{r} = \frac{\omega}{c}\left(\cos(\varphi)z + \sin(\varphi)x\right)$$

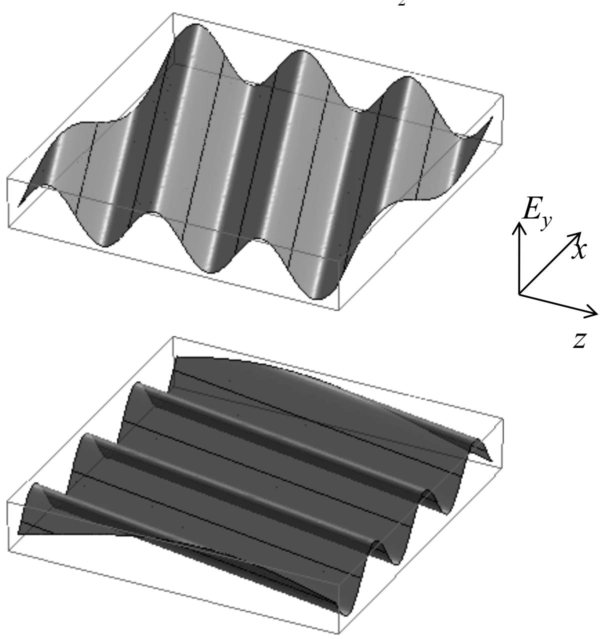
Wave vector $\vec{k}$:
the direction of $\vec{k}$ is the direction of propagation,
the length of $\vec{k}$ is the phase shift per unit length.
$\vec{k}$ behaves like a vector.

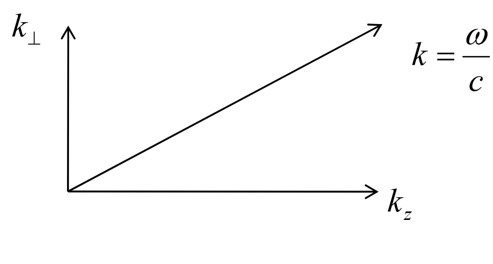


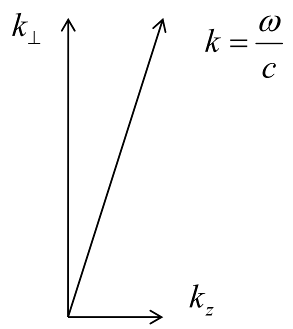


Wave length, phase velocity

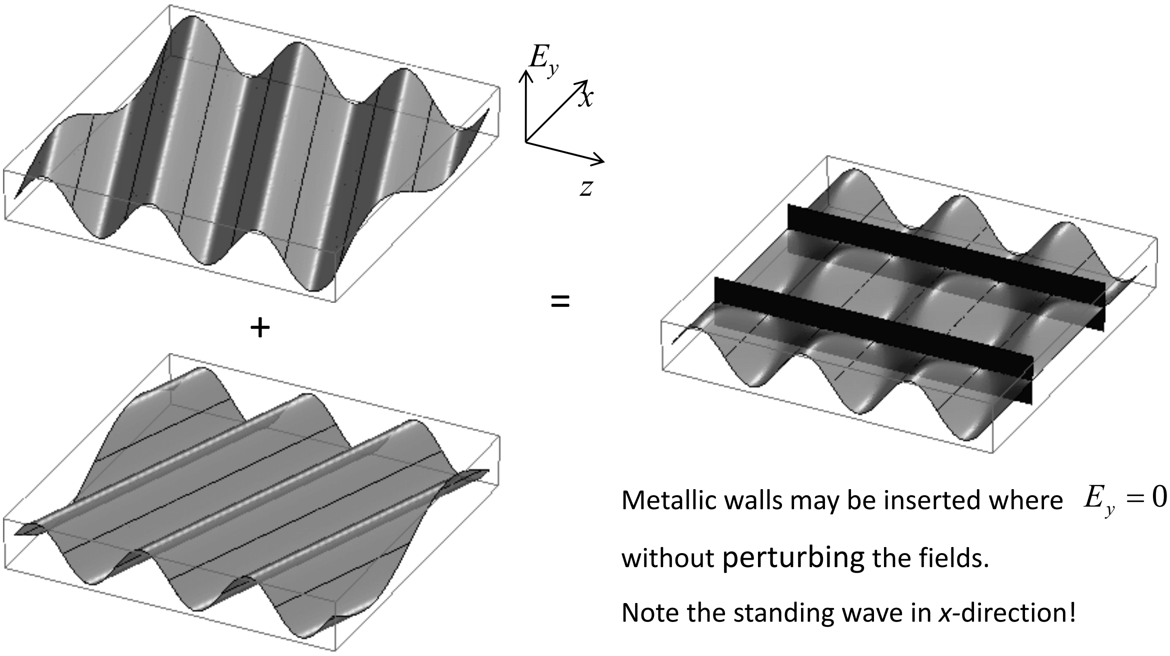
The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\,\lambda_z$







Superposition of 2 homogeneous plane waves



Metallic walls may be inserted where $E_y = 0$

without perturbing the fields.

Note the standing wave in *x*-direction!

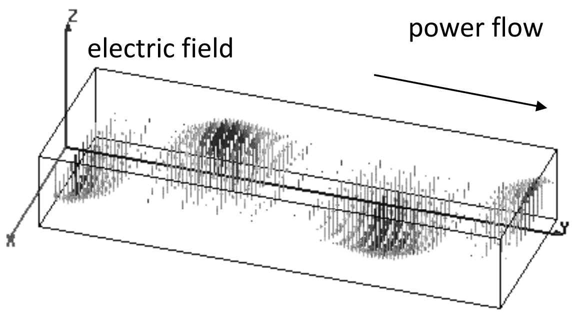
This way one gets a hollow rectangular waveguide

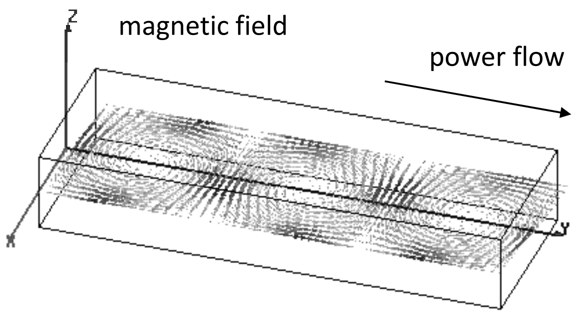
Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

Example: "S-band" : 2.6 GHz ... 3.95 GHz,
Waveguide type WR284 (2.84" wide),
dimensions: 72.14 mm x 34.04 mm.
Operated at f = 3 GHz.

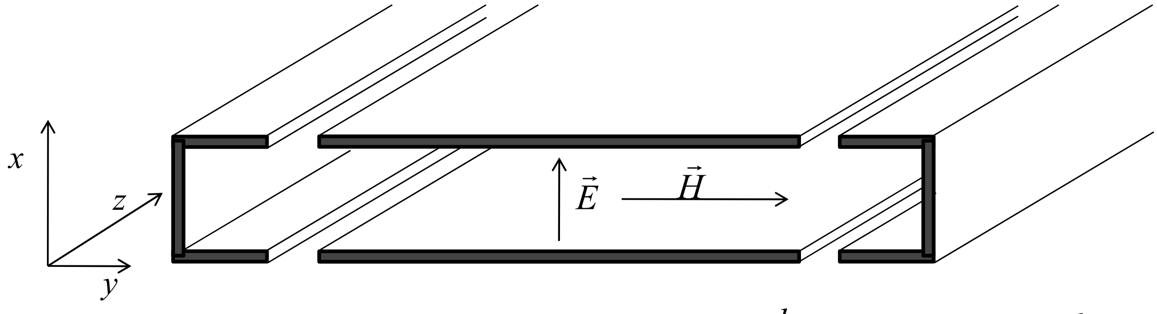
power flow: $$\frac{1}{2}\mathrm{Re}\left\{ \iint_{\substack{\text{cross}\\ \text{section}}} \vec{E} \times \vec{H}^{*} \cdot \mathrm{d}\,\vec{A} \right\}$$





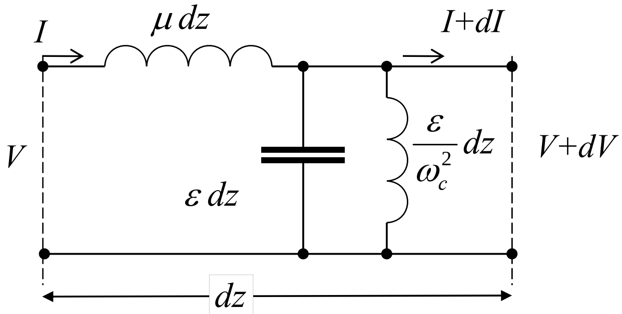
From parallel plate TL to waveguide

- The confinement by the sidewalls leads to a **parallel inductivity** in the TL equation (and equivalent circuit) for TE modes of a rectangular waveguide:

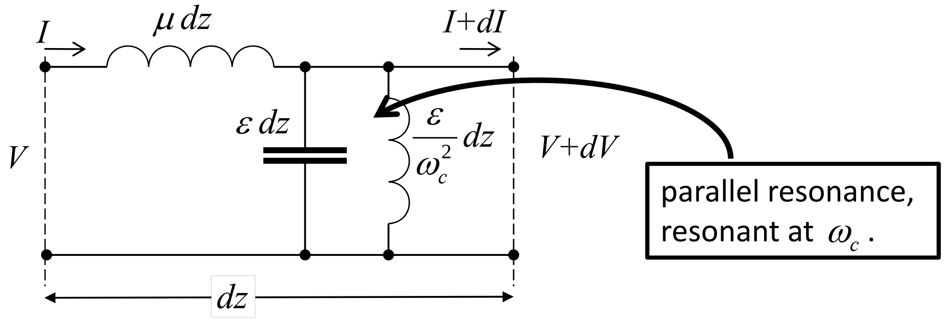


$$\frac{d}{dz}V + j\omega\mu I = 0$$

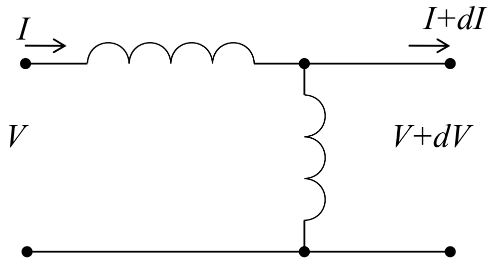
$$\frac{d}{dz}I + j\omega\varepsilon\left(1-\left(\frac{\omega_c}{\omega}\right)^2\right)V = 0$$



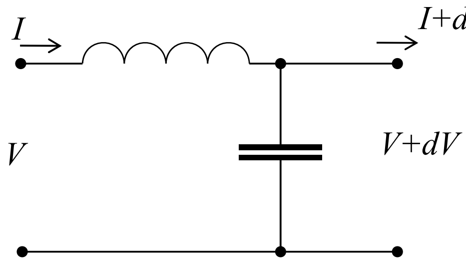
Cutoff frequency (TE)



Below cutoff: parallel L dominates



Above cutoff: parallel C dominates

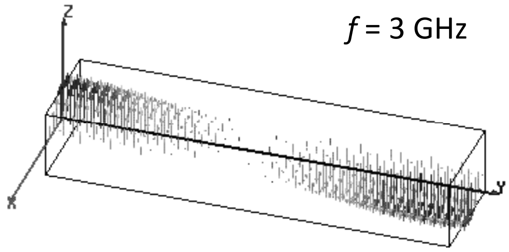


Waveguide dispersion

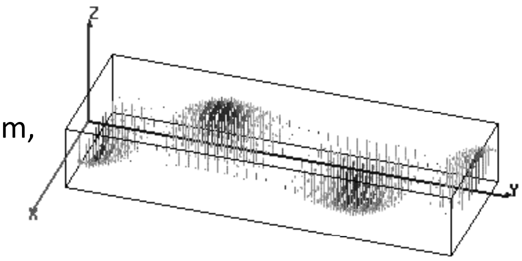
What happens with different waveguide dimensions (different width *a*)?

f = 3 GHz

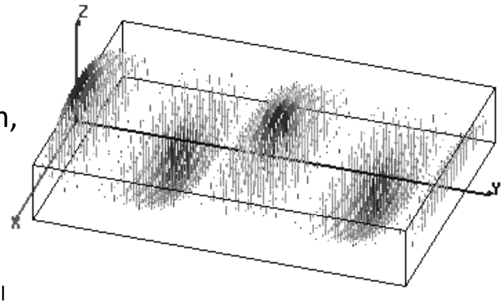
1:
a = 52 mm,
f/f_c = 1.04

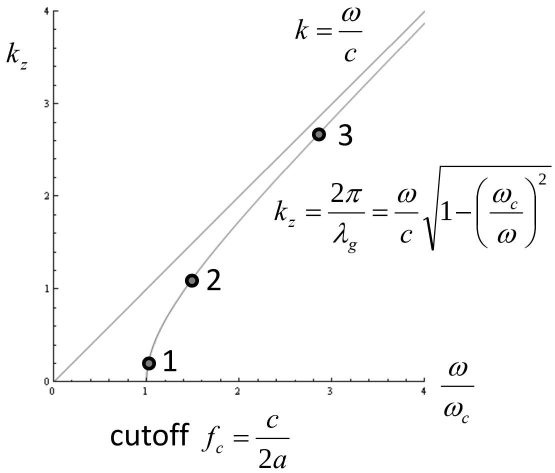


2:
a = 72.14 mm,
f/f_c = 1.44



3:
a = 144.3 mm,
f/f_c = 2.88



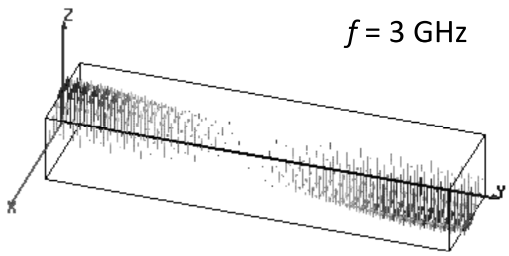


Phase velocity

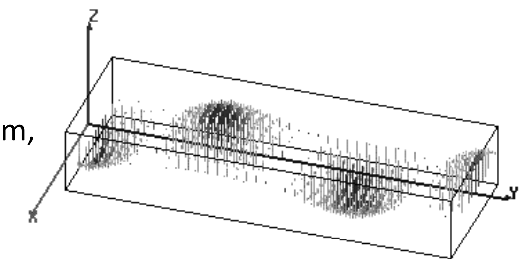
The phase velocity is the speed with which the crest or a zero-crossing travels in *z*-direction.
Note on the three animations on the right that, at constant *f*, it is $\propto \lambda_g$.
Note that at $f = f_c$, $v_{\varphi,z} = \infty$!
With $f \to \infty$, $v_{\varphi,z} \to c$!

f = 3 GHz

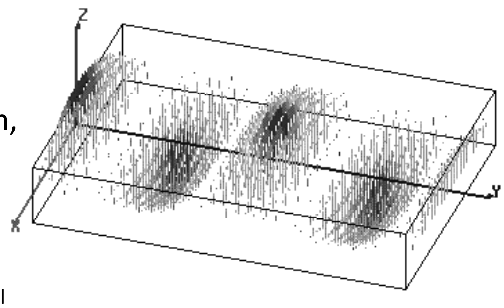
1:
a = 52 mm,
f/f_c = 1.04

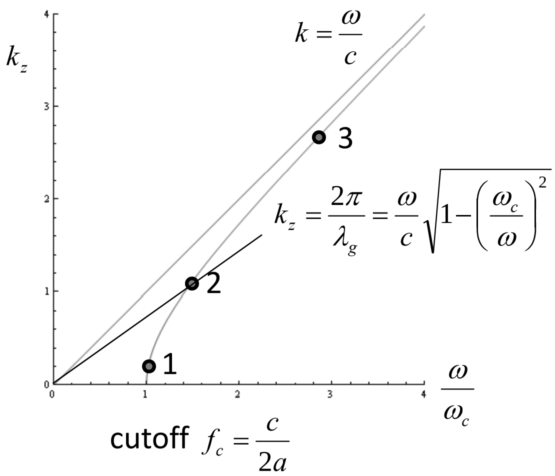


2:
a = 72.14 mm,
f/f_c = 1.44

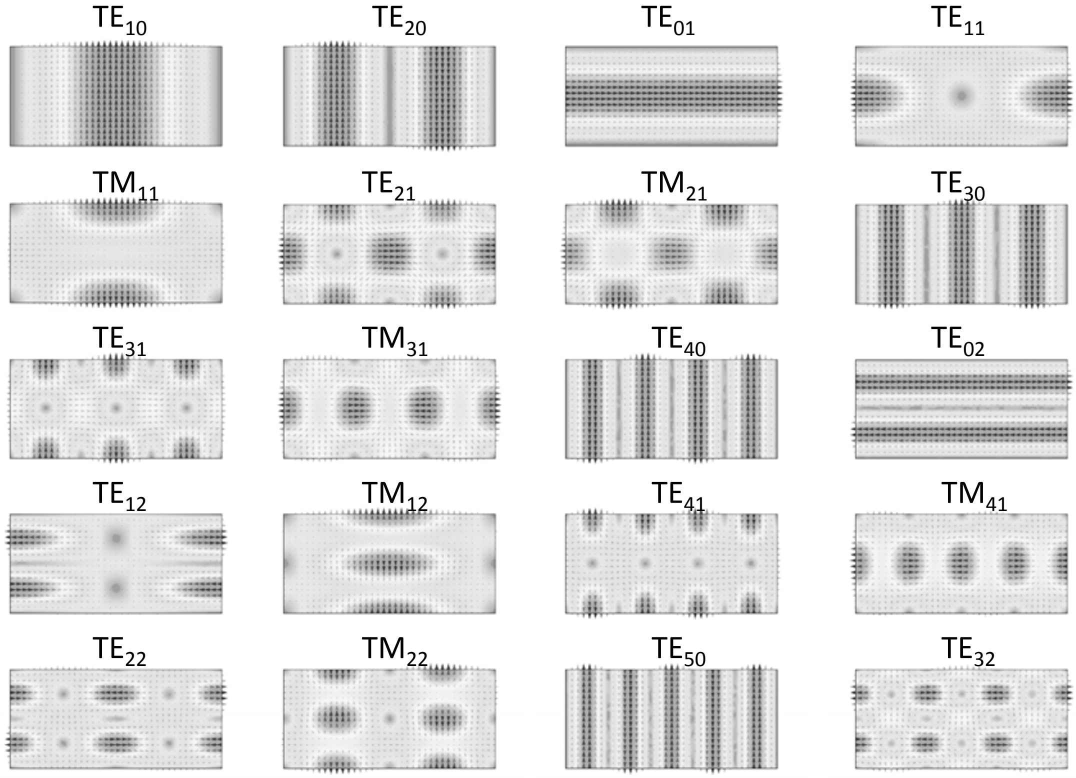


3:
a = 144.3 mm,
f/f_c = 2.88



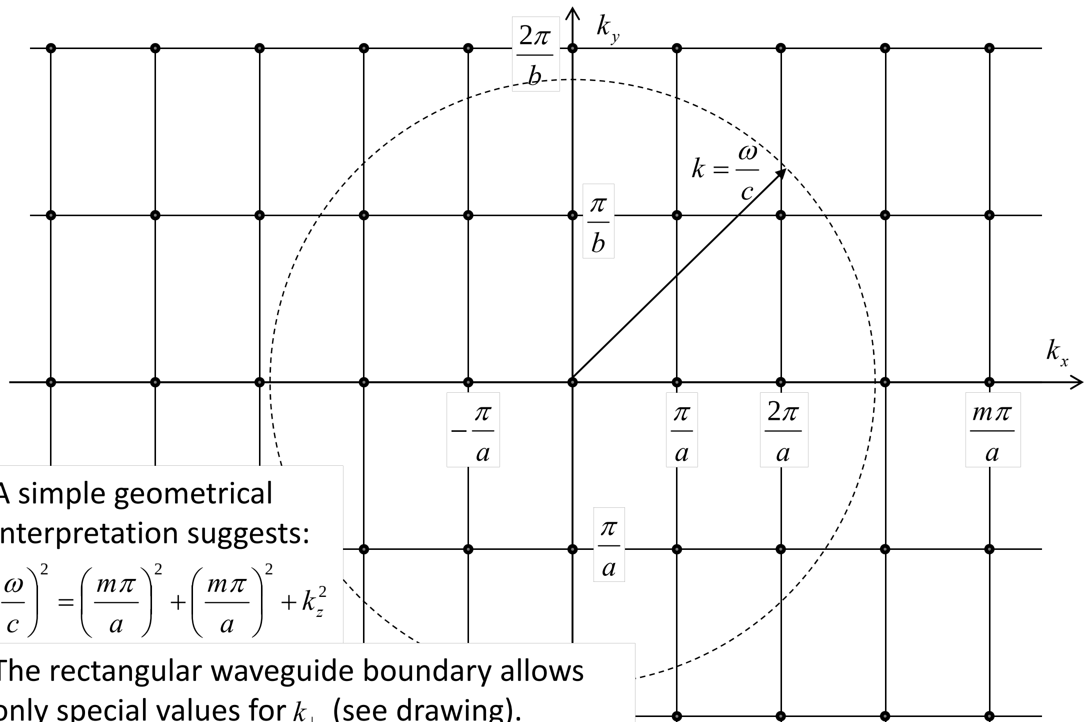


Rectangular waveguide modes



plotted: *E*-field

Constraining the wave vector



A simple geometrical interpretation suggests:

$$\left(\frac{\omega}{c}\right)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{m\pi}{a}\right)^2 + k_z^2$$

The rectangular waveguide boundary allows only special values for $k_\perp$ (see drawing). Only modes inside the circle $k = \frac{\omega}{c}$ can propagate.

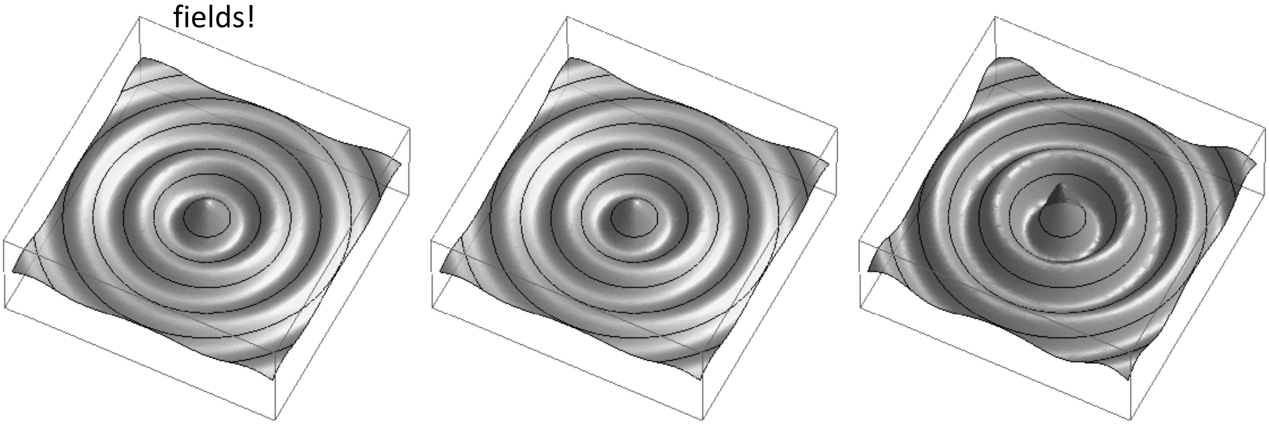
Note: This can be extended to 3 dimensions to give the resonance frequencies of a box resonator!

Radial waves

Also radial waves may be interpreted as superpositions of plane waves.

The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.

At the location of the black line, metallic walls do not perturb these fields!



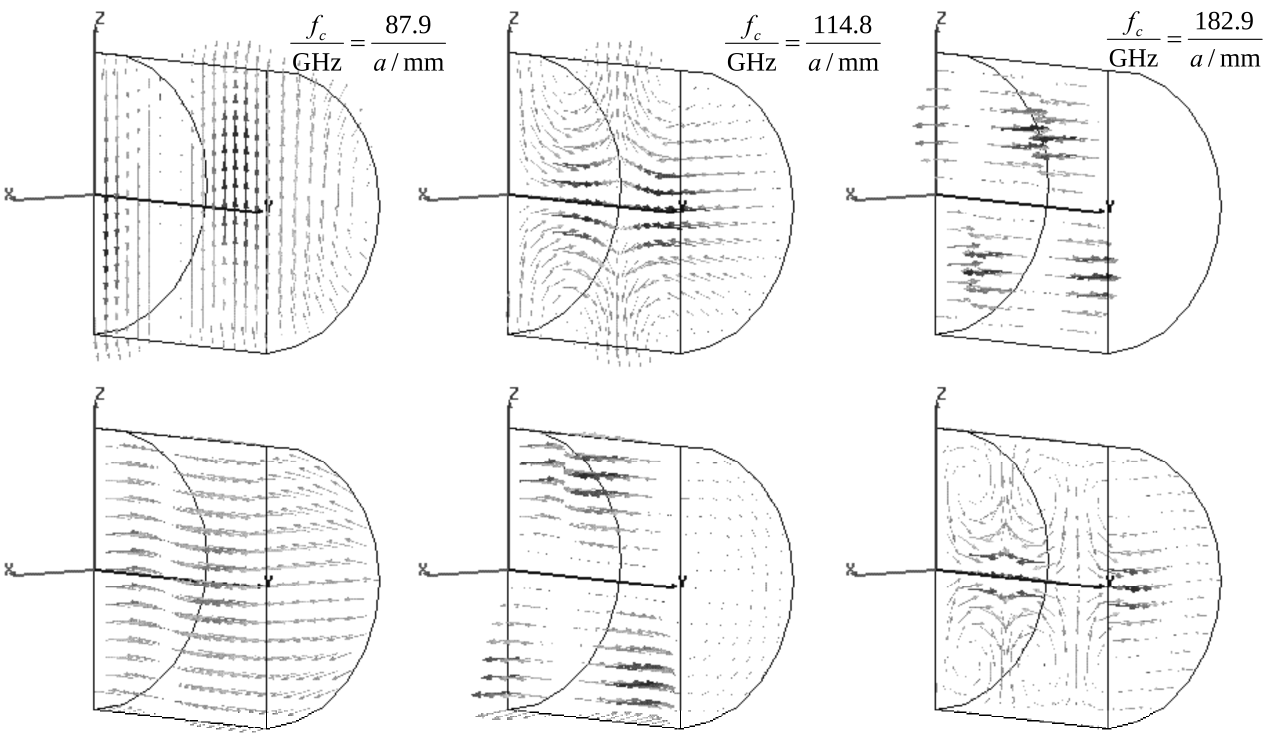
$E_z \propto H_n^{(2)}(k_\rho \rho)\cos(n\varphi)$ $E_z \propto H_n^{(1)}(k_\rho \rho)\cos(n\varphi)$ $E_z \propto J_n(k_\rho \rho)\cos(n\varphi)$

Round waveguide

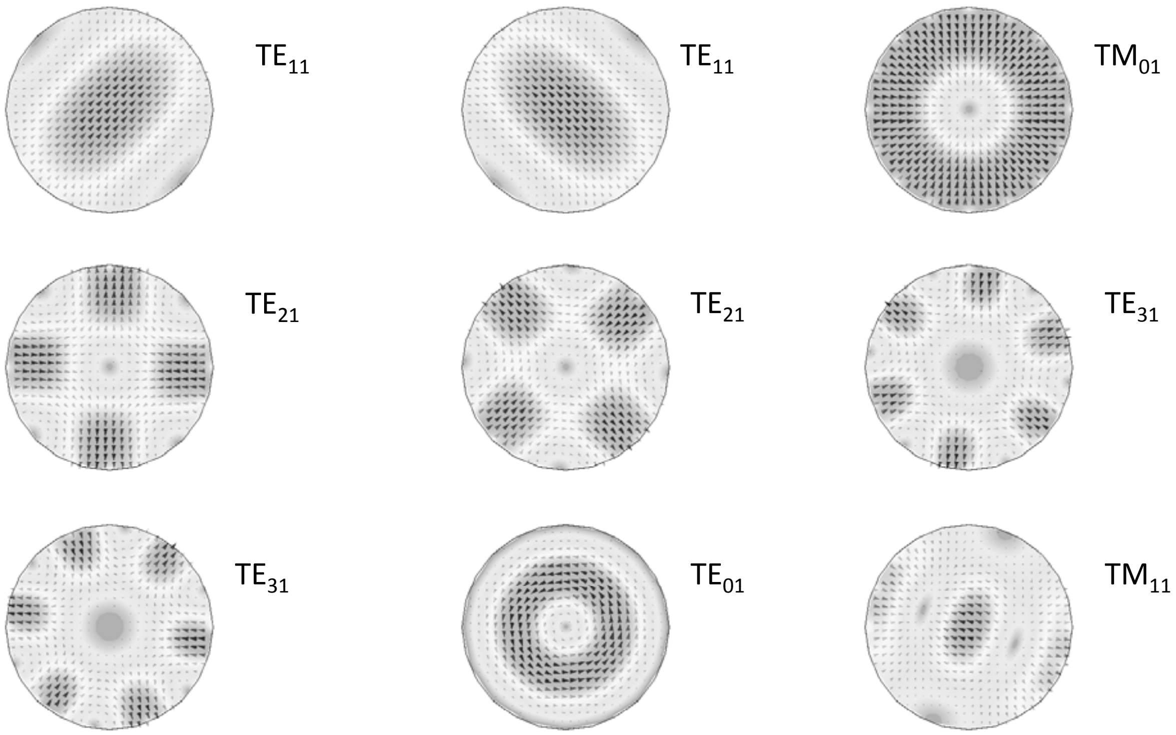
f/f_c = 1.44

TE_{11} – fundamental TM_{01} – axial field TE_{01} – low loss

$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}}$ $\frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}}$ $\frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$



Circular waveguide modes



plotted: *E*-field

General waveguide equations:

Transverse wave equation (membrane equation): $\Delta T + \left(\frac{\omega_c}{\omega}\right)^2 T = 0$

	TE (or H) modes	TM (or E) modes
boundary condition:	$\vec{n} \cdot \nabla T = 0$	$T = 0$

longitudinal wave equations (transmission line equations):

$$\frac{d}{dz} V + \gamma Z_0 I = 0$$

$$\frac{d}{dz} I + \frac{\gamma}{Z_0} V = 0$$

propagation constant: $\gamma = j \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$

	TE (or H) modes	TM (or E) modes
characteristic impedance:	$Z_0 = \frac{j\omega\mu}{\gamma}$	$Z_0 = \frac{\gamma}{j\omega\varepsilon}$
ortho-normal eigenvectors:	$\vec{e} = \vec{u}_z \times \nabla T$	$\vec{e} = -\nabla T$

transverse fields:

$$\vec{E} = V \vec{e}$$

$$\vec{H} = I \vec{u}_z \times \vec{e}$$

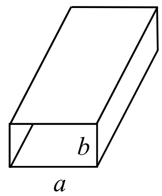
	TE (or H) modes	TM (or E) modes
longitudinal field:	$H_z = \left(\frac{\omega_c}{\omega}\right)^2 \frac{T V}{j\omega\mu}$	$E_z = \left(\frac{\omega_c}{\omega}\right)^2 \frac{T I}{j\omega\varepsilon}$

Fifth International Accelerator School for Linear Colliders, Villars 2010

Rectangular waveguide: transverse eigenfunctions

TE (H) modes: $$T_{mn}^{(H)} = \frac{1}{\pi}\sqrt{\frac{ab\varepsilon_m\varepsilon_n}{(mb)^2+(na)^2}}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

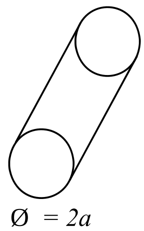
TM (E) modes: $$T_{mn}^{(E)} = \frac{2}{\pi}\sqrt{\frac{ab}{(mb)^2+(na)^2}}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$



$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

Round waveguide: transverse eigenfunctions

TE (H) modes: $$T_{mn}^{(H)} = \sqrt{\frac{\varepsilon_m}{\pi\left(\chi_{mn}'^2-m^2\right)}}\frac{J_m\left(\chi_{mn}'\frac{\rho}{a}\right)}{J_m\left(\chi_{mn}'\right)}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$$



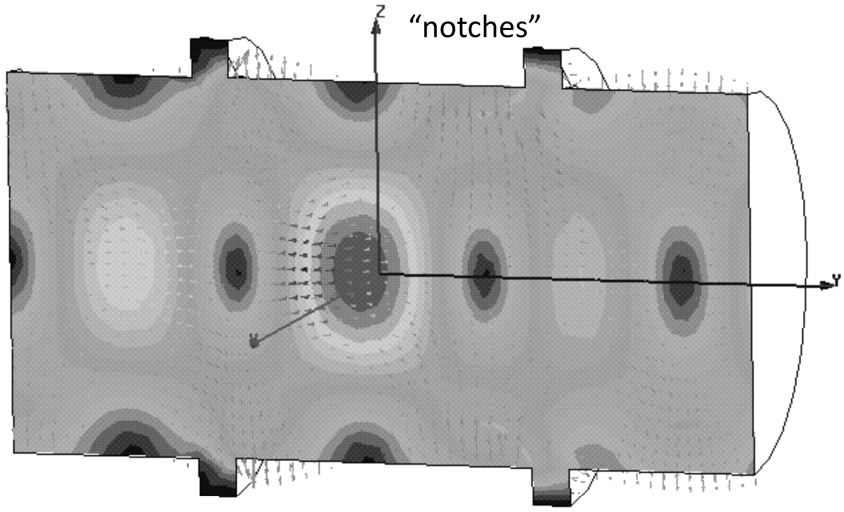
TM (E) modes: $$T_{mn}^{(E)} = \sqrt{\frac{\varepsilon_m}{\pi}}\frac{J_m\left(\chi_{mn}\frac{\rho}{a}\right)}{\chi_{mn}J_{m-1}(\chi_{mn})}\begin{Bmatrix}\sin(m\varphi)\\ \cos(m\varphi)\end{Bmatrix}$$

$$\frac{\omega_c}{c} = \frac{\chi_{mn}}{a}$$

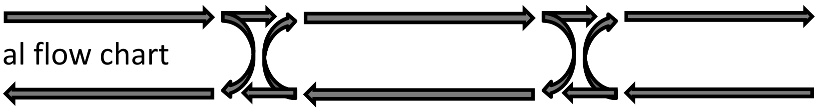
where $$\varepsilon_i = \begin{cases}1 & for \quad i=0\\ 2 & for \quad i\neq 0\end{cases}$$

CAVITIES

Waveguide perturbed by notches



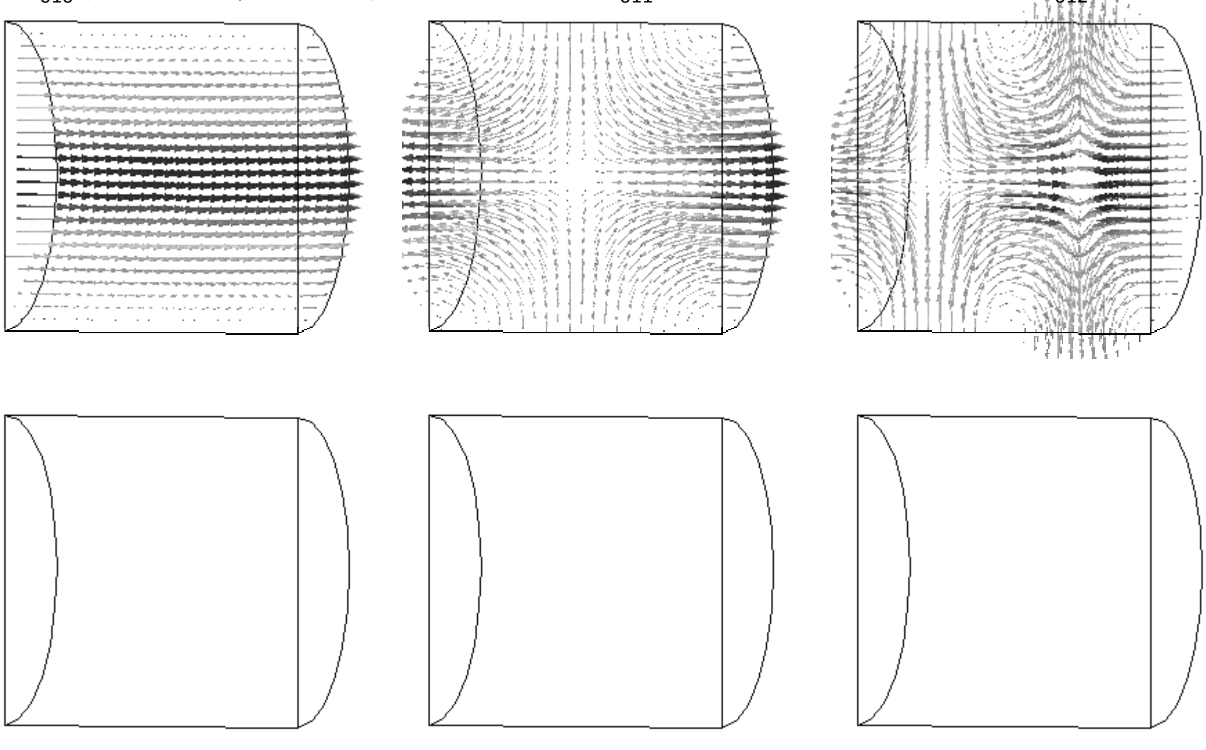
Signal flow chart



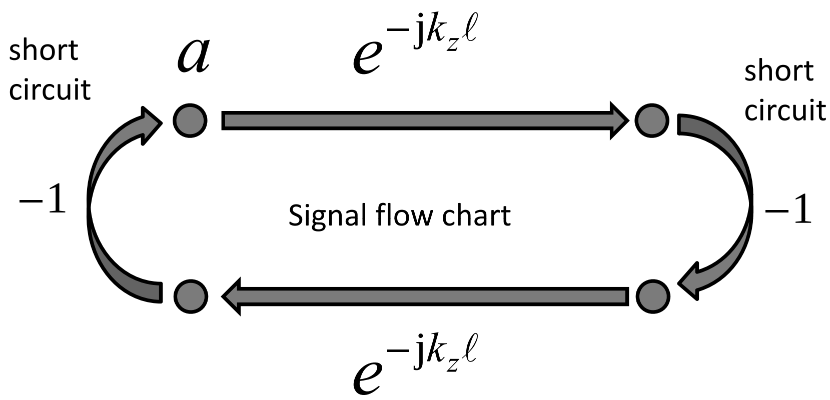
Reflections from notches lead to a superimposed standing wave pattern.
“Trapped mode”!

Short-circuited waveguide

TM_{010} (no axial dependence) TM_{011} TM_{012}



Single WG mode between two shorts



This again is an eigenvalue equation for field amplitude *a*:

$$a = e^{-jk_z 2\ell} a$$

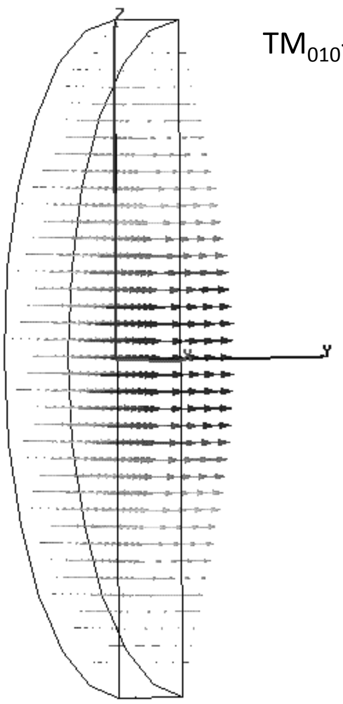
Non-vanishing solutions exist for $2k_z\ell = 2\pi\, m$:

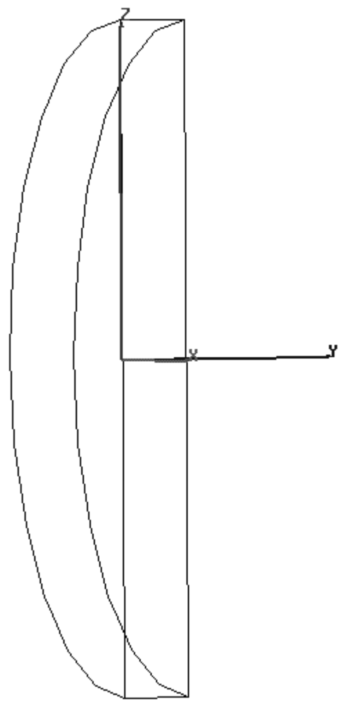
With $k_z = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(c\frac{m}{2\ell}\right)^2$

Simple pillbox

(only 1/2 shown)

TM_{010}-mode

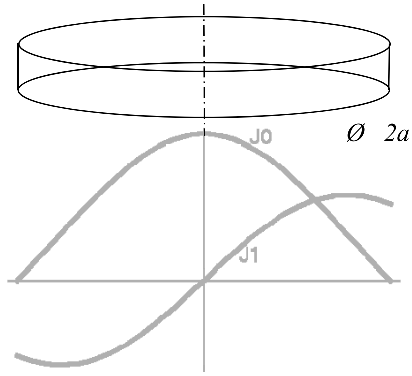




electric field (purely axial) magnetic field (purely azimuthal)

Pillbox cavity field (w/o beam tube)

$$T(\rho,\varphi)=\sqrt{\frac{1}{\pi}}\,\frac{\mathrm{J}_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01}\,\mathrm{J}_1\left(\frac{\chi_{01}}{a}\right)}\qquad \chi_{01}=2.40483\ldots$$



The only non-vanishing field components :

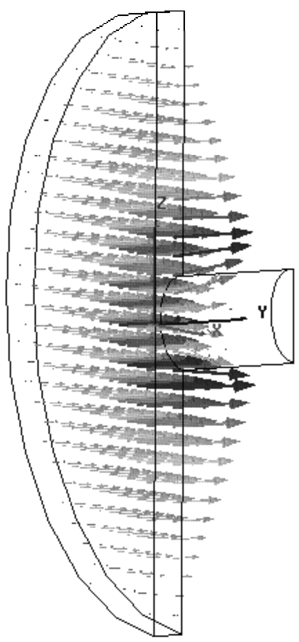
$$E_z=\frac{1}{\mathrm{j}\omega\varepsilon_0}\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\;\frac{\mathrm{J}_0\left(\frac{\chi_{01}\rho}{a}\right)}{a\,\mathrm{J}_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi=\mu_0\sqrt{\frac{1}{\pi}}\;\frac{\mathrm{J}_1\left(\frac{\chi_{01}\rho}{a}\right)}{a\,\mathrm{J}_1\left(\frac{\chi_{01}}{a}\right)}$$

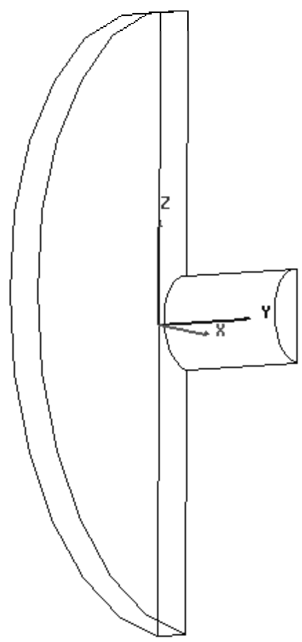
$$\boxed{\begin{array}{l}
\omega_0\big|_{pillbox}=\dfrac{\chi_{01}\,c}{a}\qquad \eta=\sqrt{\dfrac{\mu_0}{\varepsilon_0}}=377\,\Omega\\[2ex]
Q\big|_{pillbox}=\dfrac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1+\dfrac{a}{h}\right)}\\[2ex]
\dfrac{R}{Q}\bigg|_{pillbox}=\dfrac{4\eta}{\chi_{01}{}^{3}\pi\,\mathrm{J}_1^2(\chi_{01})}\;\dfrac{\sin^2\left(\dfrac{\chi_{01}}{2}\dfrac{h}{a}\right)}{h/a}
\end{array}}$$

Pillbox with beam pipe TM_{010}-mode (only 1/4 shown)

One needs a hole for the beam pipe – circular waveguide below cutoff



electric field

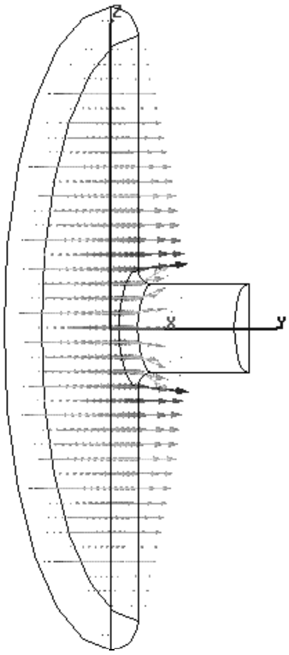


magnetic field

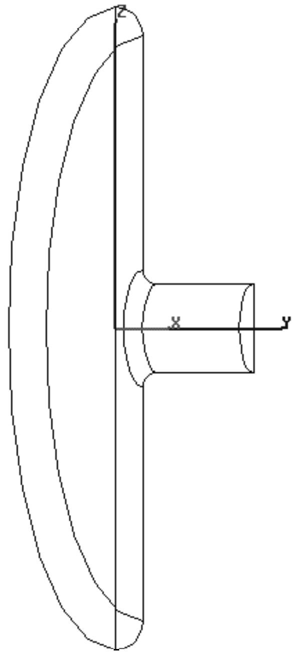
A more practical pillbox cavity

TM_{010}-mode (only 1/4 shown)

Round of sharp edges
(field enhancement!)



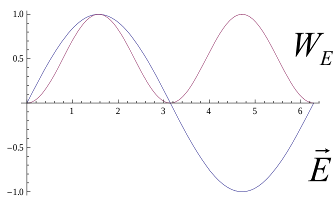
electric field



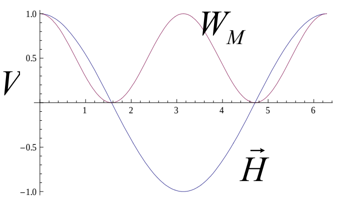
magnetic field

Stored energy

The energy stored in the electric field is $\iiint_{cavity} \frac{\varepsilon}{2} \left|\vec{E}\right|^2 \mathrm{d}V$



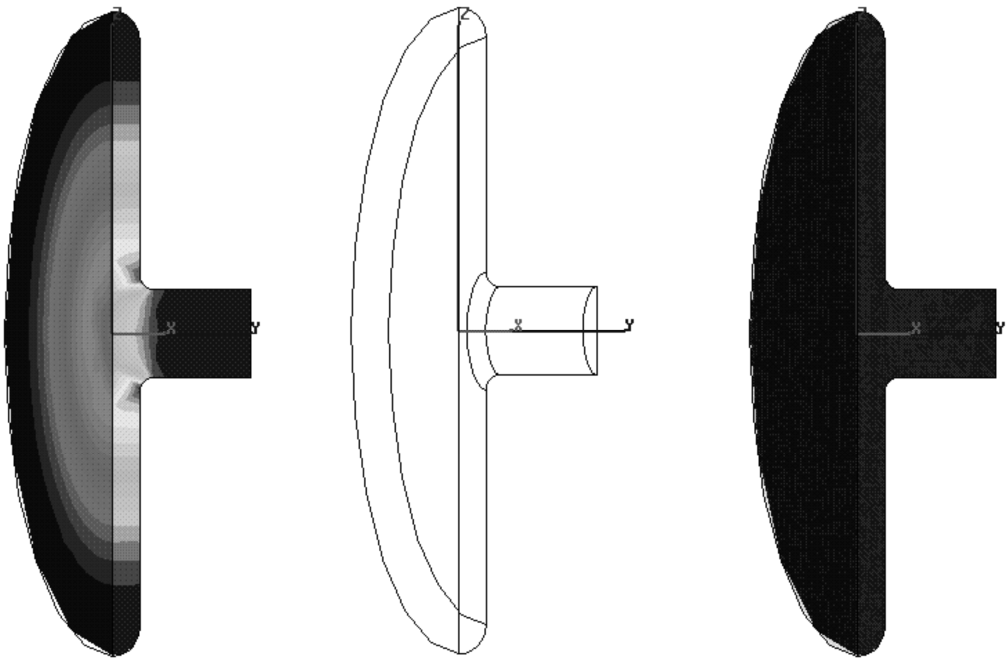
The energy stored in the magnetic field is $\iiint_{cavity} \frac{\mu}{2} \left|\vec{H}\right|^2 \mathrm{d}V$



Since $\vec{E}$ and $\vec{H}$ are 90° out of phase, the stored energy continuously swaps from electric energy to magnetic energy. On average, electric and magnetic energy must be equal.
The (imaginary part of the) Poynting vector describes this energy flux.

In steady state, the total stored energy $W = \iiint_{cavity} \left(\frac{\varepsilon}{2} \left|\vec{E}\right|^2 + \frac{\mu}{2} \left|\vec{H}\right|^2 \right) \mathrm{d}V$ is constant in time.

Stored energy & Poynting vector



electric field energy Poynting vector magnetic field energy

Losses & *Q* factor

The losses P_{loss} are proportional to the stored energy W.

The cavity quality factor Q is defined as the ratio $Q = \frac{\omega_0 W}{P_{loss}}$.

In a vacuum cavity, losses are dominated by the ohmic losses due to the finite conductivity of the cavity walls.
If the losses are small, one can calculate them with a **perturbation method**:

- The tangential magnetic field at the surface leads to a surface current.
- This current will see a wall resistance $R_A = \sqrt{\frac{\omega\mu}{2\sigma}}$
- { R_A is related to the skin depth δ by $\delta\,\sigma\,R_A = 1$. }
- The cavity losses are given by $P_{loss} = \iint_{wall} R_A \left|H_t\right|^2 \mathrm{d}A$
- If other loss mechanisms are present, losses must be added. Consequently, the inverses of the Q 's must be added!

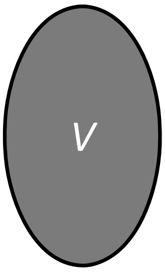
Small boundary perturbation – tuning

Another application of the perturbation method is to analyse the sensitivity to (small) surface geometry perturbations.

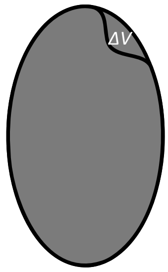
- This is relevant to understand the effect of fabrication tolerances.
- Intentional surface perturbation can be used to tune the cavity.

The basic idea of the perturbation theory is use a known solution (in this case the unperturbed cavity) and assume that the deviation from it is only small. We just used this to calculate the losses (assuming H_t would be that without losses).

The result of this calculation leads to a convenient expression for the detuning:



$$\frac{\omega-\omega_0}{\omega_0}=\frac{\iiint\limits_{\Delta V}\left(\mu|H_0|^2-\varepsilon|E_0|^2\right)\mathrm{d}V}{\iiint\limits_{V}\left(\mu|H_0|^2+\varepsilon|E_0|^2\right)\mathrm{d}V}$$



unperturbed

ω_0

"Slater-theorem"

perturbed

ω

Interaction integral (1)

- Particle dynamics

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

$$\vec{v}=\frac{\vec{p}}{m\gamma}$$

- EM Fields

$$\nabla\times\vec{B}-\frac{1}{c^2}\frac{\partial}{\partial t}\vec{E}=\mu_0\vec{J} \qquad \nabla\cdot\vec{B}=0$$

$$\nabla\times\vec{E}+\frac{\partial}{\partial t}\vec{B}=0 \qquad \nabla\cdot\vec{E}=\mu_0 c^2\rho$$

Energy exchange field – particle:

$$W\,\mathrm{d}W=c^2\vec{p}\cdot\mathrm{d}\vec{p}=qc^2\vec{p}\cdot\left(\vec{E}+\vec{v}\times\vec{B}\right)\,\mathrm{d}t=qc^2\vec{p}\cdot\vec{E}\mathrm{d}t$$

$$\mathrm{d}W=q\vec{v}\cdot\vec{E}\mathrm{d}t$$

For a particle density this becomes a volume integral

$$P_{field\to particles}=q\iiint n(x,y,z)\vec{v}\cdot\vec{E}\,\mathrm{d}V$$

For a particle beam moving in z-direction with constant speed v, the transverse density can be separated out and replaced by a current

$$I_B(z,t)=\iint\vec{J}\cdot\vec{u}_z\mathrm{d}A=q\,v\,n_z(vt-z)$$

Interaction integral (2)

$$P_{field \to particles} = q \iiint n(x,y,z) \vec{v} \cdot \vec{E}\, \mathrm{d}V$$
$$P_{field \to particles} = q\, v \int n_z(vt - z) E_z dz = \int I_B(z,t) E_z dz$$

With the FT of the current $\int_{-\infty}^{\infty} I_B(z,t) e^{-j\omega t} = qv \int_{-\infty}^{\infty} n_z(z - vt) e^{-j\omega t} = I_0(\omega) e^{-j\frac{\omega}{v} z}$

we get for the power exchange field → particle in ω-domain:

$$\boxed{P = I_0^*(\omega) \int_{gap} E_z e^{j\frac{\omega}{v} z} dz}$$

Acceleration voltage & *R*-upon-*Q*

I define $V_{acc} = \int E_z\, e^{j\frac{\omega}{\beta c} z} dz$. The exponential factor accounts for the variation of the field while particles with velocity βc are traversing the gap (see next page).

With this definition, V_{acc} is generally complex – this becomes important with more than one gap. For the time being we are only interested in $|V_{acc}|$.

From the previous page one concludes that $P = V_{acc}\, I_0^*(\omega)$.

Attention, different definitions are used!

The square of the acceleration voltage is proportional to the stored energy W. The proportionality constant defines the quantity called *R*-upon-*Q*:

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\, \omega_0\, W}$$

Attention, also here different definitions are used!

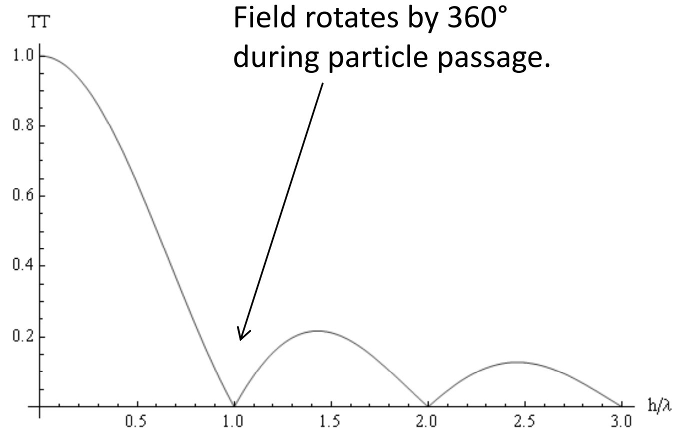
Transit time factor

The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see.

$$TT = \frac{|V_{acc}|}{\left|\int E_z \mathrm{d}z\right|} = \frac{\left|\int E_z e^{j\frac{\omega}{\beta c}z} \mathrm{d}z\right|}{\left|\int E_z \mathrm{d}z\right|}$$

The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length) h is:

$$TT = \sin\left(\frac{\chi_{01}h}{2a}\right) \Big/ \left(\frac{\chi_{01}h}{2a}\right)$$



Shunt impedance

The square of the acceleration voltage is proportional to the power loss P_{loss}.

The proportionality constant defines the quantity "shunt impedance"

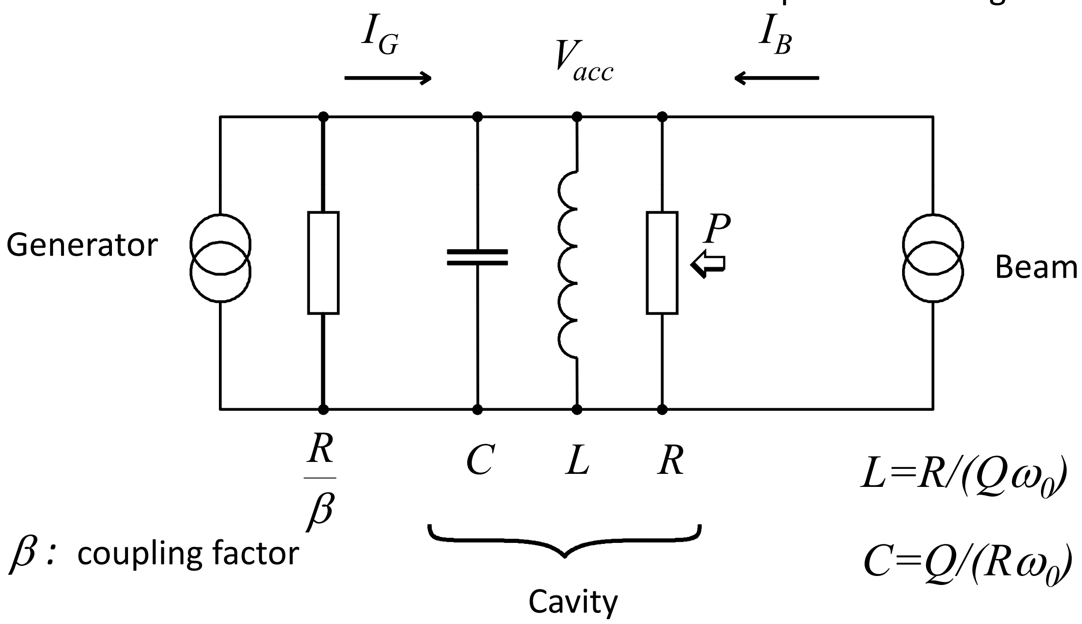
$$R = \frac{|V_{acc}|^2}{2\,P_{loss}}$$

Attention, also here different definitions are used!

Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.

Cavity equivalent circuit

Simplification: single mode



$L=R/(Q\omega_0)$

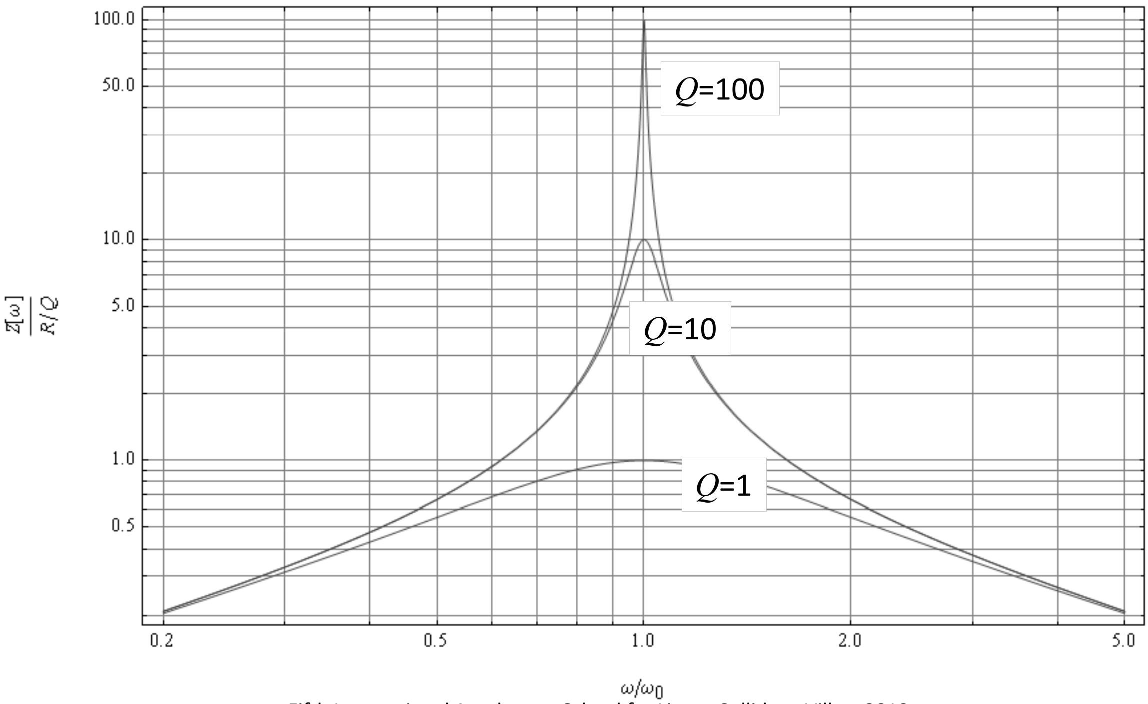
$C=Q/(R\omega_0)$

β : coupling factor

R: Shunt impedance $\sqrt{L/C}$: *R*-upon-*Q*

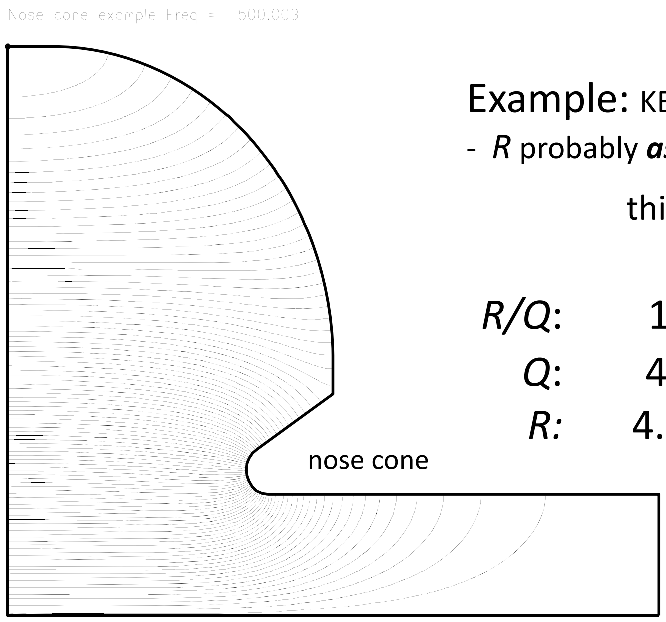
Resonance

A high Q cavity allows to use a resonance phenomenon to create high voltage with lower power – at the expense of reduced bandwidth!



Reentrant cavity

Nose cones increase transit time factor, round outer shape minimizes losses.



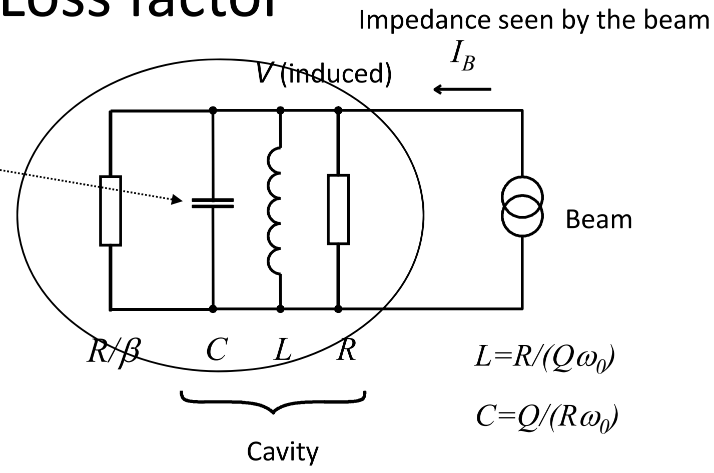
Example: KEK photon factory 500 MHz

- *R* probably ***as good as it gets*** -

	this cavity	optimized pillbox
R/Q:	111 Ω	107.5 Ω
Q:	44270	41630
R:	4.9 MΩ	4.47 MΩ

Loss factor

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{|V_{acc}|^2}{4W} = \frac{1}{2C}$$

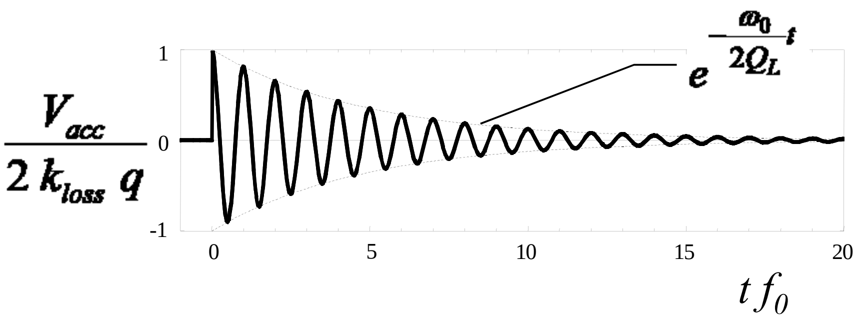


$L=R/(Q\omega_0)$

$C=Q/(R\omega_0)$

Energy deposited by a single charge *q*: $k_{loss}q^2$

Voltage induced by a single charge *q:*



Summary: relations between V_{acc}, W, P_{loss}

R*-upon-*Q

$$\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{2\,\omega_0 W}$$

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{4\,W}$$

gap voltage

V_{acc}

Shunt impedance

$$R = \frac{\left|V_{acc}\right|^2}{2\,P_{loss}}$$

Energy stored inside the cavity

W

$$Q = \frac{\omega_0 W}{P_{loss}}$$

Power lost in the cavity walls

P_{loss}

***Q* factor**

Beam loading – RF to beam efficiency

The beam current "loads" the generator, in the equivalent circuit this appears as a resistance in parallel to the shunt impedance.

If the generator is matched to the unloaded cavity, beam loading will cause the accelerating voltage to decrease.

The power absorbed by the beam is $-\frac{1}{2}\mathrm{Re}\left\{V_{acc}\, I_B^*\right\}$,

the power loss $P_{loss} = \frac{\left|V_{acc}\right|^2}{2\,R}$.

For high efficiency, beam loading should be high.

The RF to beam efficiency is $\eta = \frac{1}{1 + \frac{V_{acc}}{R\left|I_B\right|}} = \frac{\left|I_B\right|}{\left|I_G\right|}$.

Much more on beam loading in A. Grudiev's lecture!

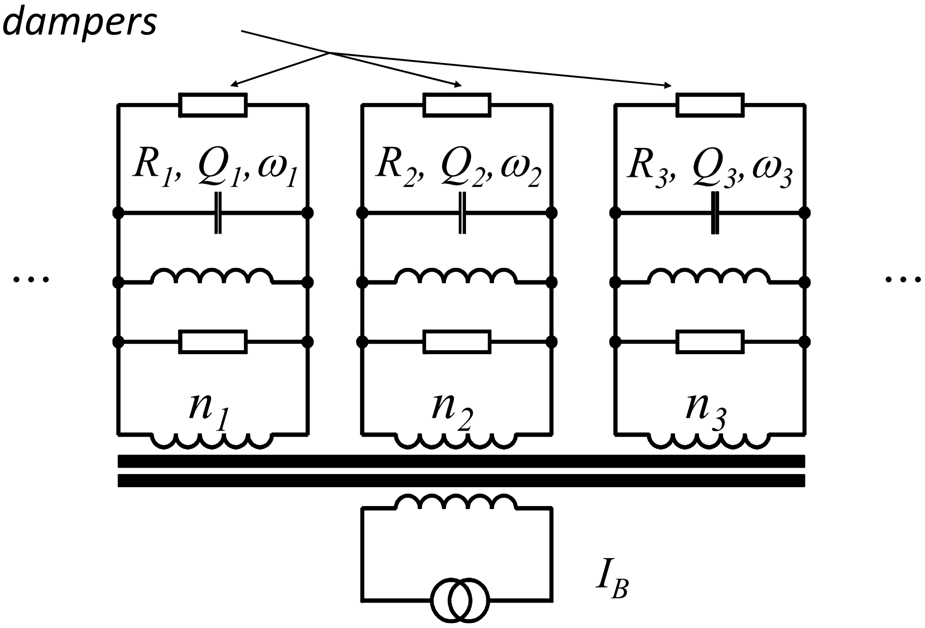
Characterizing cavities

- Resonance frequency $$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$
- Transit time factor $$\frac{\left|\int E_z e^{j\frac{\omega}{\beta c}z} \mathrm{d}z\right|}{\left|\int E_z \mathrm{d}z\right|}$$
  field varies while particle is traversing the gap

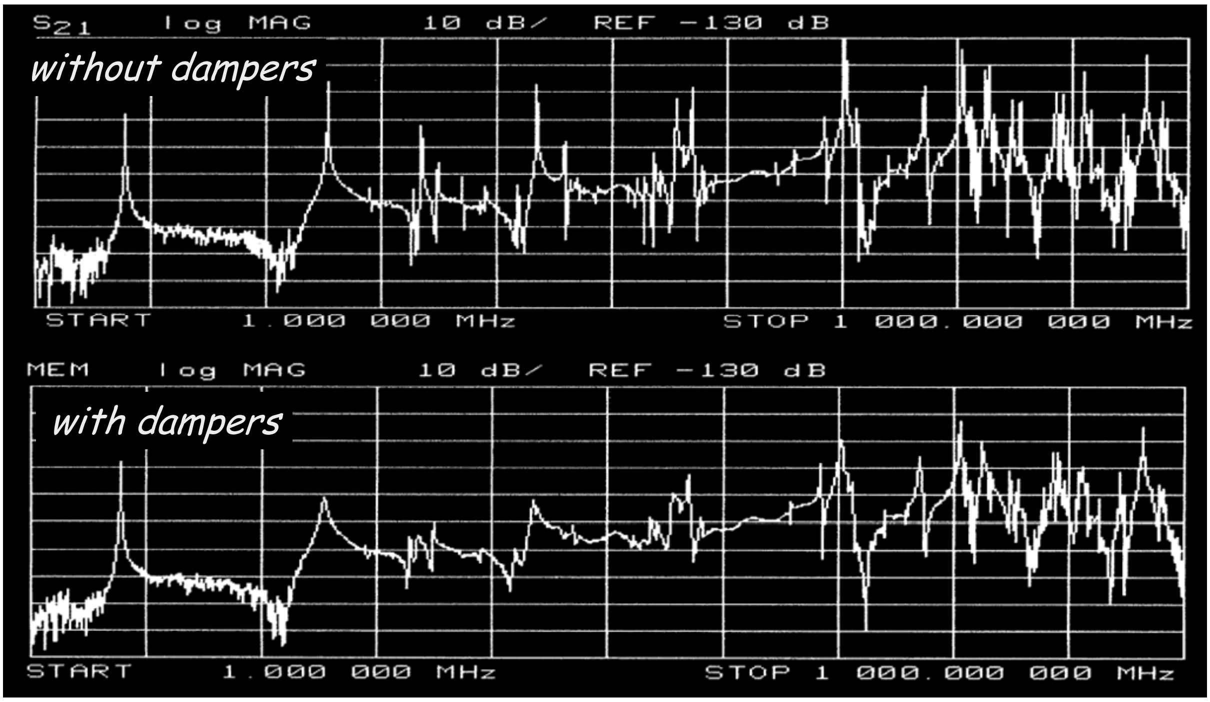
	Circuit definition	Linac definition
Shunt impedance gap voltage – power relation	$\lvert V_{acc}\rvert^2 = 2\,R\,P_{loss}$	$\lvert V_{acc}\rvert^2 = R\,P_{loss}$
Q factor	$\omega_0 W = Q P_{loss}$	
R/Q independent of losses – only geometry!	$\frac{R}{Q} = \frac{\lvert V_{acc}\rvert^2}{2\omega_0 W} = \sqrt{\frac{L}{C}}$	$\frac{R}{Q} = \frac{\lvert V_{acc}\rvert^2}{\omega_0 W}$
loss factor	$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\lvert V_{acc}\rvert^2}{4W}$	$k_{loss} = \frac{\omega_0}{4}\frac{R}{Q} = \frac{\lvert V_{acc}\rvert^2}{4W}$

Higher order modes

external dampers

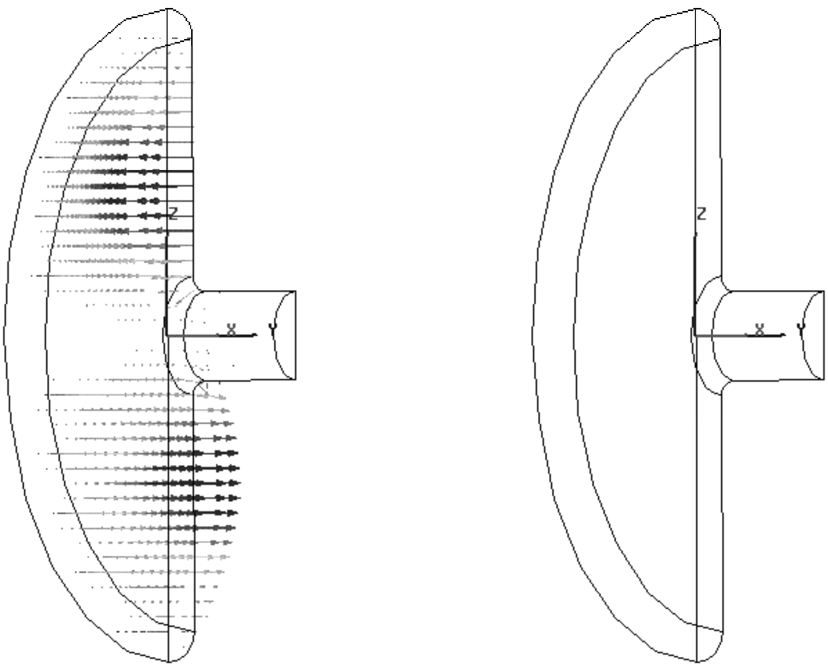


Higher order modes (measured spectrum)



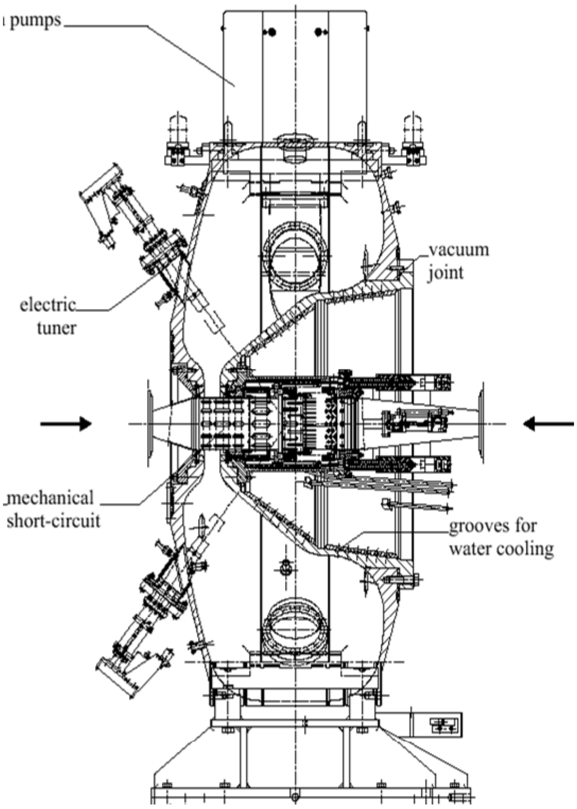
Pillbox: dipole mode

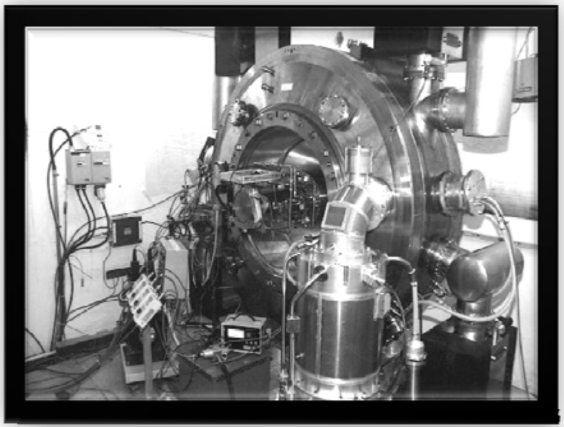
TM_{110}-mode (only 1/4 shown)

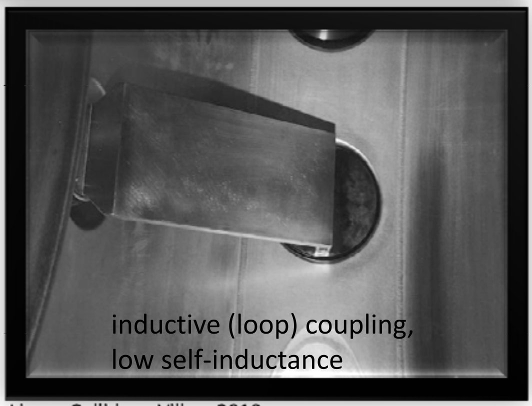


electric field magnetic field

CERN/PS 80 MHz cavity (for LHC)





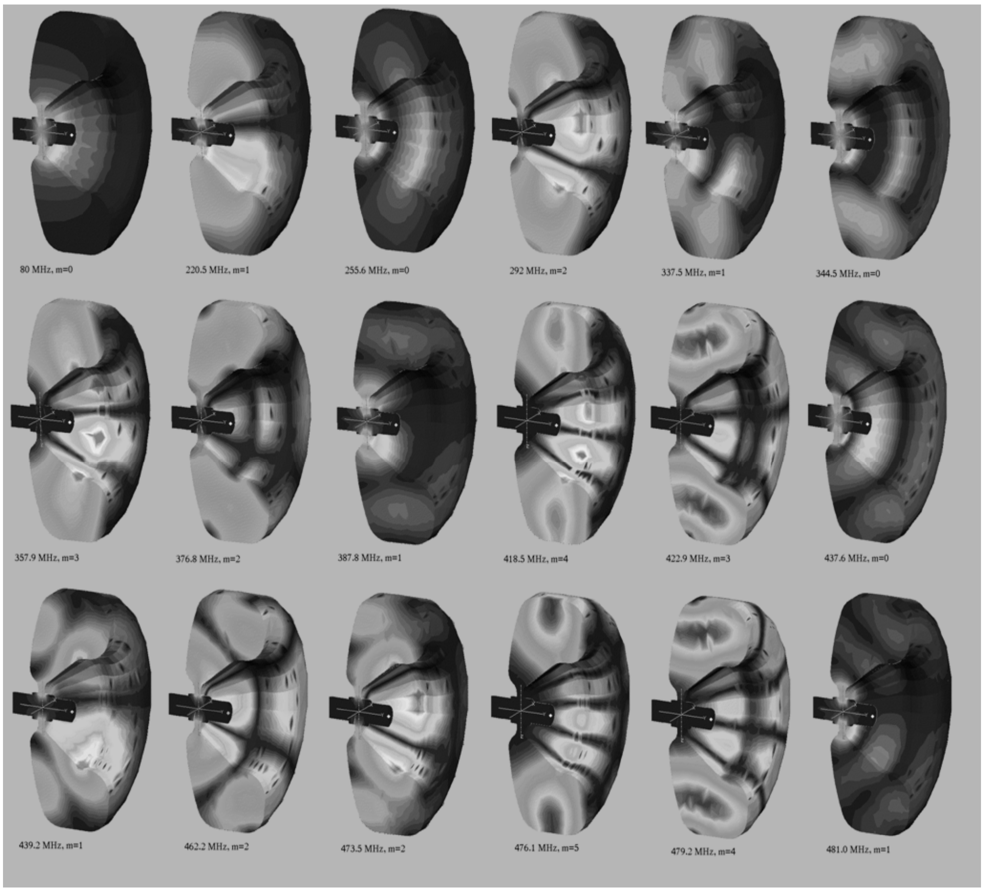


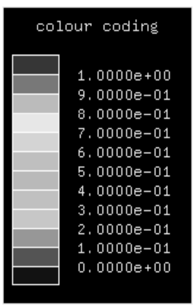
Higher order modes

Example shown: 80 MHz cavity PS for LHC.

Color-coded:

$$\left|\vec{E}\right|$$

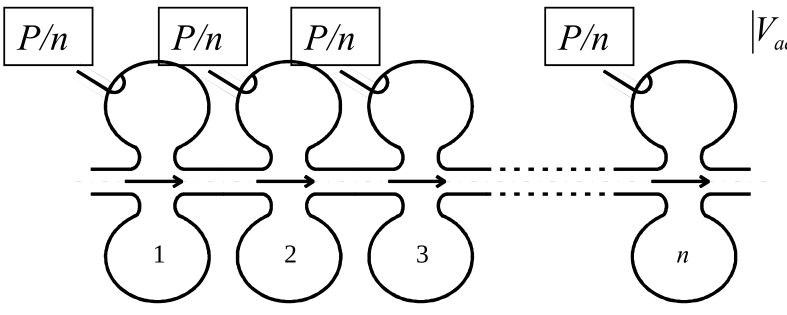




MULTICELL ACCELERATORS

What do you gain with many gaps?

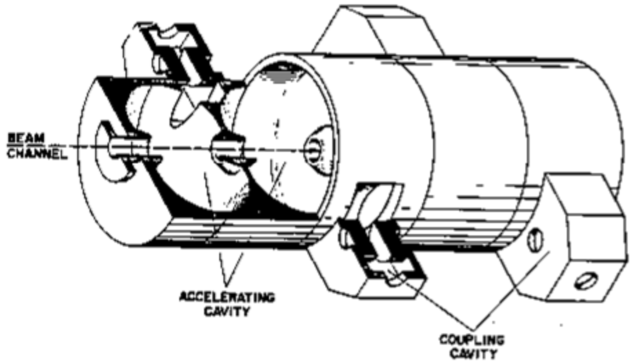
- The R/Q of a single gap cavity is limited to some 200 Ω (Linac).
  Now consider to distribute the available power to n identical cavities: each will receive P/n, thus produce an accelerating voltage of $\sqrt{2RP/n}$.
  The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR .



$$|V_{acc}| = n\sqrt{2RP/n} = \sqrt{2(nR)P}$$

Standing wave multicell cavity

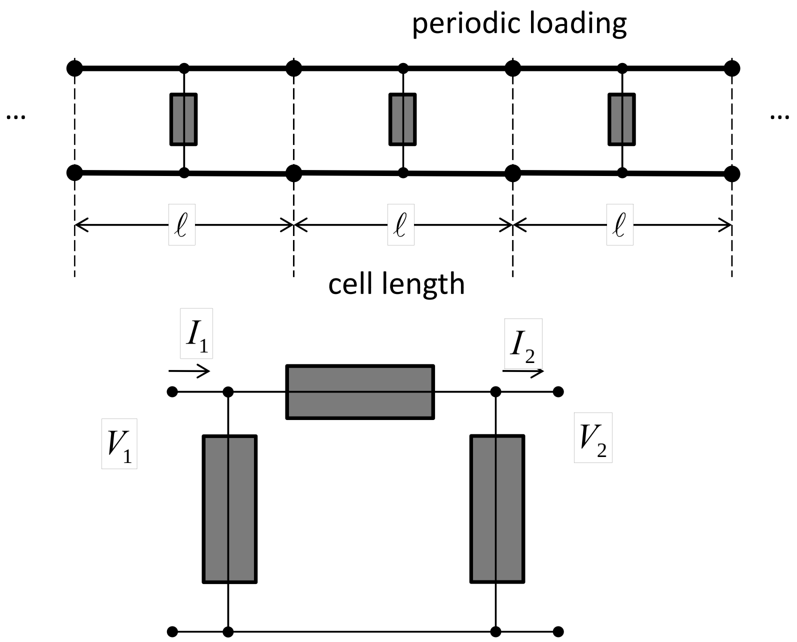
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).
- Coupled cavity accelerating structure (side coupled)



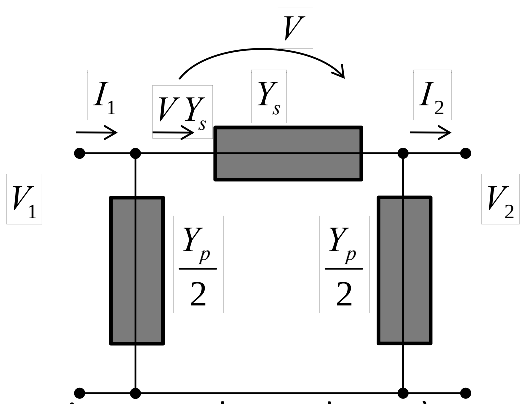
- The phase relation between gaps is important!

Travelling wave structures

- Periodic structures – simple model:



Ansatz



- *Floquet*'s theorem:
  Fields at "2" are equal to fields at "1" except for a complex factor:

$$V_2 = V_1\, e^{-j\phi}$$
$$I_2 = I_1\, e^{-j\phi}$$

(here for simplicity assumed lossless meaning pure phase advance)

- Equations (this is again the search of an eigenvector!)

$$V_2 = V_1\, e^{-j\phi}$$
$$I_2 = I_1\, e^{-j\phi}$$
$$V_1 = V + V_2$$
$$I_1 = V_1 \frac{Y_p}{2} + V\, Y_s$$
$$I_2 = -V_2 \frac{Y_p}{2} + V\, Y_s$$

$$0 = V_1 \left(\left(1 - e^{-j\phi}\right)^2 Y_s - \left(1 + e^{-j\phi}\right) \frac{Y_p}{2} \right)$$

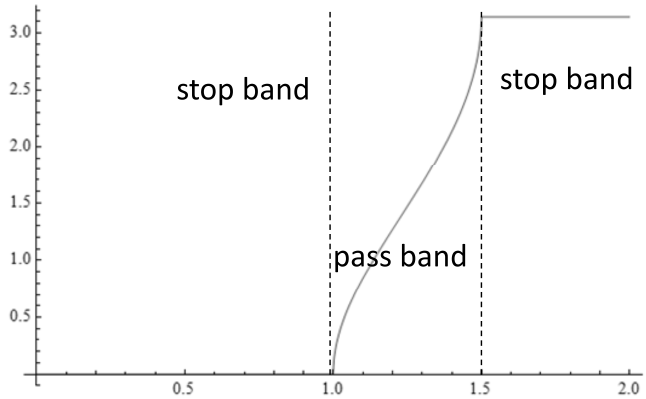
As an example, I took Y_p to be a parallel resonance and Y_s to be an inductivity:

Solution for this simple case

Equivalent circuit: π-type. Parallel $\frac{y_p}{2}$ both at input and at output, series y_s between those. Below, the series y_s consists of only an inductivity, whereas y_p is a parallel resonance (determining the lower cut-off).

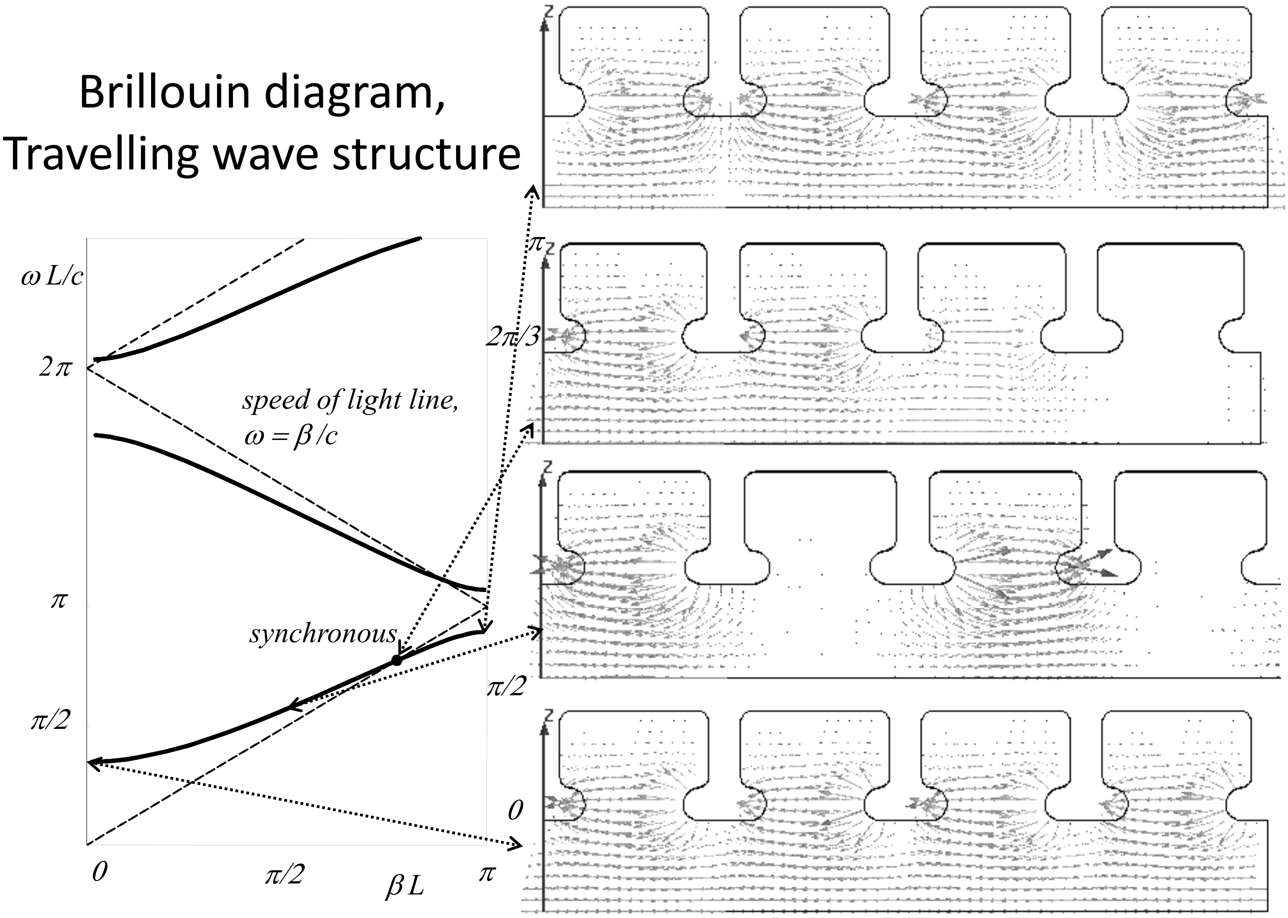
```
In[36]:= Off[Solve::ifun];
         eqns = {ys == (ωπ^2 - ω0^2)/(j 4 ω rq0 ω0), yp == j 1/rq0 (ω/ω0 - ω0/ω), v1 == v + v1 e^g, yp/2 v1 + ys v == -yp/2 v1 + ys v e^-g};
         solution = Solve[eqns, {g, v1, ys, yp}];

In[39]:= Block[{ωπ = 1.5, rq0 = 1, ω0 = 1.}, Plot[Im[g /. solution[[1]]], {ω, 0, 2}, PlotRange → All]]
```

Out[39]=



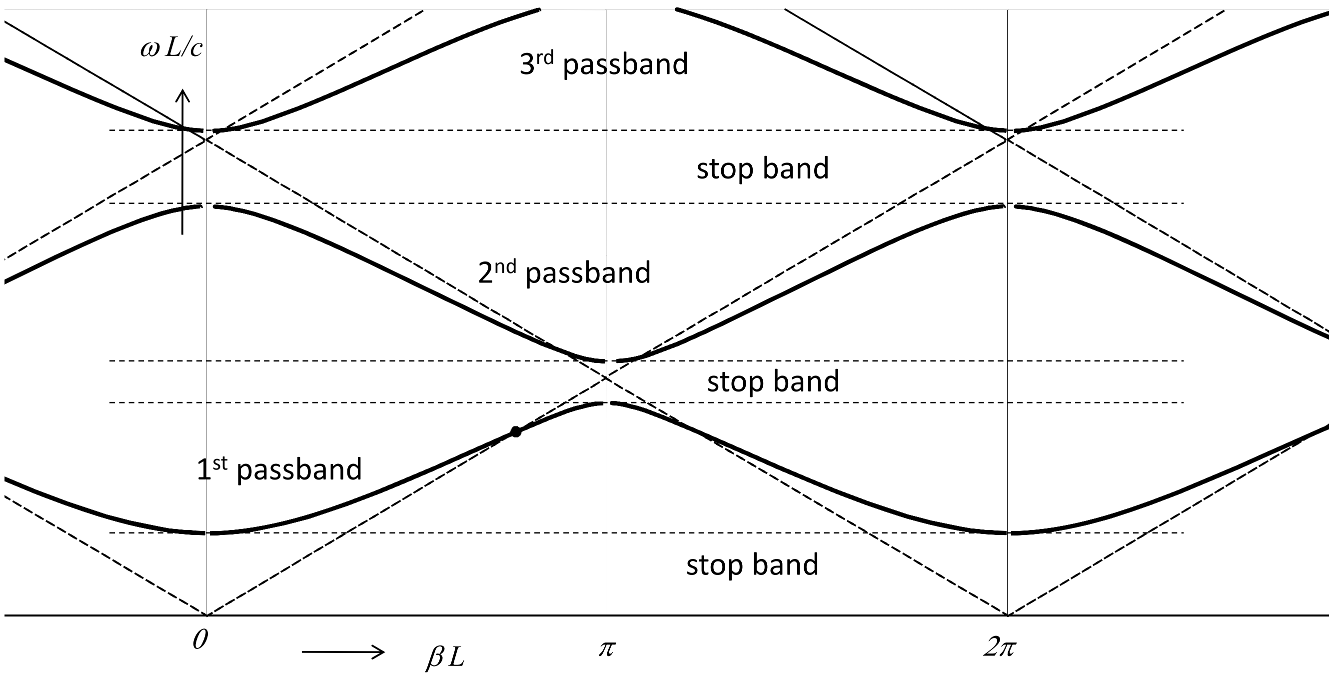
- In the stop bands, the real part of g is non-vanishing – it describes (strong) damping (like a waveguide below cutoff!)
- Imaginary g means propagation!
- This is the *Brillouin* diagram – normally $\varphi=\mathrm{Im}\{g\}$ *is* ploted vs. ω.

Brillouin diagram, Travelling wave structure

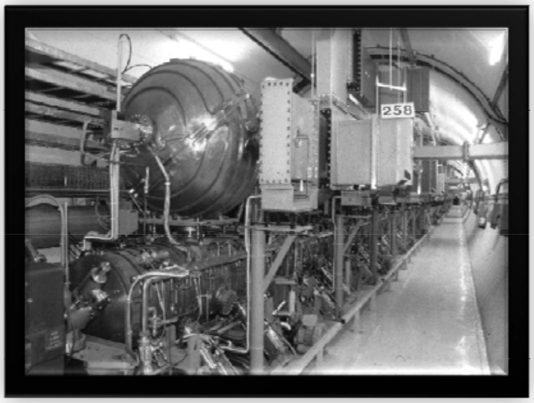


Brillouin diagram

I was only plotting the range *0* to π, but the *Brillouin* diagram is itself periodic in φ!

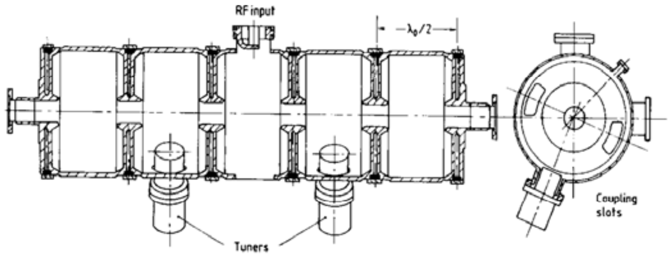


Examples of cavities

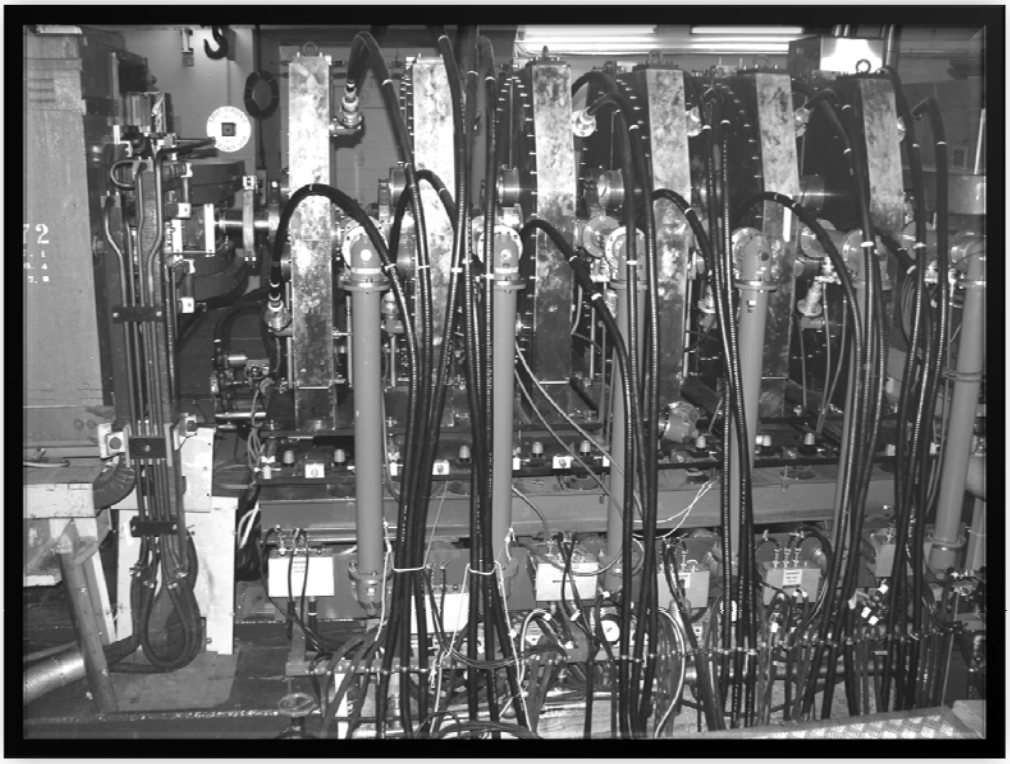


PEP II cavity
476 MHz, single cell,
1 MV gap with 150 kW,
strong HOM damping,

LEP normal-conducting Cu RF cavities,
350 MHz. 5 cell standing wave + spherical cavity
for energy storage, 3 MV

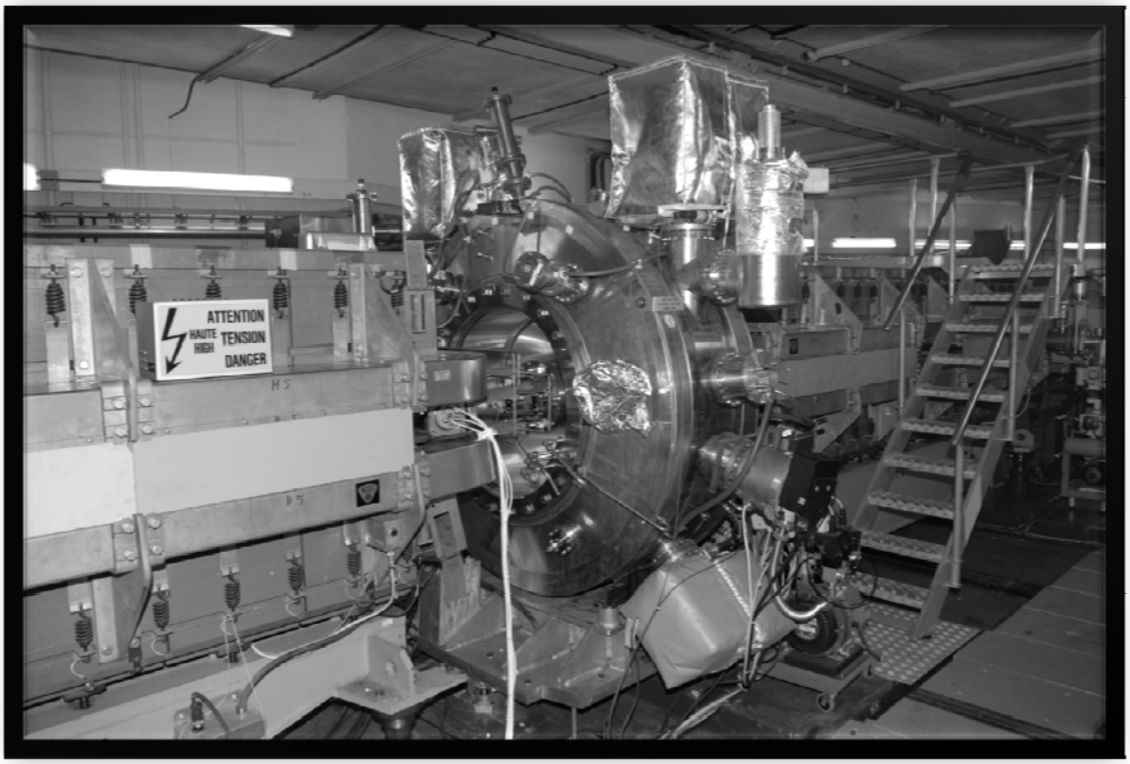


CERN PS 200 MHz cavities

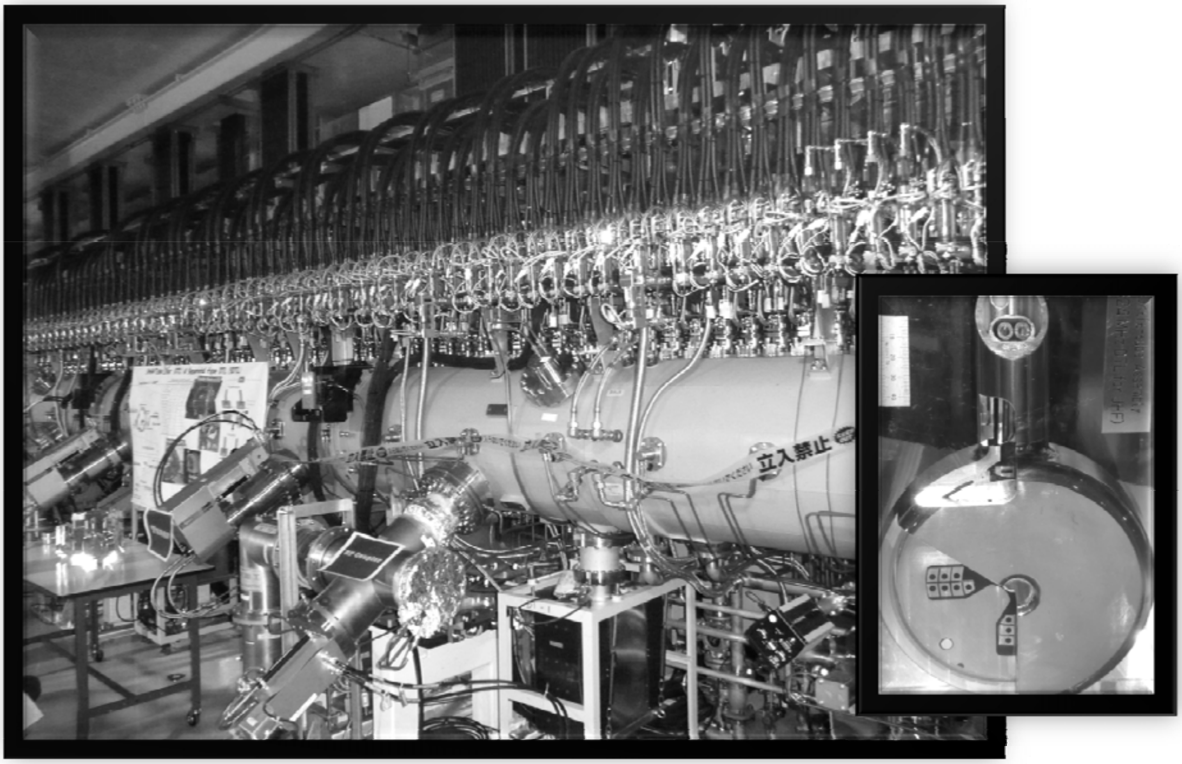


PS 19 MHz cavity (prototype, photo: 1966)

CERN PS 80 MHz Cavity (1997)



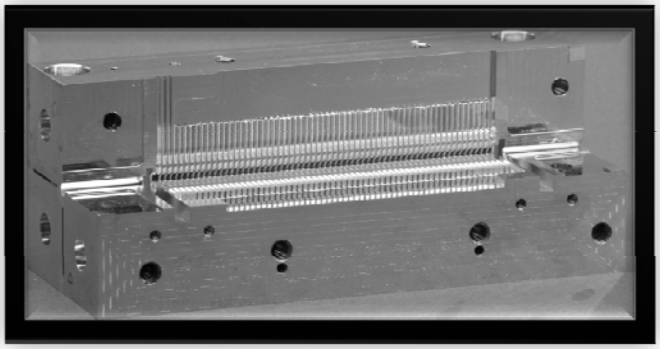
Drift-tube linac (JPARC JHF, 324 MHz)



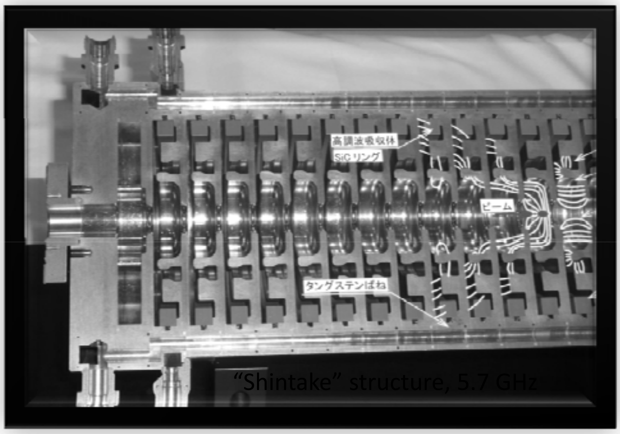
CERN SPS 200 MHz TW cavity

Travelling wave cavities

CLIC “T18”, 12 GHz

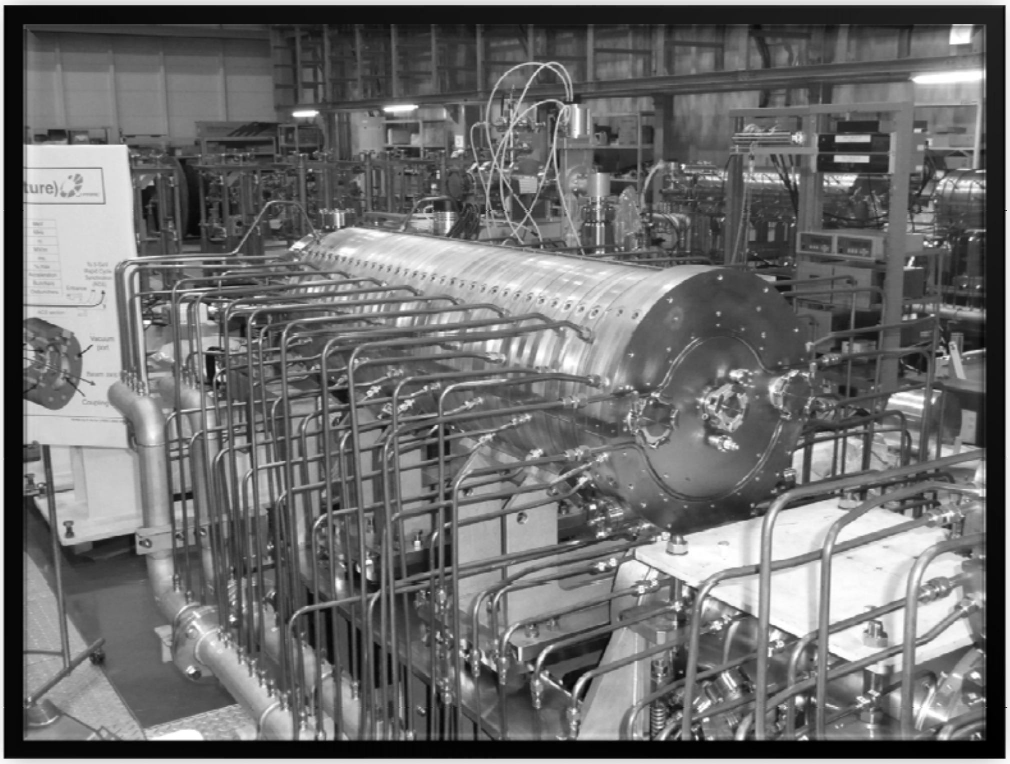


CLIC “HDS”, 12 GHz



“Shintake” structure, 5.7 GHz

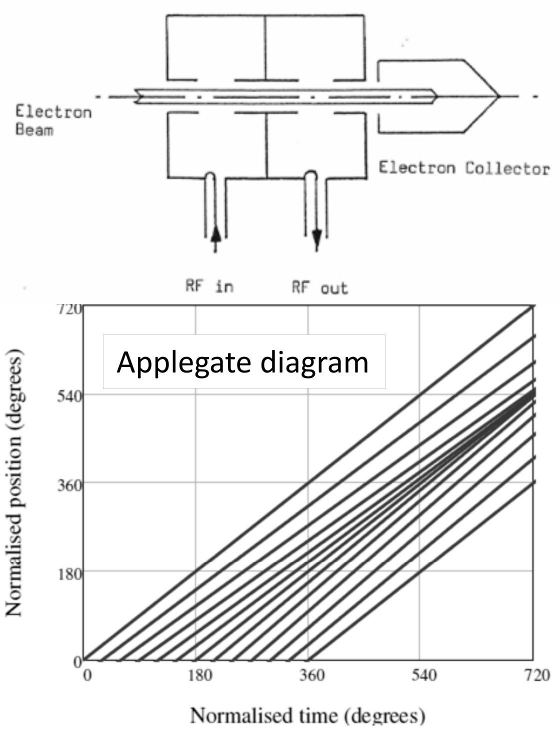
Side-coupled cavity (JHF, 972 MHz)



KLYSTRONS

Principle of a klystron

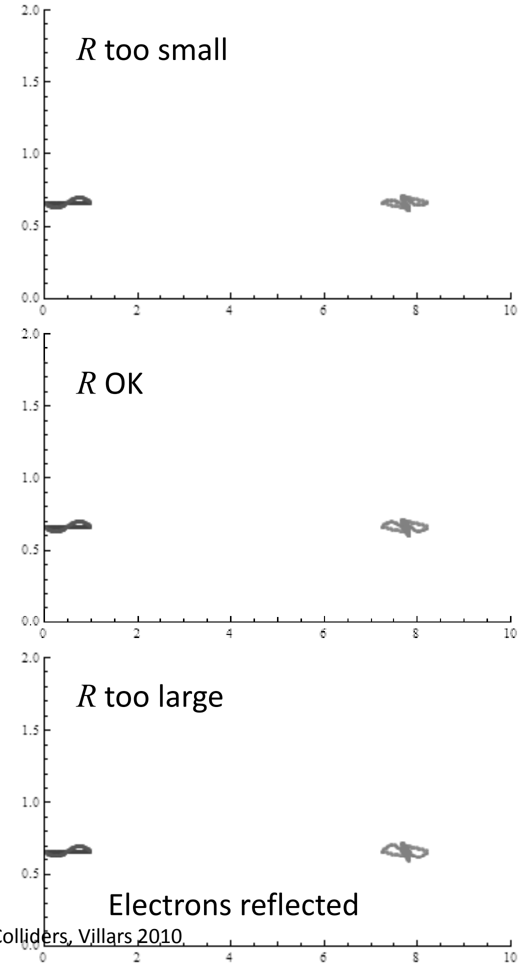
- An un-modulated electron beam passes through a cavity resonator with RF input,
- Electrons accelerated or retarded according to the phase of the gap voltage: the beam is <u>velocity modulated</u>,
- As the beam drifts downstream bunches of electrons are formed as shown in the *Applegate diagram*,
- An output cavity placed downstream extracts RF power (by its impedance)
- This is a simple <u>2-cavity klystron</u>.
- In the following animation, I simulate the output cavity as a simple RLC-circuit.
- If possible, I will present the simulation in Mathematica directly.



Thanks: Richard Carter/U Lancaster

Some simple simulation

- The simple example shown here shows the electron momenta vs. time of arrival (in rad).
- The beam enters the 1st cavity with constant momentum (0.663).
- The input voltage modulation leads to the magenta momentum distribution.
- After time 8, the beam arrives at the output cavity (green) – the velocity modulation has lead to density modulation.
- The beam then is slowed down in the output cavity; for the animation on the right the phase of the output cavity impedance is varied.

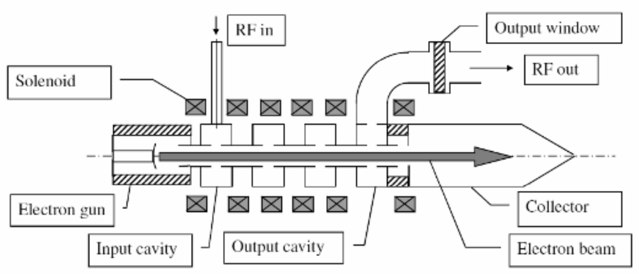


Multi-cavity klystron

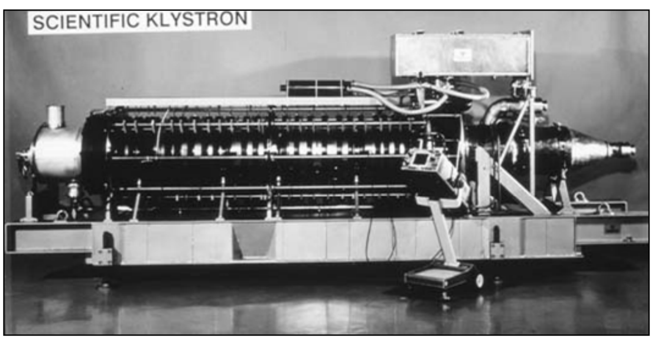
- Additional cavities are used to increase gain, efficiency and bandwidth,
- Bunches are formed by the first (N-1) cavities,
- Power is extracted by the N-th cavity
- Electron gun is a space-charge limited diode with *perveance* k given by

$$k = I_{DC} \cdot V_{DC}^{-3/2}$$

- k is typically in the range $0.3 \cdots 2 \cdot 10^{-6} \frac{A}{V^{3/2}}$.
- Beam is confined by an axial magnetic field, either with solenoids or with periodic permanent magnets (PPM)

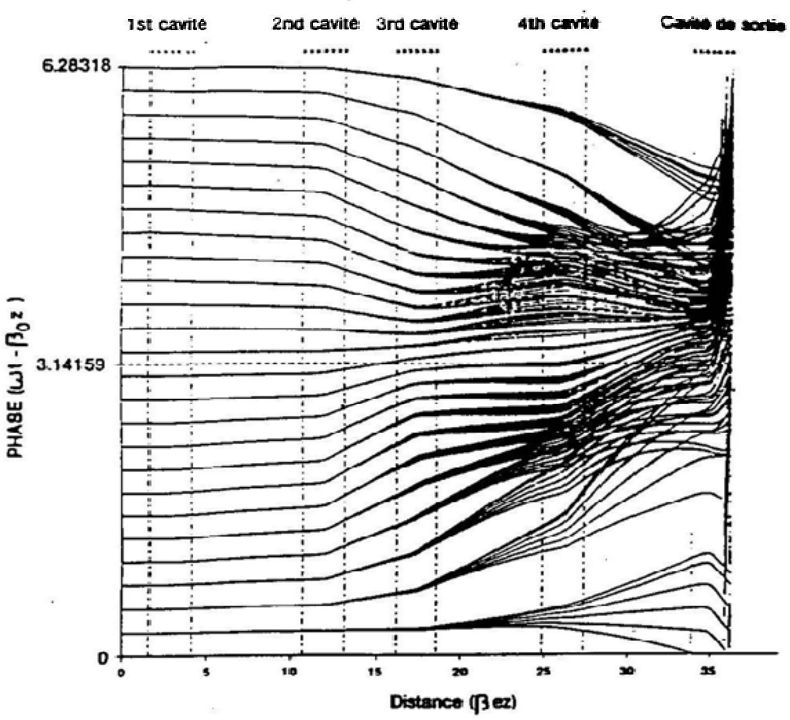


Thanks: Richard Carter/U Lancaster



LEP 1.3 MW CW klystron

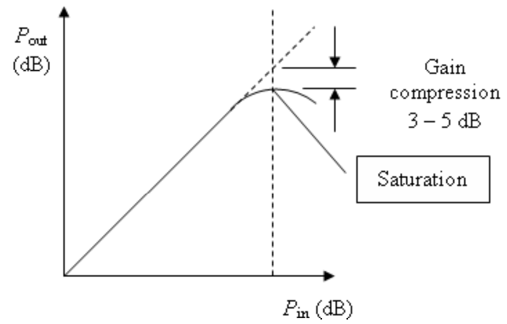
A real Applegate diagram



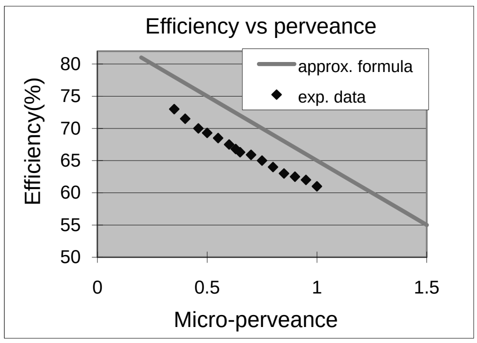
- Distance and time axes exchanged,
- Average beam velocity subtracted,
- Intermediate cavities detuned to maximise bunching,
- Cavity #3 is a second harmonic cavity,
- Space-charge repulsion in last drift section limits bunching and efficiency.
- Electrons enter output gap .

Output saturation, efficiency limits

- Non-linear effects limit the power at high drive levels and the output power saturates,
- Electrons must have residual energy > 0.1 V_0 to drift clear of the output gap and avoid reflection,
- RF beam current (I_1) increases as bunch length decreases.
  - Theoretical maximum $I_1 = 2 \cdot I_{DC}$ (Dirac δ) when space-charge is low,
  - Maximum I_1 decreases with increasing space charge.
- Higher harmonic cavities used to inprove bunching,
- Maximum possible efficiency with harmonic cavity is approximately $\eta_{el} \approx 0.85 - 0.2 \cdot 10^6\, k$.
- Efficiency decreases with increasing frequency because of increased losses and design trade-offs.



Thanks: Richard Carter/U Lancaster



Thanks: Shigeki Fukuda/KEK

Effect of output (mis-)match

- Reflected power changes the effective impedance of the output cavity and thus the amplitude and/or phase of the output gap voltage,
- The *Rieke* diagram shows output power as a function of match at the output flange,
- Shaded regions forbidden because of voltage breakdown and/or electron reflection,
- Output mismatch can also cause:
  - Output window failure
  - Output waveguide arcs
- A Circulator is needed to protect against reflected power

Rieke diagram

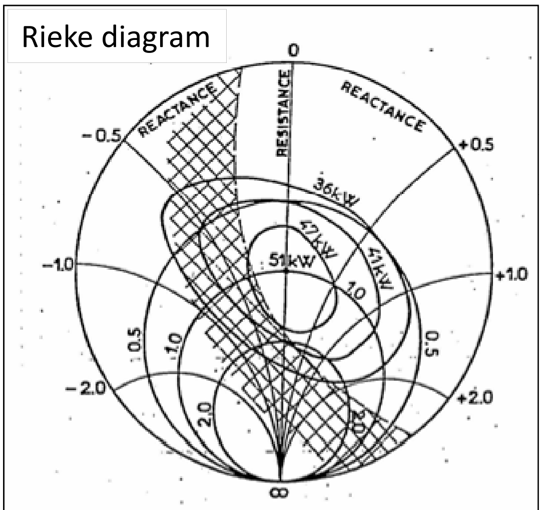
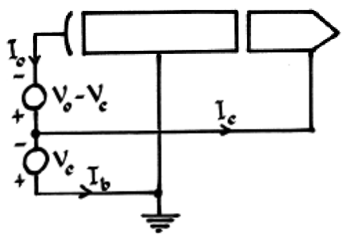
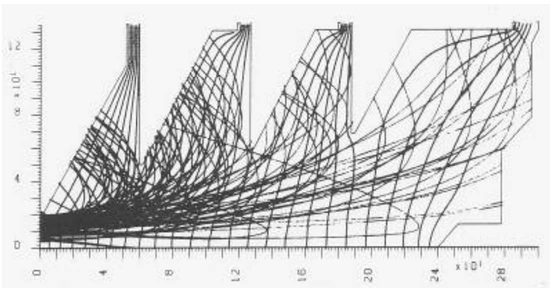


Image courtesy of Thales Electron Devices,
Thanks to Richard Carter/U Lancaster.

Depressed collector

- To increase electronic efficiency, one can send the unspent beam against a decelerating voltage to recuperate its energy.





Thanks: Richard Carter/U Lancaster

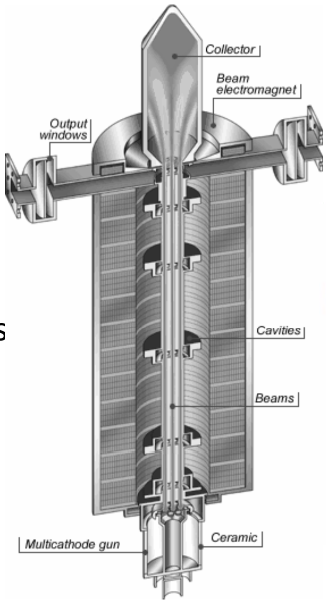
$$P_{DC} = I_C\left(V_0 - V_C\right) + I_b V_0 = I_0 V_0 - I_C V_C$$

$$\eta = \frac{P_{RF}}{I_0 V_0 - I_C V_C}$$

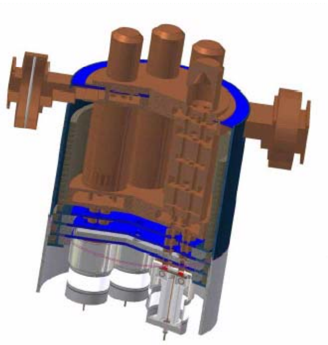
- Efficiency increases with number of stages: realistic maximum is 4 – 5 stages
- Adds to the complexity and cost of the tube!

Multi-beam klystron

- For high efficiency and high power, high voltage is desirable; very high voltages are difficult to handle! (Say < 150 kV is OK)
- High efficiency requires small perveance, which means (with given beam voltage) low current - this limits the power!
- Splitting the beam in many beamlets, the effect of space charge forces can be kept small.
- With n beamlets, the effective perveance is $k_{eff} = k/n$.
- The idea can be extended to multi-multi beam klystrons.

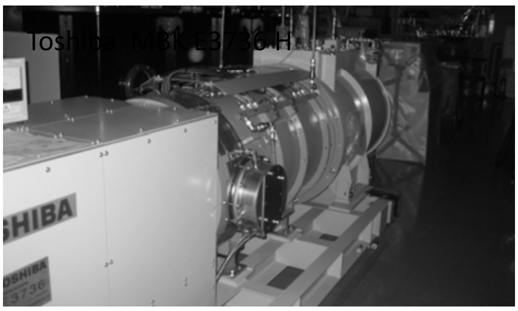


Thales 7 beam MBK,
1.3 GHz, 10 MW, 1.4 ms,
η = 65 %, 115 kV



6x6 beam MMBK idea by Shigeki Fukuda/KEK

The state of the art – ILC/X-FEL MBKs



Measured data:

- CPI: VKL-8301B (6 beam): 1.3 GHz, 10.2 MW, 1.4 ms, 10 Hz, 66.3 %, 49.3 dB gain
- Thales: TH 1801 (7 beam): 1.3 GHz, 10.1 MW, 1.4 ms, 10 Hz, 63 %, 48 dB gain
- Toshiba: E3736 (6 beam): 1.3 GHz, 10.4 MW, 1.4 ms, 10 Hz, 66 %, 49 dB gain

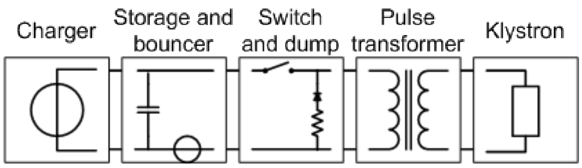
PULSE COMPRESSION

Pulse Compression

- Often one needs very short RF pulses of very high peak power.
- This is particularly true for normal-conducting RF, where energy cannot be stored economically over a long time.
- To obtain short RF pulses, many different techniques of compression have been developed – many of them are used simultaneously.
- Different methods of compression:
  - Before the RF: Klystron supply: the modulator
  - RF pulse compression
  - compression methods involving the (drive) beam

Classic modulator

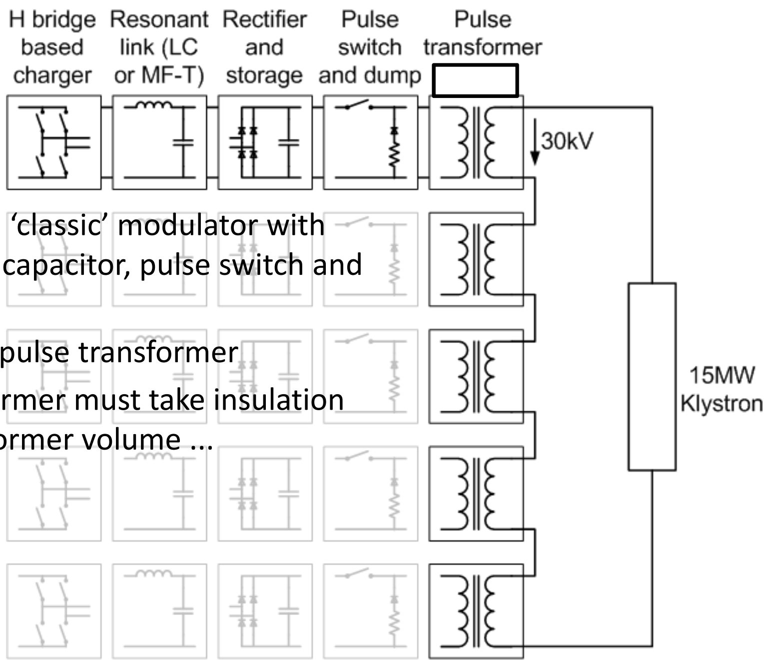
- Charger: Classical resonant topology for charging the capacitor.
- Storage capacitor: The pulsed power is collected by an intermediate storage capacitor before being transmitted through the switch.
- Bouncer – voltage droop compensation: Voltage compensator for the droop occurring in the storage capacitor during the pulse discharge.
- Switch: High voltage, high current solid state switch.
- Pulse transformer: The pulse is generated at high current lower voltage at the primary side of the pulse transformer. 150 kV is reached at the secondary side.



Thanks: David Nisbet/CERN

- Compression: The charger supply charges the capacitor slowly – the klystron extracts the stored energy rapidly.

Modular modulator

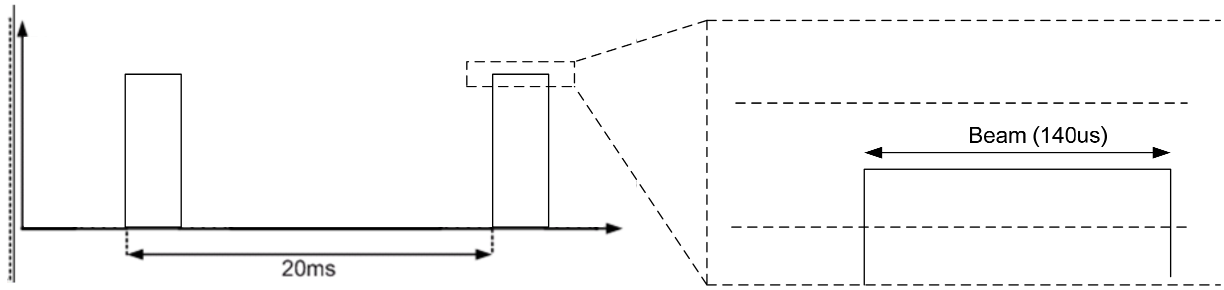


- Modular approach based on a 'classic' modulator with charger, intermediate storage capacitor, pulse switch and pulse transformer.
- Need to design a fast enough pulse transformer
- Dimensioning of pulse transformer must take insulation voltage into account -> transformer volume ...

Thanks: David Nisbet/CERN

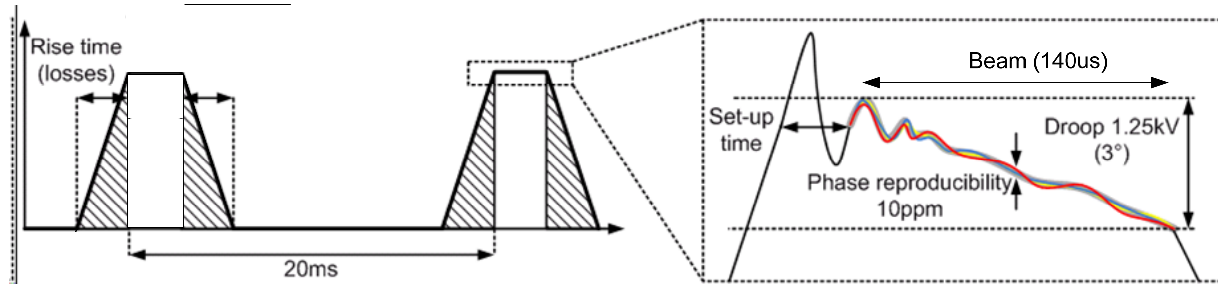
Modulators in real life:

The perfect pulse



Thanks: David Nisbet/CERN

The real world



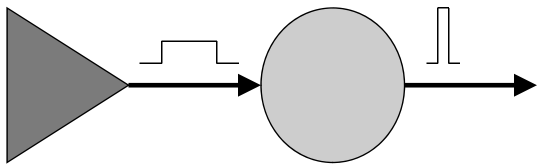
A modern modulator

ScandiNova's K2-SYSTEM for PSI; 351kV / 416A

RF Pulse compression

Why and when do we need RF pulse compression?

- For the same average power it is easier to build RF power amplifier (klystron) with moderate peak power and long RF pulses than the device with short pulses and high peak power. The same is true for the klystron modulator. On the other hand, NC accelerating structures normally need relatively short RF pulses, much shorter (factor 3-10) shorter than a convenient klystron pulse.

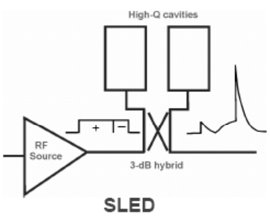


- RF pulse compression is a technique which allows to **increase the peak RF power** at the expense of **RF pulse length reduction**. The method has a limited efficiency. First due to the ohmic losses in the RF storage element and second, if the system parameters are kept constant during the pulse (passive method), because of the transient processes in a system which leads to some reflection.

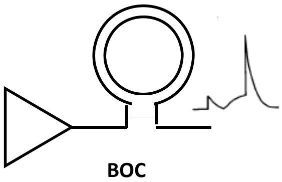
The "Zoo" of different RF pulse compression schemes

Classified by RF energy storage element(s):

Cavity(ies)

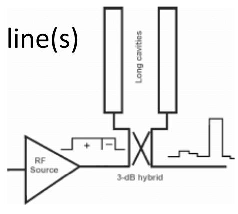


SLED

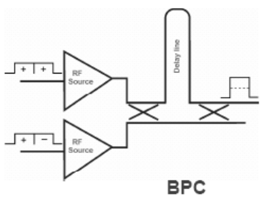


BOC

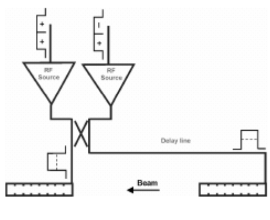
Delay line(s)



SLED-II



BPC



DLDS

Classified by operational regime:

- **Active:** The properties of the RF storage network (coupling/phase) is controlled externally. It allows to increase efficiency.
- **Passive:** The RF network is LTI – compression by phase/amplitude modulation of the RF power source

Remarks:
Concerning active systems: After decade of development, the powerful enough active elements are still not available.

Concerning delay lines:
The compressed pulse length is proportional to the length of the delay line. At low frequencies/long pulses, SLED-II, BPC and DLDS become impractically long.

Pulse compression history (1)

1973

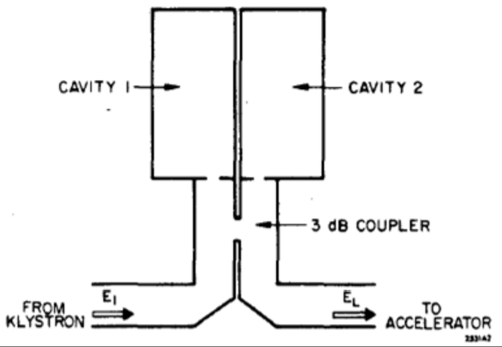
SLAC-TN-73-15
P. B. Wilson
December 1973

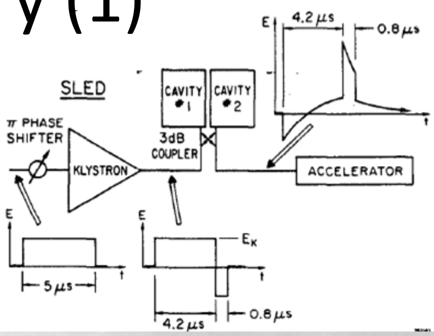
SLED:

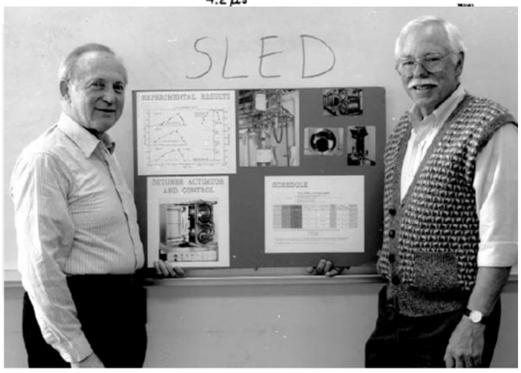
A Method for Doubling SLAC's Energy

In the course of making measurements on superconducting cavities, it is a common observation that the power radiated from a cavity that is heavily overcoupled approaches four times the incident generator power immediately after the generator has been switched off.

Normally this radiated power travels as a reverse wave back toward the generator. There are, however, several microwave networks which can direct this radiated power into an external load; for instance, two identical cavities attached to a 3-db coupler, as shown in Fig. 1.







David Farkas (left) and Perry Wilson received a 1991 Institute of Electrical and Electronics Engineers (IEEE) Particle Accelerator Technology prize for their invention and implementation of the SLAC Energy Development (SLED) radio-frequency pulse compression system. SLED boosts klystron peak power, increasing the accelerator gradient.

Pulse compression history (2)

1985

Table 1

Diameter	:	44.35 cm
Length	:	58.5 cm
Diameter of tuning piston	:	16.7 cm
Displacement	:	10 mm
Diameter of coupling holes	:	28.5 mm
Distance from cylinder axis	:	39.7 mm
Working frequency	:	2998.55 MHz
Resonance type	:	H 0 3 8
Cavity Q - theoretical	:	207000
measured	:	180000
Coupling factor β	:	9...11

CERN/PS 87-45 (RF)
March 1987

DESIGN CONSIDERATIONS, CONSTRUCTION AND PERFORMANCE OF A SLED-TYPE RADIOFREQUENCY PULSE COMPRESSOR USING VERY HIGH Q CYLINDRICAL CAVITIES

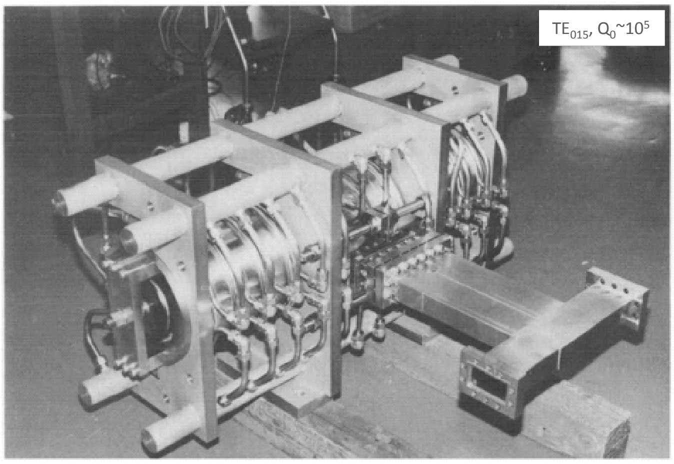
A. Fiebig, R. Hohbach, P. Marchand and J.O. Pearce

LIPS ("LEP Injector Power Saver"), still in operation today in CTF3

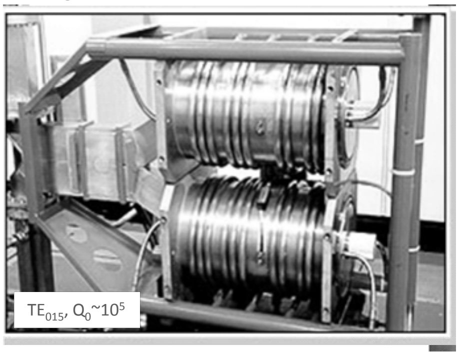
Pulse compression history (3)

1992

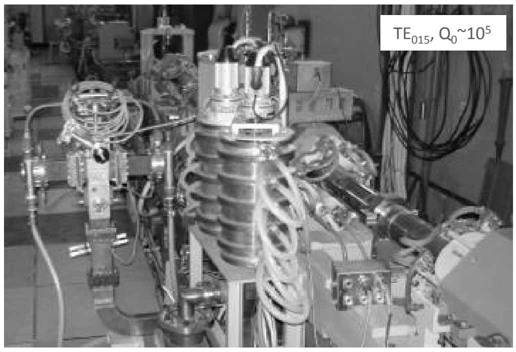
KEK, ATF&KEKB, Japan 1992.



Pohang Accelerator Lab., Korea, ~1994



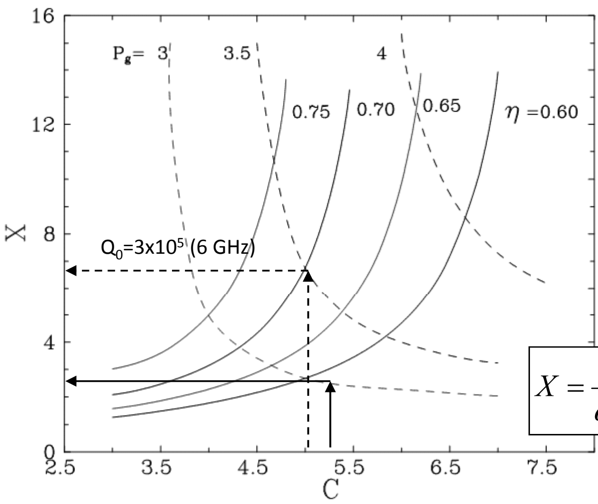
BINP, Novosibirsk, Russia, ~ 2000



Over the last 35 year,s hundreds of S-band SLED type pulse compressors are successfully built worldwide – many are still in operation today.

The limit of SLED type pulse compressors

SLED power gain/efficiency plot



C: temporal compression,
P_g: power gain
η: target efficiency (specified),

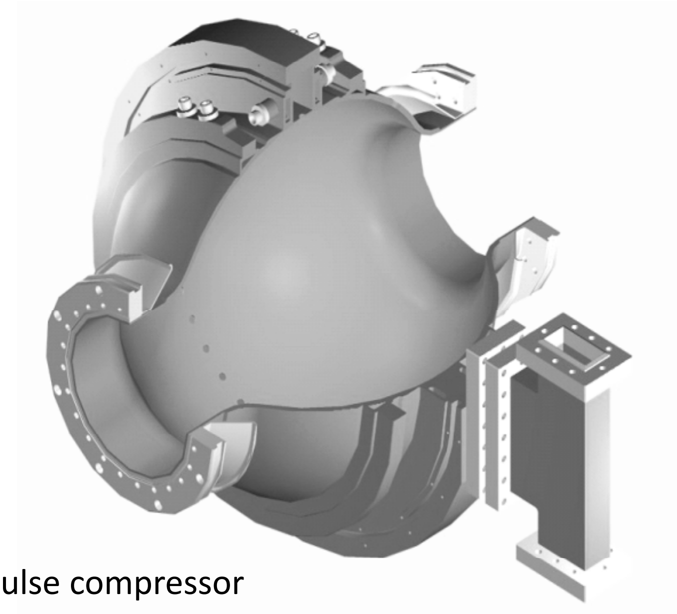
$$P_g = C\,\eta$$

X: damping time in units of the klystron pulse length – a measure for the necessary Q_0

$$X = \frac{2Q_0}{\omega T_{KL}} \qquad C = \frac{T_{KL}}{T_{out}}$$

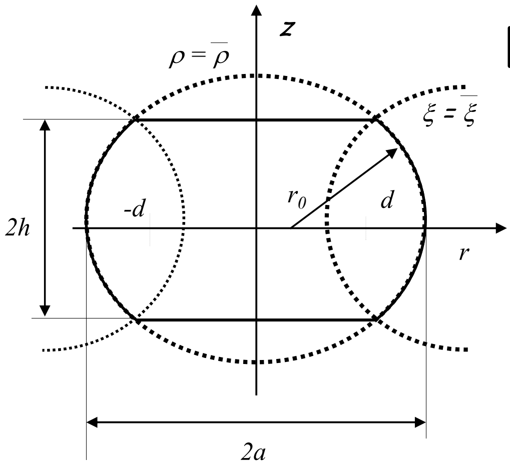
Exercise:
F=6 GHz, T_{KL}=2500 ns, T_{out} = 500 ns (C=5), target efficiency 70% (Pg=3.5)
X=6.7. To satisfy, the cavity unloaded Q-factor should be: 3.2×10^5

SLED type PC – the next generation: BOC



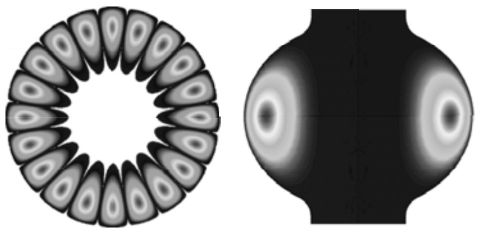
Barrel
Open
Cavity RF pulse compressor

Barrel cavity: some theory



Cavity profile: $z = \sqrt{a r_0 \left\{ 1 - \left(\frac{r}{a} \right)^2 \right\}}$

Whispering gallery mode:



The eigenfrequency of the barrel cavity with TM_{mnq} oscillation is the solution of:

$$ka = \nu_{mn} + \frac{(q - 1/2)\alpha}{\sin\theta}$$

ν_{mn} is a root of the Bessel function; for large m it can be approximated as:

$$\nu^{\rho}_{mn} = m - \mu t_n^0 \qquad (n = 1, 2, \ldots),$$

$$-t_n^0 = [(n - 0.25)1.5\pi]^{2/3}, \qquad \mu = \left(\frac{m}{2}\right)^{1/3}.$$

The optimal radius r_0 is reached when the external caustic has the smallest height, it is given by : $r_0 = 2a\sin^2\theta$
where α and θ are derived from:

$$\sin\alpha = \sqrt{\frac{a}{r_0}}\sin\theta \qquad \cos\theta = \frac{m}{\nu_{mn}}$$

Finally the height of the external caustic and Q-factor of the cavity are:

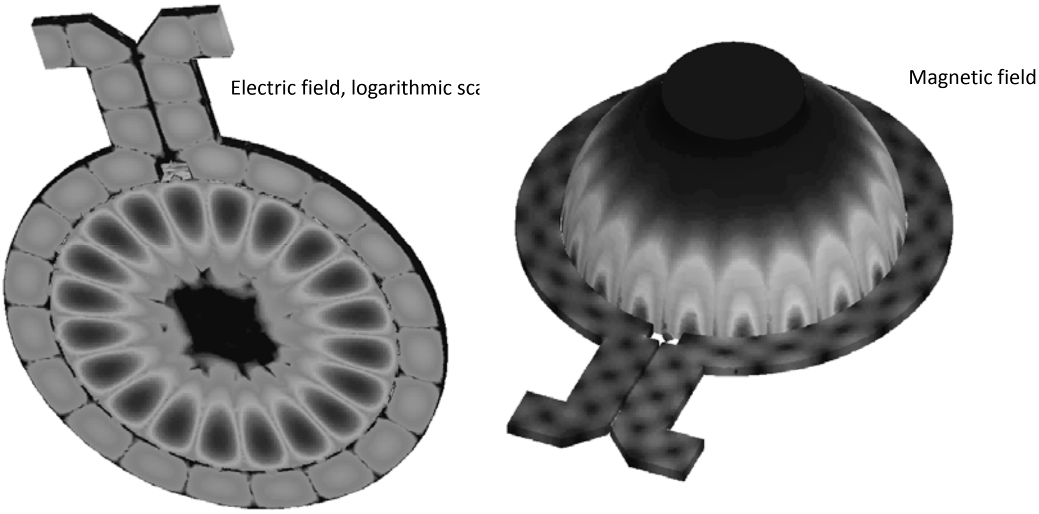
$$z_{q-1} = 2\sqrt{(q - 1/2)\frac{a\sin\theta}{k\sin 2\alpha}}$$

$$\boxed{Q_E = \frac{a}{\sigma_s}}$$

for Copper:

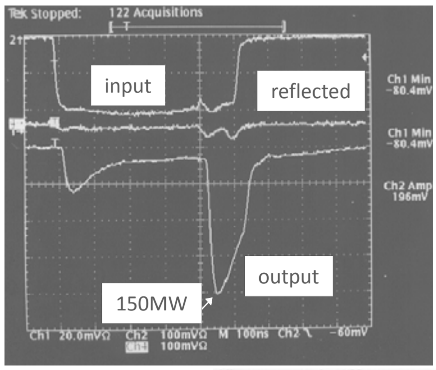
$$a = Q_0\sqrt{\lambda}\; 4\times10^{-6}$$

BOC operating mode ($TM_{10,1,1}$)

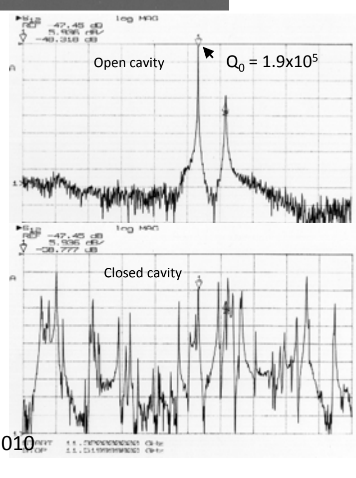


BOC exploits the properties of the cavity operating mode and in particular operates in the regime of a resonant **rotating wave**. This is implemented by exciting the mode through many coupling holes in the common wall between the cavity and waveguide feeder. The waveguide width is chosen so that the angular phase velocities in both the waveguide and the cavity are equal. To provide best matching, the distance between coupling holes is a quarter of a wavelength.

The concept of the BOC (originally VPM – VLEPP Power Multiplier) was proposed in 1990 (Balakin, Syratchev). In 1994 the fist X-band VPM was tested at KEK.



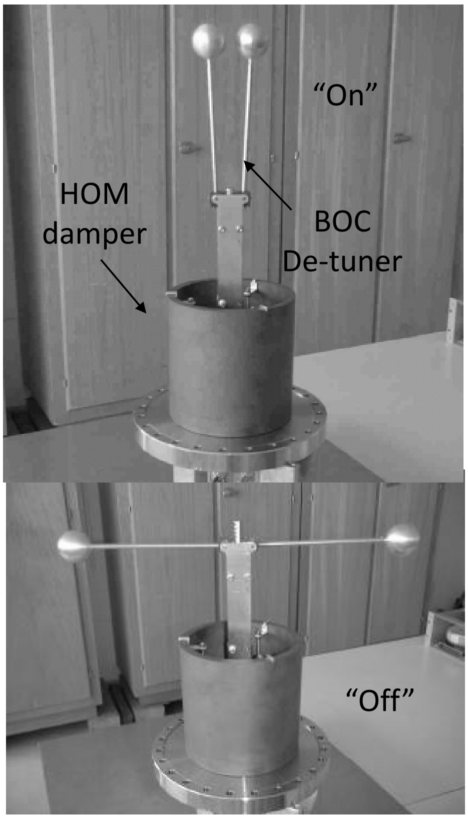
Test cavity ($TM_{25,1,1}$)
Diameter 0.263 m

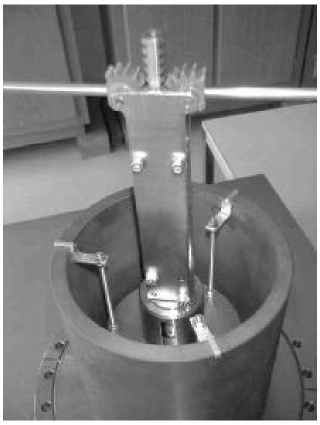


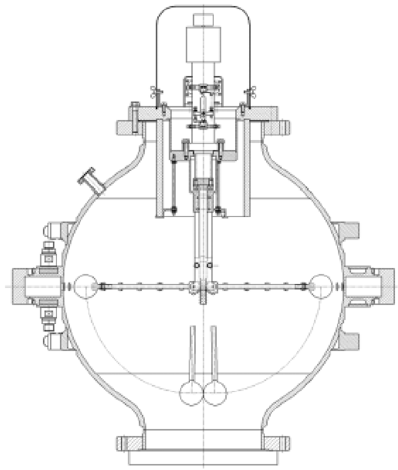
3 GHz BOC RF pulse compressor for CTF3 (2000)

Internal view
of coupling holes

BOC detuning and HOM damping

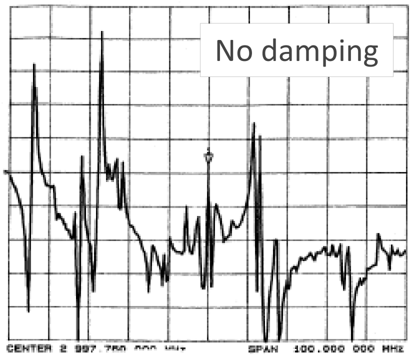


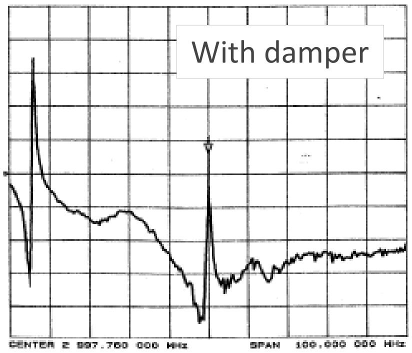




BOC characteristics

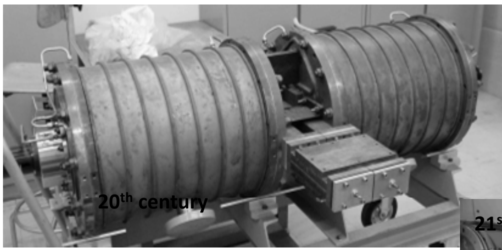
BOC spectra (span 100 MHz)

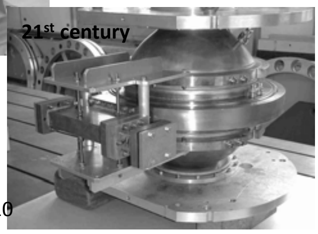




CTF3 BOC parameters:
Operating mode: rotating $TM_{10,1,1}$
Cavity diameter: 502 mm
Cavity wall curvature: 239 mm
Waveguide width: 62 mm
Coupling holes: 39+1, Ø13.8 mm

Cavity frequency: 2.99855 GHz
(at temperature 24.2 ^{0}C)
Unloaded Q- factor: 185 000
(~ 95% theoretical value)
Coupling factor: 6.5





Variations of BOC

Microwave Pulses Compressed in a Barrel-Shaped Resonator with Screw Corrugation

Yu. Yu. Danilov, S. V. Kuzikov, V. G. Pavel'ev, and Yu. I. Koshurinov
Institute of Applied Physics, Russian Academy of Sciences, Nizhni Novgorod, Russia
e-mail: danilov@appl.sci-nnov.ru
Received July 7, 2000

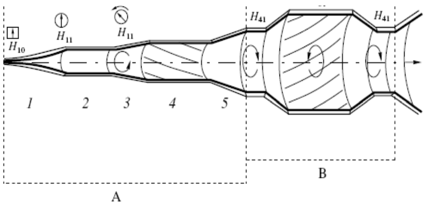


Fig. 1. Schematic diagram of the compressor: (A) mode converter; (B) circular resonator (see the text for explanations).

developed with GYCOM,
Nizhny Novgorod, Russia

Whispering Gallery Pulse Compressor

J.Hirshfield[1], S.V.Kuzikov[2], M.I.Petelin[2], V.G.Pavelyev[3]

[1]Omega-P, New Haven, [2]Institute of Applied Physics, [3]University of Nizhny Novgorod, Russia, [2]Institute of Applied Physics, Nizhny Novgorod, Russia, [3]University of Nizhny Novgorod, Russia

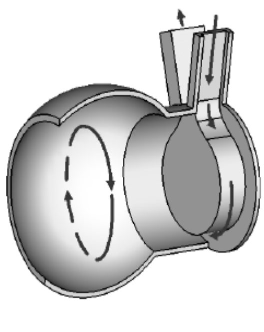
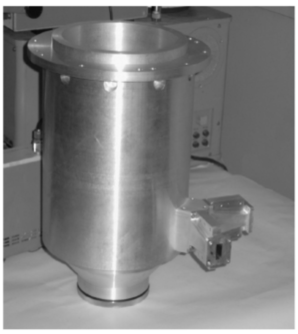
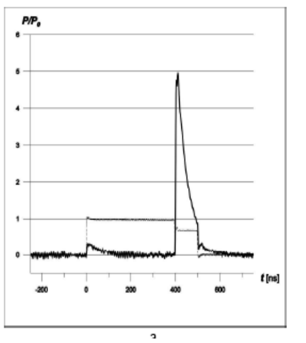
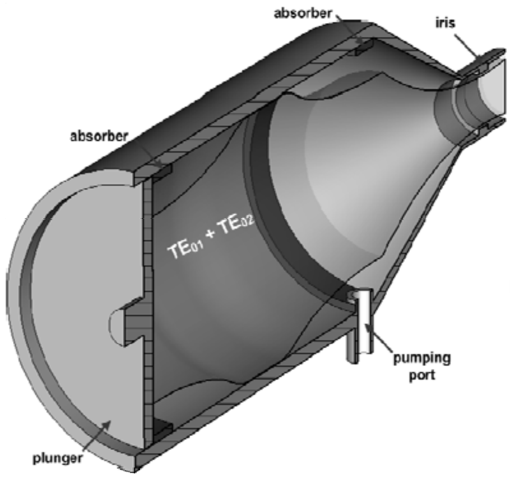


FIGURE 1. Tunnel-feed whispering-gallery pulse compressor.

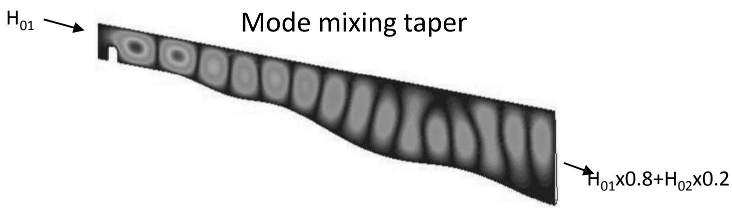




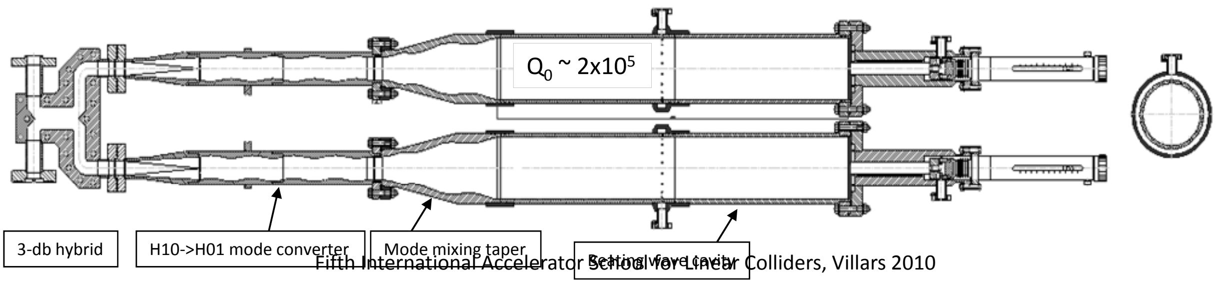
3^{rd} generation of the SLED type pulse compressor with beating modes in a cavity (BMC). 2008 (GYCOM, Russia).



This approach allows for the very high Q-factor values (big volume), while the spurious modes are sufficiently damped. It uses a controlled mixture of modes.



The 12 GHz version of the BMC have been ordered by CERN.



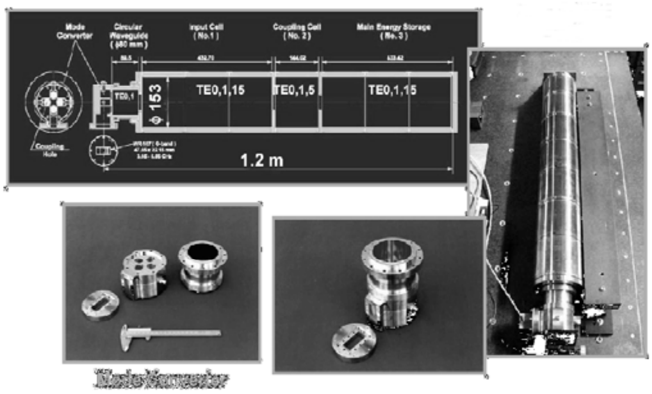
Coupled cavities for pulse flattening

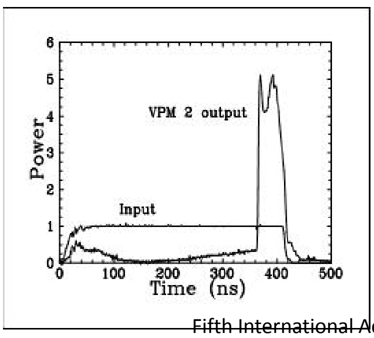
Syratchev, VLEPP 1991

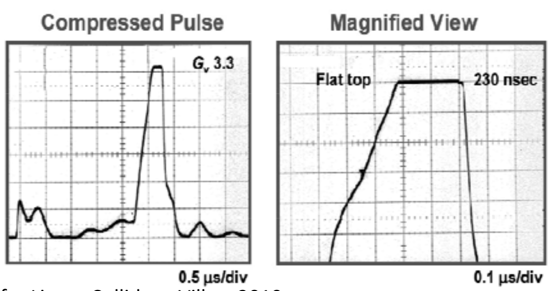
Proceedings of APAC 2004, Gyeongju, Korea

THE C-BAND (5712-MHZ) RF SYSTEM FOR e^+e^- LINEAR COLLIDER

H. Matsumoto†, Shigeru Takeda, S. S. Win, M. Yoshida, KEK, Tsukuba Japan
H. Baba, T. Shintake, SCSS Group, RIKEN, Harima Japan





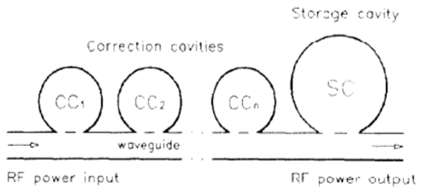


Chain of cavities

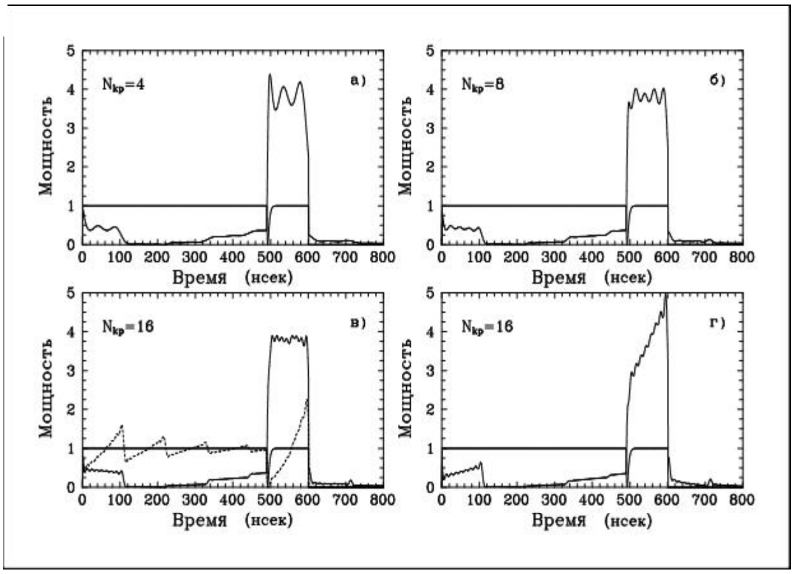
Pulse Shape Correction for RF Pulse Compression System

S.Y.Kazakov
142284, Branch INP, Protvino, Russia

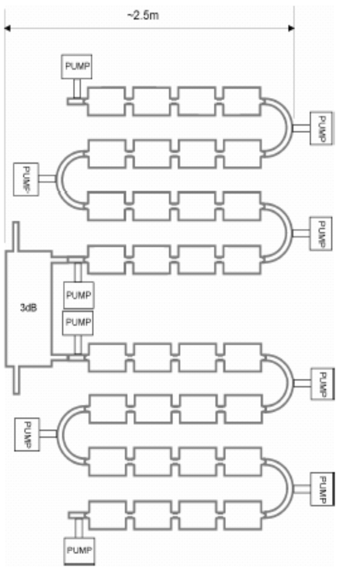
EPAC 92

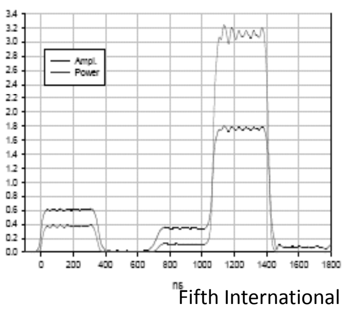


The chain of individual cavities coupled to the common waveguide can mimic the portion of the spectrum of the long delay line! The advantage is that the Q factor/size of the Corrector Cavities (CC) can be ~10 times smaller than that of Storage Cavity (SC).



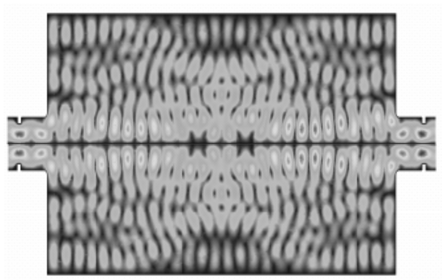
240 ns, 16 Cells SLED II



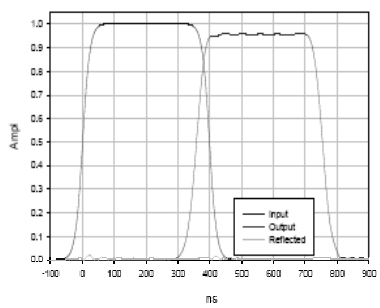


Even more…(S. Kazakov)

X-band Single multi-moded delay cell



400 ns, 32 cells (21.33 ns/m)



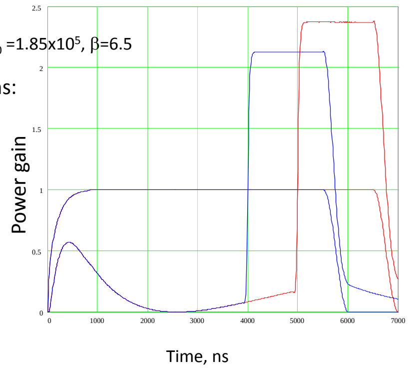
The length of the delay line build of these cells will be 6.4 times shorter compared to the circular waveguide!

How to make SLED pulses flat? (1)

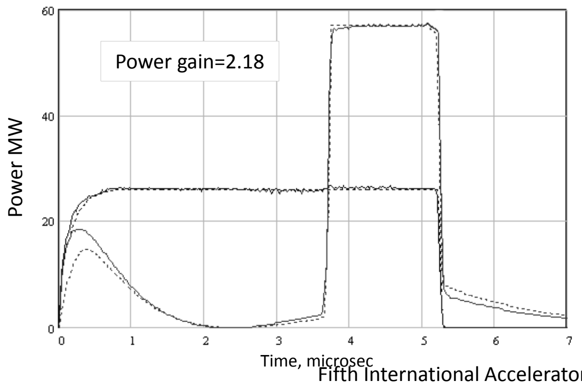
Method one: RF phase modulation

Cavity: Q_0 =1.85x10^5, β=6.5

Simulations:

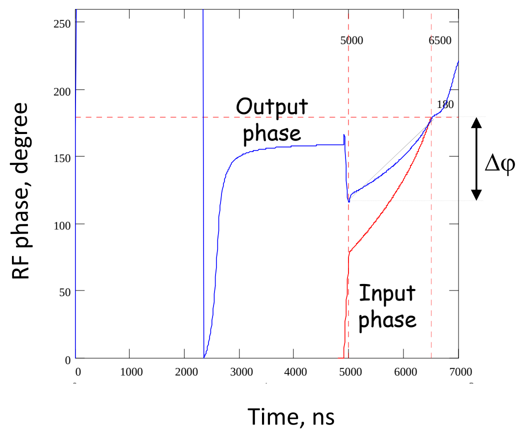


BOC high power tests:

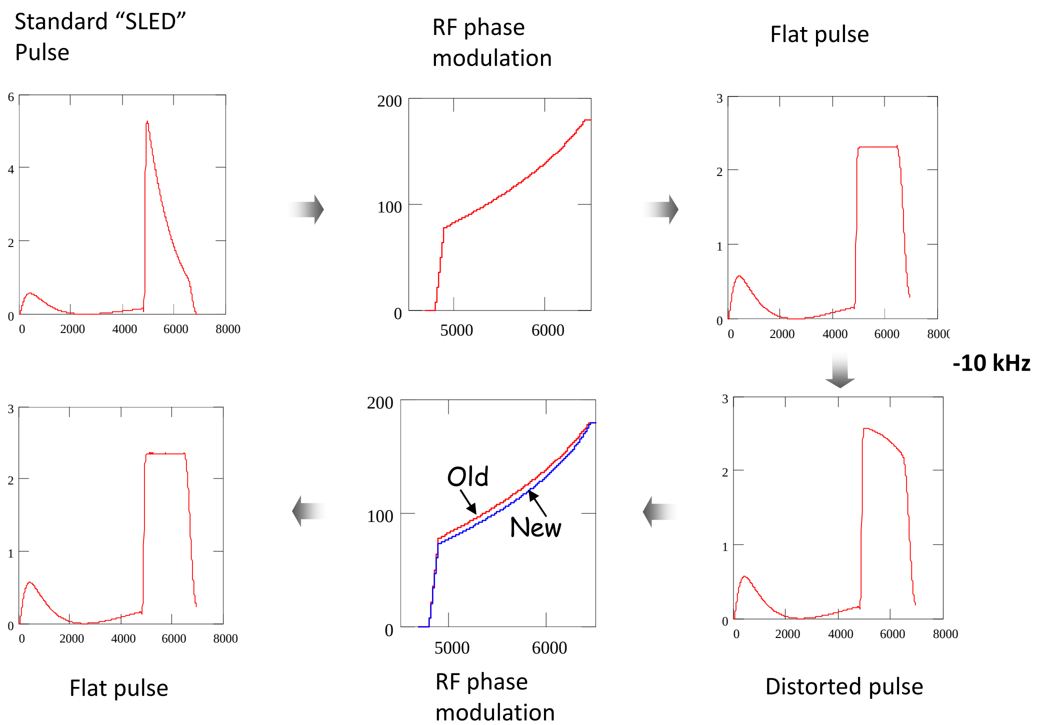


The linear part of the phase slop will be compensated with the frequency shift:

$$\pm\Delta\omega T_{out} = \pm\Delta\varphi$$

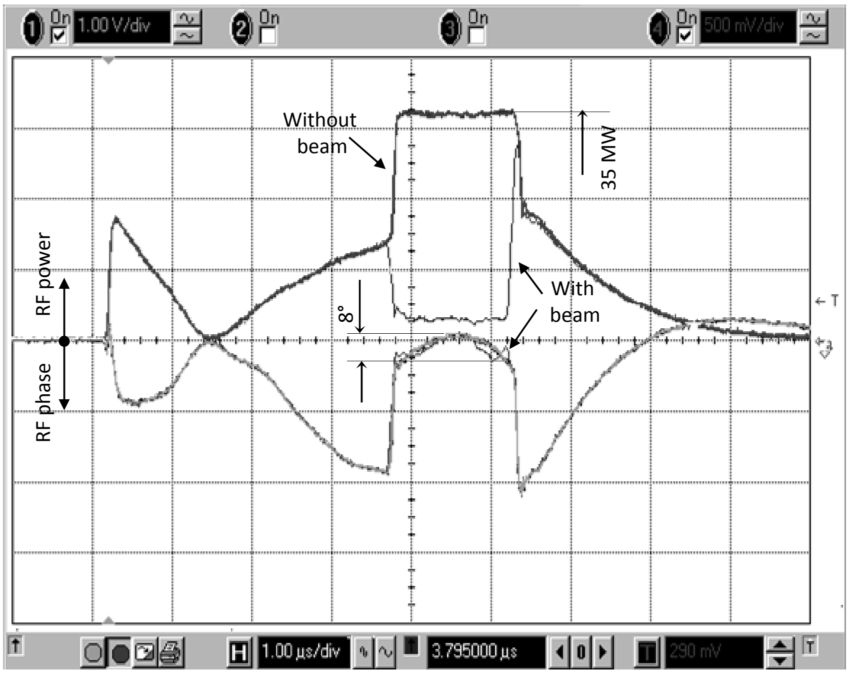


Phase modulation at work



CTF3 BOC pulse compressor

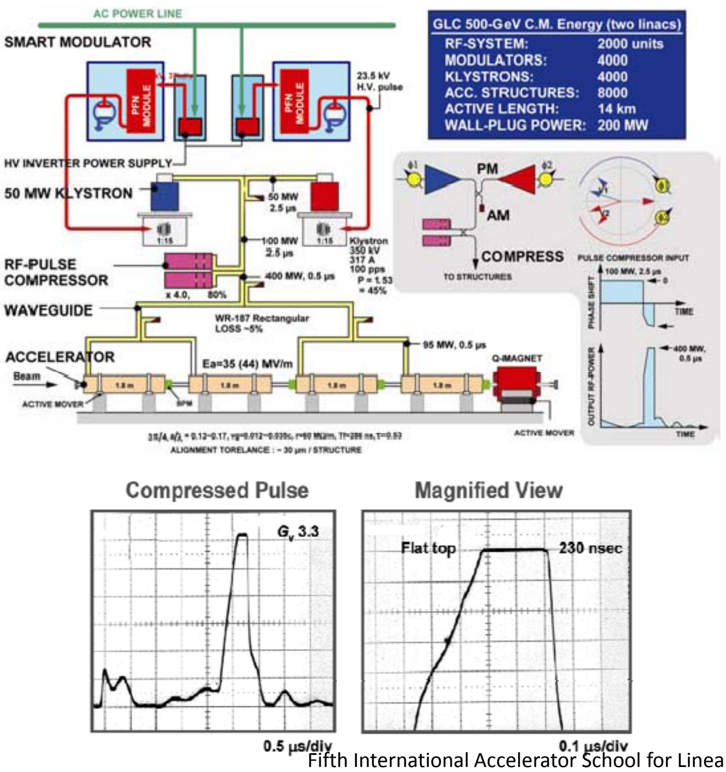
Full beam loading operation (3.5 A) with phase modulation.



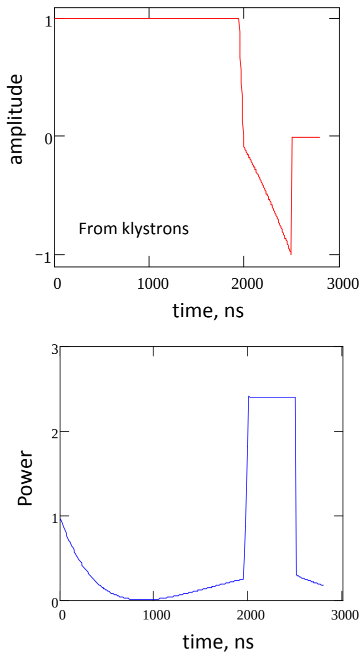
Currently ten 3 GHz RF stations at CTF3 reliably operate in this regime.

How to make SLED pulses flat? (2)

Method two: RF amplitude modulation



Single cavities



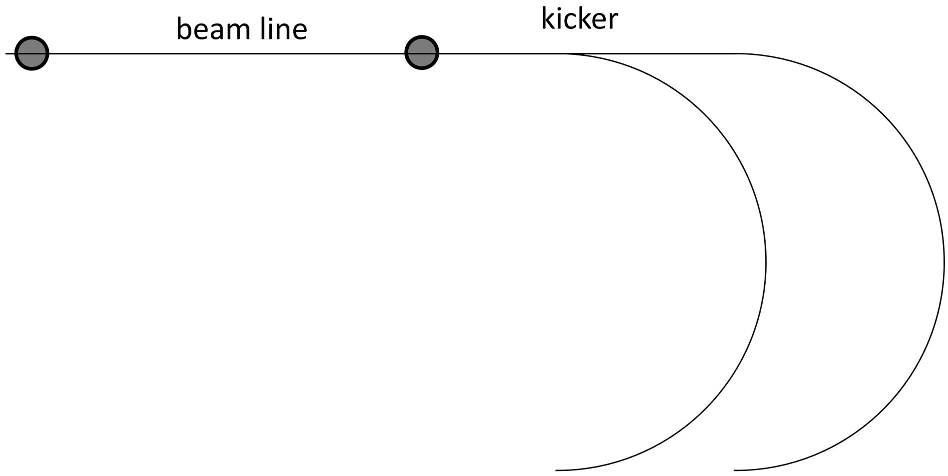
This method is used in BEPC, Beijing

Pulse compression involving beams

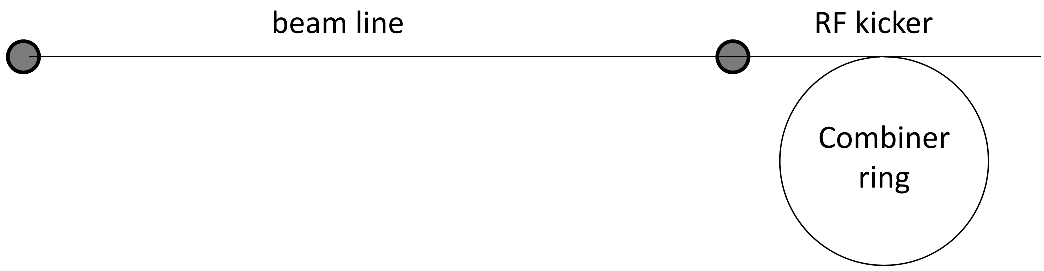
- The (passive) RF pulse compression schemes (SLED & delay lines) have limited efficiency.
- For large compression ratios these schemes can be discarded.
- Particle beams can store energy – in principle without any loss (cf. storage rings)
- Acceleration increases beam energy, deceleration decreases it.
- If acceleration and deceleration can be obtained with high efficiency, this scheme becomes attractive.
- This leads to the "**Two-beam scheme**", used e.g. in CLIC.
- CLIC compresses pulses with the drive beam by a factor 600!

Beam Compression in return lines

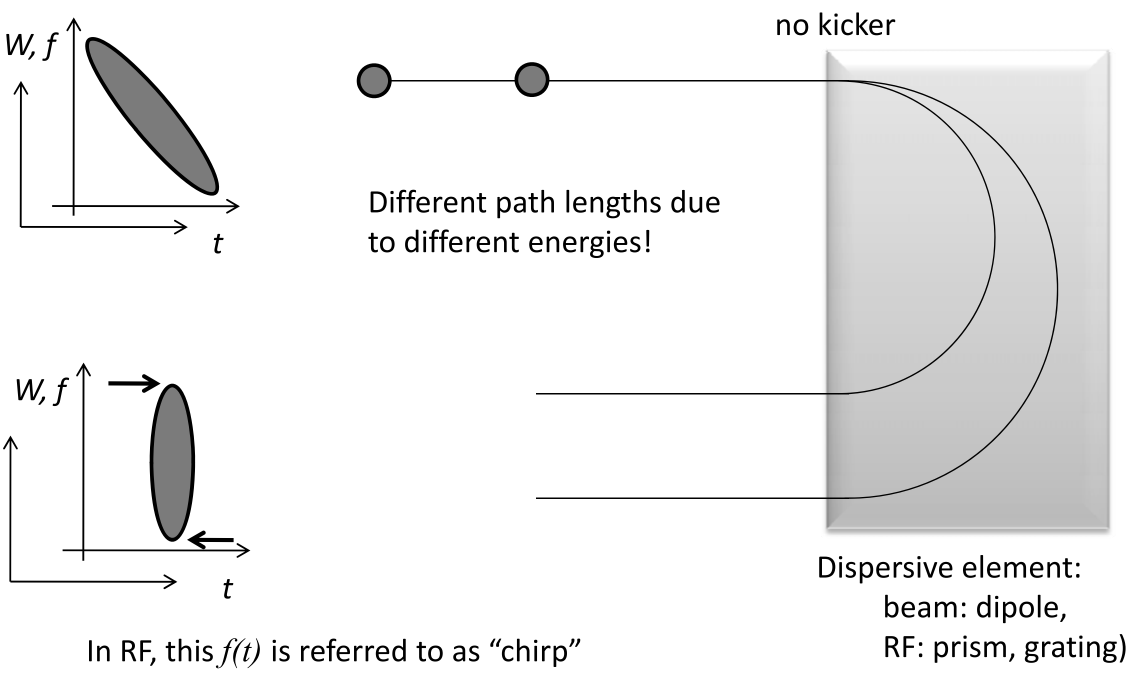
Different path lengths allow compression



Principle of a combiner ring

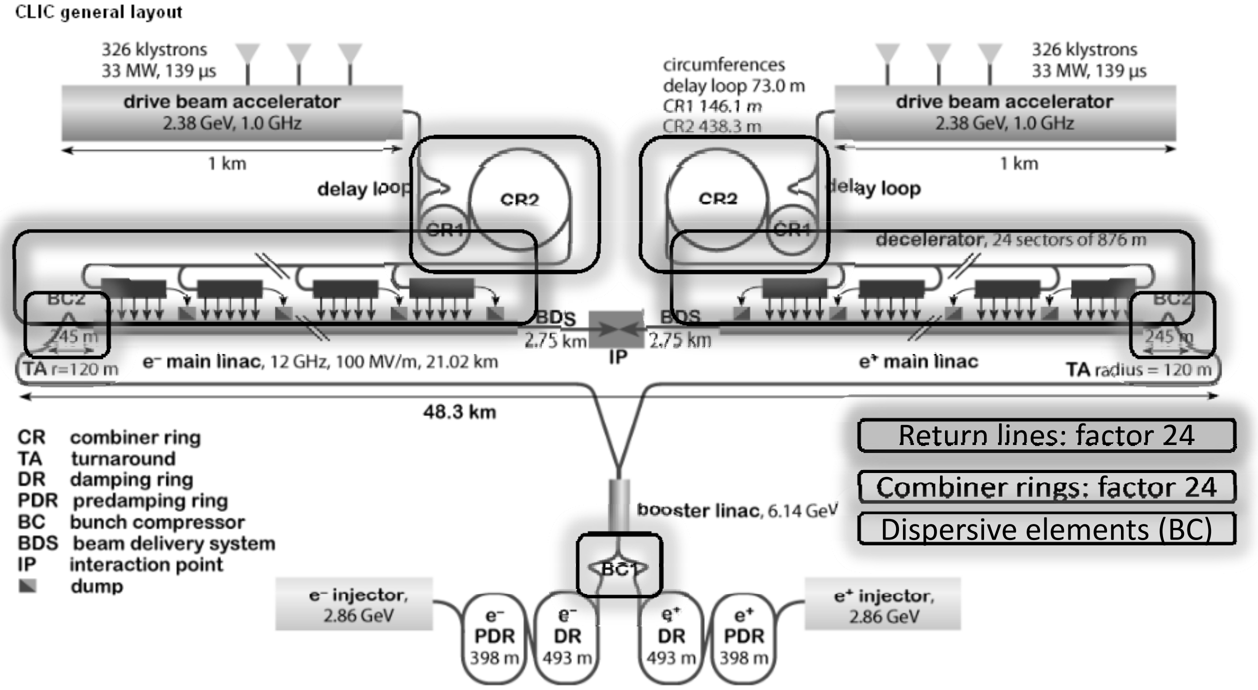


Compression via dispersive element



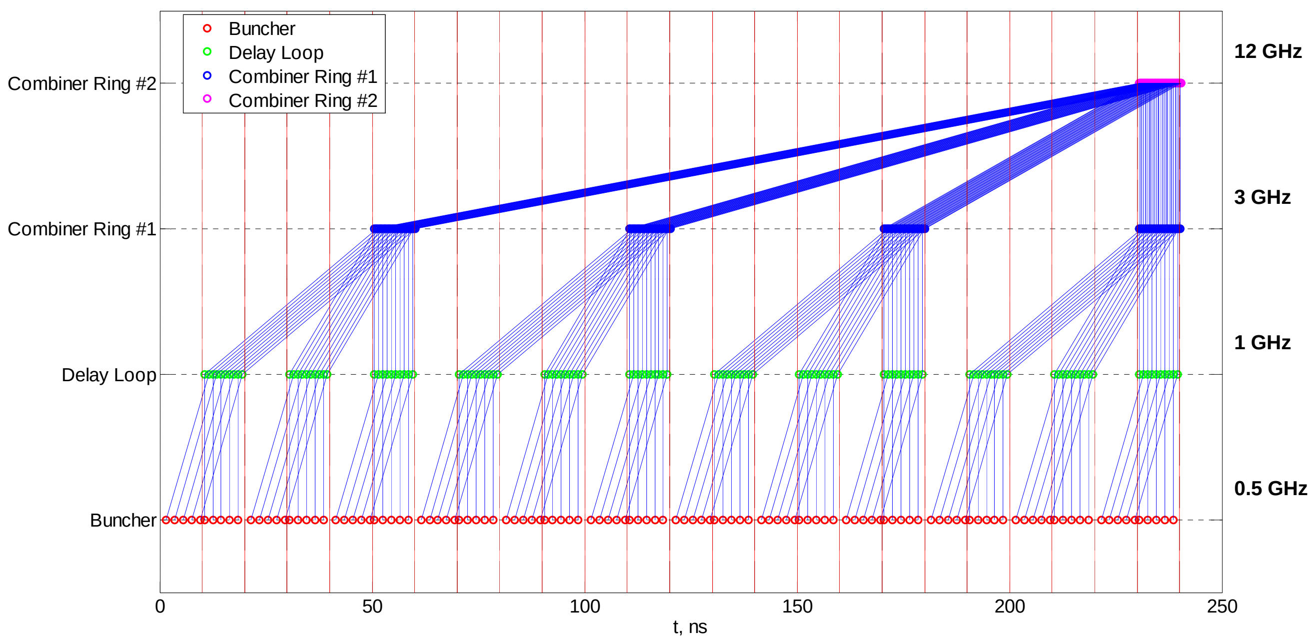
In RF, this *f(t)* is referred to as “chirp”

The CLIC 2-beam scheme



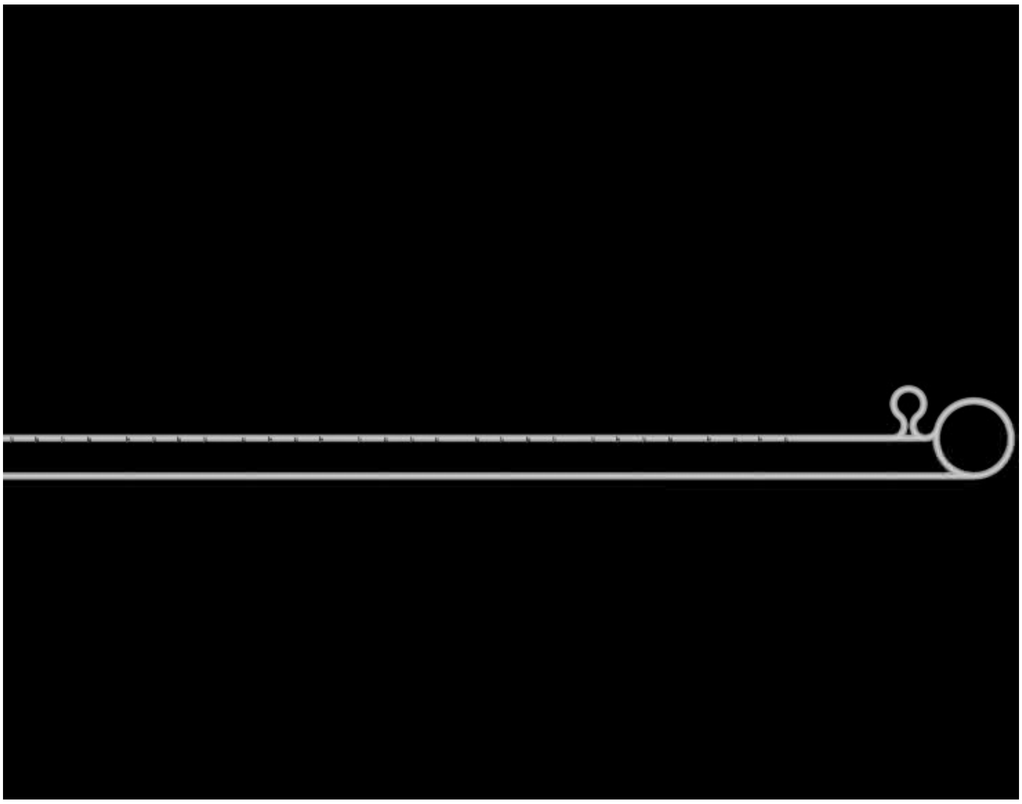
Drive Beam Combination Steps

$f_{beam} = 4 * 3 * 2 * f_{initial}$ the first factor 24



Thanks to
Oleksiy Kononenko

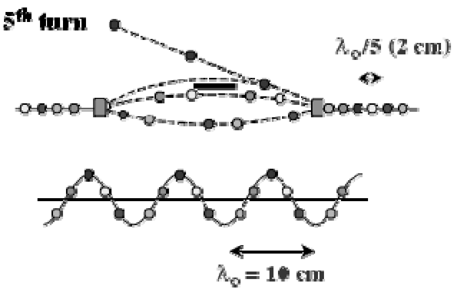
Delay loop and combiner ring animation



Animation by Alexandra Andersson/CERN

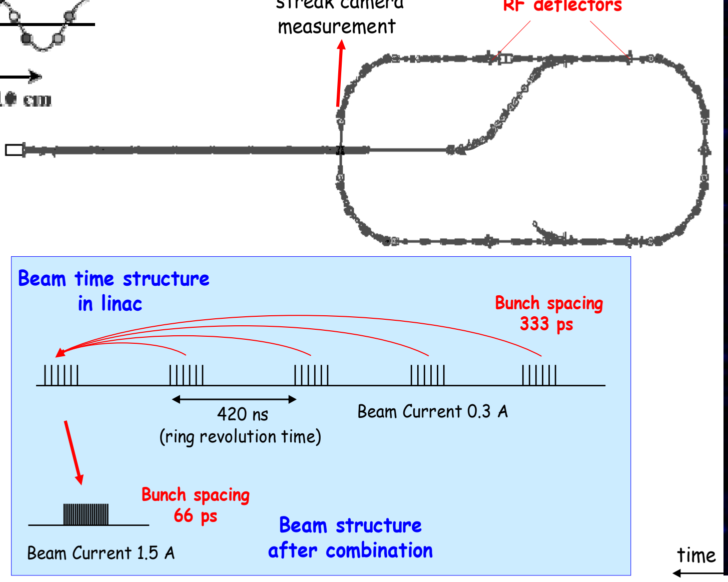
Demonstration of frequency multiplication

Combination factor 5

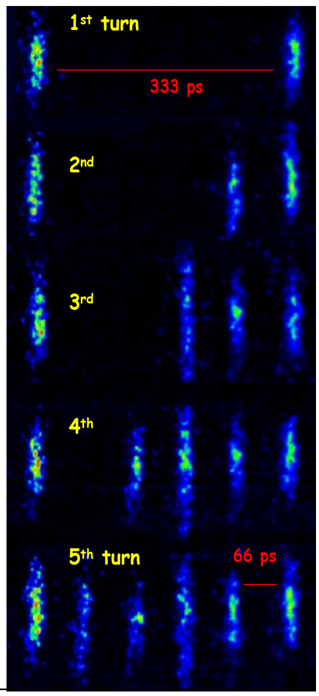


CTF3 - PRELIMINARY PHASE 2001/2002

Successful low-charge demonstration of electron pulse combination and bunch frequency multiplication by up to factor 5



Streak camera image of beam time structure evolution



Drive beam recombination tested in CTF3

- combined operation of Delay Loop and Combiner Ring (factor 8 combination)
- ~26 A combination reached, nominal 140 ns pulse length
- => **Full drive beam generation**, main goal of 2009, **achieved**