Muon Colliders

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School

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Contents

1 DEFINITIONS AND UNIT CONVENTIONS

Units

When discussing the motion of particles in magnetic fields, I will use MKS units, but this means that momentum, energy, and mass are in Joules and kilograms, rather than in the familiar 'electron Volts'. To make the conversion easy, I will introduce these quantities in the forms: $[pc/e]$, $[E/e]$, and $[mc^2/e]$, respectively. Each of these expressions are then in units of straight Volts corresponding to the values of p, E and m expressed in electron Volts. For instance, I will write, for the bending radius in a field B:

$$\rho \ = \ \frac{[pc/e]}{B\ c}$$

meaning that the radius for a 3 GeV/c particle in 5 Tesla is

$$\rho \ = \ \frac{3\ 10^9}{5\ \times\ 3\ 10^8} \ = \ 2m$$

Emittance

$$\epsilon_n \;=\; \text{normalized emittance} \;=\; \frac{\text{Phase Space Area}}{\pi \text{ m c}}$$

The phase space can be transverse: p_x vs x, p_y vs y, or longitudinal Δp_z vs z, where Δp_z and z are with respect to the moving bunch center.

If x and p_x are both Gaussian and uncorrelated, then the area is that of an upright ellipse, and:

$$\epsilon_\perp = \frac{\pi\ \sigma_{p_\perp}\sigma_x}{\pi\ mc} \;=\; (\gamma\beta_v)\sigma_\theta\sigma_x \qquad (\pi\ m\ rad) \tag{1}$$

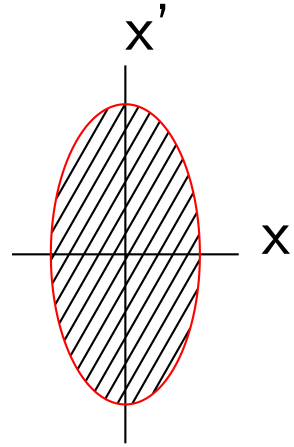
$$\epsilon_\parallel = \frac{\pi\ \sigma_{p_\parallel}\sigma_z}{\pi\ mc} \;=\; (\gamma\beta_v)\frac{\sigma_p}{p}\ \sigma_z \qquad (\pi\ m\ rad) \tag{2}$$

$$\epsilon_6 = \epsilon_\perp^2\ \ \epsilon_\parallel \qquad (\pi\ m)^3 \tag{3}$$

The subscript v on β_v indicates that $\beta_v = v/c$. The π, added to the dimension, is a reminder that the emittance is phase space/π. Un-normalize emittances $\epsilon = \sigma_\theta\sigma_x$ (without the $\beta_v\gamma$), are often used, but whenever I use an emittance, it is always assumed to be "normalized". Emittances are also sometimes quoted defining ellipses with 95% of Gaussian beams.

$$\epsilon_{95\%} \;\approx\; 6 \;\times\; \epsilon_{rms}$$

$\beta_\perp$ of Beam



For an upright phase ellipse in x' vs x,

$$\beta_\perp = \left(\frac{\text{width}}{\text{height}} \quad \text{of phase ellipse} \right) = \frac{\sigma_x}{\sigma_\theta} \tag{4}$$

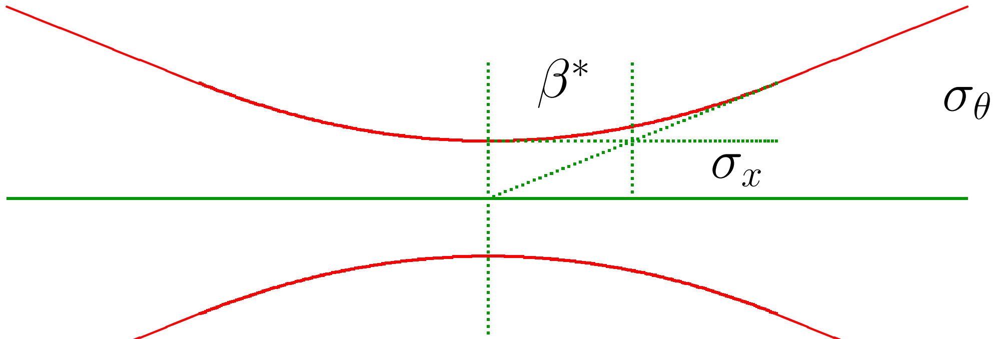
Then, using the emittance definition:

$$\sigma_x = \sqrt{\epsilon_\perp \, \beta_\perp \, \frac{1}{\beta_v \gamma}} \tag{5}$$

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp} \, \frac{1}{\beta_v \gamma}} \tag{6}$$

$\beta_{lattice}$ can also be defined for a repeating lattice, where it is that β_{beam} that is matched to the lattice. Equation 5, but not eq. 6 are valid even when the ellipse is tilted.

$\beta_\perp$ (or β^*) beam at focus



$$\sigma_x = \sigma_o \sqrt{1 + \left(\frac{z}{\beta^*}\right)^2}$$

From eqiation 5

$$\beta_x = \beta^* \left(1 + \left(\frac{z}{\beta^*}\right)^2\right)$$

β^* is like a depth of focus

As $z \to \infty$

$$\sigma_x \to \frac{\sigma_o \; z}{\beta^*}$$

giving an angular spread of

$$\theta = \frac{\sigma_o}{\beta^*}$$

as above in eq.4

$\beta_\perp$ of a Lattice

$\beta_\perp$ above was defined by the beam, but a lattice can have a β_o that may or may not "match" the $\beta_\perp$ of the beam.

e,g. if a continuous inward focusing force, then there is a PERIODIC solution:

$$\frac{d^2u}{dz^2} = -k\ u \qquad u = A\ \sin\left(\frac{z}{\beta_o}\right) \qquad u' = \frac{A}{\beta_o}\ \cos\left(\frac{z}{\beta_o}\right)$$

where $\beta_o = 1/\sqrt{k} \qquad \lambda = 2\pi\ \beta_o$

In the u' vs. u plane, this motion is also an ellipse with

$$\frac{\text{width}}{\text{height}} \text{ of elliptical motion in phase space} \quad = \frac{\hat{u}}{\hat{u'}} = \beta_o$$

If we have many particles with $\beta_\perp(\text{beam}) = \beta_o(\text{lattice})$ then all particles move arround the ellipse, the shape, and thus $\beta_\perp(\text{beam})$ remains constant, and the beam is "matched" to this lattice.

If the beam's $\beta_\perp(\text{beam}) \neq \beta_{\text{o}}$ of the lattice then $\beta_\perp(\text{beam}$ of the beam oscillates about $\beta_o(\text{lattice})$: often refered to as a "beta beat".

2 WHY CONSIDER A MUON COLLIDER

Why are leptons (e.g. e or μ) 'better' than protons

- Protons are made of many pieces (quarks and gluons)
- Each carries only a fraction of th proton energy
- Fundamental interactions occur only between these individual pieces
- And the interaction energies are only fractions ($\approx 1/10$) of the proton energies

- Leptons (e's and μ's) are point like
- Their interaction energies are their whole energies

$$\mathrm{E}(3\ \mathrm{TeV}\ e^+e^-\ \ \mathrm{CLIC\ or}\ \mu^+\mu^-) \ \equiv\ 2\times \mathrm{E}(14\ \mathrm{TeV}\ p\bar{p}\ \ \mathrm{LHC})$$

- In addition the energy and quantum state is known for e^+e^-or $\mu^+\mu^-$ but unknown for the parton-parton interaction with protons

S-Channel advantage of muons over electrons

- When all the collision energy $\longrightarrow$ a single state, it is called the "S-Channel"
- A particularly interesting S-Channel interaction would be

$$e^+e^- \;\longrightarrow\; Higgs \qquad or \qquad \mu^+\mu^- \;\longrightarrow\; Higgs$$

The cross sections σ for these interactions

$$\sigma \;\propto\; m^2$$

so

$$\sigma(e^+e^- \;\longrightarrow\; H) \;\approx\; 40,000 \;\times\; \sigma(\mu^+\mu^- \;\longrightarrow\; H)$$

Muons generate less 'Beamstrahlung'

- When high energy electrons in one bunch pass through the other bunch they see the EM fields of the other moving bunch
- These fields are enough to generate synchrotron radiation (called beamstahlung)
- So the energy of the collision is not so well known
$$\sigma_E \approx 30\% \text{ (at 3 TeV } e^+e^- \text{ CLIC)}$$
- And the luminosity at the requires energey is less
$$\mathcal{L} \approx 1/3 \text{ (for } E\pm 1\% \text{ at 3 TeV CLIC)}$$

- But for muons the synchrotron radiation ($\propto 1/m^3$) is negligible
- This could be a particular advantage for $\mu^+\mu^- \rightarrow H$ because with a narrow enough σ_E one could measure the width of a narrow Higgs

Why are Linear colliders linear?

- Earlier electon positron colliders (LEP), like proton colliders, were rings
- But proposed high energy electron colliders are linear

WHY

- Synchrotron radiation of particles bent in the ring magnetic field

$$\Delta E(\text{per turn}) = \left(\frac{4\pi\ mc^2}{3}\right)\ \left(\frac{r_o}{\rho}\right)\ \beta_v^3\gamma^4 \quad \approx\propto \quad B\ \gamma^3$$

- For electons (m$\approx$ 0.5 MeV) this becomes untenable for $E >> 0.1$ TeV
- Above this (LEP's) energy, electron colliders must be linear

- But for muons (m$\approx$ 100 MeV) rings are ok up to around 20 TeV equivaalent to a proton collider of 200 TeV

The advantages of rings

- Muon go round a ring many times
  - Muons live 2 μ seconds at the speed of light that is only 150 m But

$$\tau_{\text{lab frame}} = \tau_{\text{rest frame}} \times \gamma$$

  - For a 1 TeV muon: $\gamma \approx$ 10,000 $\tau \approx 20$ msec they go 1500 km
  - For $< B >$=10 T, a 1 TeV ring will have a circumference of

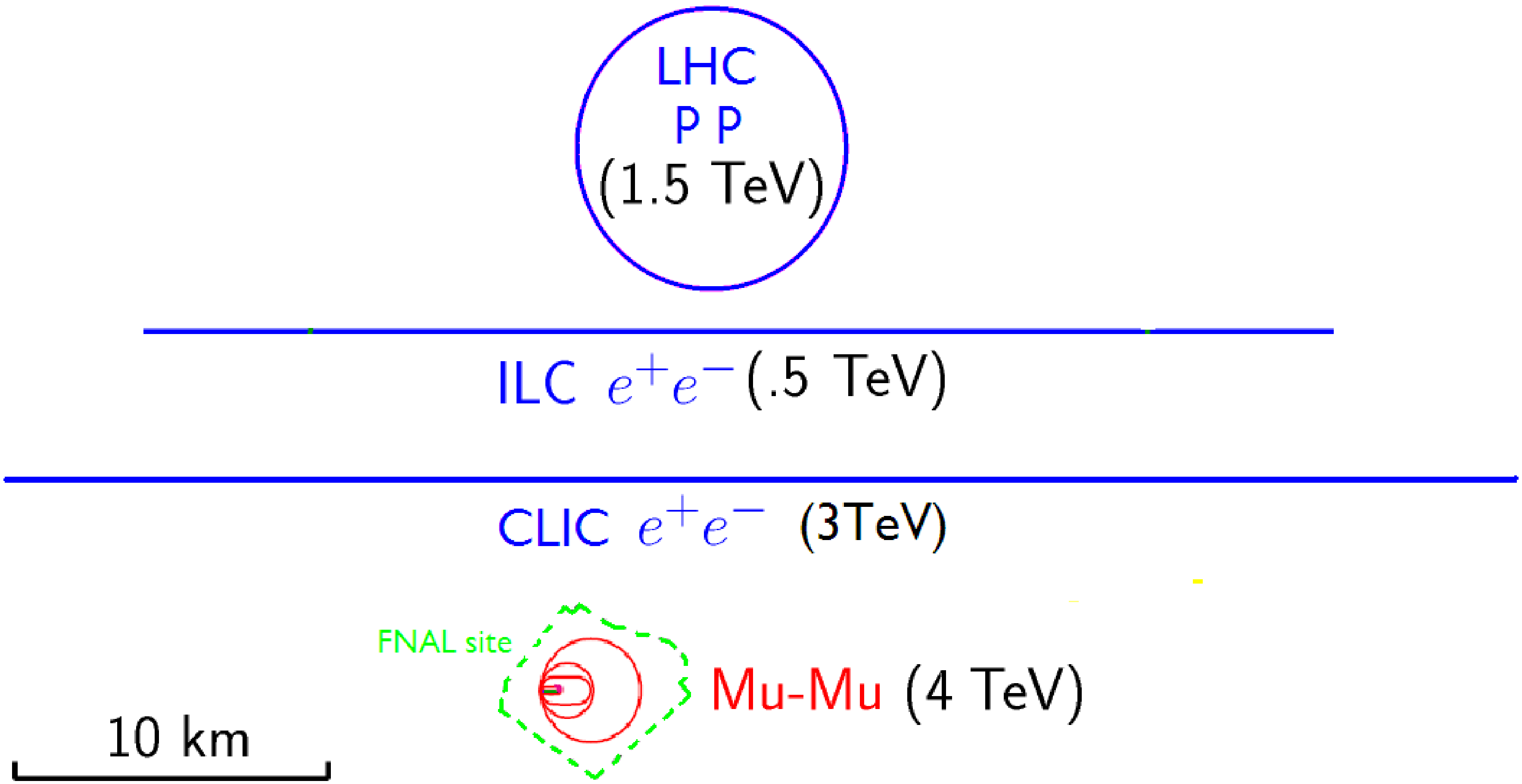
$$C = \frac{2\pi\ [pc/e]}{c\ B} = \frac{2\pi\ 10^{12}}{3\ 10^{8}\ 10} = 2 \quad \text{km}$$

    so they will go round , on average, 1500/2=700 times
  - For the same luminosity, the spot is 700 times larger than in a linear collider $\rightarrow$ easier tolerances

- There can be 2 or more Detectors
  giving an even larger total luminosity gain

- Acceleraqtion must also be fast, in a number of turns $<<$ 700
  still much easier than in the single pass required for e^+e^-

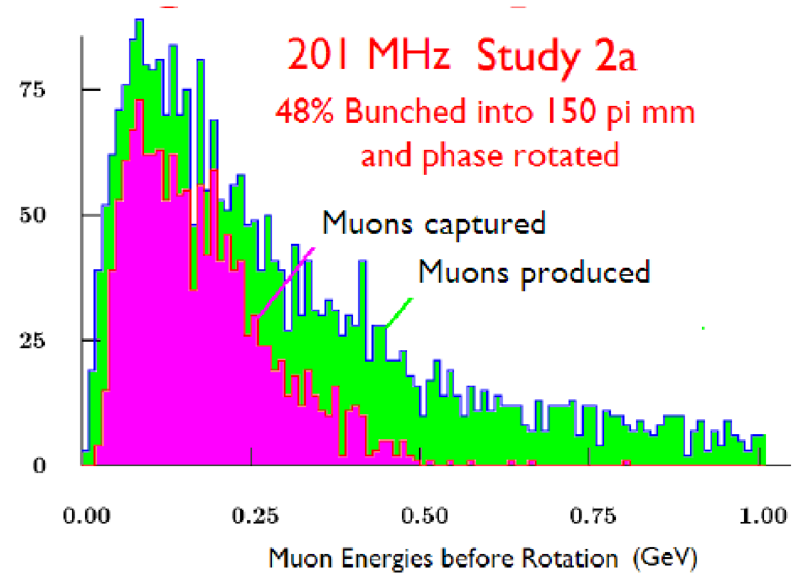
So they are much smaller



And hopefully cheaper

Why NOT a $\mu^+\mu^-$ collider

- Make muons from the decay of pions
- With pions made from protons on a target
- To avoid excessive proton power, we must capture a large fraction of pions made
  - Use a high field solenoid
    Captures most transverse momenta
  - Use Phase rotation
    Captures most longitudinal moments



- The phase space of the pions is now very large:
  - a transverse emittance of 20 pi mm and
  - a longitudinal emittance of 2 pi m
- These emittances must be somehow be cooled by
  - $\approx$ 1000 in each transverse direction and
  - 40 in longitudinal direction
- A factor of over 10^7 !

Cooling Methods

- Electrons are typically cooled (damped) by synchrotron radiation
  but muons radiate too little ($\Delta E \ \propto \ 1/m^3$)
- Protons are typically cooled by a comoving cold electron beam
  too slow
- Or by stochastic methods
  too slow and only works for low intensities ($\tau \ \propto \ 1/\sqrt{N}$)
- Ionization cooling is probably the only hope
- Although optical stochastic cooling after ionization cooling might be useful for very high energies

Luminosity Dependence

$$\mathcal{L} \; = \; n_{\rm turns} \; f_{\rm bunch} \; \frac{N_\mu^2}{4\pi\sigma_\perp^2} \qquad\qquad \Delta\nu \propto \frac{N_\mu}{\epsilon_\perp}$$

$$\mathcal{L} \; \propto \; B_{\rm ring} \; P_{\rm beam} \;\; \Delta\nu \; \frac{1}{\beta^*}$$

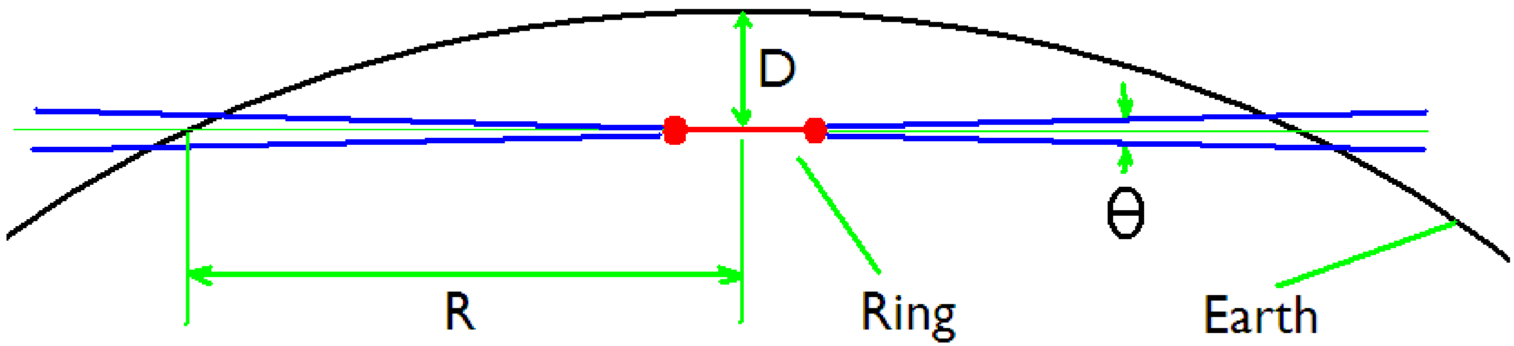
- Higher $\mathcal{L}/P_{\rm beam}$ requires lower β^* or correction of $\Delta\nu$
- Lower emittances do not directly improve Luminosity/Power
- But for fixed $\Delta\nu$, $\epsilon_\perp$ must be pretty small to avoid N_μ becoming unreasonable

For the next page:

$$\mathcal{L} \; \propto \; < B_{\rm ring} > \; I_{\rm beam} \; \gamma \; \frac{\Delta\nu}{\beta^*}$$

$$I_\mu \; \propto \; \frac{\mathcal{L} \; \beta^*}{< {\rm B} > \;\; \gamma \; \Delta\nu}$$

Neutrino Radiation Constraint



$$\text{Radiation} \;\propto\; \frac{E_\mu \; I_\mu \; \sigma_\nu}{\theta \;\; R^2} \;\propto\; \frac{I_\mu \; \gamma^3}{D} \;\propto\; \frac{\mathcal{L} \;\; \beta^*}{\Delta\nu \quad <\mathrm{B}>} \;\; \frac{\gamma^2}{D}$$

For fixed $\Delta\nu$, β^* and $< B >$; and $\mathcal{L} \propto \gamma^2$:

$$\text{Radiation} \;\; \propto \;\; \frac{\gamma^4}{D}$$

For D=135 m R=40 Km for 4 TeV example
For D=540 m R=80 Km for 8 TeV example
OK up to 8 TeV, but a problem higher

Conclusions on 'Why a muon collider"

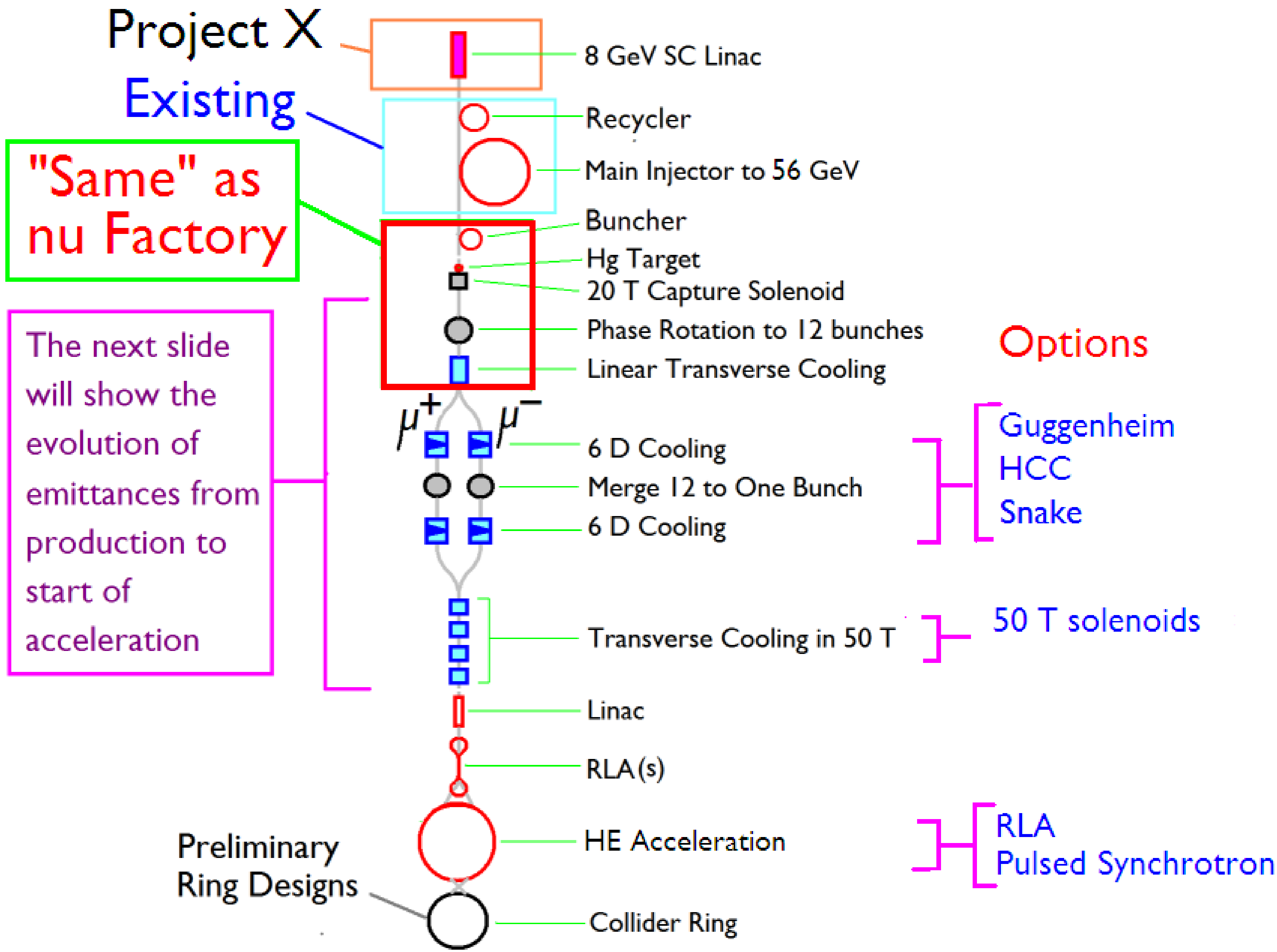
- Point like interactions as in linear e^+e^-
  effective energy 10 times hadron machines
- Negligible synchrotron radiation:
  - Acceleration in rings
  - Small footprint
  - Less rf
  - Hopefully cheaper
- Collider is a Ring $\approx$ 1000 crossings per bunch
  - Larger spot
  - Easier tolerances
  - 2 or more Detectors
- Negligible Beamstrahlung Narrow energy spread
- 40,000 greater S channel Higgs Enabling study of widths
- But serious challenge to cool the muons by $\gg 10^7$ times
- Neutrino radiation a significant problem at very high energies
- CLIC better understood, but may not be affordable

3 CURRENT BASELINE DESIGNS

Parameters

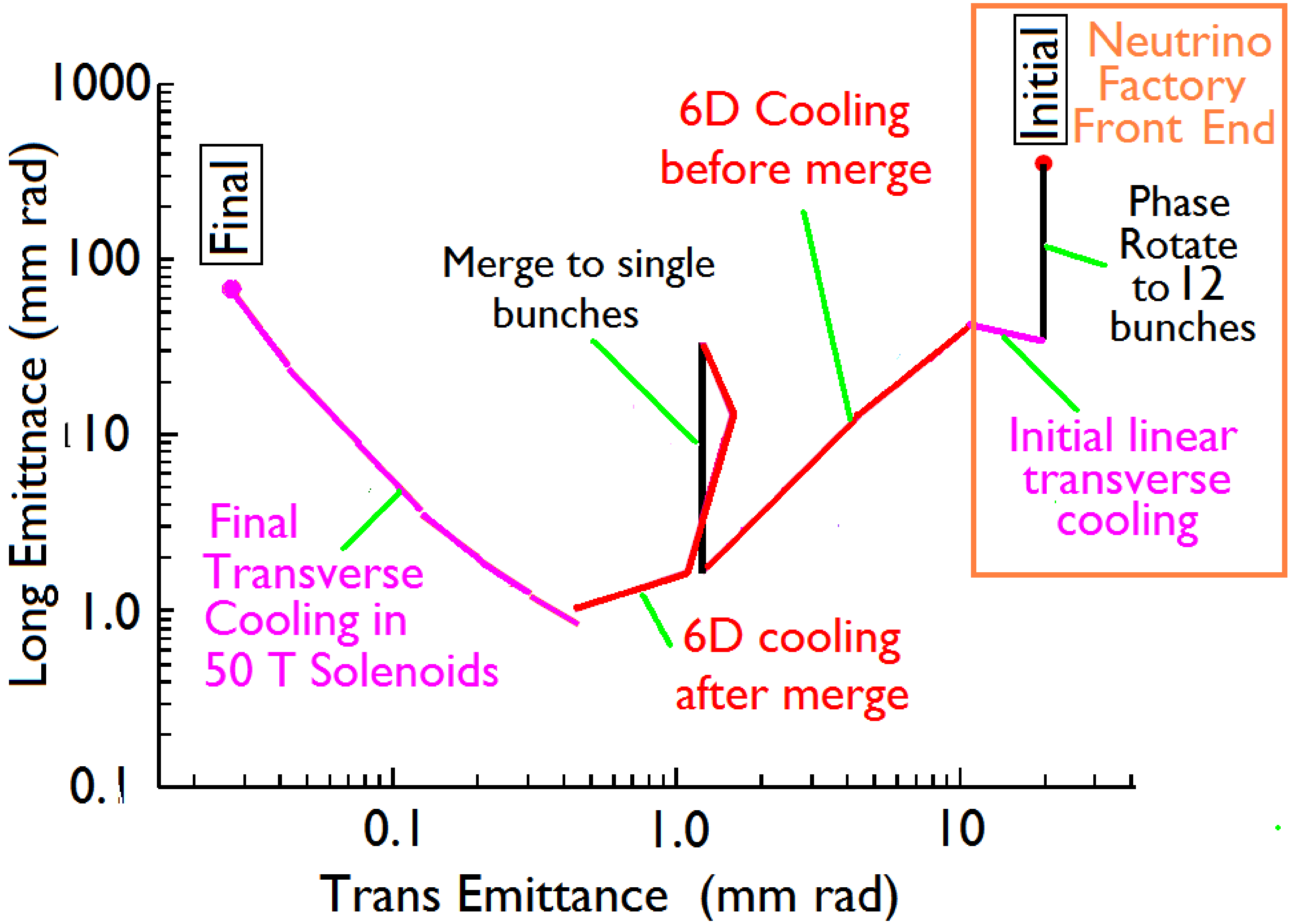
C of m Energy	1.5	4	TeV
Luminosity	1	4	10^{34} cm^{2}sec^{-1}
Muons/bunch	2	2	10^{12}
Ring circumference	3	8.1	km
β^* at IP $= \sigma_z$	10	3	mm
rms momentum spread	0.1	0.12	%
Required depth for ν rad	13	135	m
Repetition Rate	12	6	Hz
Proton Driver power	$\approx$4	$\approx$ 1.8	MW
Muon Trans Emittance	25	25	pi mm mrad
Muon Long Emittance	72,000	72,000	pi mm mrad

- Based on real Collider Ring designs, though both have problems
- Emittance and bunch intensity requirement same for both examples
- 4 TeV luminosity comparable to CLIC's
- Depth for ν radiation keeps off site dose $<$ 1 mrem/year



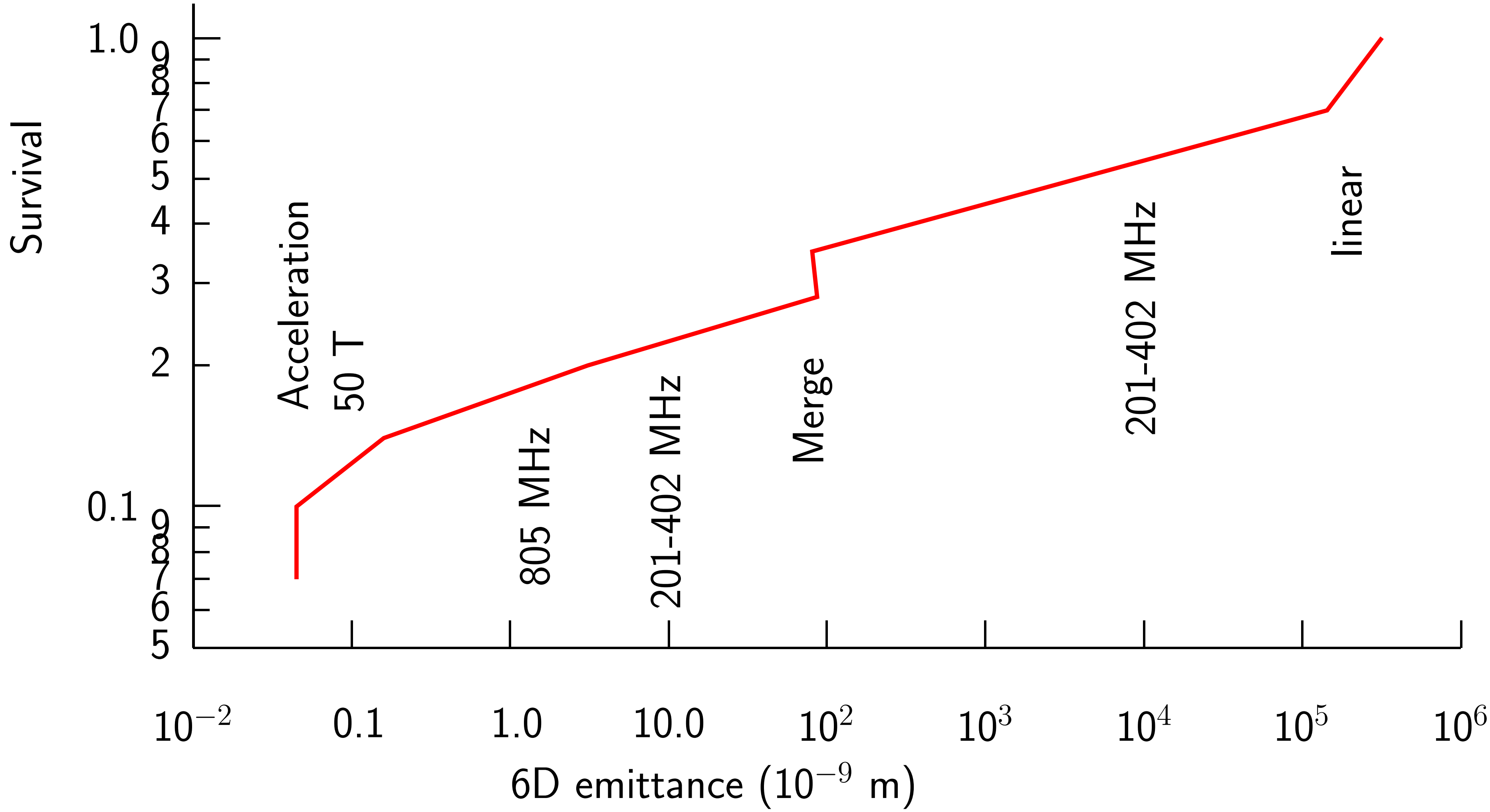
Project X
Existing
"Same" as nu Factory
The next slide will show the evolution of emittances from production to start of acceleration
8 GeV SC Linac
Recycler
Main Injector to 56 GeV
Buncher
Hg Target
20 T Capture Solenoid
Phase Rotation to 12 bunches
Linear Transverse Cooling
Options
μ^+
μ^-
6 D Cooling
Merge 12 to One Bunch
6 D Cooling
Guggenheim
HCC
Snake
Transverse Cooling in 50 T
50 T solenoids
Linac
RLA(s)
HE Acceleration
RLA
Pulsed Synchrotron
Preliminary Ring Designs
Collider Ring

Emittances vs. Stage



- Every stage simulated at some level,
- But with many caveats

Estimated losses vs 6D emittance



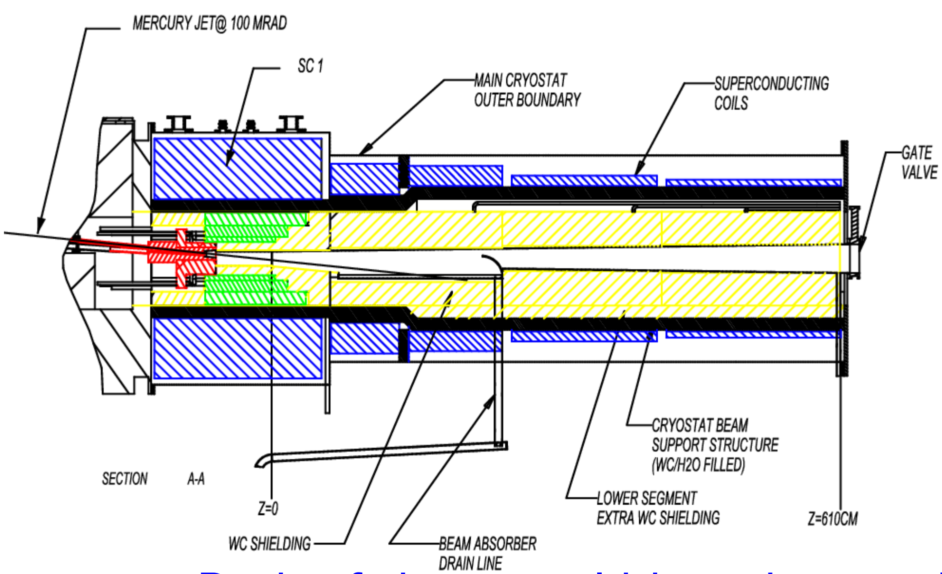
- Only 7% survive
- And even this estimate did not fully include matching losses
- This means that the initial pion, and thus proton, bunches must be intense
- Much more intense than IDS specification for a Neutrino Factory

Proton driver

- Project X (8 GeV H^- linac),
- Together with accumulation in the Re-cycler
- And acceleration to 56 GeV in the Main Injector
- Could provide the required 12 Hz protons with power = 4 MW
- This driver could meet Factory requirement, but the reverse need not be true

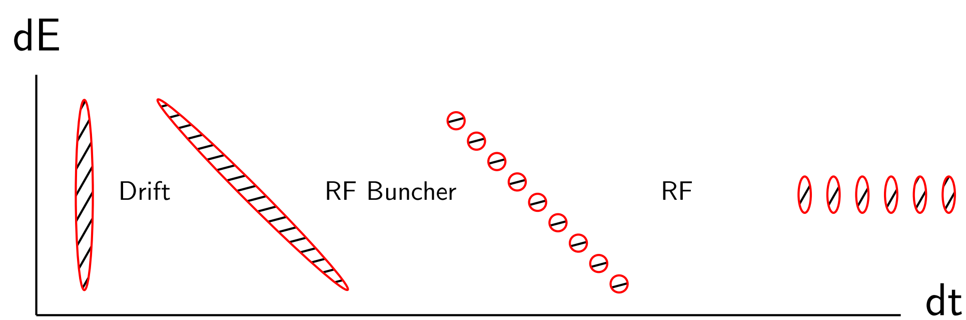
Target and Capture

Mercury Jet Target, 20 T capture
Adiabatic taper to 2 T



Phase Rotation

Drifts & Multiple frequency rf
to Bunch, then Rotate



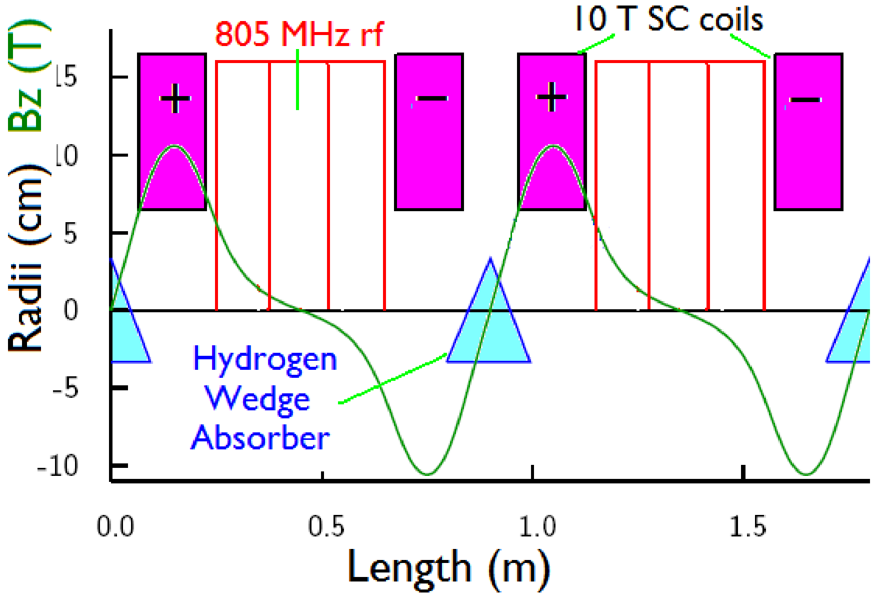
Both of these would be substantially the same as for a Neutrino Factory

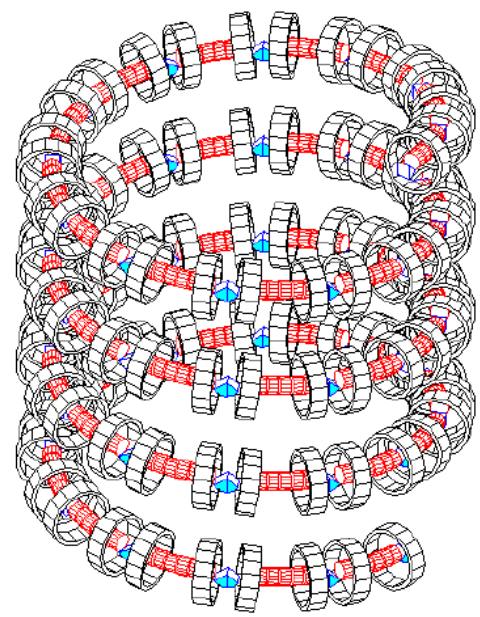
6D Cooling Several methods under study

a) "Guggenheim" Lattice (baseline)

- Lattice arranged as 'Guggenheim' upward helix
- Bending gives dispersion
- Higher momenta pass through longer paths in wedge absorbers giving momentum cooling (emittance exchange)
- Starting at 201 MHz and 3 T, ending at 805 MHz and 10 T
- Snake is similar but with alternating bends

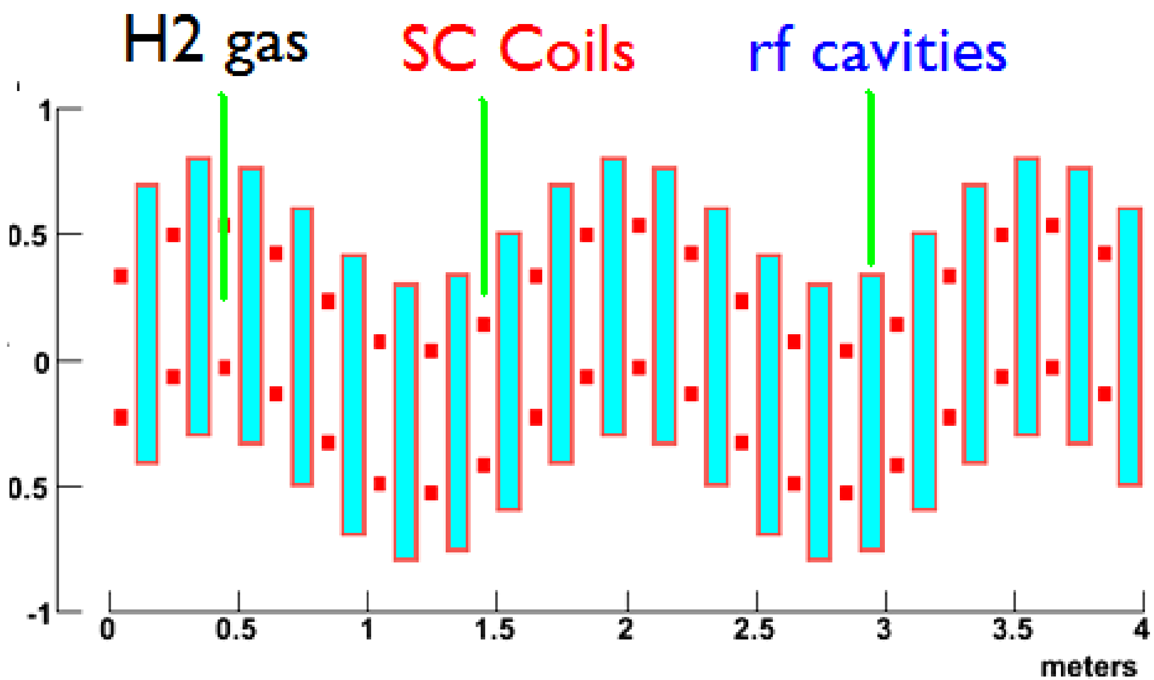
e.g. 805 MHz 10 T cooling to 400 mm mrad





b) Helical Cooling Channel (HCC)

- Muons move in helical paths in high pressure hydrogen gas
- Higher momentum tracks have longer trajectories giving momentum cooling (emittance exchange)



- Initial Bz=4.3 T
- Final Bz=13.6 T
- But final $\epsilon_\perp$=900 mm mrad c.f. 400 mm mrad in baseline scheme

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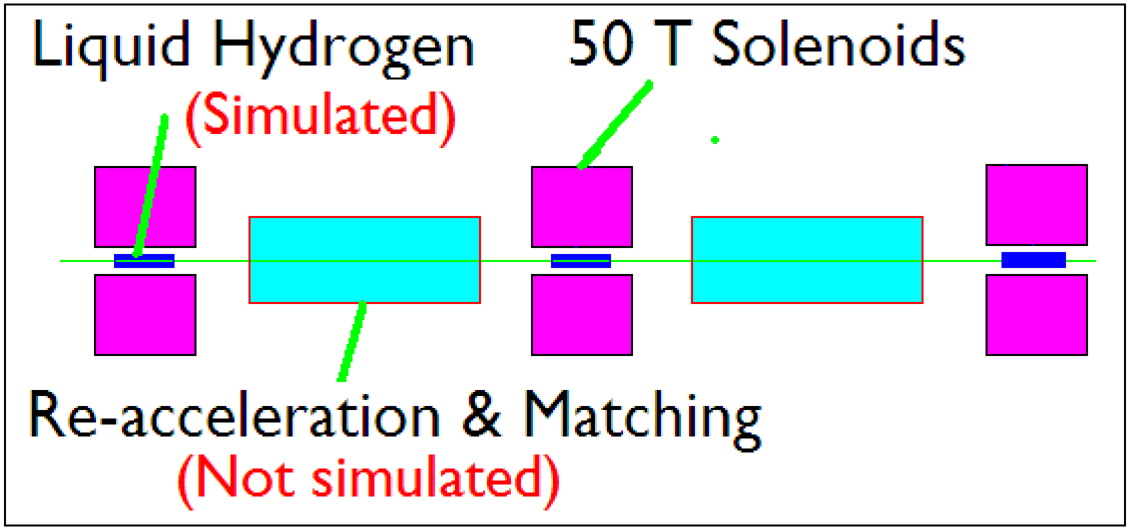
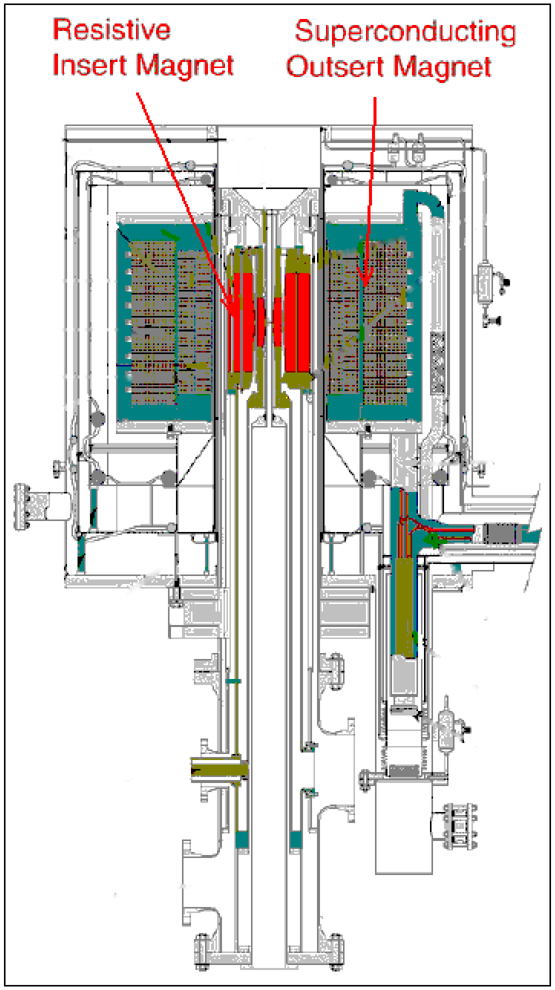
Engineering integration of rf not well defined
Possible problem of rf breakdown with intense muon beam transit

Final Transverse Cooling in High Field Solenoids

- Lower momenta allow transverse cooling to required low transverse emittance, but long emittance rises: Effectively reverse emittance exchange





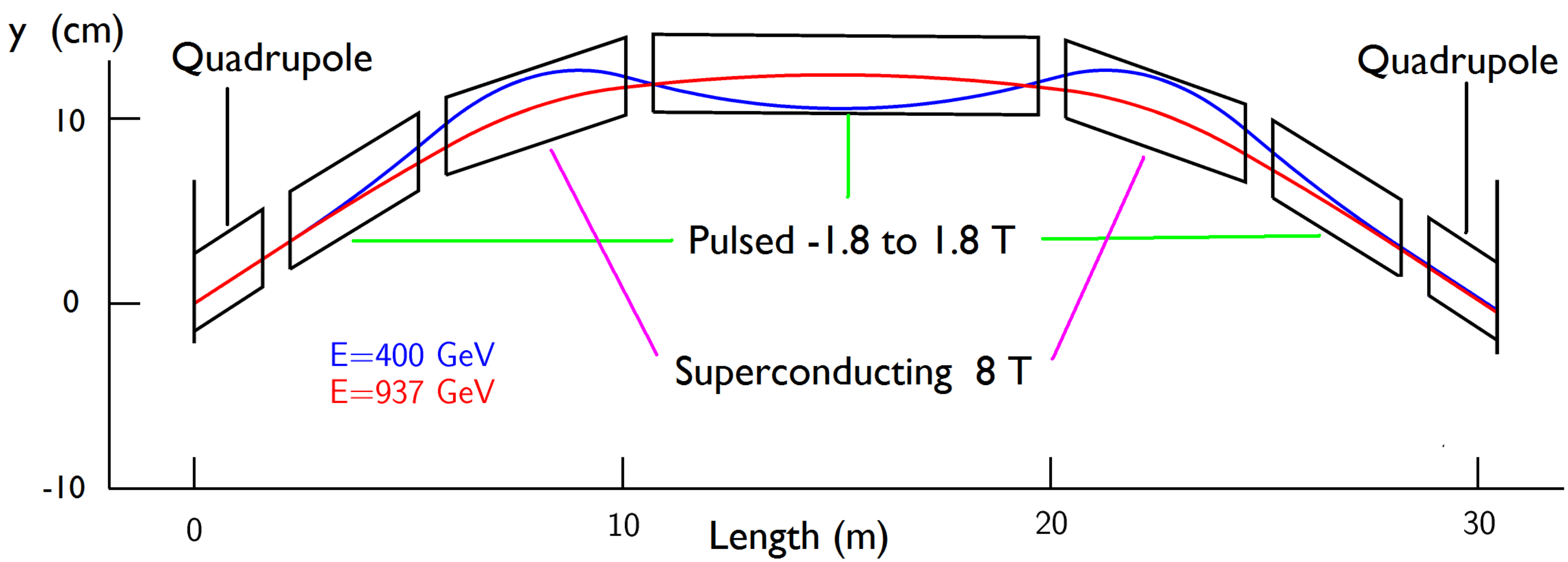
NHMFL 45 T Hybrid Magnet

- Need five 50 T solenoids
- ICOOL Simulation of cooling in solenoids
- Simulation of re-acceleration/matching started

- 45/50 T Solenoids ?
  - 45 T hybrid at NHMFL, but uses 25W
  - Could achieve 50T with 37 MW
  - 30 T all HTS designed at NHMFL

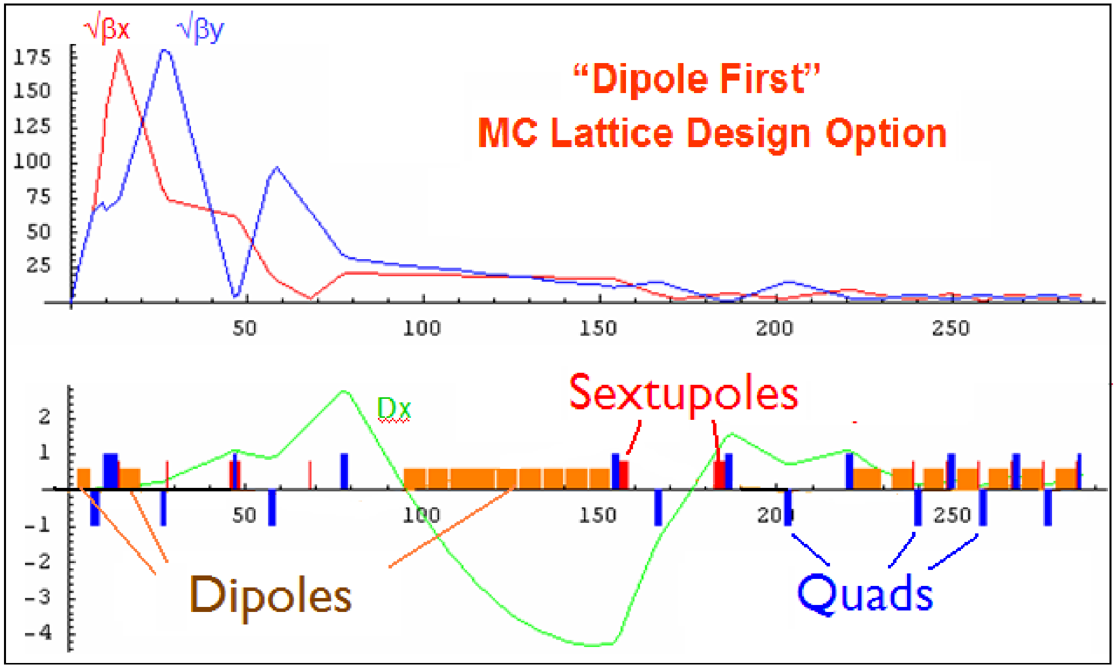
Acceleration

- Sufficiently rapid acceleration is straightforward in Linacs and Recirculating linear accelerators (RLAs)
  Using ILC-like 1.3 GHz rf
- Lower cost solution would use Pulsed Synchrotrons
- Pulsed synchrotron 30 to 400 GeV
  (in Tevatron tunnel)
- Hybrid SC & pulsed magnet synchrotron 400-900 GeV
  (in Tevatron tunnel)



Collider Ring

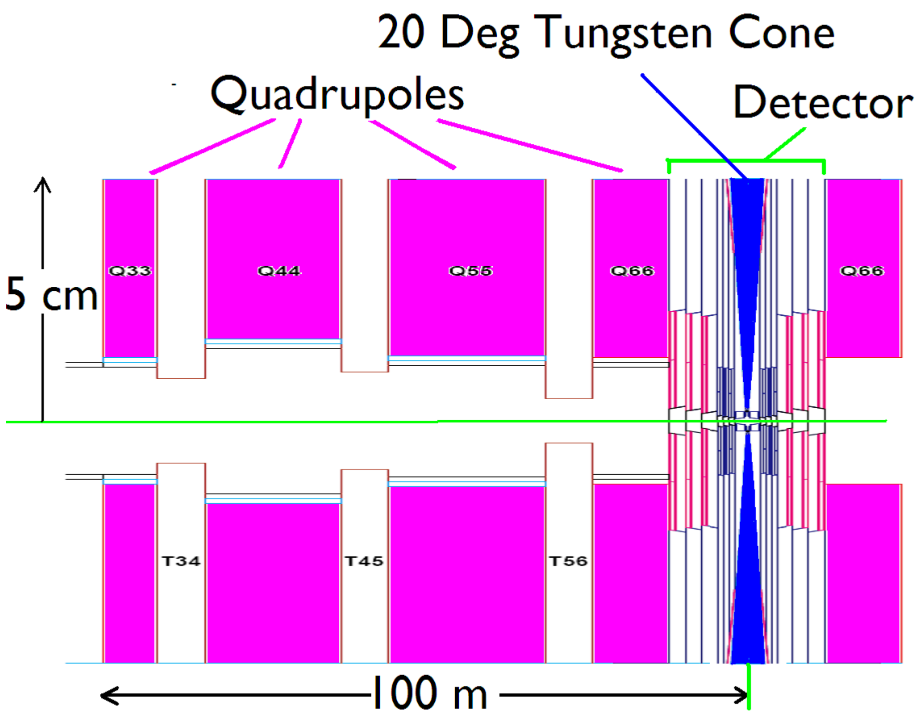
- 1.5 TeV (c of m) Design



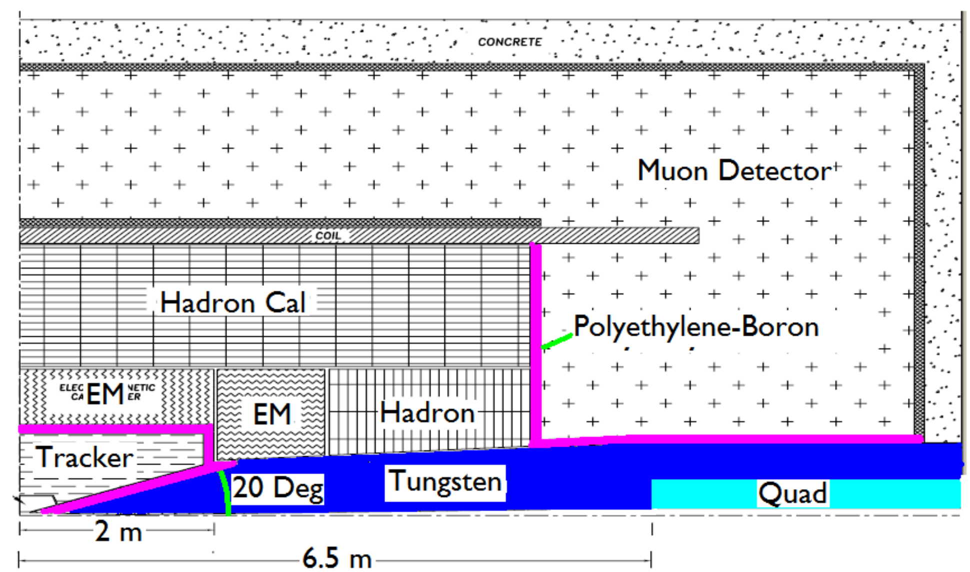
  - Nearly meets requirements
  - But early dipole may deflect unacceptable background into detector

- 4 TeV (c of m) 1996 design by Oide
  - Meets requirements in ideal simulation
  - But is too sensitive to errors to be realistic
- The experts believe that the required rings should be possible

Detector From 1996 Study of 4 TeV Collider



Shielding



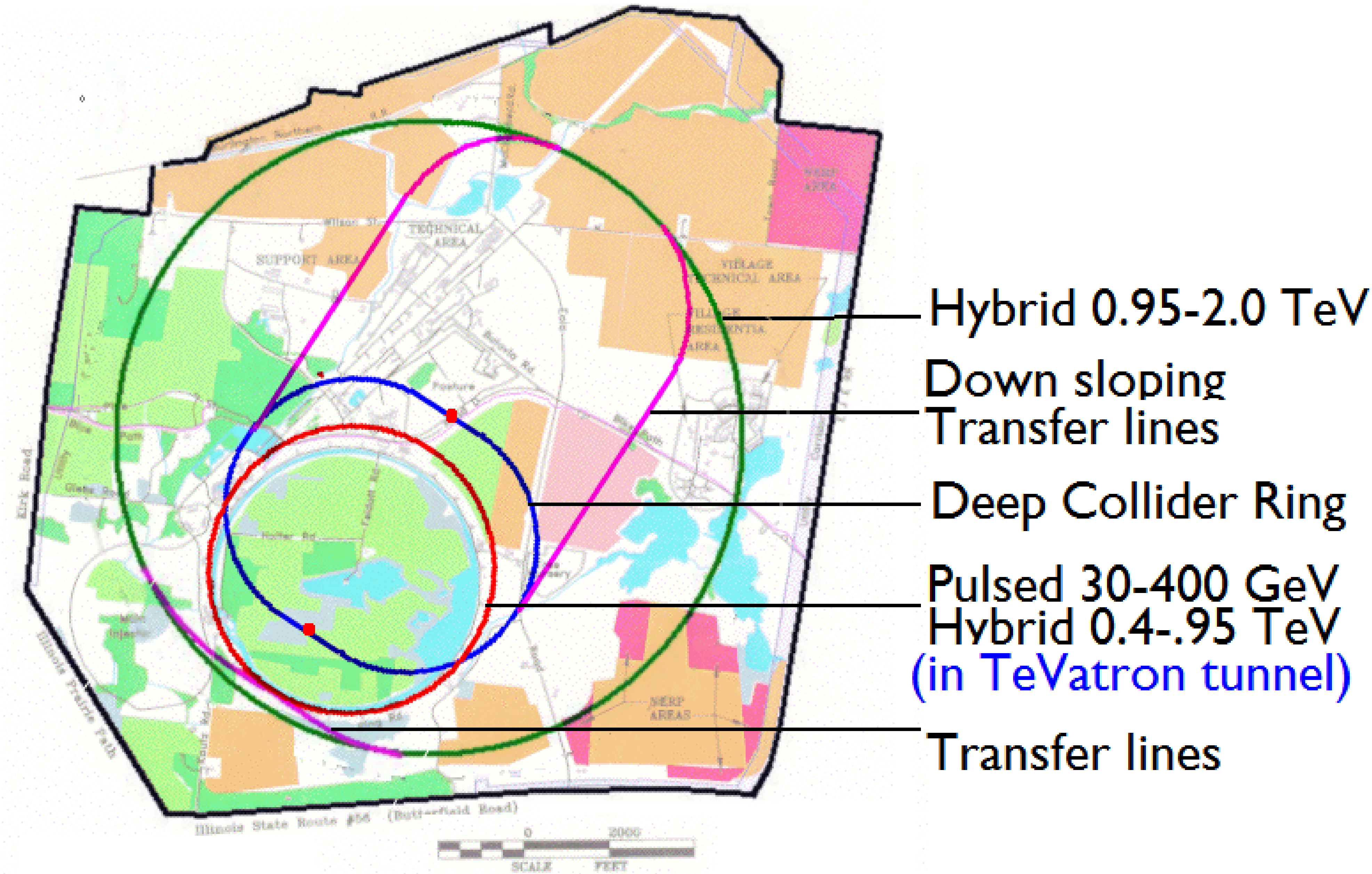
Detector

- Sophisticated shielding designed in 1996 4 TeV Study
- GEANT simulations then indicated acceptable backgrounds
- Would be less of a problem now with finer pixel detectors

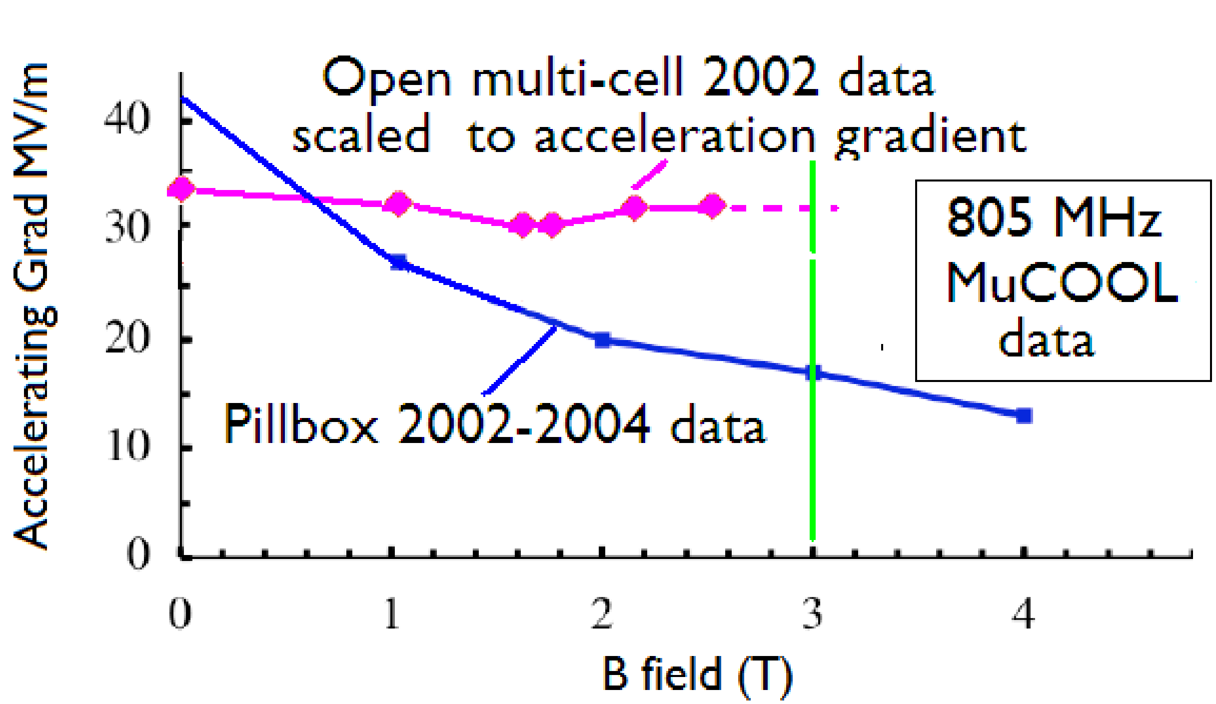
BUT

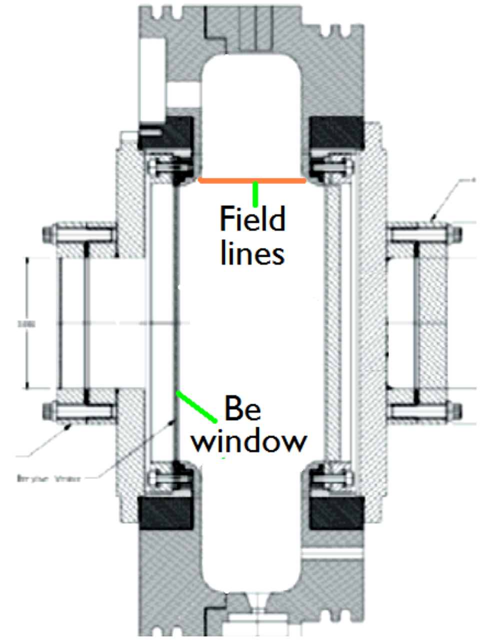
- Tungsten shielding takes up 20 degree cone

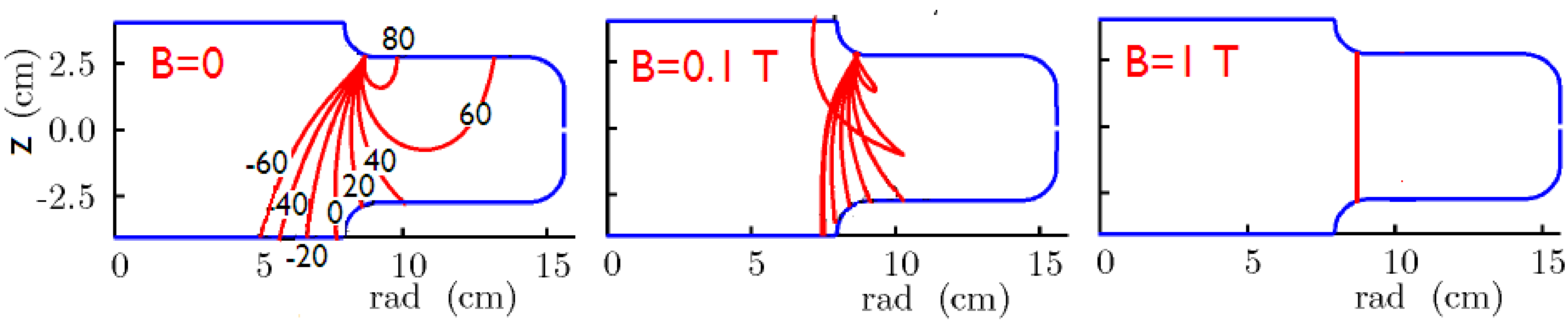
Layout of 4 TeV Collider using pulsed synchrotrons



Technical challenge: rf breakdown in magnetic fields

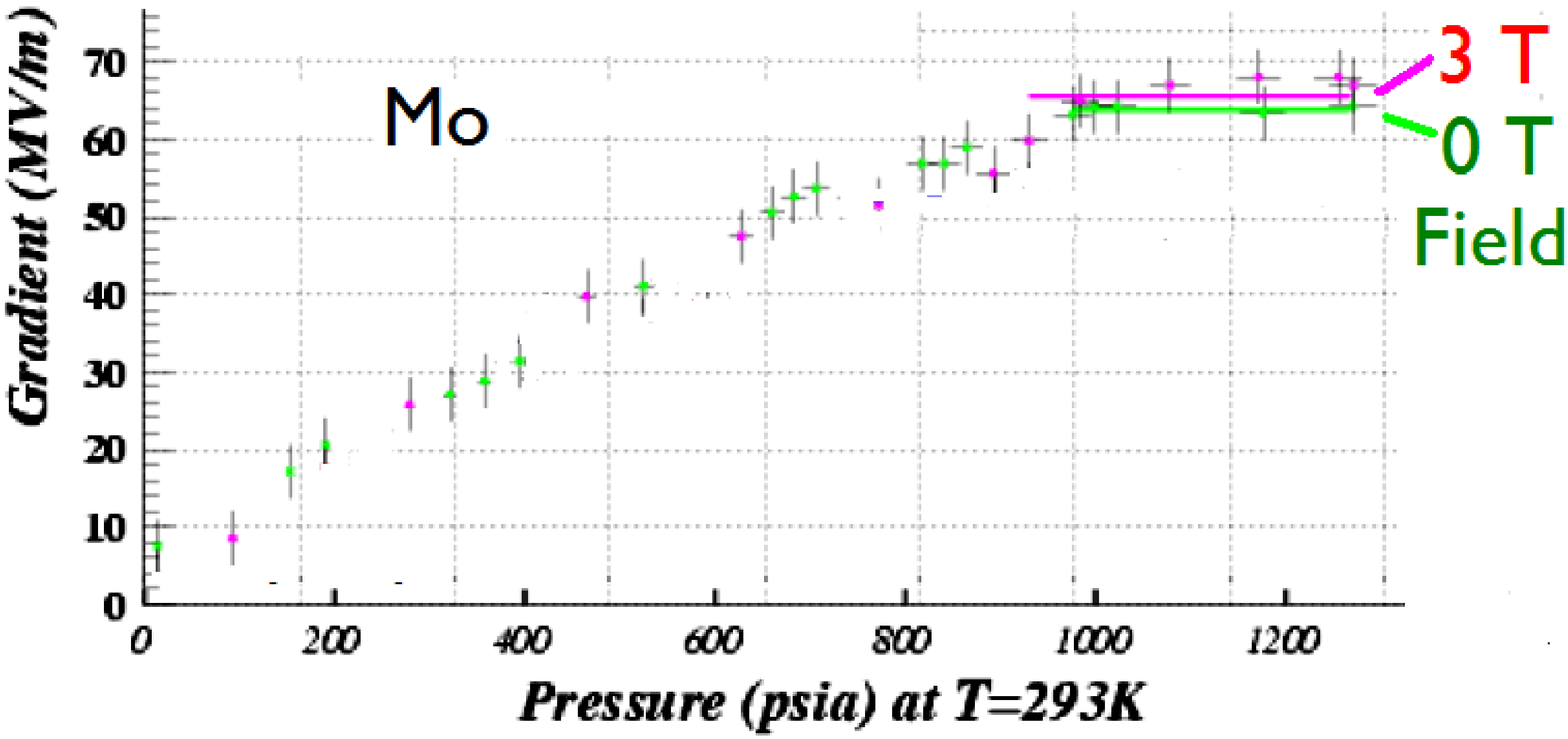






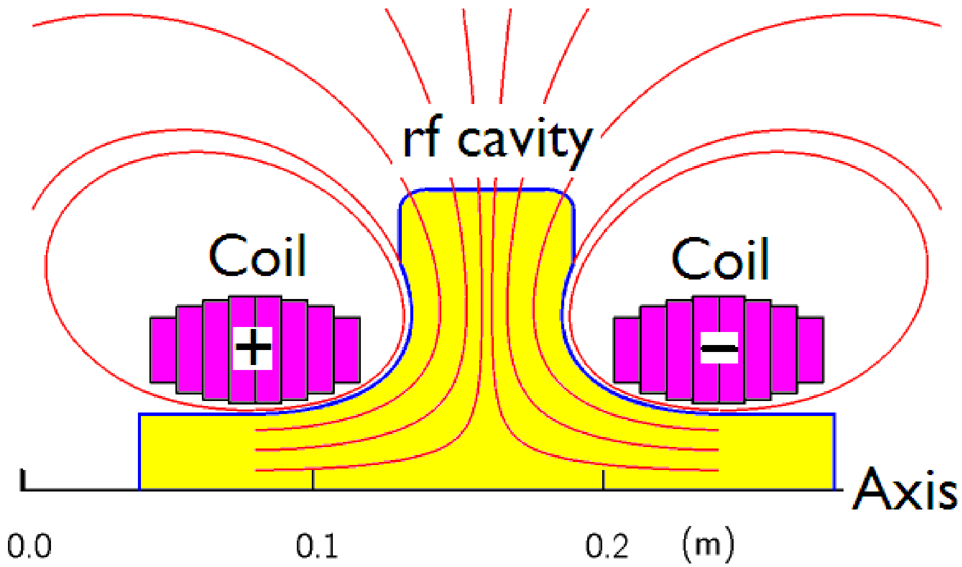
1. "Dark Current" electrons accelerated and focused by magnetic field
2. Melt small spots causing breakdown

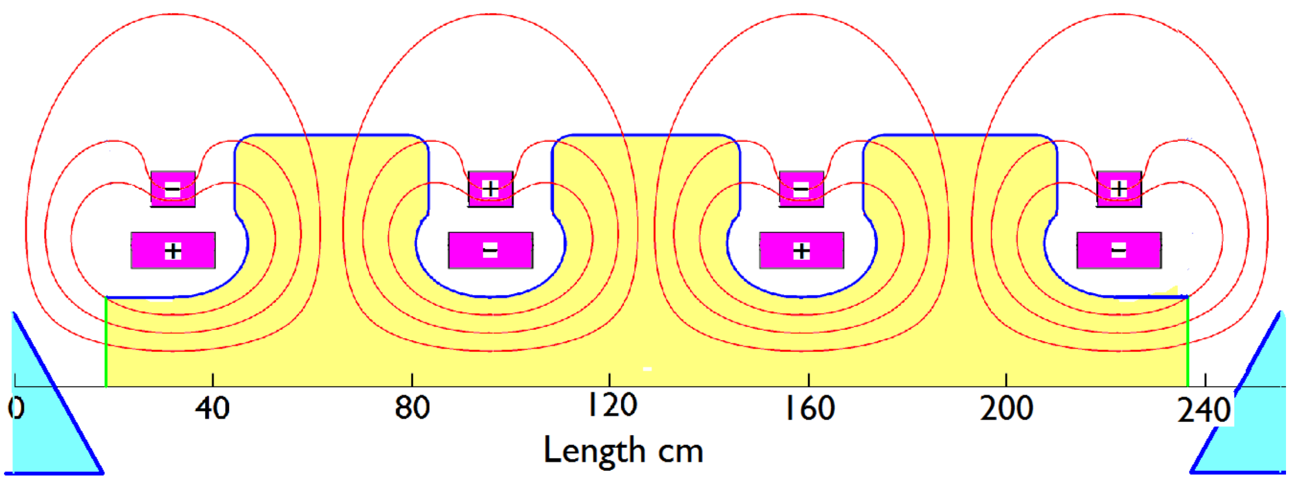
Solution ? Gas filled cavities show no such effect



- But a beam passing through may cause breakdown or rapid loss of rf (e.g. Q=2)
- Also not suitable at lower betas since hydrogen will cause Coulomb scattering

Alternative: Magnetic Insulation





- All tracks return to the surface & Energies very low
- No dark current, No X-Rays, no danger of melting surfaces

- Adding second coil improves rf cavity efficiency

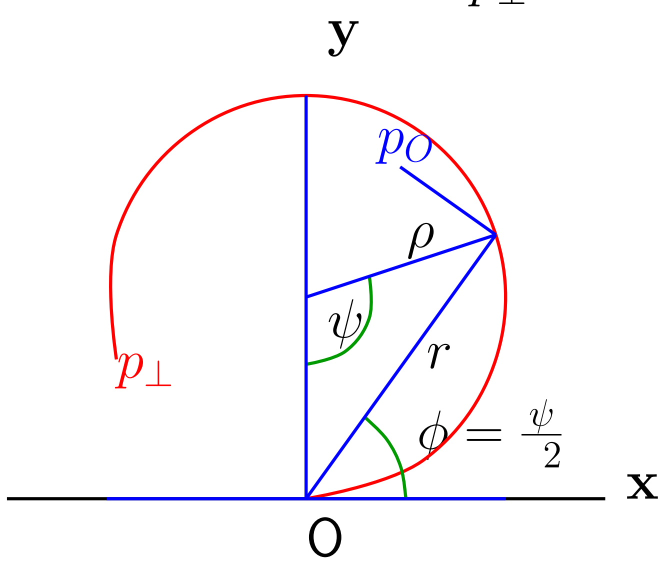
Conclusion on Baseline design

- All stages for a "baseline" design have been simulated at some level
- Matching and tapering of 6D cooling remains to be designed
- Matching and re acceleration in the final 50 T transverse cooling is under study
- Collider ring designs exist for both 1.5 TeV and 4 TeV colliders although both still have problems
- Detector design and shielding has been studied and looks OK

4 SOLENOID FOCUSING

Motion in Long Solenoid

Consider motion in a fixed axial filed B_z, starting on the axis O with finate transverse momentum $p_\perp$ i.e. with initial angular momentum=0.



$$\rho = \frac{[pc/e]_\perp}{c\; B_z} \tag{7}$$

$$x = \rho \sin(\psi)$$

$$y = \rho\; (1 - \cos(\psi))$$

$$\frac{dz}{d\psi} = \rho\; \frac{p_z}{p_\perp}$$

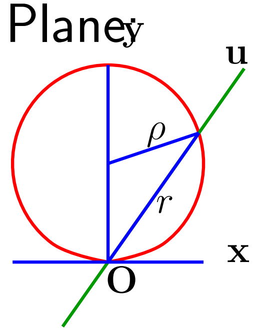
For $\psi < 180^o \quad \phi < 90^o$:

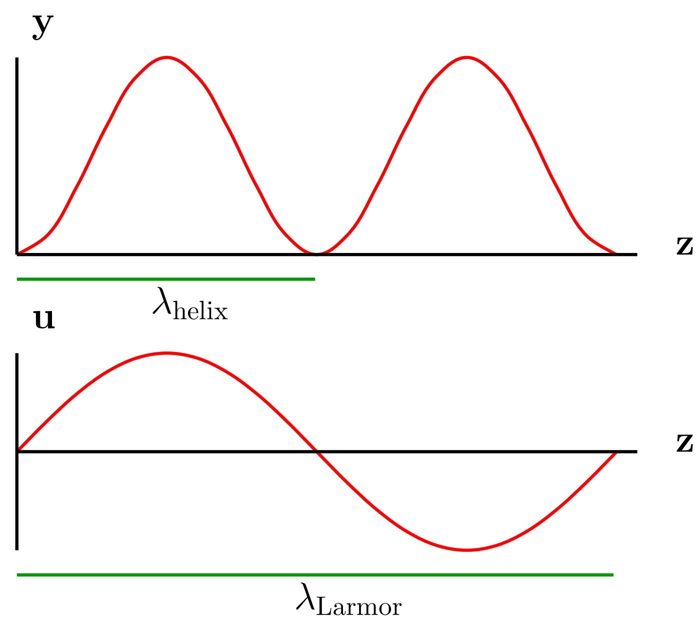
$$r = 2\rho\; \sin\left(\frac{\psi}{2}\right) = 2\rho \sin(\phi)$$

$$\frac{dz}{d\phi} = 2\; \rho\; \frac{p_z}{p_\perp}$$

Larmor Plane

If The center of the solenoid magnet is at O, then consider a plane that contains this axis and the particle. This, for a particle with initally no angular momentum, is the 'Larmor Plane





$$u \;=\; 2\rho\;\sin(\phi) \tag{8}$$

$$\lambda_{\rm Helix} \;=\; 2\pi\,\frac{dz}{d\psi} \;=\; 2\pi\;\rho\;\frac{p_z}{p_\perp} \;=\; 2\pi\;\frac{[pc/e]_z}{c\;B_z}$$

$$\lambda_{\rm Larmor} \;=\; 2\pi\,\frac{dz}{d\phi} \;=\; 2\pi\;2\rho\;\frac{p_z}{p_\perp} \;=\; 4\pi\;\frac{[pc/e]_z}{c\;B_z}$$

The lattice parameter β_o is defined in the Larmor frame, so

$$\beta_o \;=\; \frac{\lambda_{Larmor}}{2\pi} \;=\; \frac{2\;[pc/e]_z}{c\;B_z} \tag{9}$$

Focusing Force

In this constant B case, the observed sinusoidal motion in the u plane is generated by a restoring force towards the axis O.

The momentum p_O about the axis O (perpendicular to the Larmor plane), using eq.7 and eq.8:

$$[p_O c/e] \;=\; [p_\perp c/e]\,\sin(\phi) \;=\; cB_z\rho\,\frac{u}{2\rho} \;=\; \frac{cB_z}{2}\,u \tag{10}$$

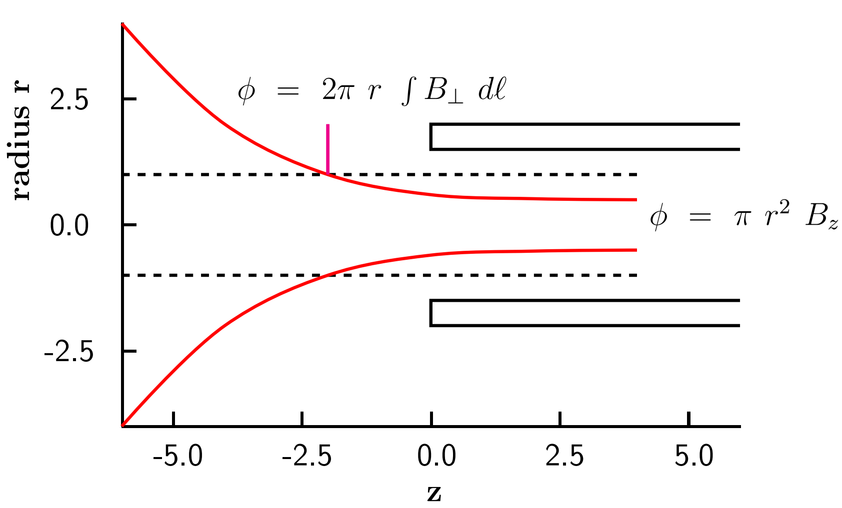
And the inward bending as this momentum crosses the B_z field is

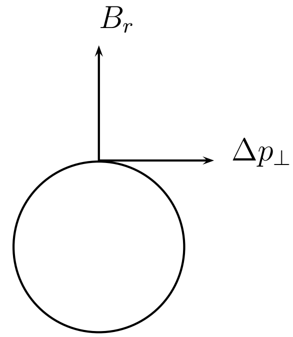
$$\frac{d^2u}{dz^2} \;=\; -\left(\frac{cB_z}{2\,[p_z c/e]}\right)^2\,u \tag{11}$$

This inward force proportional to the distance u from the axis is an ideal focusing force

Note: the focusing "Force" $\propto\; B_z^2$ so it works the same for either sign, and $\propto\; 1/p_z^2$. Whereas in a quadrupole the force $\propto\; 1/p$ So solenoids are not good for high p, but beat quads at low p.

Entering a solenoid





$$\Delta[pc/e]_\perp \;=\; \int B_r\; dz \;=\; \frac{B_z\; r\; c}{2} \tag{12}$$

So for our case with zero initial transverse momentum,

$$[pc/e]_\perp \;=\; \int B_r\; dz \;=\; \frac{B_z\; r\; c}{2}$$

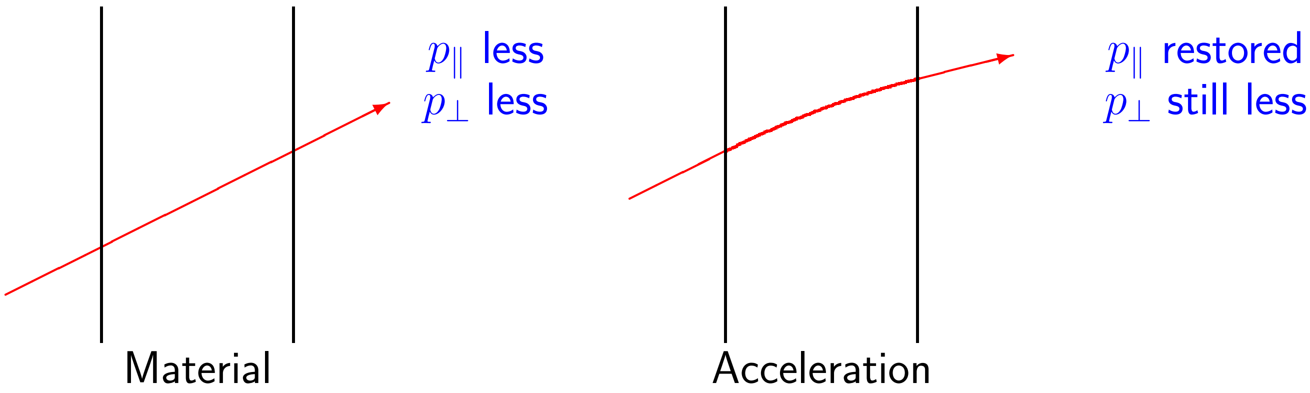
Which is the same as eq.10, and will lead to the same inward bending, as when the particle started inside the field.

In fact eq.11 is true no matter how the axial field varies

Conclusion on solenoid focusing

- In a uniform solenoid field a particle moves in a helix of wavelength λ_{helix}
- But in the rotating larmor plane it oscillates with wavelength $\lambda_{\mathrm{larmor}} = 2\,\lambda_{\mathrm{helix}}$
- Even with non uniform fields, motion in the larmor plane:
  - Focus is always towards the axis
  - With a 'force' $\propto \; B^2/p^2$
  - If a particle starts in the Larmor plane, it stays in that plane
- Since a quadrupole focuses with a 'force' $\propto B/p$ instead of 'force' $\propto B^2/p^2$, the solenoid is always stronger at low enough momenta
  and Solenoids focus in both planes, whereas quadrupoles focus in one and defocus in the other

5 TRANSVERSE IONIZATION COOLING



Cooling rate vs. Energy

$$\text{(eq 1)} \quad \epsilon_{x,y} = \gamma\beta_v\, \sigma_\theta\, \sigma_{x,y}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ are reduced. So the fractional cooling $d\epsilon\,/\epsilon$ is (using eq.??):

$$\frac{d\epsilon}{\epsilon} = \frac{dp}{p} = \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{13}$$

which, for a given energy change, strongly favors cooling at low energy.

Heating Terms

$$\epsilon_{x,y} = \gamma\beta_v\ \sigma_\theta\ \sigma_{x,y}$$

Between scatters the drift conserves emittance (Liouiville).
When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$\begin{aligned}
\Delta(\epsilon_{x,y})^2 &= \gamma^2\beta_v^2\ \sigma_{x,y}^2\Delta(\sigma_\theta^2) \\
2\epsilon\ \Delta\epsilon &= \gamma^2\beta_v^2 \left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right)\ \Delta(\sigma_\theta^2) \\
\Delta\epsilon &= \frac{\beta_\perp\gamma\beta_v}{2}\ \Delta(\sigma_\theta^2)
\end{aligned}$$

e.g. from Particle data booklet $\quad \Delta(\sigma_\theta^2) \approx \left(\frac{14.1\ 10^6}{[pc/e]\beta_v}\right)^2\ \frac{\Delta s}{L_R}$

$$\Delta\epsilon = \frac{\beta_\perp}{\gamma\beta_v^3}\ \Delta E\ \left(\left(\frac{14.1\ 10^6}{2[mc^2/e]_\mu}\right)^2\ \frac{1}{L_R dE/ds}\right)$$

Defining

$$C(mat, E) = \frac{1}{2}\left(\frac{14.1\ 10^6}{[mc^2/e]_\mu}\right)^2\ \frac{1}{L_R\ d\gamma/ds} \tag{14}$$

$$\text{then} \qquad \frac{\Delta\epsilon}{\epsilon} = dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat, E) \tag{15}$$

Equilibrium emittance

Equating this with equation 13

$$dE \frac{1}{\beta_v^2 \, E} = dE \frac{\beta_\perp}{\epsilon \gamma \beta_v^3} \; C(mat, E)$$

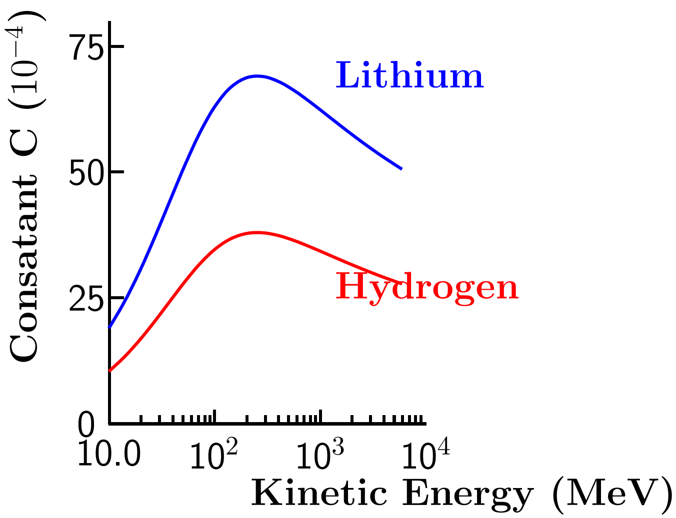
gives the equilibrium emittance ϵ_o :

$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{\beta_v} \; C(mat, E) \qquad (16)$$

At energies for minimum ionization loss:

material	T ^{o}K	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

As a function of energy:



Clearly Liquid Hydrogen is far the best material, but has cryogenic and safety complications, and requires windows which will significantly degrade the performance. At lower energies C is much lower but there is then longitudinal (dp/p) heating.

Rate of Cooling

$$\frac{d\epsilon}{\epsilon} = \left(1 - \frac{\epsilon_{\rm min}}{\epsilon}\right) \frac{dp}{p} \tag{17}$$

Beam Divergence Angles

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp \, \beta_v \gamma}}$$

so, from equation 16, for a beam in equilibrium

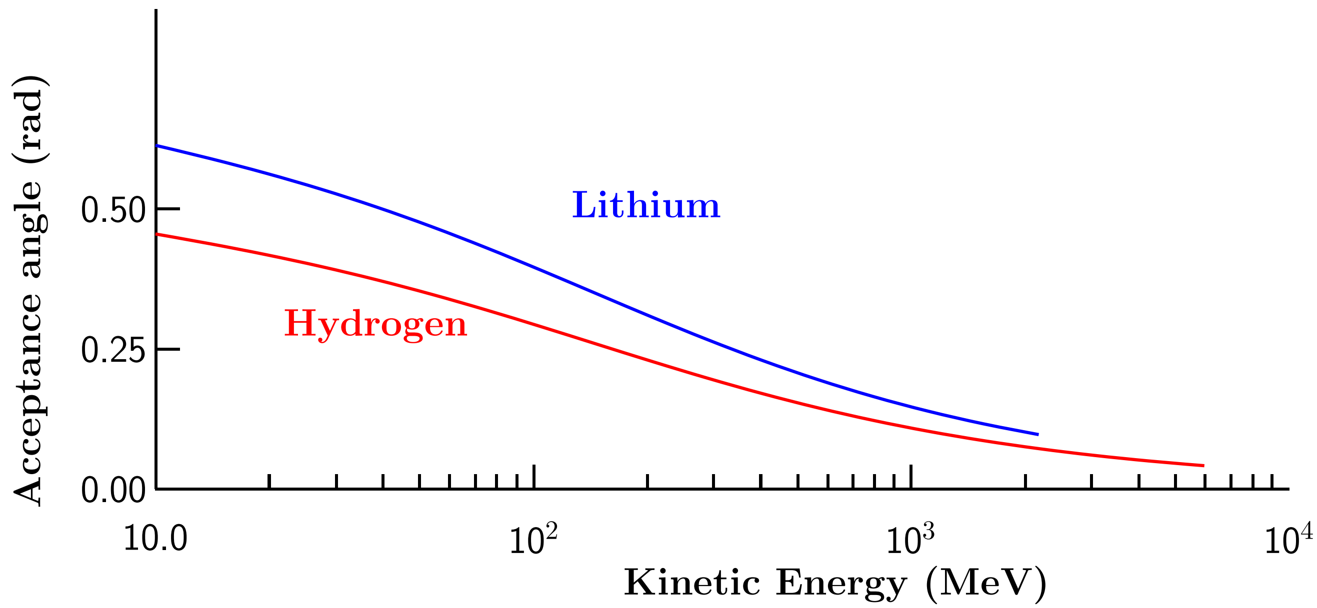
$$\sigma_\theta = \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}}$$

and for 50 % of maximum cooling rate and an aperture at 3 σ, the angular aperture $\mathcal{A}$ of the system must be

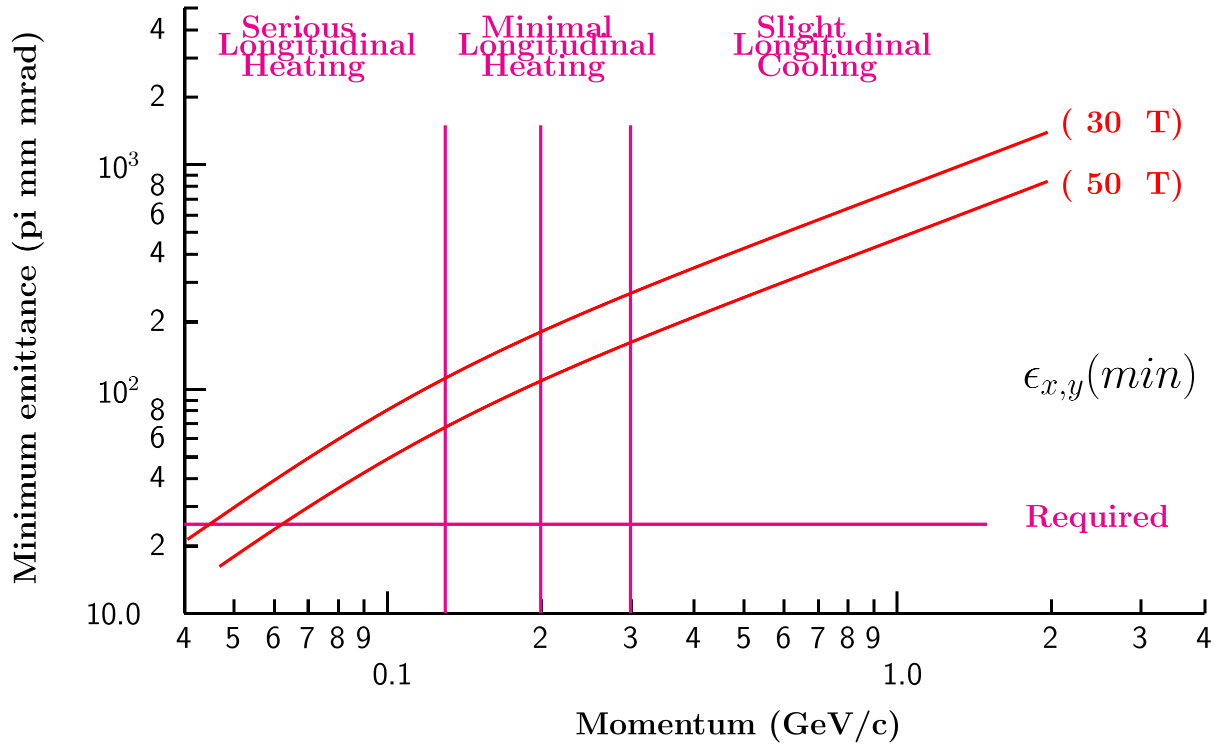
$$\mathcal{A} = 3\sqrt{2} \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \tag{18}$$

Apertures for hydrogen and lithium are plotted vs. energy below. These are very large angles, and if we limit apertures to less than 0.3, then this requirement sets lower energy limits of about 100 MeV ($\approx$ 170 MeV/c) for Lithium, and about 25 MeV ($\approx$ 75 MeV/c) for hydrogen.

$\theta = 0.3$ may be about as large as is possible in a lattice, but larger angles may be sustainable in a continuous focusing system such as a lens or solenoid. is optimistic, as we will see in the tutorial.



Focusing as a function of the beam momentum



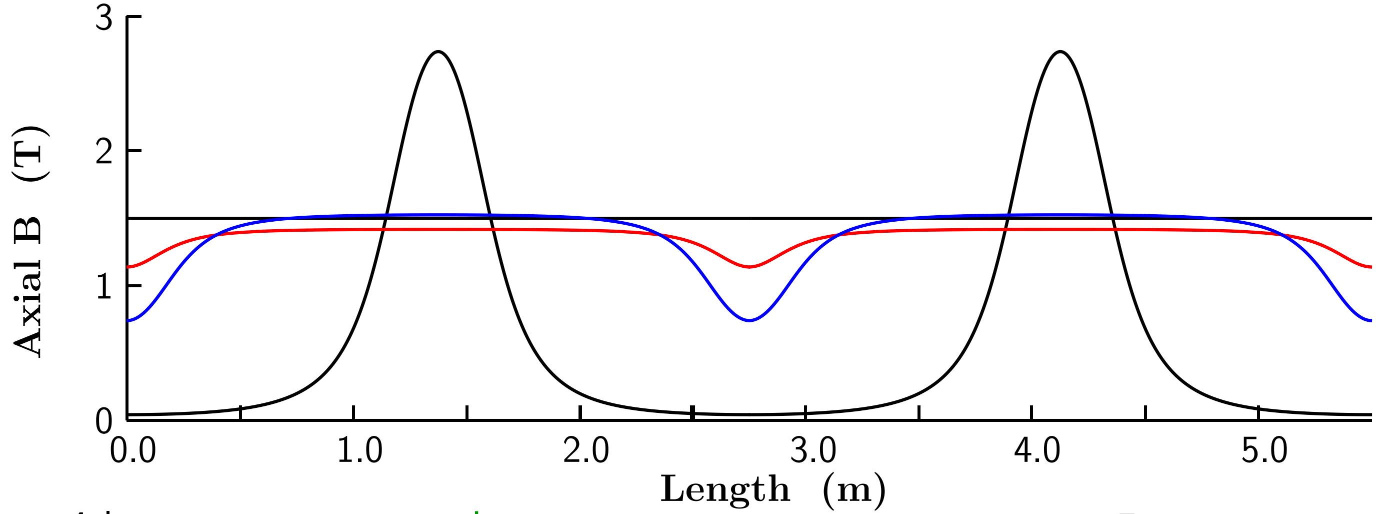
$$\beta_\perp = \frac{2\ [pc/e]}{c\ B_{sol}}$$

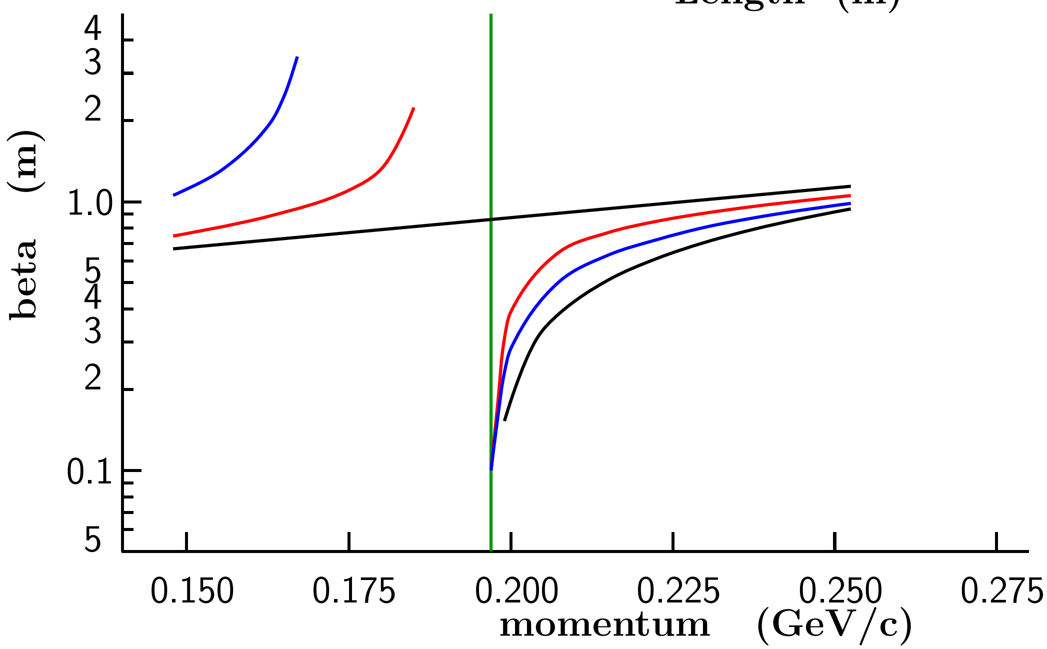
$$\epsilon_{x,y}(min) = C(mat, E)\ \frac{2\ \gamma\ [mc^2/e]_\mu}{B_{sol}\ c} \tag{19}$$

We see that at momenta where longitudinal emittance is not blown up ($\approx$ 200 MeV/c) then even at 50 T the minimum emittance
is $\approx$ 100 μm $>>$ required 25 μm

But if we allow longitudinal heating and use very low momenta (45-62 MeV/c or 9-17 MeV) the muon collider requirements can be met

Decreasing beta in Solenoids by adding periodicity

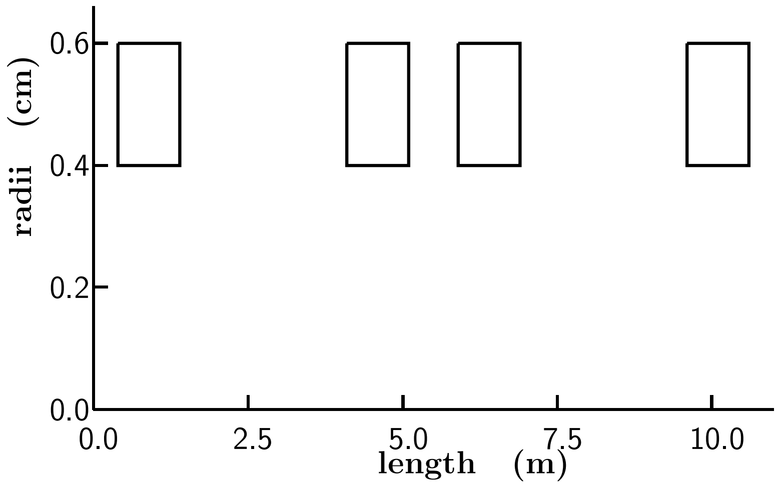


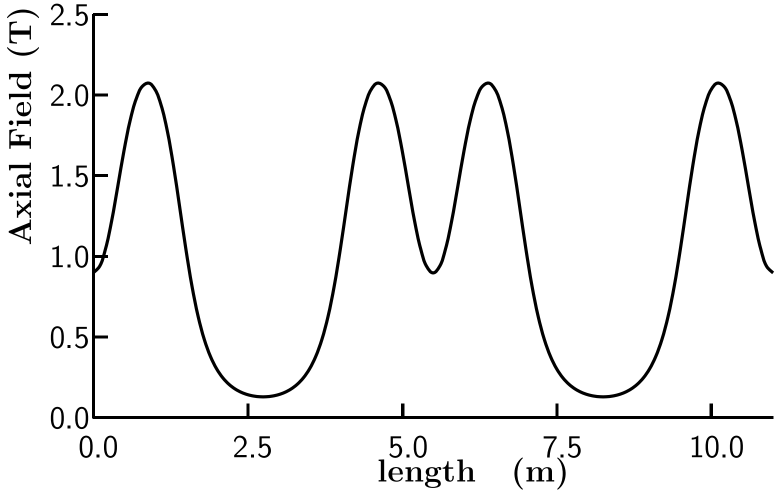


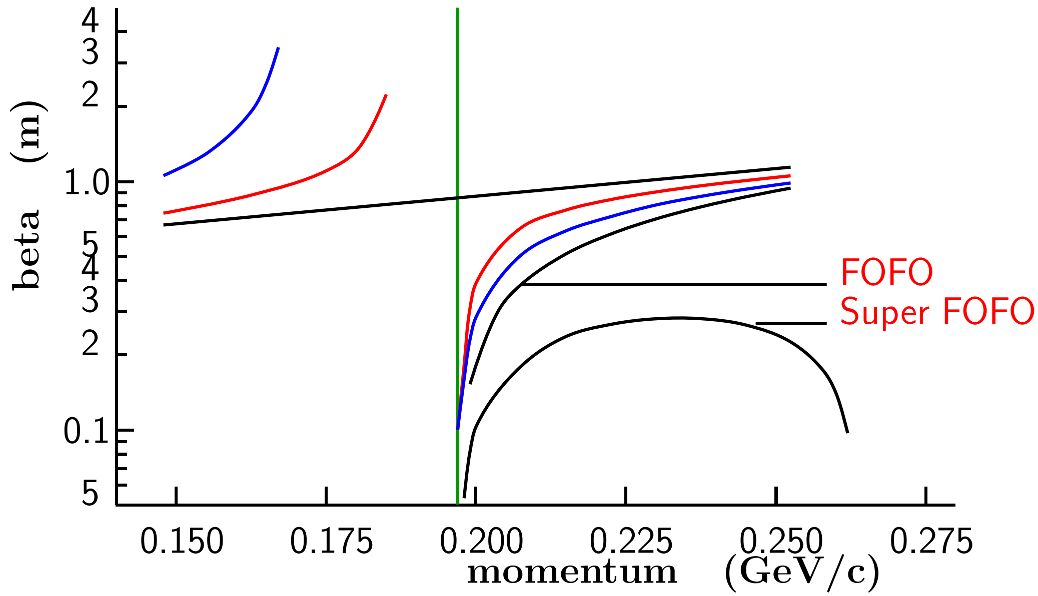
- Determination of lattice betas
  - Track single near paraxial particle through many cells
  - plot θ_x vs x after each cell
  - fit ellipse: $\beta_{x,y}$ $=$ A((x) / A(θ_x)
- Resonances introduced
- Betas reduced locally
- Momentum acceptance small

Super FOFO

Double periodicity

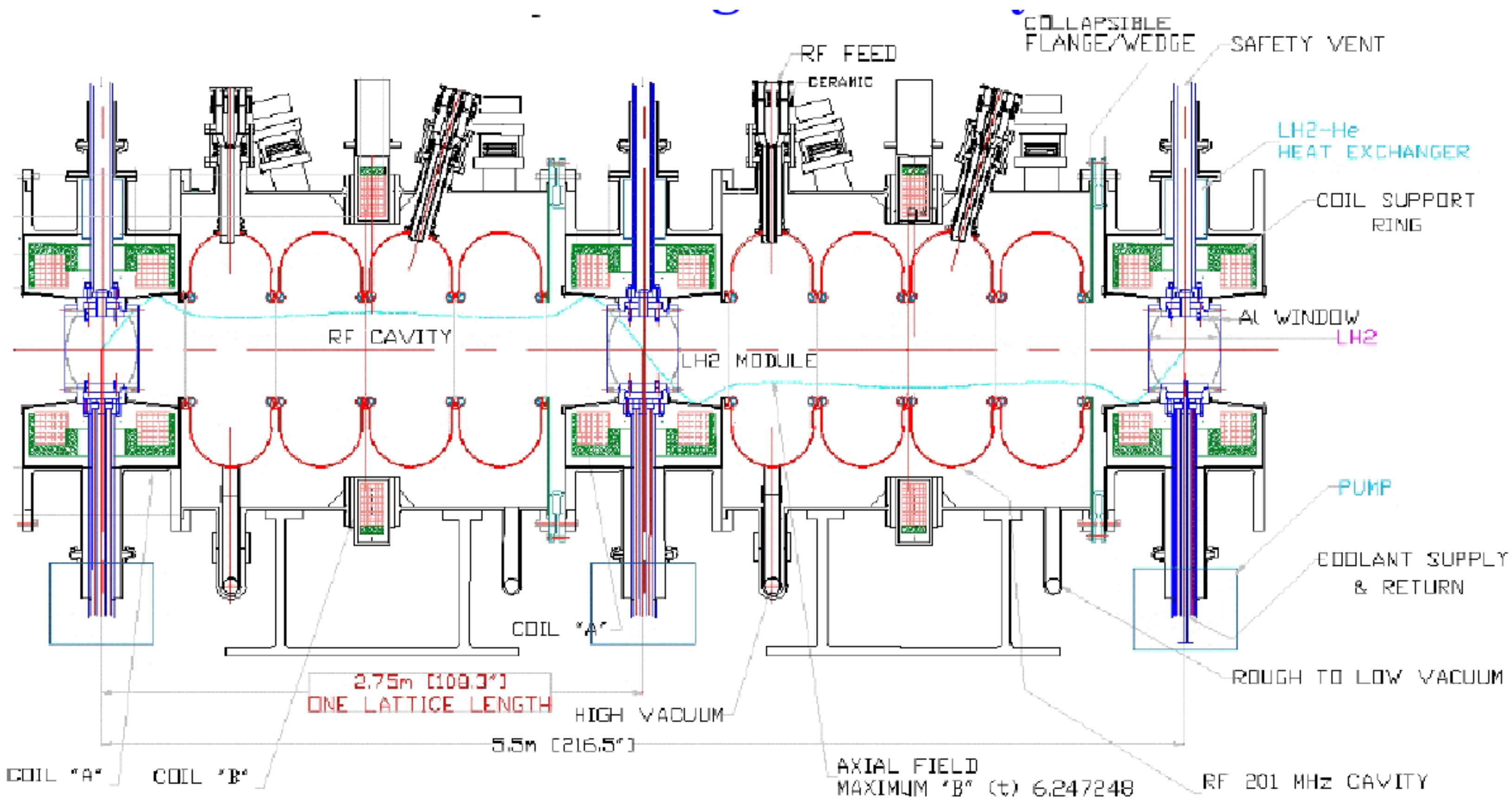






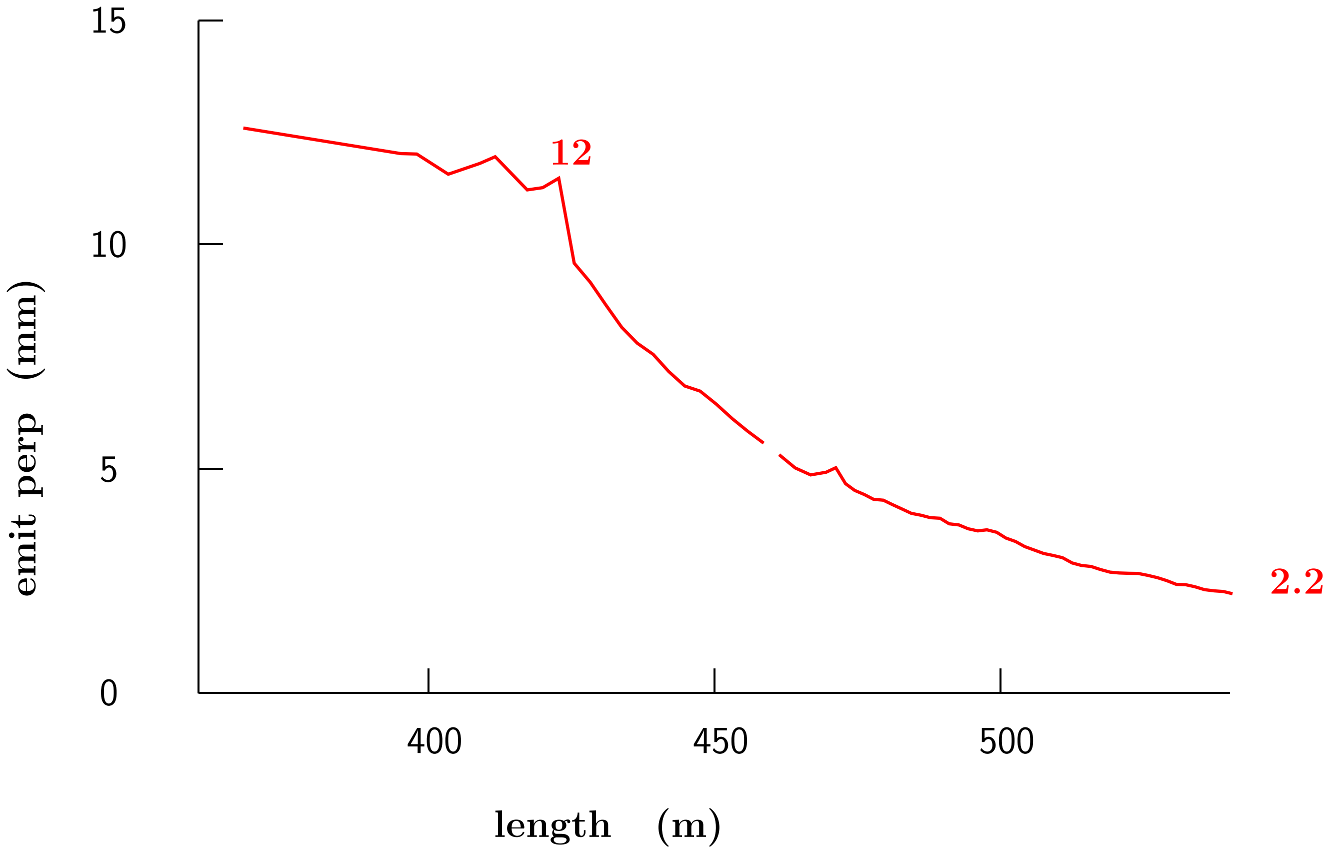
- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

SFOFO Lattice Engineering Study 2 at Start of Cooling



- This is the lattice to be tested in Muon Ionization Cooling Experiment (MICE) at RAL
- In study 2 the lattice is modified vs. length to lower $\beta_\perp$ as ϵ falls
  This keeps σ_θ and ϵ/ϵ_o more or less constant, thus maintains cooling rate

Performance



Conclusion on transverse cooling

- Hydrogen (gas or liquid) is the best material to use
- Cooling requires very large angular acceptances -
- Only realistically possible in solenoid focused systems
- Adding periodicity lowers the $\beta_\perp$ for a given solenoid field
- But periodicity does reduce acceptance
- Final cooling to 25 μm possible at 50 T and low energies
  but longitudinal emittance then rises rapidly

- The biggest technical problem is rf breakdown in magnetic fields
  but solutions are being studied

- Cooling to lower emittances would allow lower N_μ for same $\Delta\nu$
  easing space charge problems in proton driver and cooling

6 LONGITUDINAL IONIZATION COOLING

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} \;=\; \frac{\frac{\Delta\ (\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{20}$$

$$J_6 = J_x + J_y + J_z \tag{21}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). Δp and p refer to the loss of momentum induced by this energy loss.

In electron synchrotrons, with no gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In muon ionization cooling, $J_x = J_y = 1$, but J_z is negative or small.

c.f. Transverse

$$\frac{\Delta\sigma_{p\perp}}{\sigma_{p\perp}} \;=\; \frac{\Delta p}{p}$$

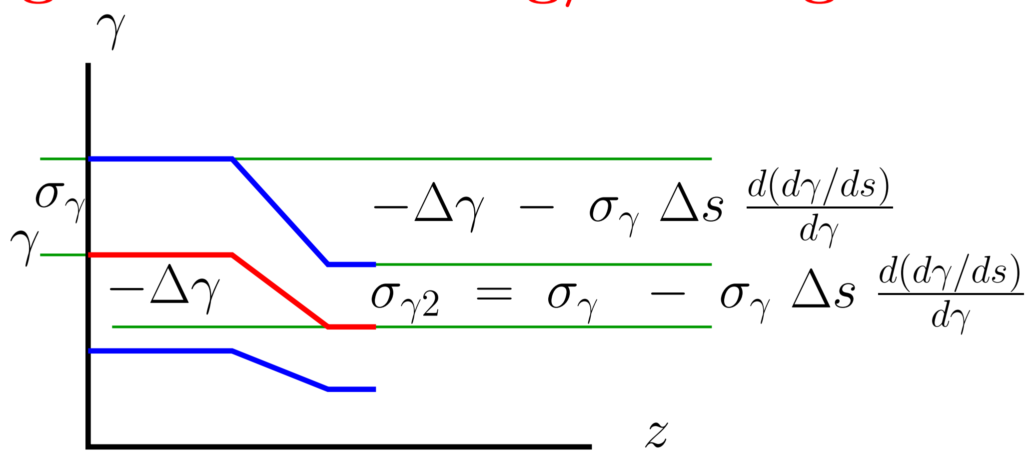
and $\sigma_{x,y}$ does not change, so

$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} \;=\; \frac{\Delta p}{p} \tag{22}$$

and thus

$$J_x \;=\; J_y \;=\; 1 \tag{23}$$

Longitudinal cooling/heating without wedges



The emittance in the longitudinal direction ϵ_z is (eq.2):

$$\epsilon_z \;=\; \gamma\beta_v \frac{\sigma_p}{p}\,\sigma_z \;=\; \frac{1}{m}\,\sigma_p\sigma_z \;=\; \frac{1}{m}\,\sigma_E\sigma_t \;=\; c\;\sigma_\gamma\;\sigma_t$$

where σ_t is the rms bunch length in time, and c is the velocity of light. Drifting between interactions will not change emittance (Louville), and an interaction will not change σ_t, so emittance change is only induced by the energy change in the interactions:

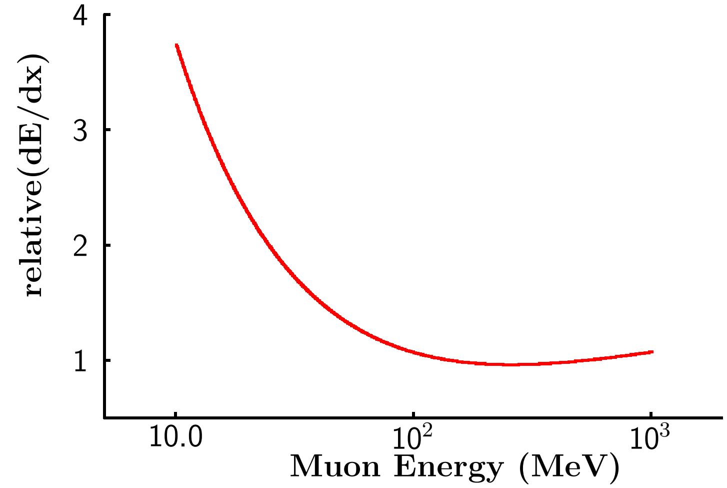
$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma} \;=\; \frac{\sigma_\gamma\;\Delta s\;\frac{d(d\gamma/ds)}{d\gamma}}{\sigma_\gamma} \;=\; \Delta s\,\frac{d(d\gamma/ds)}{d\gamma}$$

and

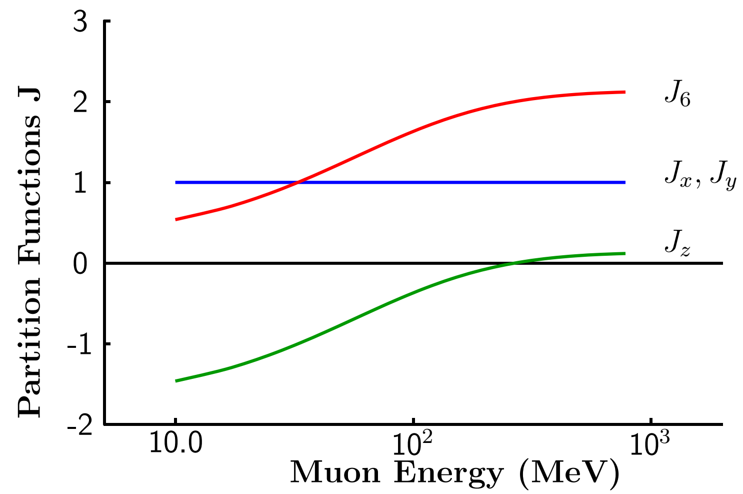
$$\frac{\Delta p}{p} \;=\; \frac{\Delta\gamma}{\beta_v^2\gamma} \;=\; \frac{\ell}{\beta_v^2\gamma}\;\left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$J_z = \frac{\frac{\Delta \epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\left(\Delta s \ \frac{d(d\gamma/ds)}{d\gamma}\right)}{\frac{\Delta s}{\beta_v^2 \gamma} \ \left(\frac{d\gamma}{ds}\right)} = \frac{\left(\beta_v^2 \ \frac{d(d\gamma/ds)}{d\gamma/\gamma}\right)}{\left(\frac{d\gamma}{ds}\right)} \tag{24}$$



A typical relative energy loss as a function of energy is shown above (this example is for Lithium). It is given approximately by:



It is seen that J_z is strongly negative at low energies (longitudinal heating), and is only barely positive at momenta above 300 MeV/c. In practice there are many reasons to cool at a moderate momentum around 250 MeV/c, where $J_z \approx 0$. However, the 6D cooling is still strong $J_6 \approx 2$.

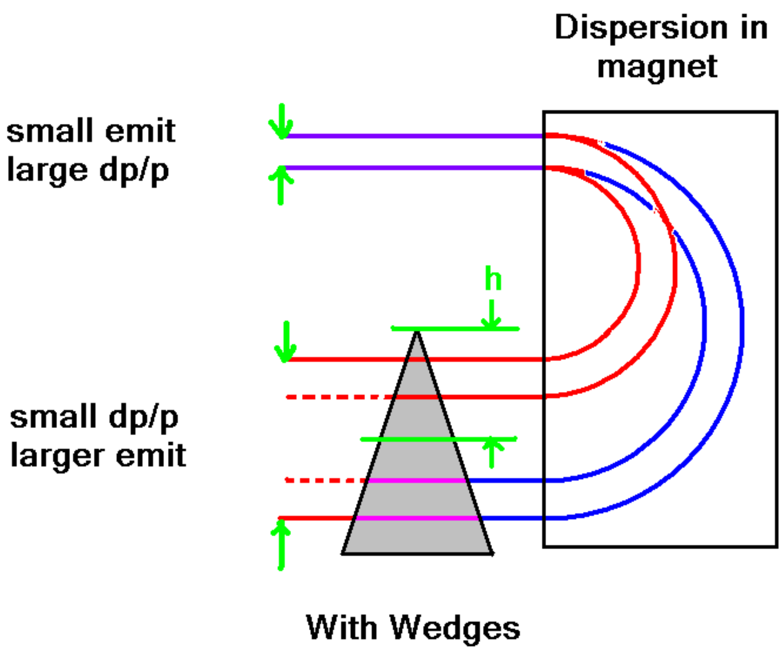
Emittance Exchange

What is needed is a method to exchange cooling between the transverse and longitudinal direction s. This is done in synchrotron cooling if focusing and bending is combined, but in this case, and in general, one can show that such mixing can only increase one J at the expense of the others: J_6 is conserved.

$$\Delta J_x + \Delta J_x + \Delta J_x = 0 \qquad (25)$$

and for typical operating momenta:

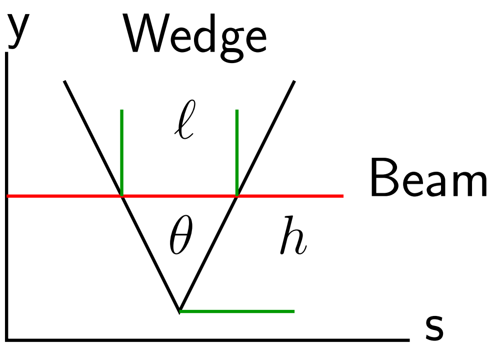
$$J_x \ + \ J_y \ + \ J_z \ = \ J_6 \ \approx \ 2.0 \qquad (26)$$

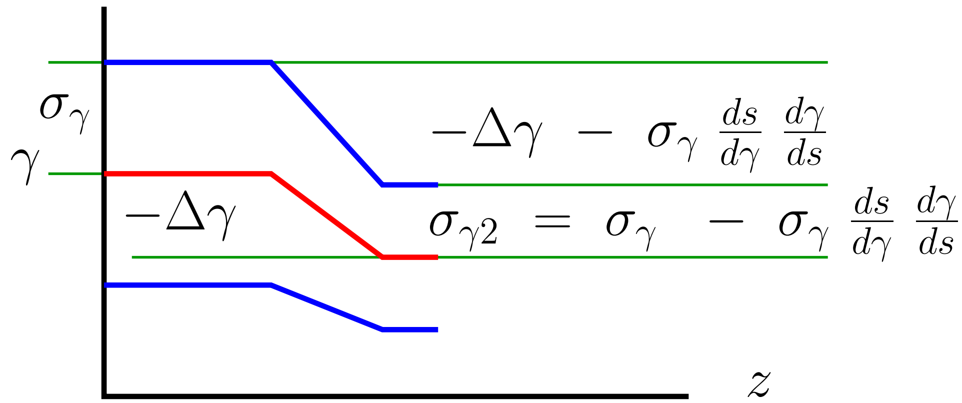


dp/p reduced	But σ_y increased
Long Emit reduced	Trans Emit Increased

"Emittance Exchange"

Longitudinal cooling with wedges and Dispersion





For a wedge with center thickness ℓ and height from center h ($2h\ \tan(\theta/2) = \ell$), in dispersion D ($D = \frac{dy}{dp/p}$: $D = \beta_v^2\ \frac{dy}{d\gamma/\gamma}$) (see fig. above):

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma} = \frac{\sigma_\gamma \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right)}{\sigma_\gamma} = \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right) = \left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\ \gamma}\left(\frac{d\gamma}{ds}\right)$$

and

$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$\Delta J_z(\text{wedge}) = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\ \gamma}\left(\frac{d\gamma}{ds}\right)}{\frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)} = \frac{D}{h} \quad (\text{for simple bend \& gas } \Delta J_z(\text{wedge}) = 1) \tag{27}$$

$$J_z = J_z(\text{no wedge}) + \Delta J_z(\text{wedge}) \tag{28}$$

Effect on transverse cooling

But from eq.25,

$$\Delta J_x + \Delta J_x + \Delta J_x = 0$$

for any finite $J_z(\mathrm{wedge})$, J_x or J_y will change in the opposite direction.

And now we have to include $J_{x,y}$ in the formulae for rate of cooling and equilibria

$$\frac{d\epsilon_{x,y}}{\epsilon_{x,y}} = J_{x,y} \frac{dp}{p}$$

$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{J_{x,y}\ \beta_v}\ C(mat, E)$$

Longitudinal Heating Terms

Since $\epsilon_z = \sigma_\gamma\ \sigma_t\ c$, and t and thus σ_t is conserved in an interaction

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

Straggling:

$$\Delta(\sigma_\gamma) \approx \frac{\Delta\sigma_\gamma^2}{2\sigma_\gamma} \approx \frac{1}{2\sigma_\gamma}\, 0.06\, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho\, \Delta s$$

$\Delta E = E\ \beta_v^2\ \frac{\Delta p}{p}$, so:

$$\Delta s = \frac{\Delta E}{dE/ds} = \frac{1}{dE/ds} E\ \beta_v^2\ \frac{\Delta p}{p}$$

so

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{0.06}{2\sigma_\gamma^2}\, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho\, \frac{\beta_v^2\ E}{dE/ds}\, \frac{\Delta p}{p}$$

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z} = -\ J_z\, \frac{dp}{p}$$

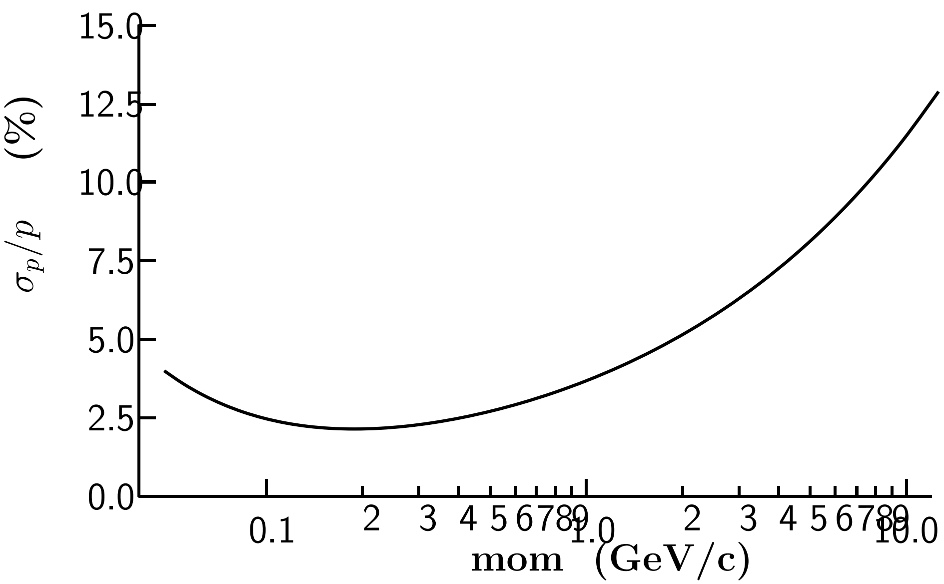
giving an equilibrium:

$$\frac{\sigma_p}{p} = \left(\left(\frac{m_e}{m_\mu}\right) \sqrt{\frac{0.06\ Z\ \rho}{2\ A\ (d\gamma/ds)}}\right) \sqrt{\frac{\gamma}{\beta_v^2} \left(1 - \frac{\beta_v^2}{2}\right) \frac{1}{J_z}} \tag{29}$$

For Hydrogen, the value of the first parenthesis is ≈1.36 %.

Without coupling, J_z is small or negative, and the equilibrium does not exist. But with equal partition functions giving $J_z \approx 2/3$ then this expression, for hydrogen, gives: the values plotted below.

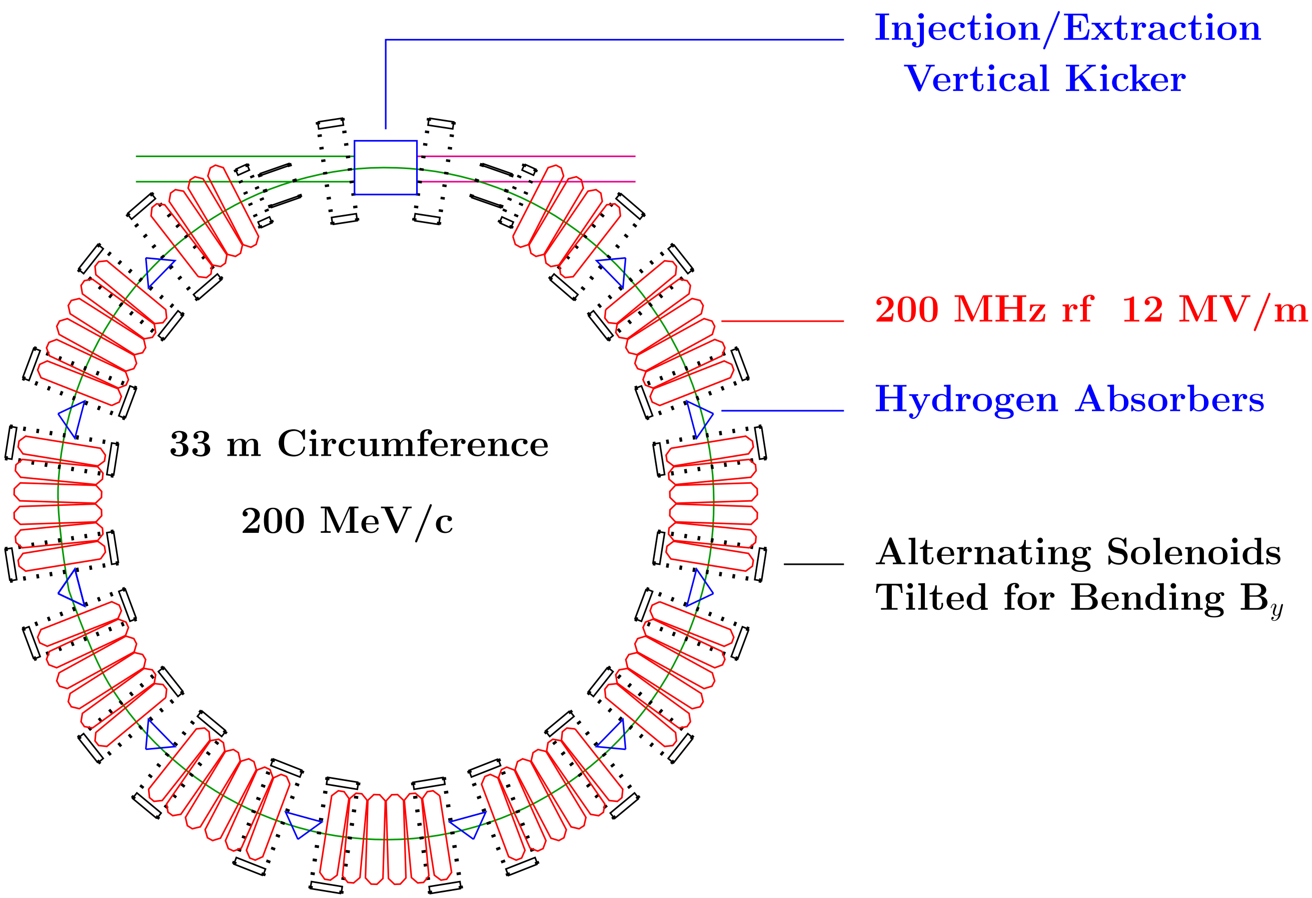
The following plot shows the dependency for hydrogen



It is seen to favor cooling at around 200 MeV/c, but has a broad minimum.

Example: RFOFO Ring

R.B. Palmer R. Fernow J. Gallardo[1], and Balbekov[2]



[1] Fernow and others: MUC-232, 265, 268, & 273
[2] V.Balbekov "Simulation of RFOFO Ring Cooler with Tilted Solenoids" MUC-CONF–0264

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Performance

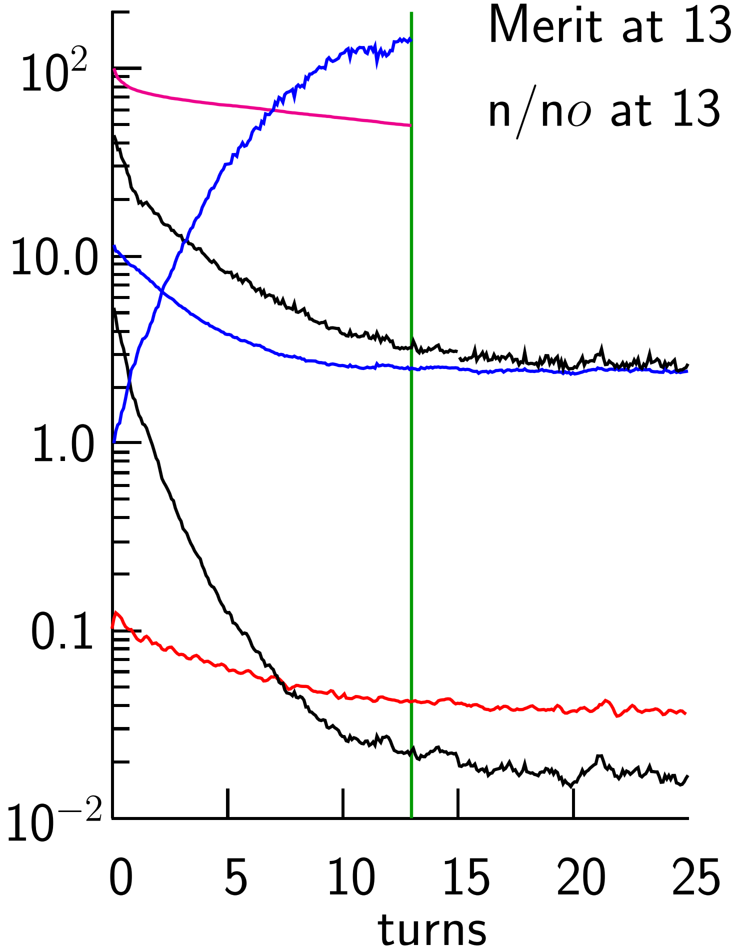
Using Real Fields, but no windows or injection insertion

$$\text{Merit} = \frac{n}{n_o}\,\frac{\epsilon_{6,o}}{\epsilon_6} = \frac{\text{Initial phase density}}{\text{final phase density}}$$

$n/n_o =$ 1543 / 4494

Merit at 13 turns 139 Falls after 13 turns from decay loss

n/no at 13 turns 0.50



$\epsilon \parallel$ 43.9 to 2.65 ($\pi\ mm$)
$\epsilon \perp$ 11.4 to 2.43 ($\pi\ mm$)

dp/p 10.2 to 3.6 %
$\epsilon 6$ 5.3 to 0.017 $(\pi\ mm)^3$

Longitudinal Cooling Conclusion

- Good cooling in 6 D in a ring
  - But injection/extraction difficult
  - Requires short bunch train
- Also good 6D cooling in HP Gas Helix (not discussed here)
  - But difficult to introduce appropriate frequency rf
  - And questions about use of gas with an ionizing beam
- Converting Ring cooler to a large Helix (Guggenheim)
  - Solves Injection/extraction problem
  - Solves bunch train length problem
  - Allows tapering to improve performance
  - But more expensive than ring