Muon Colliders

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Contents

1 INTRODUCTION **2**

2 CURRENT BASELINE DESIGNS **13**

3 R&D AND EXPERIMENTS **27**

4 DEFINITIONS AND UNIT CONVENTIONS **34**

5 SOLENOID FOCUSING **38**

6 TRANSVERSE IONIZATION COOLING **66**

7 LONGITUDINAL IONIZATION COOLING **79**

1 INTRODUCTION

Why a muon collider ?

Leptons (e.g. e or μ) are 'better' than protons

- Protons are made of many pieces (quarks and gluons)
- Each carries only a fraction of th proton energy
- Fundamental interactions occur between these individual pieces
- And the interaction energies are only fractions ($\approx 1/10$) of the proton energies

- Leptons (e's and μ's) are point like
- Their interaction energies are their whole energies

$$\text{E(3 TeV } e^+e^- \text{ CLIC or } \mu^+\mu^-) \ \equiv \ 2 \times \text{E(14 TeV } p\bar{p} \text{ LHC)}$$

- The energy and quantum state is known for e^+e^-or $\mu^+\mu^-$ but unknown for the parton-parton interaction with protons

S-Channel advantage of muons over electrons

- When all the collision energy $\longrightarrow$ a single state,
  it is called the "S-Channel"
- A particularly interesting S-Channel interaction would be

$$e^+e^- \ \rightarrow \ Higgs \qquad\qquad or \qquad\qquad \mu^+\mu^- \ \rightarrow \ Higgs$$

The cross sections σ for these interactions

$$\sigma \ \propto \ m^2$$

so

$$\sigma(e^+e^- \ \rightarrow \ H) \ \approx \ 40,000 \ \times \ \sigma(\mu^+\mu^- \ \rightarrow \ H)$$

Muons generate less 'Beamstrahlung'

- When high energy electrons in one bunch pass through the other bunch they see the EM fields of the other moving bunch
- These fields are enough to generate synchrotron radiation (called beamstahlung)
- So the energy of the collision is not so well known
  $$\sigma_E \approx 30\% \text{ (at 3 TeV } e^+e^- \text{ CLIC)}$$
- And the luminosity at the requires energy is less
  $$\mathcal{L} \approx 1/3 \text{ (for } E \pm 1\% \text{ at 3 TeV CLIC)}$$
- For muons: synchrotron radiation ($\propto 1/m^3$) is negligible giving:
  $$\sigma_E \approx 0.1\ \%$$
- This could be a particular advantage for $\mu^+\mu^- \rightarrow H$ because with a narrow enough σ_E one could measure the width of a narrow Higgs

Why are Linear colliders linear?

- Earlier election positron colliders, like proton colliders, were rings
  But proposed high energy electron colliders are linear

- Synchrotron radiation of particles bent in the ring magnetic field

$$\Delta E(\text{per turn}) \;=\; \left(\frac{4\pi\ mc^2}{3}\right)\ \left(\frac{r_o}{\rho}\right)\ \beta_v^3\gamma^4 \tag{1}$$

$$\rho \;\propto\; \frac{\beta\gamma}{B} \tag{2}$$

$$\Delta E(\text{per turn}) \;\approx\!\propto\;\; B\ \gamma^3 \tag{3}$$

- For electrons (m$\approx$ 0.5 MeV) this is untenable for $E >> 0.1$ TeV
- Above this (LEP's) energy, electron colliders must be linear
- But for muons (m$\approx$ 100 MeV) rings are ok up to around 20 TeV
  equivalent to a proton collider of 200 TeV

The advantages of rings

- A 1 TeV muon: lifetime = $\gamma\tau \approx 10,000 \times 2\mu s \approx 20$ msec goes 1500 km
- For $<B>$=10 T, a 1 TeV ring will have a circumference of

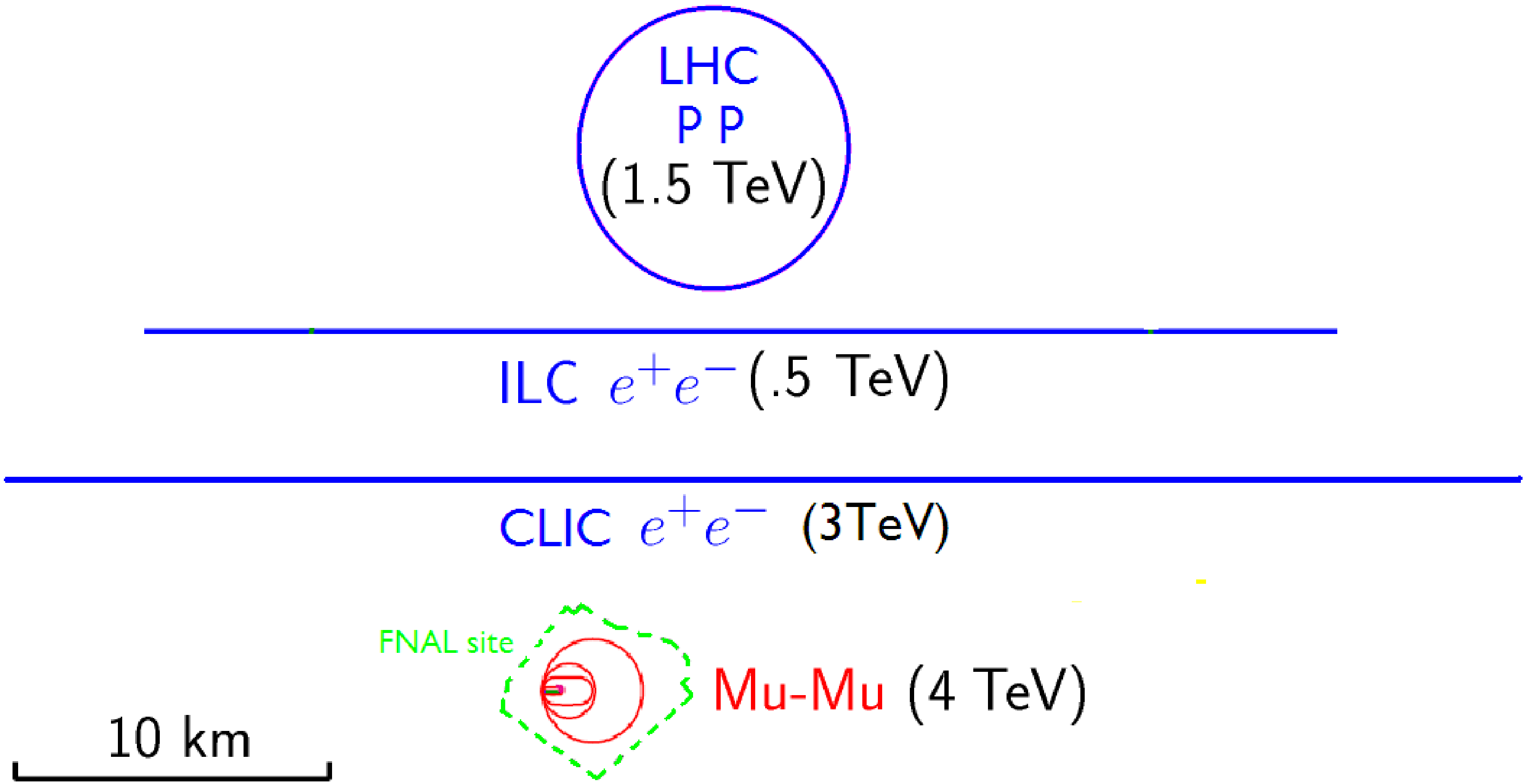
$$C = \frac{2\pi\ [pc/e]}{c\ B} = \frac{2\pi\ 10^{12}}{3\ 10^{8}\ 10} = 2 \quad \mathrm{km}$$

so they will go round , on average, 1500/2=700 times

- Spot much larger than linear collider's $\rightarrow$ easier tolerances
- Beam, and wall, power can be less than for e^+e^-
- There can be 2 or more Detectors

- Acceleration must also be fast, in a number of turns $<<$ 700 still much easier than in the single pass required for e^+e^-

So they are much smaller



And hopefully cheaper

Luminosity Dependence

$$\mathcal{L} = n_{\rm turns}\ f_{\rm bunch}\ \frac{N_\mu^2}{4\pi\sigma_\perp^2} \tag{4}$$

$$\Delta\nu = \frac{N r_o\ \beta^*}{4\pi\gamma\sigma_\perp} = \frac{r_o}{4\pi}\frac{N_\mu}{\epsilon_\perp} \tag{5}$$

$\epsilon_\perp$ is the normalized rms emittance

$$\mathcal{L} \propto B_{\rm ring}\ P_{\rm beam}\ \ \Delta\nu\ \frac{1}{\beta^*}$$

- Lower emittances do not directly improve Luminosity/Power

The same luminosity easy with $\mu - p$

- Probably with another ring
- The event rate per bunch crossing is now significant but $\ll$ LHC

Why NOT a $\mu^+\mu^-$ collider

- Make muons from the decay of pions
- With pions made from protons on a target
- To avoid excessive proton power, we must capture a large fraction of pions made

- Capture both forward and backward decays and loses polarization
- The phase space of the pions is now very large:
  - a transverse emittance of 20 pi mm and
  - a longitudinal emittance of 2 pi m

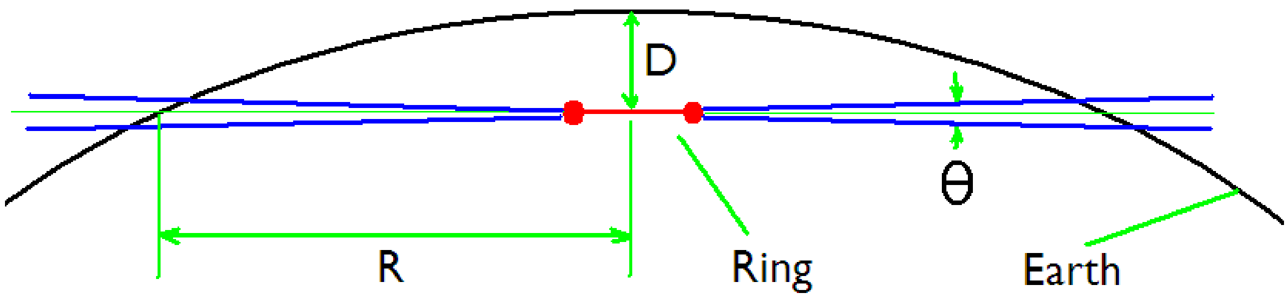
- Emittances must be somehow be cooled by a factor $\approx\ 10^7$!
  - $\approx$ 1000 in each transverse direction and
  - $\approx$ 40 in longitudinal direction

Cooling Methods

- Electrons are typically cooled (damped) by synchrotron radiation but muons radiate too little ($\Delta E \ \propto \ 1/m^3$)
- Protons are typically cooled by:
  - a co-moving cold electron beam   too slow
  - Or by stochastic methods   too slow

- Ionization cooling   is probably the only hope
- Although optical stochastic cooling has been studied   does not look good

Neutrino Radiation Constraint



$$\text{Radiation} \;\propto\; \frac{E_\mu \; I_\mu \; \sigma_\nu}{\theta \;\; R^2} \;\propto\; \frac{I_\mu \; \gamma^3}{D}$$

$$\text{Radiation} \;\propto\; \frac{\mathcal{L} \; \beta_\perp}{\Delta\nu \;\; < \mathrm{B} >} \; \frac{\gamma^2}{D} \tag{6}$$

For fixed $\Delta\nu$, $\beta_\perp$ and $< B >$; and $\mathcal{L} \propto \gamma^2$:

$$\text{Radiation} \;\propto\; \frac{\beta_\perp}{\Delta\nu \;\; < B > \;\; D} \; \gamma^4 \tag{7}$$

For 3 TeV: D=135 m R=40 Km $\beta_\perp$=5 mm

Conclusions on 'Why a muon collider"

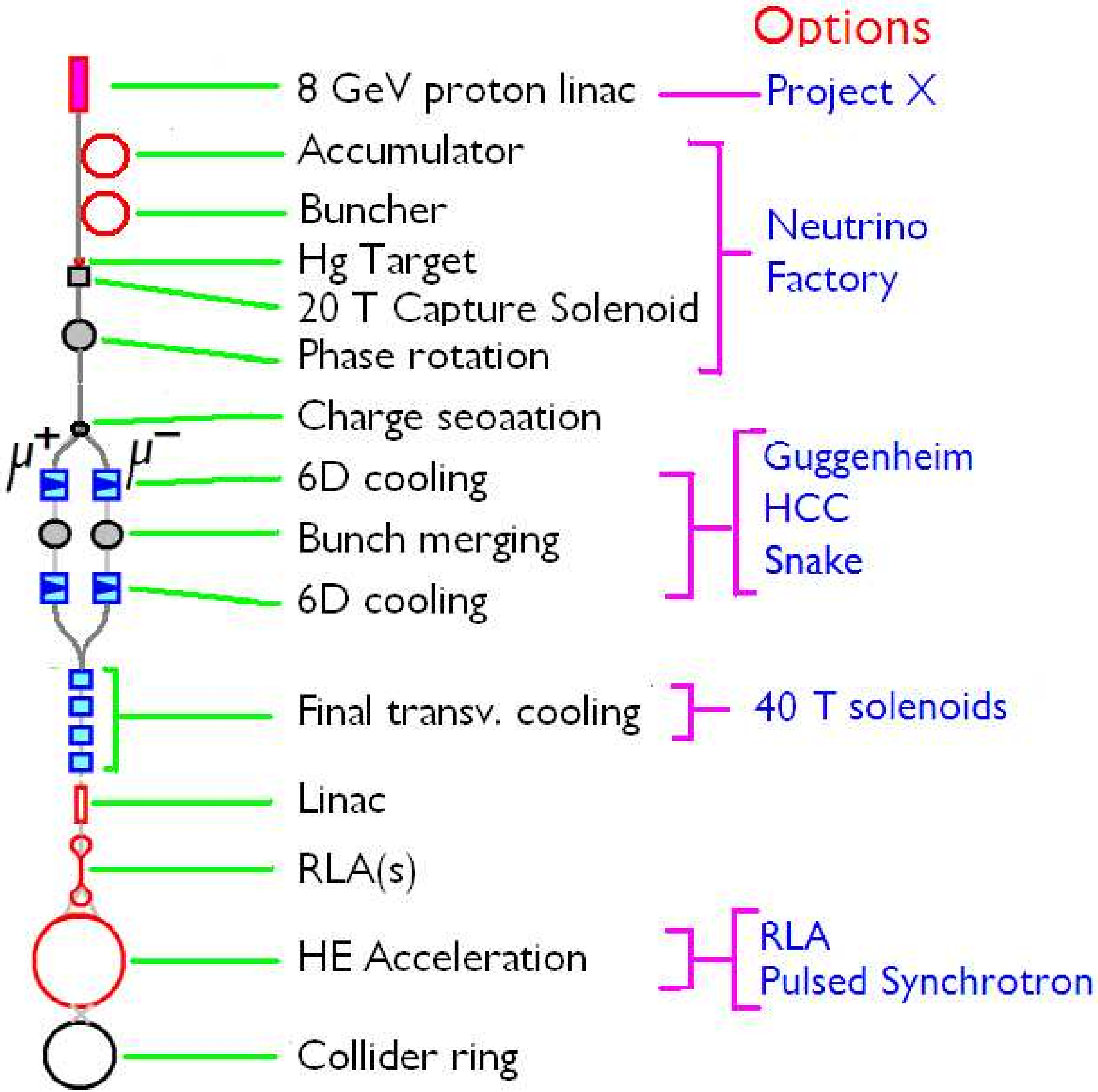
- Point like interactions as in linear e^+e^-
  effective energy 10 times hadron machines
- Negligible synchrotron radiation $\rightarrow$ Acceleration in rings
  - Less rf   Hopefully cheaper
- Collider is a Ring $\approx$ 1000 crossings per bunch
  - Larger spot $\rightarrow$ Easier tolerances
  - 2 or more Detectors
  - Small footprint   Hopefully cheaper
- Negligible Beamstrahlung   Narrow energy spread
- 40,000 greater S channel Higgs   Enabling study of widths
- But serious challenge to cool the muons by $\gg 10^7$ times
- Neutrino radiation a significant problem at very high energies
- CLIC better understood, but is it affordable?

2 CURRENT BASELINE DESIGNS

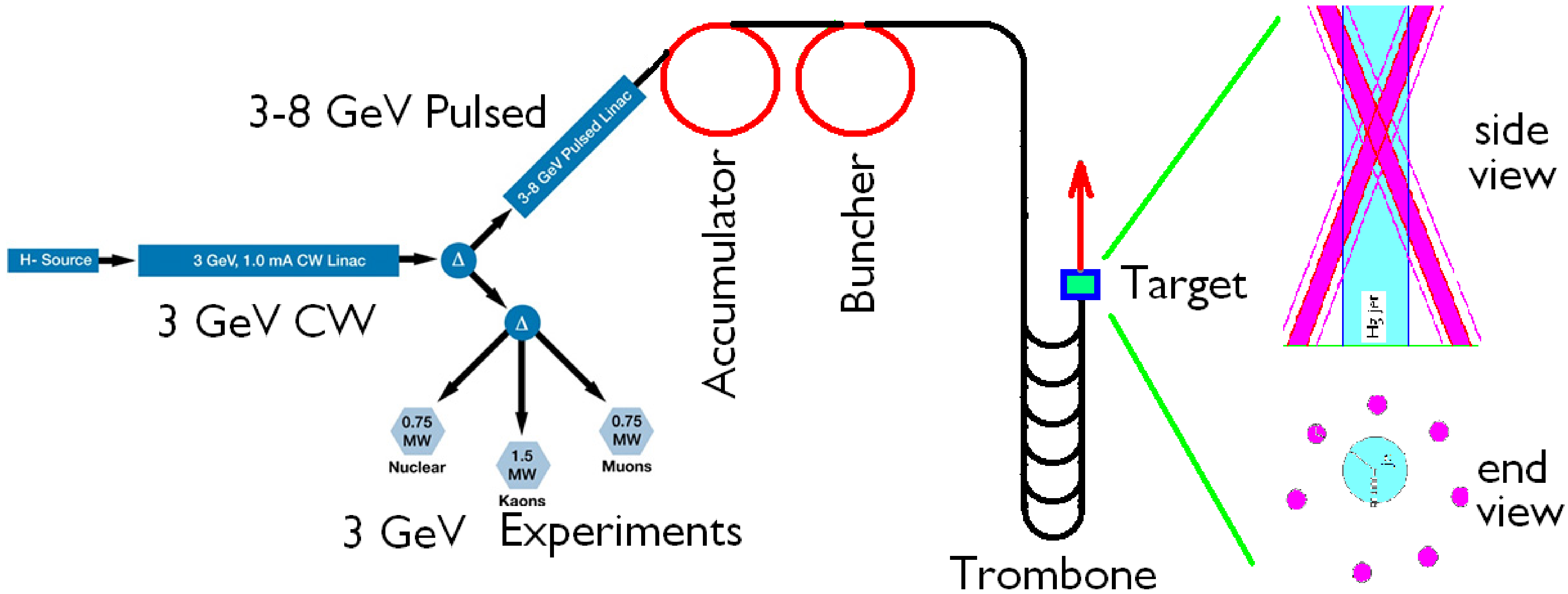
C of m Energy	1.5	3	TeV
Luminosity	1	4	10^{34} cm^{2}sec^{-1}
Muons/bunch	2	2	10^{12}
Total muon Power	7.2	11.5	MW
Ring $<$bending field$>$	6.04	8.4	T
Ring circumference	2.6	4.5	km
β^* at IP $= \sigma_z$	10	5	mm
rms momentum spread	0.1	0.1	%
Depth	135	135	m
Repetition Rate	15	12	Hz
Proton Driver power	4	3.2	MW
Muon Trans Emittance	25	25	pi μm
Muon Long Emittance	72,000	72,000	μm

- Emittance & bunch intensities same for both examples
- 3 TeV luminosity 2 $\times$ CLIC's (for dE/E $<$ 1%)

Schemati


Options
8 GeV proton linac
Project X
Accumulator
Buncher
Hg Target
20 T Capture Solenoid
Phase rotation
Neutrino
Factory
Charge seoaation
μ^+
μ^-
6D cooling
Bunch merging
6D cooling
Guggenheim
HCC
Snake
Final transv. cooling
40 T solenoids
Linac
RLA(s)
HE Acceleration
RLA
Pulsed Synchrotron
Collider ring

Proton Driver e.g. Project X

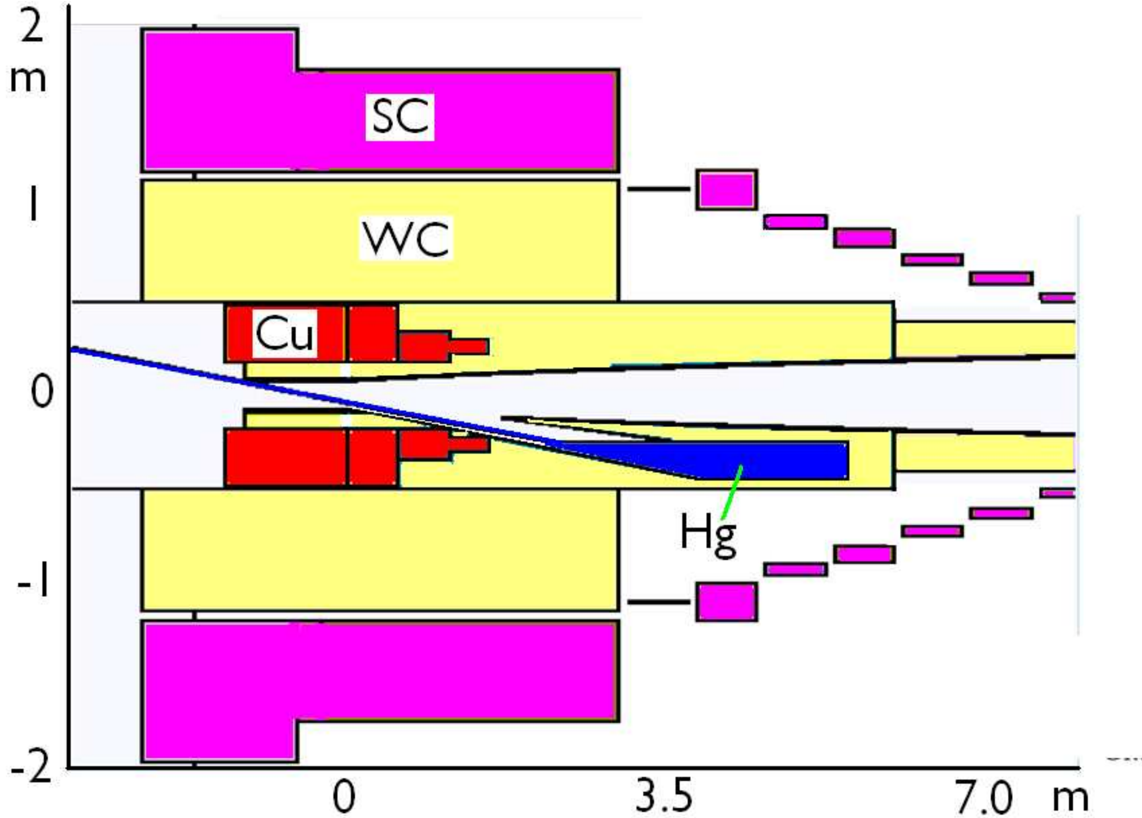


Task Force on Project X upgrades Gollwitzer

- Upgrade CW linac to 5 mA
- 3-8 GeV Pulsed Linac
- Accumulator, Buncher, and Trombone (Ankenbrandt)

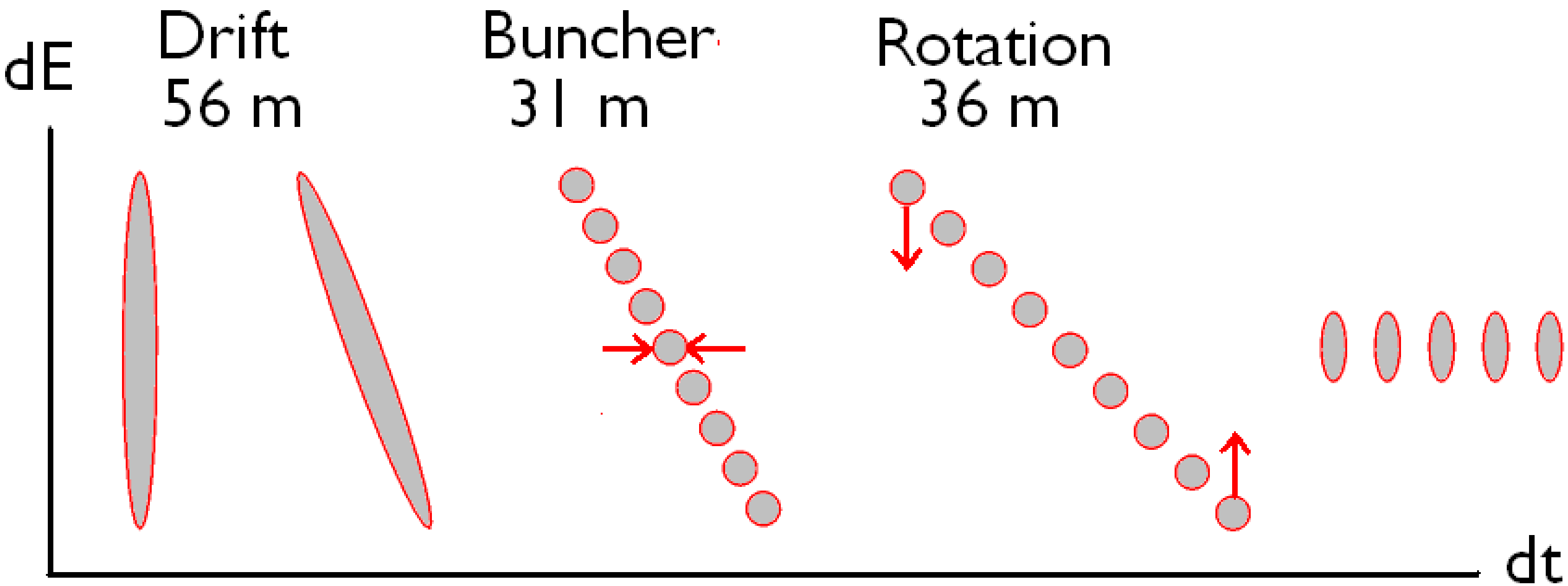
Target & Capture

20 T Hybrid with increased Shielding



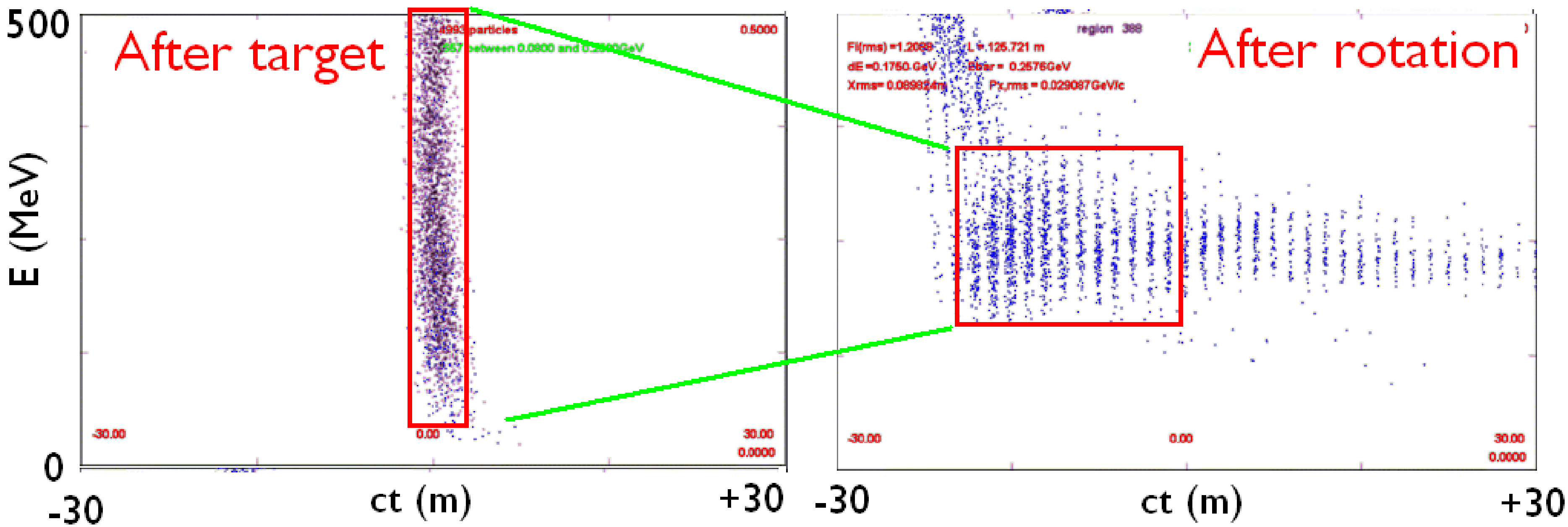
- Copper coil gives 6 T
- Super-conducting solenoid give 14 T, tapering to 3 T
- Tungsten Carbide in water shielding for 4 MW 8 GeV beam
  Cu coil uses 15 MW SC coil OD is 4 m

Phase Rotation→12 bunches
(David Neuffer)



- ΔE small Δt $\rightarrow$ small ΔE larger Δt

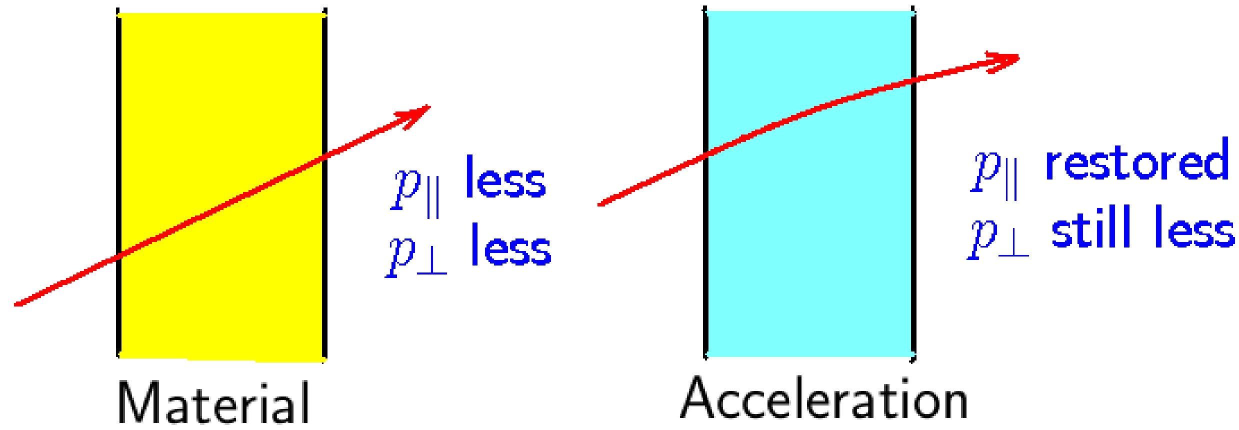
Simulation



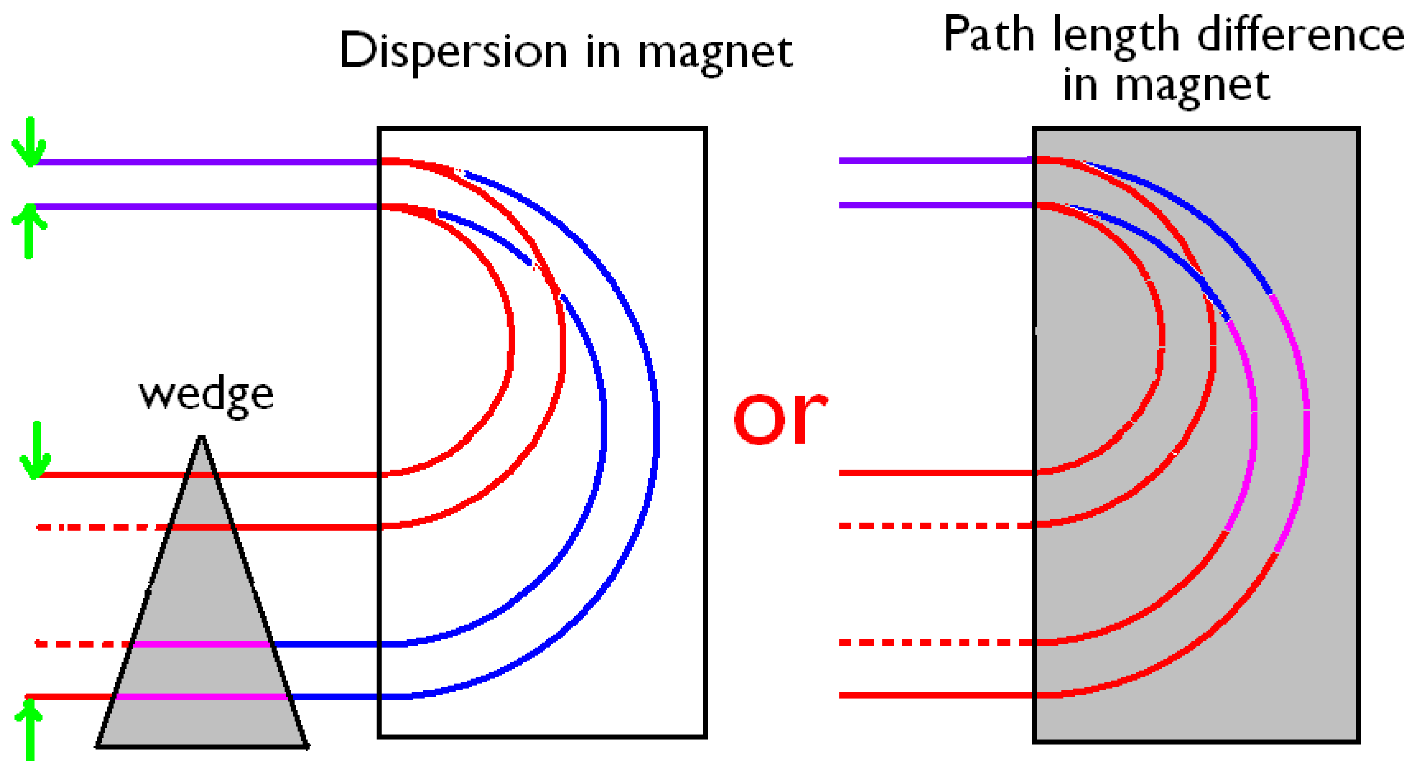
Captures $\approx$ 48%
of longitudinal phase space

Ionization Cooling

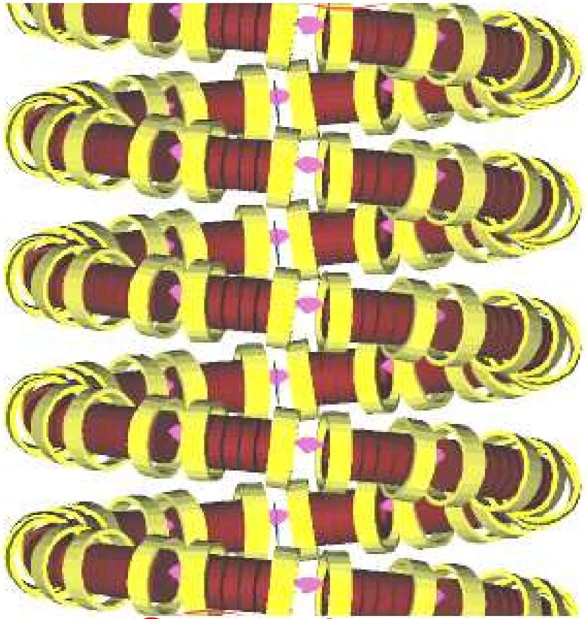
Transverse
(4D)



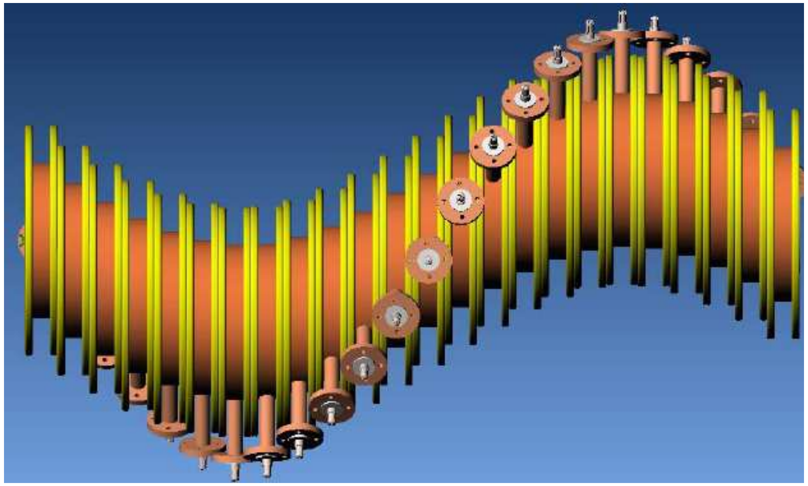
Longitudinal
(6D)



3 candidate 6D cooling lattices

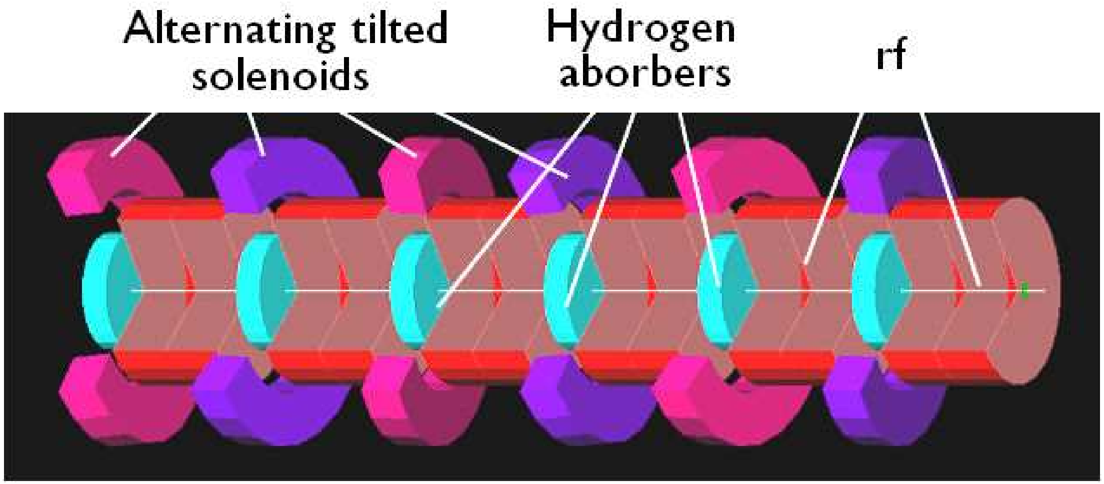


Guggenheim



Helical Cooling Channel

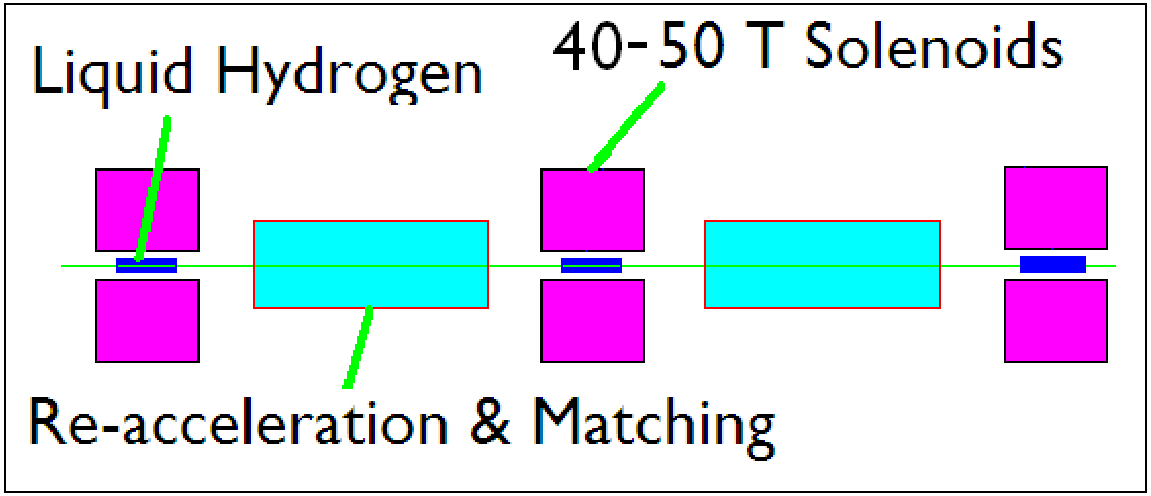
Snake



- All simulated All have problems/limitations
- I will use Guggenheim as example

Final Transverse Cooling in High Field Solenoids

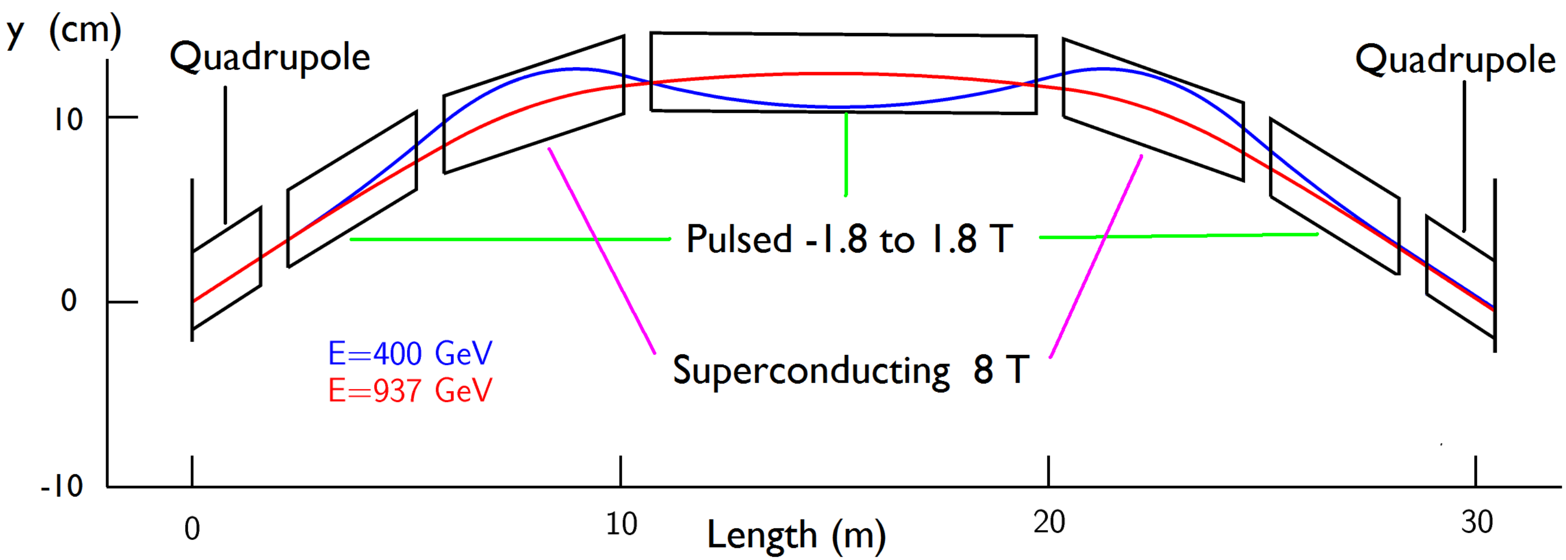
- Lower momenta allow transverse cooling to required low transverse emittance, but long emittance rises: Effectively reverse emittance exchange



- Need 12 40 T (or more 30 T) solenoids
- ICOOL Simulation of cooling in solenoids
- Simulation of re-acceleration/matching started
  - 45 T hybrid at NHMFL, but uses 25W
  - 33 T all SC under construction at NHMFL
  - 40 T 'experiment' under construction

Acceleration

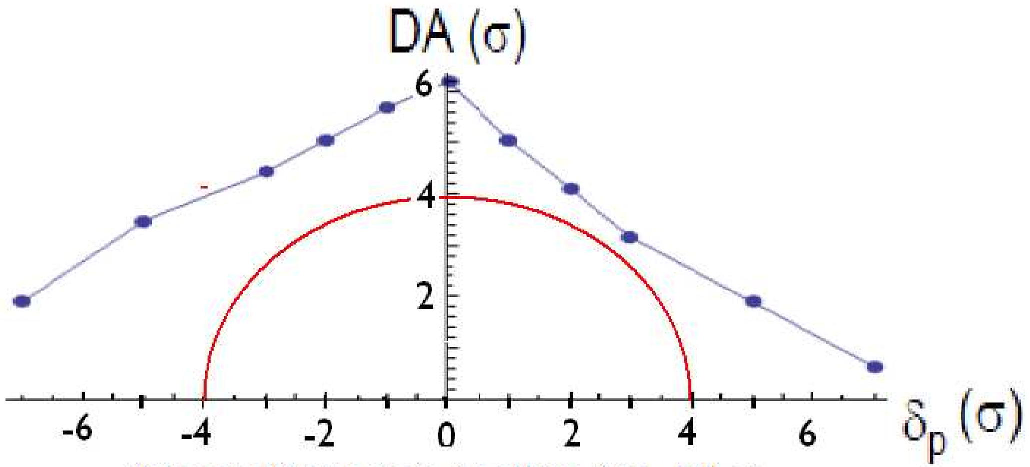
- Sufficiently rapid acceleration is straightforward in Linacs and Recirculating linear accelerators (RLAs)
  Using ILC-like 1.3 GHz rf
- Lower cost solution would use Pulsed Synchrotrons
- Pulsed synchrotron 30 to 400 GeV
  (in Tevatron tunnel)
- Hybrid SC & pulsed magnet synchrotron 400-900 GeV
  (in Tevatron tunnel)



Collider Ring

- 1.5 TeV (c of m) Design

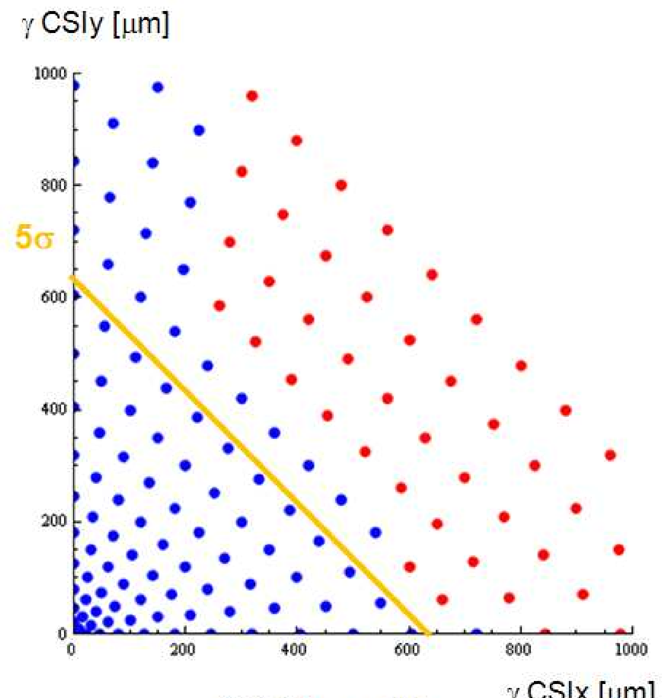
Meets requirements
at 1.5 TeV



"Diagonal" Dynamic Aperture (Ax=Ay) vs. (constant) momentum deviation in the presence of beam-beam effect (ξ = 0.09/IP) for normalised emittance $\varepsilon_{\perp N}$= 25 μm

- 3 TeV (c of m) Design

Less studied
but appears good



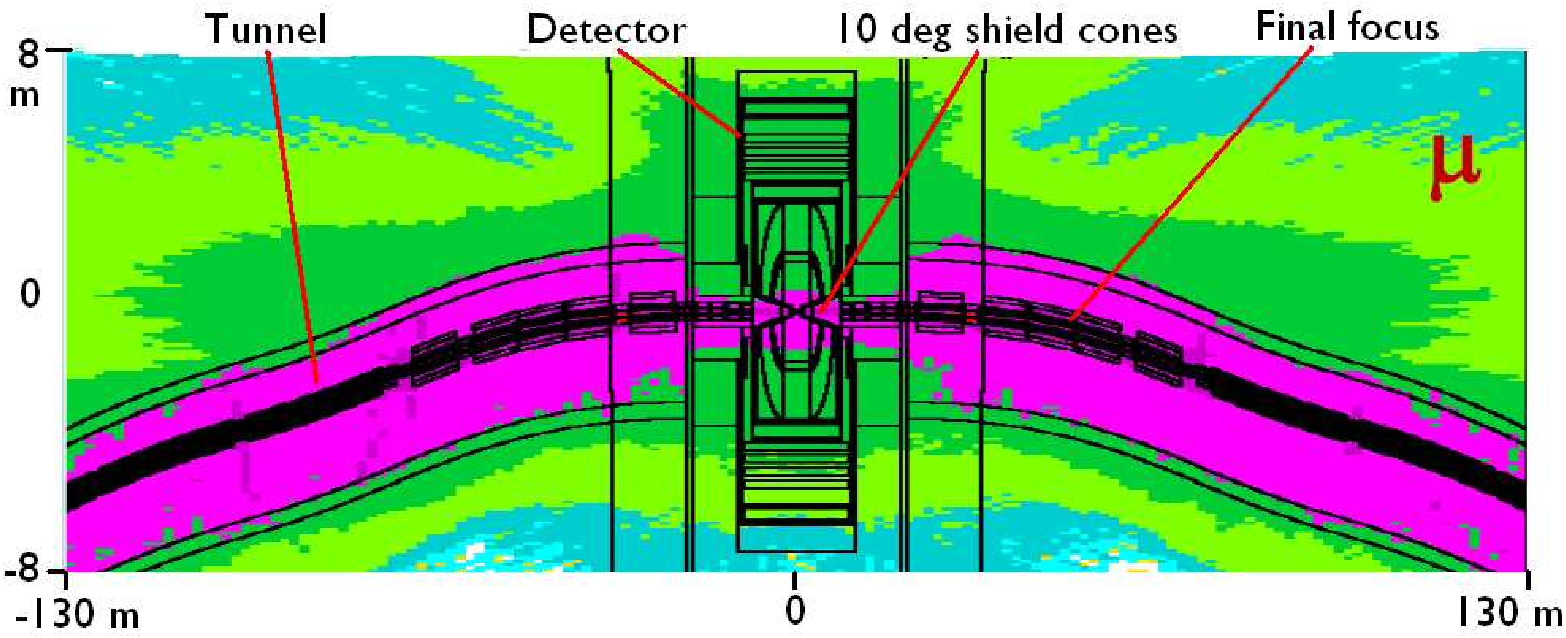
1024 turns DA

ESTIMATED WALL POWER

	Len	Static	Dynamic	—	—	—	Tot
		4^o	rf	PS	4^o	20^o	
	m	MW	MW	MW	MW	MW	MW
p Driver (SC linac)							(20)
Target and taper	16			15.0	0.4		15.4
Decay and phase rot	95	0.1	0.8		4.5		5.4
Charge separation	14						
6D cooling before merge	222	0.6	7.2		6.8	6.1	20.7
Merge	115	0.2	1.4				1.6
6D cooling after merge	428	0.7	2.8			2.6	6.1
Final 4D cooling	78	0.1	1.5			0.1	1.7
NC RF acceleration	104	0.1	4.1				4.2
SC RF linac	140	0.1	3.4				3.5
SC RF RLAs	10400	9.1	19.5				28.6
SC RF RCSs	12566	11.3	11.8				23.1
Collider ring	2600	2.3		3.0	10		15.3
Totals	26777	24.6	52.5	18.0	21.7	8.8	145.6

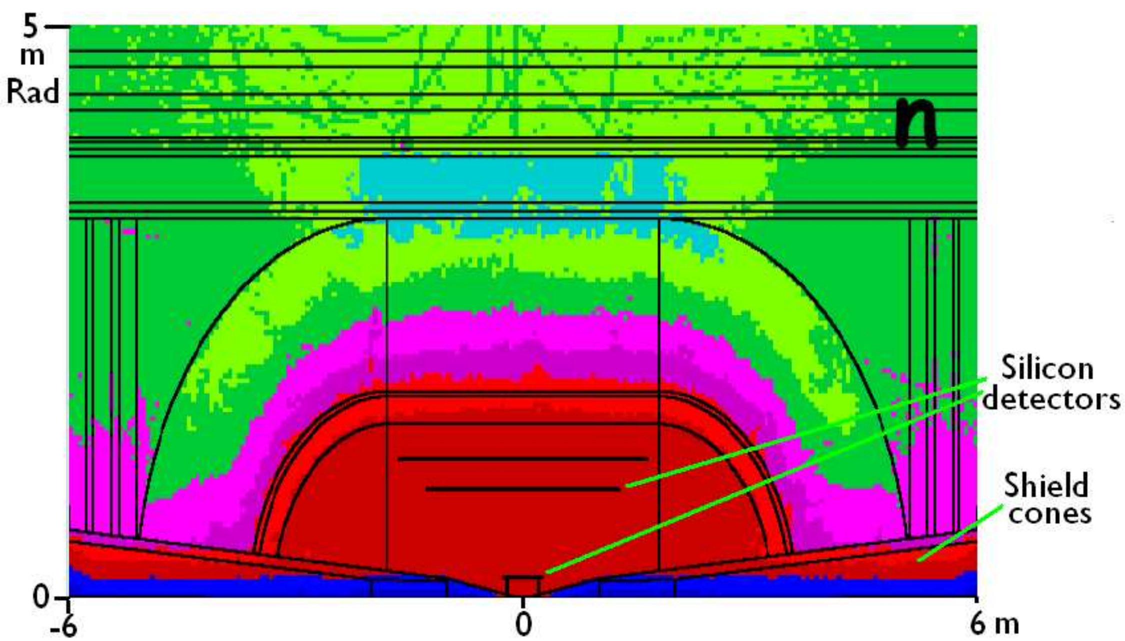
- Similar calculations for 3 TeV give Wall power = 159 MW
- NOT INCLUDING Detector, air conditioning, lighting etc but still much less than CLIC

Detector Shielding

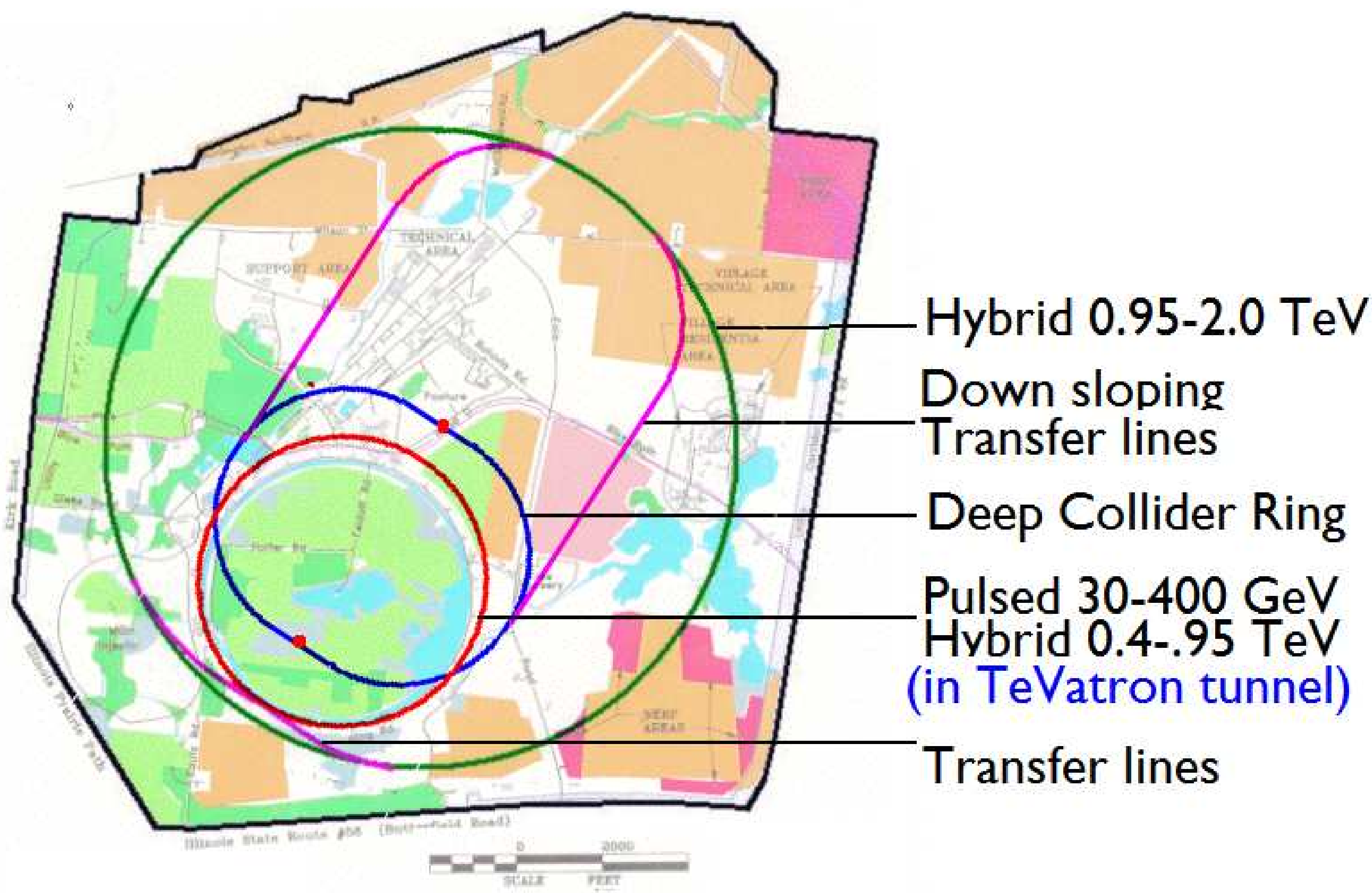


Fluence at first silicon tracker 10% of LHC (at 10^{34} $\mathrm{cm}^{-2}\mathrm{sec}^{-1}$)

Worse than e^+e^- but appears acceptable

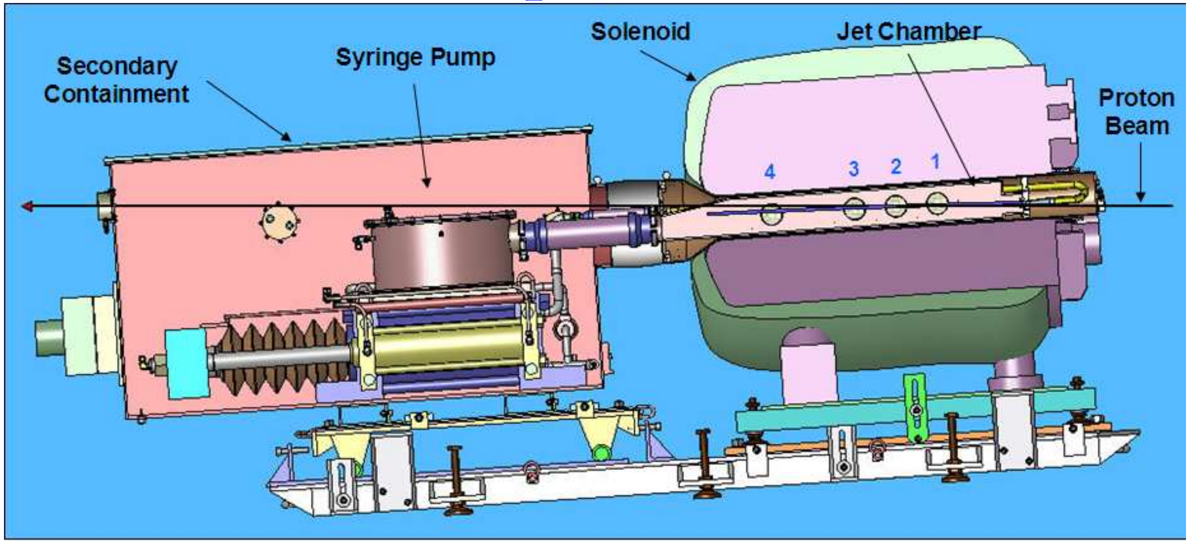


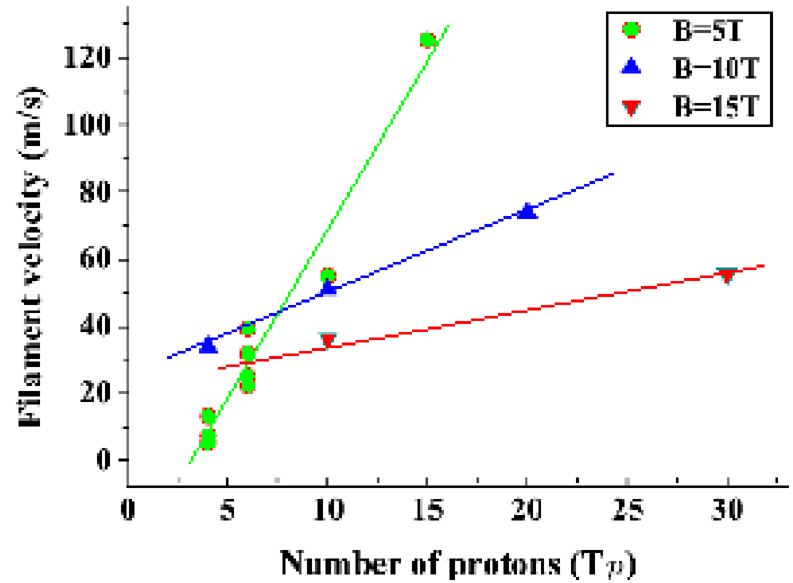
Layout of 3 TeV Collider using pulsed synchrotrons



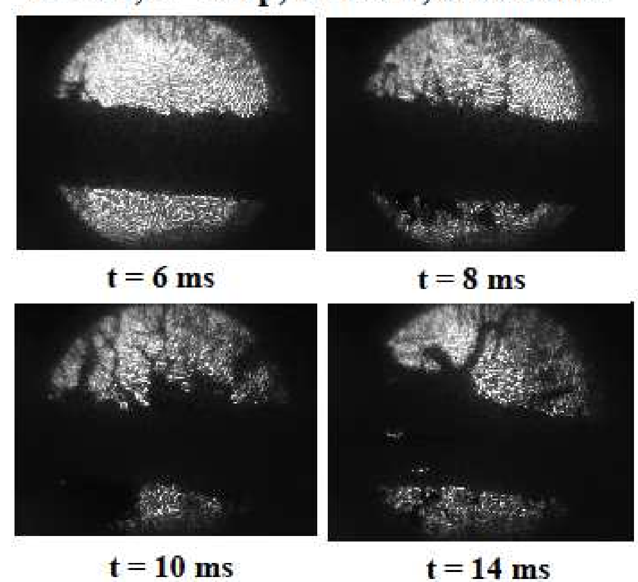
3 R&D AND EXPERIMENTS

MERIT Experiment at CERN





Images of Jet Flow at Viewport 3,
B=10T, N=10Tp, L=17cm, 2ms/frame

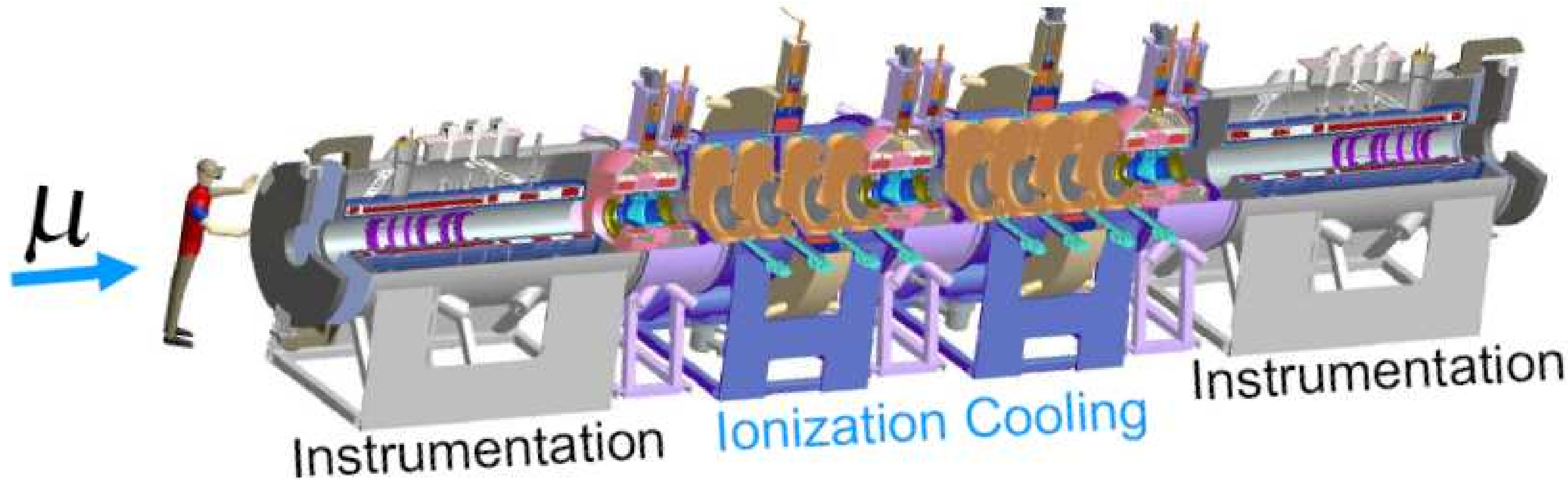


- 15 T pulsed magnet
- 1 cm rad mercury jet
- Up to 30 Tp   cf 40 Tp at 56 GeV
- Magnet lowers splash velocities
- Density persists for 100 micro sec
- No problems found

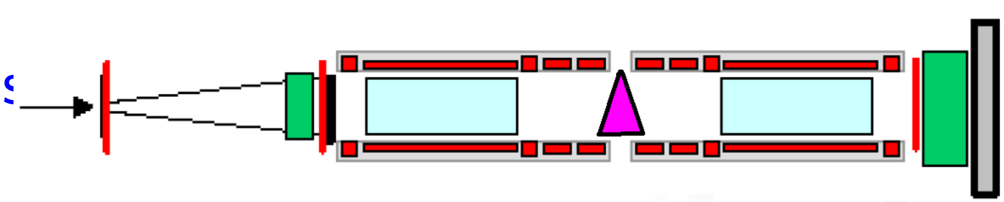
2) Muon Ionization Cooling Experiment (MICE)

International collaboration at RAL, US, UK, Japan (Blondel)

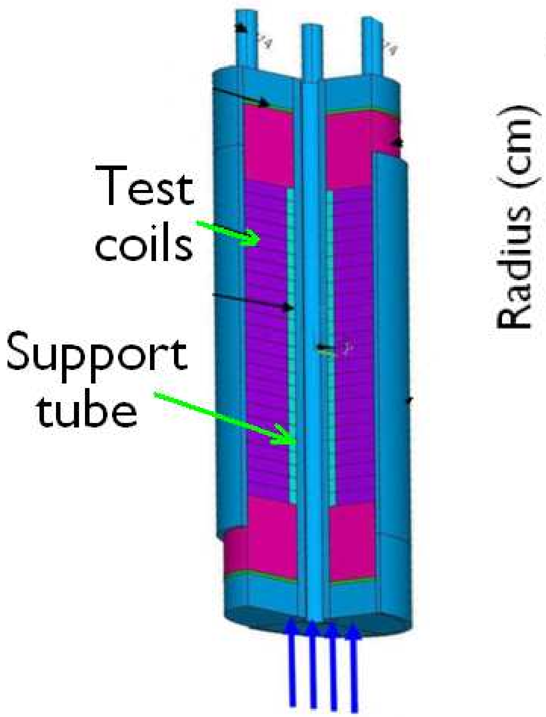
- Will demonstrate transverse cooling in liquid hydrogen, including rf re-acceleration
- Uses a different version of 'Guggenheim' lattice

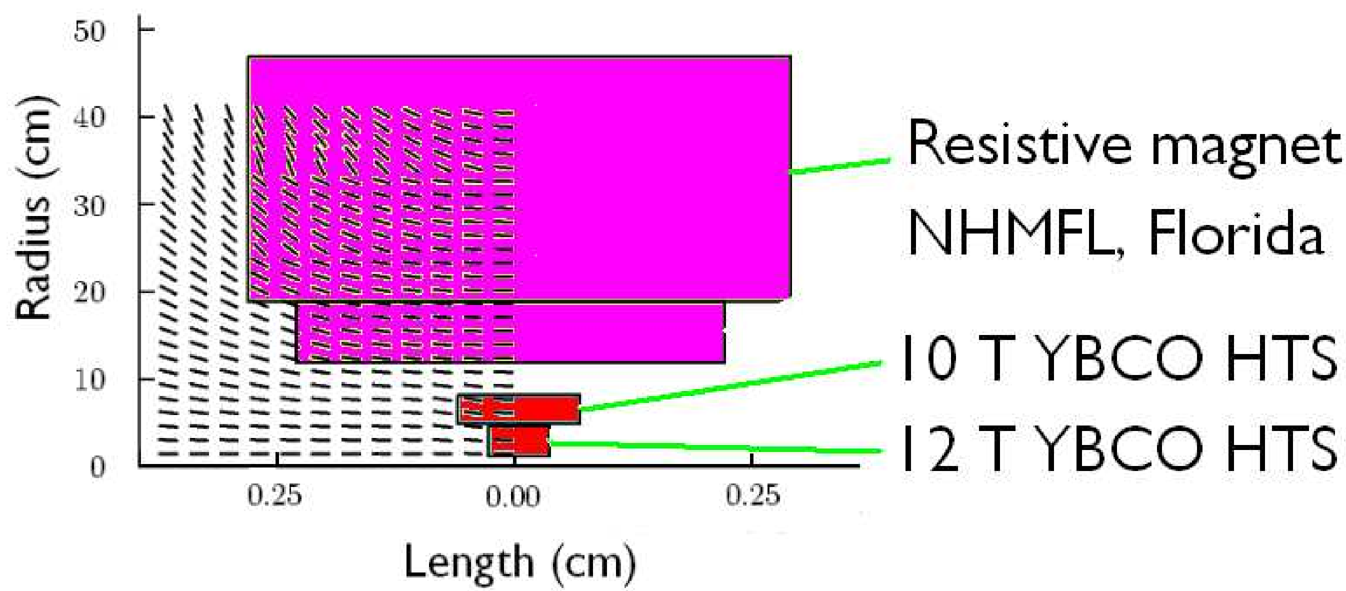


- Early Experiment to demonstrate Emittance Exchange
  - Dispersion by weighting
  - Cooling in all dimensions
  - But no re-acceleration



HTS R&D towards a 40 T solenoid





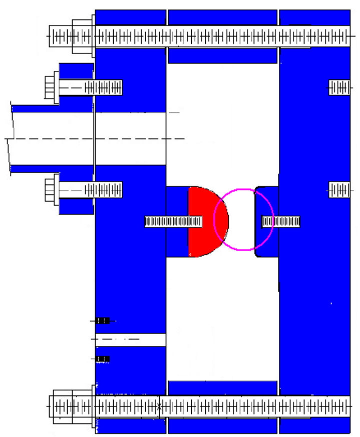
- FNAL program
- Multiple small coils
- In 12 T facility
- Fields up to 25 T

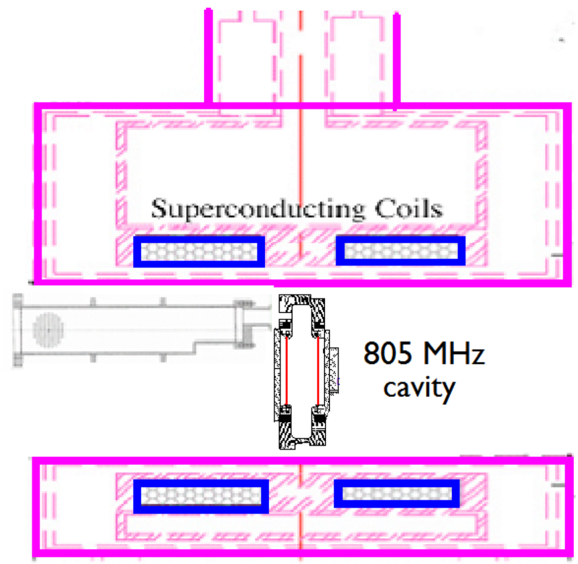
- BNL/PBL Program (SBIR)
- Test HTS coils under construction
- 12 + 10 T = 22 T stand alone
- Approx 40 T in 19 T NHMFL magnet
- Design for 19 T NbTi + Nb_3Sn design is straightforward

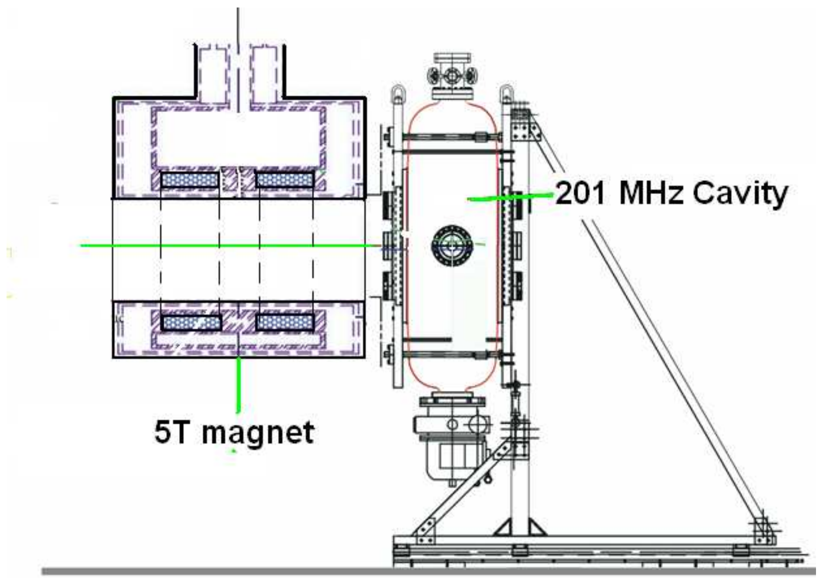
MuCool, and MuCool Test Area (MTA) at FNAL

International collaboration US, UK, Japan (Bross)

- Liquid hydrogen absorber tested
- Open & pillbox 805 MHz cavities in magnetic fields to 4 T
- 201 MHz cavity tested to magnetic field of 0.7 T Later to 2T
- High pressure H2 gas 805 MHz pillbox cavity tested
- 805 MHz gas Cavity with proton beam

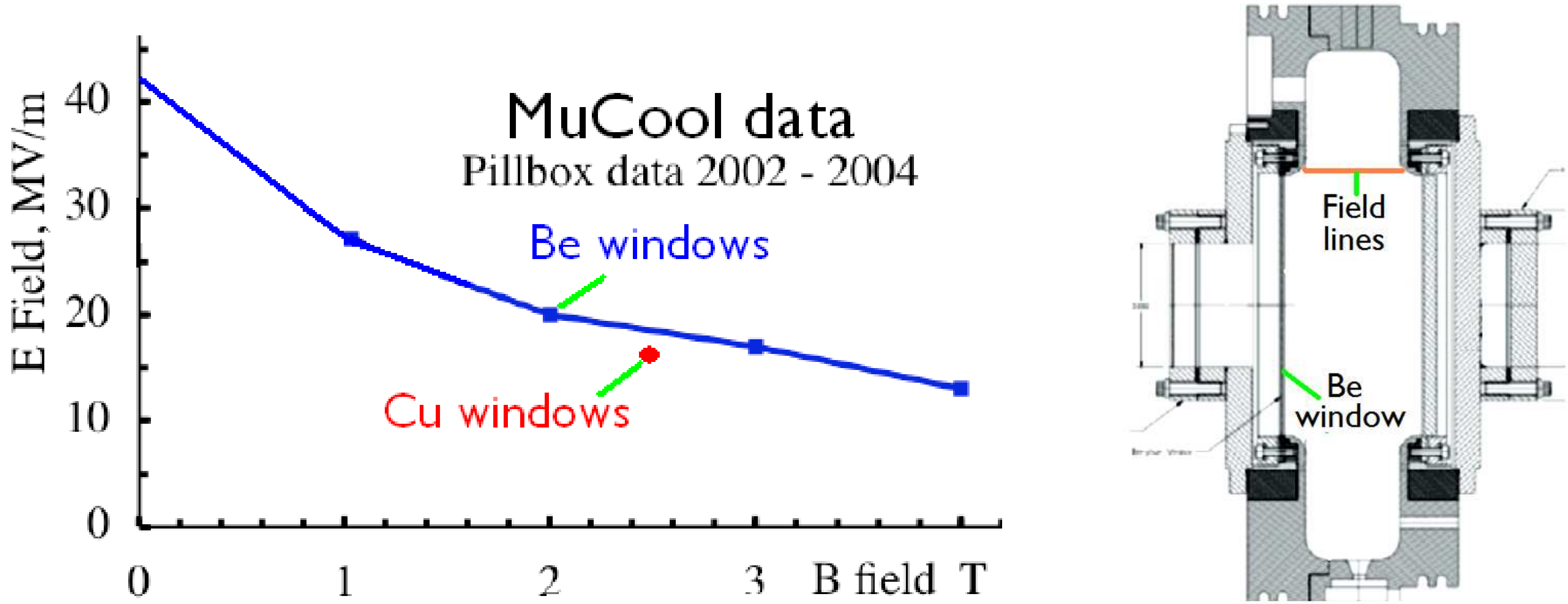


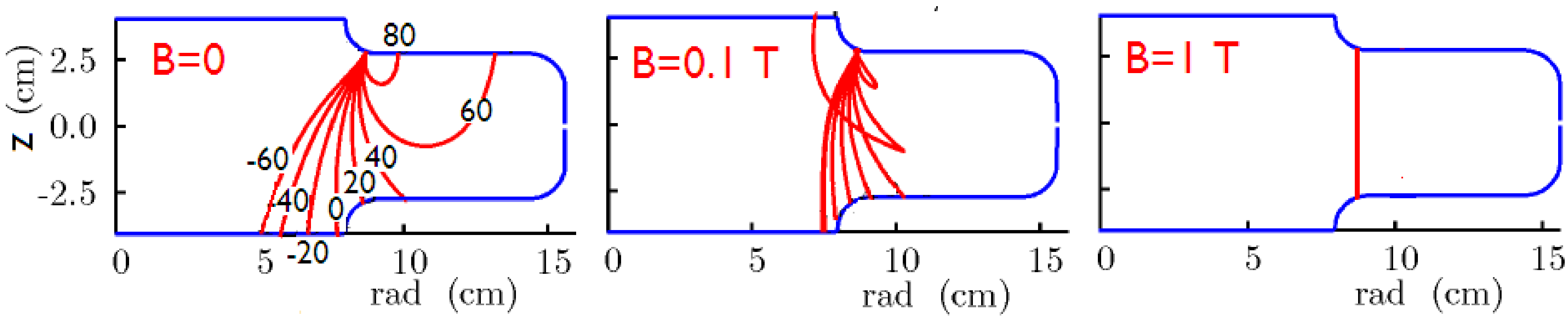




HP Gas cavity   805 MHz in 4 T magnet   201 MHz

Technical challenge: rf breakdown in magnetic fields





1. "Dark Current" electrons accelerated and focused by magnetic field
2. Damage spots by thermal fatigue causing breakdown

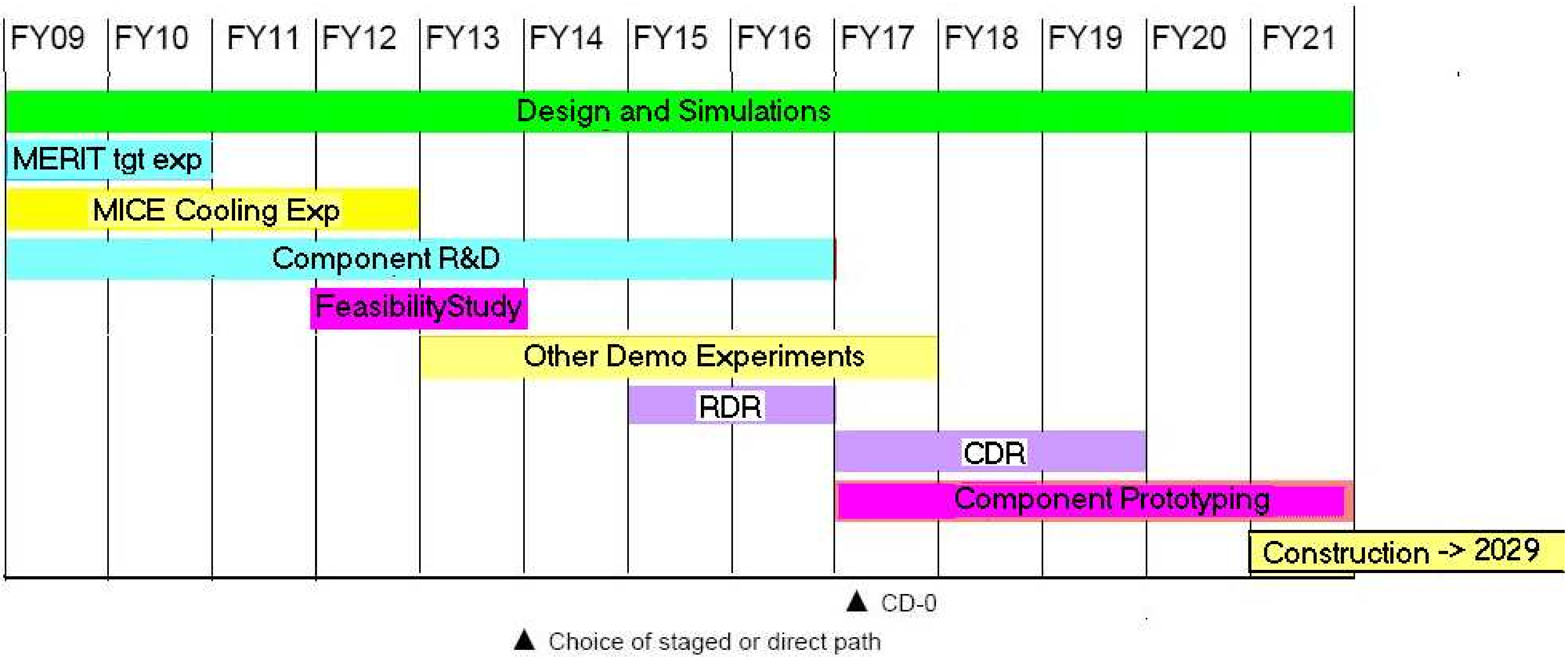
Conclusion on Baseline design

- All stages for a "baseline" design have been simulated at some level
- Matching and tapering of 6D cooling remains to be designed
- Good collider ring design exist for both 1.5 TeV
- Initial 3 TeV design
- Detector design and shielding has been studied in 1996 and now restarted

- The biggest technical challenge is rf breakdown in magnetic fields but multiple solutions are under study

Muon Accelerator Program (MAP) submitted to DoE

Administered by FNAL, but National Program, with International Collaboration (Interim Director: Steve Geer)



Expecting funding 10M\$ $\rightarrow$ 16 M\$

2012 preliminary funding at 12 M\$

Units

When discussing the motion of particles in magnetic fields, I will use MKS units, but this means that momentum, energy, and mass are in Joules and kilograms, rather than in the familiar 'electron Volts'. To make the conversion easy, I will introduce these quantities in the forms: $[pc/e]$, $[E/e]$, and $[mc^2/e]$, respectively. Each of these expressions are then in units of straight Volts corresponding to the values of p, E and m expressed in electron Volts. For instance, I will write, for the bending radius in a field B:

$$\rho = \frac{[pc/e]}{B\ c} \tag{8}$$

meaning that the radius for a 3 GeV/c particle in 5 Tesla is

$$\rho = \frac{3\ 10^9}{5 \times 3\ 10^8} = 2m$$

Emittance

Emittances will always be assumed to be normalized rms values

$$\epsilon \;=\; \text{normalized emittance} \;=\; \frac{[\text{Phase Space Area c/e}]}{\pi\ [\text{mc}^2/\text{e}]} \tag{9}$$

The phase space can be transverse: p_x vs x, p_y vs y, or Δp_z vs z, where Δp_z and z are with respect to the moving bunch center. If x and p_x are both Gaussian and uncorrelated, then:

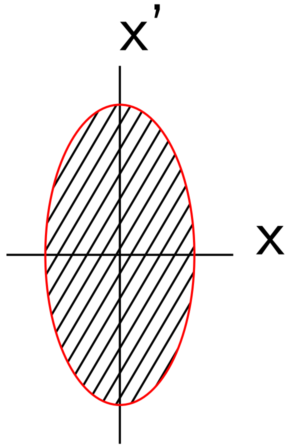
$$\epsilon_\perp = \frac{\pi\ \sigma_{[pc/e]_\perp}\sigma_x}{\pi\ [mc^2/e]} \;=\; (\gamma\beta_v)\sigma_\theta\sigma_x \qquad (m) \tag{10}$$

$$\epsilon_\parallel = \frac{\pi\ \sigma_{[pc/e]_\parallel}\sigma_z}{\pi\ [mc^2/e]} \;=\; (\gamma\beta_v)\frac{\sigma_p}{p}\ \sigma_z \qquad (m) \tag{11}$$

$$\epsilon_6 = \epsilon_\perp^2\ \ \epsilon_\parallel \qquad (\ m)^3 \tag{12}$$

The subscript v on β_v indicates that $\beta_v = v/c$.
Un-normalize emittances $\epsilon_o = \sigma_\theta\sigma_x$, are often used, but not here.

$\beta_\perp$ of Beam



For an upright phase ellipse in x' vs x,

$$\beta_\perp \;=\; \left(\frac{\text{width}}{\text{height}} \quad \text{of phase ellipse}\right) \;=\; \frac{\sigma_x}{\sigma_\theta} \tag{13}$$

Then, using the emittance definition:

$$\sigma_x \;=\; \sqrt{\epsilon_\perp\ \beta_\perp\ \frac{1}{\beta_v \gamma}} \tag{14}$$

$$\sigma_\theta \;=\; \sqrt{\frac{\epsilon_\perp}{\beta_\perp}\ \frac{1}{\beta_v \gamma}} \tag{15}$$

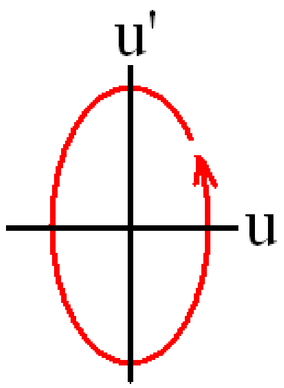
$\beta_{lattice}$ can also be defined for a repeating lattice, where it is that β_{beam} that is matched to the lattice. Equation 14, but not eq. 15 are valid even when the ellipse is tilted.

$\beta_\perp$ of a Lattice

$\beta_\perp$ above was defined by the beam, but a lattice or ring has a β_o that may or may not "match" the $\beta_\perp$ of the beam.

e,g. if a continuous inward focusing force, then there is a periodic solution:

$$u = A \sin\left(\frac{z}{\beta_o}\right) \qquad u' = \frac{A}{\beta_o} \cos\left(\frac{z}{\beta_o}\right)$$



In the u' vs. u plane, this motion is also an ellipse with

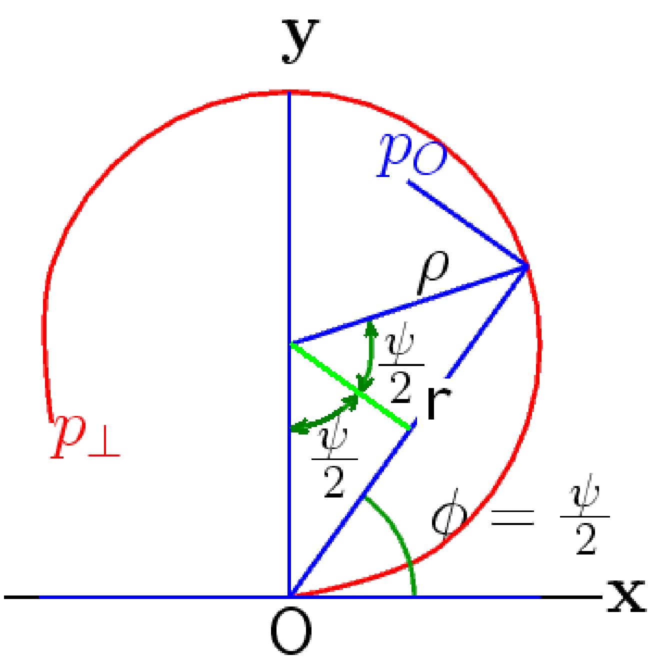
$$\frac{\text{width}}{\text{height}} = \frac{\hat{u}}{\hat{u'}} = \beta_o$$

If we have many particles with $\beta_\perp(\text{beam}) = \beta_o(\text{lattice})$ then all particles move arround the ellipse, the shape, and thus $\beta_\perp(\text{beam})$ remains constant, and the beam is "matched" to this lattice.

If the beam's $\beta_\perp(\text{beam}) \neq \beta_\text{o}$ of the lattice then $\beta_\perp(\text{beam})$ oscillates about $\beta_o(\text{lattice})$: often refered to as a "beta beat".

5 SOLENOID FOCUSING

1) x, y motion in Long Solenoid (B_Z =constant)



Consider motion in a fixed axial filed B_z, starting on the axis O with finite transverse momentum $p_\perp$ i.e. with initial angular momentum=0.

$$\rho = \frac{[pc/e]_\perp}{c\, B_z} \quad (16)$$

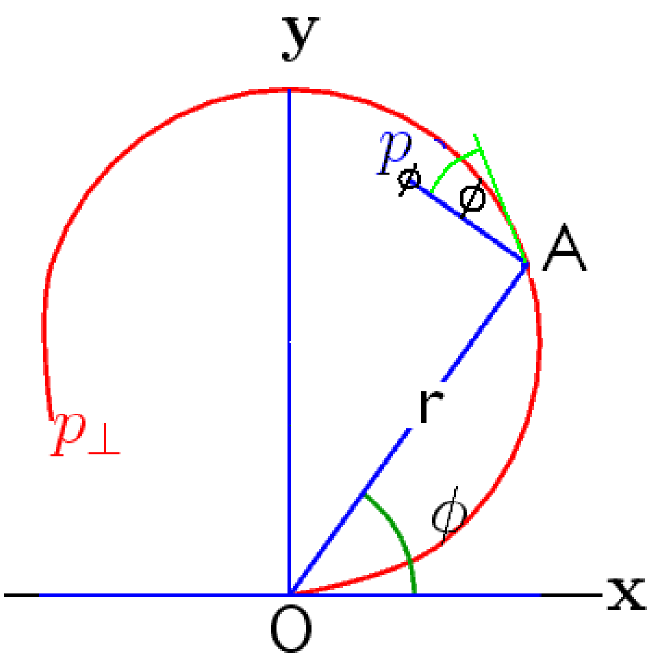
$$x = \rho \sin(\psi)$$

$$y = \rho\,(1 - \cos(\psi))$$

$$r = 2\rho\, \sin\left(\frac{\psi}{2}\right) = 2\rho \sin(\phi) \quad (17)$$

r, like x is sinusoidal, but at half the frequency

2) x, y motion in Long Solenoid (B_Z =constant)



$$p_\phi \;=\; -p_\perp \;\sin(\phi)$$

and from eq. 17

$$r \;=\; 2\;\rho\;\sin(\phi)$$

$$p_\phi \;=\; -p_\perp \,\frac{r}{2\;\rho}$$

and from eq. 16

$$\rho \;=\; \frac{[pc/e]_\perp}{c\;B_z}$$

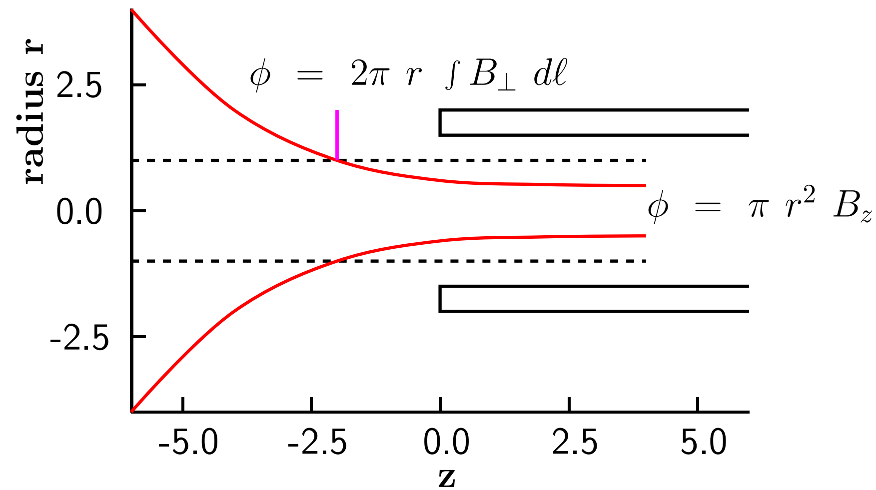
giving

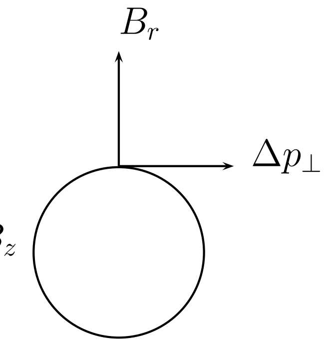
$$[pc/e]_\phi \;=\; -\frac{r\;c}{2}\,B_z \qquad (18)$$

Entering solenoid

$$\Delta[pc/e]_\phi \;=\; \int B_r\, dz$$

$$=\; -\frac{r\;c}{2}\,\Delta B_z \qquad (19)$$





This is exact, so if the particle has no initial angular momentum

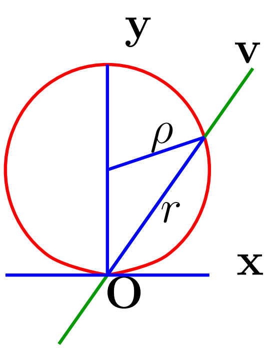
$$[pc/e]_\phi \;=\; -\frac{r\;c}{2}\,B_z \qquad (20)$$

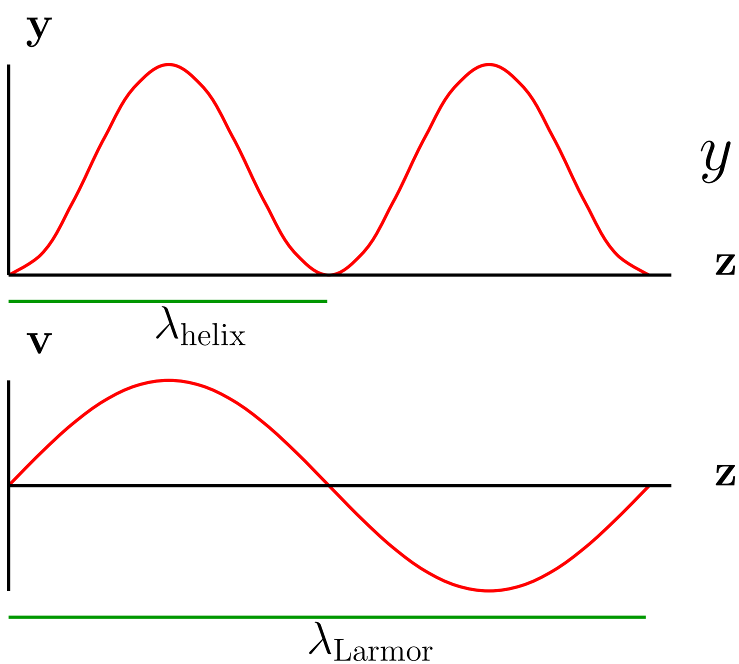
This is exactly that needed (18) to make a helix that passes through the axis O

If we define a coordinate system u, v that is rotated about the axis by the above angle ϕ, then in that frame a particle starting without angular momentum and $u = 0, \dot{u} = 0$ remains in the plane $u = 0$ plane. This is the Larmor frame.

e.g. For fixed B_z

If The center of the solenoid magnet is at O, then consider a plane that contains this axis and the particle. This, for a particle with initially no angular momentum, is the 'Larmor Plane:



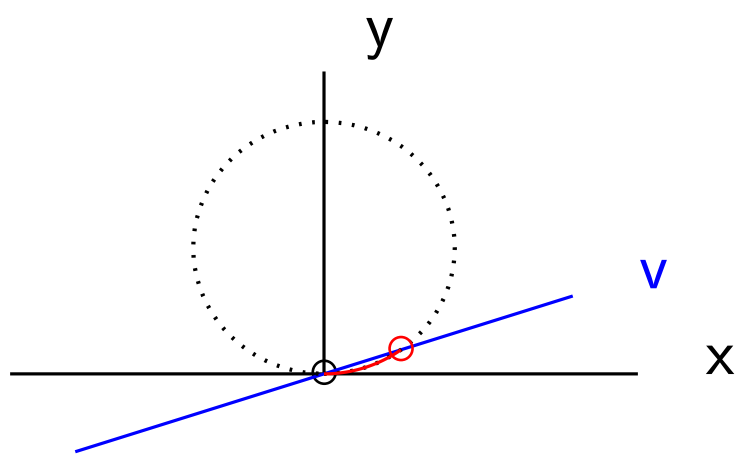


$$y = \rho\,(1 - cos(\psi)) \tag{21}$$

$$v = 2\rho\ \sin(\phi) \tag{22}$$

$$\frac{\lambda_{\text{Helix}}}{2\pi} = \frac{dz}{d\psi} = \rho\,\frac{p_z}{p_\perp} = \frac{[pc/e]_z}{c\ B_z} \tag{23}$$

$$\frac{\lambda_{\text{Larmor}}}{2\pi} = \beta_{\text{lattice}} = \frac{dz}{d\phi} = 2\rho\,\frac{p_z}{p_\perp} = 2\,\frac{[pc/e]_z}{c\ B_z} \tag{24}$$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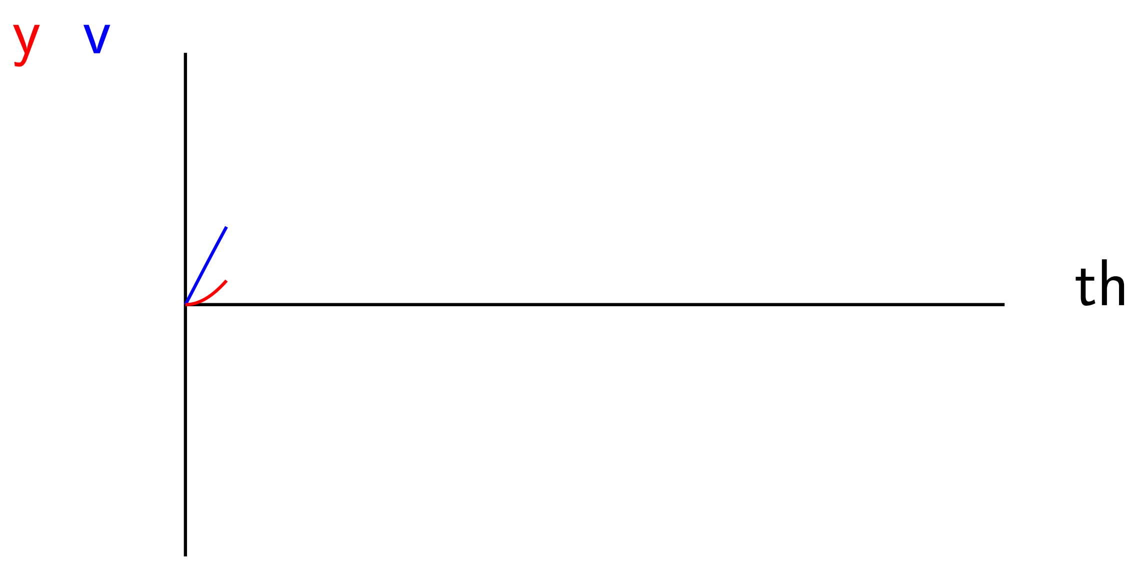
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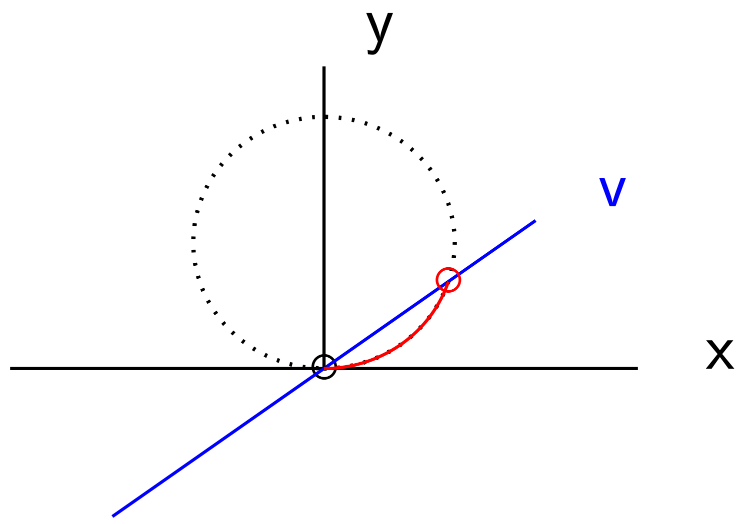
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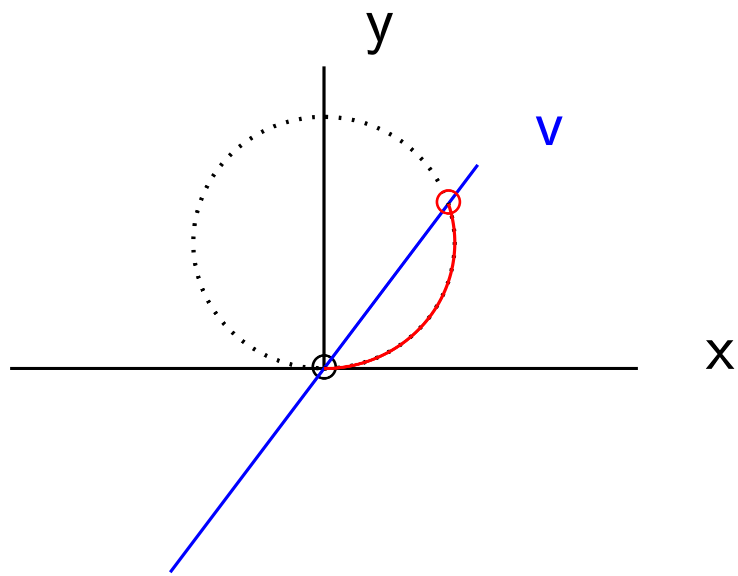
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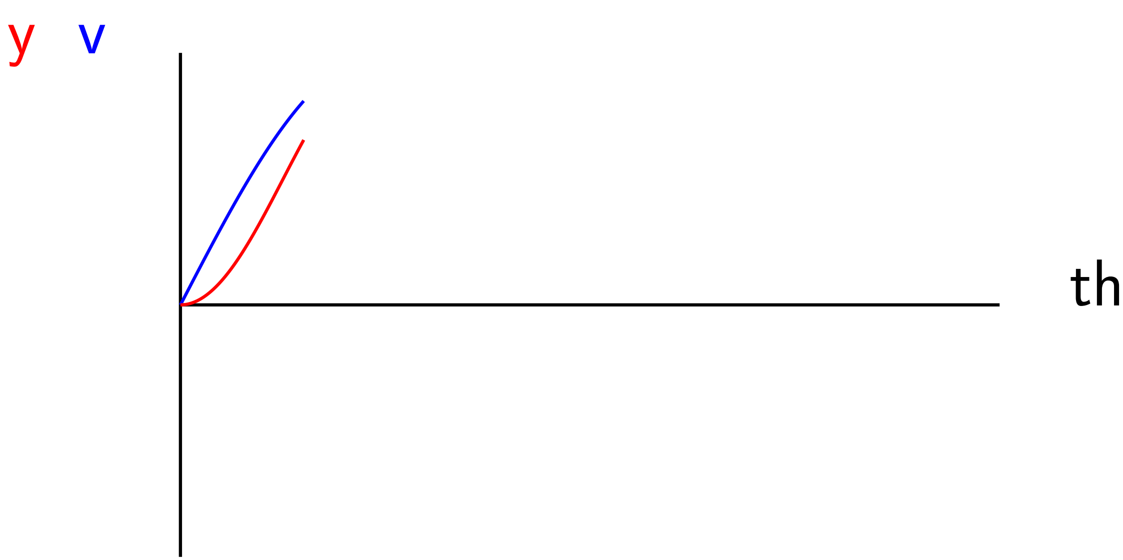
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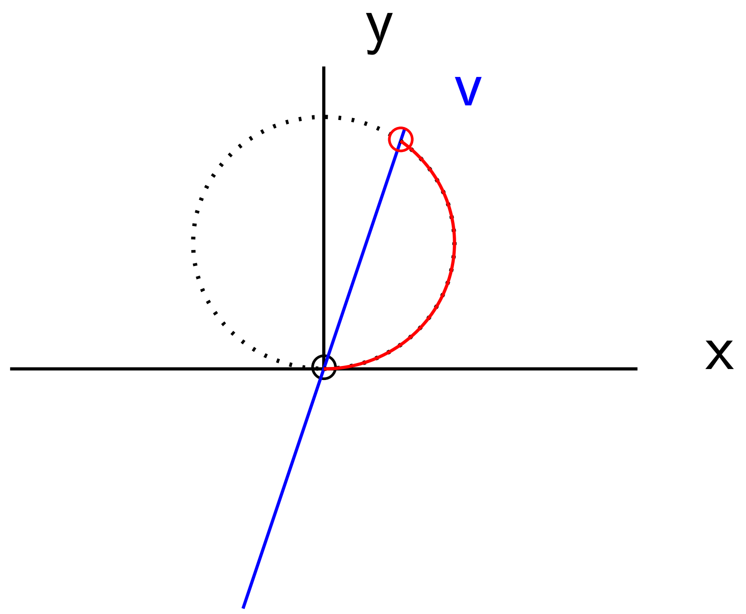
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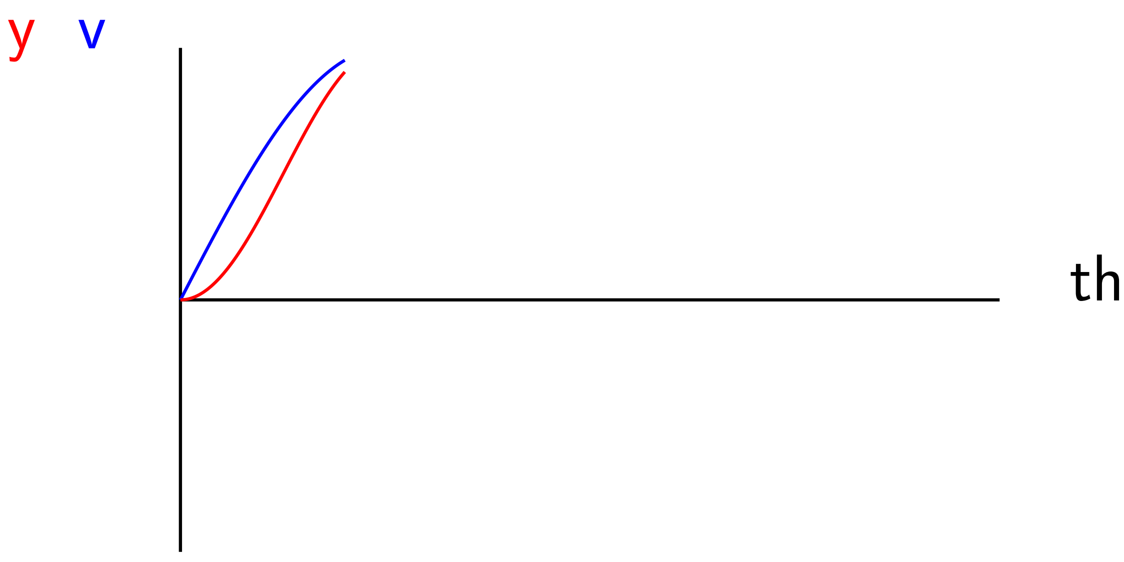
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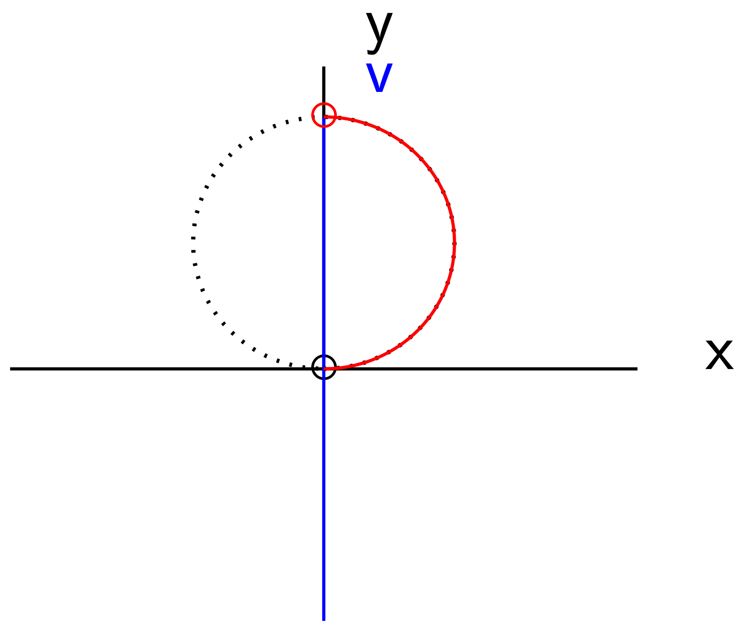
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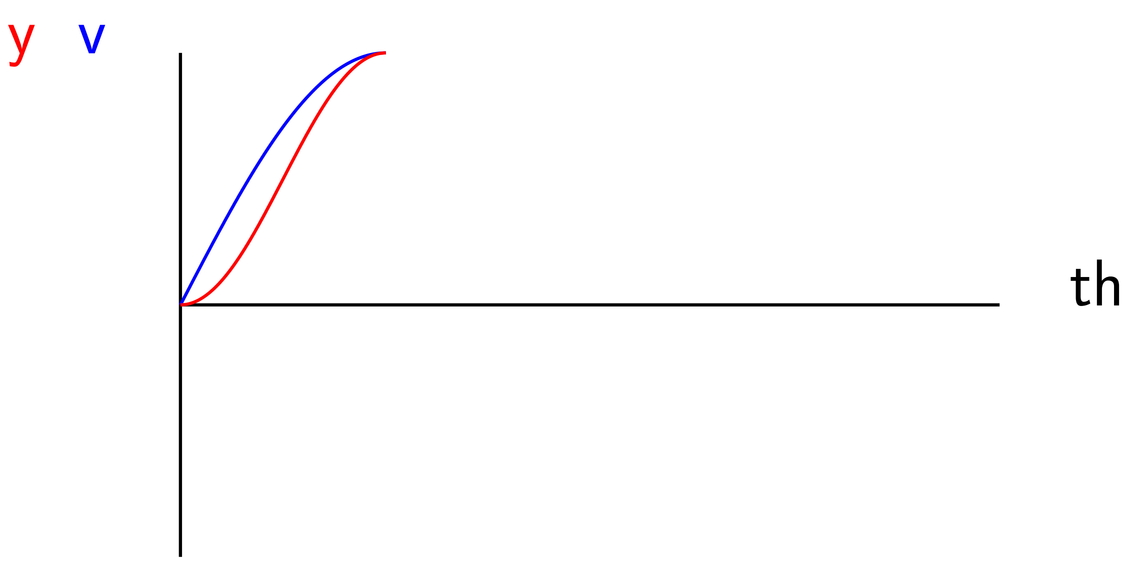
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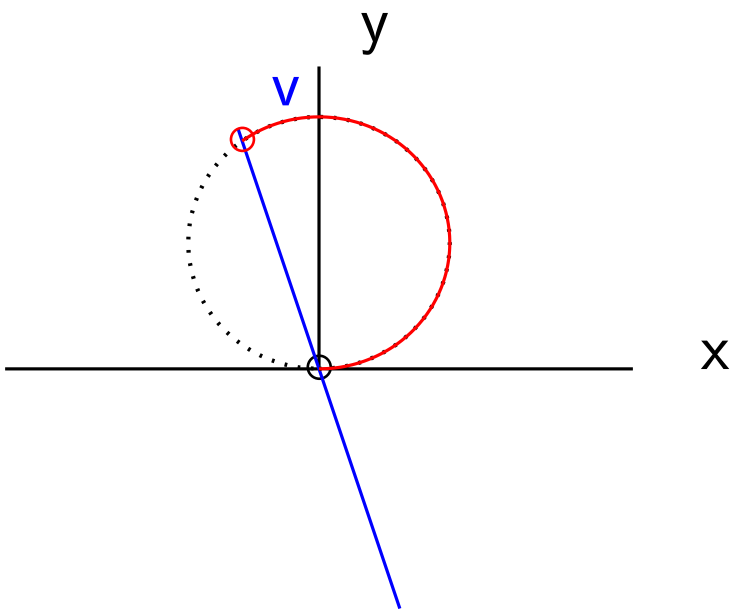
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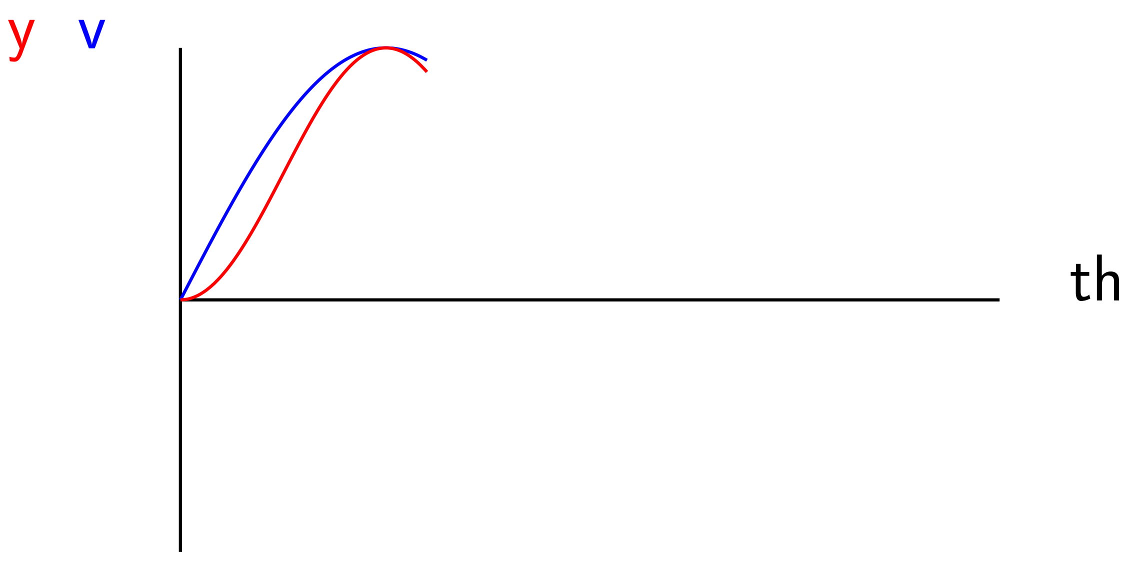
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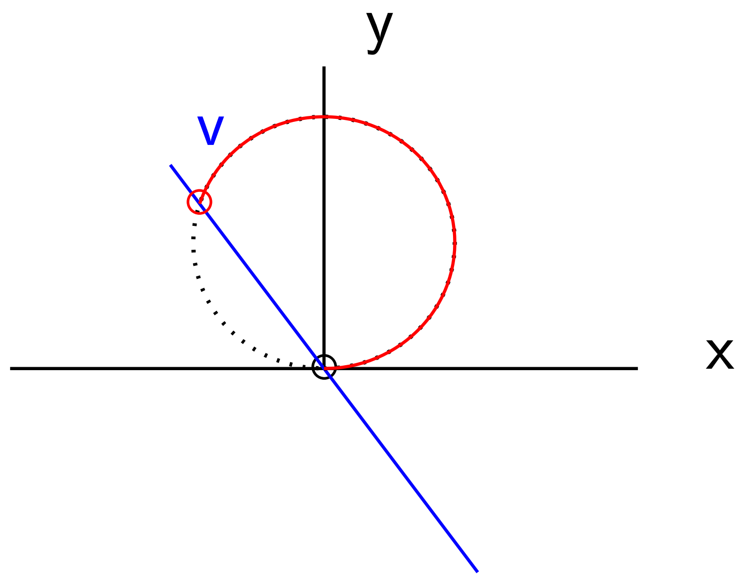
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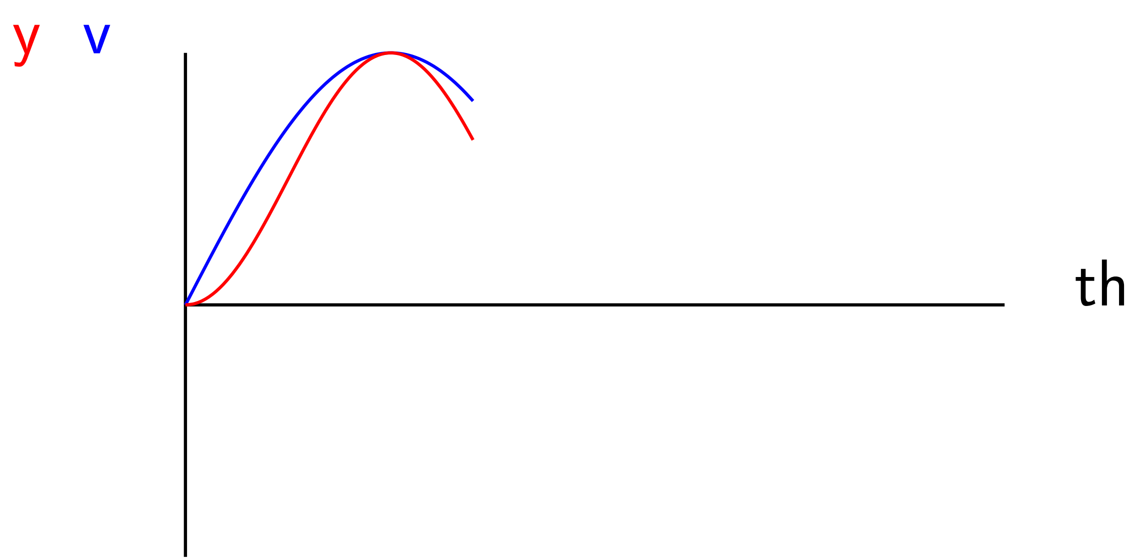
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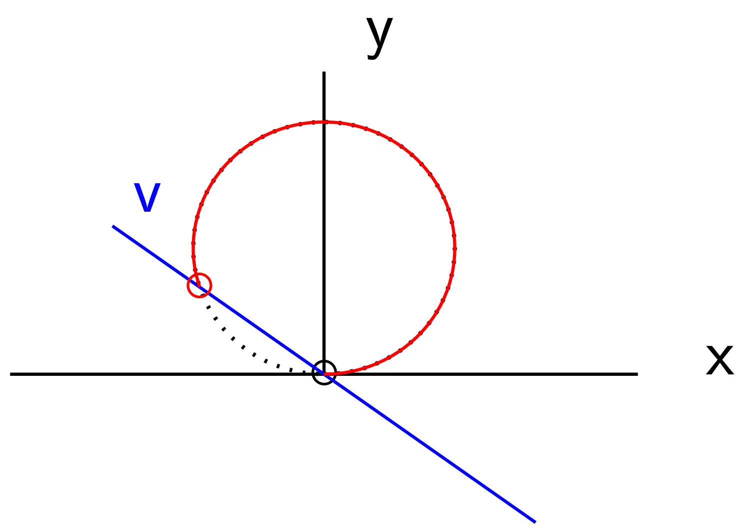
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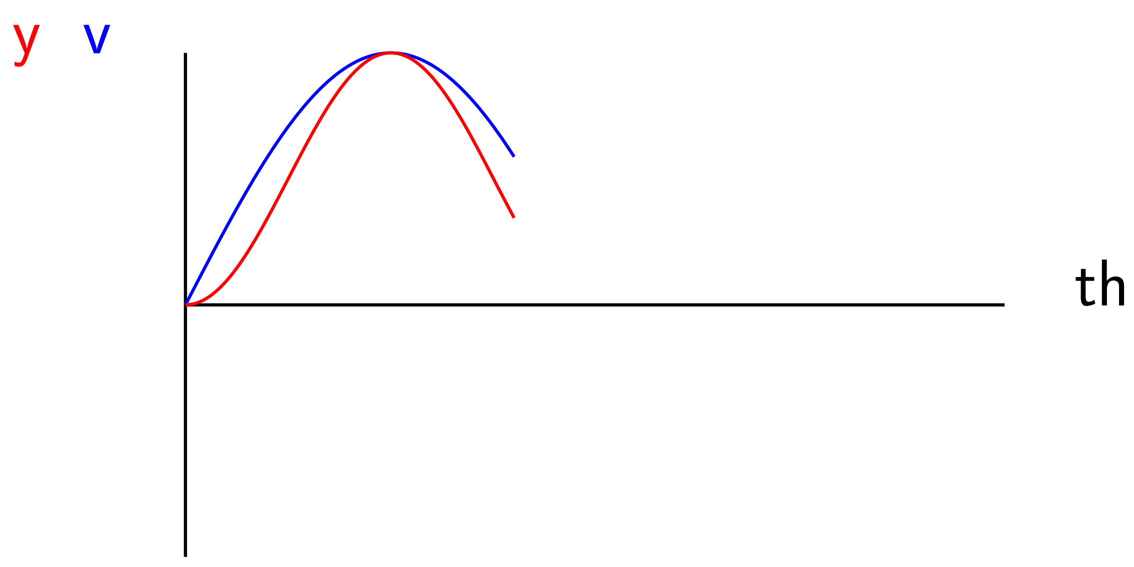
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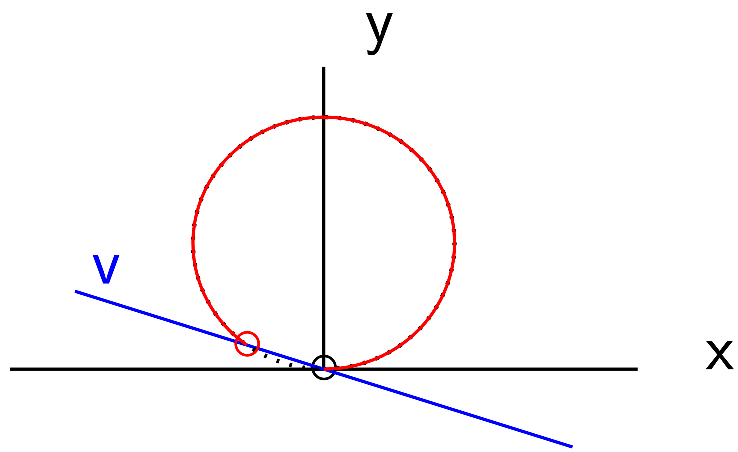
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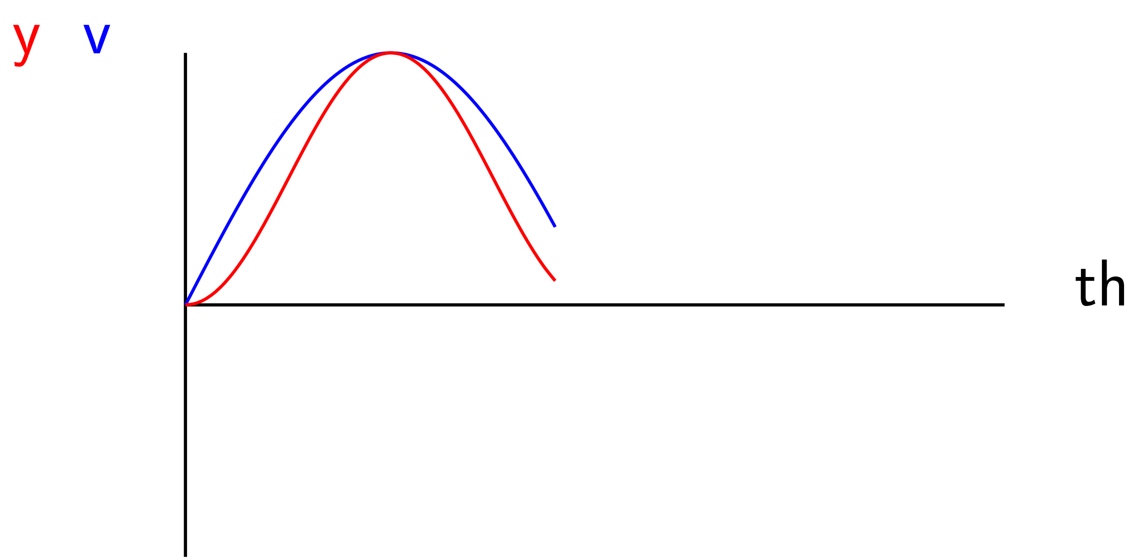
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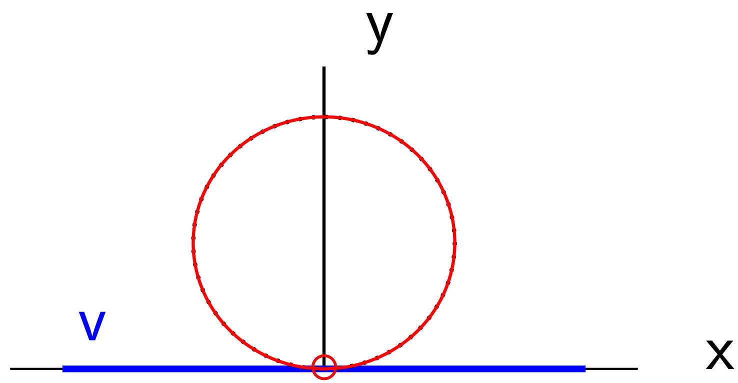
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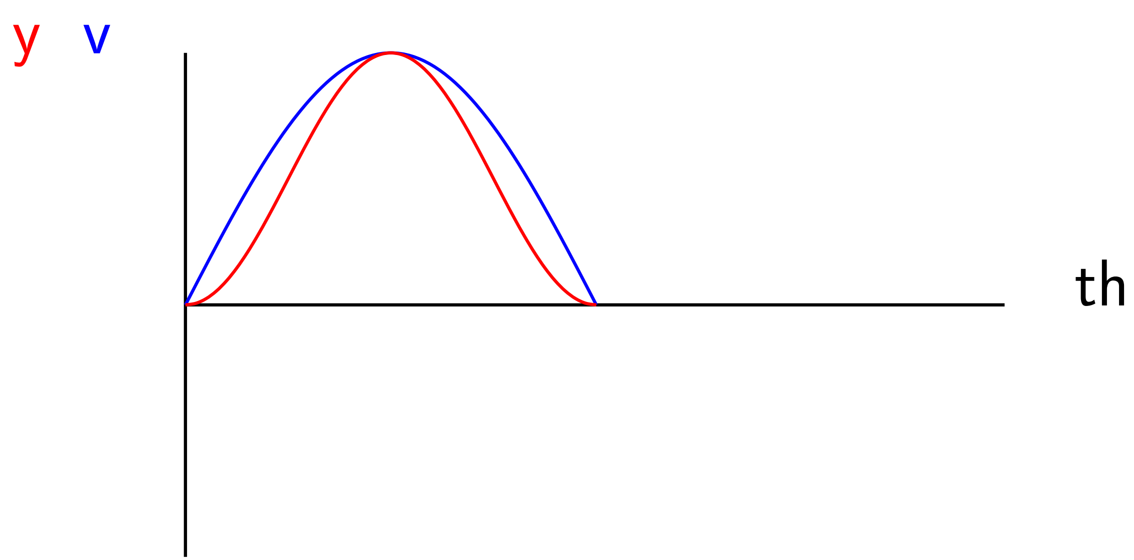
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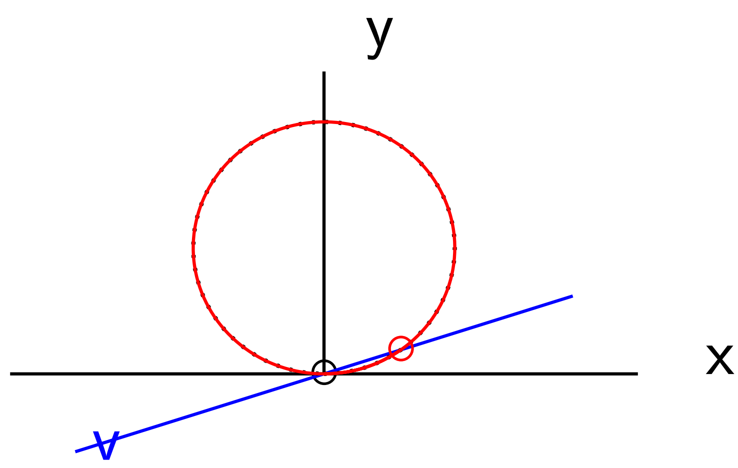
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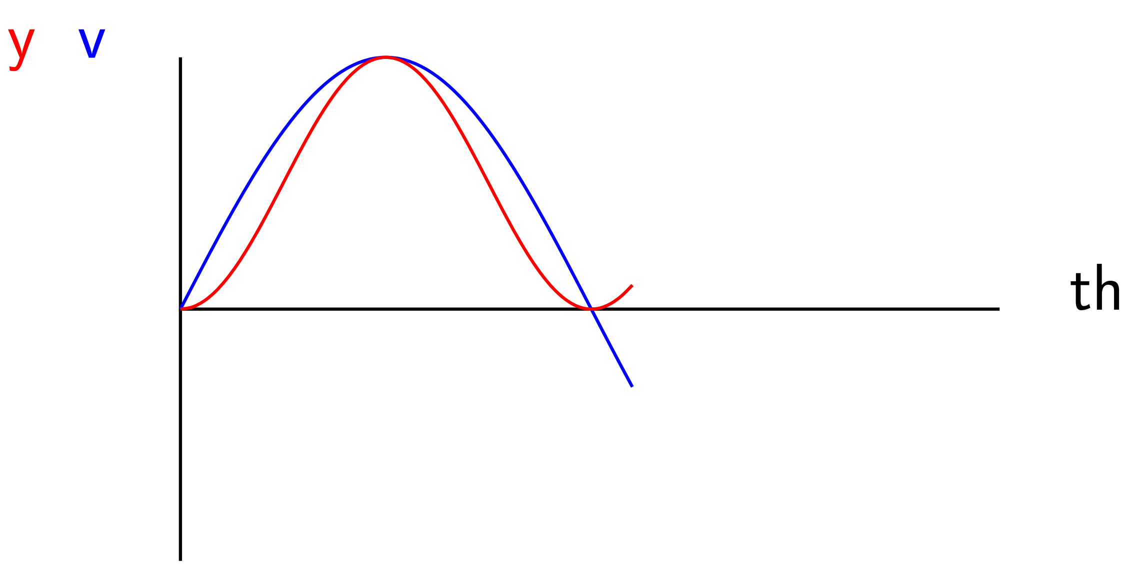
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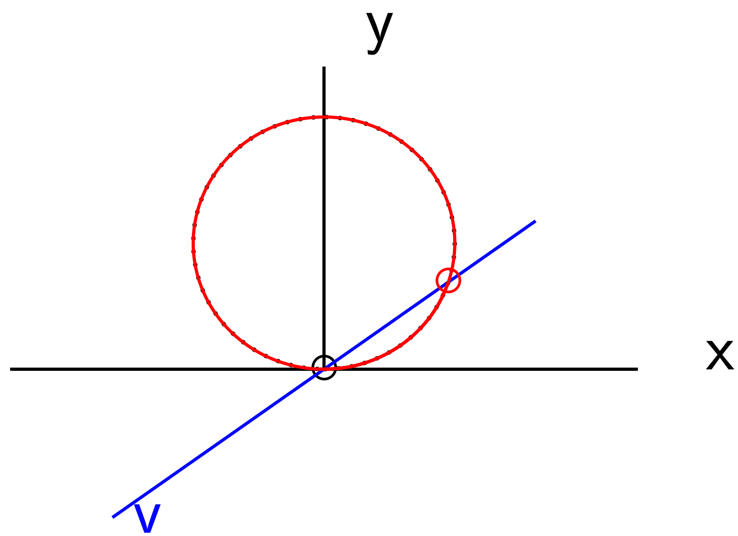
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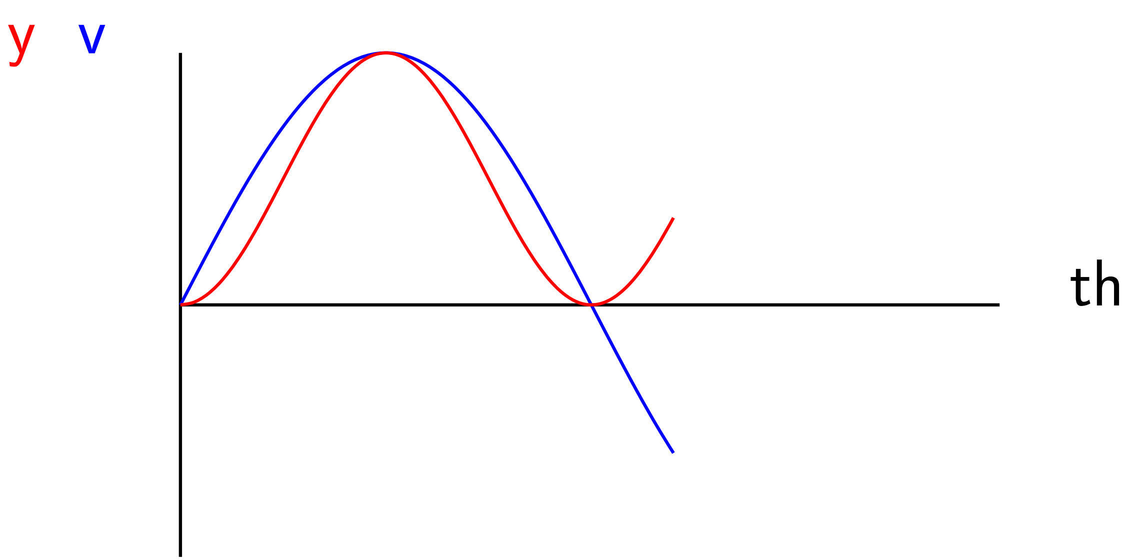
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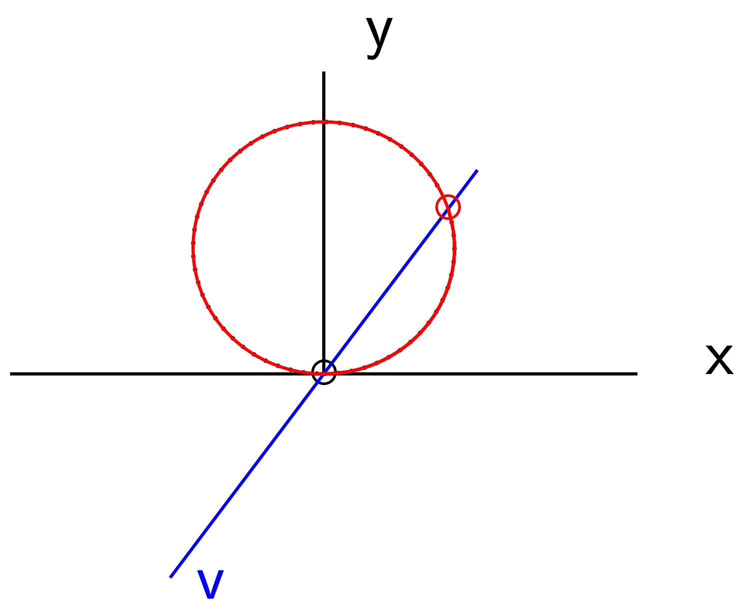
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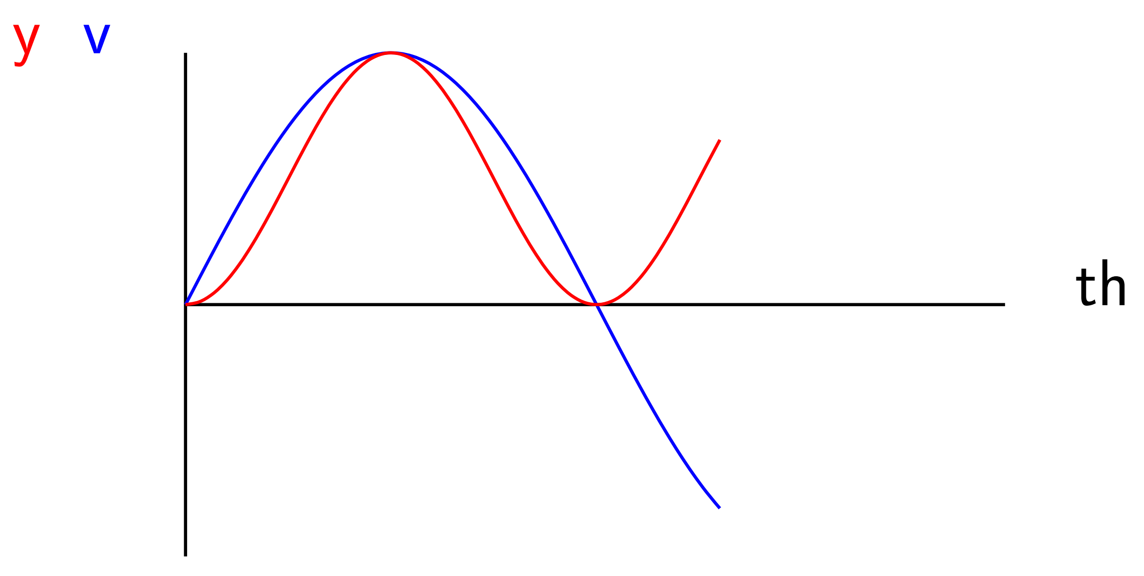
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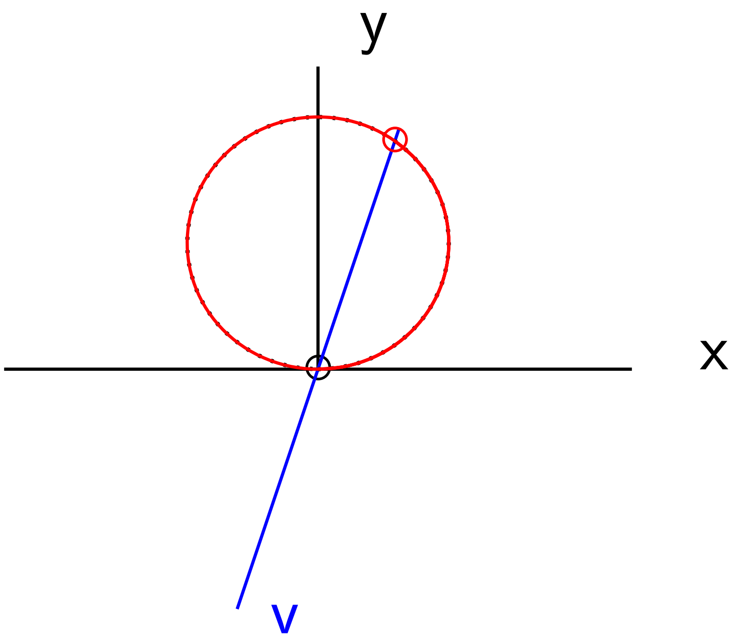
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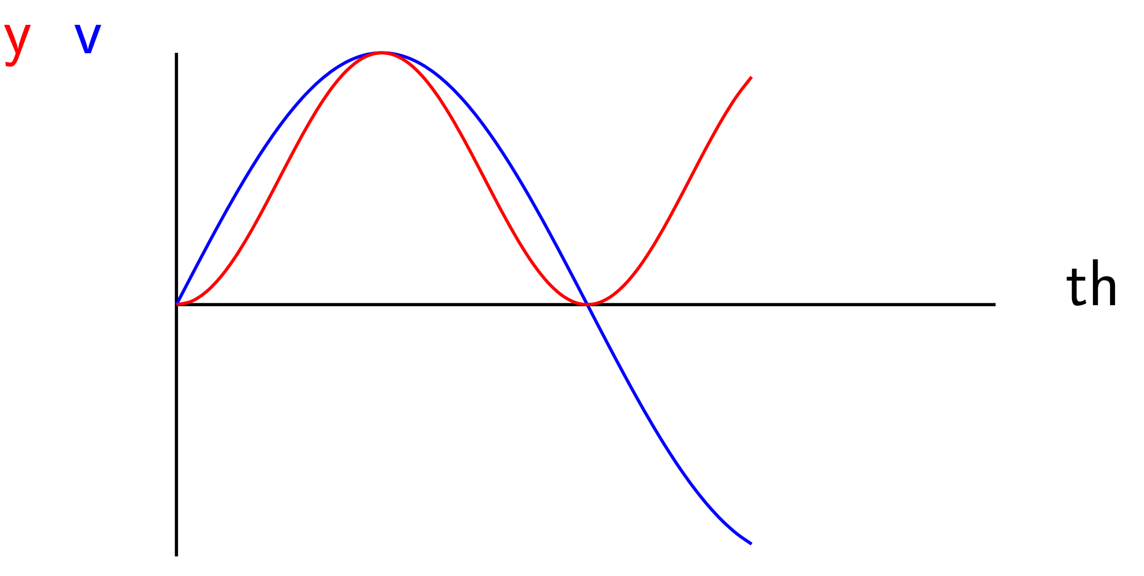
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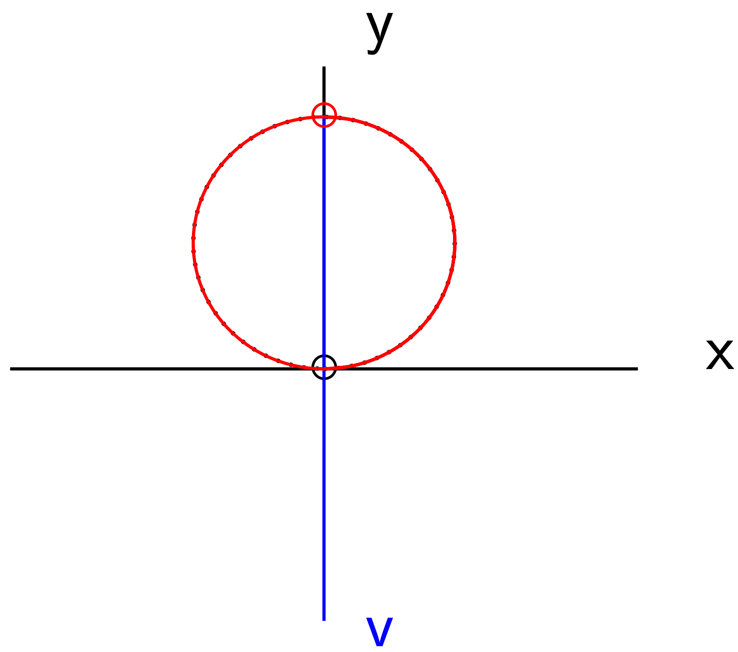
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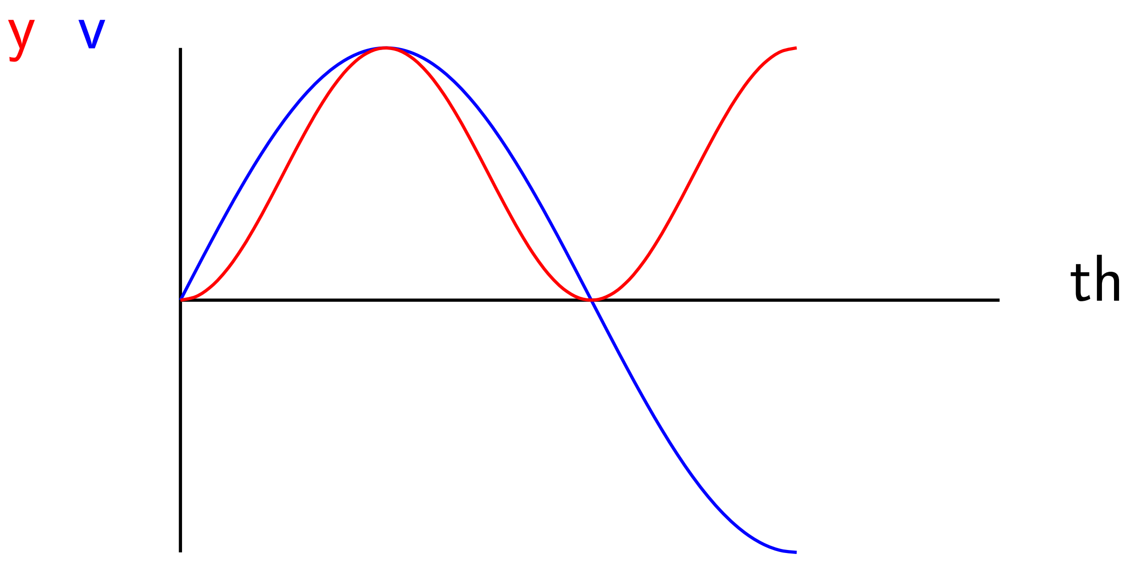
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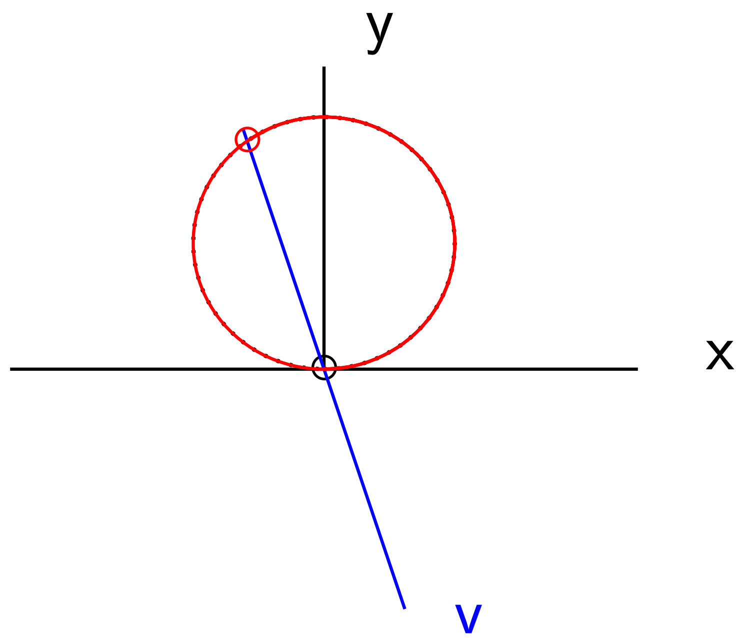
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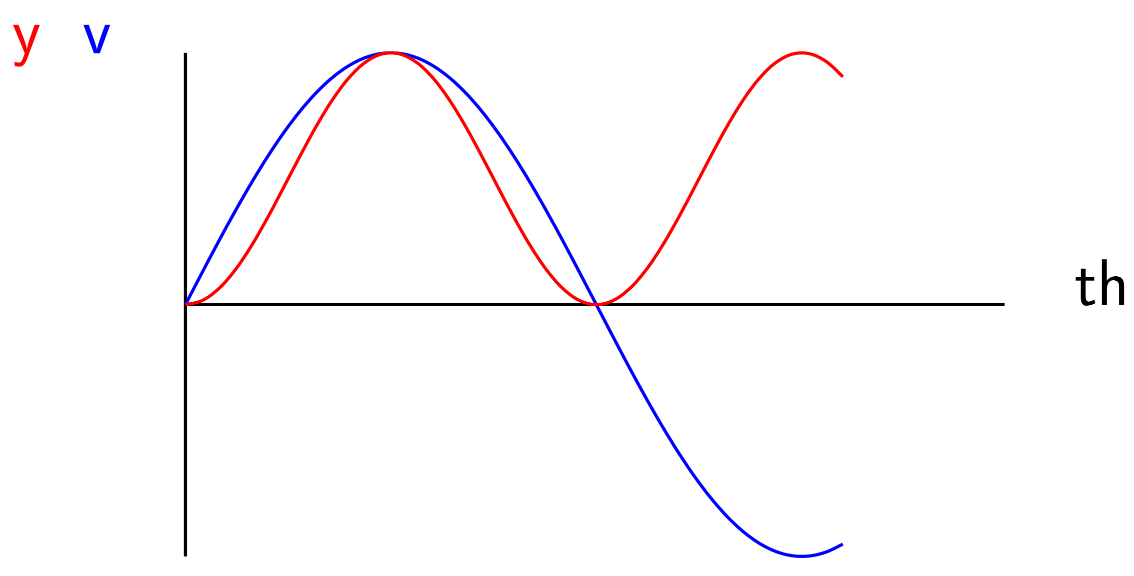
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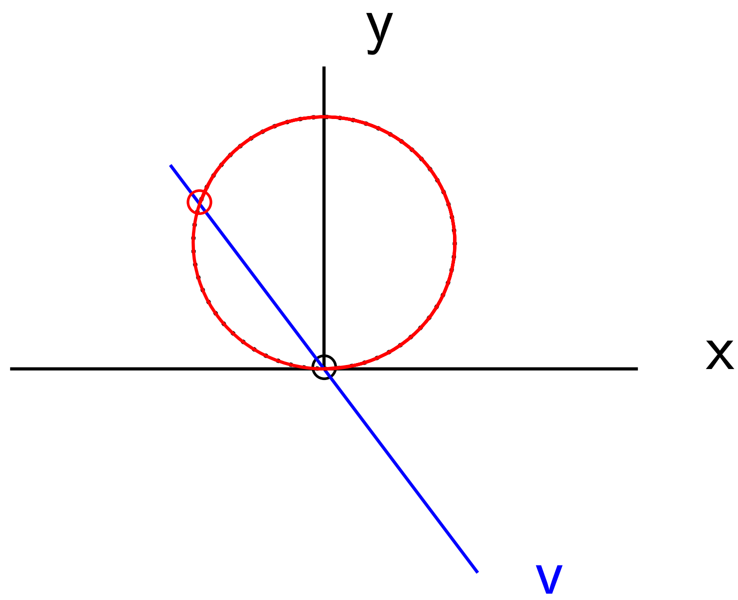
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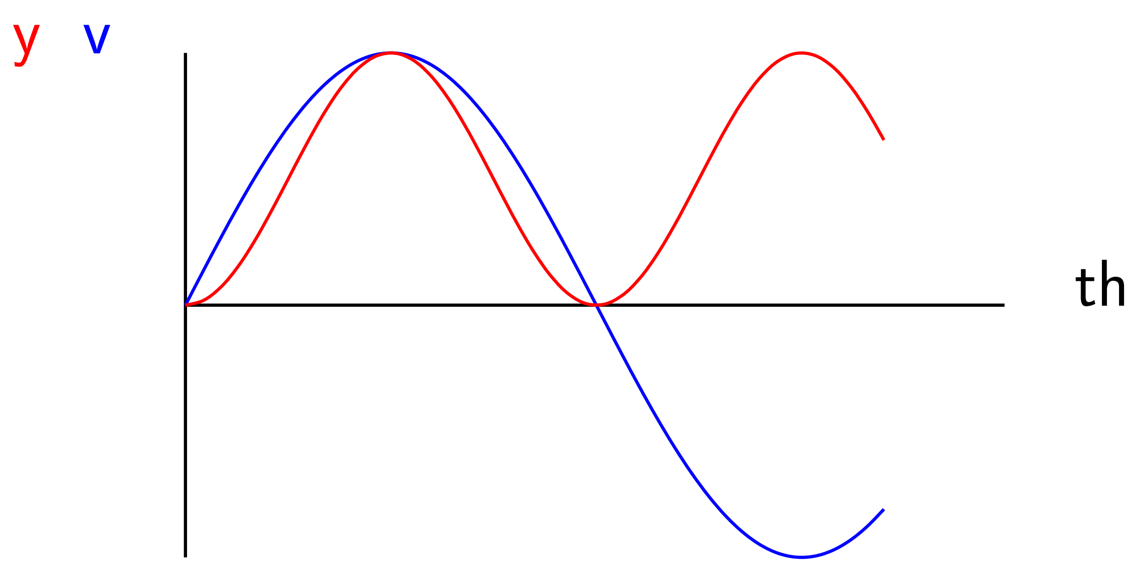
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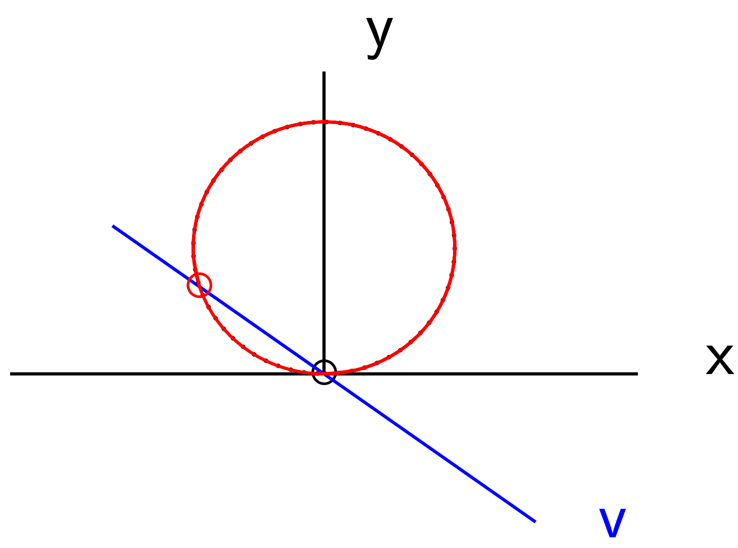
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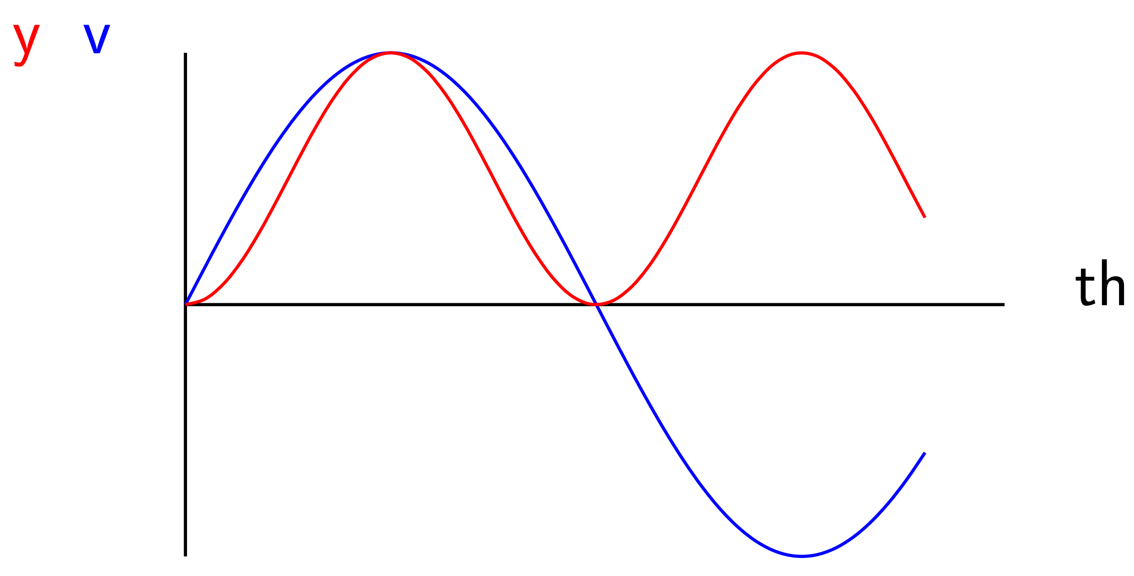
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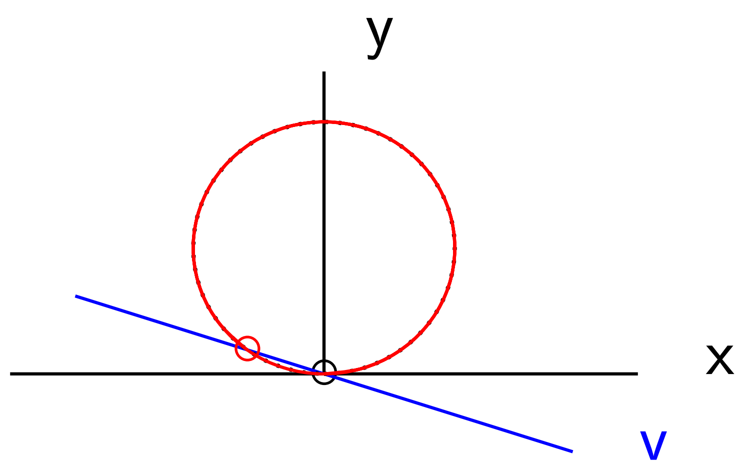
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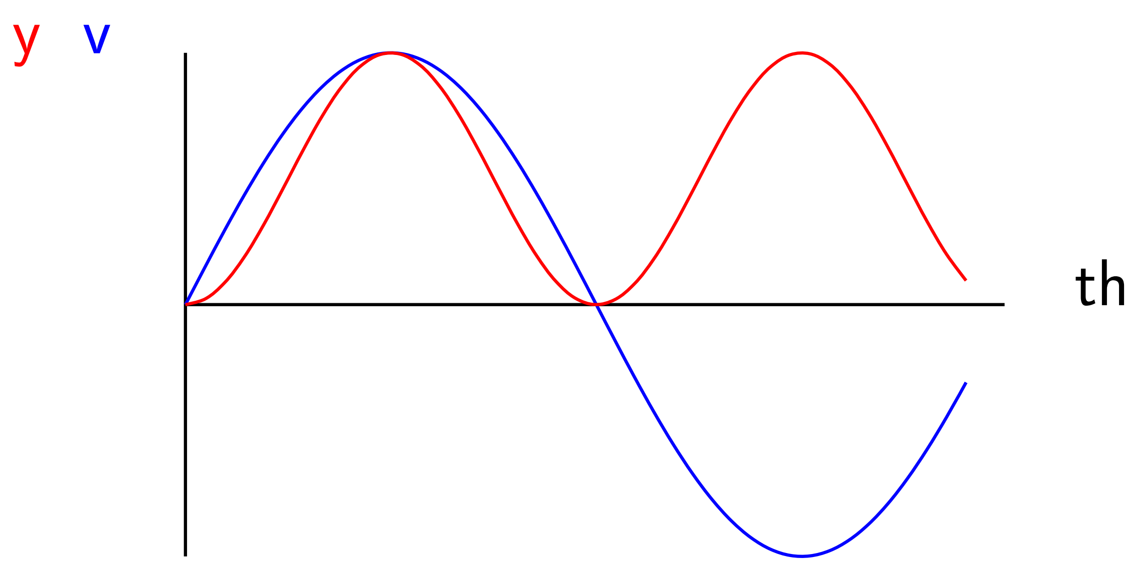
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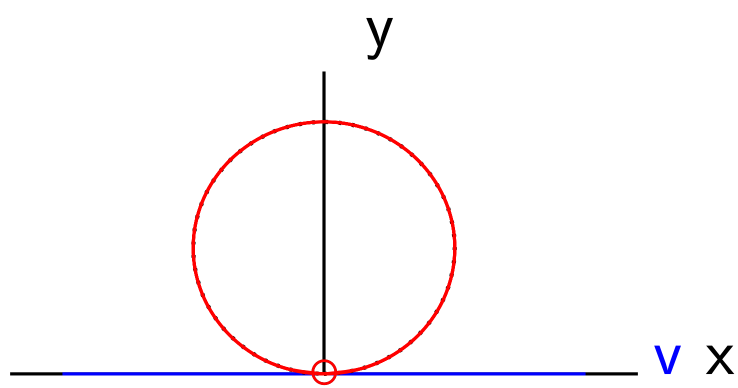
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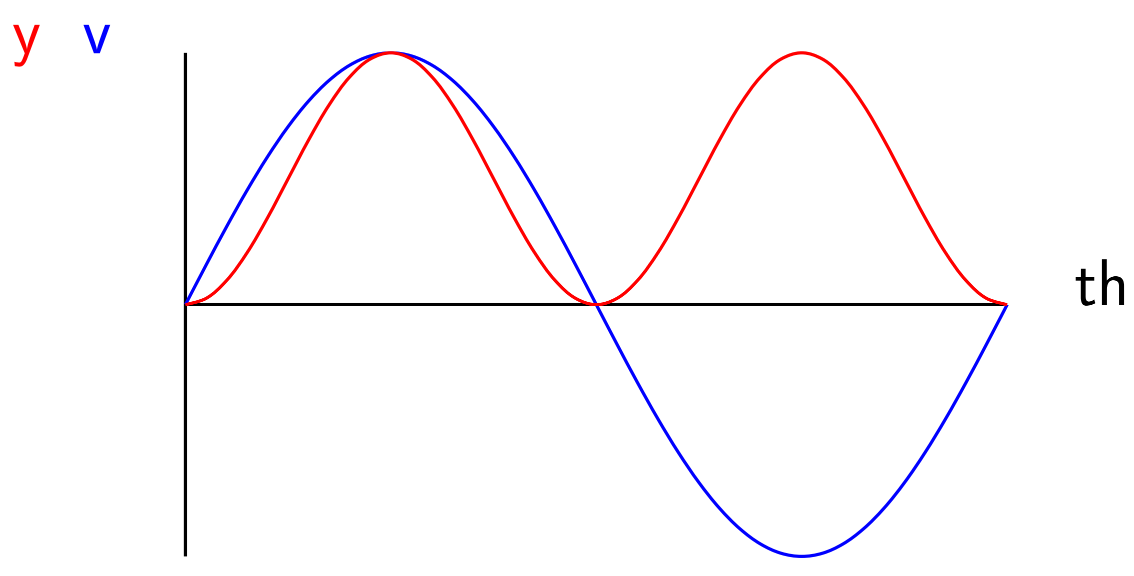
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Larmor Theorem

Motion of a charged particle in any axial symmetric solenoid fields $B_z(z)$ is given by that of a particle moving with the same p_z in a $u,\ v$ frame rotating about that axis by

$$\frac{d\phi}{dz} = -\frac{c\ B_z}{2\ [pc/e]_z}$$

under a focusing 'force' towards the axis giving bending

$$\frac{1}{\eta} = \frac{d^2r}{dz^2} = -\left(\frac{cB_z}{2\ [pc/e]}\right)^2 r$$

r is the distance to the axis and $[pc/e]$ is the momentum component perpendicular to r

Compared with quadrupole $$\frac{1}{\eta} = \frac{d^2r}{dz^2} = -\left(\frac{G\ c}{[pc/e]}\right) r$$

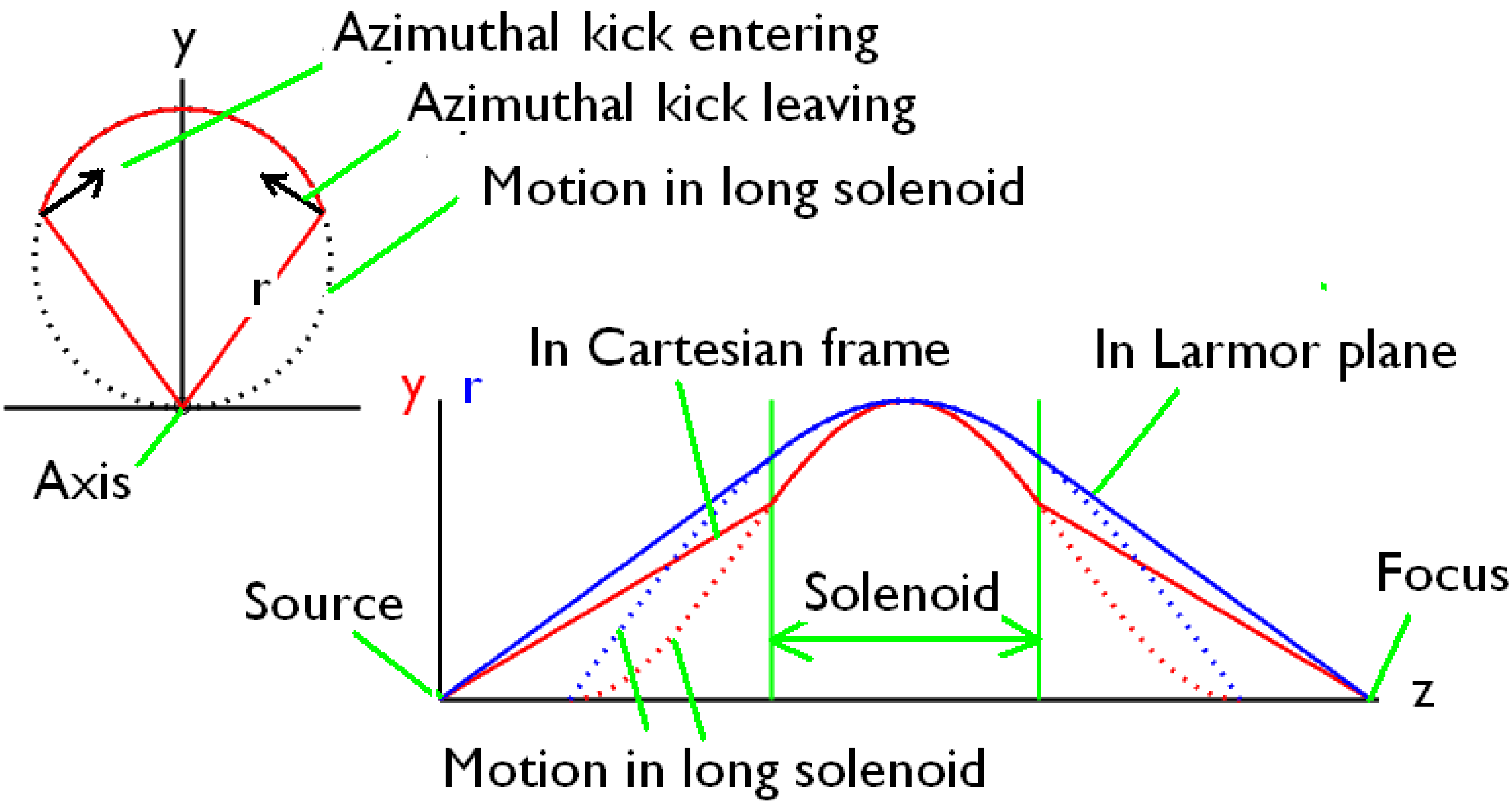
Solenoid focusing $\propto B^2/p^2$

Independent of sign   Stronger at low momenta

Example of thin solenoid focus

Treating azimuthal kicks on entering and leaving as delta functions

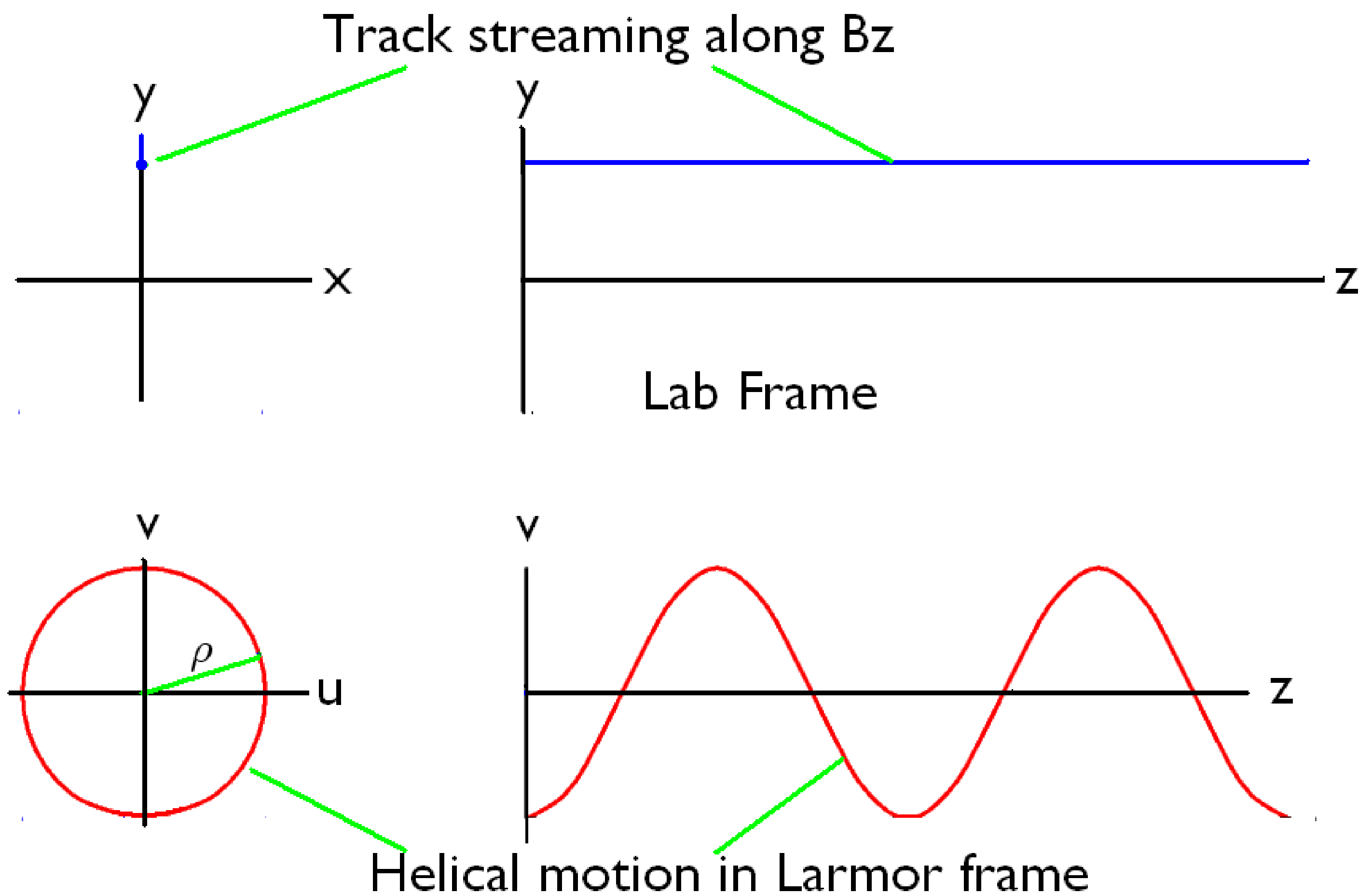
Doing it right only rounds the corners



In Cartesian we see kinks from azymuthal kicks   In Larmor we see pure focusing

Example of streaming down field lines

It seems bat first hard to see how particles streaming down the filed lines are being focused

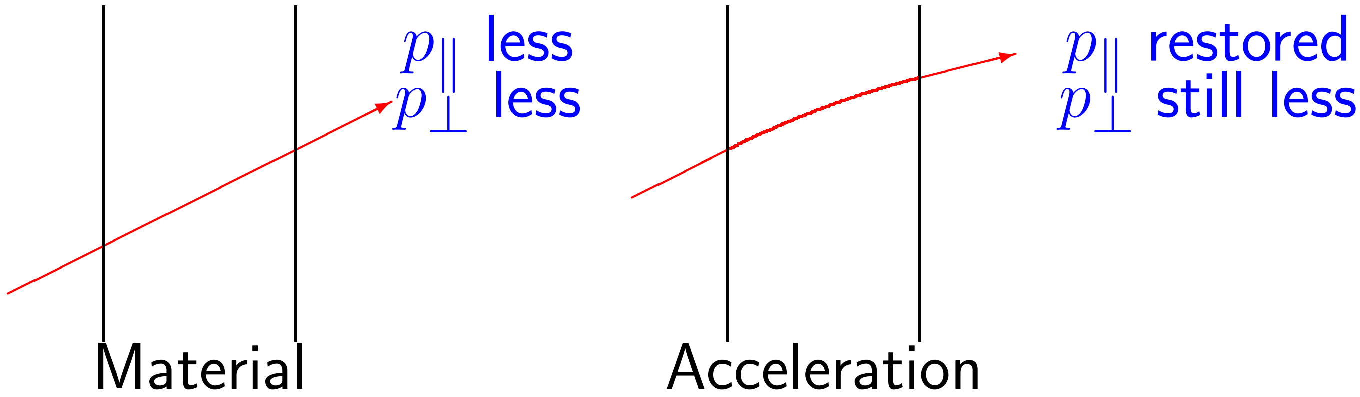


The angular rotation in Larmor frame plus focusing 'force' give helical motion

Conclusion on solenoid focusing

- In long solenoid: particle moves in a helix of wavelength $\lambda_{\rm helix}$
- In Larmor plane: oscillates with wavelength $\lambda_{\rm larmor} = 2\,\lambda_{\rm helix}$
- Even with non uniform fields, motion in the larmor plane:
  - Focus is always towards the axis
  - With a 'force' $\propto B^2/p^2$
  - If a particle starts in the Larmor plane, it stays in that plane
- Since a solenoid focuses with a 'force' $\propto B^2/p^2$, compared with a quadrupole 'force' $\propto B/p$, the solenoid is always stronger at a low enough momenta
- Solenoids focus in both planes, whereas quadrupoles focus in one and defocus in the other
- A solenoid can focus very large transverse emittances, with angles of a radian or more, which makes solenoids the preferred focusing in ionization cooling

6 TRANSVERSE IONIZATION COOLING



Cooling rate vs. Energy

$$\text{(eq 10)} \quad \epsilon_{x,y} = \gamma\beta_v \, \sigma_\theta \, \sigma_{x,y}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ are reduced. So the fractional cooling $d\epsilon\,/\epsilon$ is (using eq.??):

$$\frac{d\epsilon}{\epsilon} = \frac{dp}{p} = \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{25}$$

which, for a given energy change, favors cooling at low energy.

Heating Terms

$$\epsilon_{x,y} = \gamma\beta_v \, \sigma_\theta \, \sigma_{x,y}$$

Between scatters the drift conserves emittance (Liouiville).

When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$\begin{aligned}
\Delta(\epsilon_{x,y})^2 &= \gamma^2\beta_v^2 \, \sigma_{x,y}^2 \Delta(\sigma_\theta^2) \\
2\epsilon \, \Delta\epsilon &= \gamma^2\beta_v^2 \left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right) \Delta(\sigma_\theta^2) \\
\Delta\epsilon &= \frac{\beta_\perp\gamma\beta_v}{2} \, \Delta(\sigma_\theta^2)
\end{aligned}$$

Coulomb Scattering

e.g. from Particle data booklet

$$\Delta(\sigma_\theta^2) \approx \left(\frac{14.1\ 10^6}{[pc/e]\beta_v}\right)^2 \frac{\Delta s}{L_R}$$

$$\Delta\epsilon \;=\; \frac{\beta_\perp}{\gamma\beta_v^3}\,\Delta E\;\left(\left(\frac{14.1\ 10^6}{2[mc^2/e]_\mu}\right)^2 \frac{1}{L_R dE/ds}\right)$$

Defining $$C(mat, E) \;=\; \frac{1}{2}\left(\frac{14.1\ 10^6}{[mc^2/e]_\mu)}\right)^2 \frac{1}{L_R\, d\gamma/ds} \tag{26}$$

$$\text{then}\qquad \frac{\Delta\epsilon}{\epsilon} \;=\; dE\;\frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\;C(mat, E) \tag{27}$$

Equilibrium emittance

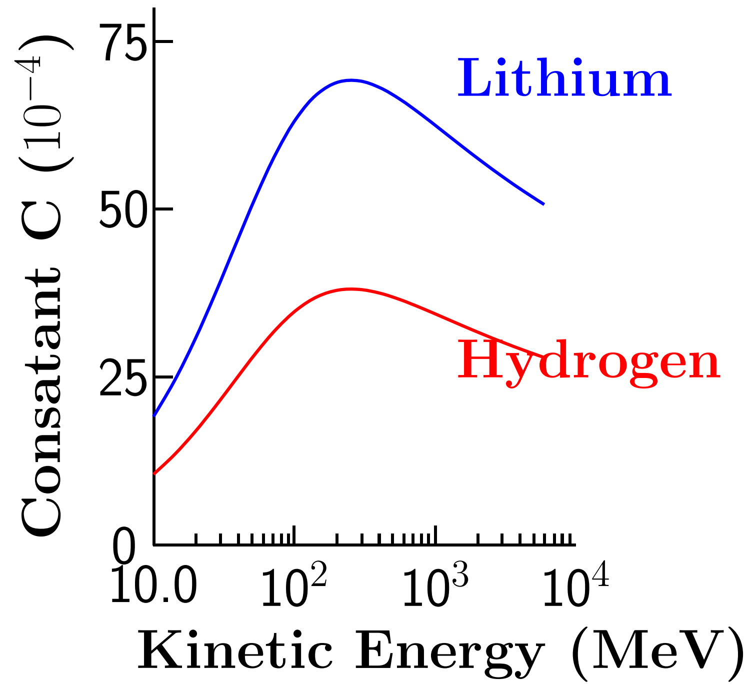
Equating with equation 25 $$dE\,\frac{1}{\beta_v^2\ E} = dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E)$$

Gives equilibrium ϵ_o : $$\epsilon_{x,y}(min) = \frac{\beta_\perp}{\beta_v}\ C(mat,E) \quad (28)$$

At minimum ionization loss:

material	T ^{o}K	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

Vs. energy:



Choice of material

Liquid Hydrogen is far the best material, but has cryogenic and safety complications, and requires windows which significantly degrade the performance. LiH is the next best and does not need windows or cryogenics.

- The IDS Neutrino Factory uses LiH, but
- The MAP Muon Collider uses Liquid or gas hydrogen

Choice of energy

At lower energies, the constant C(mat,E) is much lower but there is then longitudinal (dp/p) heating.

- For the Neutrino factory and initial Collider cooling, we use near minimum ionizing (130 MeV)
- For Final cooling we let the energy drop to $\approx$ 10 MeV

Rate of Cooling

$$\frac{d\epsilon}{\epsilon} = \left(1 - \frac{\epsilon_{\min}}{\epsilon}\right) \frac{dp}{p} \tag{29}$$

Choice of β

One might think one should keep $\epsilon_{\min} \ll \epsilon$, but this generally gives problems from non-linearities with the required large beam divergence angles σ_θ required.

Beam Divergence Angles

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp \, \beta_v \gamma}}$$

so, from equation 28, for a beam in equilibrium

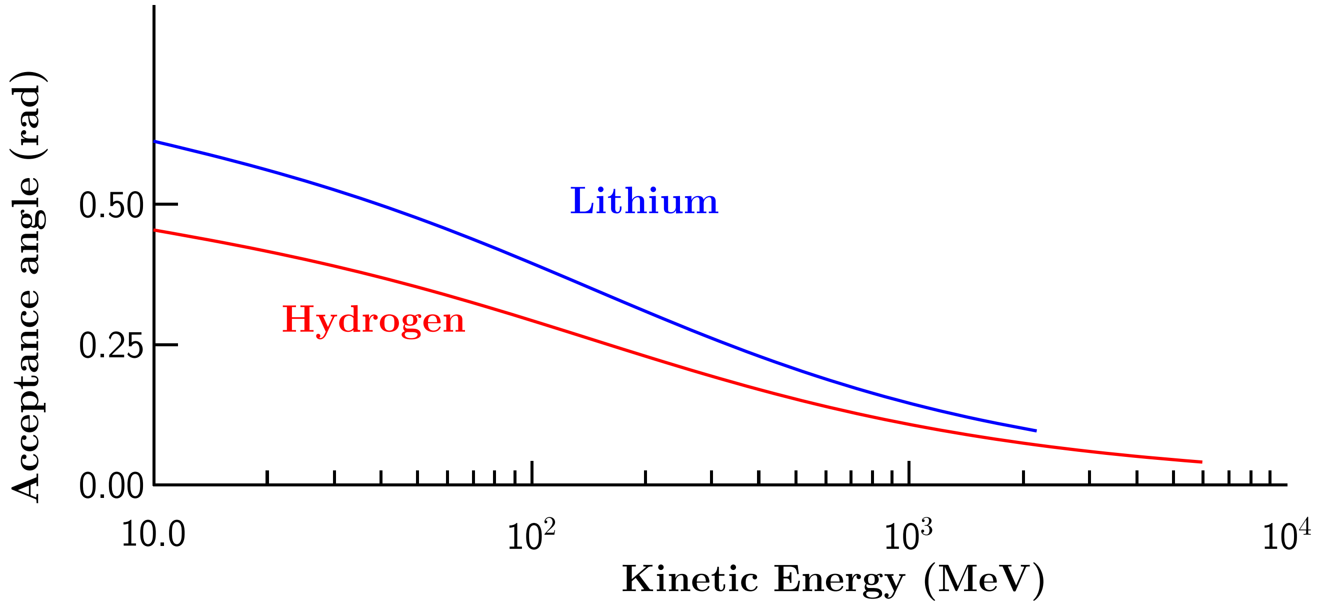
$$\sigma_\theta = \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \tag{30}$$

and for 50 % of maximum cooling rate and an aperture at 3 σ, the angular aperture $\mathcal{A}$ of the system must be

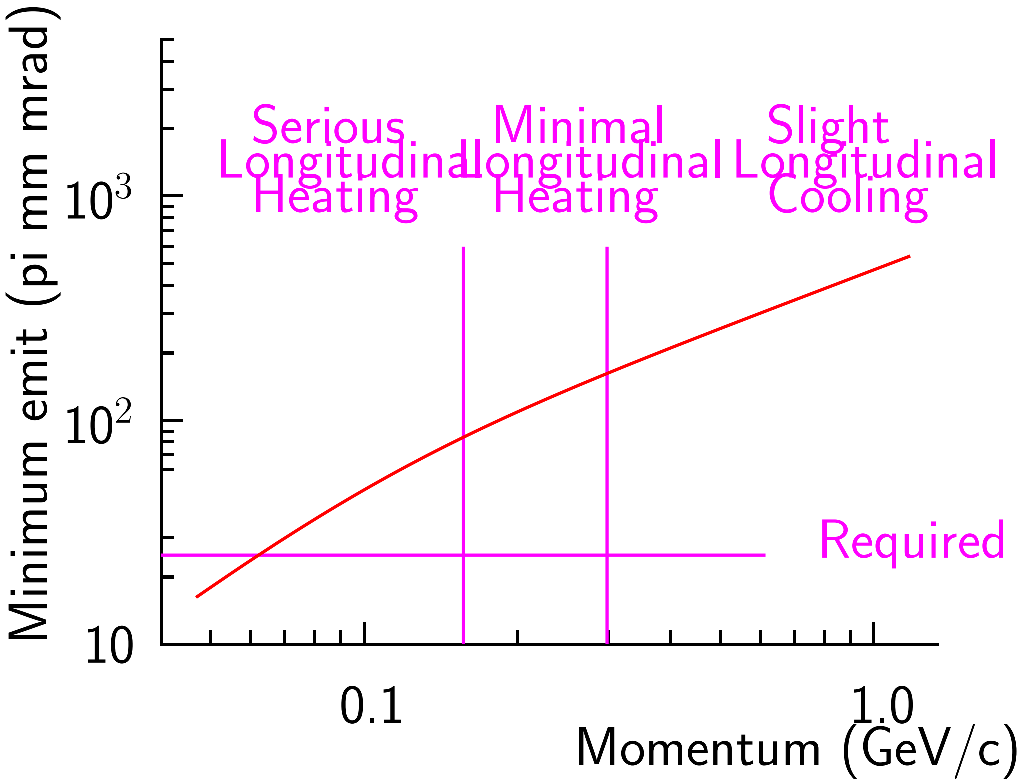
$$\mathcal{A} = 3\sqrt{2}\,\sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \tag{31}$$

Apertures for hydrogen and lithium are plotted vs. energy below. These are very large angles, and if we limit apertures to less than 0.3, then this requirement sets lower energy limits of about 100 MeV ($\approx$ 170 MeV/c) for Lithium, and about 25 MeV ($\approx$ 75 MeV/c) for hydrogen.

$\theta = 0.3$ may be about as large as is possible in a lattice, but larger angles may be sustainable in a continuous focusing system such as a lens or solenoid. is optimistic, as we will see in the tutorial.



Focusing as a function of the beam momentum



From eq.24

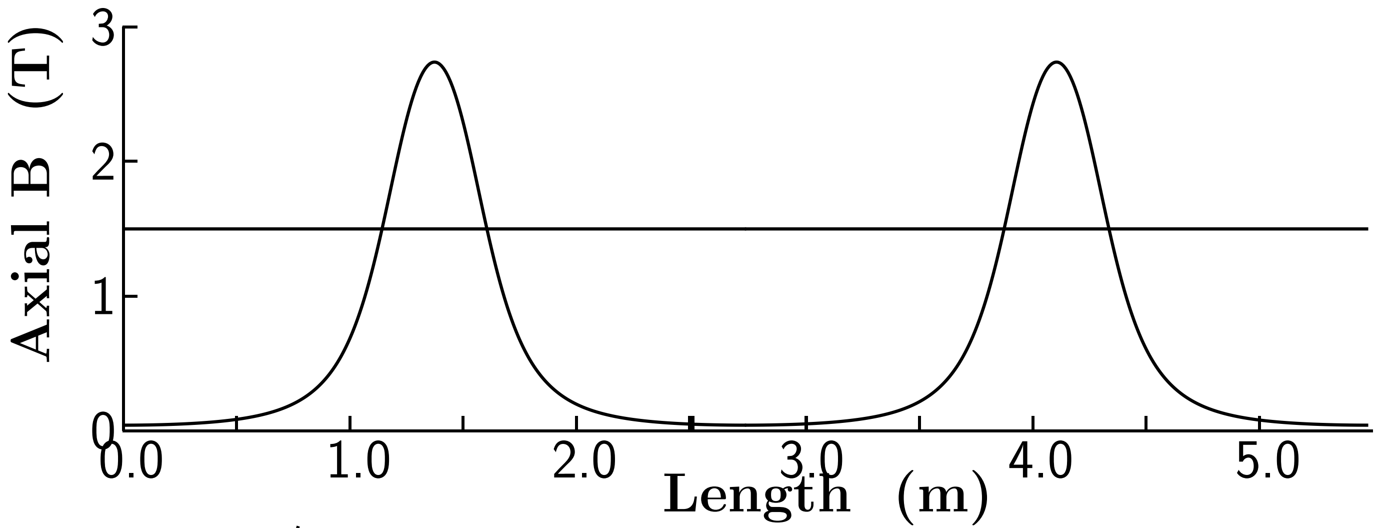
$$\beta_\perp = \frac{2\ [pc/e]}{c\ B_{sol}}$$

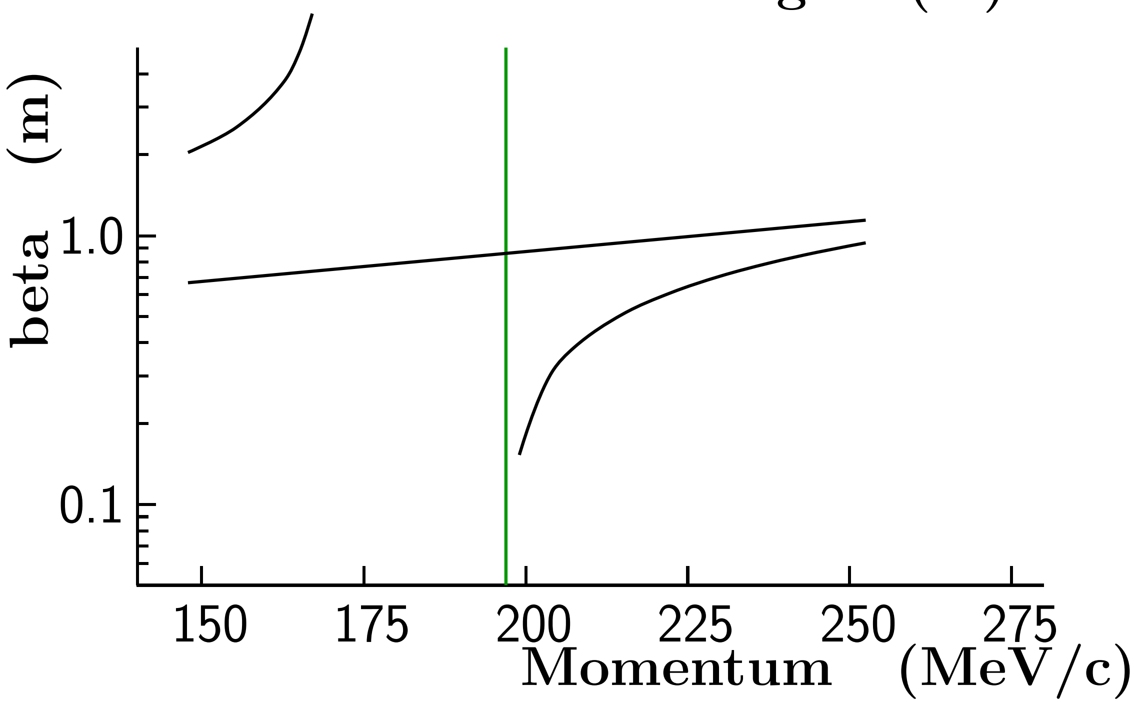
$$\epsilon_{x,y}(min) = C(mat, E)\ \frac{2\ \gamma\ [mc^2/e]_\mu}{B_{sol}\ c} \qquad (32)$$

We see that at momenta where longitudinal emittance is not blown up ($\approx$ 200 MeV/c) then even at 40 T the minimum emittance is $\approx$ 100 μm $>>$ required 25 μm

But if we allow longitudinal heating and use very low momenta (45-62 MeV/c or 9-17 MeV) the collider requirements can be met

Decreasing beta in Solenoids by adding periodicity



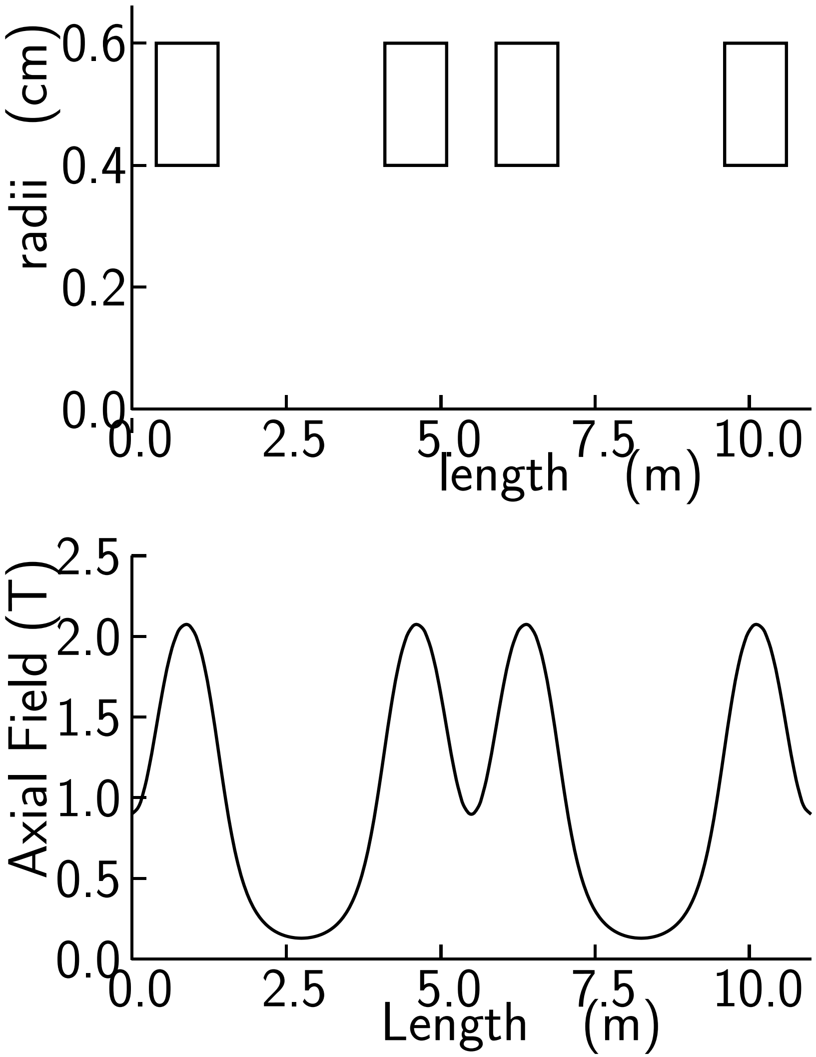


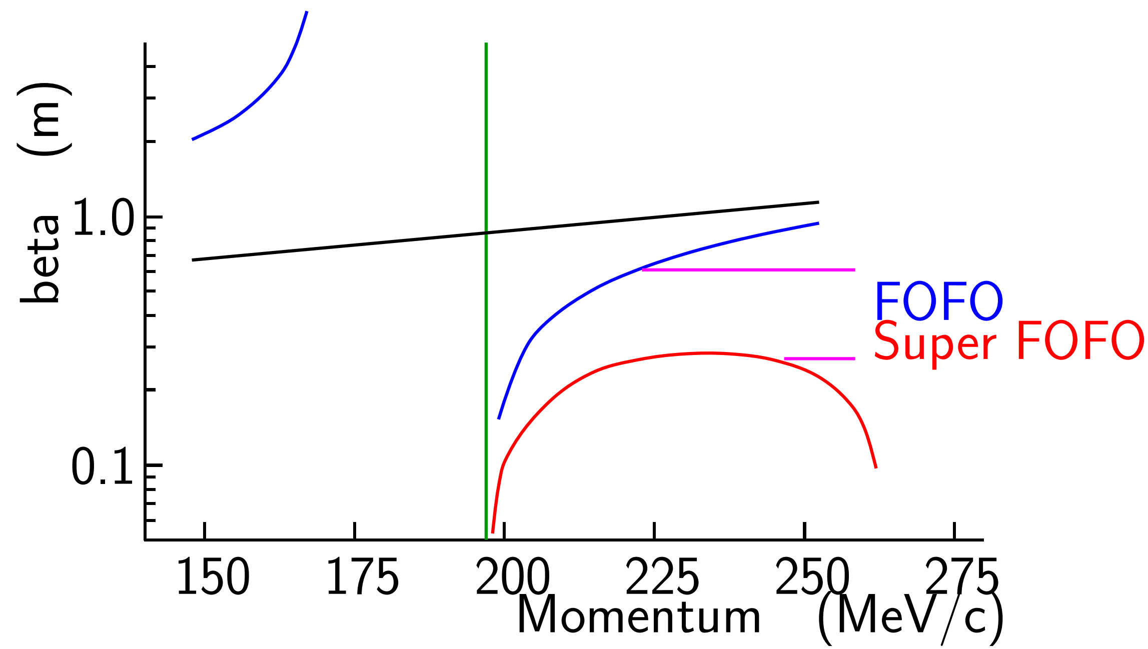
- Resonances introduced
- Betas reduced locally
- But only over small momentum range

Solenoid fields are alternated to avoid a buildup of angular momentum

Super FOFO

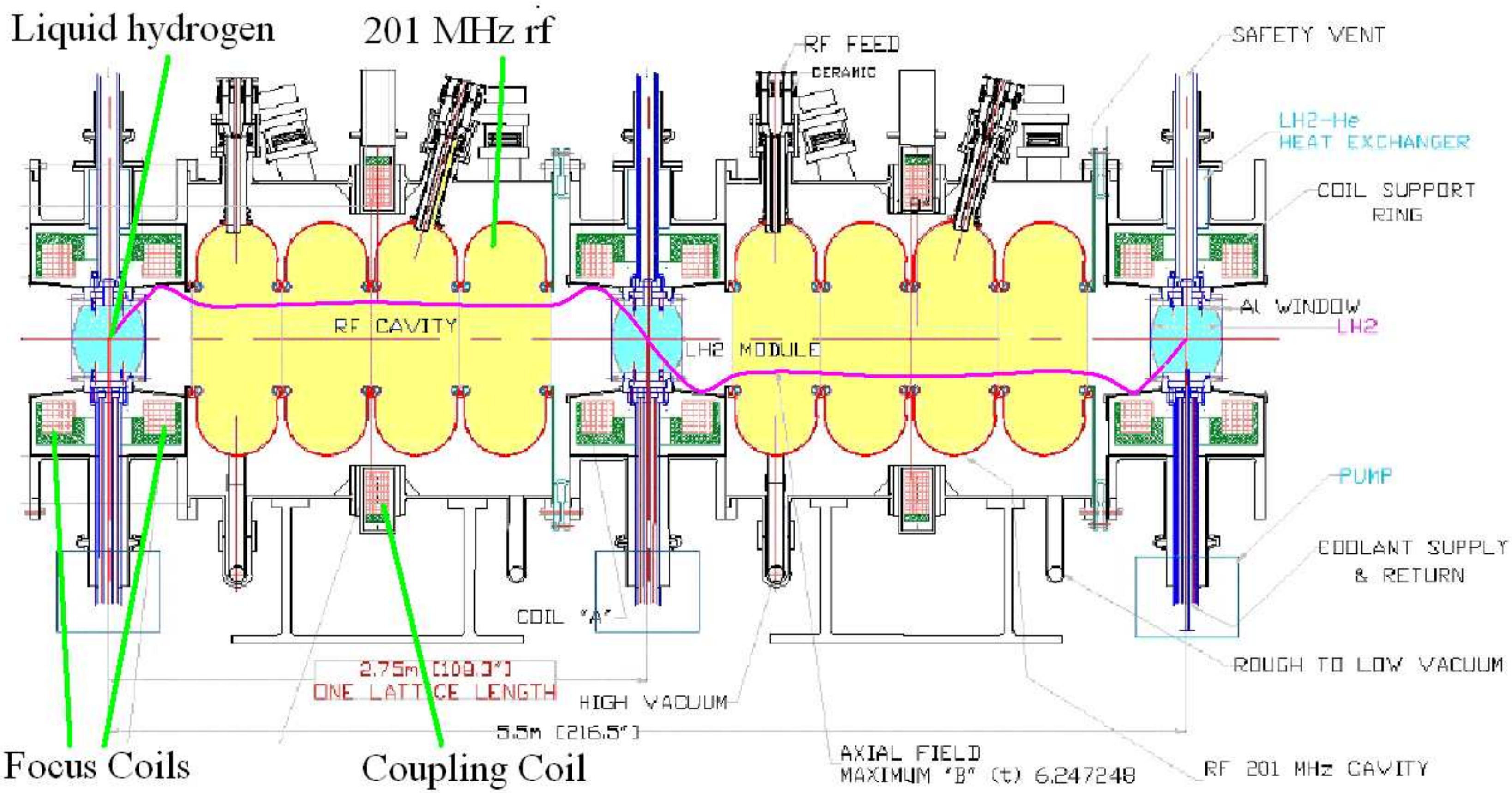
Double periodicity





- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

SFOFO Lattice Engineering
Study 2 at Start of Cooling



- This is the lattice to be tested in Muon Ionization Cooling Experiment (MICE) at RAL
- Study 2 the lattice is modified vs. length to lower $\beta_\perp$ as ϵ falls, keeping σ_θ and ϵ/ϵ_o more constant, thus maintains cooling rate

Conclusion on transverse cooling

- Hydrogen (gas or liquid) is the best material to use
- Cooling requires very large angular acceptances -
- Only realistically possible in solenoid focused systems
- Adding periodicity lowers the $\beta_\perp$ for a given solenoid field
- But periodicity does reduce momentum acceptance
- Final cooling to 25 μm possible at 40 T and low energies
  but longitudinal emittance then rises

7 LONGITUDINAL IONIZATION COOLING

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} = \frac{\frac{\Delta\ (\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{33}$$

$$J_6 = J_x + J_y + J_z \tag{34}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). Δp and p refer to the loss of momentum induced by this energy loss.

In electron synchrotrons, with no gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In muon ionization cooling, $J_x = J_y = 1$, but J_z is negative or small.

Transverse cooling with $J_{x,y} \neq 1$

From last lecture:

$$\frac{\Delta\sigma_{p\perp}}{\sigma_{p\perp}} = \frac{\Delta p}{p}$$

and $\sigma_{x,y}$ does not change, so

$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} = \frac{\Delta p}{p}$$

and thus

$$J_x = J_y = 1 \tag{35}$$

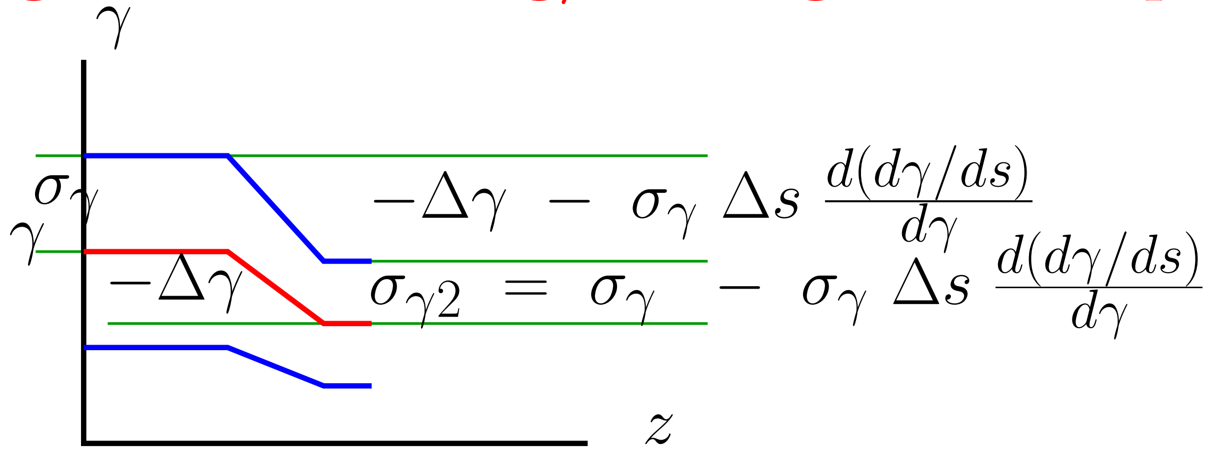
But if $J_{x,y} \neq 1$

$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} = \frac{1}{J_{x,y}} \frac{\Delta p}{p} \tag{36}$$

and

$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{J_{x,y}\ \beta_v}\ C(mat, E) \tag{37}$$

Longitudinal cooling/heating from shape of dE/dx



The emittance in the longitudinal direction ϵ_z is (eq.11):

$$\epsilon_z \;=\; \gamma\beta_v\,\frac{\sigma_p}{p}\,\sigma_z \;=\; \frac{1}{m}\,\sigma_p\sigma_z \;=\; \frac{1}{m}\,\sigma_E\sigma_t \;=\; c\,\sigma_\gamma\,\sigma_t$$

where σ_t is the rms bunch length in time, and c is the velocity of light. Drifting between interactions will not change emittance (Louville), and an interaction will not change σ_t, so emittance change is only induced by the energy change in the interactions:

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma} \;=\; \frac{\sigma_\gamma\,\Delta s\,\frac{d(d\gamma/ds)}{d\gamma}}{\sigma_\gamma} \;=\; \Delta s\,\frac{d(d\gamma/ds)}{d\gamma}$$

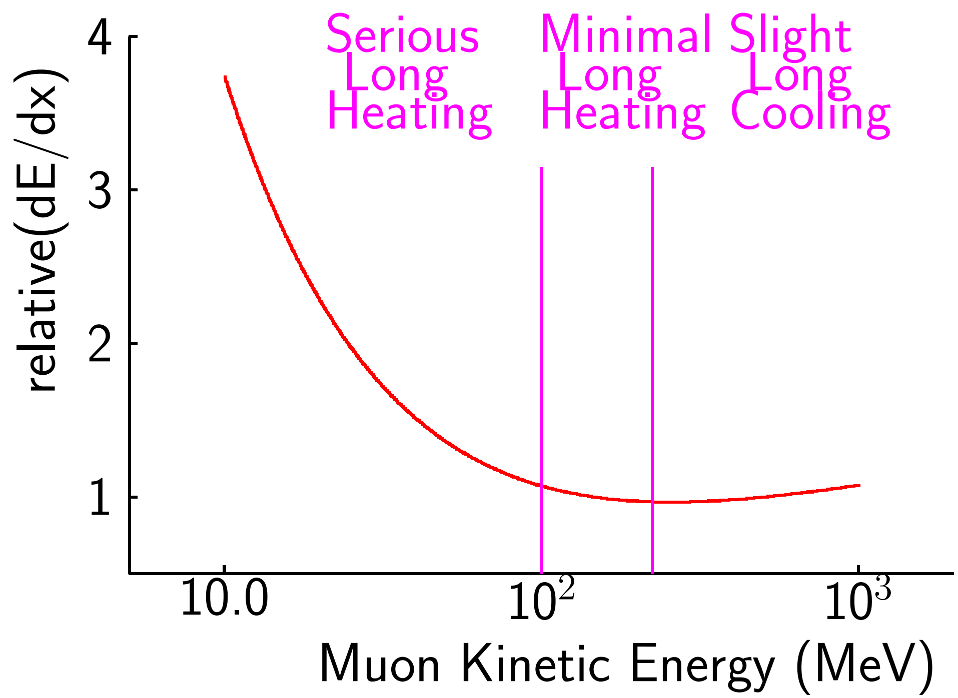
and

$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

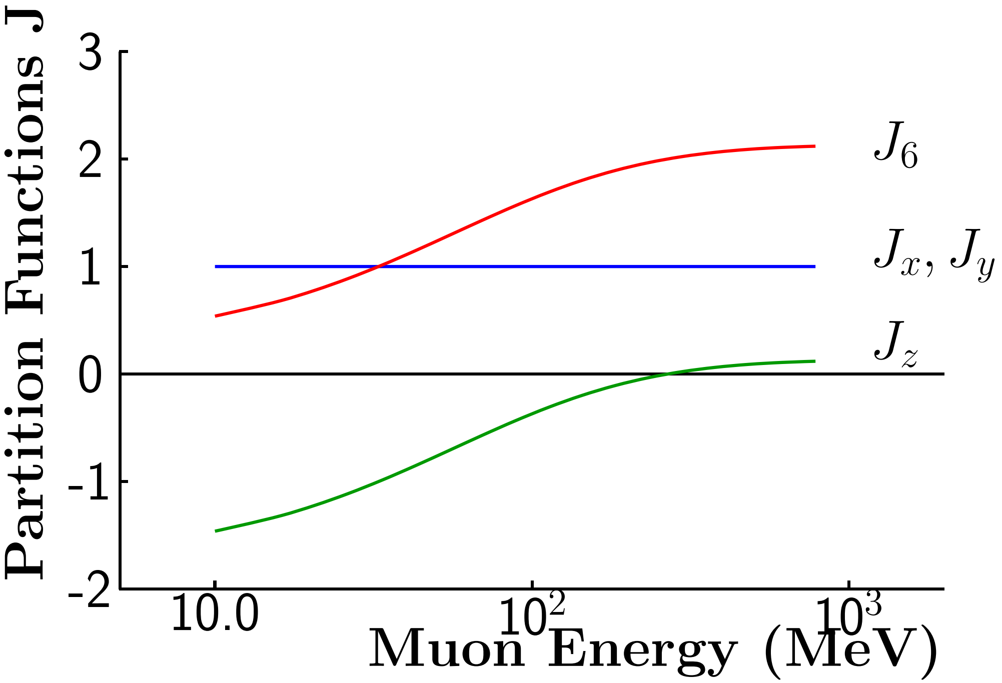
$$J_z = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\left(\Delta s\ \frac{d(d\gamma/ds)}{d\gamma}\right)}{\frac{\Delta s}{\beta_v^2\gamma}\ \left(\frac{d\gamma}{ds}\right)} = \frac{\left(\beta_v^2\ \frac{d(d\gamma/ds)}{d\gamma/\gamma}\right)}{\left(\frac{d\gamma}{ds}\right)} \quad (38)$$

A typical relative energy loss as a function of energy is shown above (this example is for Lithium).



Jx, Jy, Jz vs energy

It is seen that J_z is strongly negative at low energies (longitudinal heating), and is only barely positive at momenta above 300 MeV.
Since more acceleration per cooling decrement at higher energies, we prefer to use of the order of 200 MeV.
Only for final cooling are we forced to very low energies.
Note however, the 6D cooling is still strong finite even at the lowest energies.



Emittance Exchange

What is needed is a method to exchange cooling between the transverse and longitudinal direction s. This is done in synchrotron cooling if focusing and bending is combined. Wedges and other tricks can do the same for muons.

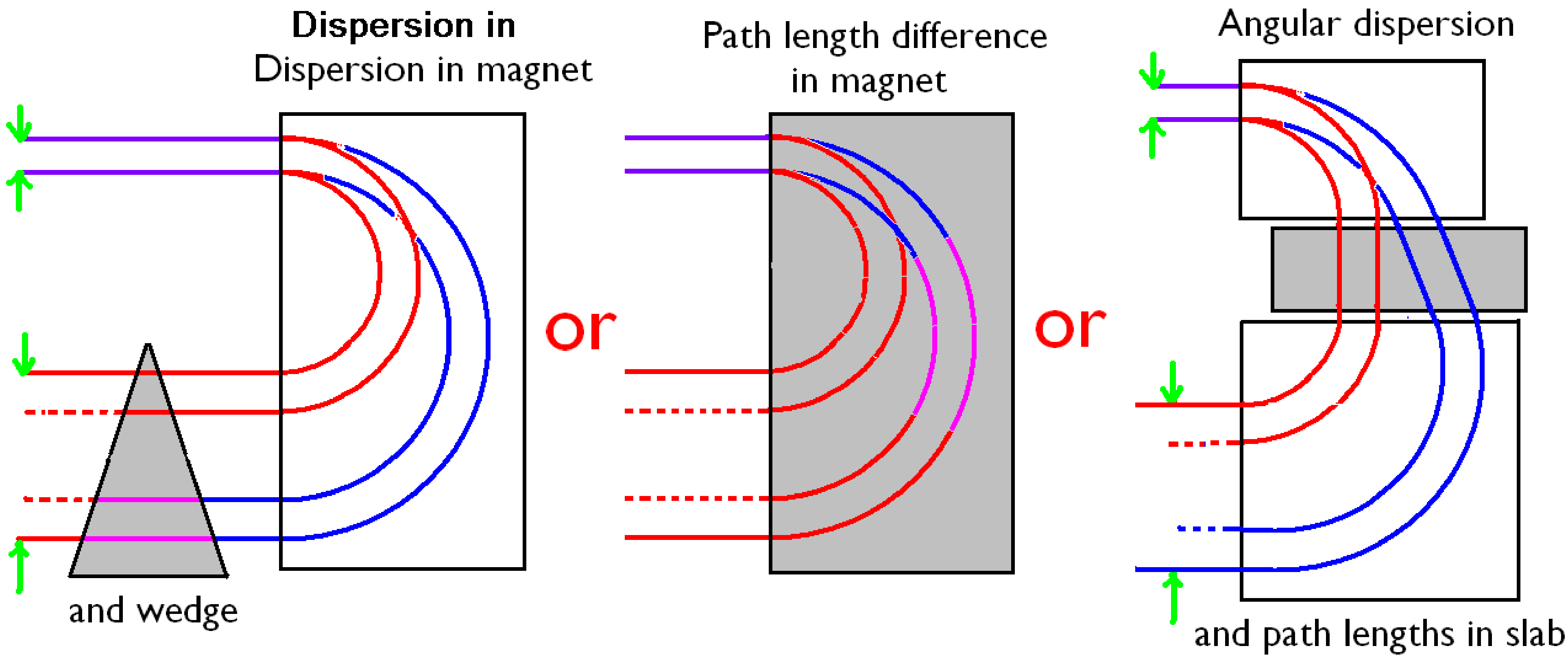
In both electron and muon cases, such mixing can only increase one J at the expense of the others: J_6 is conserved.

$$\Delta J_x + \Delta J_x + \Delta J_x = 0 \tag{39}$$

and for typical operating momenta:

$$J_x \ + \ J_y \ + \ J_z \ = \ J_6 \ \approx \ 2.0 \tag{40}$$

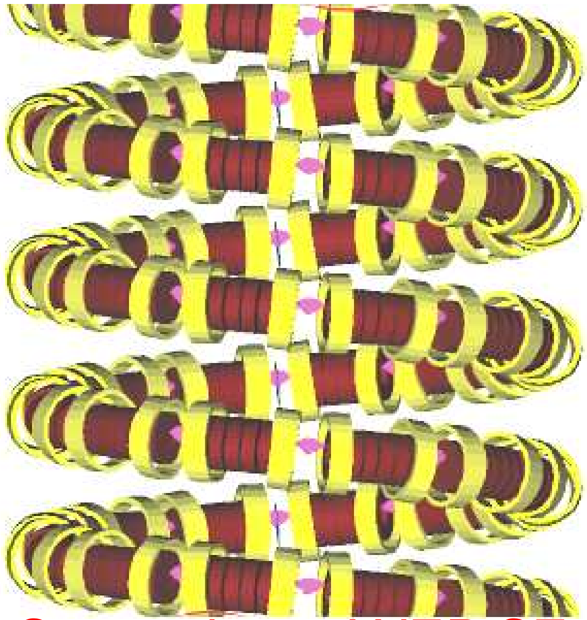
methods to exchange emittances for muons



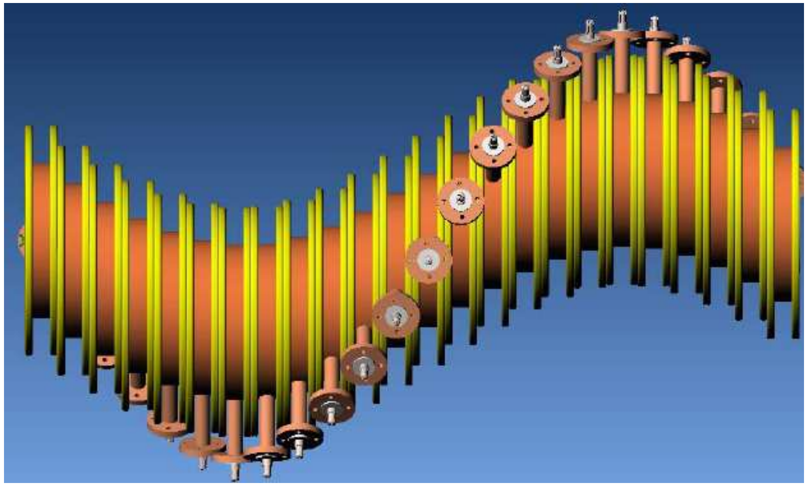
dp/p reduced But σ_y increased

Long Emit reduced Trans Emit Increased

3 candidate 6D cooling lattices

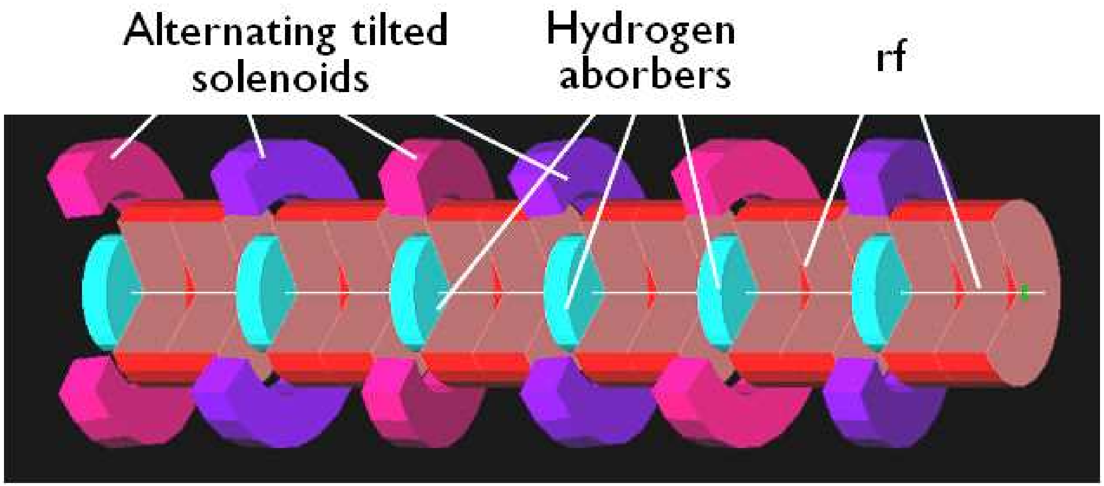


Guggenheim WEDGE



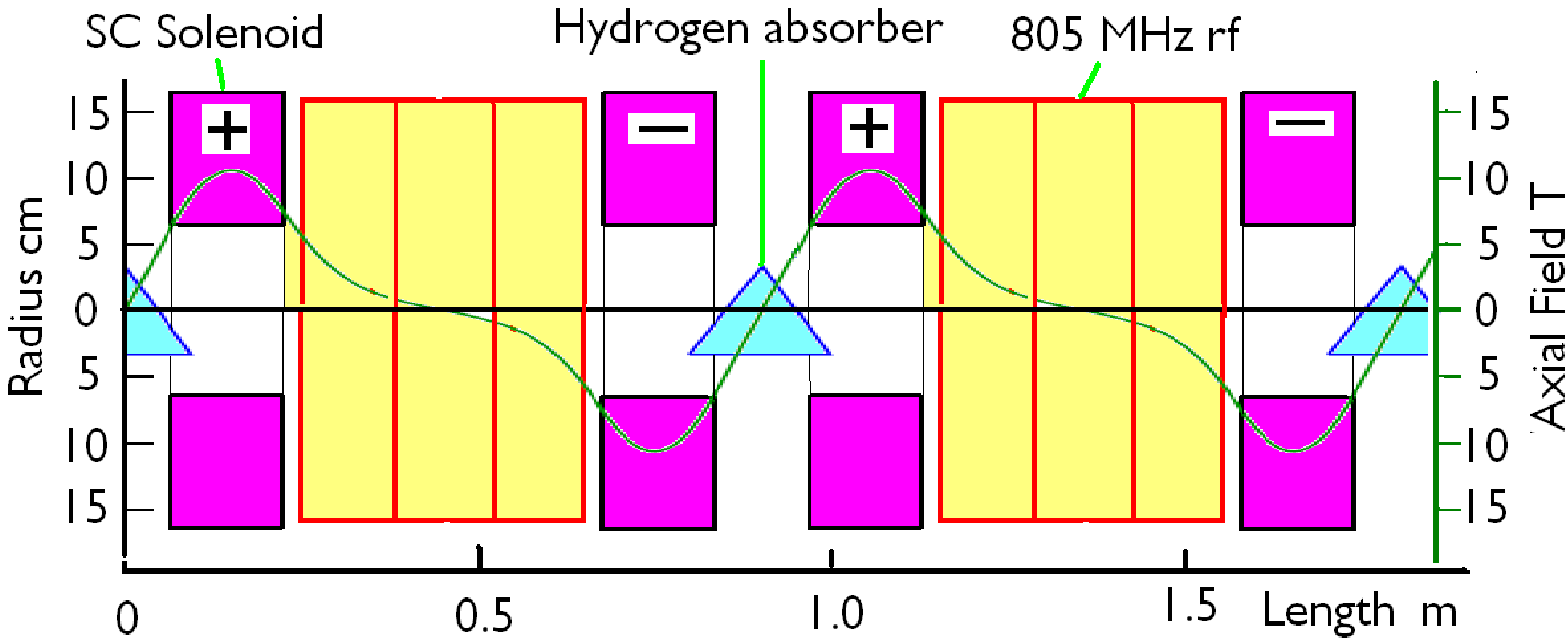
Helical Cooling Channel PATH

Snake SLAB



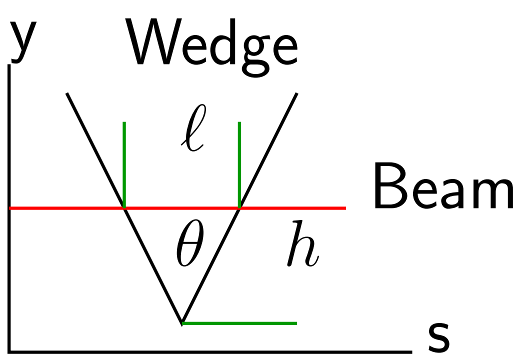
- All simulated   All have problems/limitations
- I will use Guggenheim as example

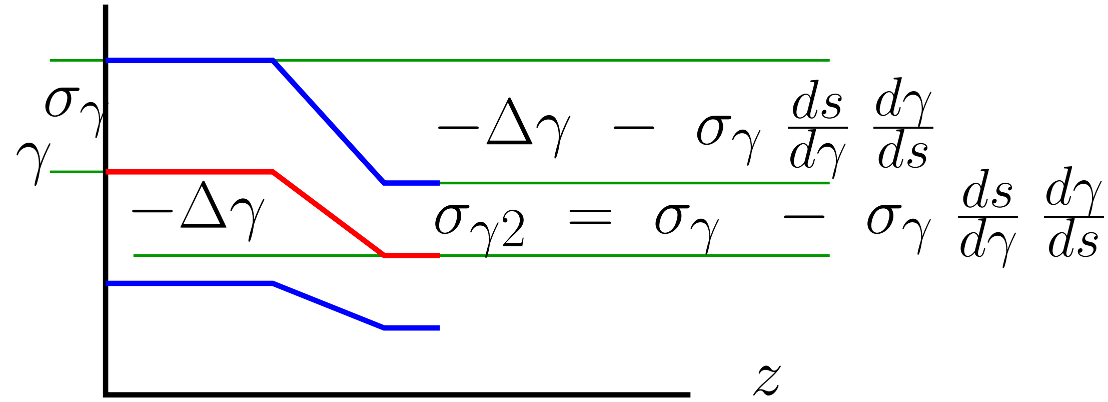
Detail of Guggenheim Lattice



- Coils are slightly tilted to generate vertical bending field
- giving dispersion at the wedge absorbers
- and generating the helical form

Longitudinal cooling with wedges and Dispersion





For a wedge with center thickness ℓ and height from center h ($2h \ \tan(\theta/2) = \ell$), in dispersion D ($D = \frac{dy}{dp/p}$ $D = \beta_v^2 \frac{dy}{d\gamma/\gamma}$) (see fig. above):

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma} = \frac{\sigma_\gamma \, \frac{ds}{d\gamma} \left(\frac{d\gamma}{ds}\right)}{\sigma_\gamma} = \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right) = \left(\frac{\ell}{h}\right) \frac{D}{\beta_v^2 \, \gamma} \left(\frac{d\gamma}{ds}\right)$$

and

$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma} \left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$\Delta J_z(\text{wedge}) \;=\; \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} \;=\; \frac{\left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\;\gamma}\;\left(\frac{d\gamma}{ds}\right)}{\frac{\ell}{\beta_v^2\gamma}\;\left(\frac{d\gamma}{ds}\right)} \;=\; \frac{D}{h} \tag{41}$$

$$J_z \;=\; J_z(\text{no wedge}) \;+\; \Delta J_z(\text{wedge}) \tag{42}$$

But from eq.39, for any finite $J_z(\text{wedge})$, J_x or J_y will change in the opposite direction.

Longitudinal Heating Terms

Since $\epsilon_z = \sigma_\gamma \ \sigma_t \ c$, and t and thus σ_t is conserved in an interaction

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

$$\text{Straggling}: \Delta(\sigma_\gamma) \approx \frac{\Delta\sigma_\gamma^2}{2\sigma_\gamma} \approx \frac{1}{2\sigma_\gamma} 0.06 \, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho \, \Delta s$$

$$\Delta E = E \, \beta_v^2 \, \frac{\Delta p}{p}, \quad \text{so}: \quad \Delta s = \frac{\Delta E}{dE/ds} = \frac{1}{dE/ds} E \, \beta_v^2 \, \frac{\Delta p}{p}$$

giving:

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{0.06}{2\sigma_\gamma^2} \, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho \, \frac{\beta_v^2 \, E}{dE/ds} \, \frac{\Delta p}{p}$$

.

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z} = -\ J_z\ \frac{dp}{p}$$

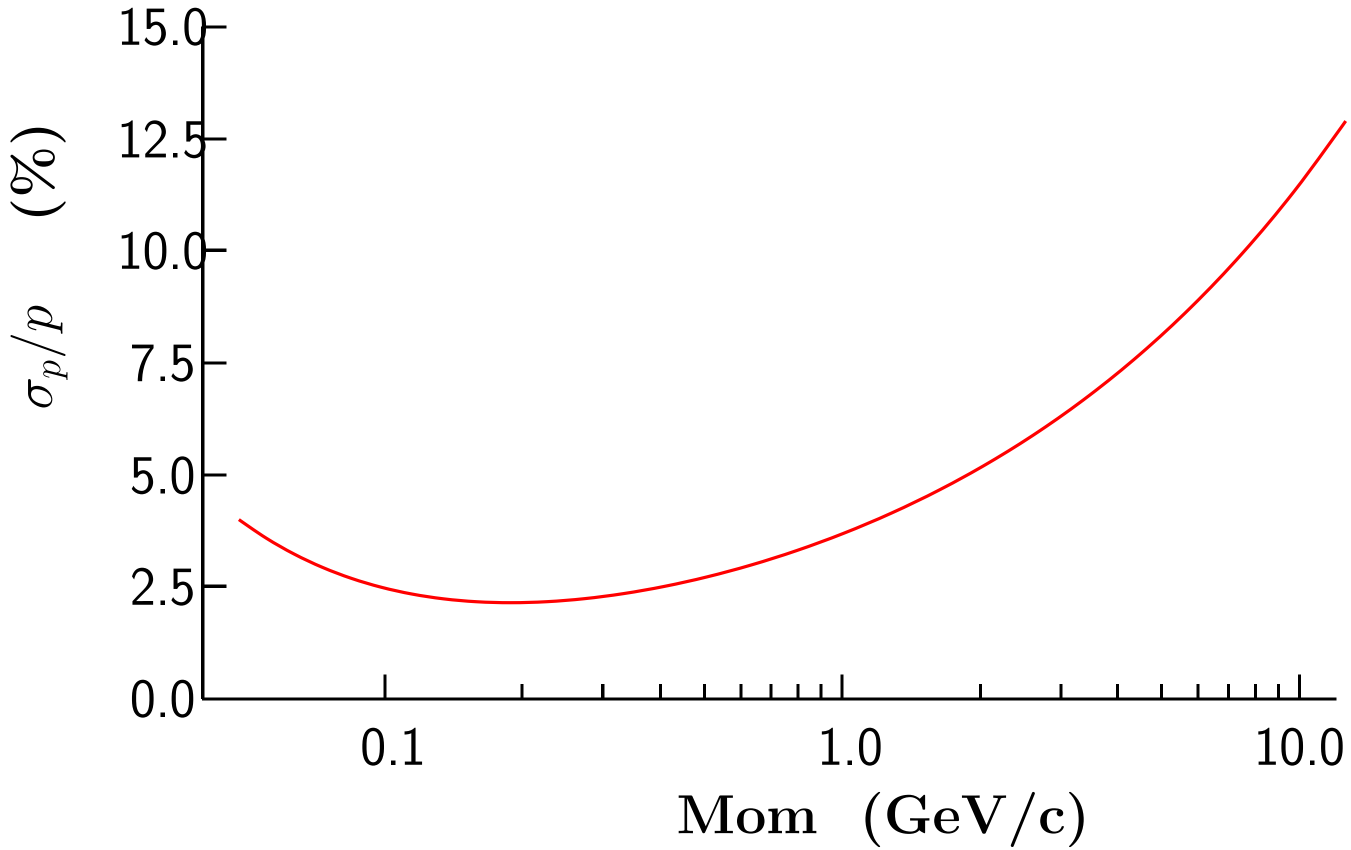
giving an equilibrium:

$$\frac{\sigma_p}{p} = \left(\frac{m_e}{m_\mu}\sqrt{\frac{0.06\ Z\ \rho}{2\ A\ (d\gamma/ds)}}\right)\ \sqrt{\frac{\gamma}{\beta_v^2}\ \left(1-\frac{\beta_v^2}{2}\right)\quad\frac{1}{J_z}} \tag{43}$$

For Hydrogen, the value of the first parenthesis is ≈1.36 %.

Without emittance exchange, J_z is small or negative, and the equilibrium does not exist. But with equal partition functions giving $J_z \approx 2/3$ then this expression, for hydrogen, gives: the values plotted below.

The following plot shows the dependency for hydrogen



It is seen to favor cooling at around 200 MeV/c, but has a broad minimum.

Longitudinal Cooling Conclusion

- Good cooling in 6 D in a ring
  - But injection/extraction difficult
  - Requires short bunch train
- Converting Ring cooler to a large Helix (Guggenheim)
  - Solves Injection/extraction problem
  - Solves bunch train length problem
  - Allows tapering to improve performance
  - But more expensive than ring
- Also good 6D cooling in HP Gas Helix (not discussed here)
  - But difficult to introduce appropriate frequency rf
  - And questions about use of gas with an ionizing beam
- 6D cooling in Snake accepts both signs
  Useful at start, but does not yet cool enough