# IMPROVED PREDICTION OF EVENT SHAPES FROM EFFECTIVE FIELD THEORY

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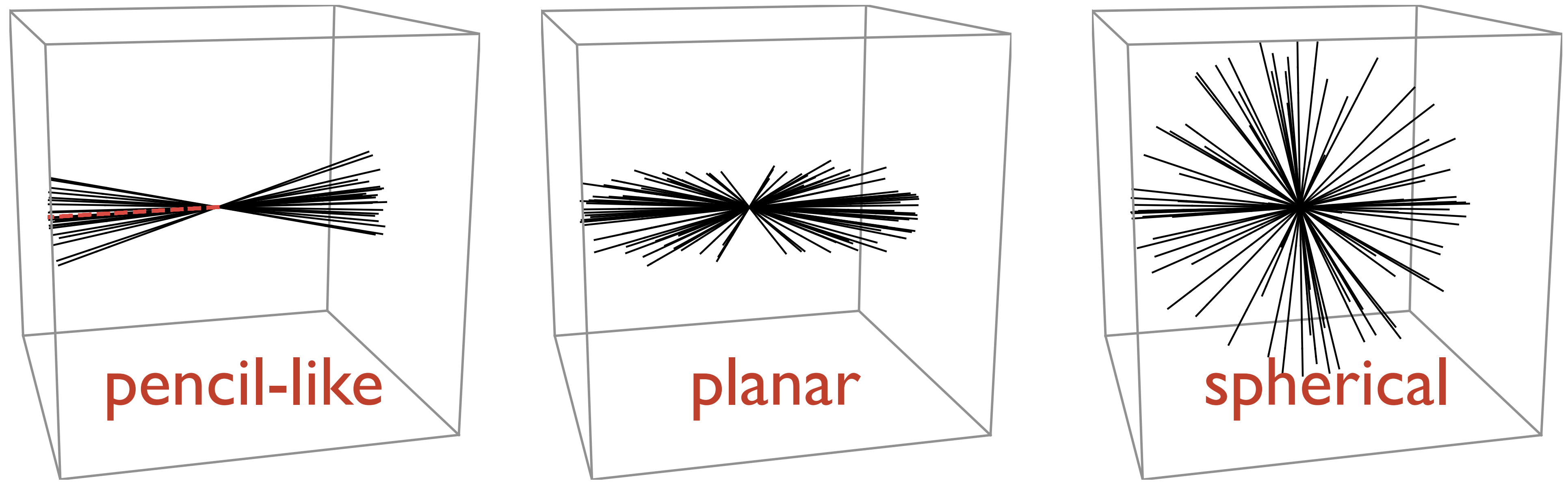
# OVERVIEW

- Event shape variables
  - Definition and experimental prospects at the ILC
- Improved theoretical prediction of thrust
  - NNLO fixed order
  - $N^3LL$ resummation by RG evolution in Soft Collinear Effective Theory (SCET)

TB and M.D. Schwartz, JHEP 0807:034,2008

- Phenomenological results
  - Precision determination of $\alpha_s$ using LEP data.
  - Bound on light gluinos
  - Comparison with MC event generators

# EVENT-SHAPE VARIABLES




- Parameterize geometric properties of energy and momentum flow in high energy collisions.
  - Inclusive observables: can be calculated in perturbation theory, hadronisation effects are suppressed at high energy.
- Canonical event shape is thrust $T$

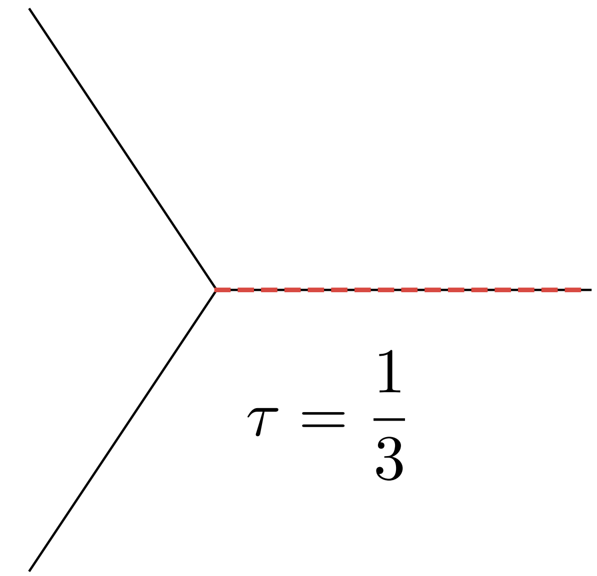
# THRUST $T$ AND THRUST AXIS $\vec{n}$.



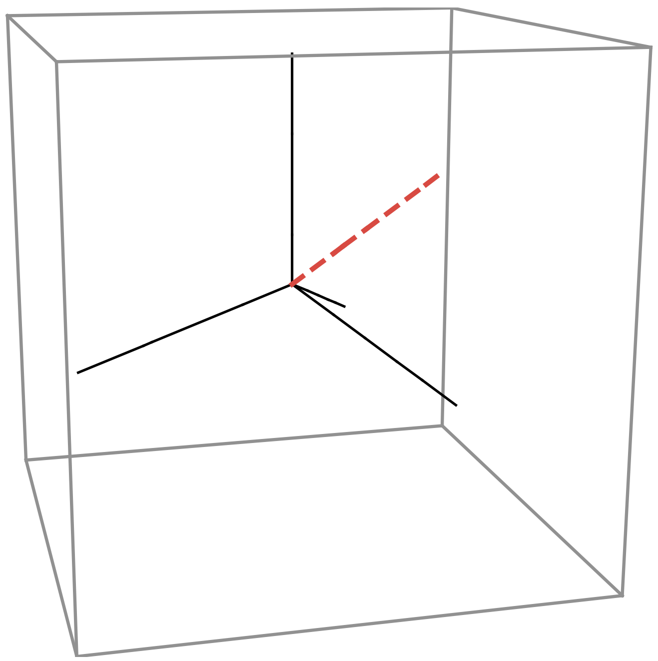
$\tau = 0$

$$T = \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{n} \cdot \vec{p_i}|$$

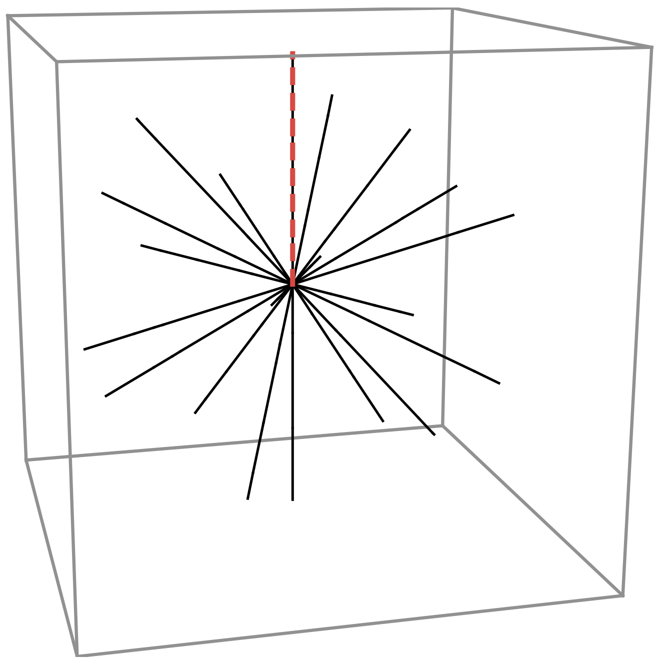
$$\tau = 1 - T$$



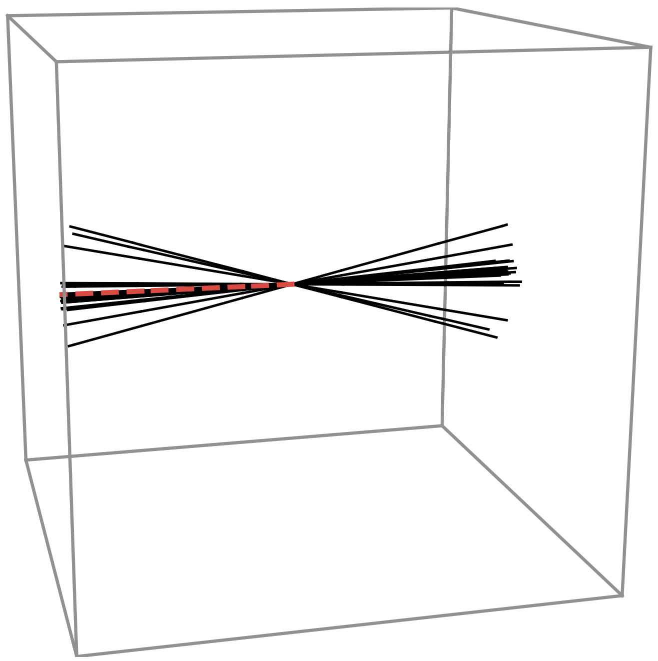
$\tau = \frac{1}{3}$



$\tau = 1 - \frac{1}{\sqrt{3}} = 0.42$

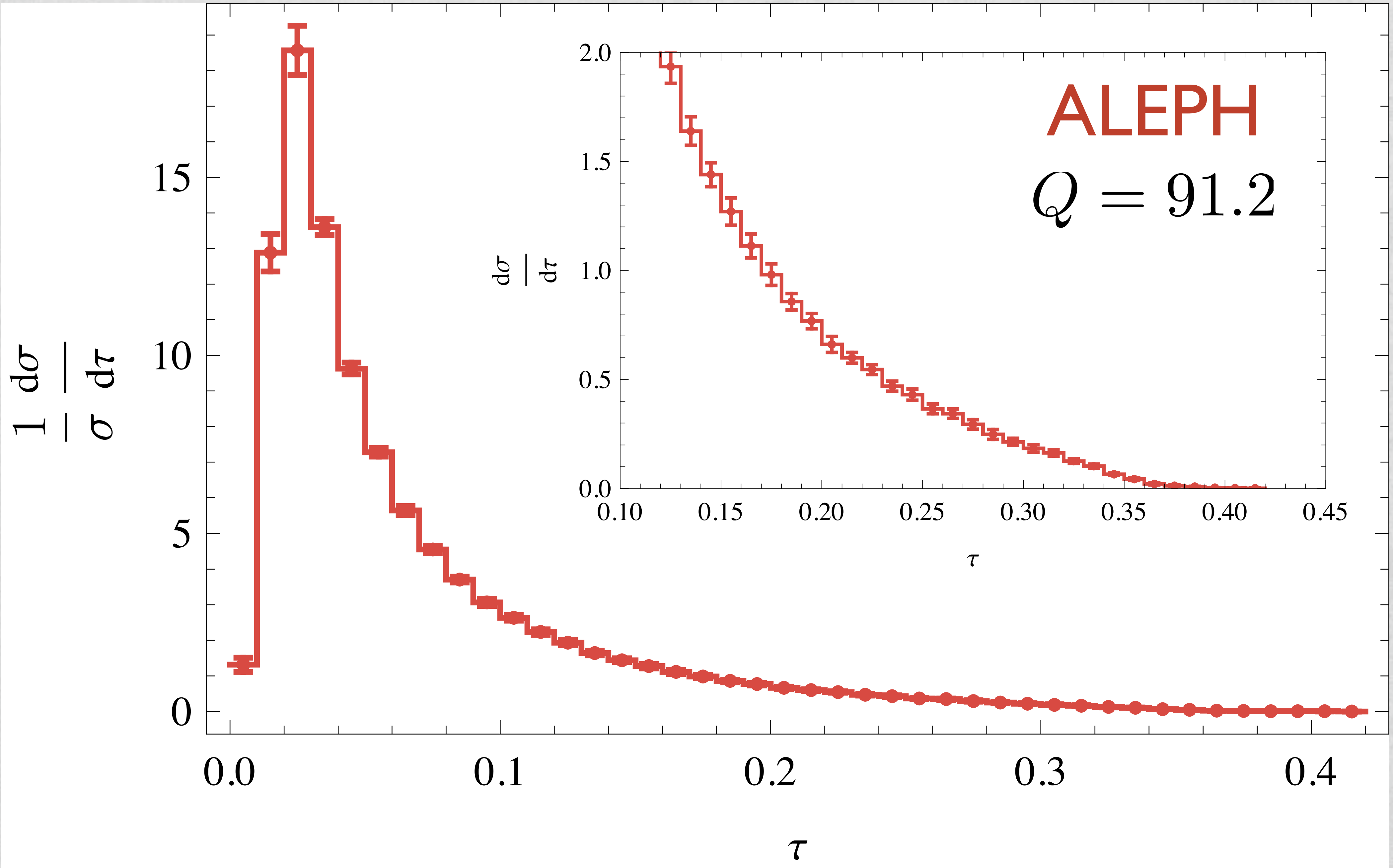


$\tau = 0.48$



$\tau = \frac{M_1^2 + M_2^2}{Q^2}$

# MEASUREMENTS OF THRUST




- Based on 300’000 events. Similar precision by the other LEP experiments.

# USE OF EVENT SHAPES

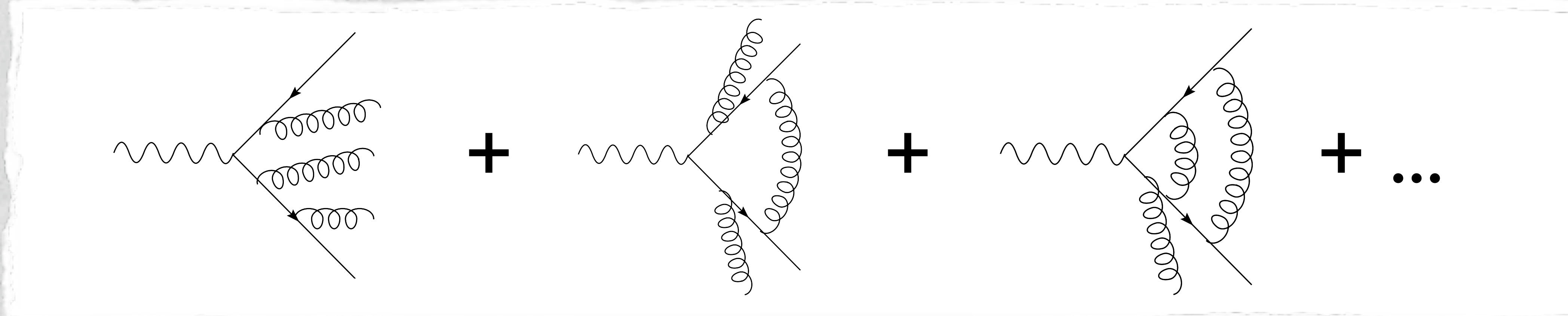
- QCD studies
  - convergence of perturbation theory, validation of shower MCs, studies of hadronisation effects
- Measurement of SM parameters
  - strong coupling constant $\alpha_s$ with $e^+e^- \to q\bar{q}$
  - top-mass with $e^+e^- \to t\bar{t}$ → Sonny Mantry's talk
- Discrimination against background
  - e.g. identification of energetic hadronic top-jets
- Search for new physics
  - e.g. search for light gluinos

# EVENT SHAPES AT THE ILC

- At design luminosity, the ILC produces few hundred thousand $e^+e^- \to q\bar{q}$ events/year
  - Statistical uncertainties on extracted value of $\alpha_s$ is below 0.5%.
  - Systematic uncertainties are expected to be ~ 1%

    Schumm '96 & Truitt '01; Burrows '01
    - "contamination" from $e^+e^- \to \left(t\bar{t}, W^+W^-, ZZ\right)$
    - luminosity spectrum
  - Note: hadronisation effects scale as ~$1/E_{\mathrm{c.m.}}$ and are thus smaller than at LEP
    - can be further constrained by varying $E_{\mathrm{c.m.}}$

# EVENT SHAPES AT NNLO

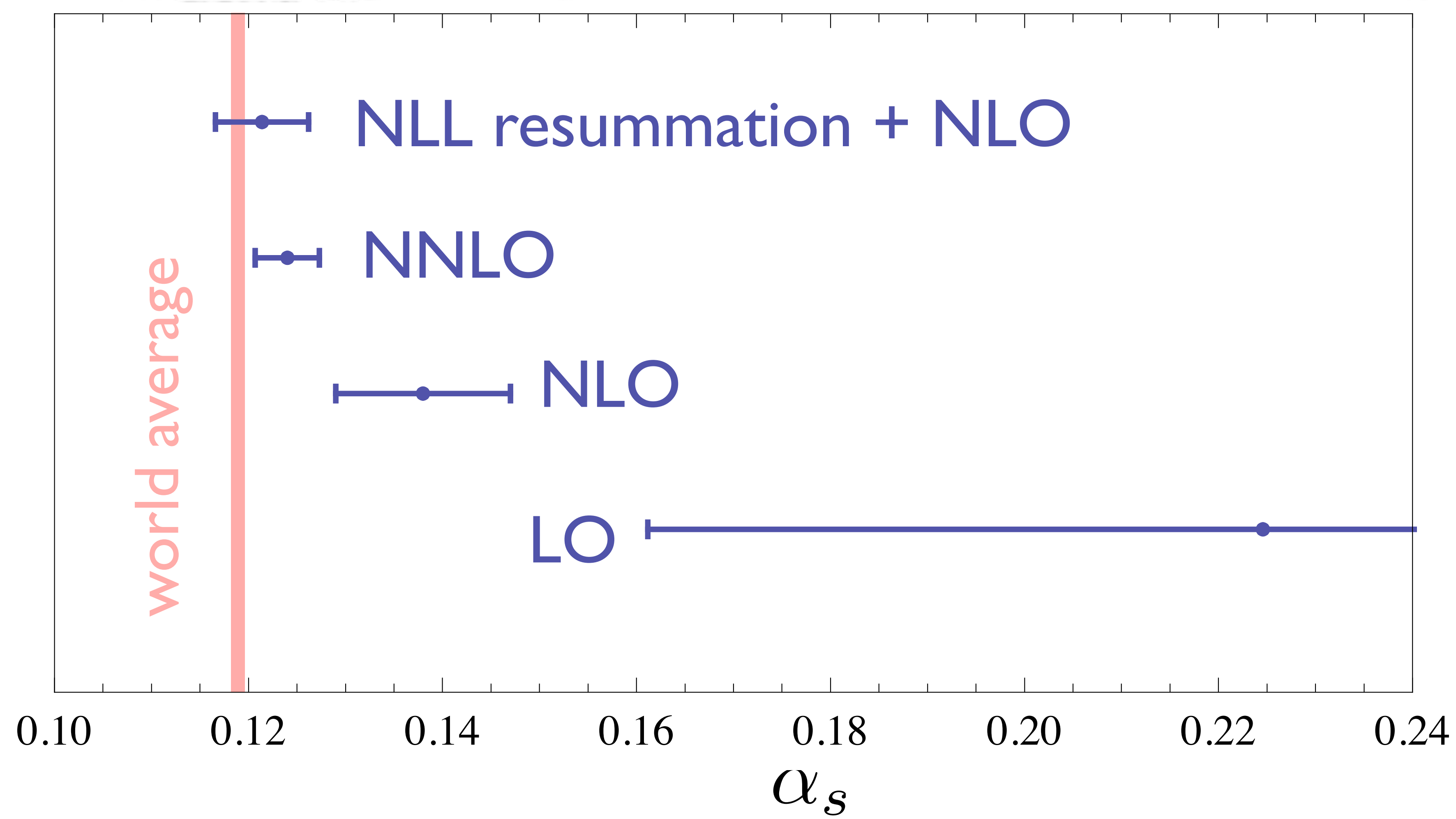



- After years of work, the NNLO calculation of $e^+e^- \to$ 3 jets has been completed.

  A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich '07

  - First time a subtraction scheme has been implemented at NNLO.
    - Real and virtual contributions are have collinear and soft divergences which cancel in the sum.
  - Implemented in fixed order event generator. Can be used for NNLO evaluation of event shapes.

# $\alpha_s$ FROM EVENT SHAPES AT LEP I




- Perturbative uncertainty dominates. At NNLO

$$\alpha_s(M_Z^2) = 0.1240 \pm 0.0008\,(\text{stat}) \pm 0.0010\,(\text{exp}) \pm 0.0011\,(\text{had}) \pm 0.0029\,(\text{theo})$$

# RESUMMATION

- All-order formalism for resummation of thrust distribution
- $N^3LL$ resummation
- Comparison with fixed order

# LOGARITHMICALLY ENHANCED CONTRIBUTIONS

- The LO thrust distribution has the form

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{2\alpha_s}{3\pi}\left[-\frac{3}{\tau}+6+9\tau+\frac{(6\tau^2-6\tau+4)}{(1-\tau)\tau}\ln\frac{1-2\tau}{\tau}\right]$$

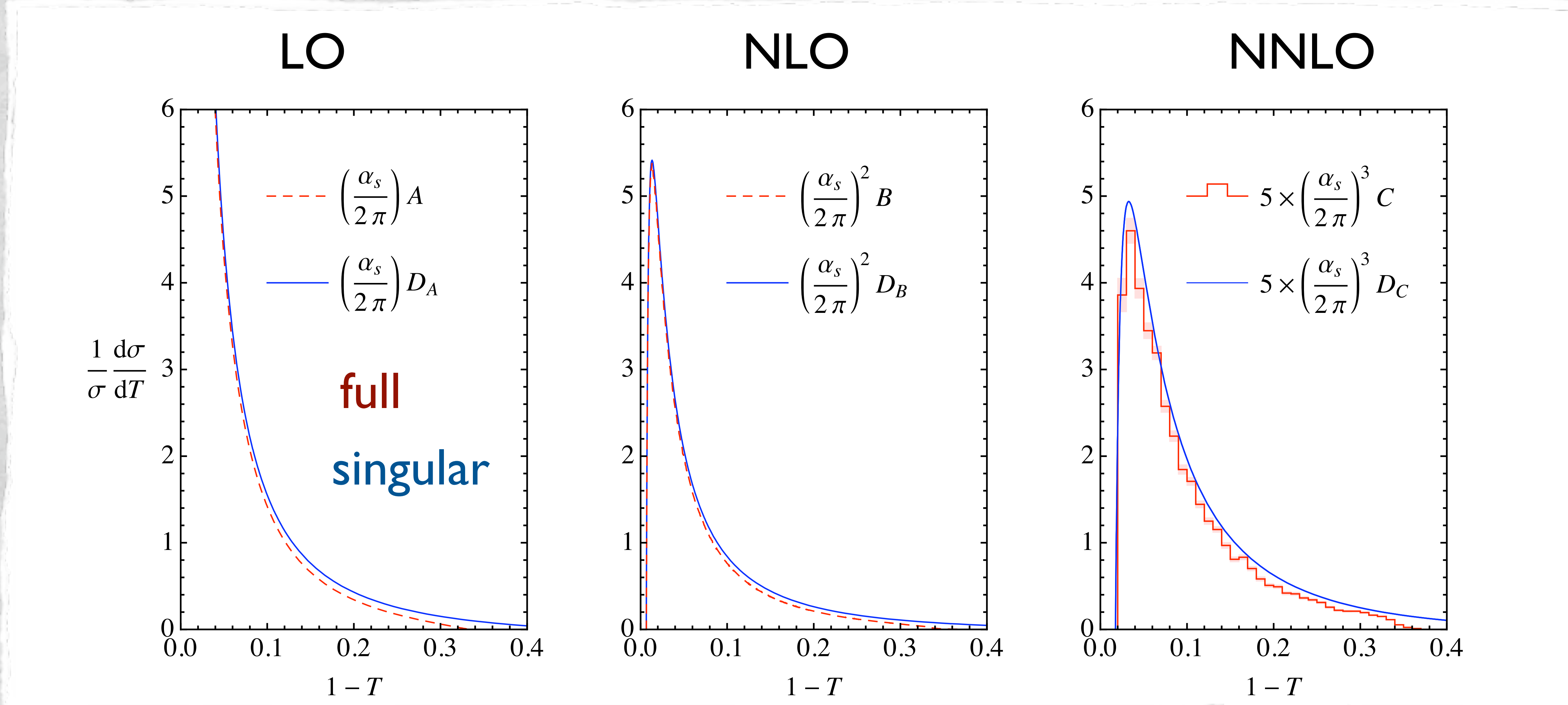
$$= \frac{2\alpha_s}{3\pi}\left[\underbrace{\frac{-4\ln\tau-3}{\tau}}_{\text{singular terms}}+d_{\text{regular}}(\tau)\right]$$

- Integral over the end-point is

$$R(\tau) = \int_0^\tau d\tau'\,\frac{1}{\sigma_0}\frac{d\sigma}{d\tau'} = \frac{2\alpha_s}{3\pi}\left[-2\ln^2\tau-3\ln\tau+\ldots\right]$$

Sudakov double logarithm

# SINGULAR TERMS DOMINATE

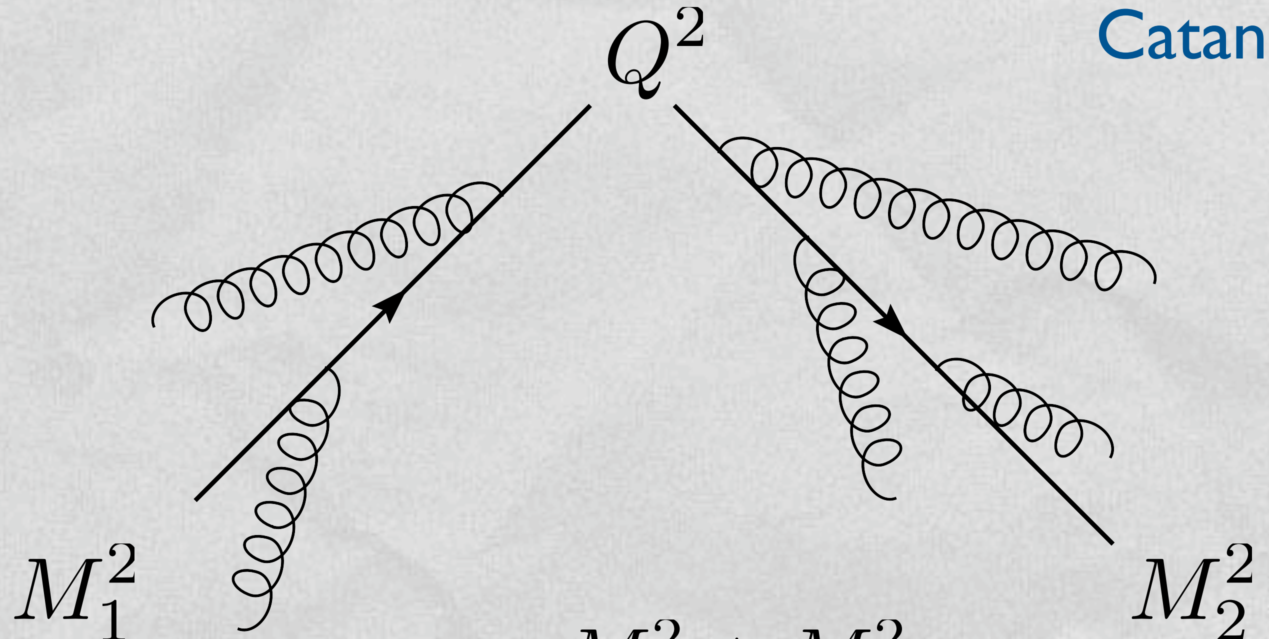



- Singular terms are predicted (and later resummed to all orders) using Soft-Collinear Effective Theory.
- Regular terms (difference of blue and red) are added back after resummation.

# RESUMMATION: THE TRADITIONAL WAY

- Logarithmically enhanced contributions lead to slow convergence of perturbation theory
- The leading logarithms (LL) $\alpha_s^n \ln^{2n} \tau$ and next-to-leading log's (NLL) $\alpha_s^n \ln^{2n-1} \tau$ can be resummed using the "coherent branching algorithm"

Catani, Trentadue, Turnock, Webber '93




$$\tau = \frac{M_1^2 + M_2^2}{Q^2}$$

NLL+NNLO calculation by T. Gehrmann, G. Luisoni and H. Stenzel, arXiv:0803.0695,

# EFFECTIVE THEORY RESUMMATION

- Using Soft Collinear Effective Theory (SCET), one can show that for $\tau \to 0$ the rate factorizes as

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2\, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T\!\left(\tau\, Q - \frac{M_1^2+M_2^2}{Q},\mu\right)$$

Fleming, Hoang, Mantry and Stewart '07
Schwartz '07
see also: Korchemsky '98; Berger, Kucs, Sterman '03

- Three relevant scales:

| $Q^2$ | $\gg$ | $M_1^2 \sim M_2^2 \sim \tau\, Q^2$ | $\gg$ | $\tau^2 Q^2$ |
|---|---|---|---|---|
| hard | | jet | | soft |

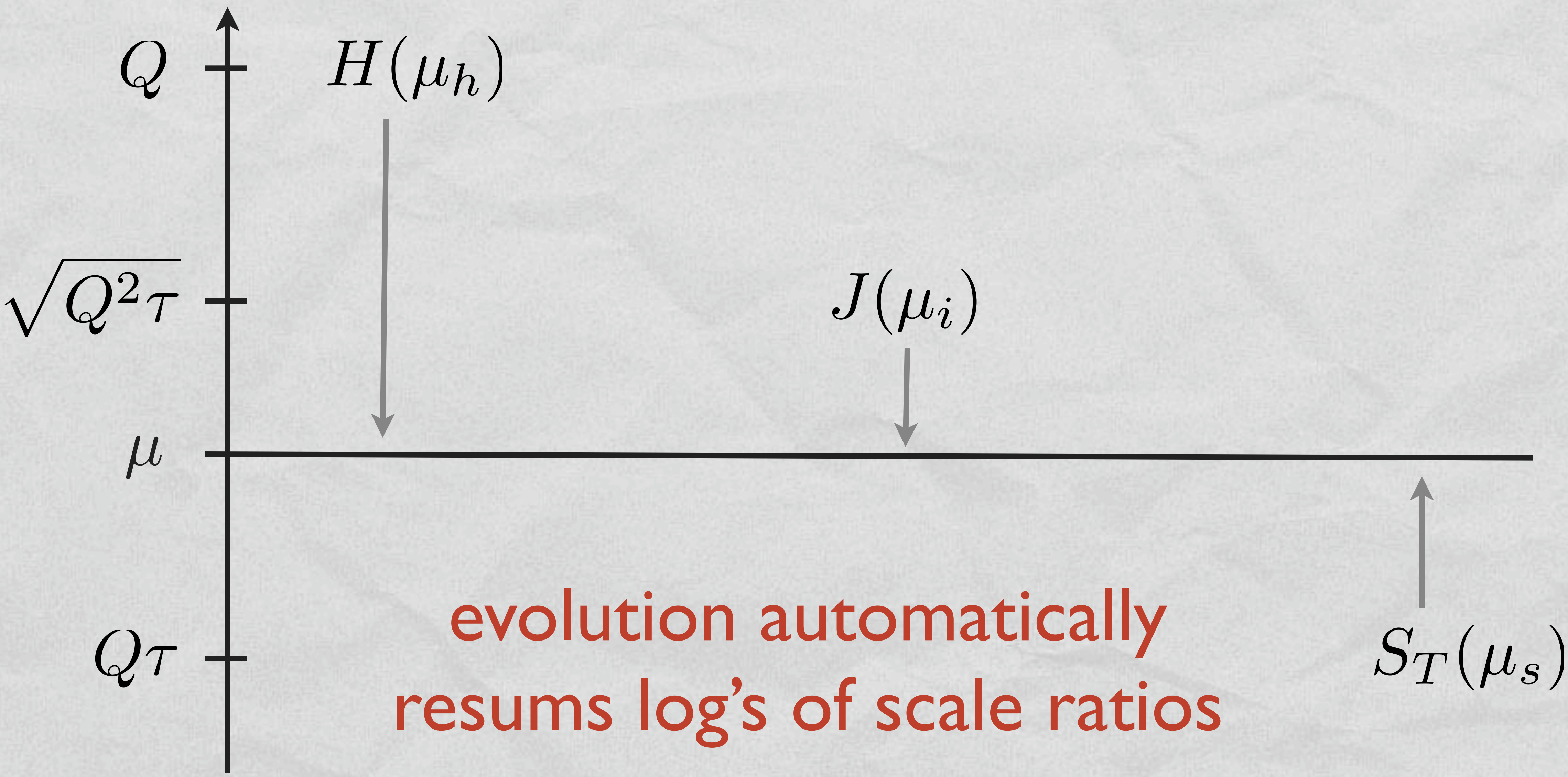
# RESUMMATION

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2\, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T(\tau\, Q - \frac{M_1^2 + M_2^2}{Q},\mu)$$

- The presence of the three separated scales leads to large perturbative logarithms.
  - Any choice of μ will produce large logarithms in either H, J or S.
- *H* and *J* are Wilson coefficients in SCET, *S* a matrix element,
  - fulfill renormalization group equation.

# RESUMMATION BY RG EVOLUTION

- Evaluate each part at its characteristic scale, evolve to common scale:



# RESUMMED THRUST DISTRIBUTION

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = U(\mu_h,\mu_i,\mu_s)\left(\frac{Q^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} H(Q^2,\mu_h)$$
$$\times\left[\widetilde{j}\left(\ln\frac{\mu_s Q}{\mu_i^2}+\partial_\eta,\mu_i\right)\right]^2 \widetilde{s}_T\left(\partial_\eta,\mu_s\right)\frac{1}{\tau}\left(\frac{\tau Q}{\mu_s}\right)^\eta \frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}$$

- $\tilde{j}$ and $\tilde{s}_T$ are Laplace transforms of $J$ and $S_T$
- $U$ is an evolution factor from solving RG eq's
- For N$^3$LL resummation, we need:
  - 4-loop $\Gamma_{\text{cusp}}$ (use Pade approx. for 4-loop term),
  - 3-loop $\gamma$'s,
  - 2-loop $H$, $\widetilde{j}$ and $\widetilde{s}$ .

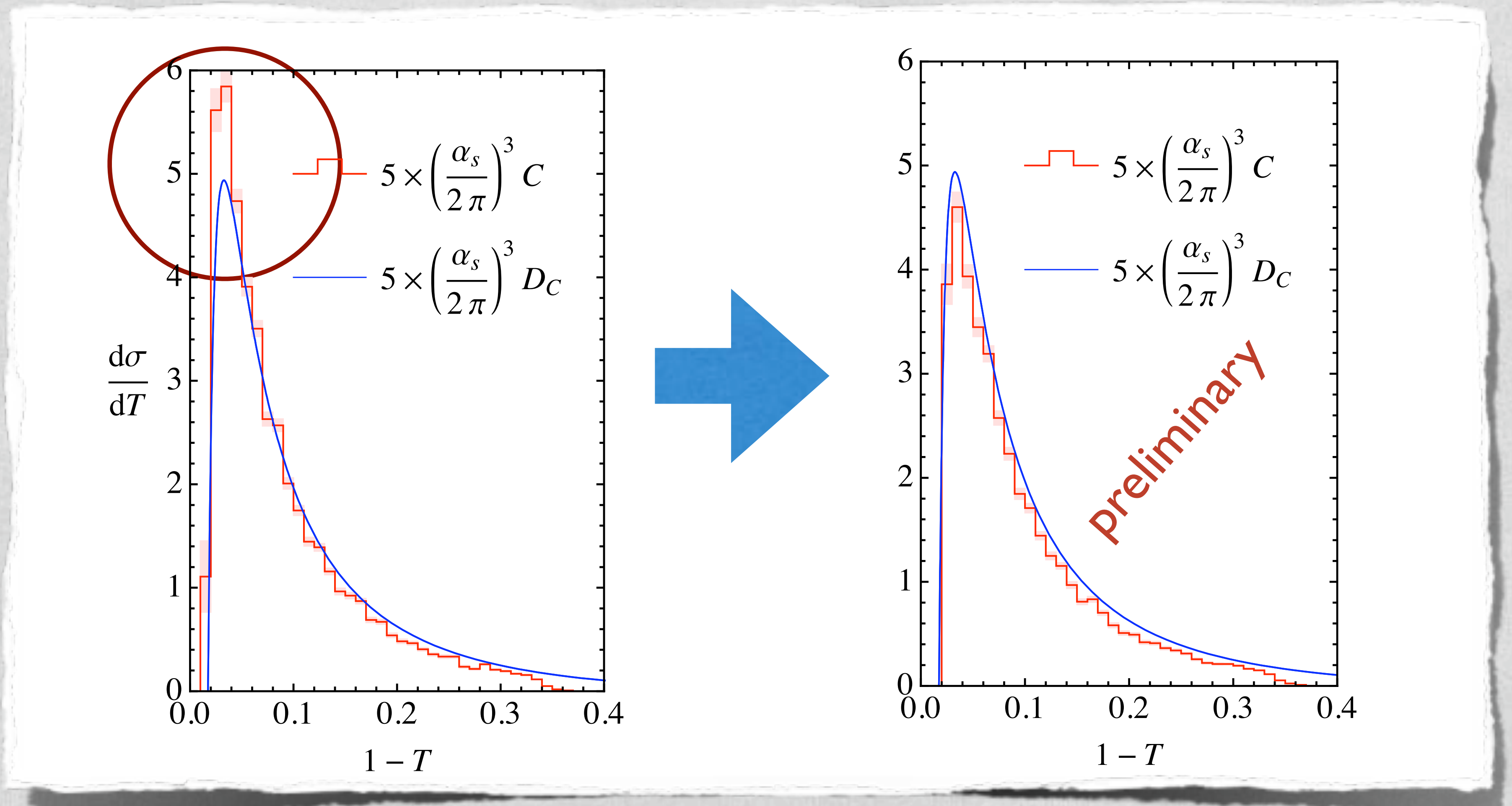
All ingredients known except
2-loop soft function.
Obtain it numerically using EVENT2

# NNLO SINGULAR TERMS

- With 2-loop *H*, *J* and *S* and 3-loop anomalous dimension we predict all singular terms at $\alpha_s^3$.
- For small $\tau$ singular terms dominate full result: check of NNLO calculation of Gehrmann et al.
- In our paper arXiv:0803.0342, we found disagreement at small $\tau$ values in 2 color structures.
- In arXiv:0807.3241 Stefan Weinzierl identified a soft divergence in one of the subtraction terms used by Gehrmann et al.
  - Affects thrust at small $\tau$. New numerical results for thrust by Weinzierl and by Gehrman et al. should soon be available.

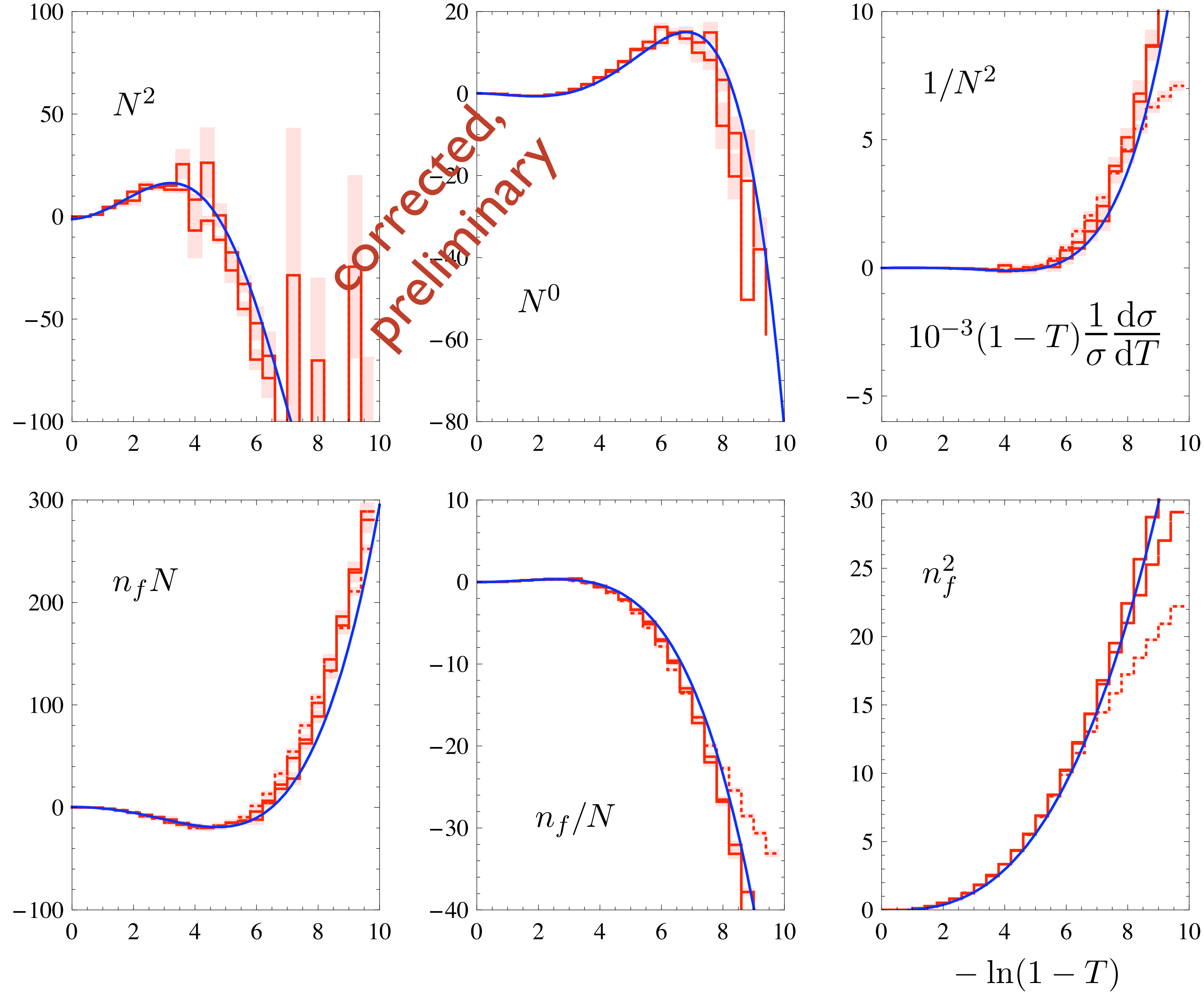
# NNLO SINGULAR TERMS

- Nice agreement with preliminary corrected results obtained from T. Gehrmann (thanks!)



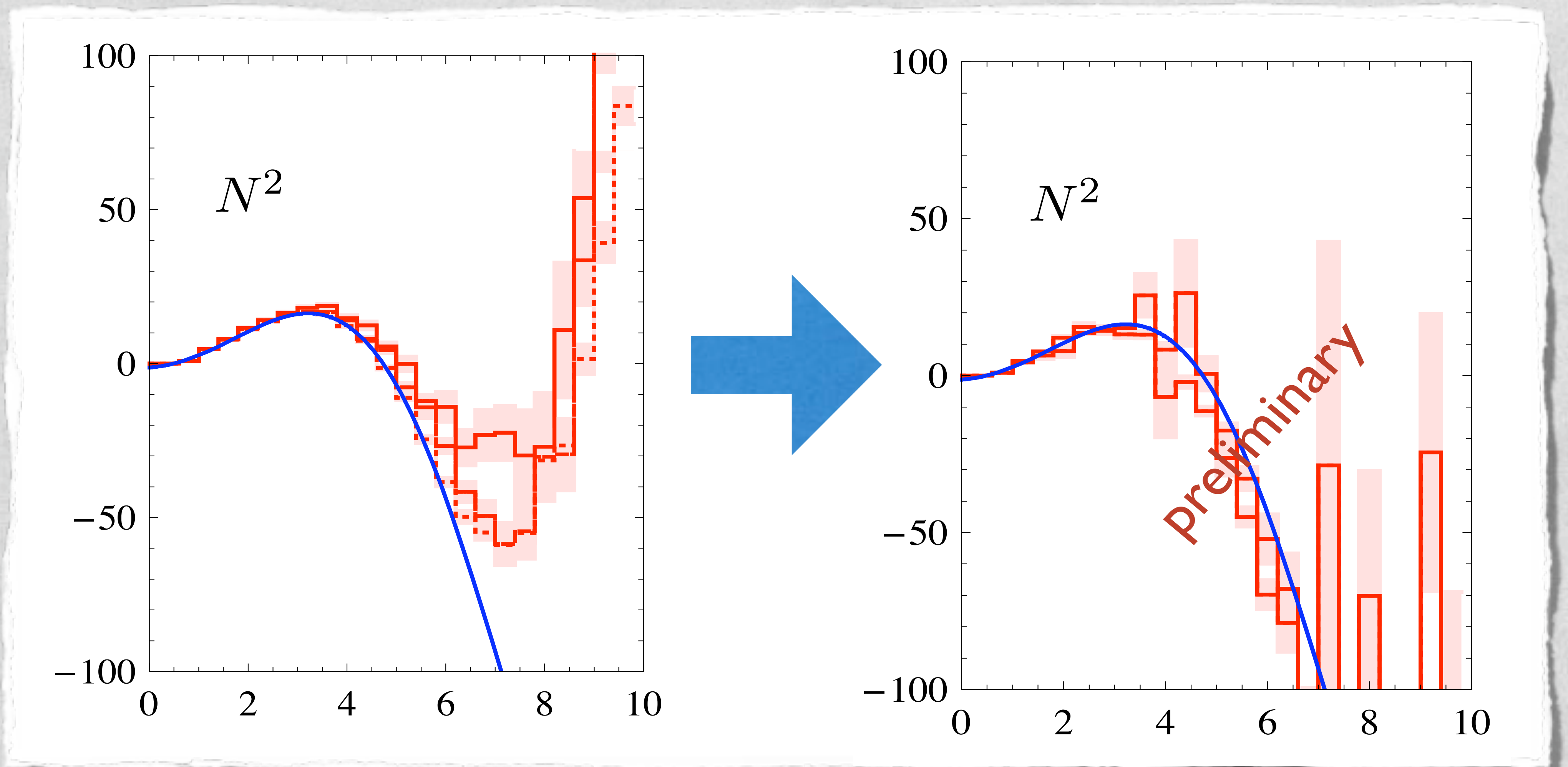

- Note: correction only affects region of very small τ
  - Should have negligible impact on $\alpha_s$ extraction.

# INDIVIDUAL COLOR STRUCTURES: SMALL $\tau$



# LEADING COLOR STRUCTURE

- Nice agreement with preliminary corrected results obtained from T. Gehrmann




- Note: only the $N_c^2$ and $N_c^0$ color structures are affected

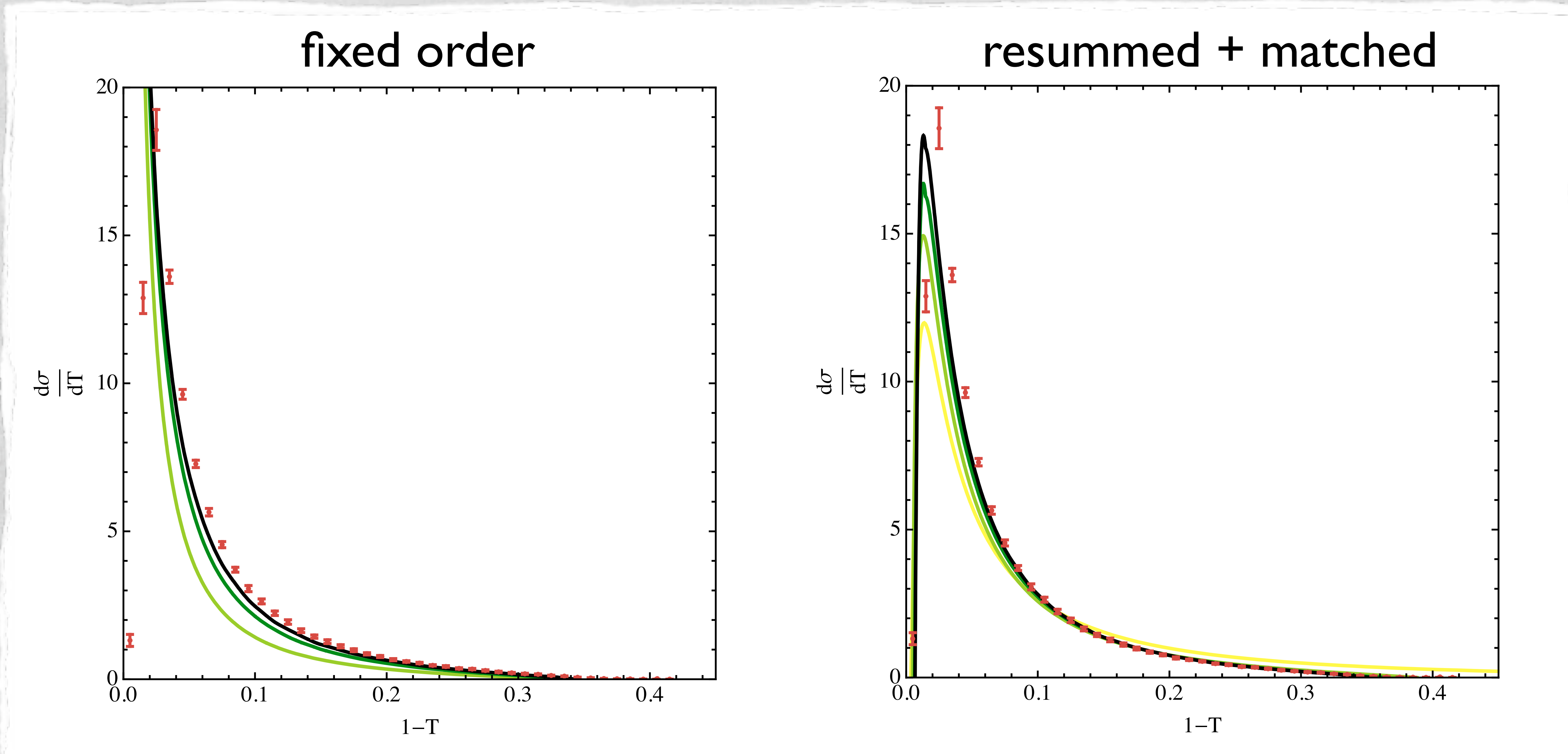
# MATCHING

- Will now combine resummation and fixed order result to obtain $\alpha_s$ from a fit to LEP data.
- Different possibilities, we use

| order | fixed-order matching | logarithmic accuracy |
|---|---|---|
| $1^{\mathrm{st}}$order | – | NLL |
| $2^{\mathrm{nd}}$order | LO | NNLL |
| $3^{\mathrm{rd}}$order | NLO | $\mathrm{N^3LL}$ |
| $4^{\mathrm{th}}$order | NNLO | $\mathrm{N^3LL}$ |

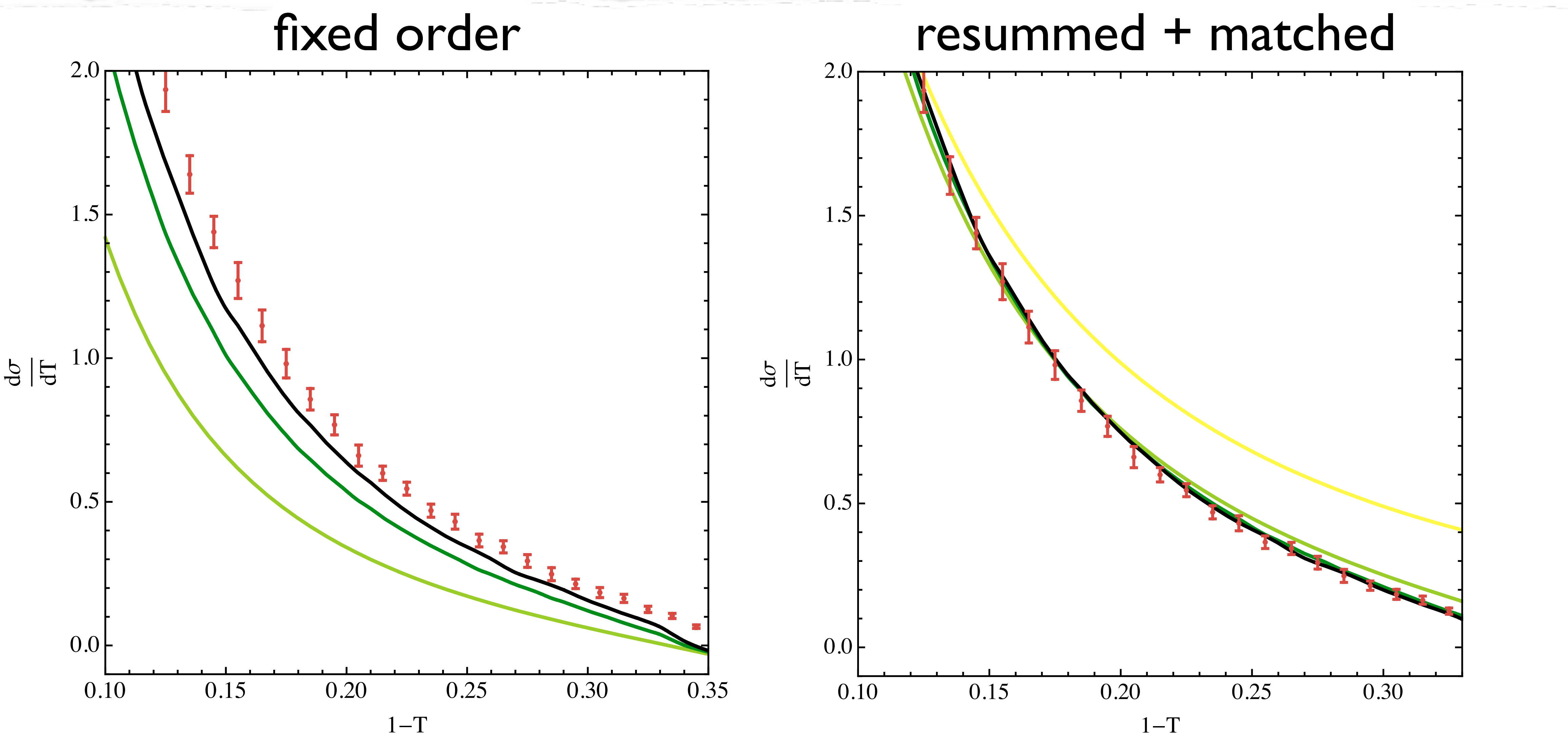
- note: previous speaker G. Luisoni used NLL+NNLO

# RESUMMED VS. FIXED ORDER




- For PDG value $\alpha_s(M_Z)$=0.1176.

# RESUMMED VS. FIXED ORDER




- For PDG value $\alpha_s(M_Z)$=0.1176
- This is the region relevant for $\alpha_s$ determination

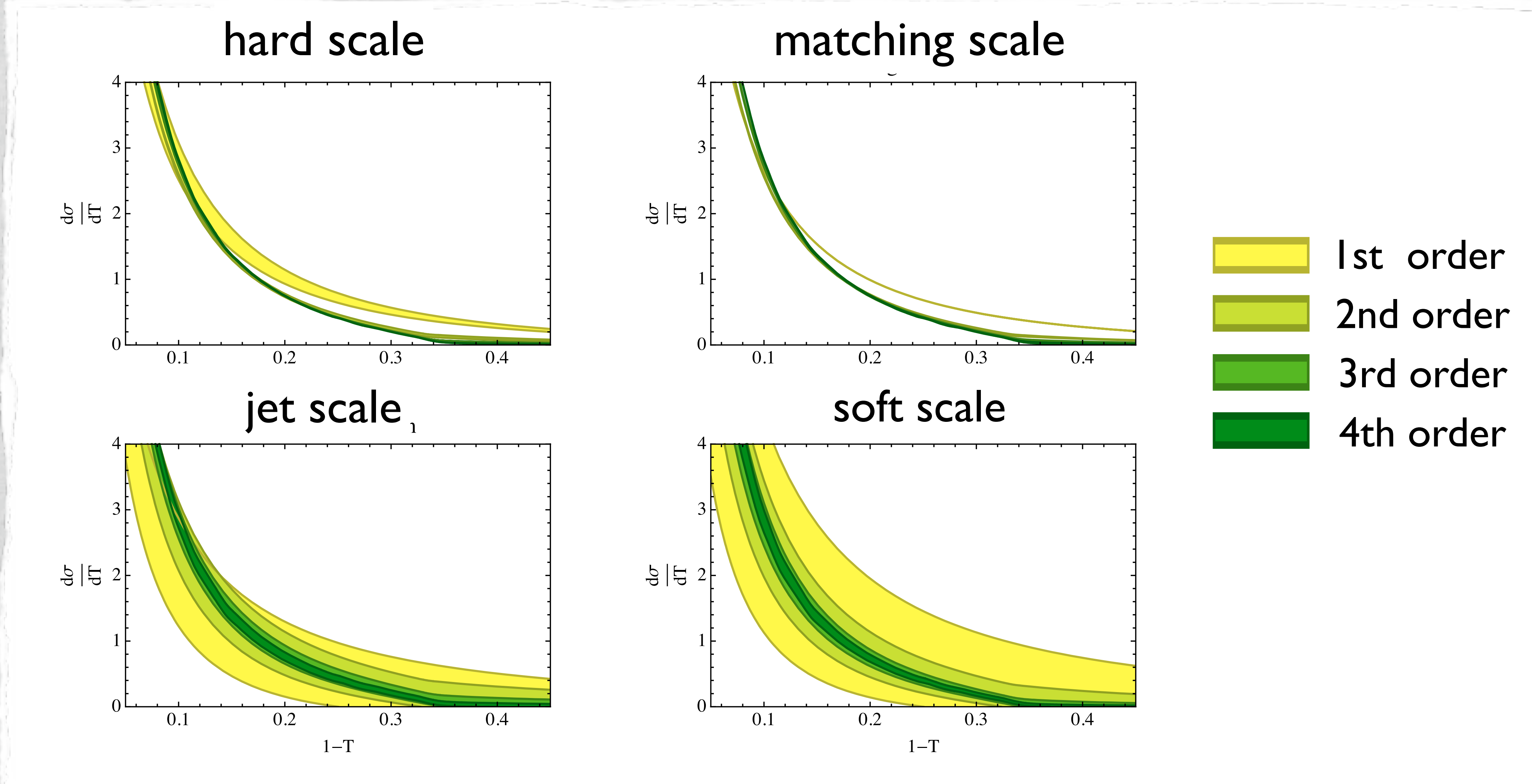
# PHENOMENOLOGICAL APPLICATIONS

- Determination of $\alpha_s$
  - Scale variation, error band method
  - Fit to ALEPH and OPAL LEP data
- Bound on light gluinos Kaplan and Schwartz '08
- Comparison with event generator results at ILC energies

# THEORETICAL UNCERTAINTY

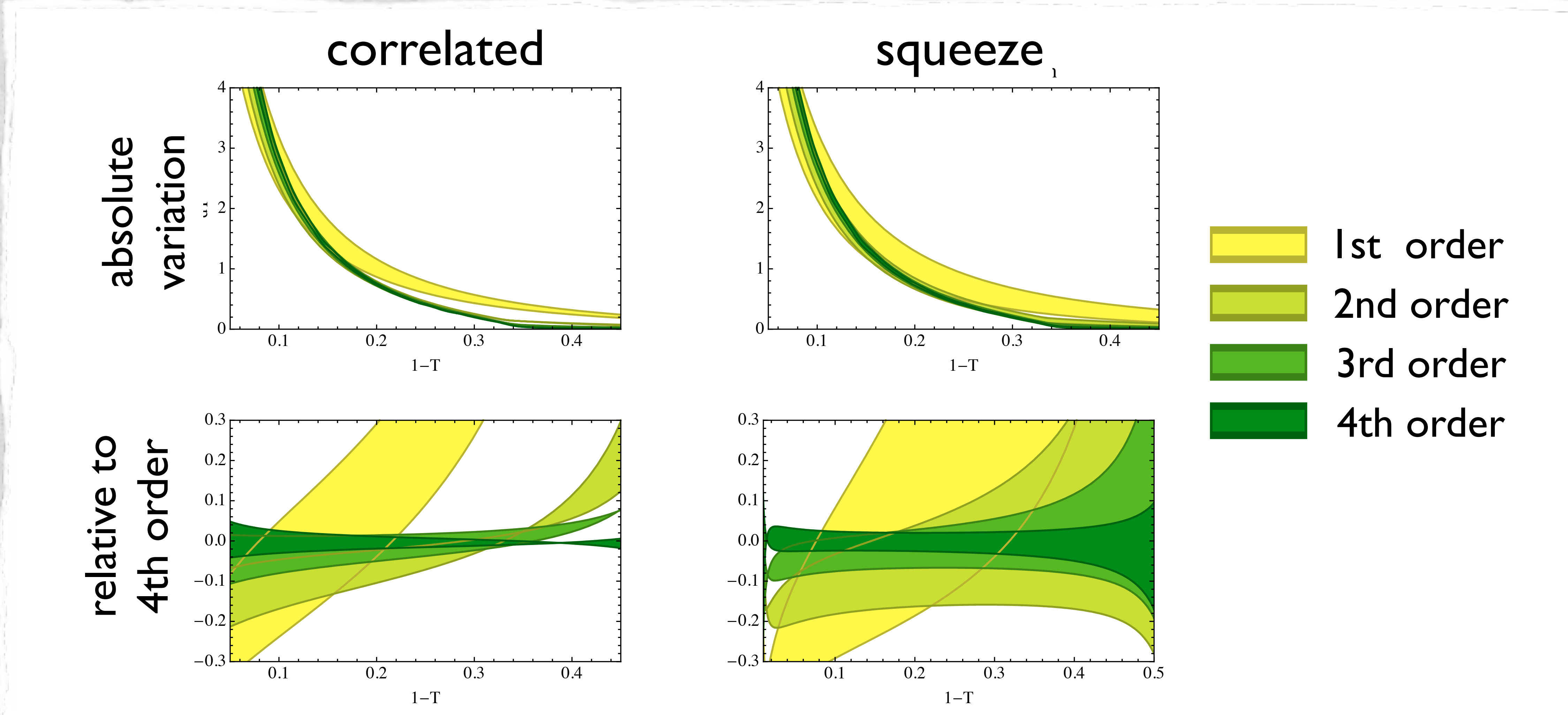
- We will assess the perturbative uncertainty in the standard way, by varying the renormalization (resp. matching) scales.
  - To the order of the calculation, the cross section is independent of these scales;
  - variation then is a measure of unknown higher order terms.
- We have four scales
  - $\mu_{\mathrm{hard}}^2 \sim Q^2$ : scale at which $H$ is evaluated
  - $\mu_{\mathrm{jet}}^2 \sim \tau\, Q^2$ : scale at which $J$ is evaluated
  - $\mu_{\mathrm{soft}}^2 \sim \tau^2\, Q^2$ : scale at which $S_T$ is evaluated
  - $\mu_{\mathrm{match}}^2$ : scale of the regular terms

# INDEPENDENT SCALE VARIATION




- Varying jet and soft scale independently by a factor 2 makes no sense at moderate $\tau$ (leads to $\mu_{soft} > \mu_{jet}$, etc.), overestimates the uncertainty.

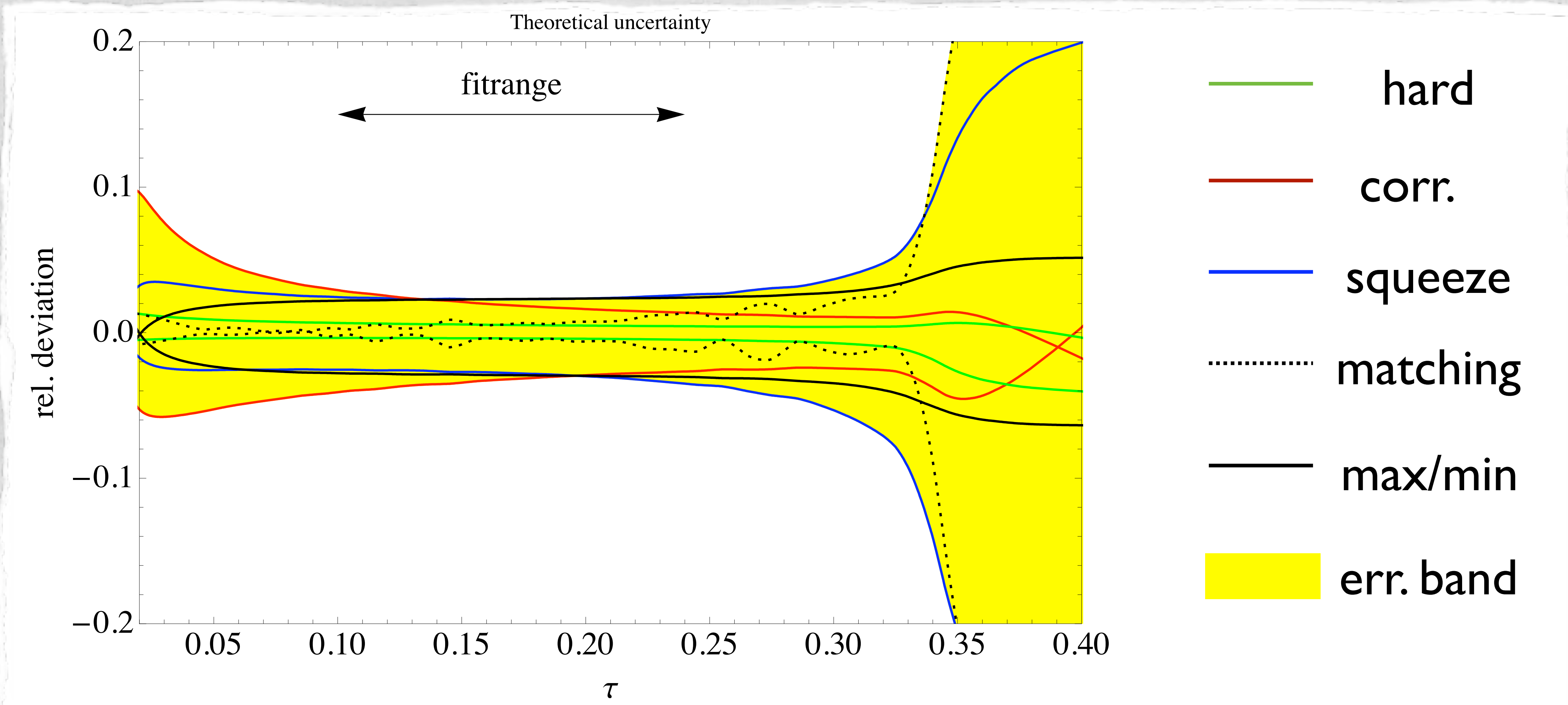
# JET AND SOFT SCALE VARIATION




- Instead of independently varying the jet and soft scales, we vary as follows
  - correlated: $\mu_{jet} \longrightarrow \alpha\, \mu_{jet}$, $\mu_{soft} \longrightarrow \alpha\, \mu_{soft}$ with $1/2 < \alpha < 2$
  - squeeze: $\mu_{jet} \longrightarrow \sqrt{\alpha}\, \mu_{jet}$, $\mu_{soft} \longrightarrow \alpha\, \mu_{soft}$ with $1/\sqrt{2} < \alpha < \sqrt{2}$

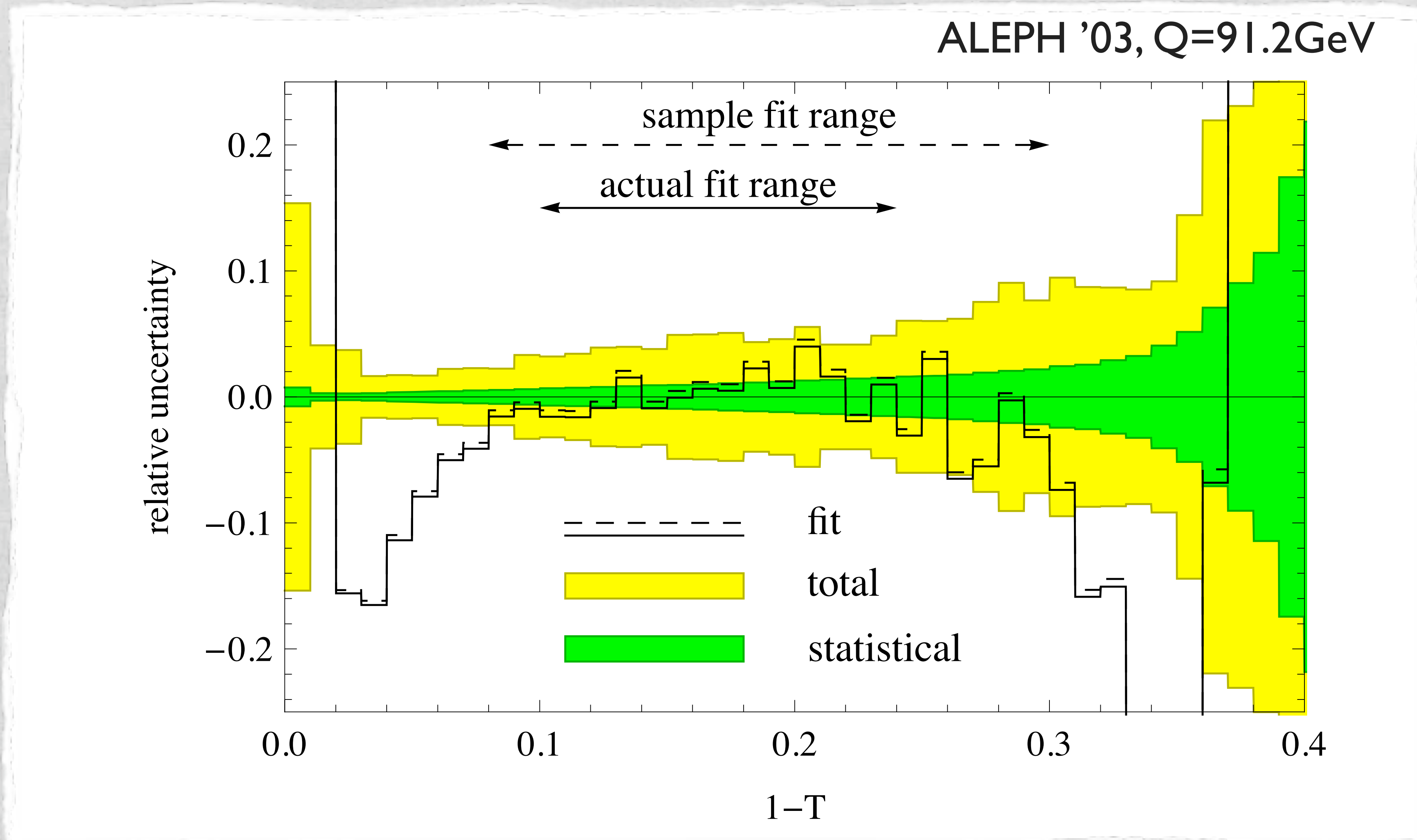
# ERROR BAND METHOD

Jones, Ford, Salam Stenzel & Wicke '03; adopted by ALEPH and OPAL



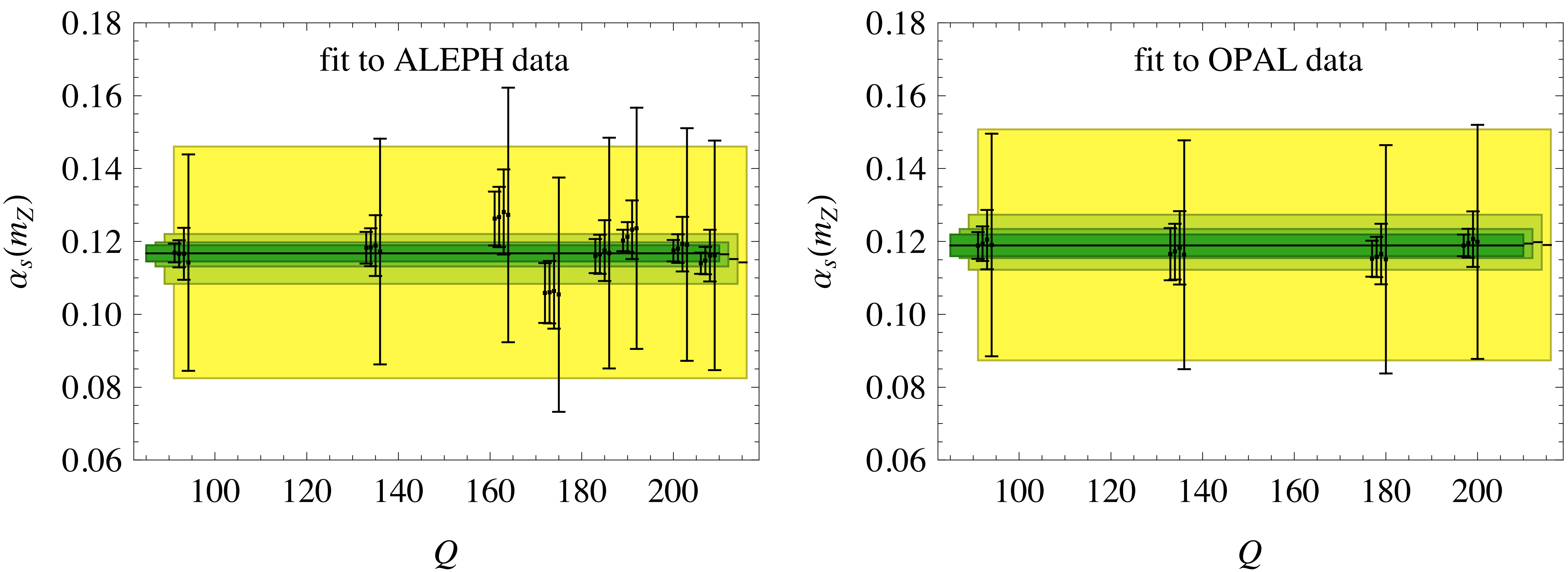

- Perform $\chi^2$-fit to the data, extract best-fit value of $\alpha_s$. Calculate maximum deviation from default distribution: "error band".
- To get theoretical uncertainty, calculate max. and min. $\alpha_s$ for which theoretical distribution lies inside the error band.

# EXPERIMENTAL UNCERTAINTY




- OPAL '05 and ALEPH '03 give results for binned thrust distributions. Do not provide correlations.
- Put only stat. err. in our $\chi^2$-fit. For each $Q$, use same fit ranges as exp. paper and use their systematic uncertainties.

# FIT RESULT

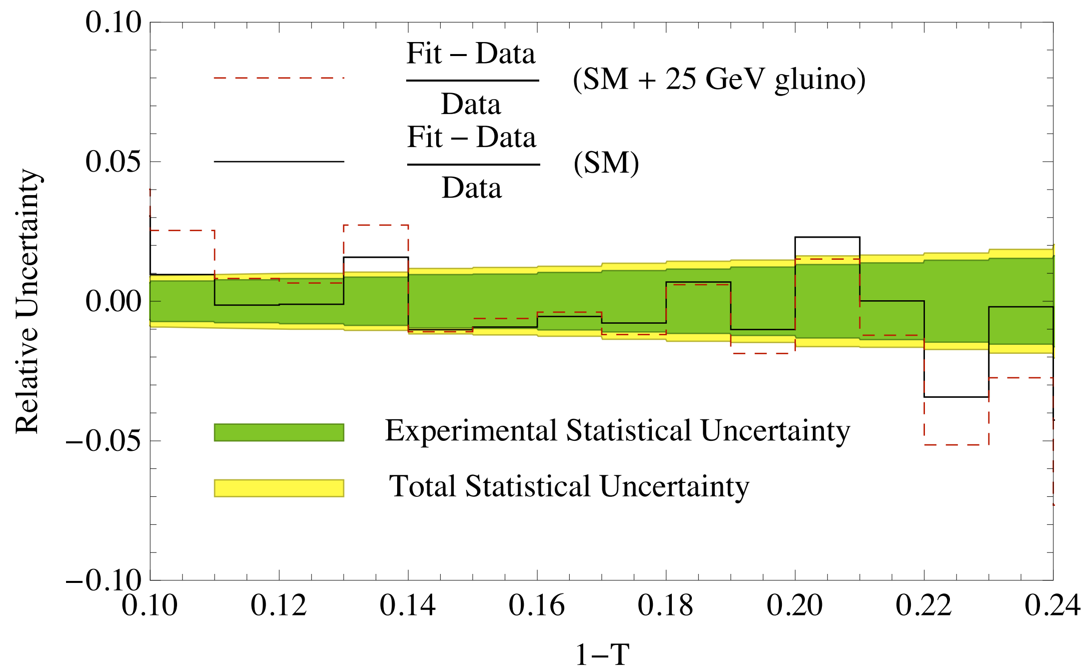


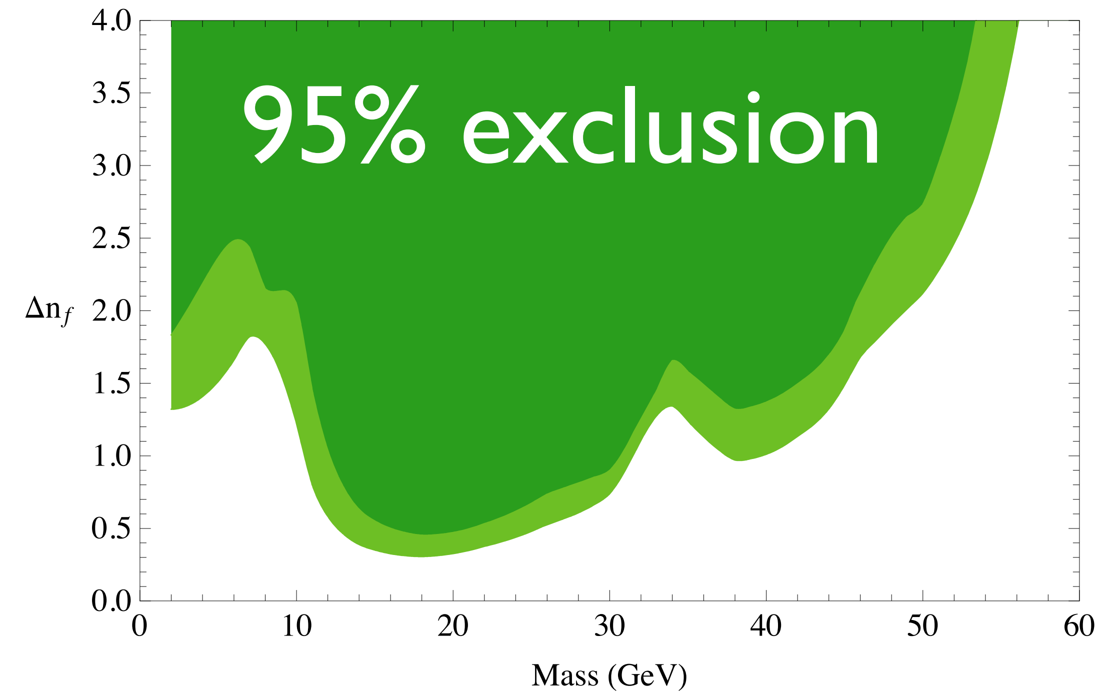

$$\boxed{\begin{aligned}\alpha_s(m_Z) &= 0.1172 \pm 0.0010(\text{stat}) \pm 0.0008(\text{sys}) \pm 0.0012(\text{had}) \pm 0.0012(\text{pert}) \\ &= 0.1172 \pm 0.0022\,.\end{aligned}}$$

[ **PDG:** $\alpha_s(m_Z) \;=\; 0.1176 \pm 0.0020$ ]

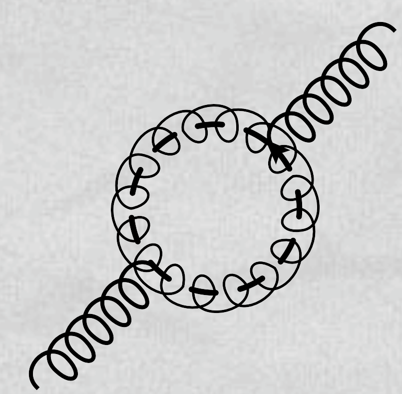
# BOUND ON LIGHT GLUINOS

Kaplan and Schwartz '08

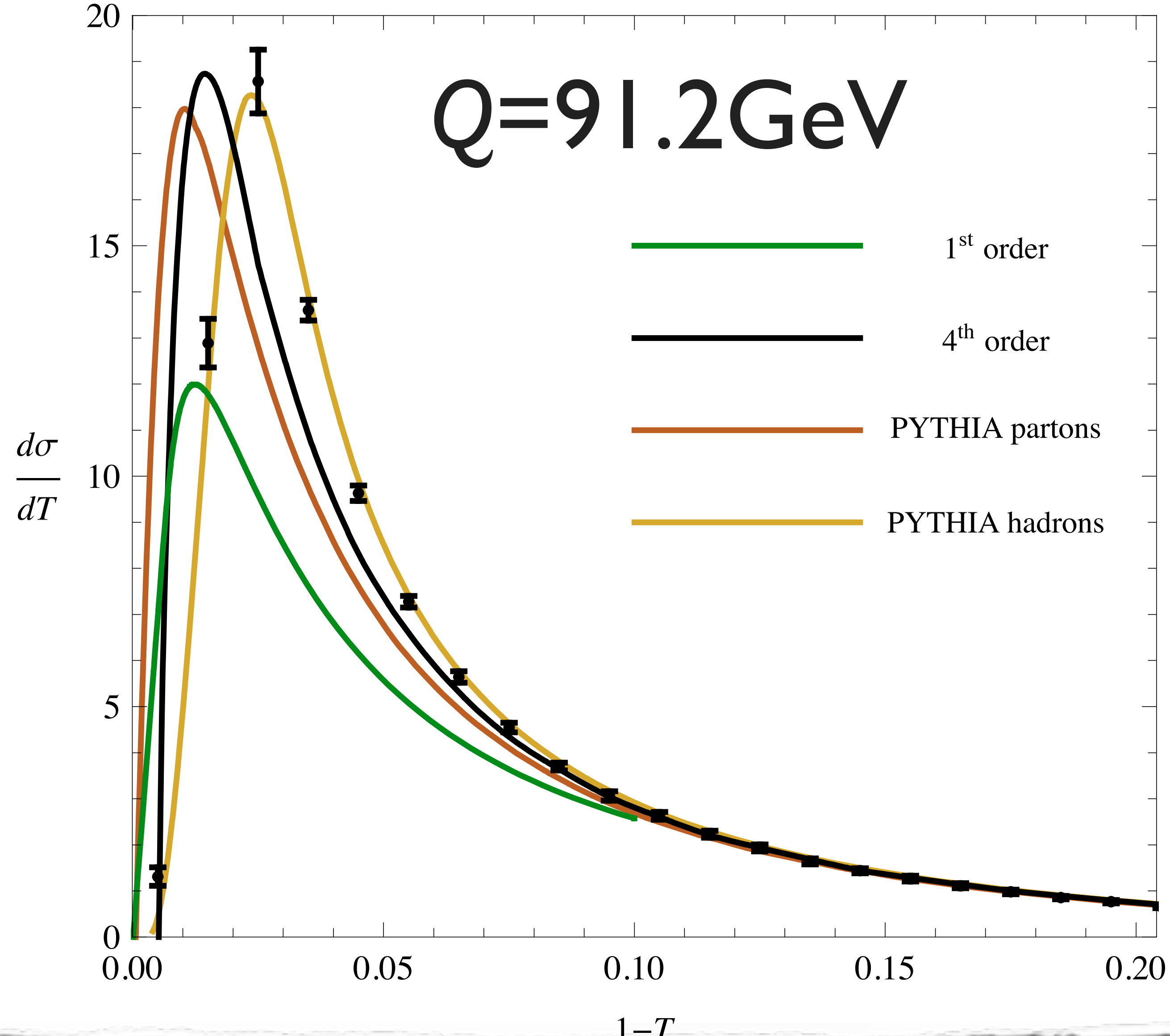



- Gluinos would affect *H*, *J* and *S* functions at the two-loop level. Leading effect is $\Delta n_f = 3$
  - in hard function *H* if $m_{\tilde{g}} < Q$ ,
  - in jet function *J* if $m_{\tilde{g}} < \sqrt{\tau} Q$ ,
  - in soft function S if $m_{\tilde{g}} < \tau Q$ ,

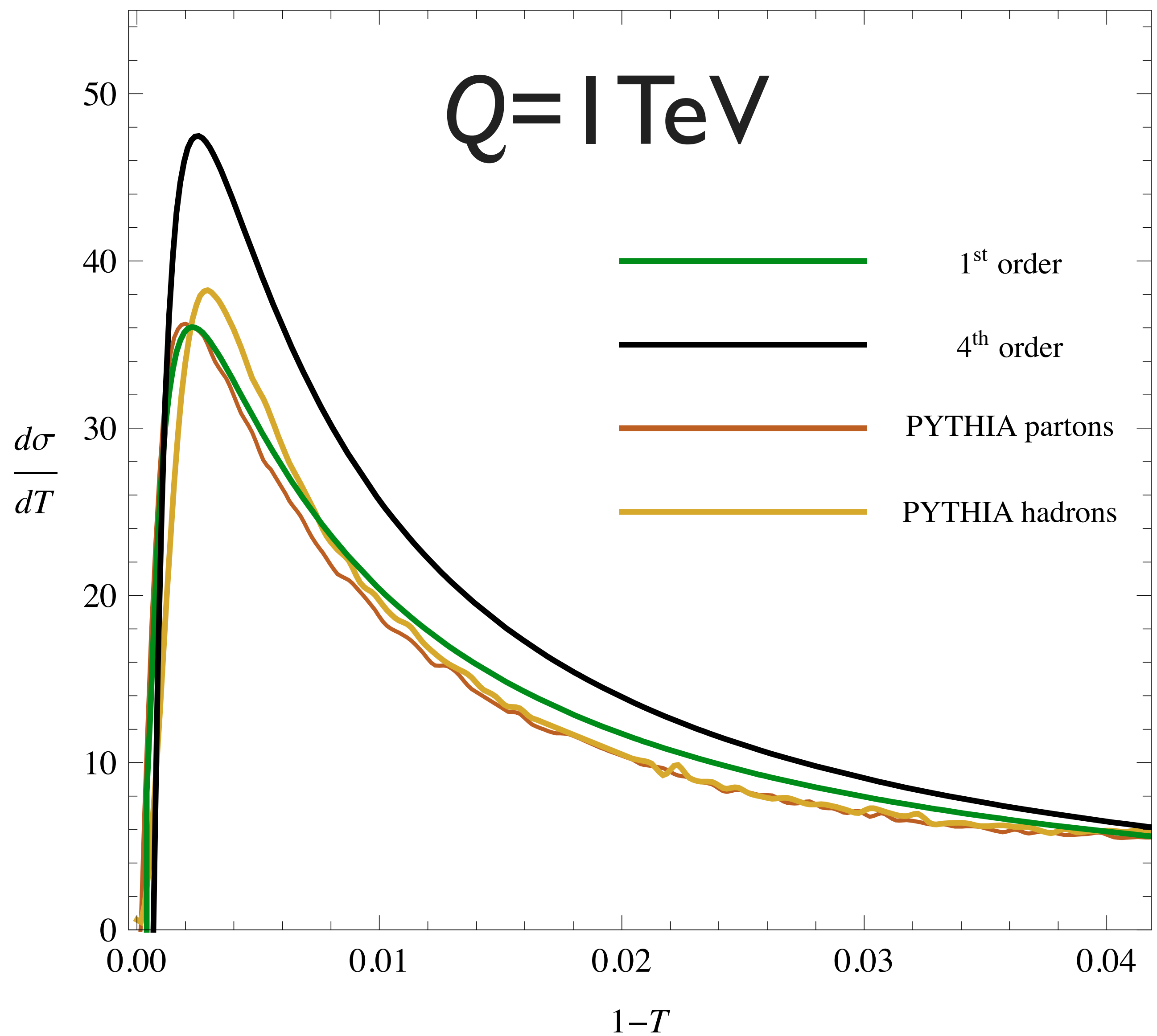


# COMPARISON WITH PYTHIA



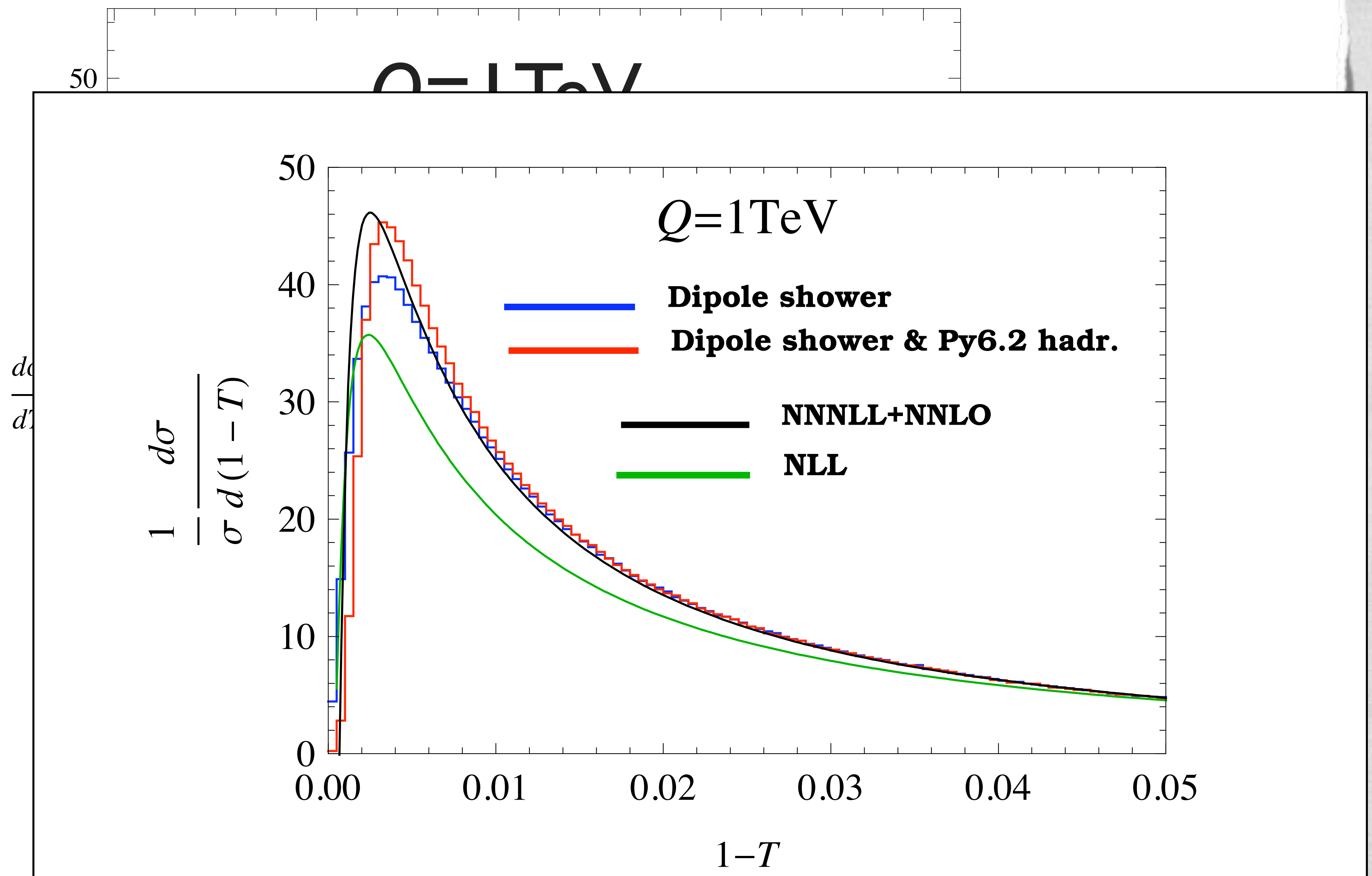

- hadronic Pythia agrees perfectly with the ALEPH data
- partonic Pythia does much better than NLL

# I TEV LEPTON COLLIDER




- Partonic Pythia now looks much more NLL like.
- Will need to retune (or redesign) the shower.
  - Can tune partonic shower to our theoretical prediction.

# 1 TEV LEPTON COLLIDER




New dipole shower by Krauss and Winter '08 gives better agreement

- Partonic Py
- Will need t
  - Can tune partonic shower to our theoretical prediction.

# SUMMARY

- Have used effective field theory methods to resum thrust distribution to $N^3LL$.
  - Traditional method works only up to NLL.
  - Logarithmically enhanced contributions dominate. Have evaluated all singular terms at $\alpha_s^3$.
    - Check of NNLO calculation of $e^+e^- \to 3$ jets.
  - Also other event shapes can be improved beyond NLL
- Extract $\alpha_s$ from a fit to LEP data:

$$\alpha_s(m_Z) = 0.1172 \pm 0.0010(\text{stat}) \pm 0.0008(\text{sys}) \pm 0.0012(\text{had}) \pm 0.0012(\text{pert})$$

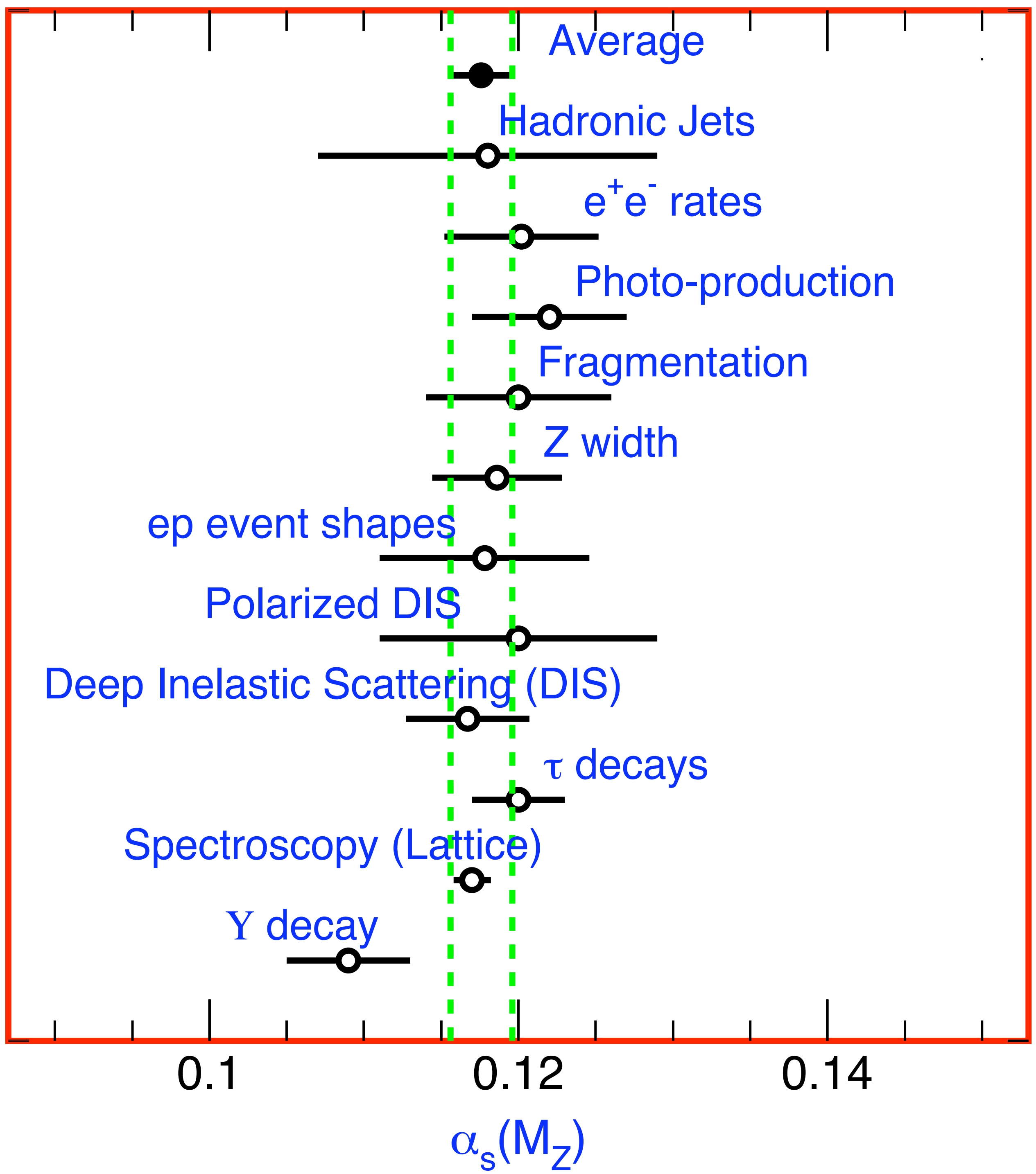
  - Most precise determination of $\alpha_s$ at high energies, agrees well with low energy determinations.
  - Theoretical accuracy matches exp. precision at the ILC

# EXTRA SLIDES

# POWER CORRECTIONS

- So far, we have not included 1/Q power corrections:
  - finite b-quark mass effects ≈ +1.5% at LEP I
    - calculated perturbatively, e.g. using NLO event generator by Nason and Oleari.
    - could perform resummation for this part, using SCET, see Sonny Mantry's talk
  - hadronisation ~ -1.5% at LEP I
    - estimated using Pythia to calculate transfer matrix
    - uncertainty is estimated by comparing Pythia to Herwig and Ariadne: 2.5% at LEP I. Now the dominant uncertainty!
    - Our precise perturbative prediction can and should be used to study hadronisation effects in more detail, using also lower energy data.

# PDG AVERAGE


Average
Hadronic Jets
e+e- rates
Photo-production
Fragmentation
Z width
ep event shapes
Polarized DIS
Deep Inelastic Scattering (DIS)
τ decays
Spectroscopy (Lattice)
Υ decay
0.1
0.12
0.14
αs(MZ)