

Dominant NNLO corrections to four-fermion production at the WW threshold

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in collaboration with M. Beneke, P. Falgari and C. Schwinn

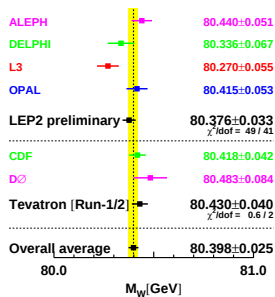
18 Nov 2008, LCWS08, University of Illinois at Chicago

Outline

- 1 Why $e^- e^+ \rightarrow 4$ fermions at WW
- 2 EFT method for NNLO corrections
- 3 Numerical results
- 4 Conclusions

W mass at LEP2

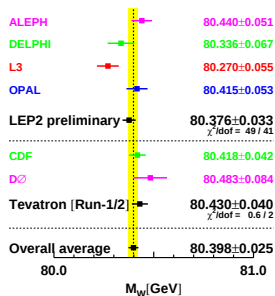
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PDG

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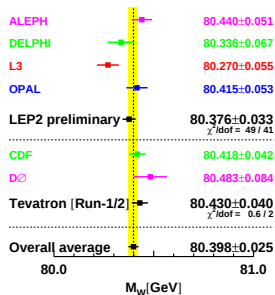


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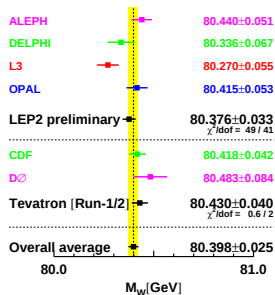


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- Foreseen improvement of 50% at the **LHC**

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W unstable $\Rightarrow$ effects due to the W width taken into account

NLO predictions for $e^- e^+ \rightarrow 4$ fermions

LEP2 analysis with YFSWW [Jadach-Placzek-Skrzypek-Ward-Was '00] and RACOONWW [Denner-Dittmaier-Roth-Wackerath '00] (selected radiative corrections to LO); ILC accuracy $\Rightarrow$ full NLO computation needed

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$\sqrt{s}$ [GeV]	Born	EFT	CMS	shift
161	107.06(4)	117.38(4)	118.12(8)	0.6 %
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EFT tailored to threshold region: simple computation, compact result, easy evaluation of dominant NNLO corrections at threshold

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Collectively denoted by δ , of the same order for power counting

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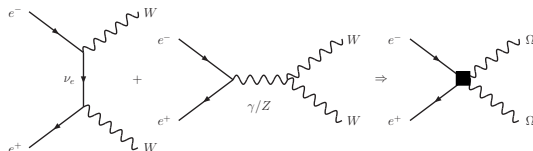
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Integrate out hard modes introducing matching coefficients

$\Rightarrow$ Short-distance physics:

- * non-resonant W's
- * light with large k^2
- * heavy particles



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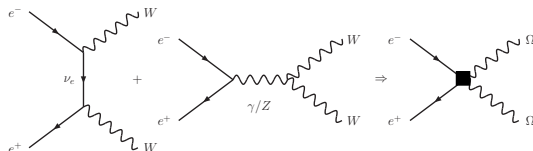
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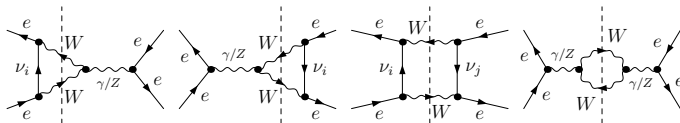
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Potential, soft and collinear modes (long-distance physics) generate radiative corrections in the EFT

Outline of the computation

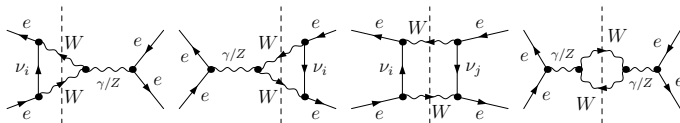
- 1) Extract the total cross section for $e^- e^+ \rightarrow 4$ fermions from the cuts of the $e^- e^+$ forward-scattering amplitude



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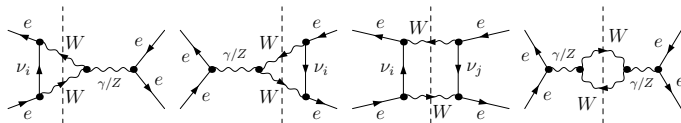


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- 3) Evaluate **loop corrections** to the EFT matrix element:

- resonant W's $k^2 - M_W^2 \sim M_W \Gamma_W$
- Coulomb and soft photons $k^2 \sim M_W \Gamma_W$ and $k^2 \sim \Gamma_W^2$
- high-energy external fermions $k^2 = 0$

Dominant NNLO corrections

Re-organization of perturbative expansion $\Rightarrow$ identify the dominant NNLO corrections from the power of δ for each EFT diagram

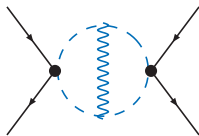
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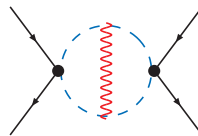
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- EFT assigns different scaling properties to diagrams with **Coulomb**- and **soft**-photon exchange



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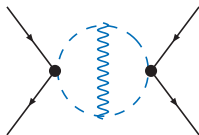
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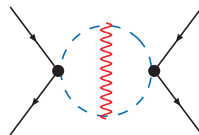
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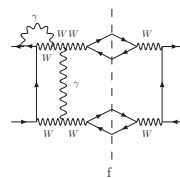
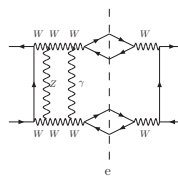
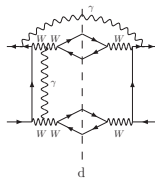
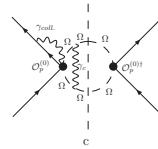
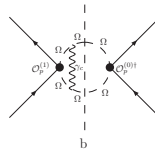
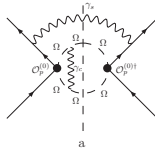


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$\Rightarrow$ NNLO: $\alpha_{ew}^2 \alpha \delta$ (dominant) and $\alpha_{ew}^2 \alpha \delta^{3/2}$ (sub-dominant)

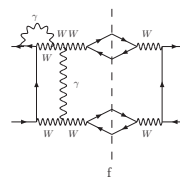
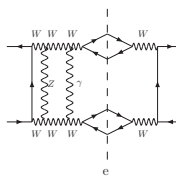
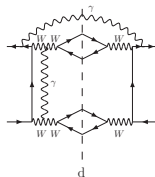
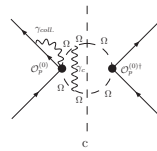
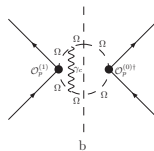
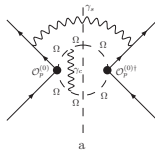
Corrections to single-Coulomb-exchange diagrams (I)

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- EFT:
- point-like interaction between leptons and resonant W's
 - bare W propagator replaced with effective re-summed one

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Two independent evaluations:

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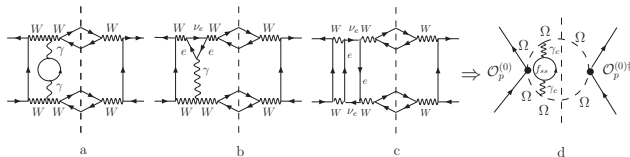
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- result sensitive to the electron mass $\Rightarrow \ln(2M_W/m_e)$
- large logarithms subtracted and re-absorbed in the ESFs $\Gamma(x)$
[Skrzypek '92, Beenakker et al. '96]

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma(x_1) \Gamma(x_2) \hat{\sigma}(x_1 x_2 s)$$

Corrections to the Coulomb potential

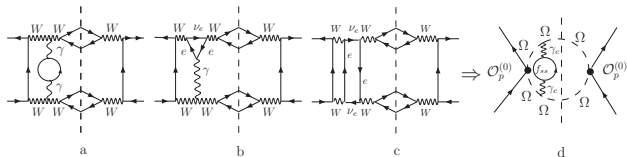
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$$\Delta\sigma_2|_{\alpha(M_Z)} = -\frac{\alpha_{ew}^2 \alpha^2}{81s} \sum_f C_f Q_f^2 \left\{ 4 \ln\left(\frac{2M_W}{M_Z}\right) \operatorname{Im} \left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right) \right] + \operatorname{Im} \left[\ln^2\left(-\frac{\mathcal{E}_W}{M_W}\right) \right] \right\}$$

$$\Delta\sigma_2|_{G_\mu} = \Delta\sigma_2|_{\alpha(M_Z)} - \frac{2\pi\alpha_{ew}^2 \alpha}{27s} \delta_{\alpha(M_Z) \rightarrow G_\mu} \operatorname{Im} \left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right) \right]$$

re-summation of large fermion-mass logs in the $\alpha(M_Z) \rightarrow G_\mu$ [Dittmaier-Krämer '01]

(as for NLO result [Beneke-Falgari-Schwinn-Signer-Zanderighi '07])

Corrections specific of the EFT

Decay corrections: flavour-specific corrections to the decay of the resonant W's to $\mu^- \bar{\nu}_\mu$ and $u\bar{d}$

$$\Delta\sigma_3 = - \left(\frac{\Gamma_{\mu^- \bar{\nu}_\mu}^{(1)}}{\Gamma_{\mu^- \bar{\nu}_\mu}^{(0)}} + \frac{\Gamma_{u\bar{d}}^{(1)}}{\Gamma_{u\bar{d}}^{(0)}} \right) \underbrace{\frac{2\pi\alpha_{ew}^2\alpha}{27s} \ln\left(-\frac{\mathcal{E}_W}{M_W}\right)}_{\text{single Coulomb}}$$

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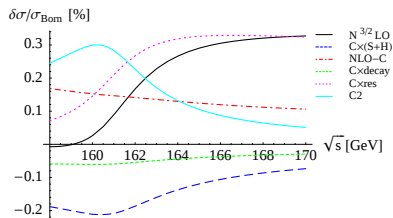
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Residue corrections: matching coefficient for the production operator dressed with **WFR factors**

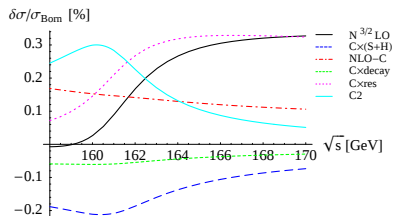
$$\Delta\sigma_4 = \frac{4\pi\alpha_{EW}^2\alpha}{27s} \frac{\Gamma_W}{M_W} \ln\left(\frac{2|\mathcal{E}_W|(\text{Re}\mathcal{E}_W + |\mathcal{E}_W|)}{\Gamma_W^2}\right)$$

Results for the total cross section



$\sqrt{s} [\text{GeV}]$	$\sigma (\text{fb})$		
	Born	+ NLO	NNLO only
158	45.64(2)	49.19(2)	0.000 [+0.00%]
161	108.60(4)	117.81(5)	0.087 [+0.06%]
164	219.7(1)	234.9(1)	0.544 [+0.18%]
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170	378.4(2)	398.0(2)	1.207 [+0.25%]

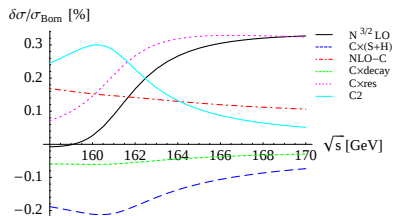
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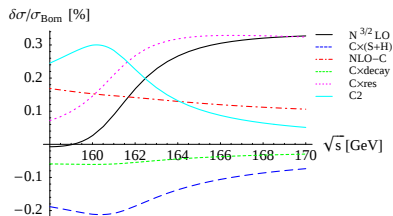
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- Individual contributions to the total cross section at threshold from -0.2% to $+0.3\%$
- Screening effect $\Rightarrow$ combined effect below the permille level
- Conversion of shift on cross section $\rightarrow$ shift on W mass $\Rightarrow$ effect of NNLO corrections below 6 MeV accuracy expected at the ILC ($\delta\sigma \sim 1\% \Rightarrow \delta M_W \sim 15$ MeV)

Effect of experimental cuts

For practical applications, [experimental cuts](#) are needed; complicated in the EFT because they introduce new scales

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Estimate of the effect of cuts using **WHIZARD** [Kilian-Ohl-Reuter '07] for the SM Born result and the **L3** cuts

Cut	$\sigma_{\text{Born}}(e^-e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d})(\text{fb})$	$\sigma_{\text{cut}}/\sigma_{\text{tot}}$
–	154.18(5)	
$ \vec{p}_\mu > 20 \text{ GeV}$	153.71(5)	99.69(5) %
$M_{\mu\nu} > 55 \text{ GeV}, 40 \text{ GeV} < M_{ij} < 120 \text{ GeV}$	150.61(5)	97.68(5) %
$\theta_{\mu j} > 15 \text{ degrees}$	149.35(5)	96.87(5) %
$ \cos \theta_\nu < 0.95$	148.28(5)	96.17(5) %
all	140.03(5)	90.82(5) %

- cut on the muon momentum negligible
- cuts on the invariant masses of pairs of decay products can be implemented in the EFT; do not affect dominant NNLO terms
- angular cuts more complicated; not relevant since NNLO corrections are very small

Conclusions

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Conclusions

- ▶ Using EFT methods for unstable particles it has been possible to evaluate all parametrically dominant NNLO corrections to four-fermion production at the WW threshold
- ▶ Even if finite-width effects are handled in a different way in the EFT respect to the full result in the CMS, the NNLO subset of corrections can be implemented in both schemes without modifications
- ▶ Each diagram contributes between -0.2% and $+0.3\%$, but large cancellations occur. The impact of the result on the W-mass determination is well below the 6 MeV goal at the ILC