

NLO QCD Corrections to $t\bar{t}Z$ Production

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The top quark

- Discovered in 1995 by CDF and D0
- Heaviest known elementary particle
 - $m_t \approx v/\sqrt{2}$: top may play a special role in EWSB.
- $t\bar{t}$ production at Tevatron depends only on top mass and color charge.
- EW properties largely unknown (e.g. $t\bar{t}\gamma$, $t\bar{t}Z$, $t\bar{t}H$ couplings)
 - sensitive to new physics (e.g. Z' , T quark, ...)
 - $t\bar{t}Z$ couplings:
 - inaccessible at Tevatron
 - can be studied at ILC via $e^+e^- \rightarrow \gamma^*/Z^* \rightarrow t\bar{t}$
 - or at LHC via $pp \rightarrow t\bar{t}Z$

Top quark electroweak couplings

- General ttZ coupling:

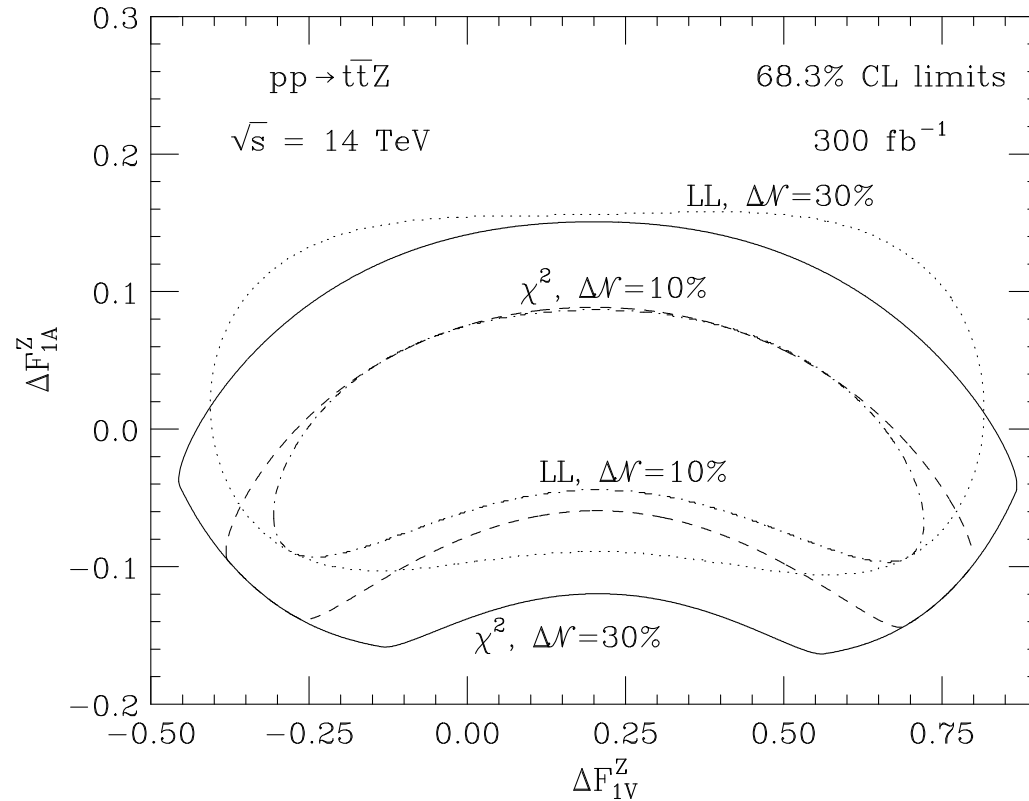
$$\Gamma_{\mu}^{ttZ}(k) = -ie \left[\gamma_{\mu} F_{1V}^Z(k^2) + \gamma_{\mu} \gamma_5 F_{1A}^Z(k^2) + \frac{i}{2m_t} \sigma_{\mu\nu} k^{\nu} F_{2V}^Z(k^2) + \frac{1}{2m_t} \sigma_{\mu\nu} \gamma_5 k^{\nu} F_{2A}^Z(k^2) \right]$$

- No direct measurements of **form factors**
- F_{1V}^Z and F_{1A}^Z tightly but indirectly constrained by LEP:

$$-0.044 \leq -\Delta F_{1A}^Z(0) [1 + 0.842 \Delta F_{1A}^Z(0)] \ln \left(\frac{\Lambda^2}{m_t^2} \right) \leq 0.065,$$

$$-0.029 \leq - \left[\Delta F_{1A}^Z(0) - \frac{3}{5} \Delta F_{1V}^Z(0) \right] \ln \left(\frac{\Lambda^2}{m_t^2} \right) \leq 0.143$$

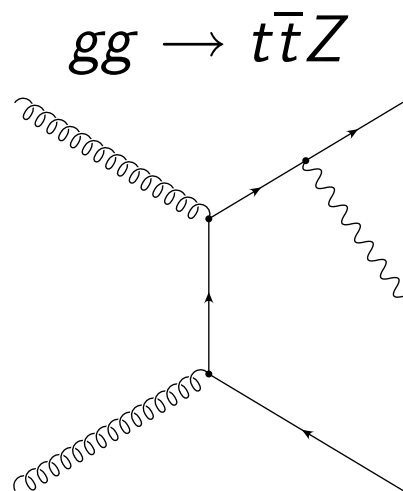
Top quark electroweak couplings



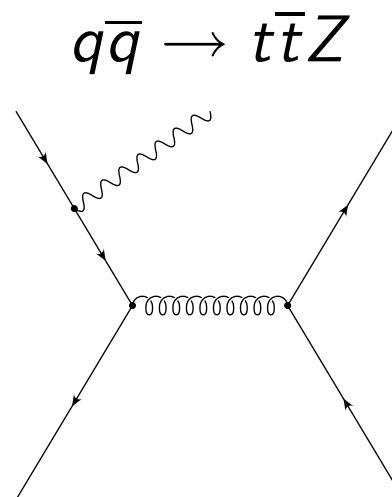
[Baur, Juste, Orr, Rainwater, 2005]

- Dominant uncertainty is signal normalization $\mathcal{N}$.
- Reducing $\Delta\mathcal{N}$ from 30% to 10% improves sensitivity by factor of 2.

$pp \rightarrow t\bar{t}Z$: LO

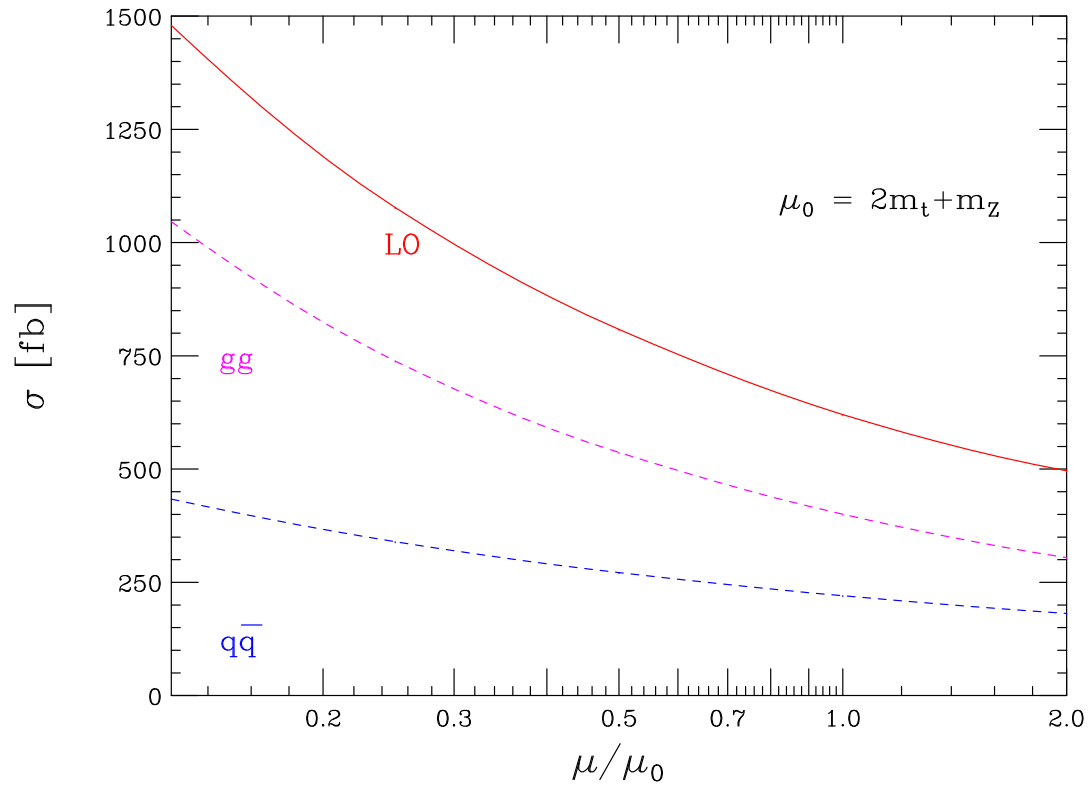


8 diagrams



4 diagrams

$pp \rightarrow t\bar{t}Z$: LO



- Significant scale dependence contributes to $\Delta\mathcal{N}$.
- NLO QCD calculation is necessary!

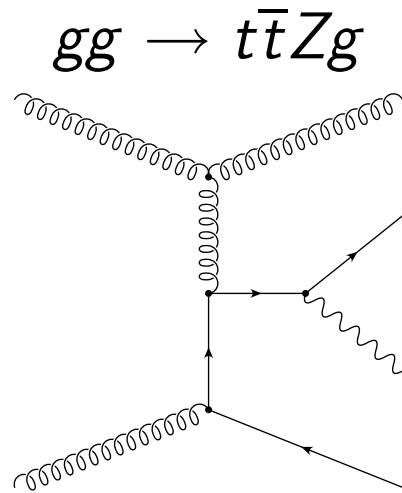
$pp \rightarrow t\bar{t}Z$: NLO

Full set of complexities that appear in multi-leg one-loop calculations:

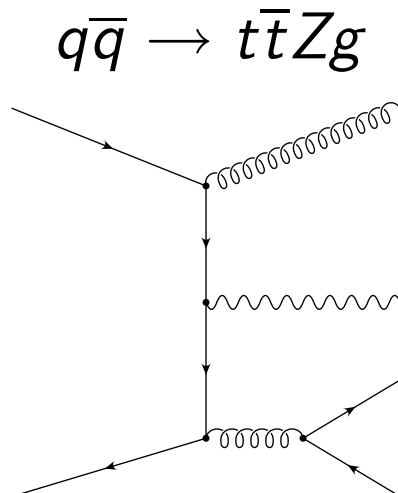
- Multiple mass scales: m_t and m_Z
- Soft and collinear singularities on boundaries of phase space and Feynman-parameter space
- Threshold singularities in interior of Feynman-parameter space
- Pentagon diagrams with 1–4 massive propagators

Regulate singularities dimensionally: $D = 4 - 2\epsilon$

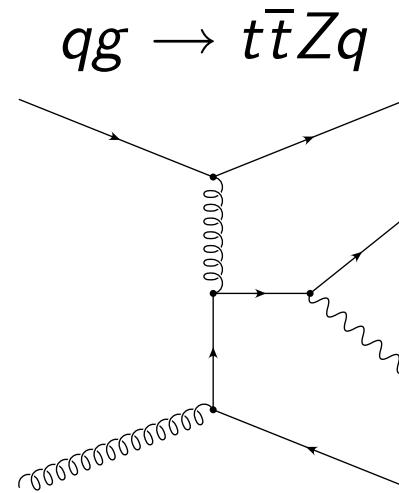
NLO: Real radiative corrections



50 diagrams



44 diagrams



24 diagrams

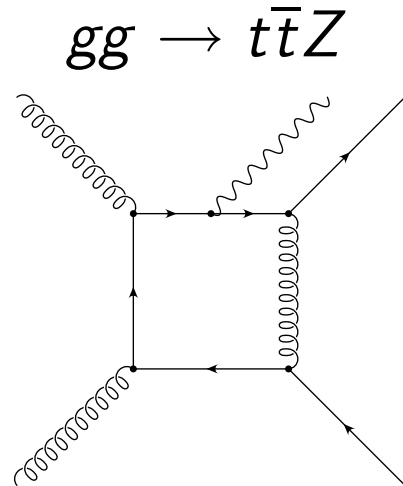
- Use **QGRAF** [Nogueira, 1993], **FORM** [Vermaseren, 2000], and **MAPLE** to calculate amplitudes and generate **FORTRAN** or **C++** code; use **CUBA** [Hahn, 2004] for numerical integration.
- Two-cutoff phase space slicing to isolate soft and collinear singularities [Harris, Owens, 2002]

Phase space slicing

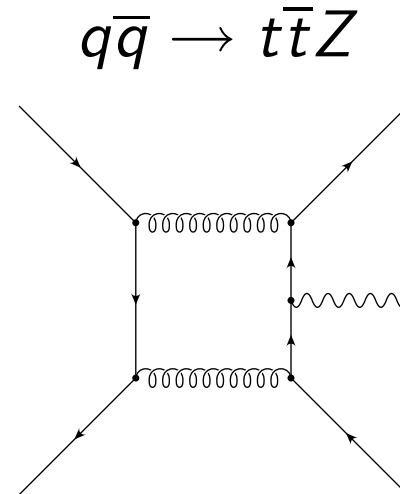
$$\begin{aligned} & \int_0^{E_{\max}} dE_g \int_{-1}^1 d \cos \theta_g \\ &= \int_0^{\delta_s E_{\max}} dE_g \int_{-1}^1 d \cos \theta_g \\ &+ \int_{\delta_s E_{\max}}^{E_{\max}} dE_g \left(\int_{-1}^{-1+\delta_c} + \int_{-1+\delta_c}^{1-\delta_c} + \int_{1-\delta_c}^1 \right) d \cos \theta_g \end{aligned}$$

- Exact matrix elements and numerical integration in hard region
- Approximate matrix elements and analytical integration in soft and colinear regions

NLO: Virtual corrections



160 diagrams



62 diagrams

- Soft and collinear singularities extracted using sector decomposition [Binoth, Heinrich, 2000]
- Threshold singularities avoided using contour deformation [Nagy, Soper, 2006; Lazopoulos, Melnikov, Petriello, 2007]

Sector decomposition

- Use $\delta(x_1 + \dots + x_n - 1)$ to form n primary sectors.
- Singularities appear when some set $\{x_i, x_j, \dots\}$ of Feynman parameters vanish.
- e.g. $\int_0^1 dx_1 \int_0^1 dx_2 (x_1 + x_2)^{-2-\epsilon}$
- Split integration region into sectors: $x_1 > x_2$ and $x_1 < x_2$
- Change variables in each sector: $x_2 \rightarrow x_1 x_2$, $x_1 \rightarrow x_1 x_2$
- Integral becomes

$$\int_0^1 dx_1 \int_0^1 dx_2 \left[x_1^{-1-\epsilon} (1 + x_2)^{-2-\epsilon} + x_2^{-1-\epsilon} (1 + x_1)^{-2-\epsilon} \right]$$
- Extract singularities using plus distributions:

$$x^{-1-\epsilon} = -\frac{1}{\epsilon} \delta(x) + \sum_{n=0}^{\infty} \frac{(-\epsilon)^n}{n!} \left(\frac{\ln^n x}{x} \right)_+$$

Contour deformation

- After sector decomposition, we have a denominator $(\Delta - i0)^{-a-b\epsilon}$, where

$$\Delta = Z + Y_i x_i + \frac{1}{2} X_{ij} x_i x_j + \frac{1}{3} W_{ijk} x_i x_j x_k + \dots$$

- Avoid threshold singularities by deforming the integration contour.
- Set $x_i = y_i - i\tau_i$, where

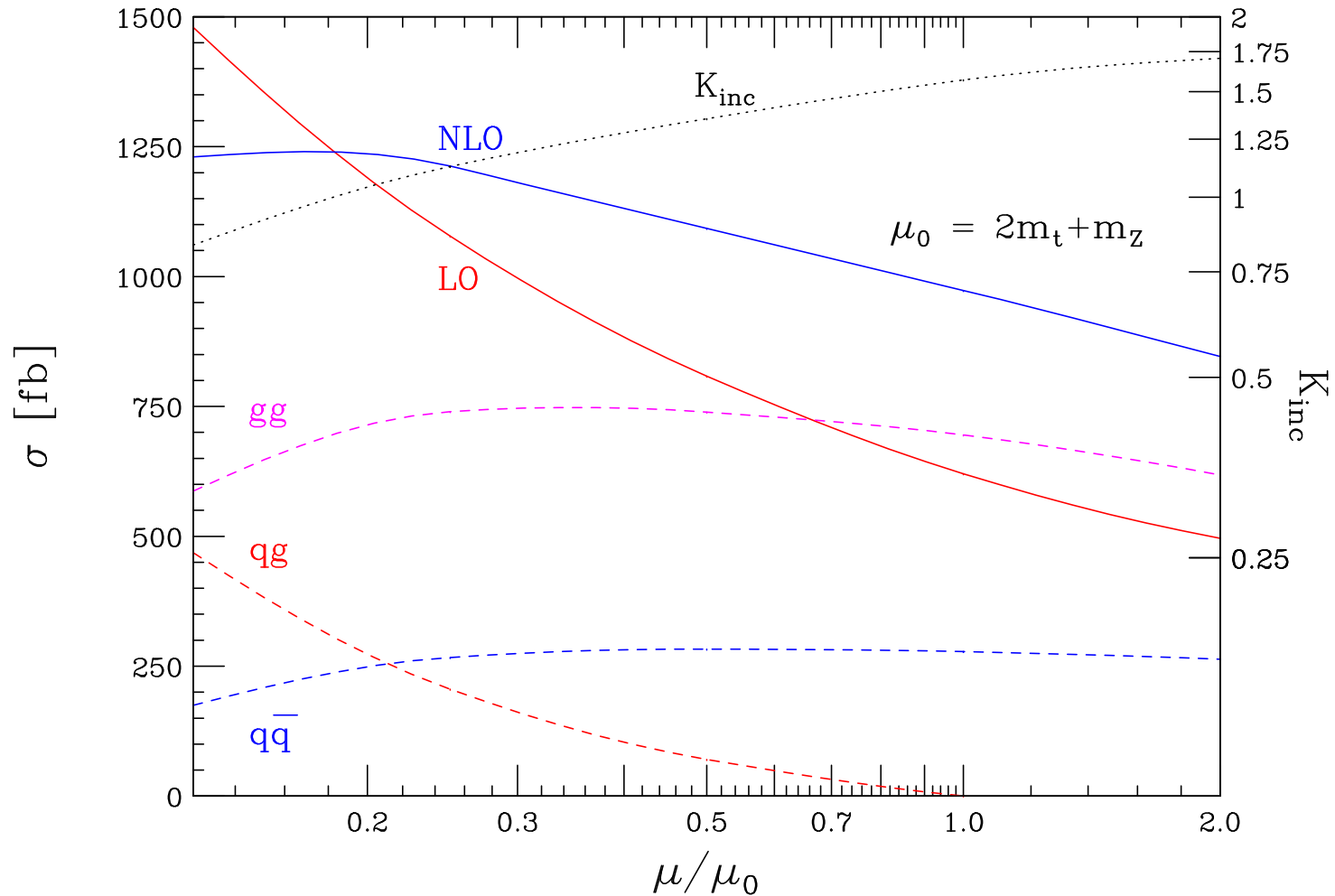
$$\tau_i = \lambda y_i (1 - y_i) [Y_i + X_{ij} y_j + W_{ijk} y_j y_k + \dots]$$

- $\text{Im } \Delta < 0$ for small λ ; we can check that we cross no poles as we increase λ .

Checks

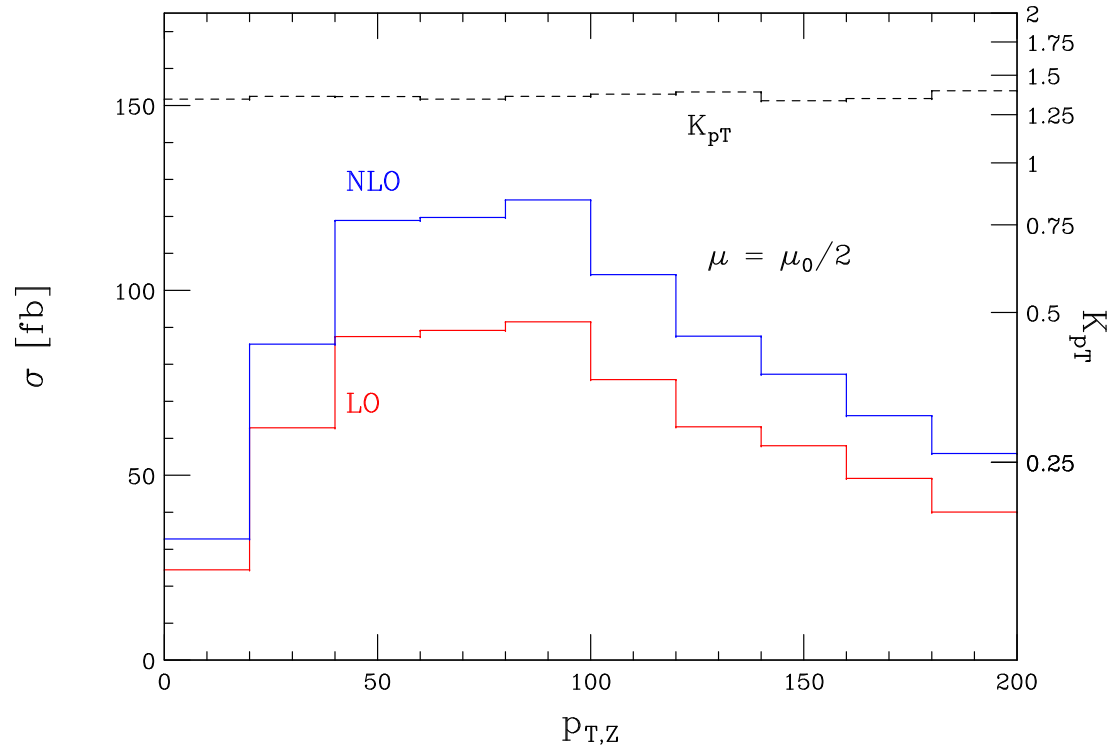
- LO result agrees with **MAD****EVENT** [Maltoni, Stelzer, 2003; Alwall *et al.*, 2007].
- Soft parts of virtual corrections agree with eikonal approximation.
- All poles in ϵ cancel.
- Virtual corrections independent of λ (magnitude of contour deformation)
- Real corrections independent of δ_s and δ_c (phase-space slicing cutoffs)
- Agreement between multiple independent codes

Results



- Vary scale from $\mu_0/4$ to μ_0 : uncertainty is $\pm 11\%$.
- $K_{\text{inc}} = 1.35$ for $\mu = \mu_0/2$.

Results



- It is easy to generate arbitrary kinematic distributions:
- e.g. $p_T(Z)$: NLO corrections change the normalization, but not the shape.

Summary

- We have computed the NLO QCD corrections to $pp \rightarrow t\bar{t}Z$.
- Automated, fully numerical approach
 - $\Rightarrow$ arbitrary kinematic distributions
- $K = 1.35$ for $\mu = (2m_t + m_Z)/2$, independent of $p_T(Z)$
- Theoretical uncertainty reduced from $\sim 30\%$ to $\pm 11\%$
 - $\Rightarrow$ improvement by factor of $1.5\text{--}2$ in measurement of ttZ couplings