$\mathcal{O}(\alpha\alpha_s)$ corrections to the $\gamma t\bar{t}$ vertex at the top quark threshold

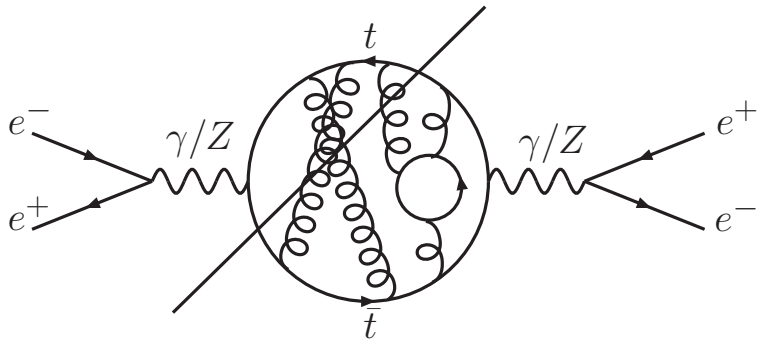
Dirk Seidel

Department of Physics
University of Alberta, Edmonton

in collaboration with Yuichiro Kiyo and Matthias Steinhauser

November 19th, 2008

$\sigma(e^+e^- \to t\bar{t}X)$ near $\sqrt{s} = 2m_t$



cross section at LO

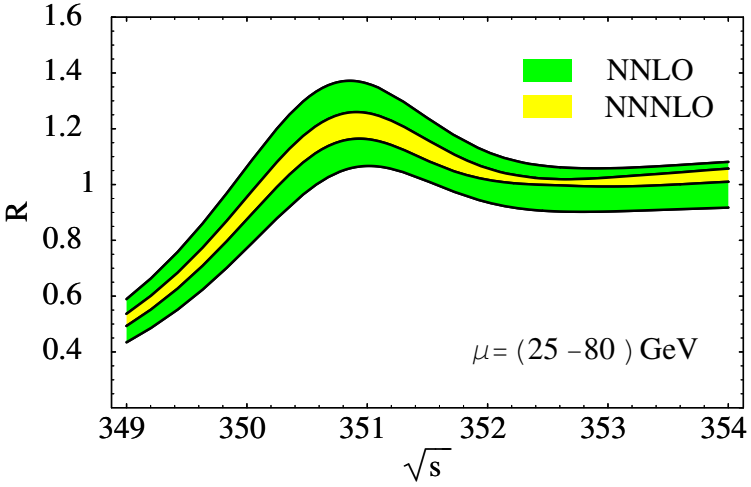
$$\sigma_{t\bar{t}}^{\mathrm{LO}} \sim L^{\mu\nu} \left(\frac{Q_e Q_t}{s} + \frac{\beta_e \beta_t}{s - M_Z^2} \right)^2 \mathrm{Im}\langle \mathrm{T} \mathrm{J}_\mu \mathrm{J}_\nu \rangle$$

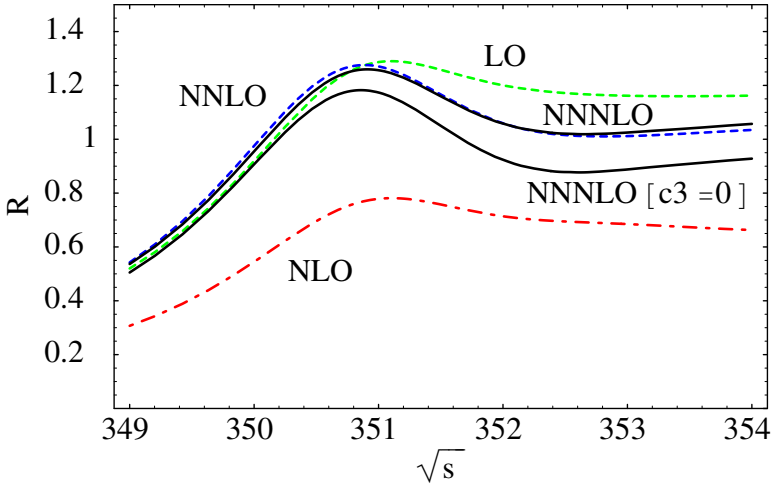
- $\Gamma_t \gg \Lambda_{\mathrm{QCD}} \to$ non-pertubative effects are strongly supressed
- simultanious determination of m_t, α_s, Γ_t [Martinez, Miquel '02]

Expansion in $\alpha_s \sim v$ at threshold (NRQCD)

- hard matching corrections
- summing up terms $\left(\frac{\alpha_s}{v}\right)^n$
  $\rightsquigarrow$ solving Schrödinger equation with static QCD–potential

$\sigma(e^+e^- \to t\bar{t})$ near $\sqrt{s} = 2m_t$





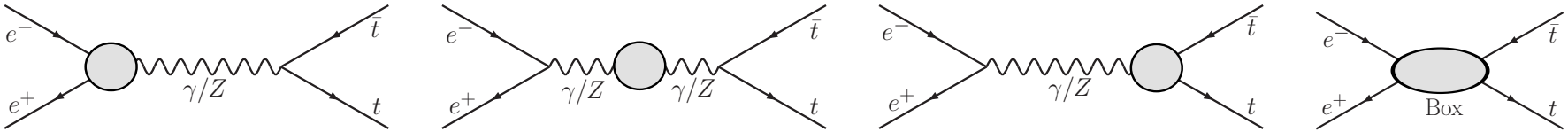
Plots from [Beneke, Kiyo, Schuller '08]

- NNLO calculations as big as NLO ones
- ILC: uncertainty < 100 MeV for m_t can be obtained
- theory: $\delta\sigma/\sigma \leq 3\% \to$ NNNLO calcualation needed
- $\alpha \sim \alpha_s^2 \to \alpha\alpha_s$ corrections are NNNLO

- QCD:
  - NNNLO bound state corrections [Beneke, Kiyo, Schuller '08] [Beneke, Kiyo, Penin '07] [Beneke, Kiyo '08]
  - 3–loop hard corrections
    - n_l [Marquard, Piclum, DS, Steinhauser '06]
    - n_h, singlet: in preparation
  - partial NNLL RG improvement [Hoang '03; Pineda, Signer '06] ($m_t \gg \mathbf{p} \simeq m_t v \gg E \simeq m_t v^2$)
- EW:
  - α [Grzadkowski, Kühn, Krawczyk, Stuart '87; Guth, Kühn '92; Hoang, Reißer '05]
  - 2-loop $\alpha\alpha_s$
    - hard Z/H and gluon [Eiras, Steinhauser '06]
    - hard W and gluon corrections [Kiyo, DS, Steinhauser '08]

Outline

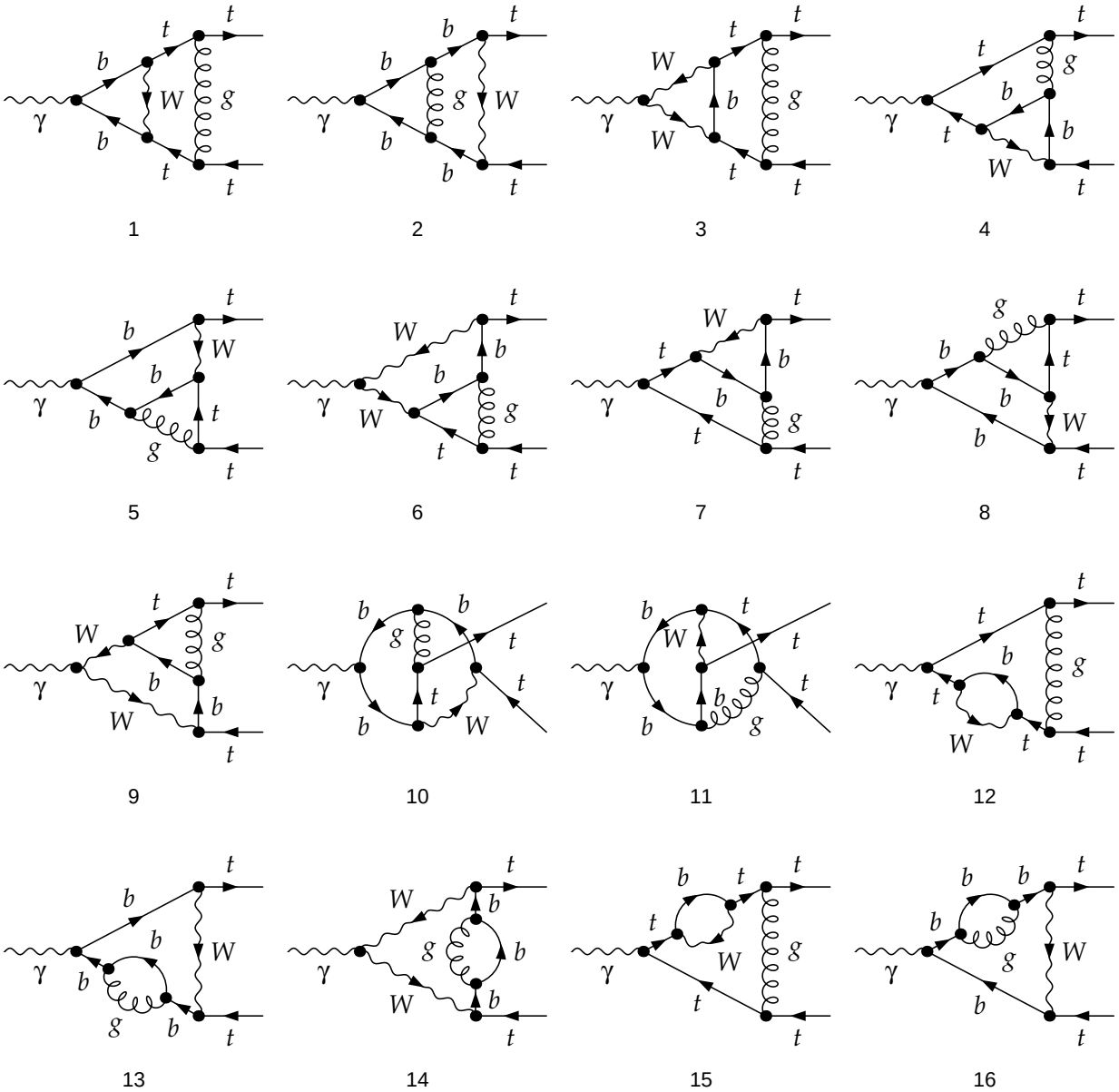
Hard corrections at $\mathcal{O}(\alpha\alpha_s)$



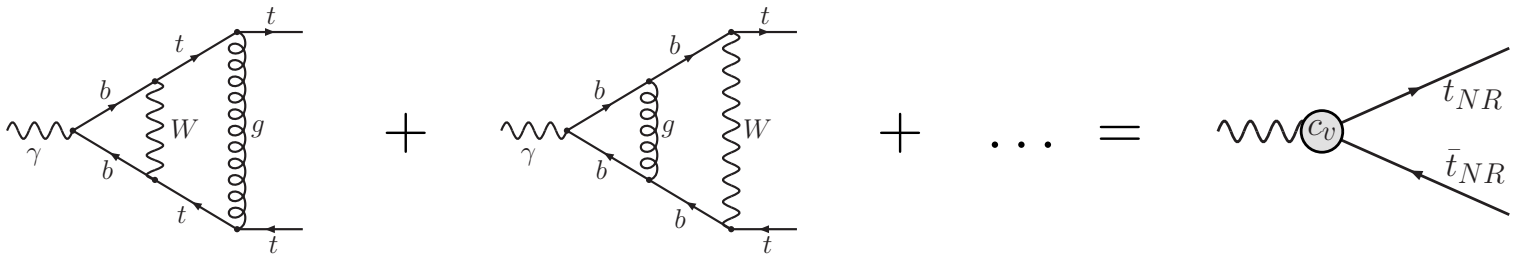
EW⊗QCD

- $\Gamma_{e^+e^-}$, $\Pi_{\gamma/Z}$: trivial/simple
- $\Gamma_{t\bar{t}}$
  - Z/H and gluon [Eiras, Steinhauser '06]
  - W and gluon: this talk
- Box contribution still needs to be calculated
- EW gauge invariant only after inclusion of boxes

Sample Feynman digrams



Matching coefficient



Matching condition

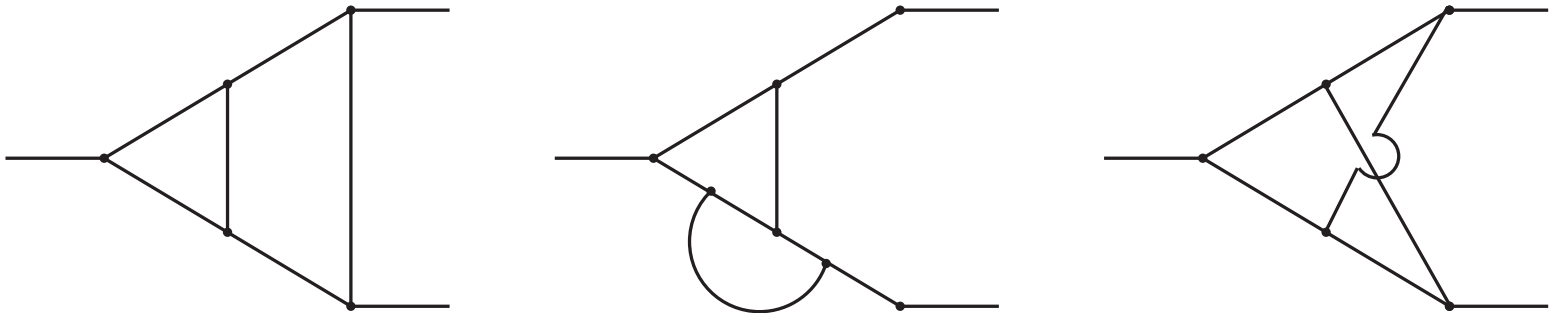
$$\Gamma^\mu = \gamma^\mu F_1 + \frac{[\gamma^\mu, \not{q}]}{4m_t} F_2 + \frac{\not{q}\, q^\mu}{q^2} F_3$$

$$\bar{t}\,\Gamma^i\, t = (\underbrace{F_1 + F_2}_{\Gamma_v})\,[\psi^\dagger \sigma^i \chi] + \ldots$$

$$Z_{\mathrm{A/Z}} Z_2^{\mathrm{OS}} \Gamma_v = c_v(M_W, m_t) \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v$$

- time like component vanishes up to higher orders in v
- threashold expansion used to identify hard contributions contained in c_v
  $\rightsquigarrow q_1^2 = q_2^2 = m_t^2,\ q^2 = (q_1 + q_2)^2 = 4m_t^2,\ q_1 = q_2 = q/2$

Generic Topologies



- after projecting out form factors: scalar integrals
- two mass scales: M_W, m_t
- many different mass configurations

Integration By Parts

$$\int d^{2d}\ell_{1,2}\,\frac{d}{d\ell_i^\mu}\,p^\mu\,I\left(M_W, m_t\right) = 0$$

- use relations to reduce integrals to master integrals

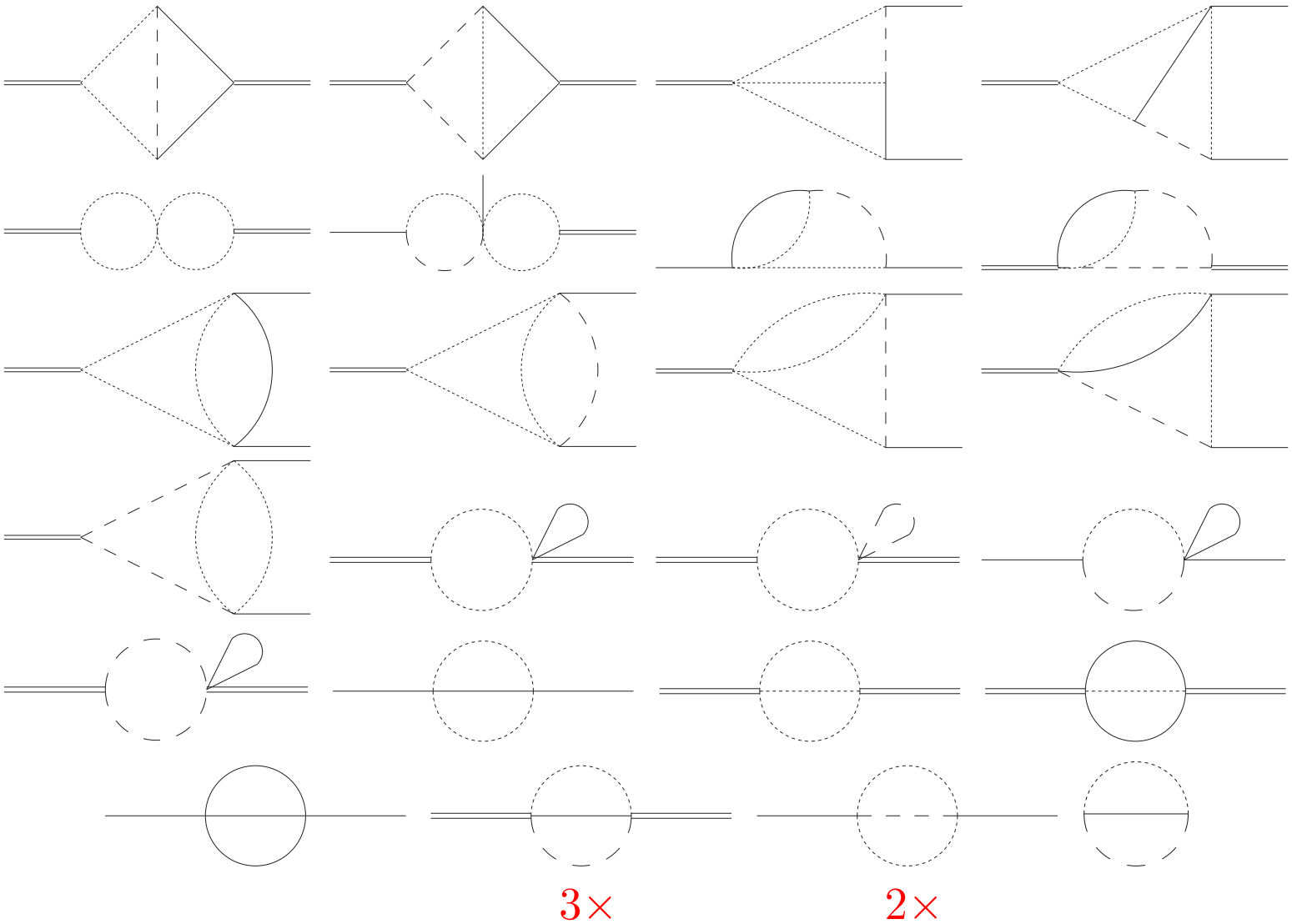
Feynman diagrams

- generated with `QGRAF` [Nogueira '91]
- various topologies are identified with
  `q2e` and `exp` [Harlander '97, Seidensticker '99]

Laporta Algorithm [Laporta '96]

- `Crusher`: Implementation written in C++ [Marquard, DS '06]
- uses `GiNaC` for simple manipulations
- coefficient simplification done with `Fermat`
  $\rightsquigarrow$ interface from [Tentyukov '06]
- automated generation of the IBP identities
- complete symmetrization of the diagrams
- use of multiprocessor environment

Master Integrals

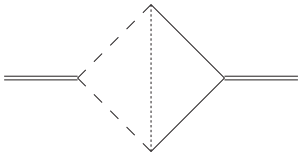
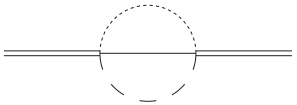


Differential Equation Method

$$\frac{d}{dz}\int d^{2d}\ell_i \frac{1}{(\ell_1^2 - z)\cdots} = \int d^{2d}\ell_i \frac{1}{(\ell_1^2 - z)^2 \cdots} + \ldots$$

$$z = \frac{M_W^2}{m_t^2}$$

$\frac{d^3}{dz^3}$  $+ \left(-\frac{1}{2(z-1)} + \frac{1}{z} - \frac{1}{2(z-9)}\right) \frac{d^2}{dz^2}$  $+ \cdots = 0$

$\frac{d}{dz}$  $= \left(\frac{5}{12(z-1)} - \frac{3}{8z} - \frac{1}{24(z-4)}\right)$ 

$+ \left(-\frac{1}{4(z-1)} + \frac{3}{8z} + \frac{1}{2(z-1)^2} - \frac{1}{8(z-4)}\right)$  $+ \ldots$

- differential equation at $d = 4$; some subtoplogies neglected
- no analytic result for MI's in terms of HPL's
- initial conditions known for some integrals

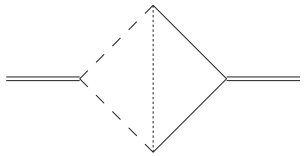
Calculation of Master Integrals as series in z

Ansatz for MI's

$$\mathrm{MI} = \sum c_{ijk} \varepsilon^i z^j \ln^k z$$

- integrals with less lines are assumed to be known
- expand differential equation in ε and z
  $\rightsquigarrow$ reduction to an algebraic system for c_{ijk}
- in every order in ε one constant c_{ijk} cannot be determined
  $\rightsquigarrow$ initial condition at $z = 0$
- expansion up to $\mathcal{O}(z^{10})$ calculated

Solution for MI

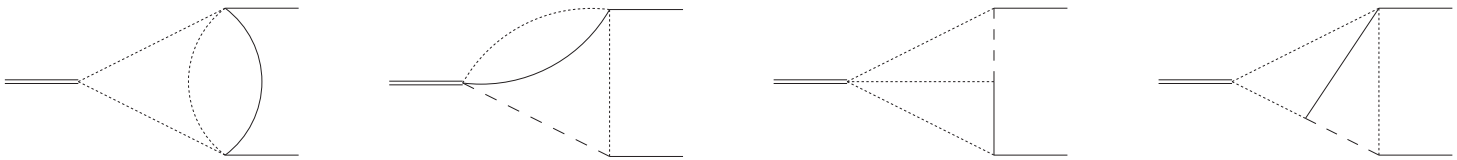


$$
\begin{aligned}
= \quad & \frac{1}{4}\pi^2 \ln 2 - 2\ln^3 2 - \frac{1}{3}\pi^2 \ln 3 + 2\ln^2 2 \ln 3 - \ln 2 \ln^2 3 + \frac{\ln^3 3}{3} \\
& - \mathsf{Li}_3(-2) + \frac{1}{2}\mathsf{Li}_3\left(\frac{1}{4}\right) - 2\mathsf{Li}_3\left(\frac{2}{3}\right) + \mathsf{Li}_3\left(\frac{3}{4}\right) + \frac{21\zeta(3)}{8} \\
& - i\pi\left(\frac{\pi^2}{12} + \frac{1}{2}\ln^2 2\right) + z\Bigg[i\pi\left(\frac{1}{2} - \frac{1}{4}\ln 2 + \frac{3}{4}\ln 3 - \frac{3}{8}\ln z\right) \\
& + \frac{3}{8}\ln 3 \ln z - \frac{3\ln^2 3}{8} + \frac{1}{4}\ln 2 \ln 3 - \frac{\ln^2 2}{4} - \ln 2 \\
& + \frac{17\pi^2}{48} - \frac{1}{8}\mathsf{Li}_2\left(\frac{3}{4}\right)\Bigg] + \mathcal{O}(z^2)
\end{aligned}
$$

- initial condition only affects leading term in z
- higher order in z: constants from solutions of subtopologies

- for two Master Integrals no initial condition is needed
- some MI's could be obtained analytically

Initial condition for MI's



- no solution for four integrals at $z = 0$ found in literature

Mellin–Barnes

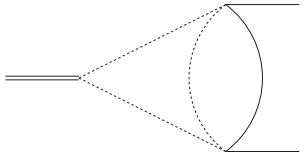
$$\frac{1}{(K-M)^\lambda} = \frac{1}{(K)^\lambda}\frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_\gamma ds \left(-\frac{M}{K}\right)^s \Gamma(-s)\Gamma(\lambda+s)$$

- trade massive propagator for massless one
- simplify Feynman integral representations
- we use `AMBRE` to get MB representation[Gluza,Kajda,Riemann '07]

$\rightsquigarrow$ at most 6-dimensional representation

- inconsistent results when using `MB.m` [Czakon '05] for different representations

Mellin Barnes integration



$$= \int_{-\frac{1}{4}-i\infty}^{-\frac{1}{4}+i\infty} \frac{dz_1}{2\pi i} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{dz_2}{2\pi i}\, e^{i\pi z_1} 4^{-z_2}$$
$$\times \frac{\Gamma(-z_1+1)\Gamma(-z_1)\left[\Gamma(-z_2+1)\right]^2 \Gamma(-z_1-z_2+2)\Gamma(z_1-z_2)\Gamma(z_2)^2}{\Gamma(-z_1+2)^2\Gamma(-2z_2+2)} + \dots$$

Asymptotic behaviour of the Γ–function

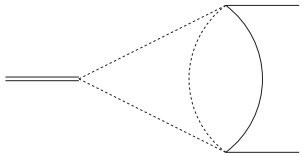
$$\Gamma(a \pm ib) \overset{b\to\infty}{\simeq} \sqrt{2\pi} e^{\pm i\frac{\pi}{4}(2a-1)} e^{\pm ib(\ln b - 1)} e^{-\frac{b\pi}{2}} b^{a-\frac{1}{2}}$$

- no exponential falloff for $\mathrm{Im}(z_2) = 0$, $\mathrm{Im}(z_1) < 0$
- power–law dropoff

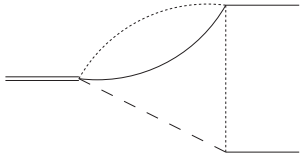
Behaviour for large negative $\mathrm{Im}(z_1)$: $|z_1|^{-\frac{17}{25}}$

After taking two residues in z_2 to the right: $|z_1|^{-\frac{67}{25}}$

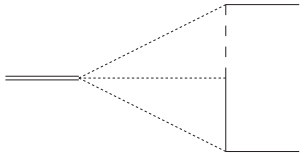
$\rightsquigarrow$ Integrand converges well enough to be integrated



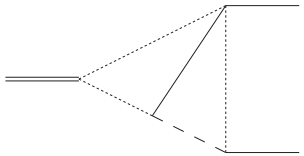
$$= \left(\frac{\mu^2}{m_t^2}\right)^{2\epsilon}\left[\frac{1}{2\epsilon^2}+\frac{1}{\epsilon}\left(\frac{5}{2}-2\ln 2+i\pi\right)-4.81543683(7)\right.$$
$$\left.+4i\pi(1-\ln 2)\right]$$



$$\overset{z=0}{=} \left(\frac{\mu^2}{m_t^2}\right)^{2\epsilon}\left[\frac{1}{2\epsilon^2}+\frac{1}{\epsilon}\left(\frac{5}{2}+i\pi\right)+\frac{19}{2}-\frac{23\pi^2}{24}-\frac{5\ln^2 2}{2}-\frac{3\ln 3}{2}\right.$$
$$+\frac{5}{2}\ln 2\ln 3-\frac{5}{4}\mathsf{Li}_2\left(\frac{3}{4}\right)+\frac{i\pi}{2}(11-5\ln 2)-\epsilon\, 16.690539(1)$$
$$-i\pi\epsilon\left(-\frac{45}{2}+\frac{35\pi^2}{24}+3\ln 2-\frac{21\ln^2 2}{4}+6\ln 3+2\mathsf{Li}_2(-2)\right.$$
$$\left.\left.-4\mathsf{Li}_2\left(\frac{1}{4}\right)\right)\right]$$

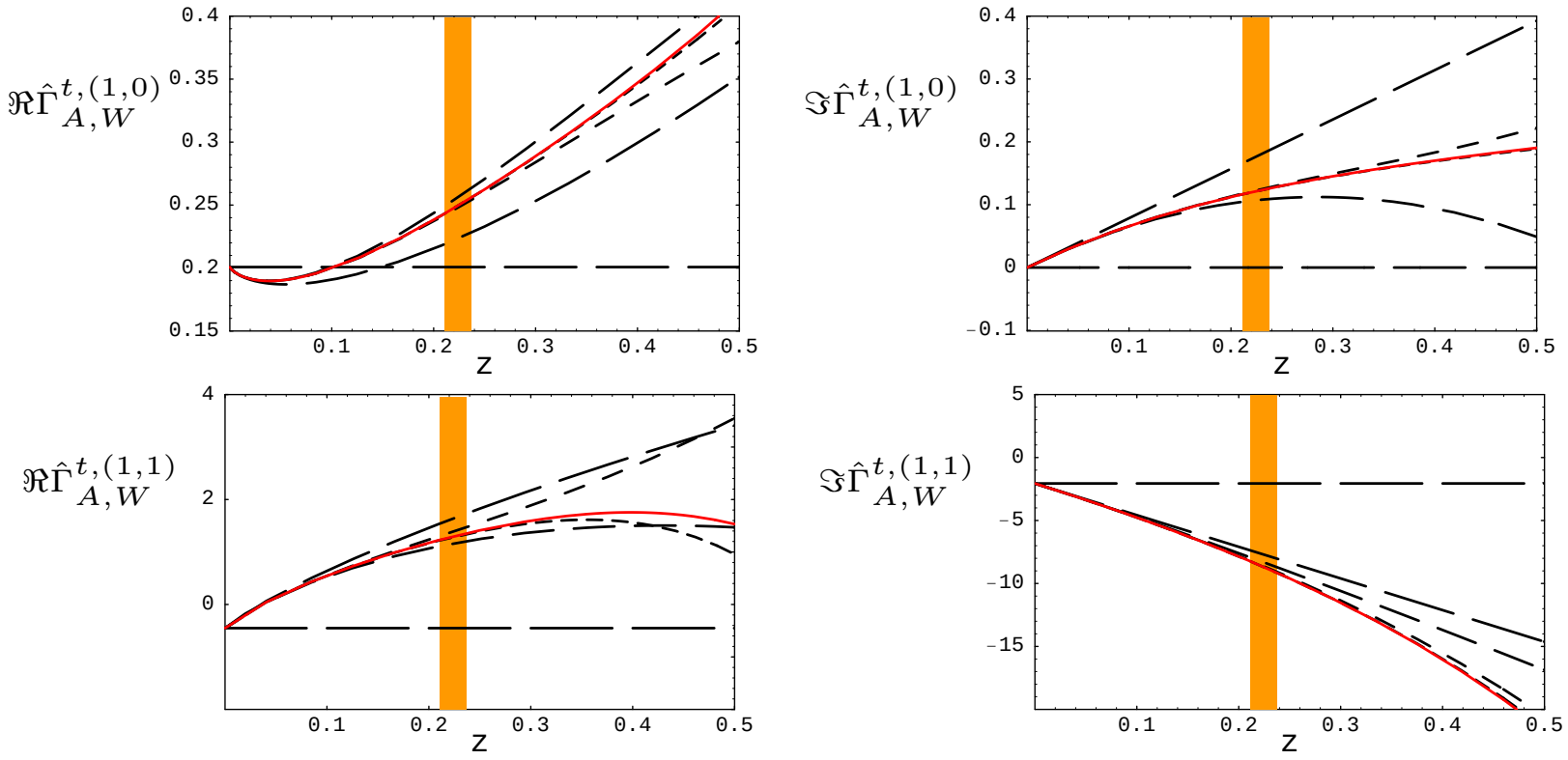


$$\overset{z=0}{=} 2.704628(4)-5.167709(4)i$$



$$\overset{z=0}{=} 2.70543(6)-1.91431(6)i$$

- numerical Monte–Carlo integration done with `DIVONNE` instead of `VEGAS`



- z-range for 165 GeV $< m_t <$ 175 GeV
- rapid convergence in the physical range
- correction from $\hat{\Gamma}^{t,(1,1)}_{A,W}$ to R: 0.1%

Comparison of known hard EW and EW×QCD corrections

$$\hat{\Gamma}^{t,(1,0)}_{A,H} = 21.1 \times 10^{-3}\ (10.6 \times 10^{-3}) \quad \text{for} \quad M_H = 120\ (200)\ \text{GeV}$$
$$\hat{\Gamma}^{t,(1,0)}_{A,W} = 3.0 \times 10^{-3}$$
$$\hat{\Gamma}^{t,(1,0)}_{A,Z} = 1.7 \times 10^{-3}$$
$$\hat{\Gamma}^{t,(1,1)}_{A,H} = -17.6 \times 10^{-3}\ (-6.6 \times 10^{-3}) \quad \text{for} \quad M_H = 120\ (200)\ \text{GeV}$$
$$\hat{\Gamma}^{t,(1,1)}_{A,W} = 0.2 \times 10^{-3}$$
$$\hat{\Gamma}^{t,(1,1)}_{A,Z} = -1.0 \times 10^{-3}$$

- strong cancellation between one and two loop contributions
- imaginary part not taken into account
  $\rightsquigarrow$ cuts not corresponding to $t\bar{t}X$ final states have to be omitted [Hoang, Reisser '04]

- last missing piece of hard $\gamma t\bar{t}$–vertex corrections at order $\alpha\alpha_s$ calculated
- perturbation theory works well in the EW–sector
- contribution amounts to 0.1% $\ll$ 3% aimed for theoretical predictions
- troublesome MI's at threashold can be handled

- two–loop box–diagrams still missing
- consistent treatment of imaginary part
- finite width effects, EW effects in NRQCD
- hard corrections at NNNLO