



# ***Top quark physics at the ILC: methods and meanings***

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# Outline

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1. Top quark properties: mass
  - What is it?
  - Precision calculations
  - Why do we care?
2. Top quark as a probe of new physics
  - Looking for deviations ( $g_{tth}$ )
  - Anomalous couplings
  - New particles
3. Where do we go from here?



# What is the top-quark mass?

Answer 1: A parameter of the Lagrangian  $L \sim m_t \bar{t}t$

Answer 2: An effective coupling between  $t$ - $t$ - $h$

$m_t = Y_t / (2\sqrt{2}G_F)^{1/2} \approx 1$  in the SM

Answer 3: The kinematic mass seen by the experiments

Right after the discovery of the top quark, Martin Smith and Scott Willenbrock asked this question about the “pole mass” of the top quark. They showed that a renormalon (the closest pole of the Borrel transform) induced an ambiguity of  $\mathcal{O}(\Lambda_{QCD})$  in the definition of the pole mass.

This led to the recommendation to use the  $\overline{\text{MS}}$  mass for top quarks as a standard.

We theorists are good at setting standards that make our life easier ... most perturbative calculations use the  $\overline{\text{MS}}$  mass for simplicity.



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We theorists are good at setting standards that make our life easier ... most perturbative calculations use the  $\overline{\text{MS}}$  mass for simplicity.

Of course mass is NOT measured directly. Instead, it affects the distribution of events that are measured, and that distribution is used to INFER the mass.

At the ILC, we tend to concentrate on the  $t\bar{t}$  cross section distribution at **threshold** and in the **continuum**.

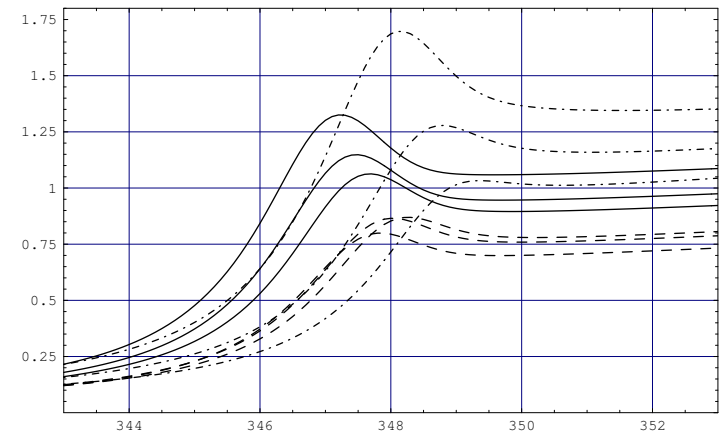


## $t\bar{t}$ threshold at a linear collider (LC)

There is a subtle question when you try to make a precision measurement of QCD:  
What mass do you use?

The pole mass is not defined beyond  $\Lambda_{\text{QCD}}$ .

In fact it is not well-defined at all, since there are no free quarks.



Yakovlev, Groote PRD63, 074012(01)



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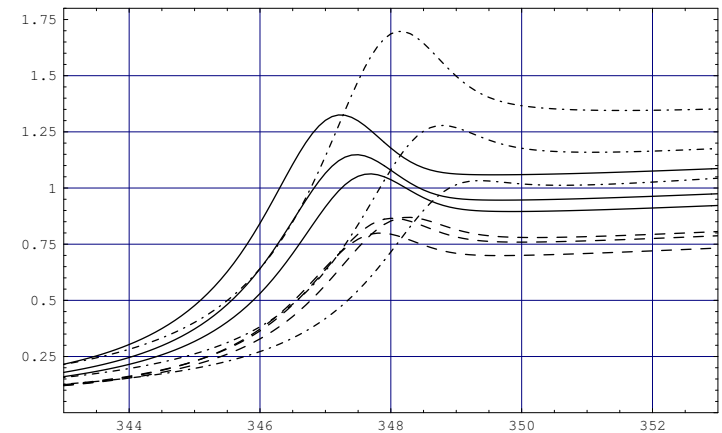
Solution: Use the 1S mass (pseudo bound state)  
There are large non-relativistic corrections

$$\sigma_{t\bar{t}} \propto v \sum \left( \frac{\alpha_s}{v} \right) \times \left\{ \frac{1}{\sum (\alpha_s \ln v)} \right\} \\ \times \left\{ \begin{array}{l} \text{LO}(1) + \text{NLO}(\alpha_s, v) + \text{NNLO}(\alpha_s^2, \alpha_s v, v^2) \\ \text{LL} + \text{NLL} + \text{NNLL} \end{array} \right\}$$

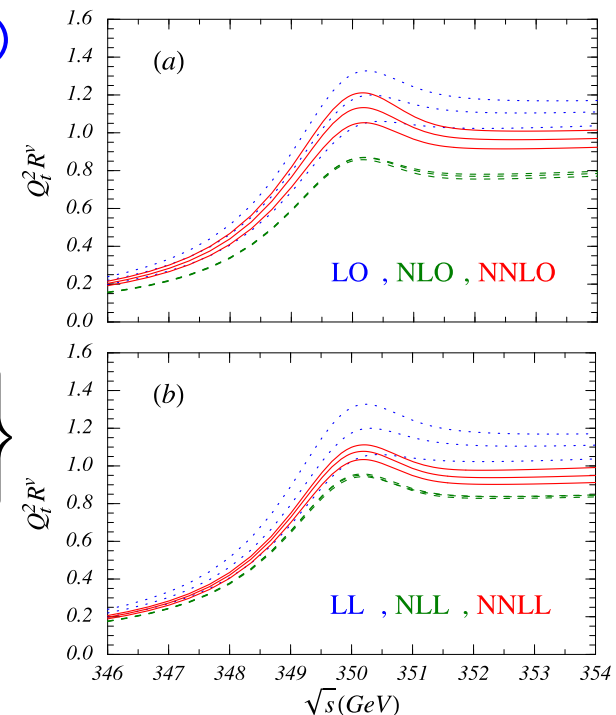
Normalization changes, but peak stable.

$\delta\sigma_{t\bar{t}}$  is  $\pm 6\%$  before ISR/beamstrahlung

$\delta m_t \sim 100 \text{ MeV}$  is attainable



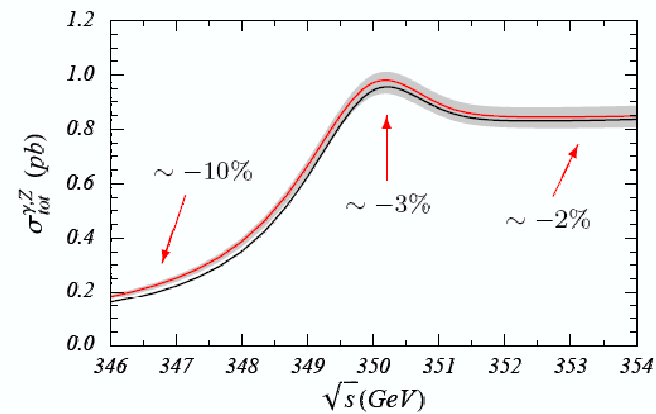
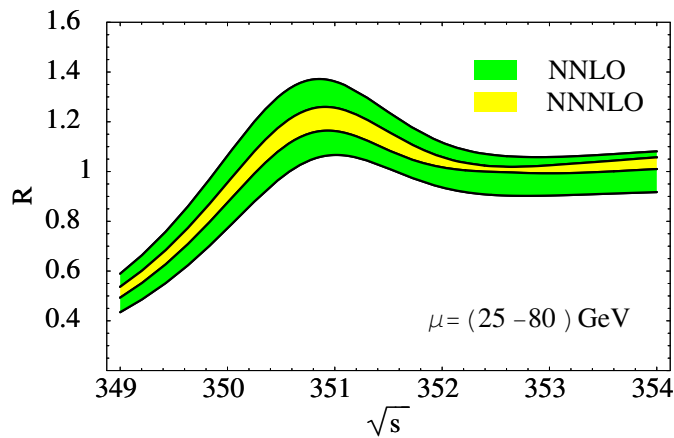
Yakovlev, Groote PRD63, 074012(01)



Hoang, Manohar, Stewart, Teubner



# $t\bar{t}$ threshold corrections



Beneke, Kiyo, Schuller ph/0801.3464

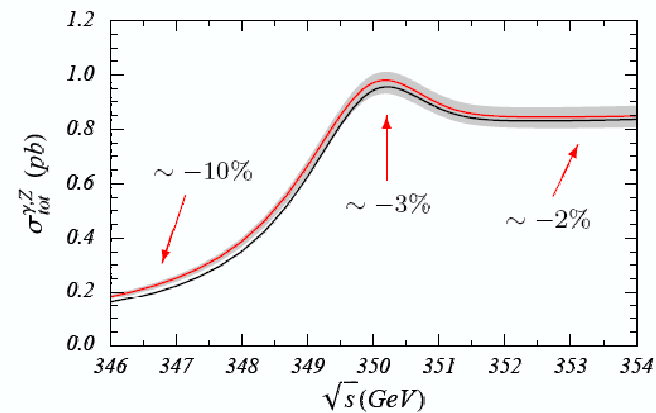
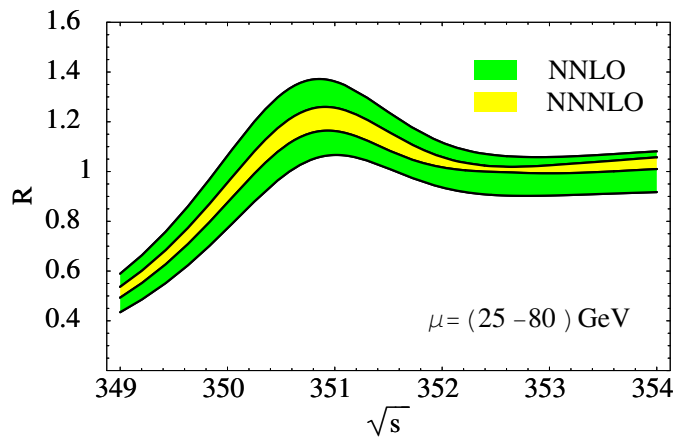
Reisser, Hoang (06)

NNLO calculation mostly done    Unstable particle effects

Order  $\alpha\alpha_s$  corrections done —  $\Delta\sigma \sim 0.1\%$  cf. Dirk Seidel talk



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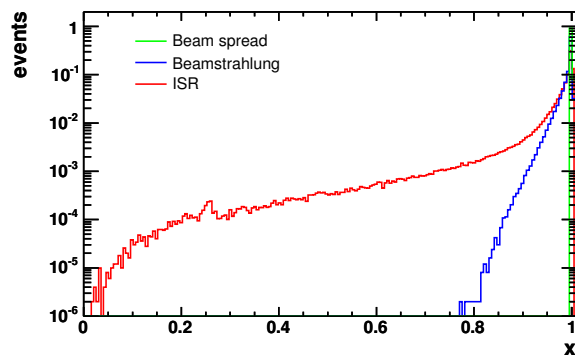
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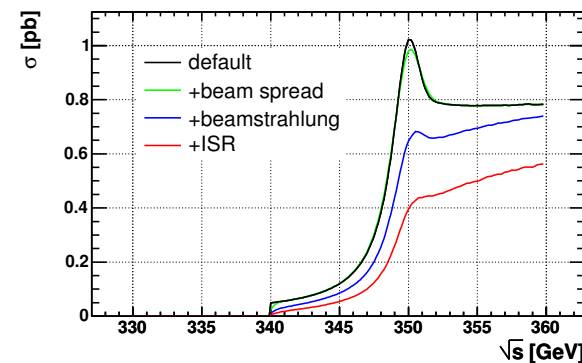
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Order  $\alpha\alpha_s$  corrections done —  $\Delta\sigma \sim 0.1\%$  cf. Dirk Seidel talk

Luminosity has huge influence on spectrum cf. Boogert talk, Gounaris



$$\sigma(s) = \int_0^1 L(x)\sigma(x, s)$$



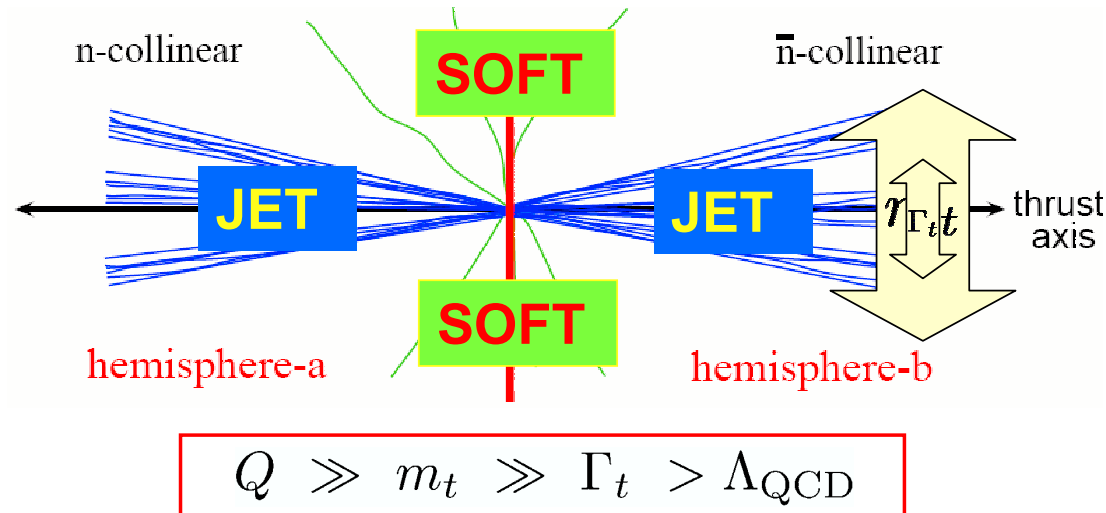
There are many effects that are large vs. 50 MeV — we need to match subtraction of ISR and ISR/FSR interference.





## $t\bar{t}$ continuum mass

We heard a fantastic talk by Sonny Mantry, about a new top-quark jet mass, so I will not repeat details.



Fleming, Hoang, Mantry, Stewart, PRD 77, 114003 (08)

Factorization of the effective field theories into hard, jet, and ultra-soft pieces was shown.

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} = \sigma_0 H_Q(Q, \mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- J_n(s_t - Q\ell^+, \mu) J_{\bar{n}}(s_{\bar{t}} - Q\ell^-, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

This utilizes the strong ordering of scales:  $Q \gg m_t \gg \Gamma_t > \Lambda_{\text{QCD}}$ .



## *What mass do we measure?*

---

The statement has been made that you measure a  $1S$  mass at threshold, and a top-jet mass in the continuum (using the new calculations).

Other masses have been mentioned:  $\overline{MS}$  mass, pole mass, (could have mentioned peak mass, Breit-Wigner mass, ...)

*Which mass do we measure?*



## What mass do we measure?

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Which mass do we measure? None of them.

We measure line-shapes or particle flow or invariant masses with cuts and ISR/FSR effects.

This leads to the following questions:

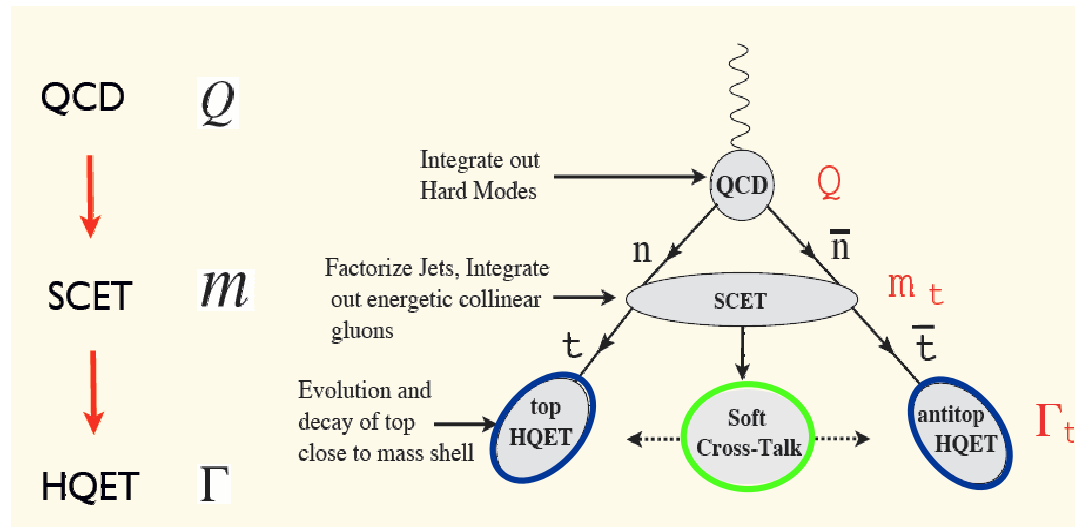
1. Do we need to work out IS subtraction terms to merge correctly with ISR estimates for incoming electrons?  
The box diagrams,  $\alpha^2$ , ... diagrams may be significant on the level of the 0.01% theory error we want for  $m_t$  or some other observable.
2. The hemisphere-like top jet definitions are nice, but does the factorization demonstrated hold in the presence of hard cuts?
3. One jet algorithm to be used is particle flow — this estimates the neutral particles from the charged ones. Does this invalidate any assumptions?

The opportunity and challenge going forward will be to ensure that the experimental and theoretical definitions agree.



## Techniques for the future (EFTs)

The new calculations this year have consisted of matching of various effective field theories to simplify the problems at each natural scale.  
(cf. event shapes, Bhabha scattering,  $WW$  threshold, top mass, etc.)



Can we attach one more EFT, e.g., SCET-II, and treat the jets and FSR consistently?

**My opinion:** In order to reach the ever higher theoretical precision in QCD demanded by the experimental results, we will move toward merged EFTs as a general course.

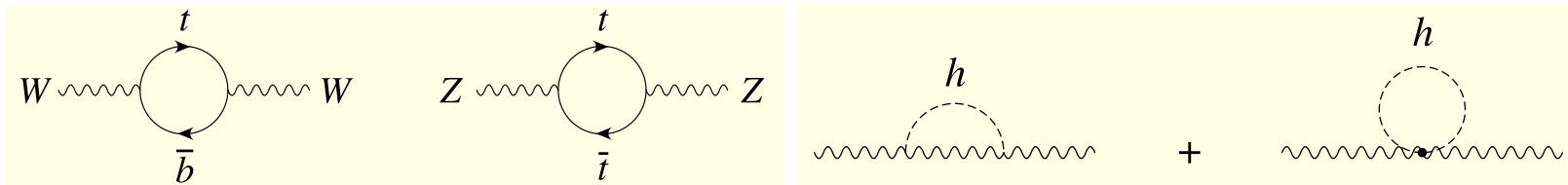
I believe the focus should shift from higher loops to external matching (i.e., getting the theorists and experimentalists to agree on IS/FS.)



# Why study the top-quark mass?

Answer: Electroweak (EW) precision physics

Both top quark and Higgs contribute at 1-loop to the  $W/Z$  propagators.



Assuming  $\alpha$ ,  $G_F$ , and  $M_Z$  as inputs,  $M_W^2$  at 1-loop is:

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W} \frac{1}{1 - \Delta r(m_t, m_H)}$$

where  $\Delta r(m_t, m_H) \approx c_t m_t^2 = c_H \ln(M_H^2/M_Z^2) + \dots$

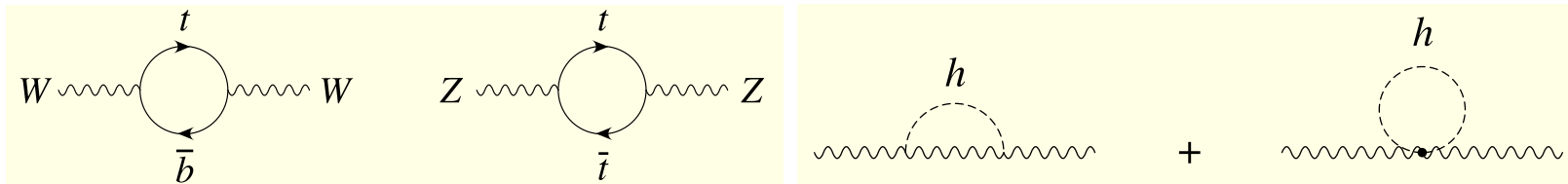
Inverting the formula provides a logarithmic constraint on  $M_H$ .



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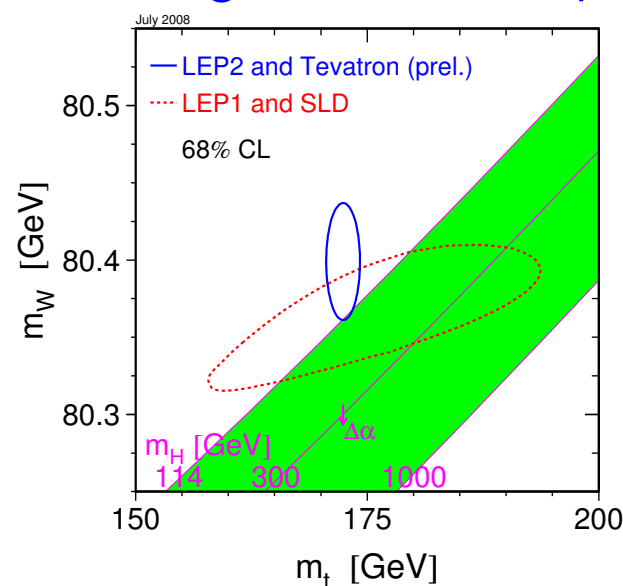


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End of Run I

$$m_t = 174.3 \pm 5.1 \text{ GeV (3\%)}$$

Early Summer 2005

$$m_t = 178.0 \pm 4.3 \text{ GeV (fishy)}$$

Late Summer 2005

$$m_t = 172.7 \pm 2.9 \text{ GeV}$$

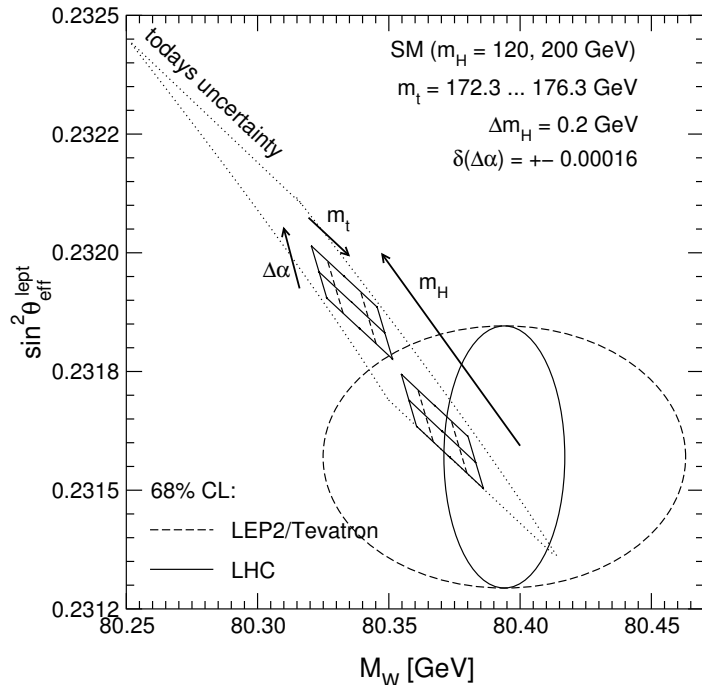
Summer 2008

$$m_t = 172.4 \pm 1.2 \text{ GeV}$$



# How well do we need to know $m_t$ ?

There is a better way than “blue band plots” to look at this in the SM.



Beneke et al., hep-ph/0003033

- Assume  $M_H$  is known.
- $M_W$  will be measured to  $\sim 20$  MeV  
 $\Rightarrow$  Need  $m_t$  to  $\sim 3$  GeV at LHC.

- A linear collider can measure  $M_W$  to  $\sim 6$  MeV.

Giga-Z can measure  $\sin^2 \theta_W \sim 10^{-5}$

cf. G. Moortgat-Pick talk for

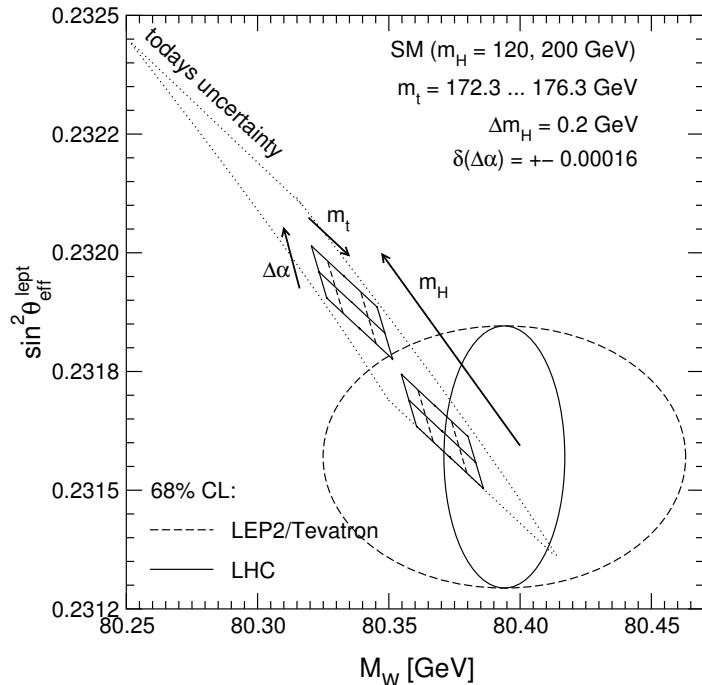
$Z$ -calibration data idea  $\Rightarrow 3 \times 10^{-5}$

$\Rightarrow$  Need  $m_t$  to  $\sim 1$  GeV.



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Beneke et al., hep-ph/0003033

At the LHC:

- Several channels can reach  $< 1$  GeV (stat.)
- To reach systematics  $< 1$  GeV use:  
 $M_{J/\Psi \ell \nu}$  w/ template for  $m_t$ . ( $\sim 300 \text{ fb}^{-1}$ )

The bottom line: We have already saturated the information we can extract about a SM Higgs from top-quark measurements given any near-term collider (i.e., LHC).

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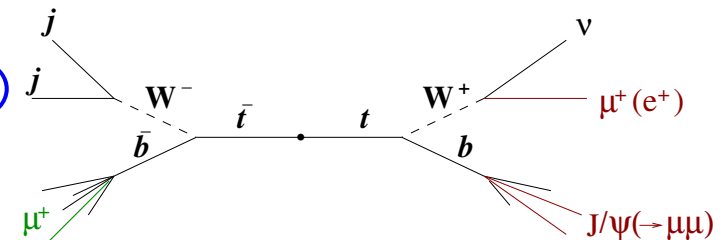
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Most excitement about Higgs production has nothing to do with the SM.

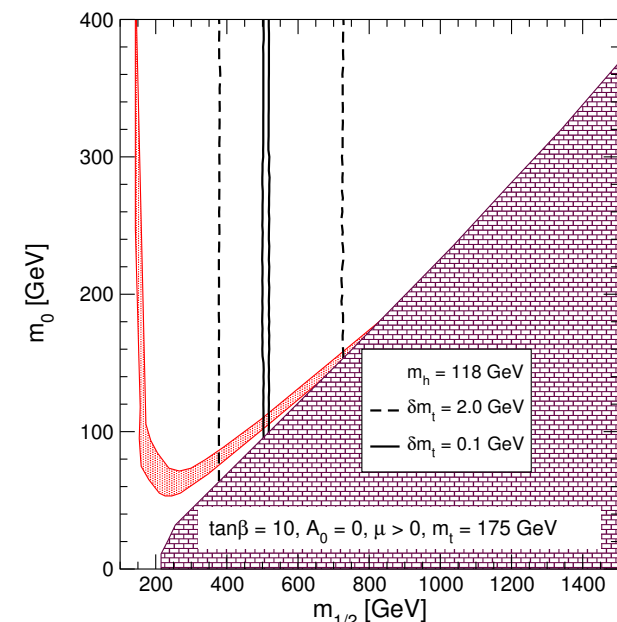
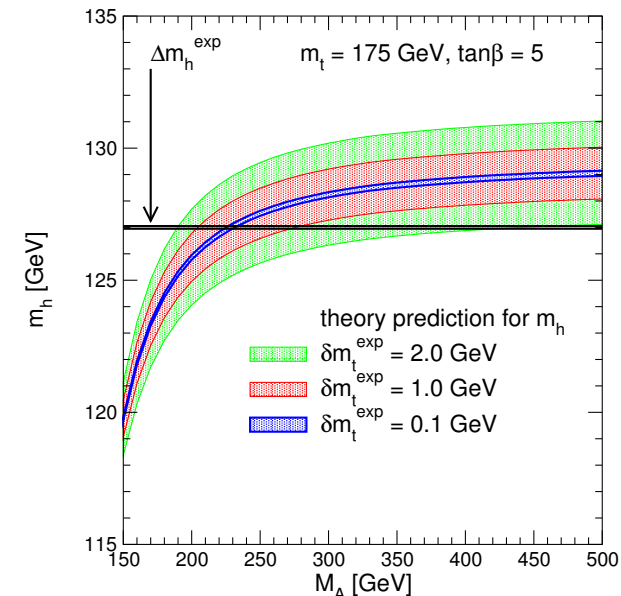
Models of new physics predict different sensitivity to the top-quark mass.

SUSY Higgs masses are VERY sensitive to the top-quark mass

$$\Delta M_H^2 \approx \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2 \beta} \ln \left( \frac{\overline{m}_t^2}{m_t^2} \right)$$

- Experimental error from LHC *may* reach  $\sim 200$  MeV (using rare decays)
- $\delta M_H \sim \delta m_t$ , so we will want  $\delta m_t \sim 100$  MeV.

If a smaller error in  $m_t$  is achieved, we gain indirect access to  $M_A$ ,  $A_t$ ,  $m_{1/2}$ , etc.





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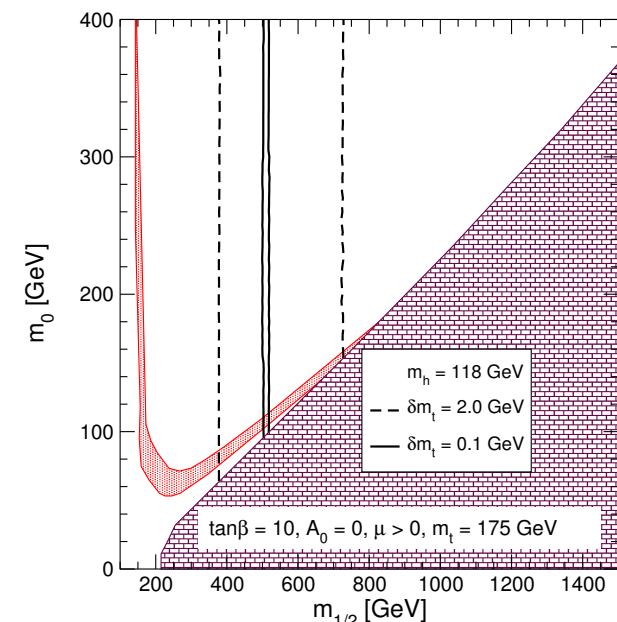
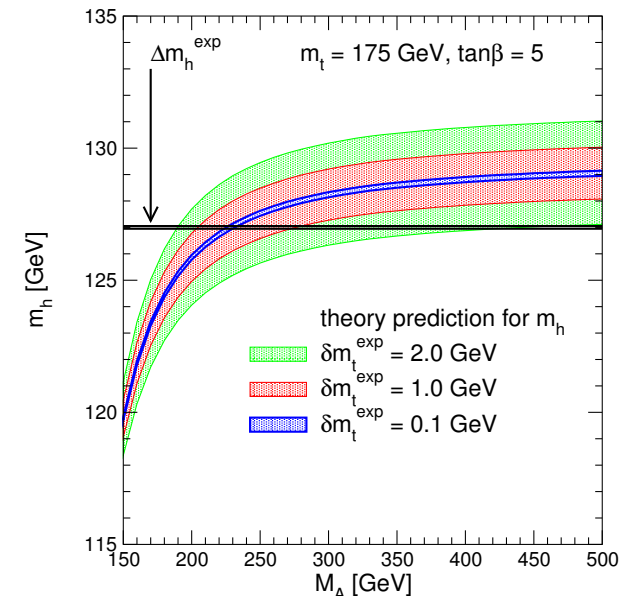
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Warning: 4-loop corrections are comparable in size.

This needs major Loopverein!

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# $t\bar{t}H$ at a linear collider and $y_t$

Calculating  $t\bar{t}H$  at an  $e^+e^-$  collider is very challenging.

- There are many 10% corrections near threshold.

- There are NLO calculations:

*You, et al., PLB 571, 85 (03)*

*Belanger, et al., PLB 571, 163 (03)*

*Denner, et al., PLB 575, 290 (03)*

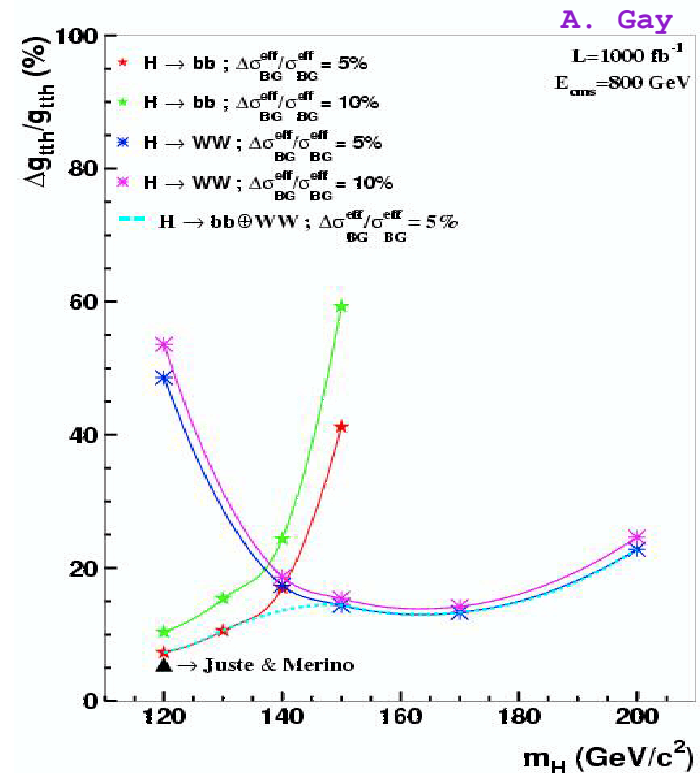
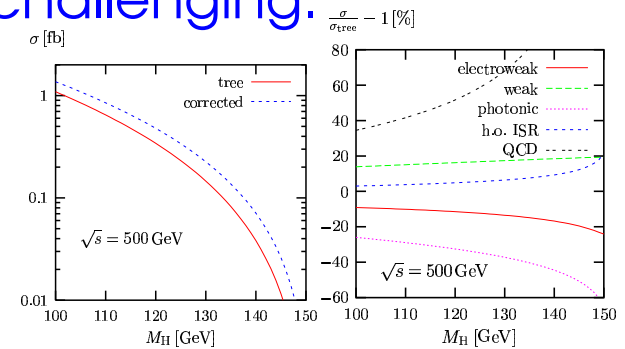
Now NLO EW+QCD  $WW/ZZ \rightarrow t\bar{t}H$

*Bouayed, Boudjema, PMCP A2,3 (08)*

**Needs > 1 TeV machine**

- SUSY corrections tend to reduce  $\sigma_{t\bar{t}H}$  another 20–30%

*J.J. Liu, et al., PRD 72, 033010 (05)*





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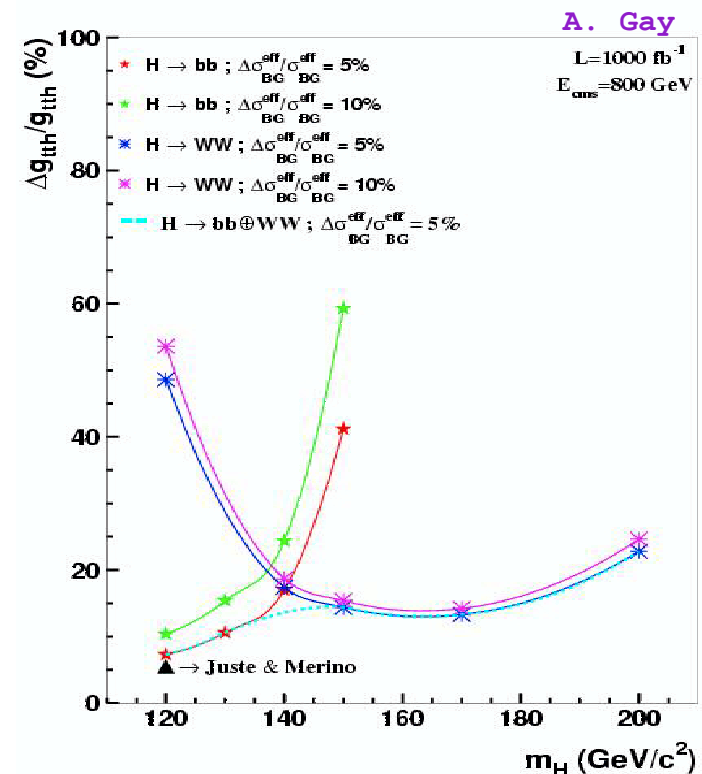
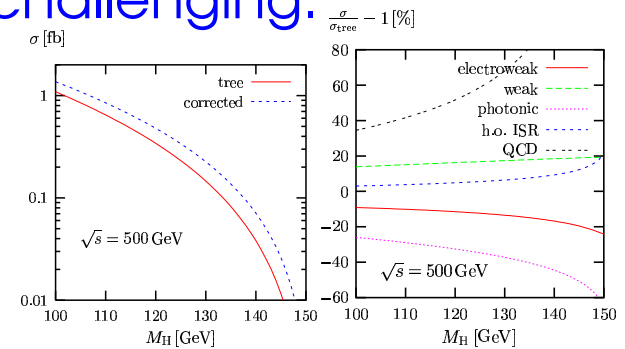
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This measurement is tenable  $\geq 800$  GeV  
with lots of luminosity, Gay  
and maybe at 500 GeV Juste

At best you get  $\pm 10\%$  if  $M_H < 180$  GeV

Bottom line: There is no known way to get  $\delta_{y_t}$  below 10%,  
and certainly not to 1%. Maybe you can figure this out.





## Anomalous couplings: $\gamma/Zt\bar{t}$

The most general  $t\bar{t}(\gamma, Z)$  couplings can be written

$$\Gamma_{t\bar{t}(\gamma, Z)}^\mu = i e \left\{ \gamma^\mu [F_{1V}^{\gamma, Z} + F_{1A}^{\gamma, Z} \gamma^5] + \frac{(p_t - p_{\bar{t}})^\mu}{2 m_t} [F_{2V}^{\gamma, Z} + F_{2A}^{\gamma, Z} \gamma^5] \right\}$$

A study of  $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^\pm + \text{jets}$  at  $\sqrt{s} = 500$  GeV predicts

ILC Design V.2; Ridani; Grzadkowski, Hioki (03)

| Coupling        | LO SM Value | $\mathcal{P}(e^-)$ | $\int \mathcal{L} dt$ (fb $^{-1}$ ) | $1\sigma$ sensitivity |
|-----------------|-------------|--------------------|-------------------------------------|-----------------------|
| $F_{1A}^\gamma$ | 0           | $\pm 0.8$          | 100                                 | 0.011                 |
| $F_{1A}^Z$      | -0.6        | -0.8               | 100                                 | 0.013                 |
| $F_{1V}^\gamma$ | 2/3         | $\pm 0.8$          | 200                                 | 0.047                 |
| $F_{1V}^Z$      | 0.2         | $\pm 0.8$          | 200                                 | 0.012                 |
| $F_{2A}^\gamma$ | 0           | +0.8               | 100                                 | 0.014                 |
| $F_{2A}^Z$      | 0           | +0.8               | 100                                 | 0.052                 |
| $F_{2V}^\gamma$ | 0           | $\pm 0.8$          | 200                                 | 0.038                 |
| $F_{2V}^Z$      | 0           | $\pm 0.8$          | 200                                 | 0.009                 |

Adding  $e^+$  polarization  $\mathcal{P}_{e^+} = 0.5$  improves  $F_{1V}^{\gamma, Z}$  and  $F_{2V}^{\gamma, Z}$  by a factor of 3.

G. Moortgat-Pick et al., [ph/0507011](https://arxiv.org/abs/hep-ph/0507011)

Increasing the C.M. energy to  $\sqrt{s} = 800$  GeV improves the limits by a factor 1.3–1.5

Bernreuther, LCWS99

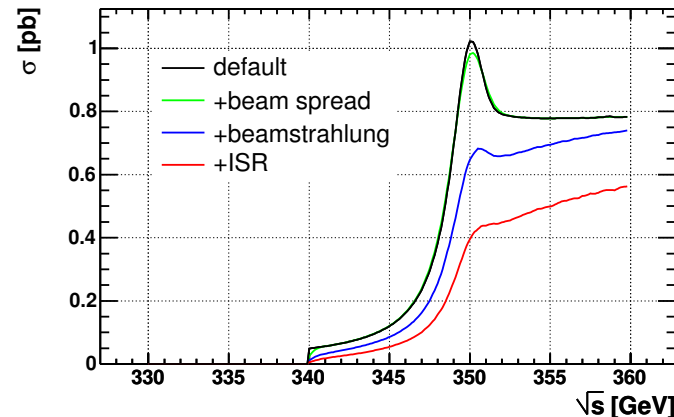
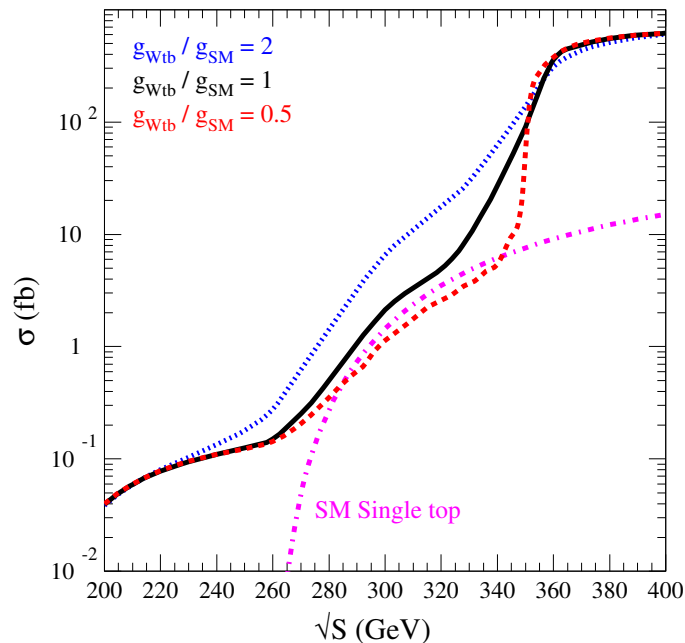


# Anomalous couplings: $Wtb$

The most general  $Wtb$  couplings can be written

$$\Gamma_{tbW}^\mu = -\frac{g}{\sqrt{2}} V_{tb} \left\{ \gamma^\mu [f_1^L P_L + f_1^R P_R] - \frac{i\sigma^{\mu\nu}}{M_W} (p_t - p_b)_\nu [f_2^L P_L + f_2^R P_R] \right\},$$

It has been proposed to measure  $f_1^L$  below  $t\bar{t}$  threshold:



cf. talk by Boogert

P. Batra, T. Tait, PRD 74, 054021 (06)

Single top contributes strongly here, and  $V_{tb}f_1^L$  can be measured to 3%.

One thing left out of this study was the ISR and beamstrahlung.

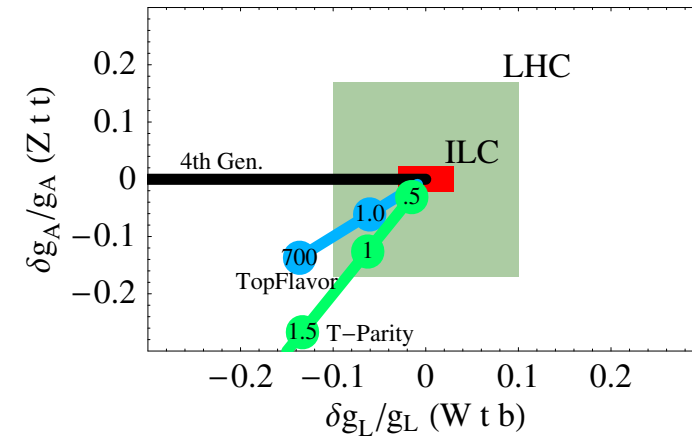
A careful study should be done just below threshold to determine just what can be observed in a realistic accelerator and detector environment.



# Anomalous couplings: $Wtb$ and more

If a 3% measurement of  $V_{tb}f_1$  is attained, you have access to a wide array of models.

- new generations
- top-flavor ( $Z'$  couples to 3rd gen.)
- Little Higgs w/ T-parity (new vector quarks mix with 3rd gen.)



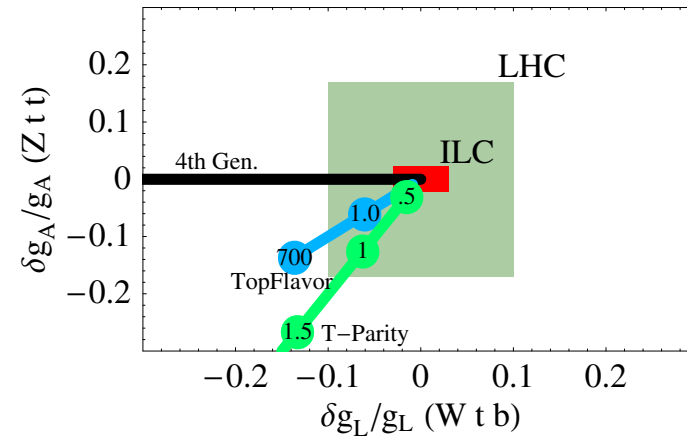
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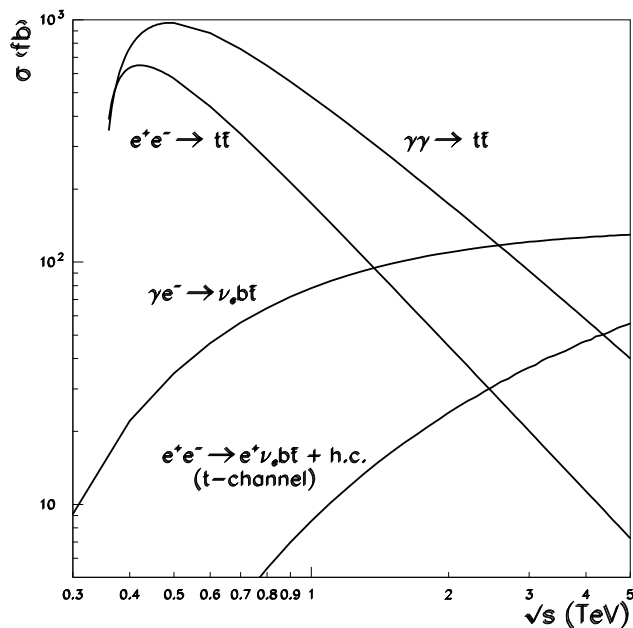
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P. Batra, T. Tait, PRD 74, 054021 (06)



Boos in ph/0410364

Single top is interesting at  $\gamma e^-$ :

$\gamma e^- \rightarrow \nu_e b t + X$  can do a lot:

- $\gamma e$  ( $\sqrt{s_{e+e^-}} = 0.5$  TeV): probe  $|F_{2L}, F_{2R}| > 0.05$
- Look for  $t \rightarrow H^+ b$
- general 4-fermion operators





# Little Higgs

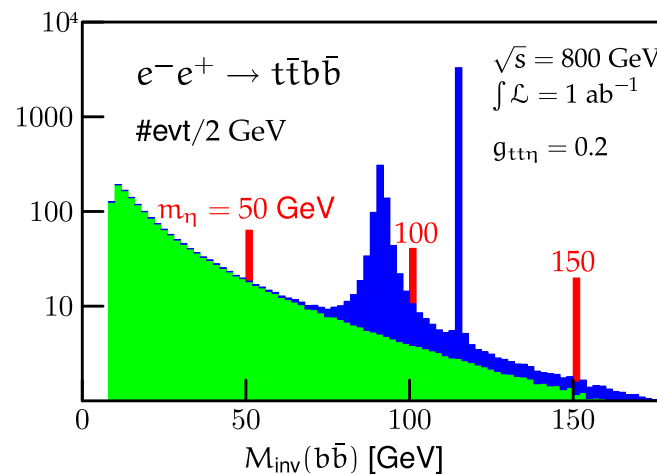
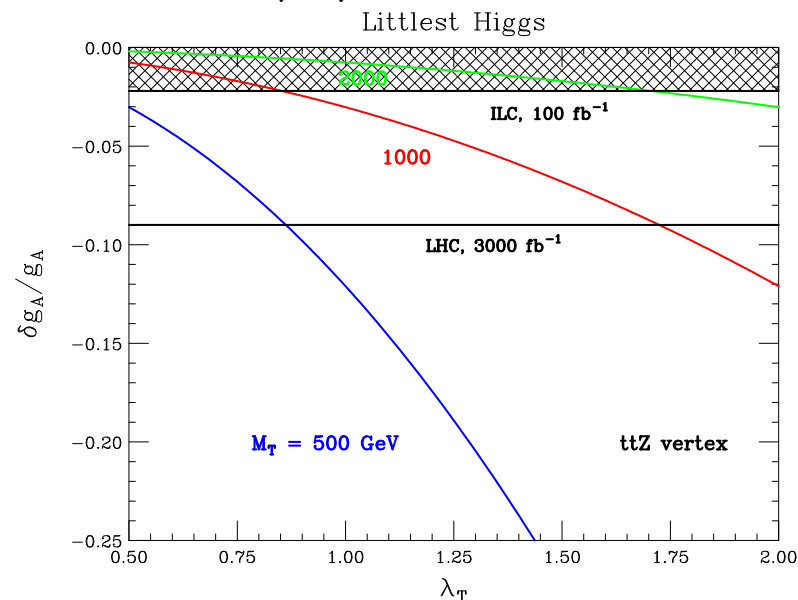
Little Higgs seeks to stabilize the Higgs vev by a collective symmetry breaking mechanism, e.g.  $[SU(2)_L \times U(1)_Y]^2 \rightarrow SU(2)_L \times U(1)_Y$ , which typically leads to several new states.

One important state is a vector-like top-quark partner  $T$ .

When T-parity is imposed,  $T$ 's are pair produced, and too heavy for ILC, BUT indirect evidence can appear in modifications of the  $Zt\bar{t}$  vertex.

At  $\sqrt{s} = 500$  GeV, w/  $500 \text{ fb}^{-1}$ ,  $T$ 's less than 1 TeV are probed!

Without T-parity, a pseudo-axion might be light enough to observe in  $e^+e^- \rightarrow t\bar{t}\eta$ ,  $\eta \rightarrow b\bar{b}$ .



Kilian, Rainwater, Reuter (06)

Berger, Perelstein, Petriello (05)



# Where do we go from here?

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## 1. Top quark properties: mass

- NNNLO calculations are mostly complete — cross some  $t$ 's
- There may be many small  $O(10 \text{ MeV})$  effects that still need to be calculated, but that may not be pressing
- The exciting prospect is to use the ILC as a foil to understand these new effective field theory combinations (SCET+HQEFT+PT+?).

## 2. Top quark as a probe of new physics

- The physics case was made a while ago  
Look for deviations in  $g_{tth}$ , anomalous couplings, new particles
- Some new ideas trickle in, e.g., unparticles
- We are really waiting for LHC data

There is an exciting program ahead in top-quark physics, and the ILC will give new meaning to precision QCD.