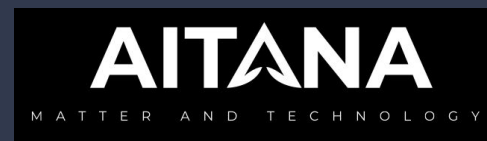
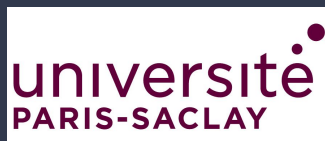


ILC-Japan Physics 3rd Working GM

$e^+e^- \rightarrow ss$ Study at 250 GeV

Yuichi Okugawa
Apr 25th, 2023



Introduction

Di-fermion Production

- Di-fermion production

- $e^+e^- \rightarrow s\bar{s}$
- CME 250 GeV.
- eL pR
- Int. Lumi. 4300 fb⁻¹

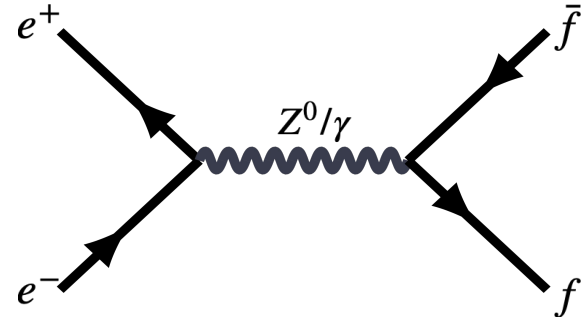
- Differential Cross Section

- Couplings can be extracted from helicity amplitudes included within the Differential Cross section

$$\frac{d\sigma}{d\cos\theta} = S(1 + \cos^2\theta) + A\cos\theta$$

- Extracted via forward-backward asymmetry. (AFB)

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B}$$



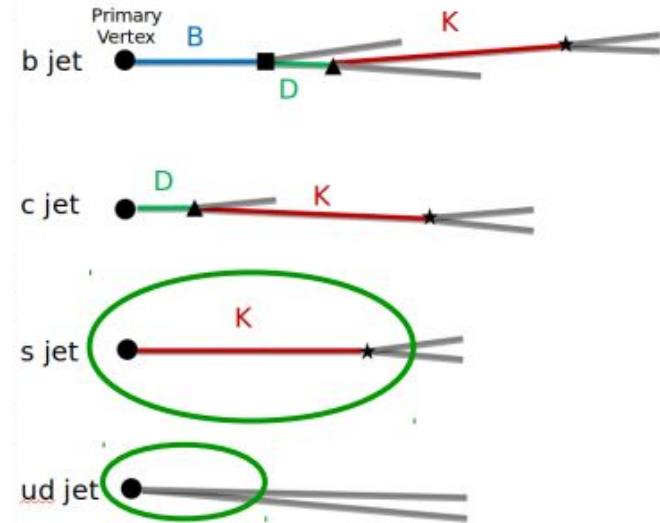
Energy	Process	Goal of measurements
91 GeV	$e^+e^- \rightarrow Z^0$	Z^0 physics and calibration
250 GeV	$e^+e^- \rightarrow Z^0H$	Higgs couplings
	$e^+e^- \rightarrow f\bar{f}$	Z^0/γ couplings
350 GeV	$e^+e^- \rightarrow t\bar{t}$	top mass precision
	$e^+e^- \rightarrow \nu\bar{\nu}H$	Higgs couplings
500 GeV	$e^+e^- \rightarrow t\bar{t}$	top couplings
	$e^+e^- \rightarrow t\bar{t}H$	Higgs-top coupling
	$e^+e^- \rightarrow Z^0HH$	Higgs self coupling
1000 GeV	$e^+e^- \rightarrow \nu\bar{\nu}HH$	Higgs self coupling

Analysis

Kinematics

- **QQbar Production**

- Background removal in $SS\bar{b}$ analysis plays a key role in precise cross section measurement.
- Light quark pair production events (uu, dd), in particular, can disguise as $SS\bar{b}$ event, which will be the source of migration and mis-measurements.
- Each process produces jets with unique characteristics:
 - **bb**: b-jets with high b-tag, based on various jet and vertex parameters. Mostly from SV.
 - **cc**: c-jets with high c-tag.
 - **ss**: Jets with kaons which carry predominant energy and momentum.
 - **uu,dd**: Jet with mixture of pions and kaons.



Taken from Slide 5 of Tomohiko Tanabe's 2020/11/24 presentation.

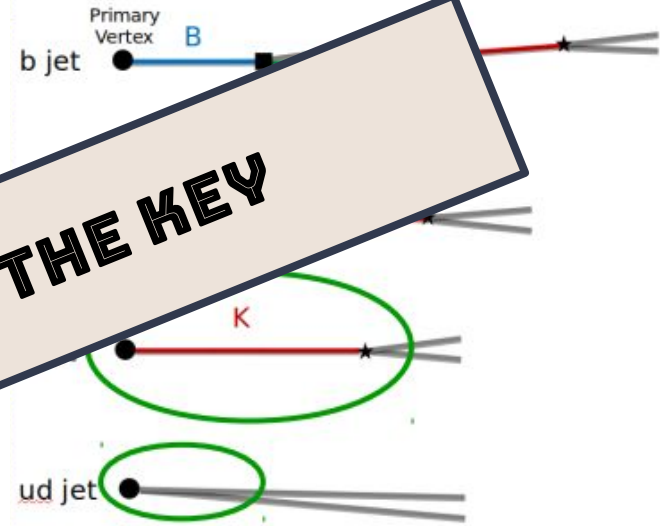
[M.Basso 2021](#)

Kinematics

- **QQbar Production**

- Background removal in $SS\bar{b}$ analysis plays a key role in precise cross section measurement.
- Light quark pair production events (uu, dd), in particular, can disguise as $SS\bar{b}$ event, which will be the source of migration and mis-measurements.
- Each process produces jets with unique kinematics
 - **bb**: b-jets with high b -tagging vertex parameters.
 - **cc**: charm jets with high c -tagging vertex parameters.
 - **ss**: strange jets with high s -tagging vertex parameters and predominant energy.
 - **uu, dd**: light quark jets with mixture of pions and kaons.

PRECISE PID IS THE KEY



Taken from Slide 5 of Tomohiko Tanabe's 2020/11/24 presentation.

[M.Basso 2021](#)

dE/dx Minimum

LPFO Selection

Charge Check

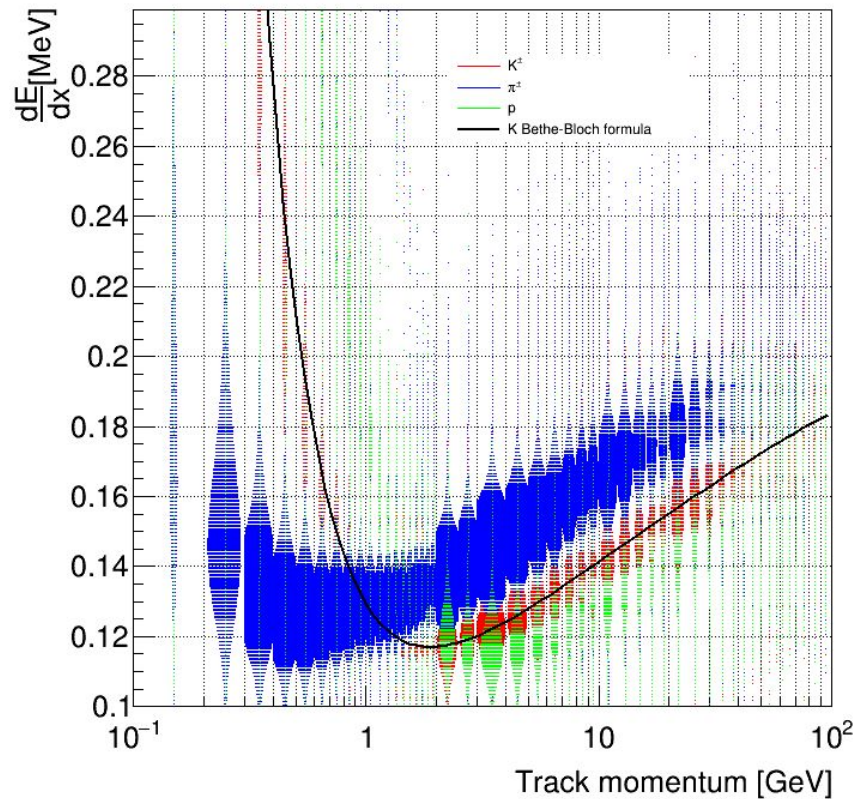
Momentum Check

TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check



dE/dx Minimum

LPFO Selection

Charge Check

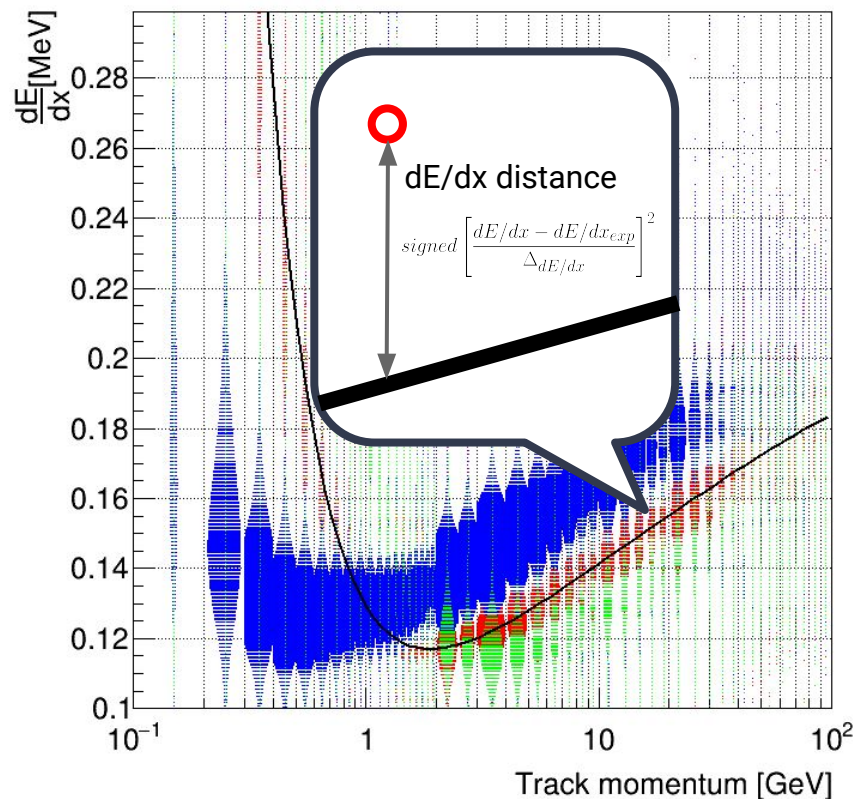
Momentum Check

TPC Hit Check

IP Check

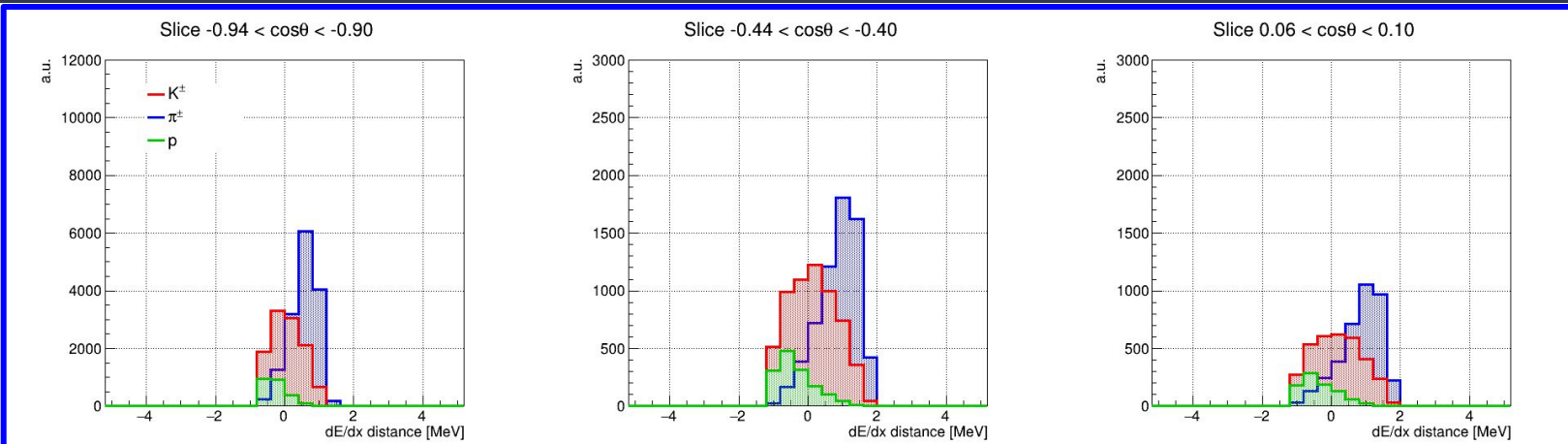
dE/dx Minimum Check

SPFO Check

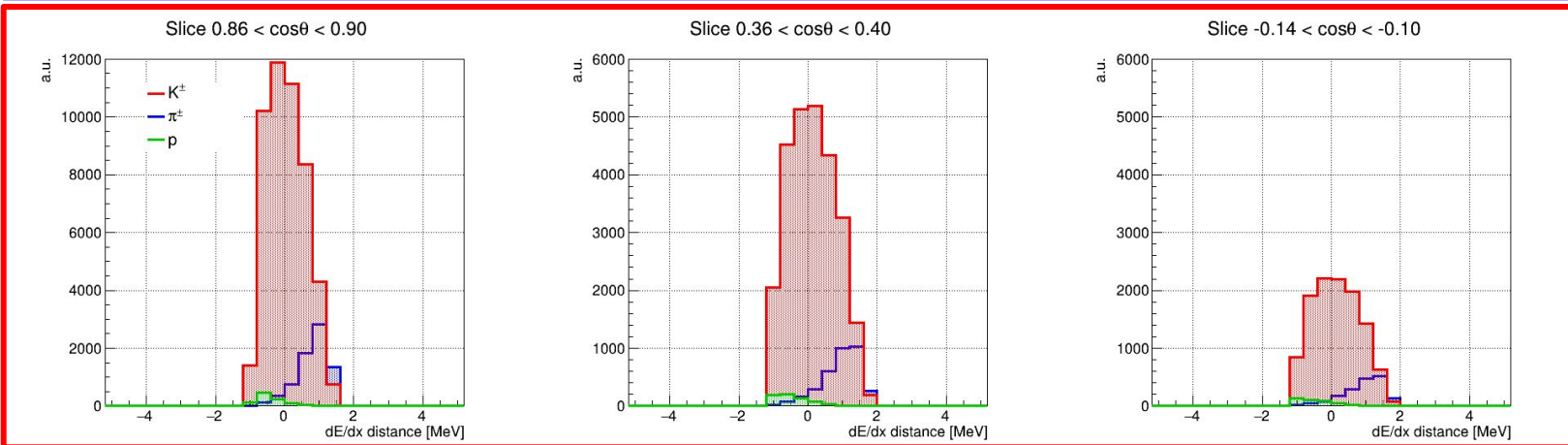


dE/dx distances (Kaon)

UU

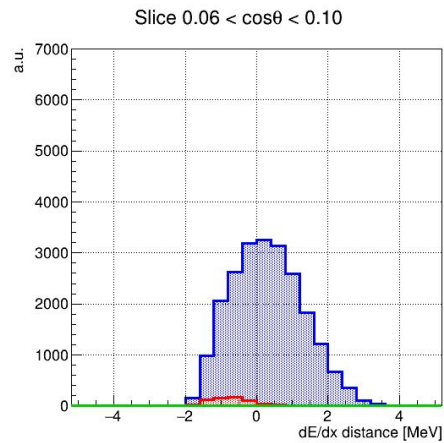
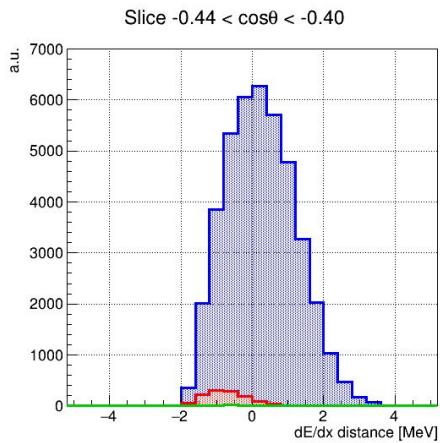
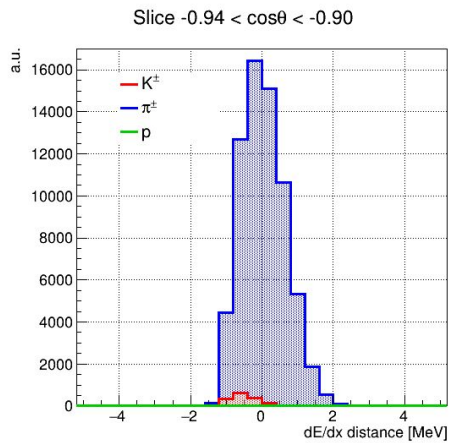


SS

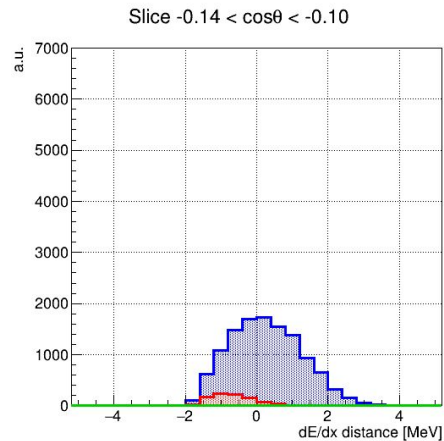
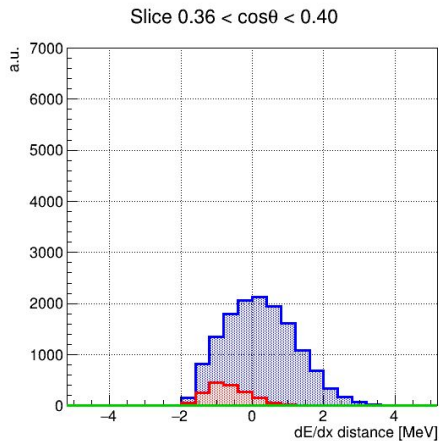
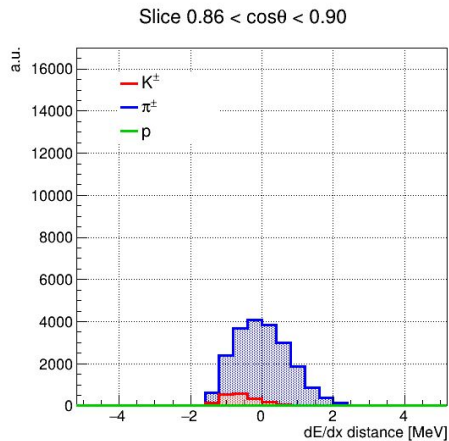


dE/dx distances (Pion)

UU



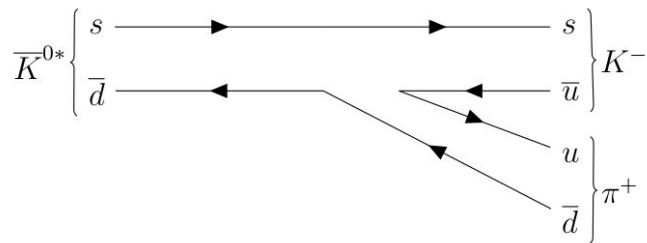
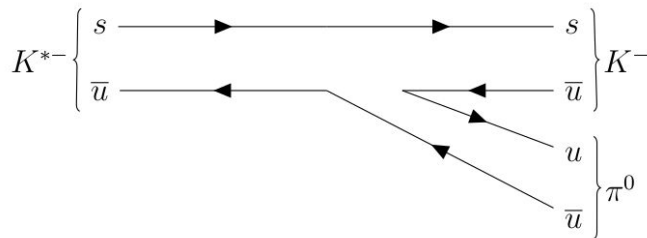
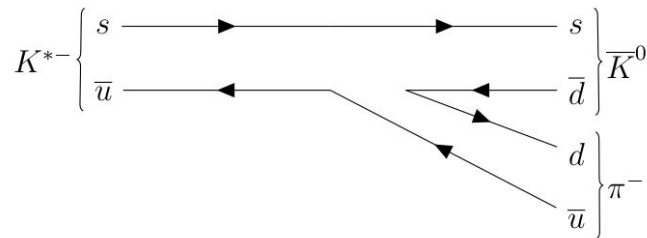
SS



Kinematics

- **Migration**

- Migration occurs when wrongly reconstructing the particle and thus identify the wrong charge, consequently measuring the opposite polar angle.
- Such migration can happen in 2 occasions:
 - Particle mis-ID
dE/dx distance method can mis-identify a PFO as another particle (e.g. K -> Pi)
 - Mis-selection
It is possible that the PFO which contains the original s-quark doesn't have the highest momentum among all other PFOs. This is often the case when there is a gluon radiation.



Kinematics

- **PFO Determination**

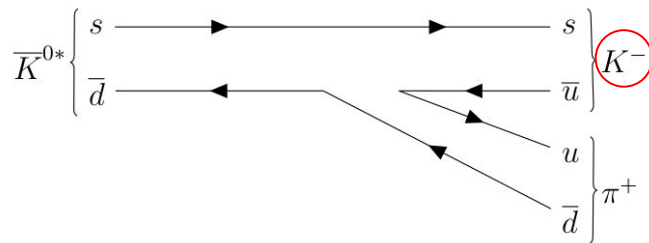
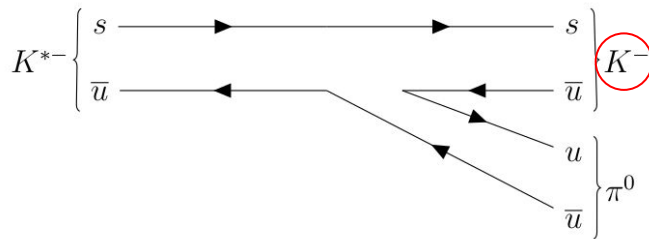
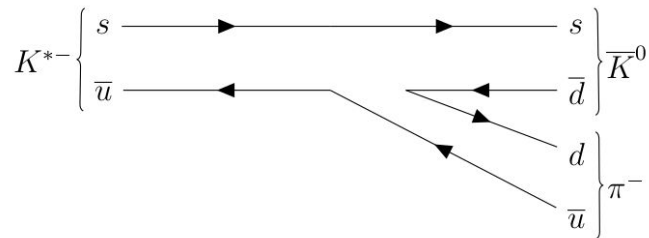
- Kaon identification

Most of energy and momentum for $s\bar{s}$ process is passed onto Kaons. These Kaons bear most of energy within a jet, thus become a leading PFO. Mis-identification of kaons with pions, will not affect the polar angle since they'll have the same charge.

- Pion identification

$S\bar{s}$ process often contains a pion with leading momentum. This is usually the result of K^* decay.

Pions can also be a signal for $s\bar{s}$, yet contamination from light quark production process is huge. Mis-identification of pions with kaons will be the source of migration since K^*0 will contain oppositely charged kaon.



Kinematics

- **PFO Determination**

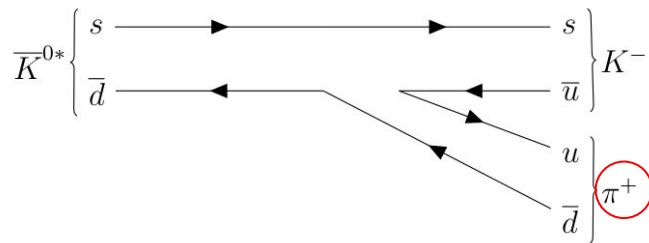
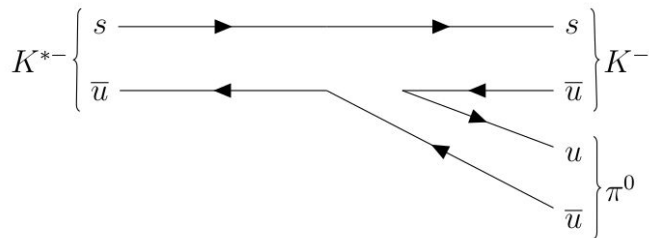
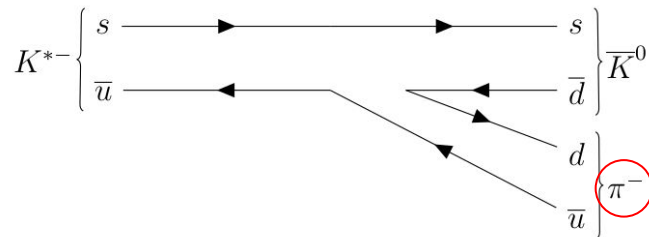
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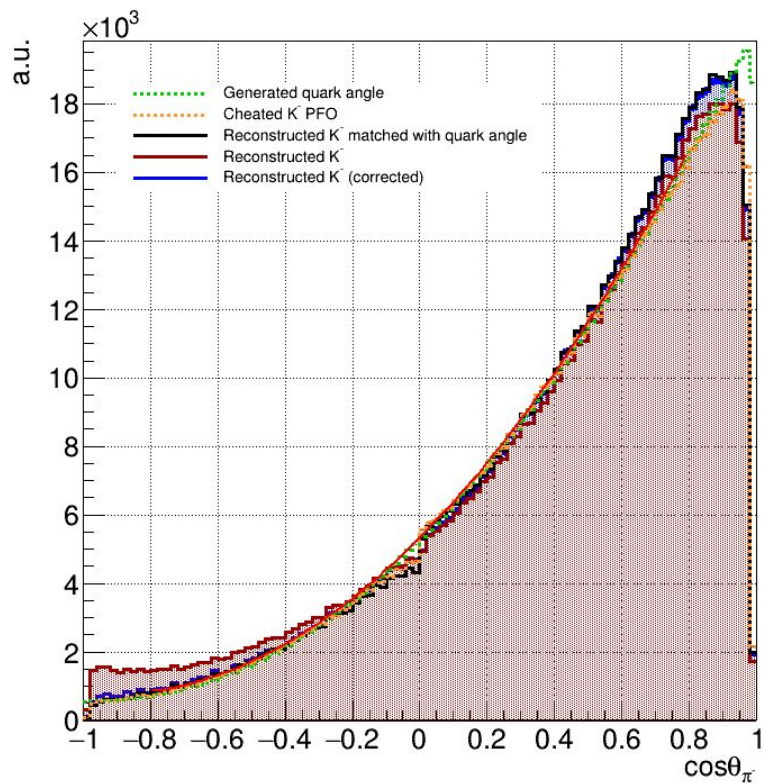
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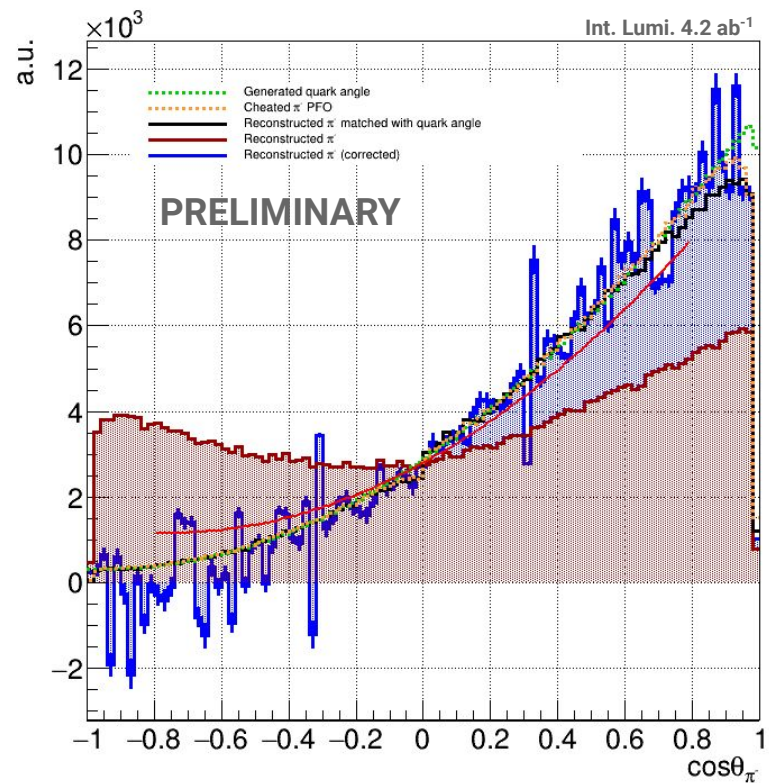


Polar Angles

ss Polar Angle

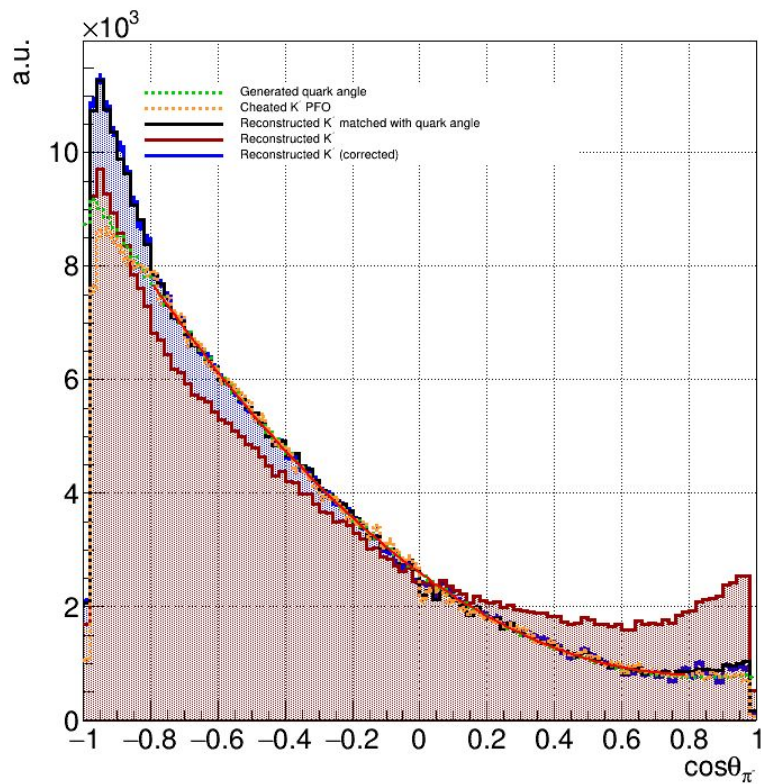


KK

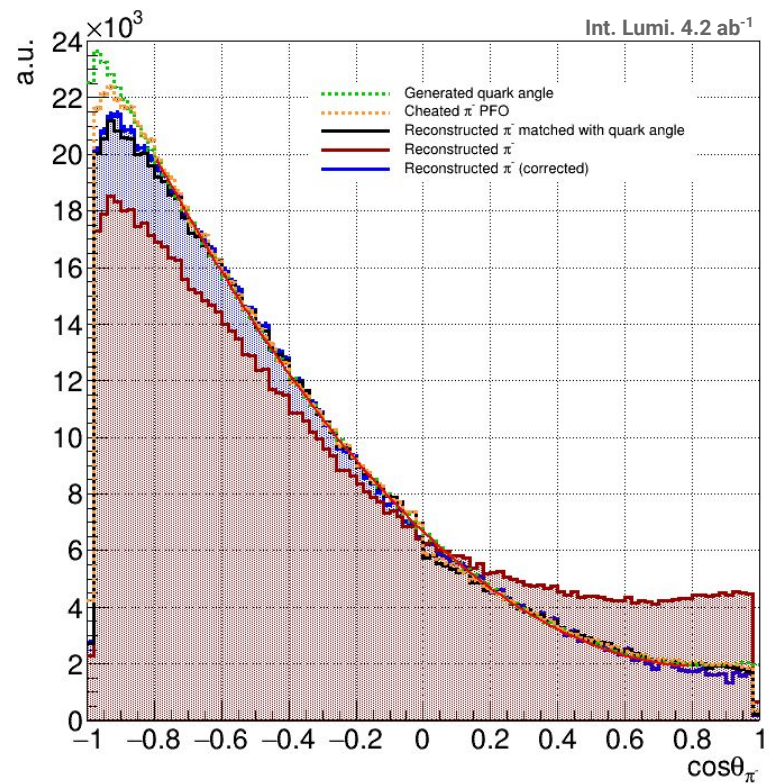


PiPi

uu Polar Angle

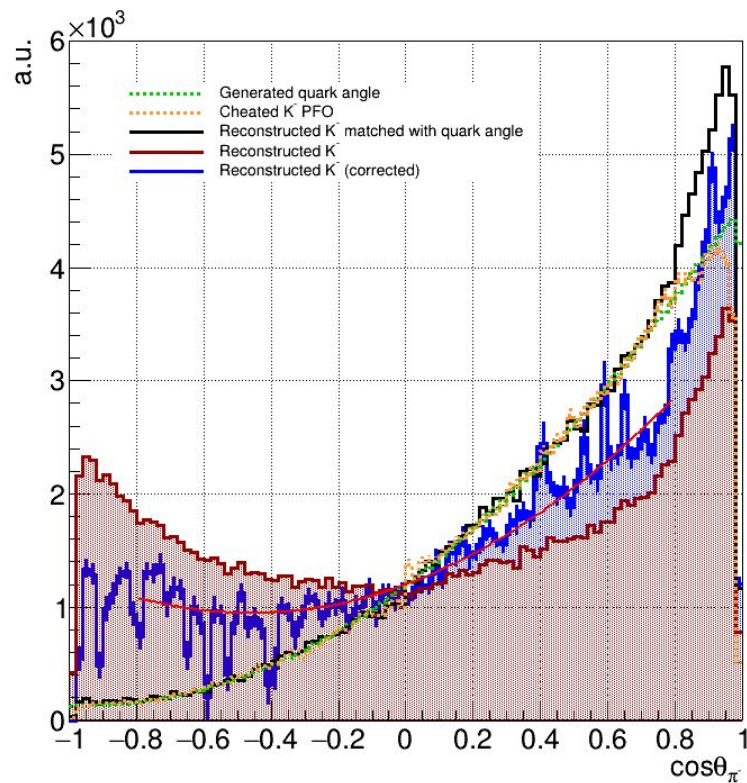


KK

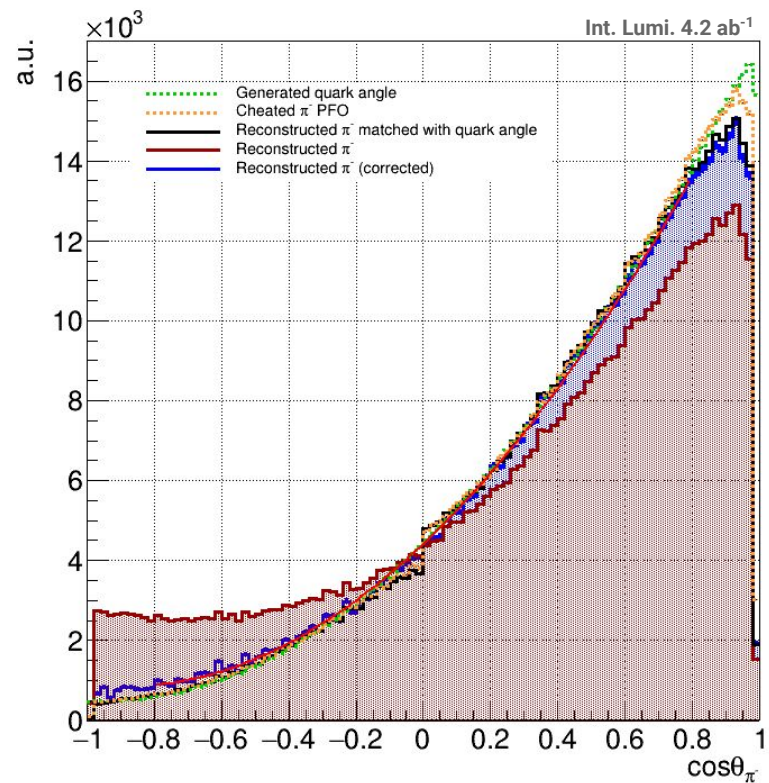


PiPi

dd Polar Angle



KK



PiPi

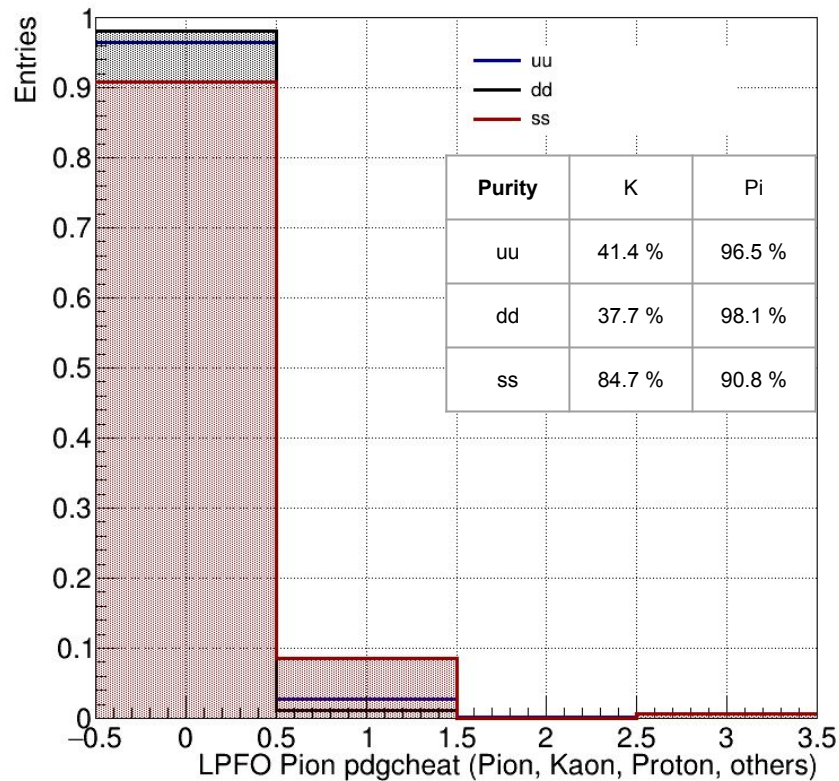
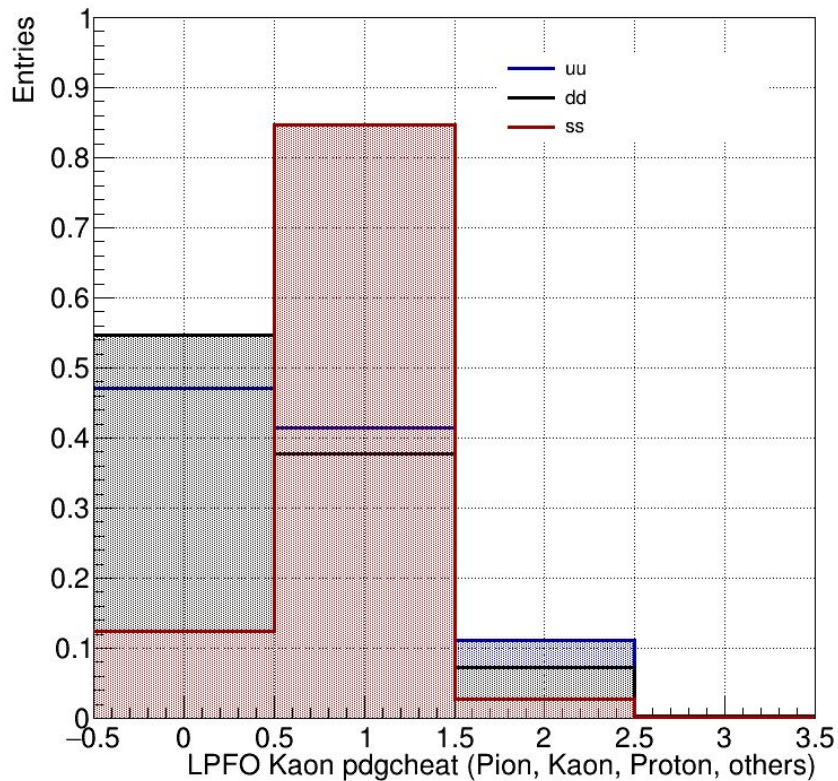
- **AFB fits**

- Individual fits were applied for uu, dd, ss samples.
- Fit parameters were determined as following:

$$\frac{d\sigma}{d\cos\theta} = S(1 + \cos^2\theta) + A\cos\theta$$

	mode	S	dS	A	dA
uu	PiPi	6.66878e+03	8.66063e+00	-1.12910e+04	2.32748e+01
dd	PiPi	4.42012e+03	7.06985e+00	7.96591e+03	1.87377e+01
ss	KK	5.34001e+03	6.68662e+00	9.89372e+03	1.68159e+01

K/Pi ID purity after LPFO reconstruction



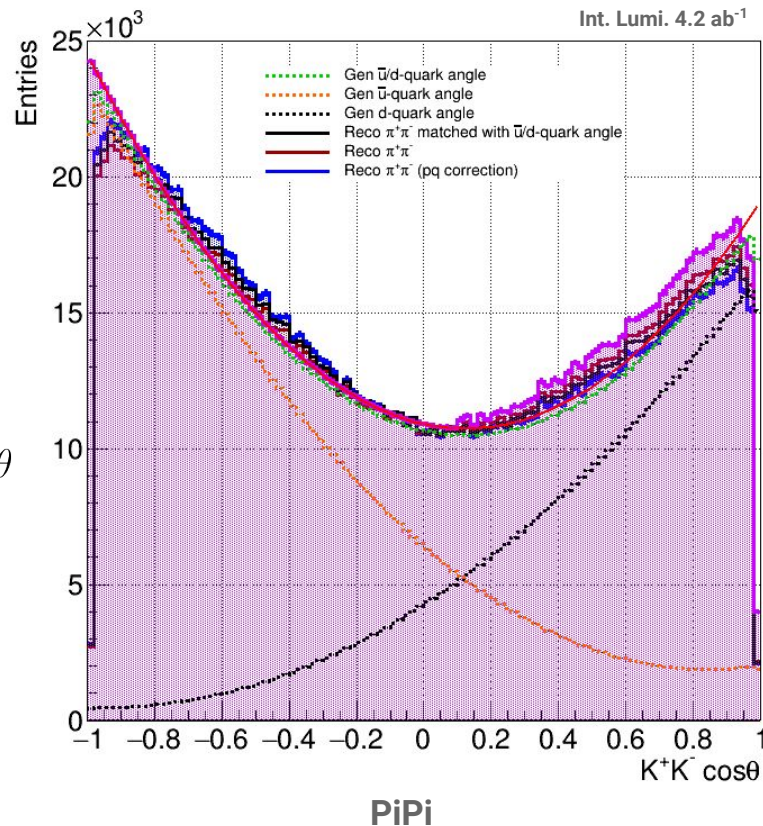
Mixture Sample

ud Polar Angle

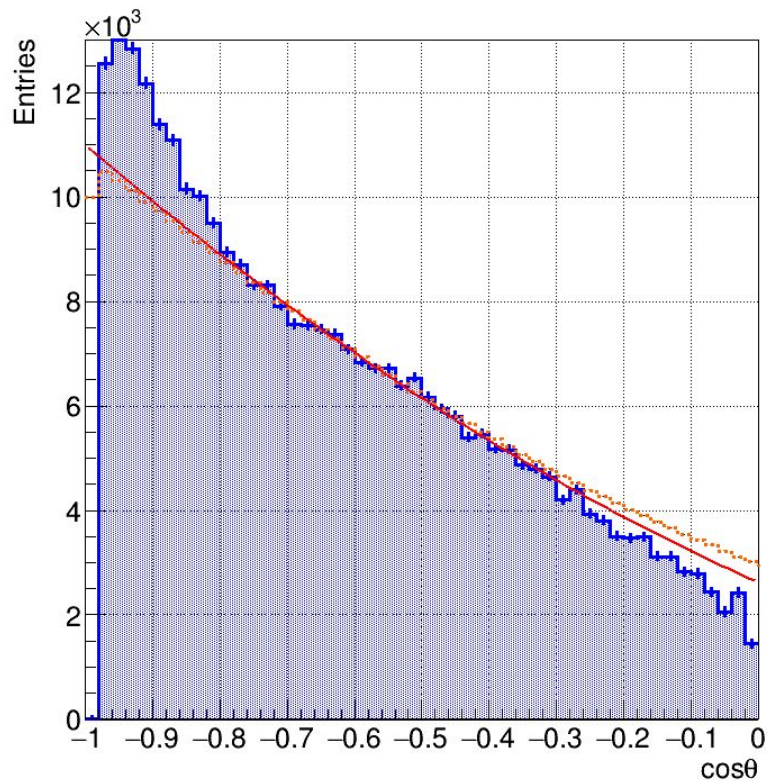
- **Background Estimation**

- uu and dd processes were mix together to see if the algorithm is able to evaluate the backgrounds.
- uu and dd are essentially inseparable, meaning that these polar angle distributions will always be treated together.
- One can obtain the uu and dd contribution from the Gen information to normalize the final fits.

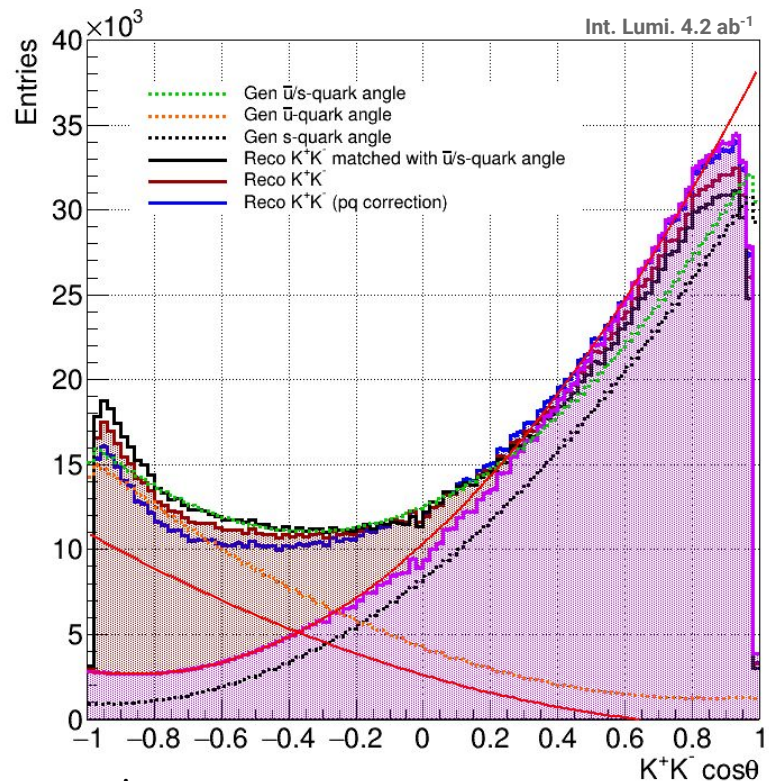
$$\frac{d\sigma}{d \cos \theta} = (S_{ss} + S_{uu,dd})(1 + \cos^2 \theta) + (A_{ss} + A_{uu,dd}) \cos \theta$$



uu,ss Polar Angle (w/o Efficiency corr.)



uu extract



us mixture

Summary

Summary & Prospects

Summary

- $S\bar{S}$ event selection
 - Kaon identification
 - Pion identification
- Fit
 - Individual fits
 - Mix sample fits
- BG mixture
 - ud mixture
 - us mixture

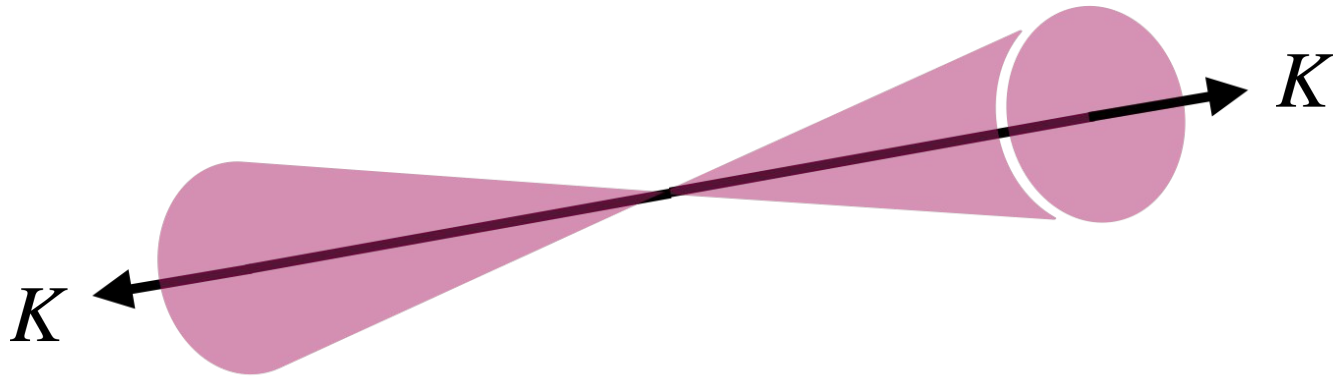
Prospects

- Mix and fit three processes ($uu/dd/ss$)
- Addition of $C\bar{C}$ background analysis
 - Ongoing with help of intern student
- Performance check for LPFO identification using:
 - Leading Kaon
 - Leading Pion

Backup Slides

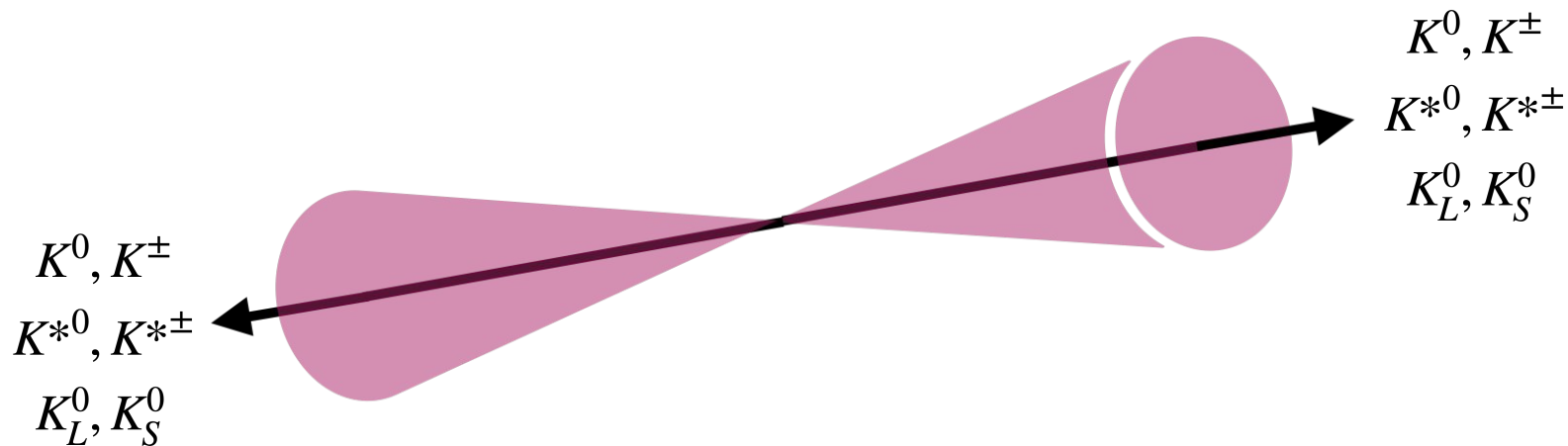
SSbar Process

$$e^+e^- \rightarrow s\bar{s}$$



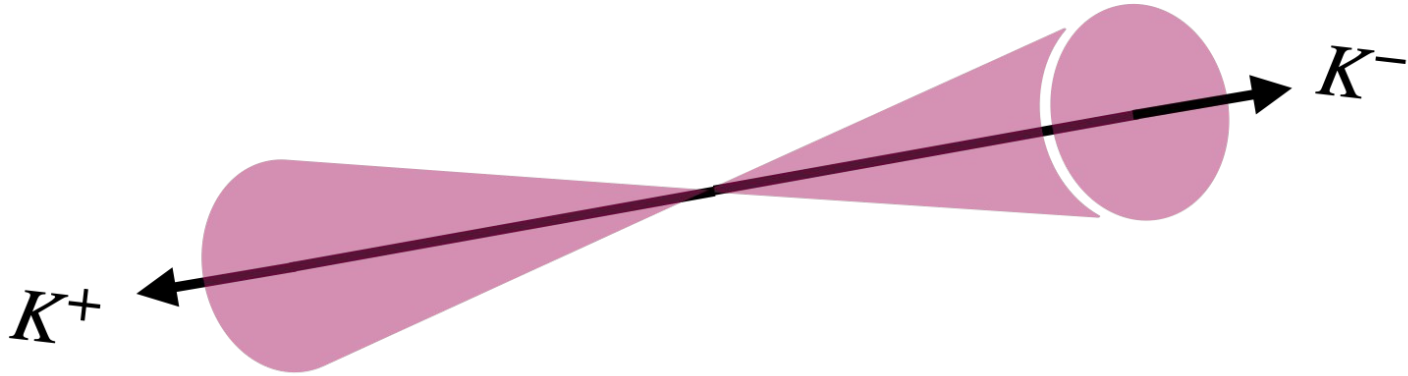
SSbar Process

$$e^+e^- \rightarrow s\bar{s}$$

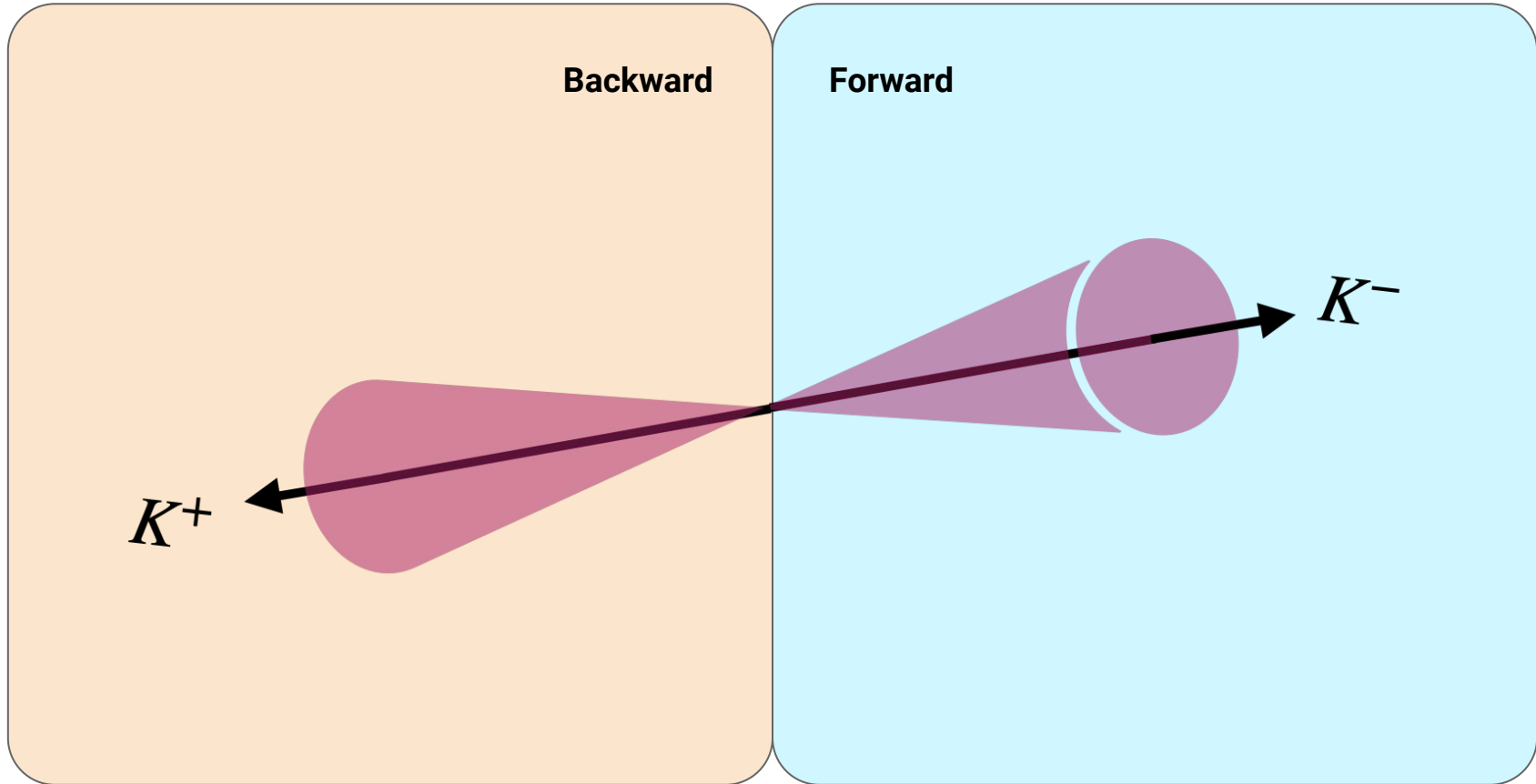


SSbar Process

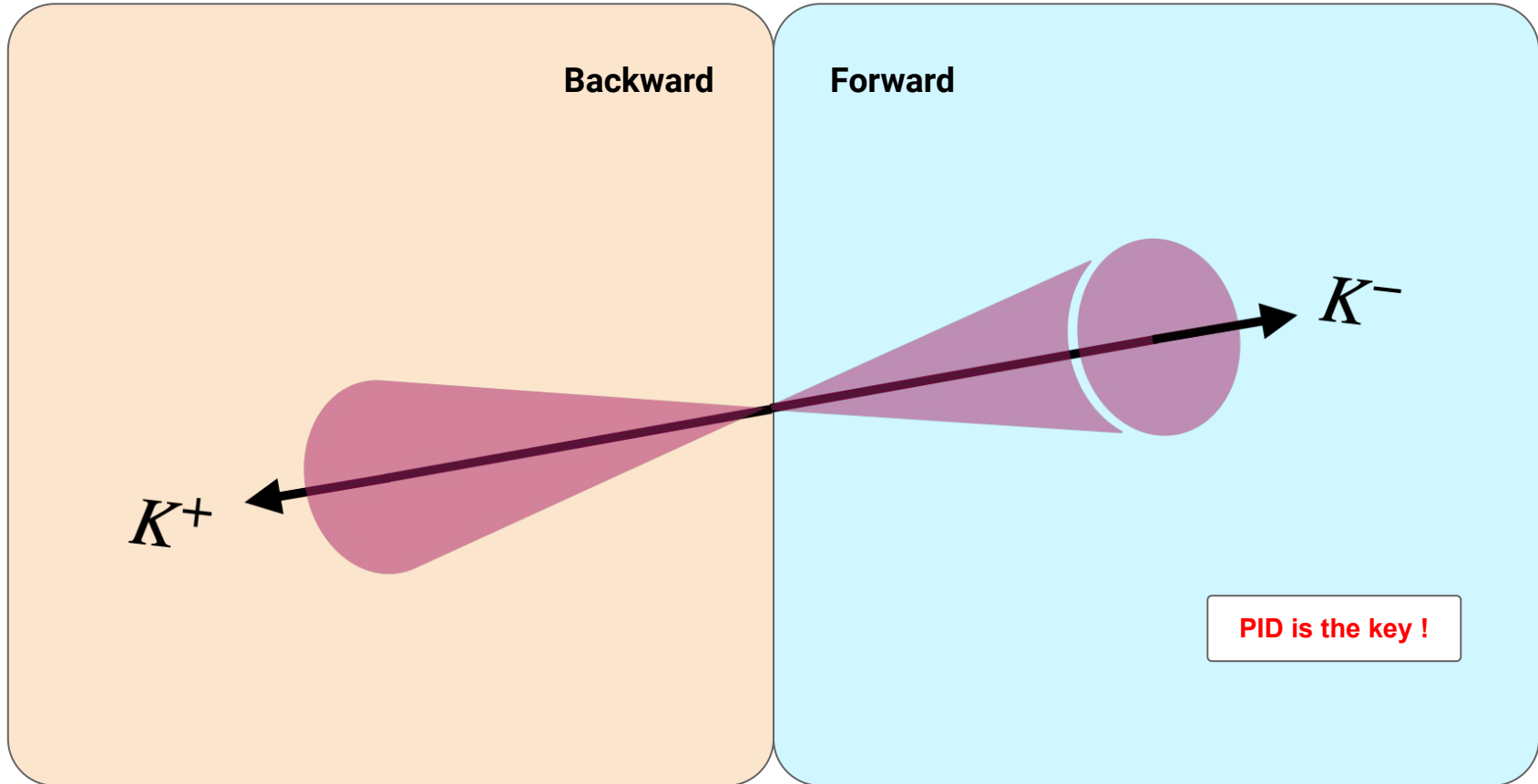
$$e^+e^- \rightarrow s\bar{s}$$



SSbar Process



SSbar Process



Event Selection



Reconstruction Steps

LPFO Selection

Charge Check

Momentum Check

TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check

Gen Signal Selection

- SSbar back-to-back

$$0.95 < \cos \theta_{S\bar{S}}$$

- Total Energy

$$120 < E_{S,\bar{S}} < 127 \text{ GeV}$$

Leading PFO

LPFO Selection

Charge Check

Momentum Check

TPC Hit Check

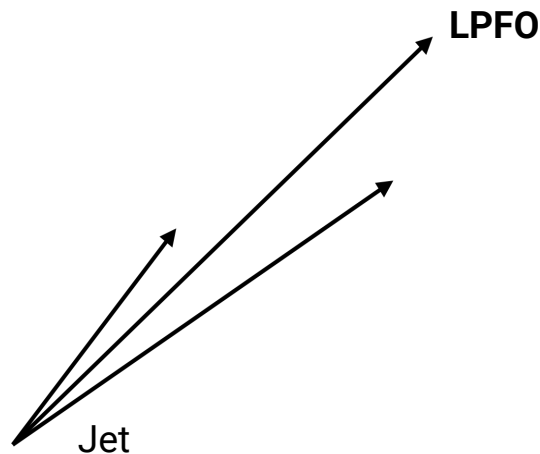
IP Check

dE/dx Minimum Check

SPFO Check

Leading PFO (LPFO)

- Particle with *highest* momentum within a Jet.
- $S\bar{S}$ typically disintegrate into a pair of energetic kaons.
- We choose LPFO among **charged PFOs** inside a jet.



Charge & Momentum

LPFO Selection

Charge Check

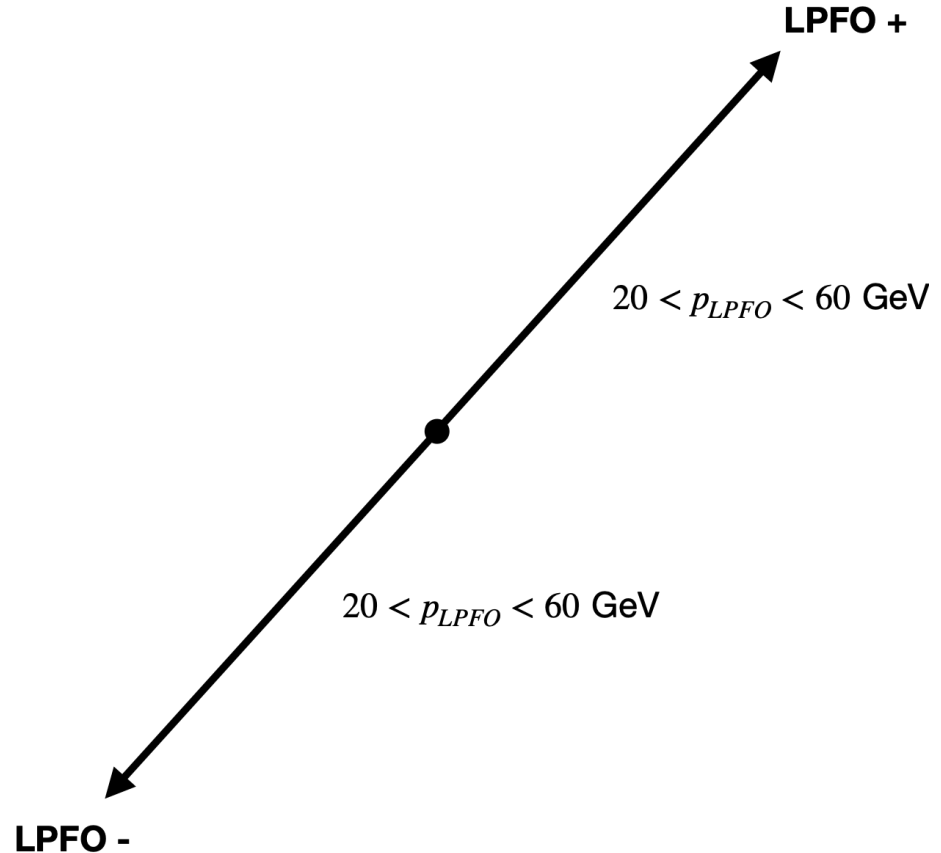
Momentum Check

TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check



TPC Hits

LPFO Selection

Charge Check

Momentum Check

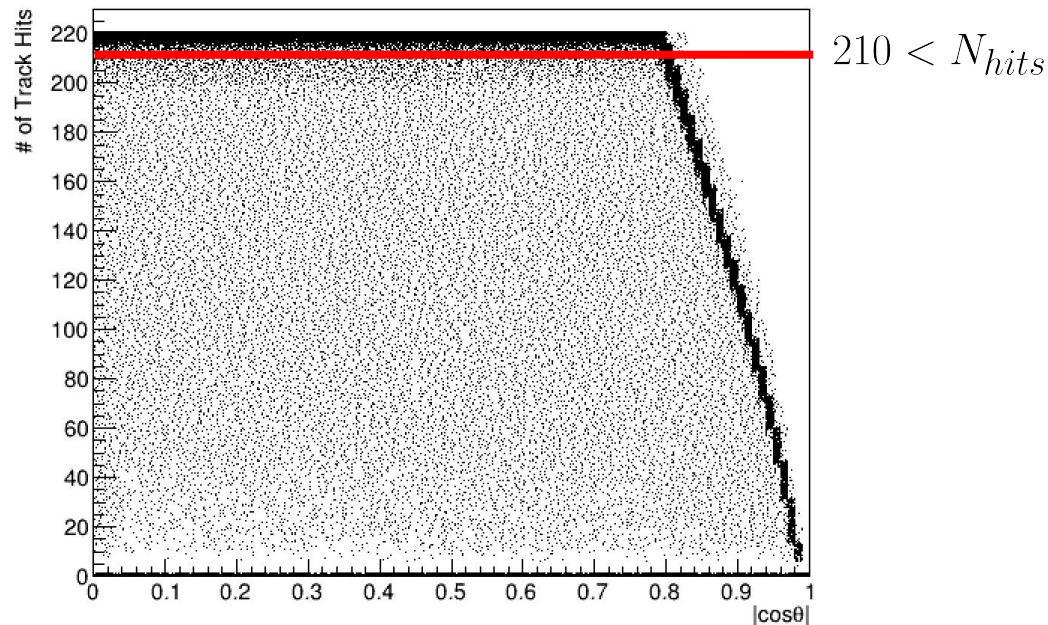
TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check

$$\sigma_{dE/dx} \propto \frac{1}{\sqrt{N_{hits}}}$$



Impact Parameter

LPFO Selection

Charge Check

Momentum Check

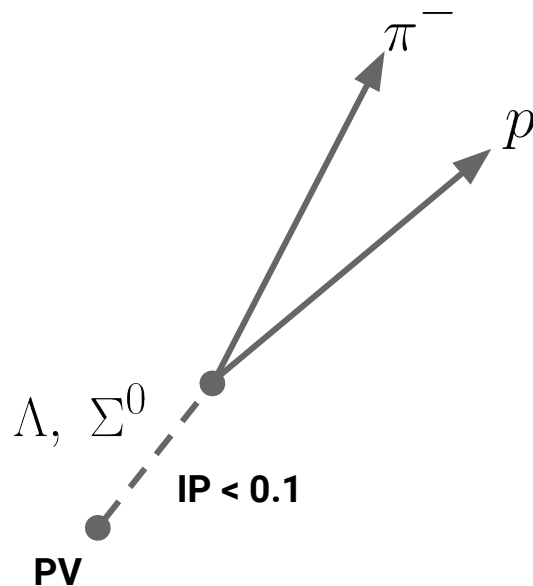
TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check

Hyperon Suppression



dE/dx Minimum

LPFO Selection

Charge Check

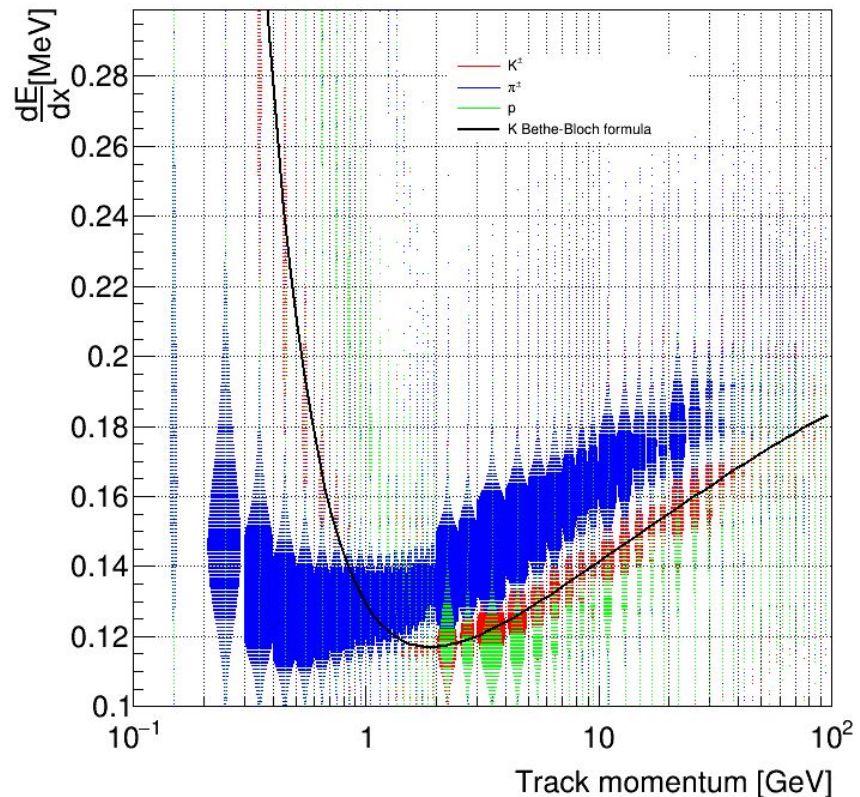
Momentum Check

TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check



dE/dx Minimum

LPFO Selection

Charge Check

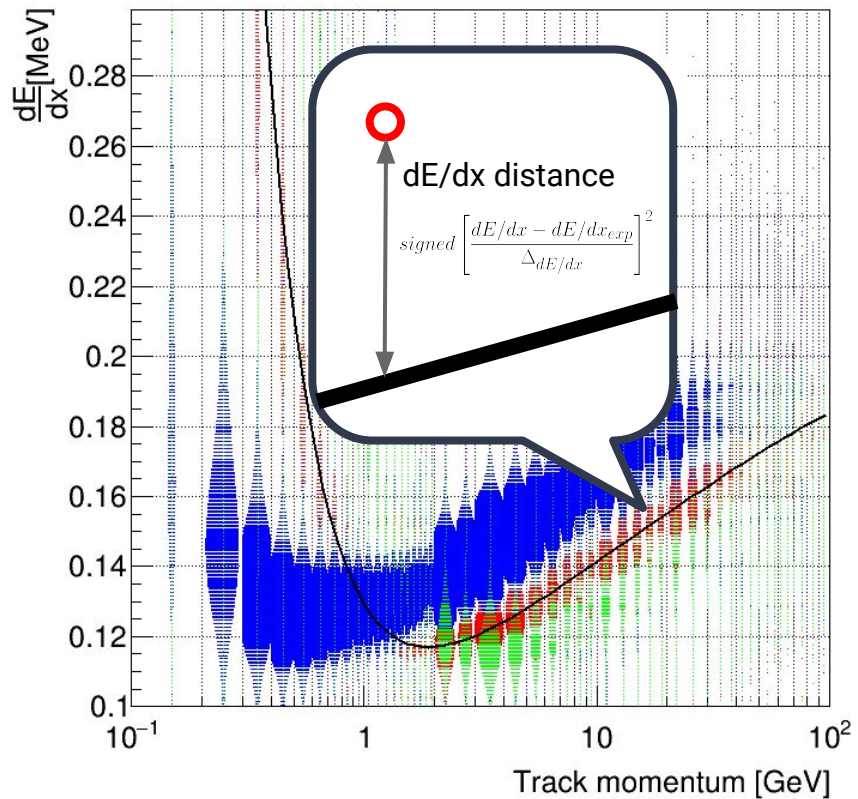
Momentum Check

TPC Hit Check

IP Check

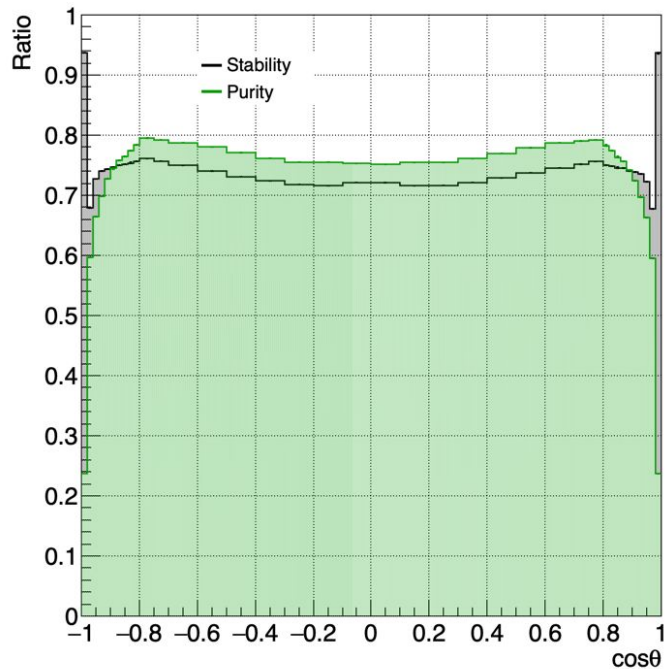
dE/dx Minimum Check

SPFO Check

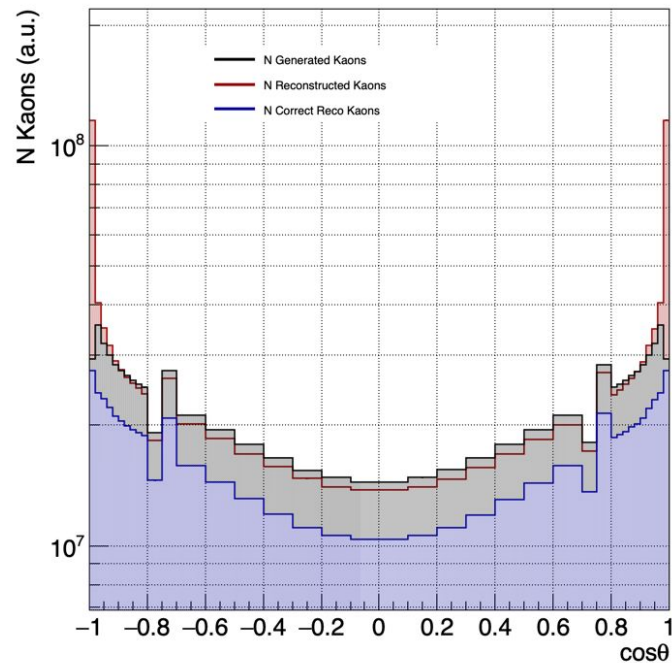


dE/dx Minimum

Using PFO cheat | $p > 5$ GeV



SPFO Check



*Definitions of stability and purity are described [here](#)

dE/dx Minimum

LPFO Selection

Charge Check

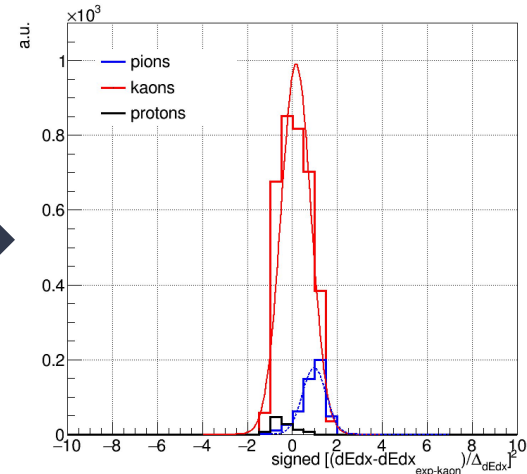
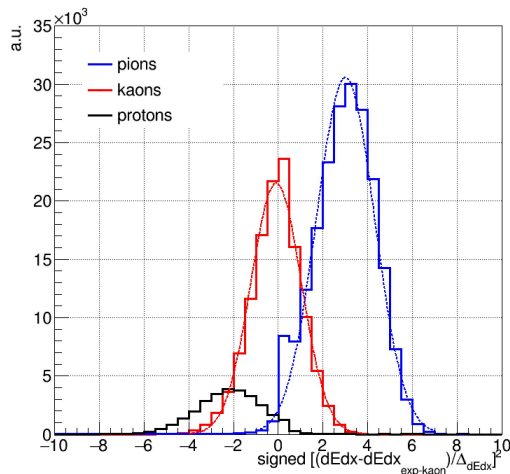
Momentum Check

TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check



dE/dx distance minimization

- k dE/dx distance $<$ π dE/dx distance
- k dE/dx distance $<$ p dE/dx distance

SPFO Check

LPFO Selection

Charge Check

Momentum Check

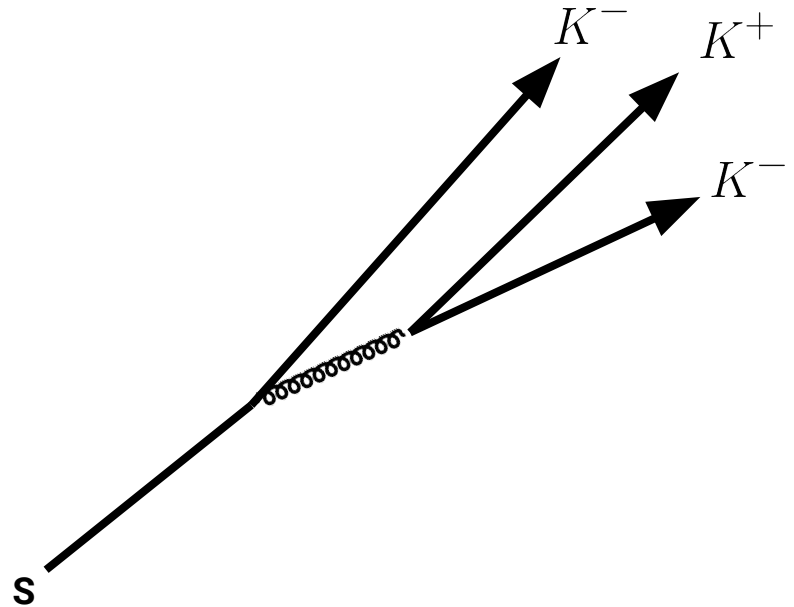
TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check

Secondary PFO (SPFO) Check



SPFO Check

LPFO Selection

Charge Check

Momentum Check

TPC Hit Check

IP Check

dE/dx Minimum Check

SPFO Check

Secondary PFO (SPFO) Check

- Find SPFO such that:
 - Charged Kaon
 - Charge must be opposite to LPFO Kaon
(same sign does not create confusion)
 - Must have least 10 GeV momentum
- If there is such SPFO -> veto

Polar Angle Result

Polar Angle Result

Fit function:

$$\frac{d\sigma}{d\cos\theta} = S(1 + \cos^2\theta) + A\cos\theta$$

Gen:

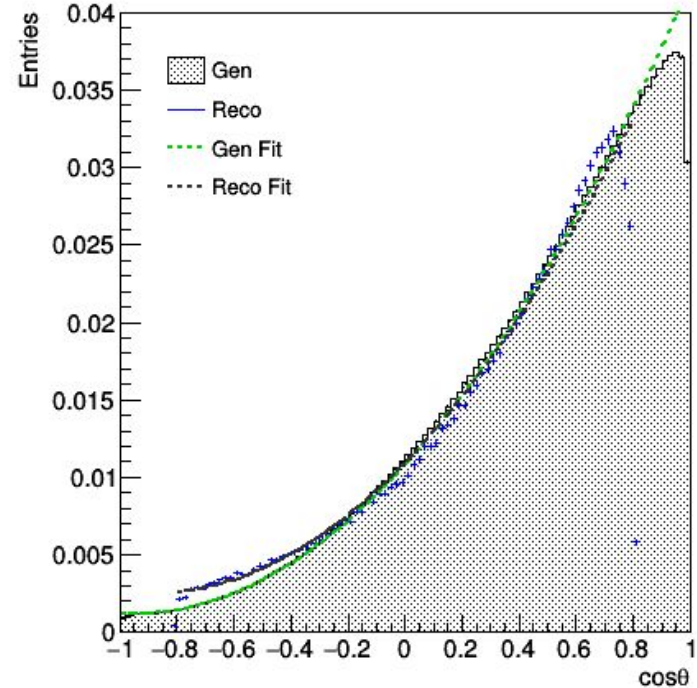
$$S = 1.07\text{E-}2 \pm 1.7\text{E-}6$$

$$A = 2.01\text{E-}2 \pm 3.7\text{E-}6$$

Reco:

$$S = 1.08\text{E-}2 \pm 1.9\text{E-}5$$

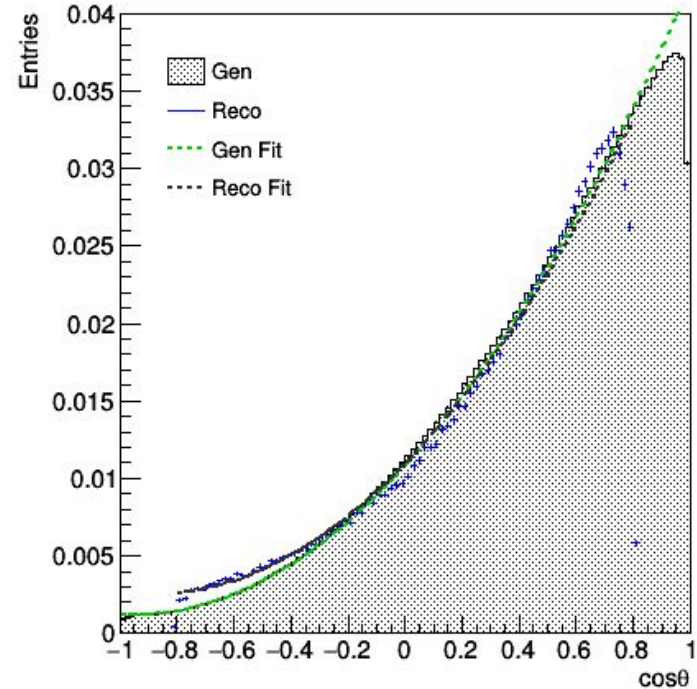
$$A = 1.90\text{E-}2 \pm 4.7\text{E-}5$$



Polar Angle Result

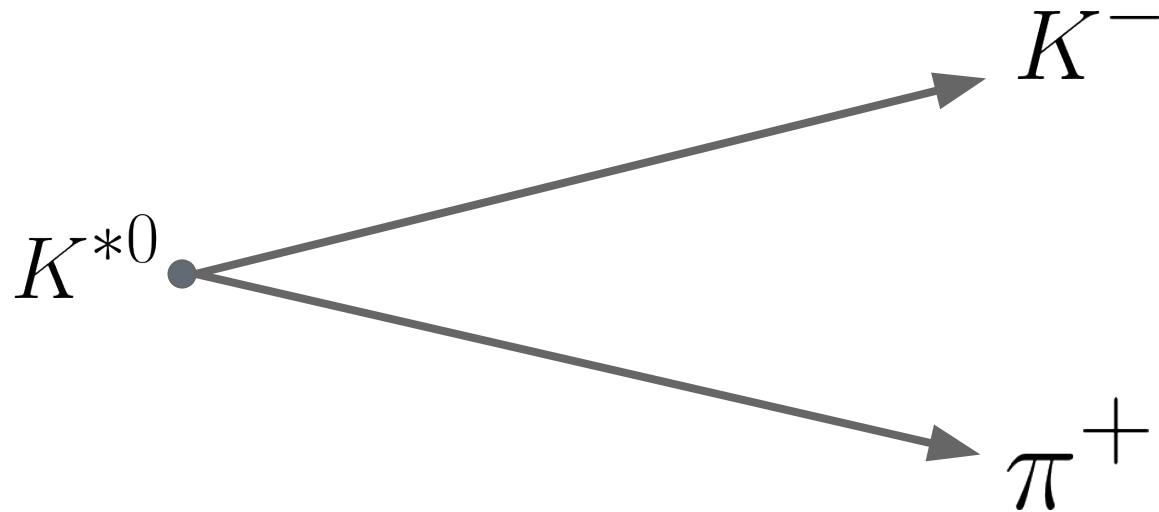
Number of Events and Efficiencies

- All data $ee \rightarrow qq$ processed
 - **26M events** (ss: ISR removed)
 - **Luminosity 4.6 ab⁻¹**
- SSbar reconstruction
 - Total valid events: 168k events
 - **Efficiency: 0.64%**

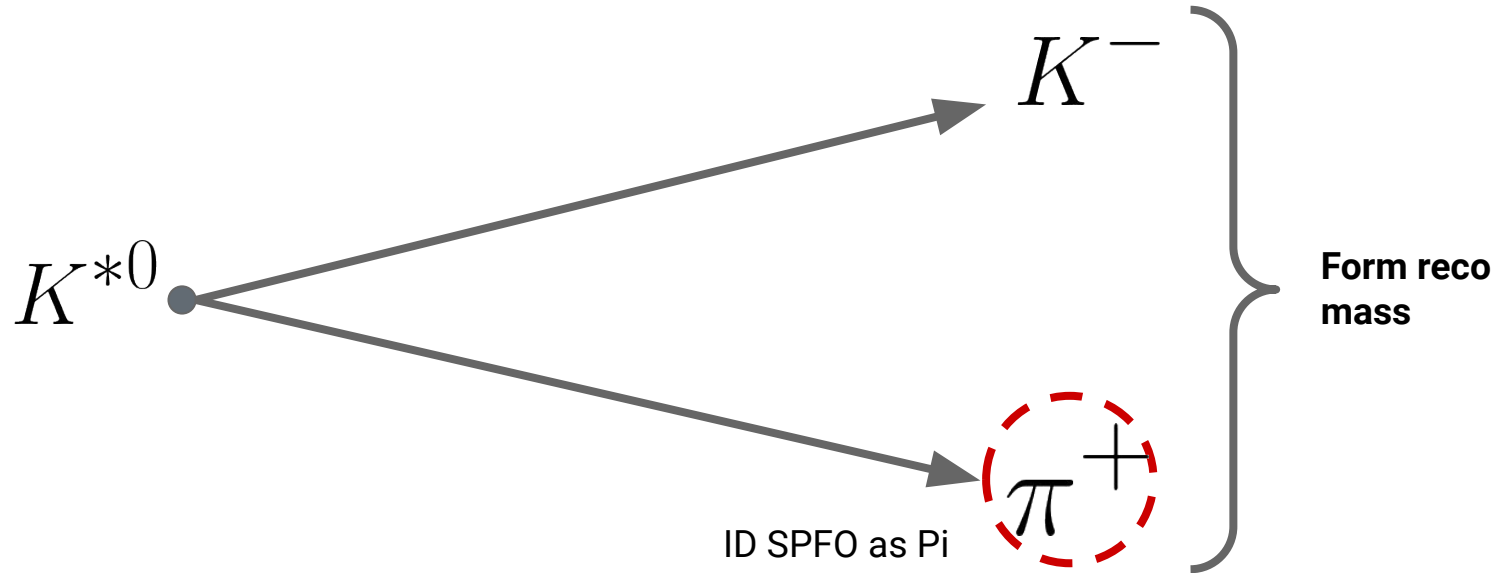


K^{*0}

K^{*0}



K^{*0}



Conditions for K*0 identification:

1. Identify Particles

- Pion as LPFO
- Kaon as SPFO



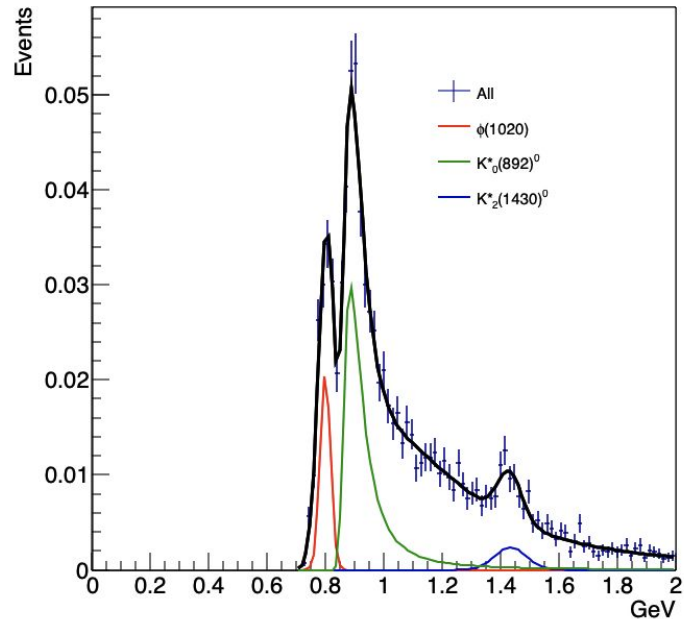
Case: if one of jets have K*0 and the other doesn't

- Check 2 LPFOs have same charges.
- Check charges of Pi-K and make sure they're opposite.
- Momentum cut on Kaon (> 10GeV) for kinematic constraints.
- Reconstruct invariant masses for all possible combinations of Pion and Kaons.

Invariant Mass

Invariant Mass Reconstruction

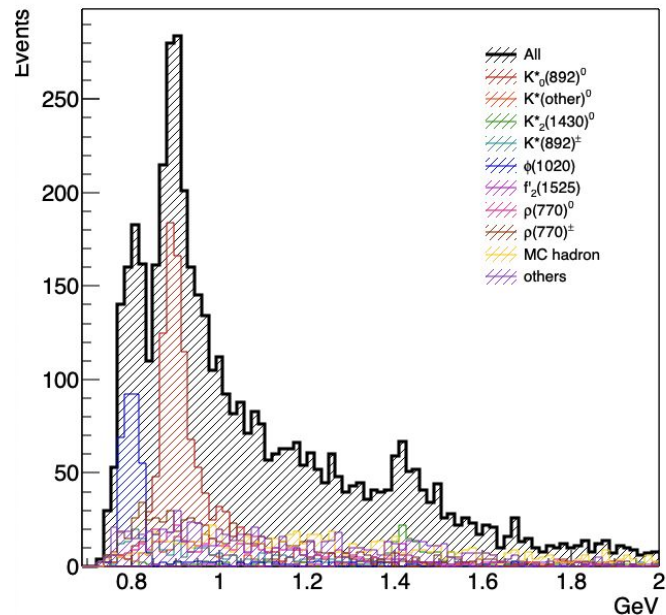
- Invariant mass plot on the right shows the combined mass of Leading Pion and Secondary Kaons.
- See clear peak at 0.8, 0.9 and 1.4 GeV
- MC parent information shows that those pions and kaons are coming from $\phi(1020)$, $K^*(892)$ and $K^*(1430)$
- Phi mass distribution might be coming from the misidentification of pions from dE/dx information?
 - Phi decay into charged kaon pair.
 - Checking the leading pion MC PID. If it is misidentification of kaon as pion, those mass should be changed to kaon mass to obtain correct reconstructed phi mass at 1020 MeV



Invariant Mass

Invariant Mass Reconstruction

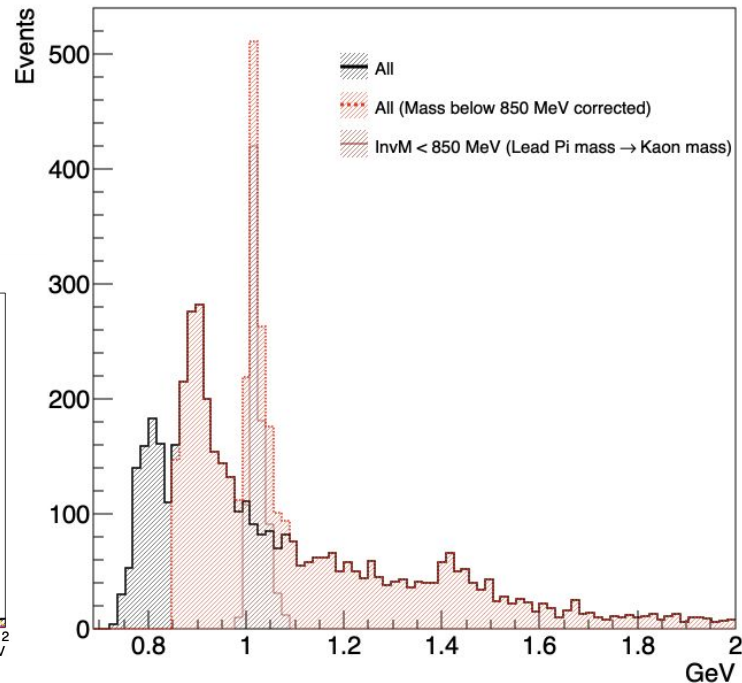
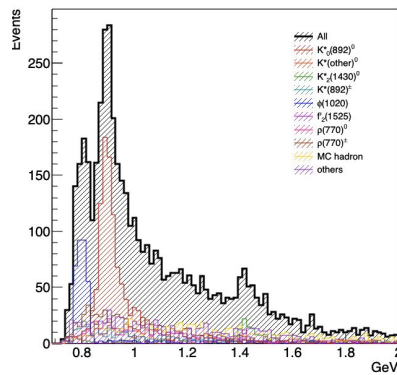
- Invariant mass plot on the right shows the combined mass of Leading Pion and Secondary Kaons.
- See clear peak at 0.8, 0.9 and 1.4 GeV
- MC parent information shows that those pions and kaons are coming from $\phi(1020)$, $K^*(892)$ and $K^*(1430)$
- Phi mass distribution might be coming from the misidentification of pions from dE/dx information?
 - Phi decay into charged kaon pair.
 - Checking the leading pion MC PID. If it is misidentification of kaon as pion, those mass should be changed to kaon mass to obtain correct reconstructed phi mass at 1020 MeV



Invariant Mass

Full Luminosity Simulation

- $\phi \rightarrow KK$
- Cutting invariant mass at 850 MeV
- Replace leading pion mass hypothesis by the kaon mass
- Extreme peak at 1020 MeV
Corresponds to ϕ mass.



Polar Angle Result for K^{*0}

Polar Angle Result (K^{*0})

Fit function:

$$\frac{d\sigma}{d\cos\theta} = S(1 + \cos^2\theta) + A\cos\theta$$

Gen:

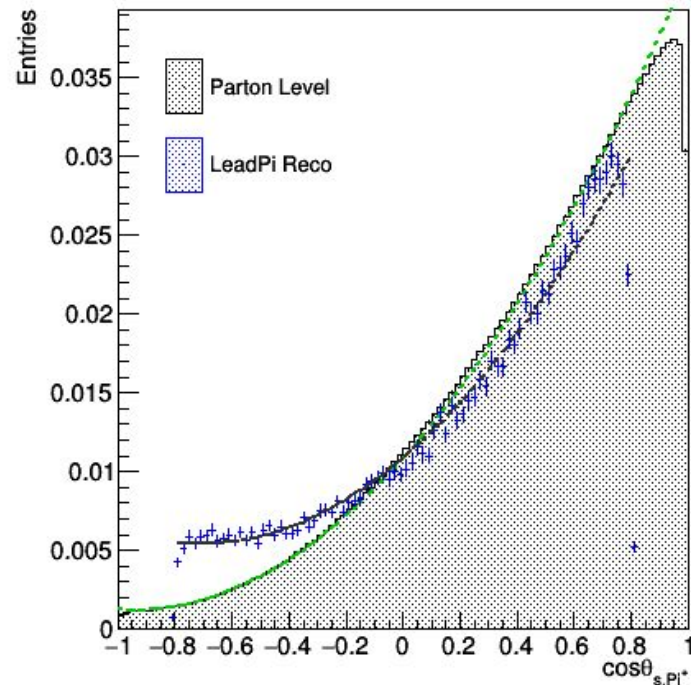
$$S = 1.07\text{E-}2 \pm 1.7\text{E-}6$$

$$A = 2.01\text{E-}2 \pm 3.7\text{E-}6$$

Reco:

$$S = 1.08\text{E-}2 \pm 4.6\text{E-}5$$

$$A = 1.53\text{E-}2 \pm 1.2\text{E-}4$$



pq method

pq method

pq calculation

- Solve :

$$N_{acc} = p^2 N + q^2 N$$

$$N_{rej} = 2pqN$$

$$1 = p + q$$

p : Probability of getting the configuration right.
 q : Probability of getting the configuration wrong.

- Solution :

$$p = \frac{N \pm \sqrt{N(N - 2N_{rej})}}{2}$$

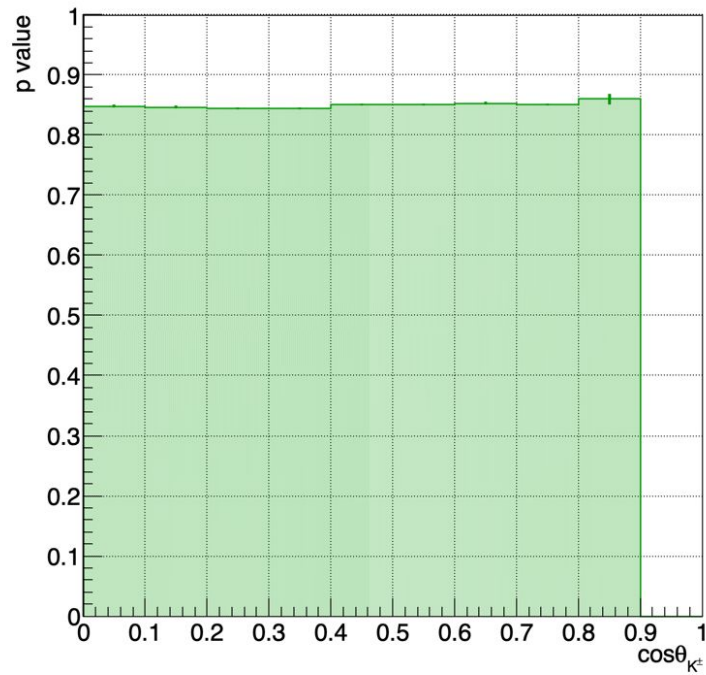
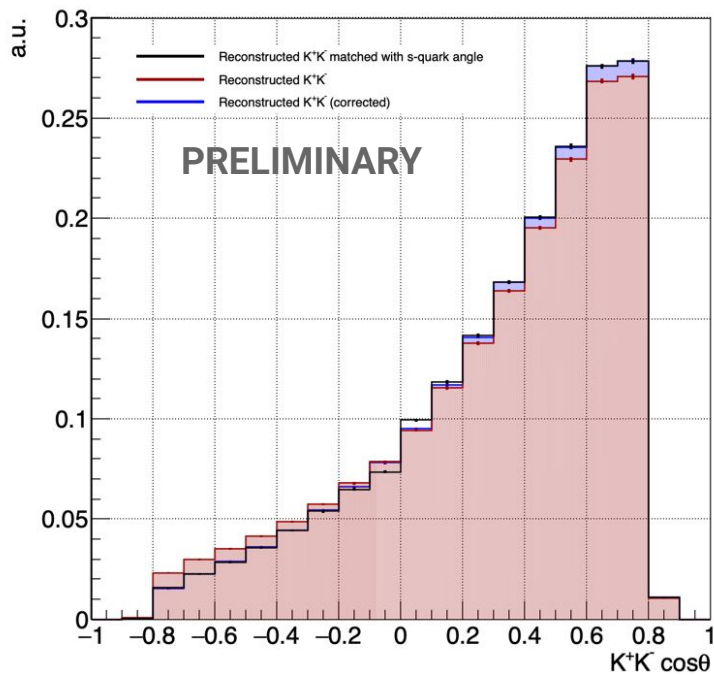
$$q = \frac{N \mp \sqrt{N(N - 2N_{rej})}}{2}$$

- Weight

- Scale each bin in AFB plot so that we will obtain N_{acc} with eq on the right.
- Take average of p values over 4 different points with polar angle value \pm stat errors

pq method (4.3 ab⁻¹)

LPFO 20.0 < p < 60.0 GeV



$N(KxK) = 161724$

Loosening Selections

Efficiency

The Main Efficiency Killer

- TPC Hits cut
 - Restricts detector acceptance region ($0.8 < |\cos\theta|$)
- Momentum cut
 - Tight cut for LPFO momentum selection ($20 < p < 60$ GeV)
- dE/dx distance selection
 - The minimum K dE/dx distance is selected.

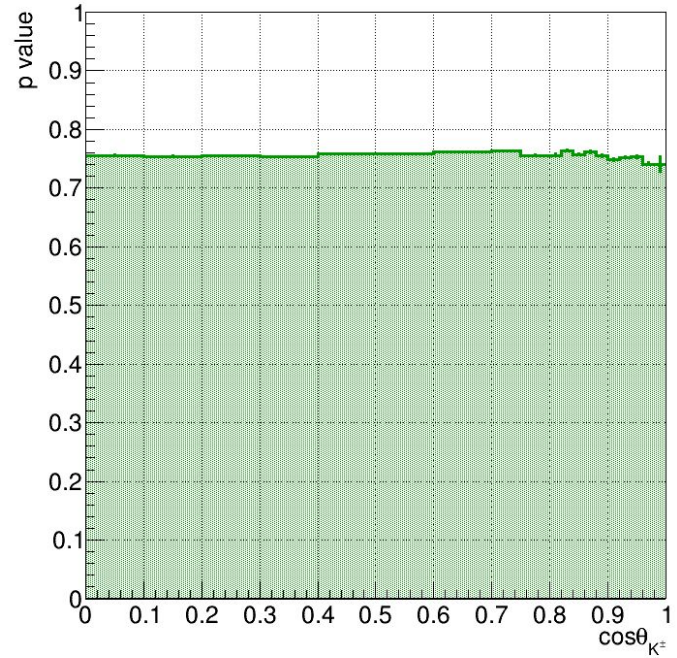
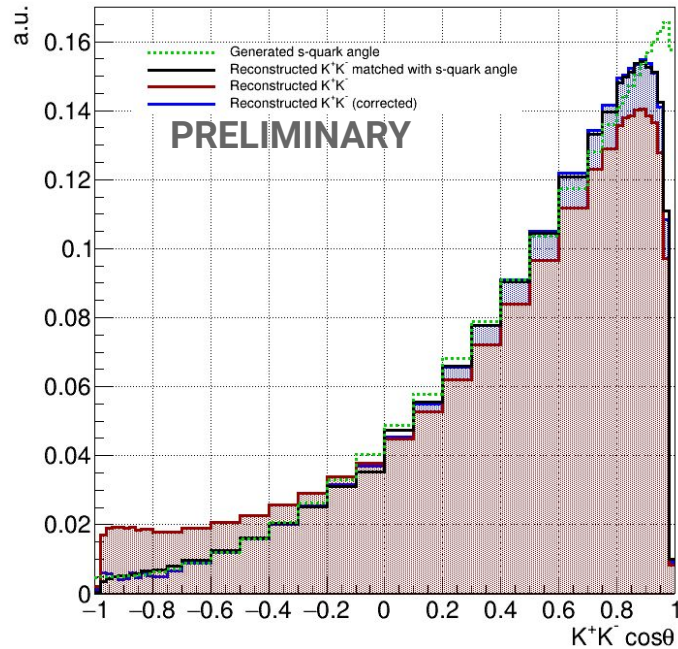
Efficiency

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pq method (4.3 ab⁻¹)

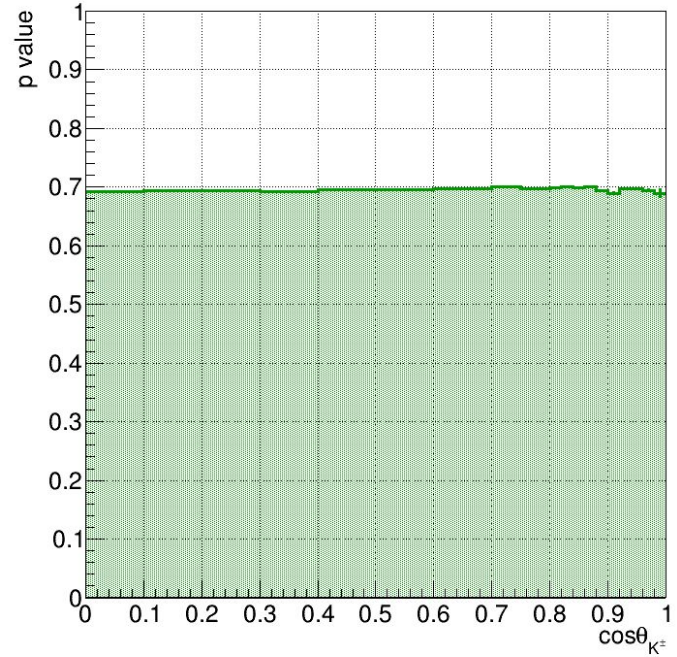
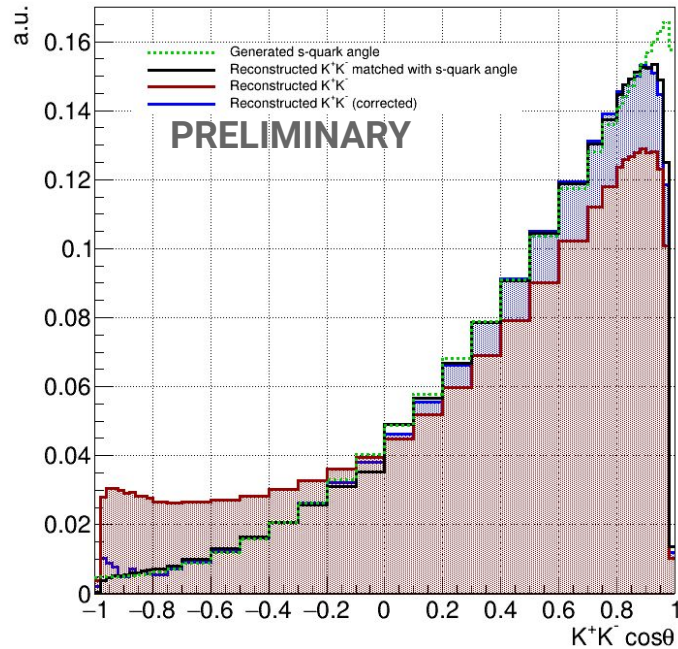
LPFO 20.0 < p < 60.0 GeV | 0 < TPC Hits



$N(KxK) = 489246$

pq method (4.3 ab⁻¹)

LPFO 10.0 < p GeV | 0 < TPC Hits



$N(KxK) = 1681872$

Summary & Prospects



Summary & Prospects

Summary

- $SS\bar{b}$ reconstruction was performed, using dE/dx distance PID.
 - Kaon identification
- Multiple methods were used to reject backgrounds and salvage rejected events.
 - K^*0 method
 - pq-method
- The pq-method has shown that the method works well for the double tagging cases such as in this $ss\bar{b}$ analysis.

Prospects

- Mix other light/heavy quark pair production events.
 - $uu/dd/cc/bb$
- Can we do better with K^*0 ?
 - Apply pq-method to the polar angle, which will give us extra handle on K^*0
- Calculation of couplings from the fitted parameters.

ISR Suppression

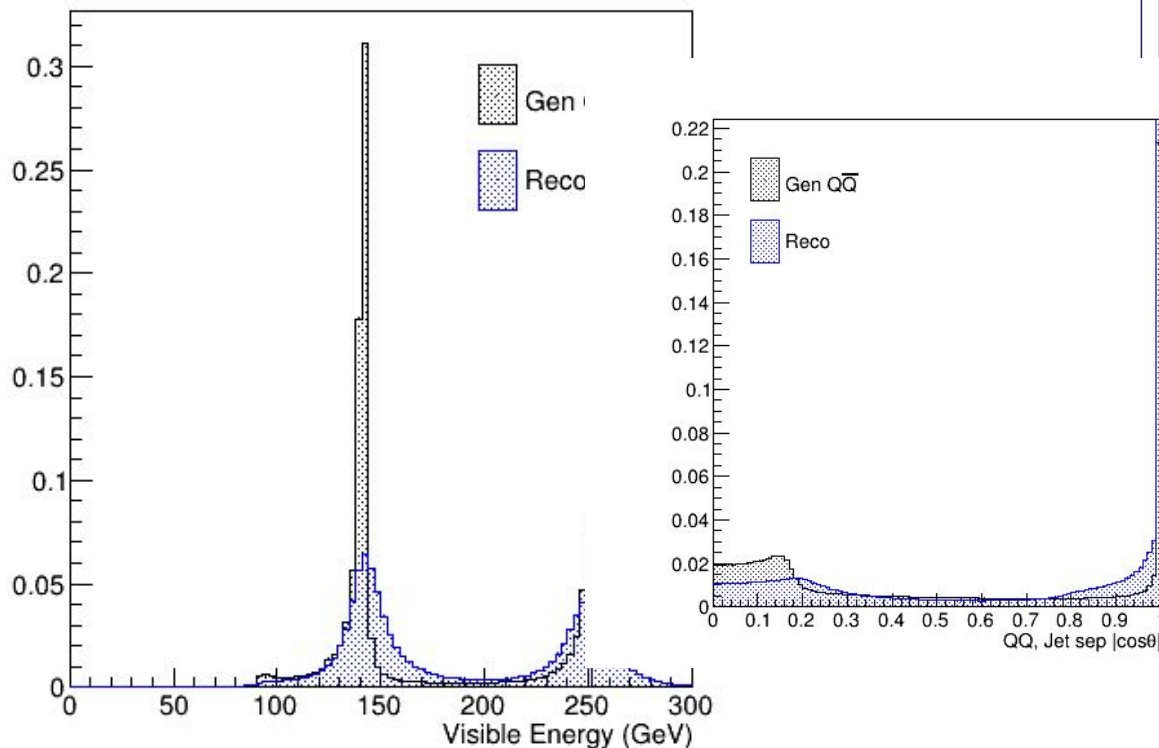
ISR Suppression

Signal Definition

- SSbar back-to-back
 $0.95 < \cos \theta_{s\bar{s}}$
- Total Energy
 $220 < E_s + E_{\bar{s}}$

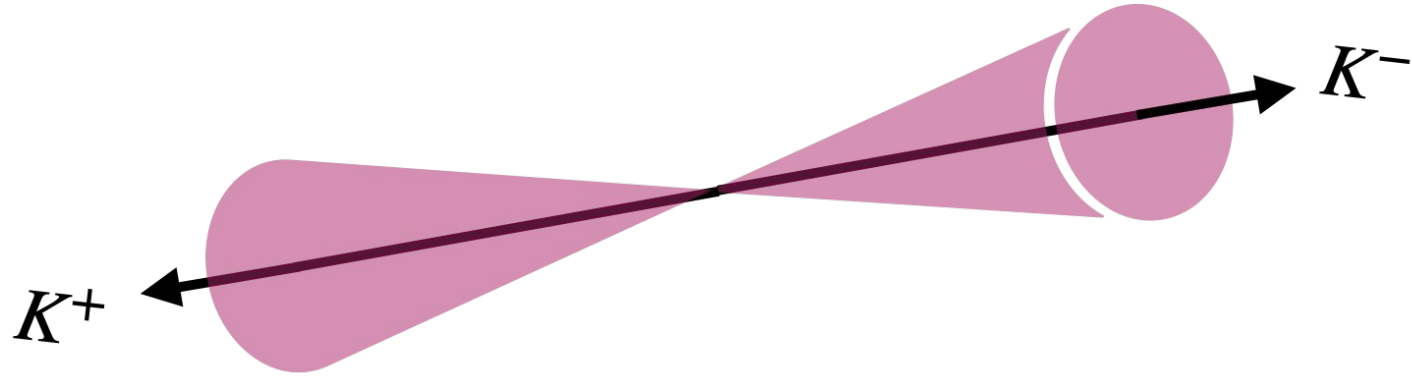
Reco ISR Removal

- LPFO back-to-back
 $0.95 < \cos \theta_{jets}$
- Total Energy
 $220 < E_{vis}$

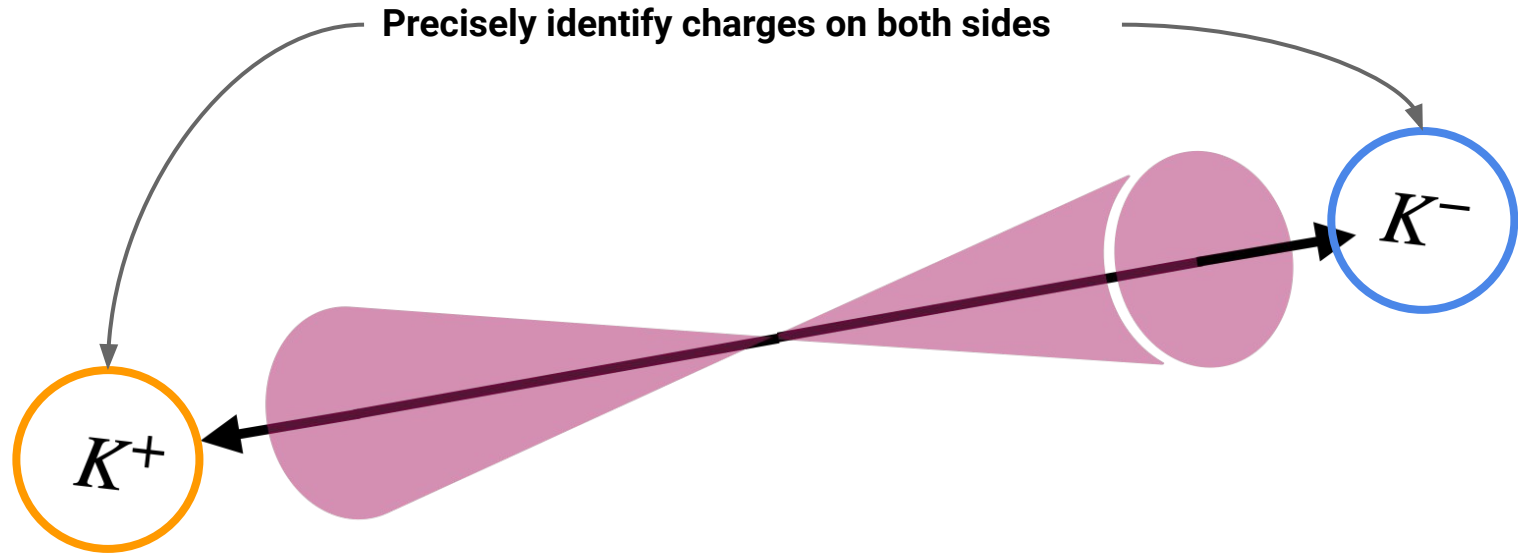


Migrated Event Analysis

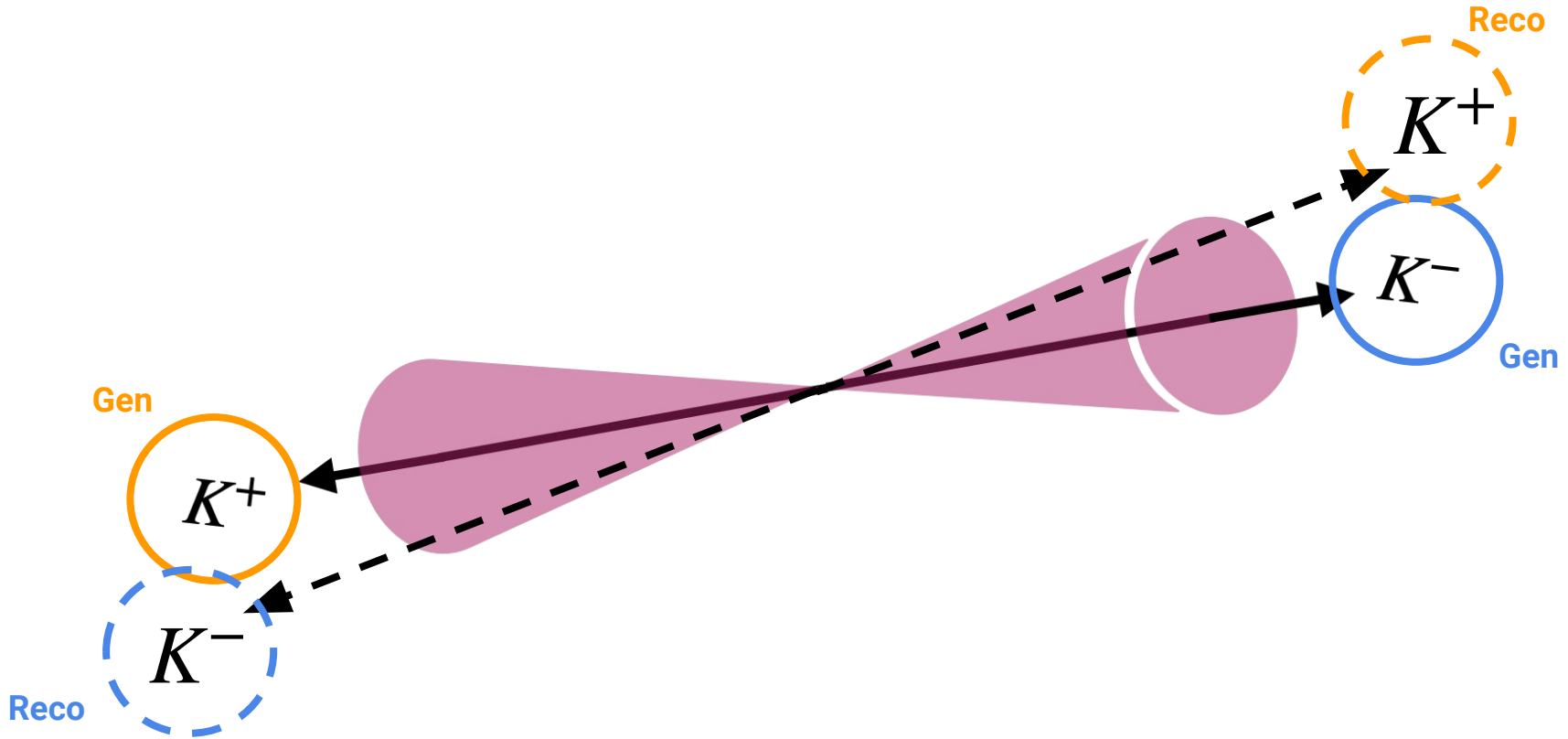
What is Migration?



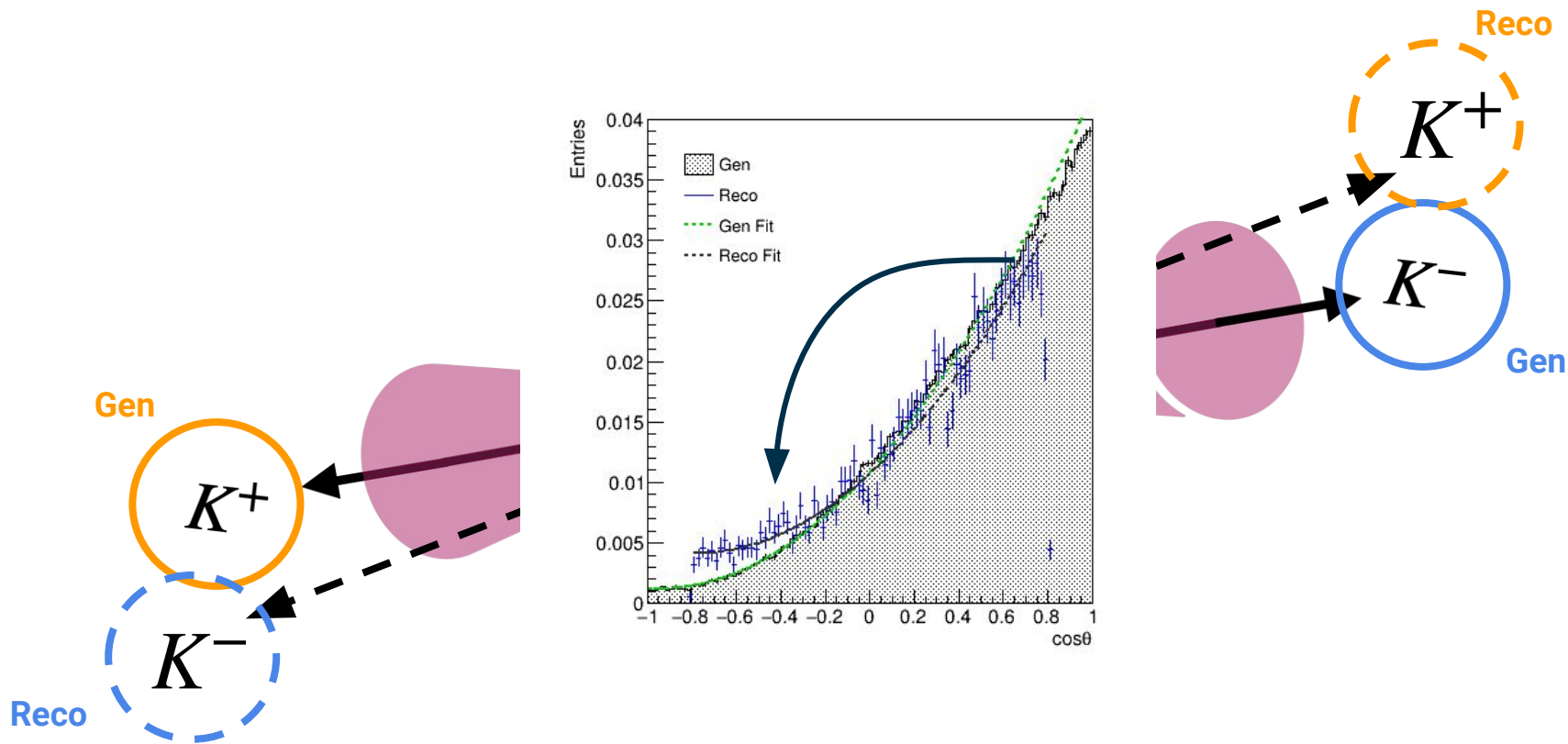
What is Migration?



What is Migration?



What is Migration?



Why Migration?

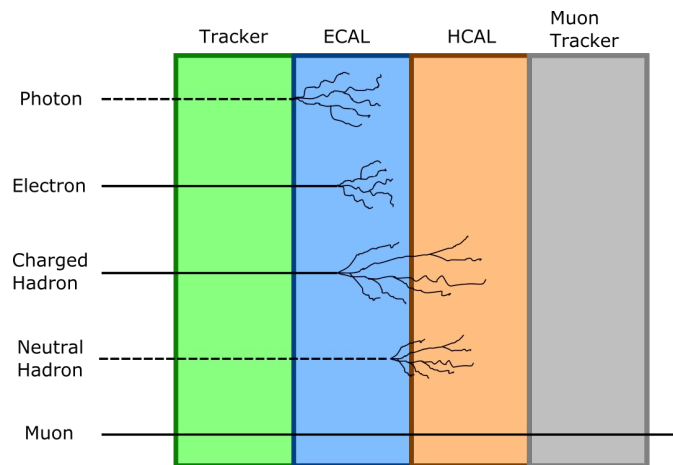


Why Migration?

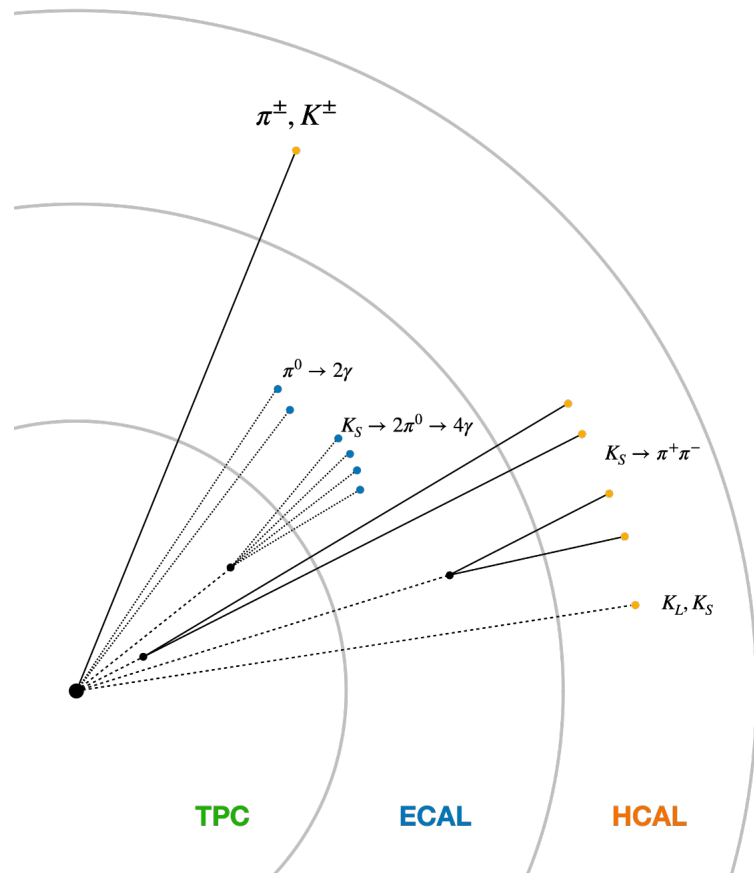
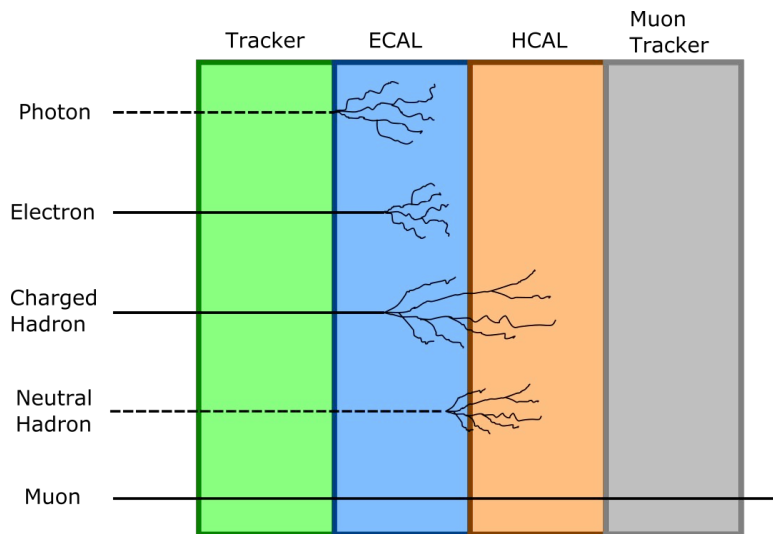


Possible types of Kaons from $s\bar{s}$ pair

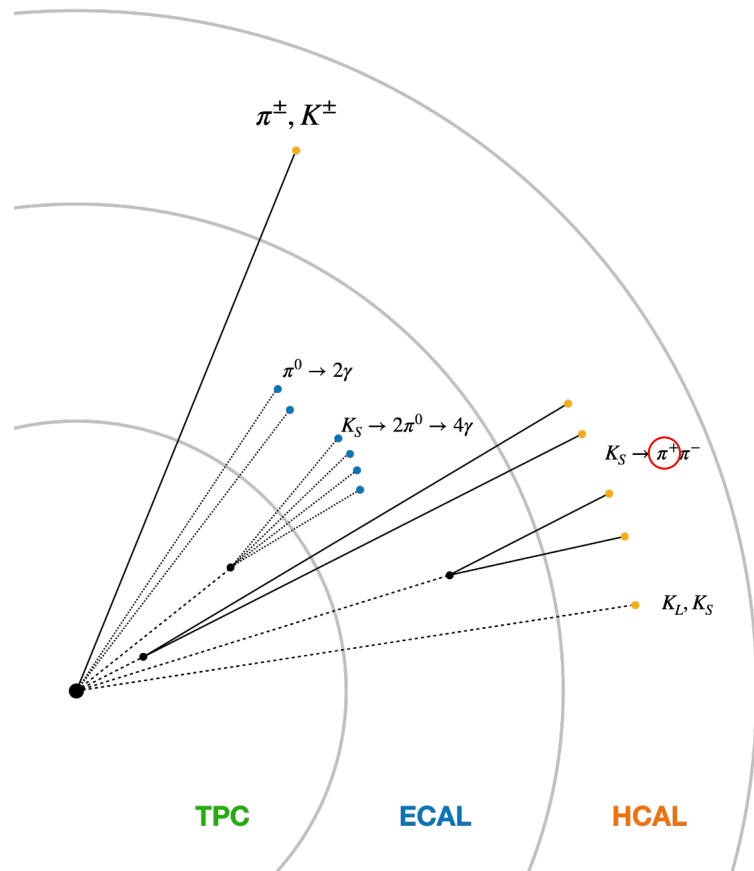
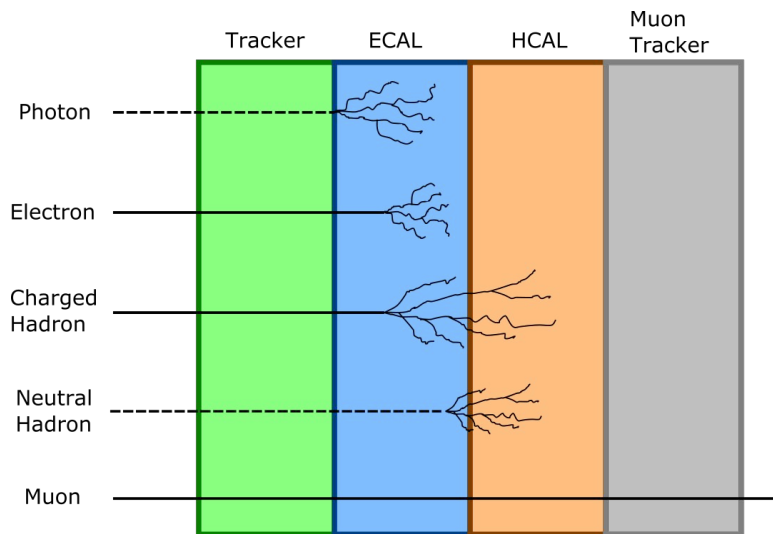
$$K^- = s\bar{u}, K^+ = \bar{s}u, K_L \approx \frac{(s\bar{d} - d\bar{s})}{\sqrt{2}}, K_S \approx \frac{(s\bar{d} + d\bar{s})}{\sqrt{2}}$$



Why Migration?



Why Migration?

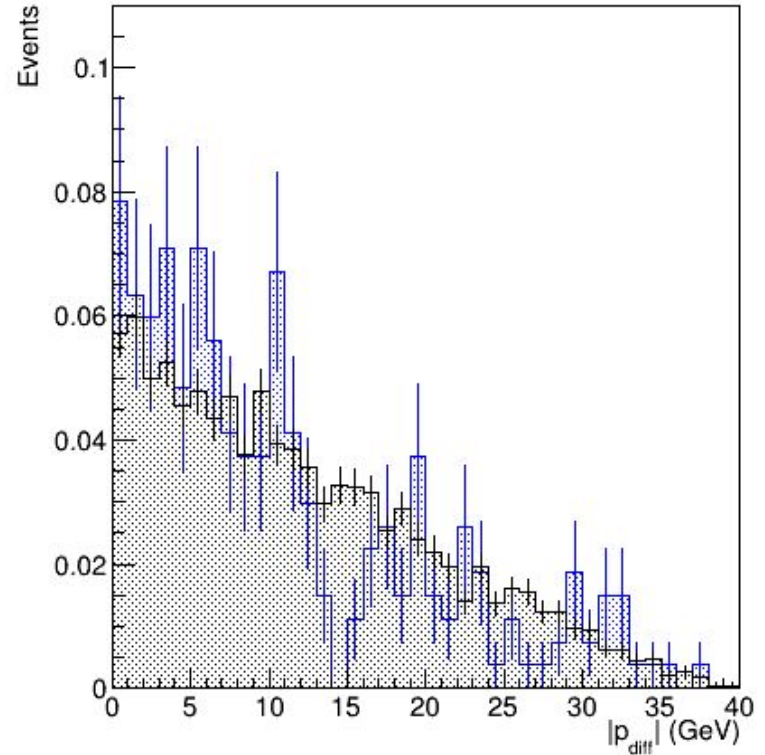


Full Stats

Migration

LPFO momentum separation

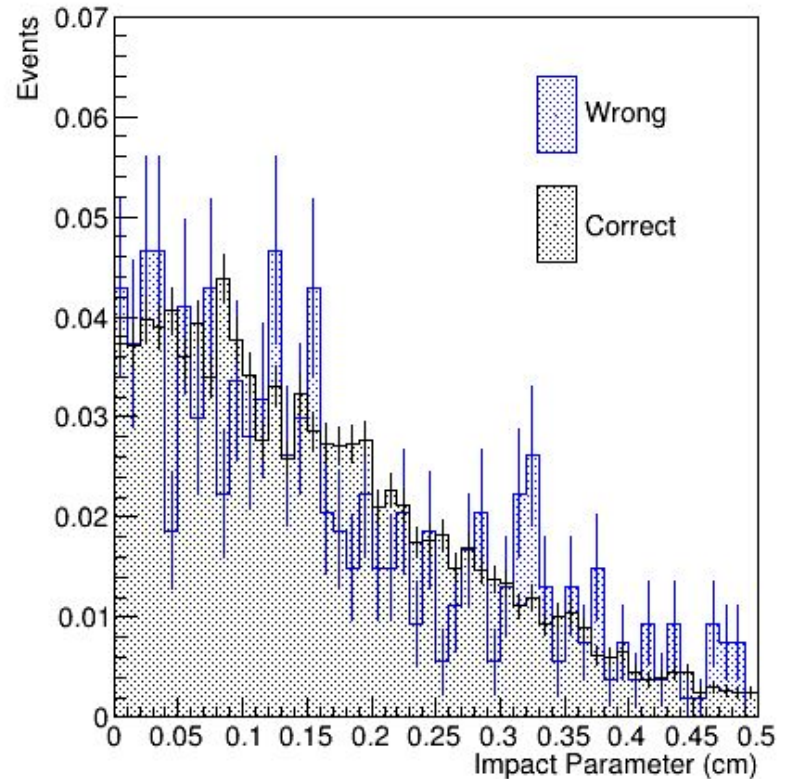
- eLpR full polarized
 - ss: 375,000 events \rightarrow 125 fb $^{-1}$
- Computation
 - LPFOp0 - LPFO p1
- Distribution at $p > 15$.



Migration

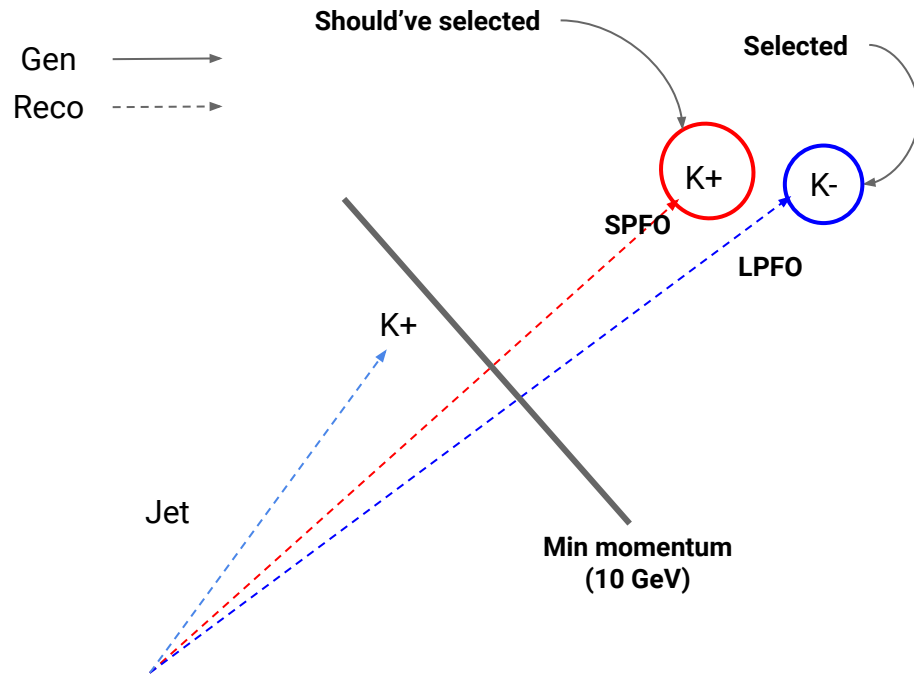
LPFO Impact Parameter

- Peak at 0.3
 - Lambda decay?
- More statistics needed?



Migration

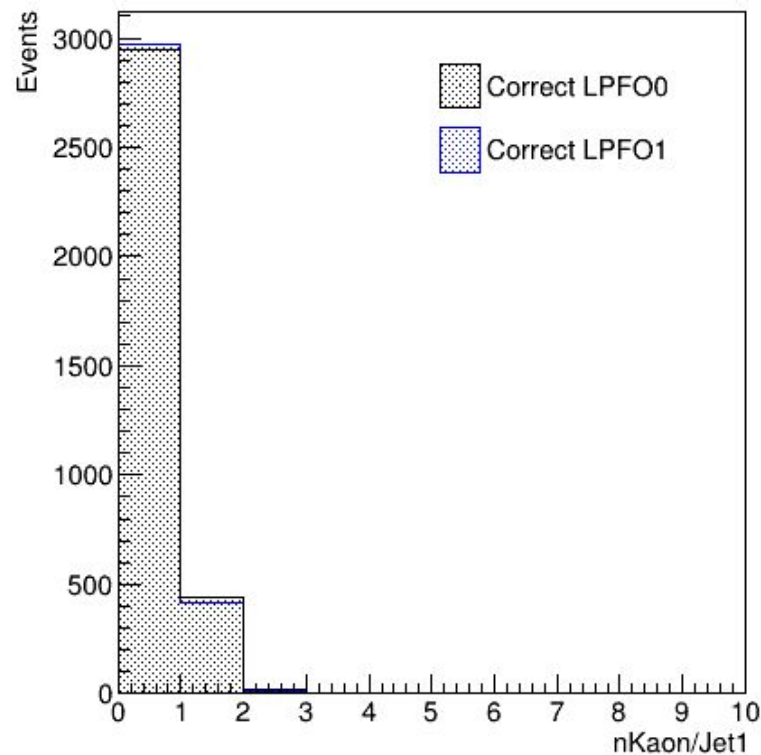
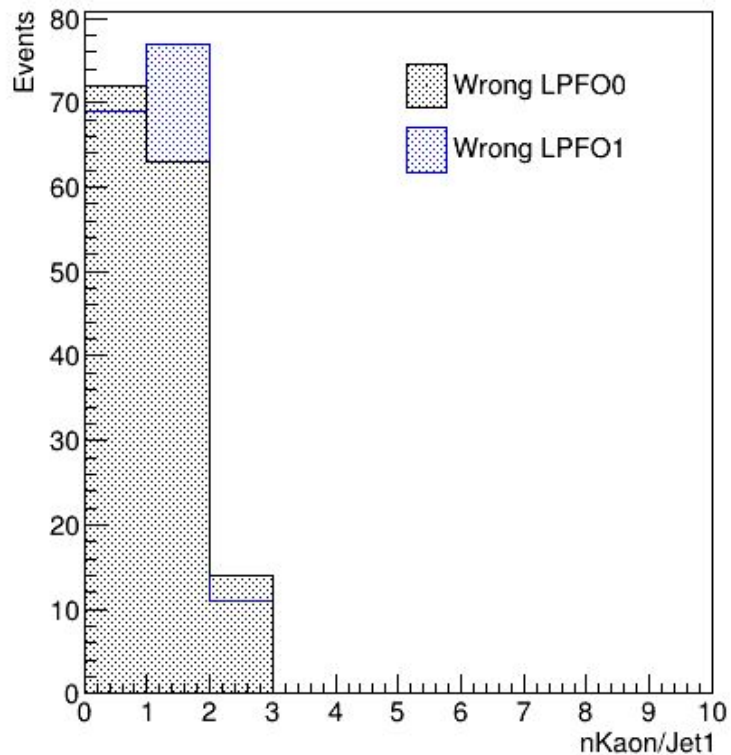
- We look for **Secondary PF0s (SPFO)** with opposite charge to LPFO.
- Wrong events should have SPFO with momentum close to LPFO. (Other **stole** original s-quark)
- Definition for **SPFO Kaon** with **opposite charge**
 - Not leading
 - LPFO is Kaon (ID MC gen partner)
 - SPFO is Kaon (ID MC gen partner)
 - Has opposite charge respect to LPFO
 - Min momentum : 10 GeV



WRONG EVENT

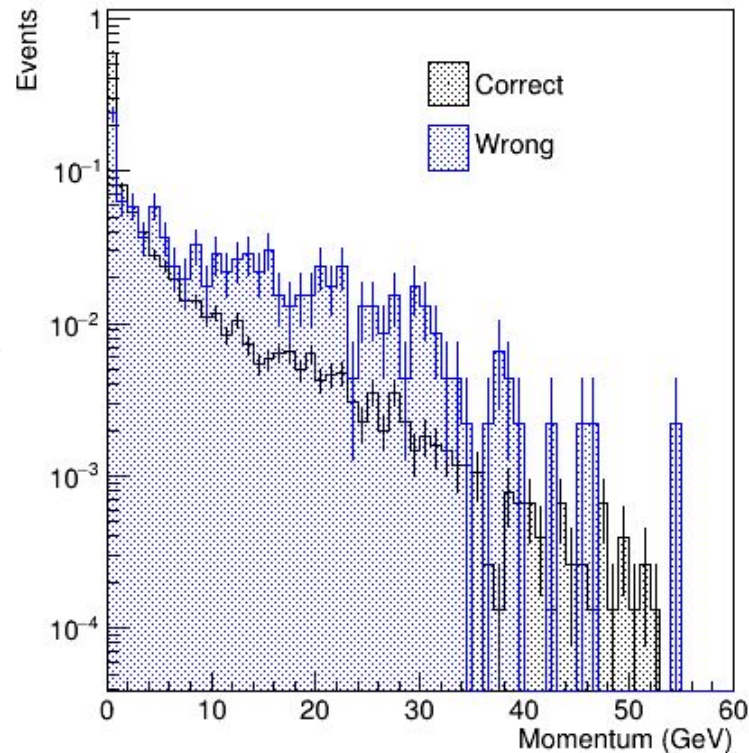
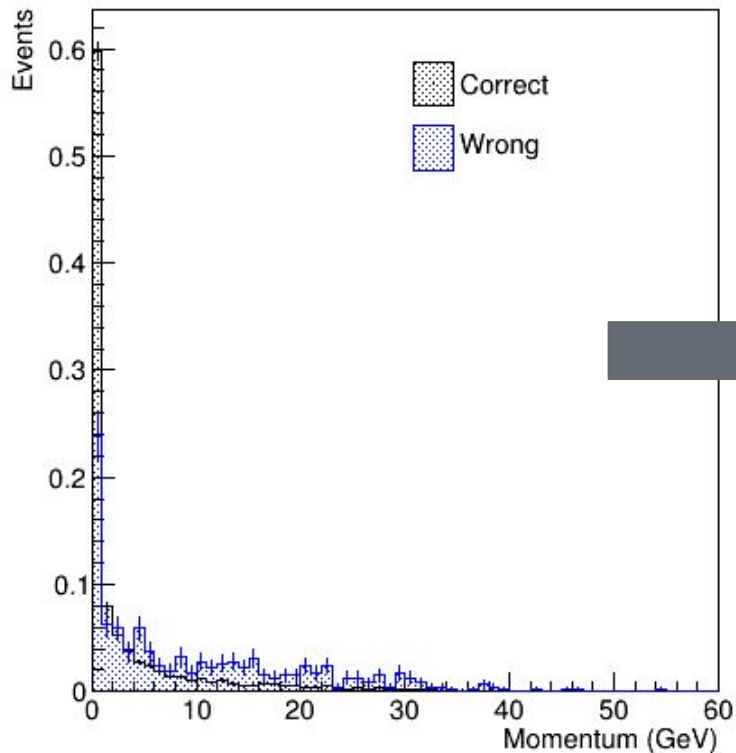
Migration

SPFO Kaon Opposite Charge Multiplicity



Migration

SPFO Kaon Opposite Charge Momentum



Selections (ss)

Cut MC

ISR suppression

- $QQ \cos \text{sep} > 0.95$
- $120 < QQ \text{ mom} < 127$

Cut PFO

General PFO

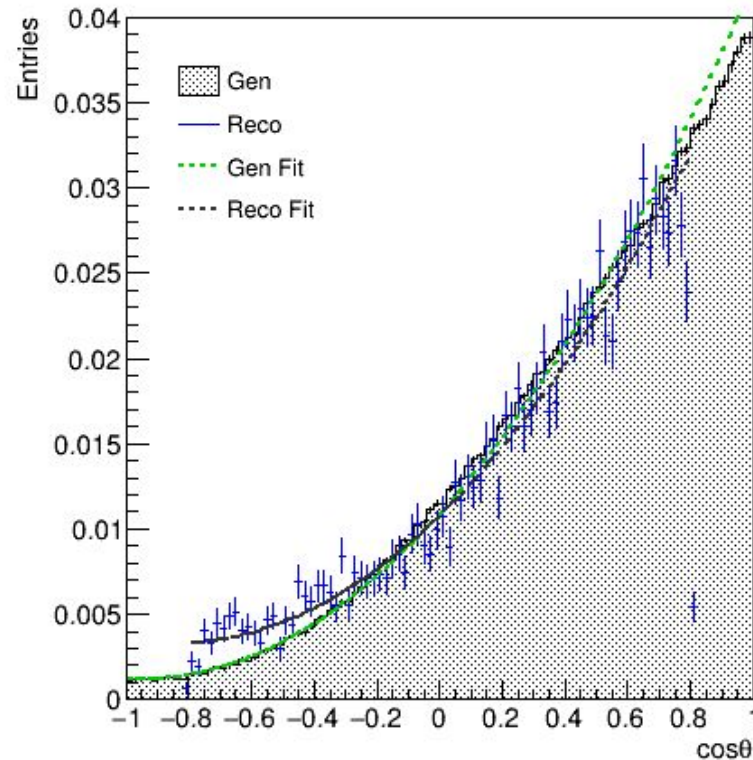
- PFO match (It should fall into either jet0 or jet1)
- # PFO tracks == 1 (more than 2 tracks cannot be associated to make 1 PFO)

Lead PFO (double tag)

- Both PFO should have momentum window
 $20 < \text{Lead PFO mom} < 60$
- Lead PFO charge \pm or $-+$
- # TPC hits **$210 < \text{Lead PFO hits}$**
- Offset cut < 1.0
- $\text{kdEdx_dist} < (\text{pdEdx_dist} \ \& \ \text{pidEdx_dist})$

Notes

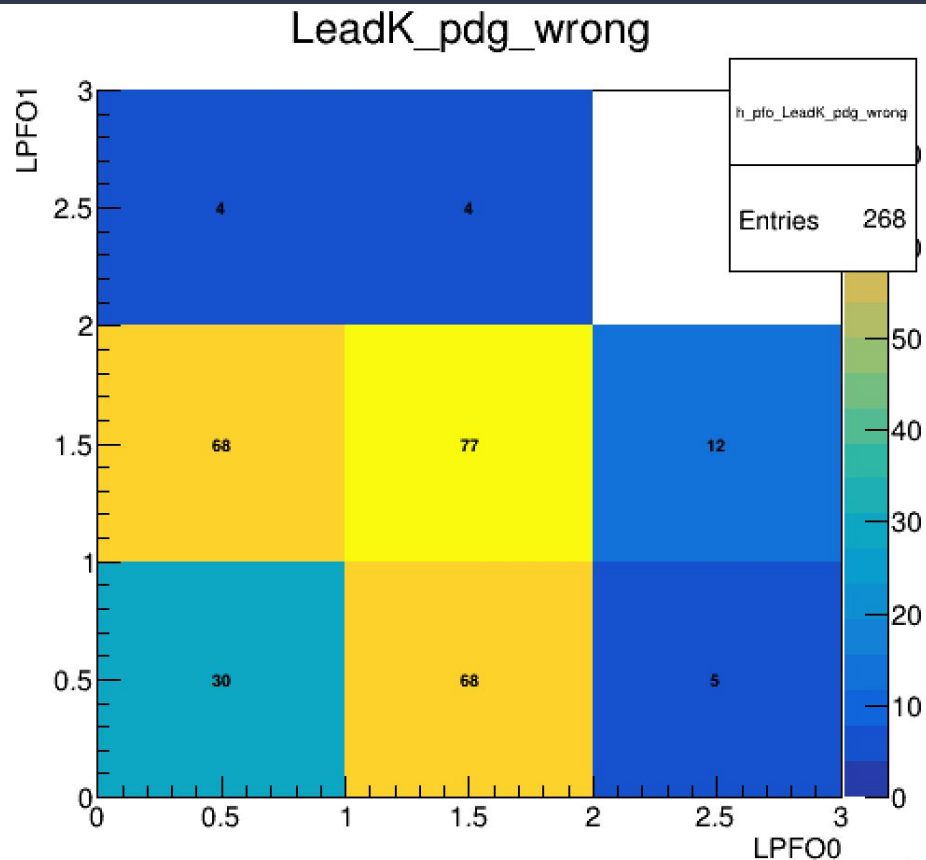
- TPC hits -> changed from base
- Normalization changed (integrate from $-0.8 < \cos < 0.8$) because of cut in # TPC hits
- Momentum window minimum changed from 10 -> 20 GeV



Migration after pcut20

Right plot shows the PDG of leading PFOs for the migrated events when the momentum of both LPFO0 && LPFO1 > 20 GeV.

Config	#Events	%
K-K	77	28.7
Pi-Pi	30	11.2
Pi-K	136	50.7
Pi-p	9	3.3
p-K	16	6.0
p-p	0	0



SPFO Removal

Selections (ss)

Cut MC

ISR suppression

- $QQ \cos \text{sep} > 0.95$
- $120 < QQ \text{ mom} < 127$

Cut PFO

General PFO

- PFO match (It should fall into either jet0 or jet1)
- # PFO tracks == 1 (more than 2 tracks cannot be associated to make 1 PFO)

Lead PFO (double tag)

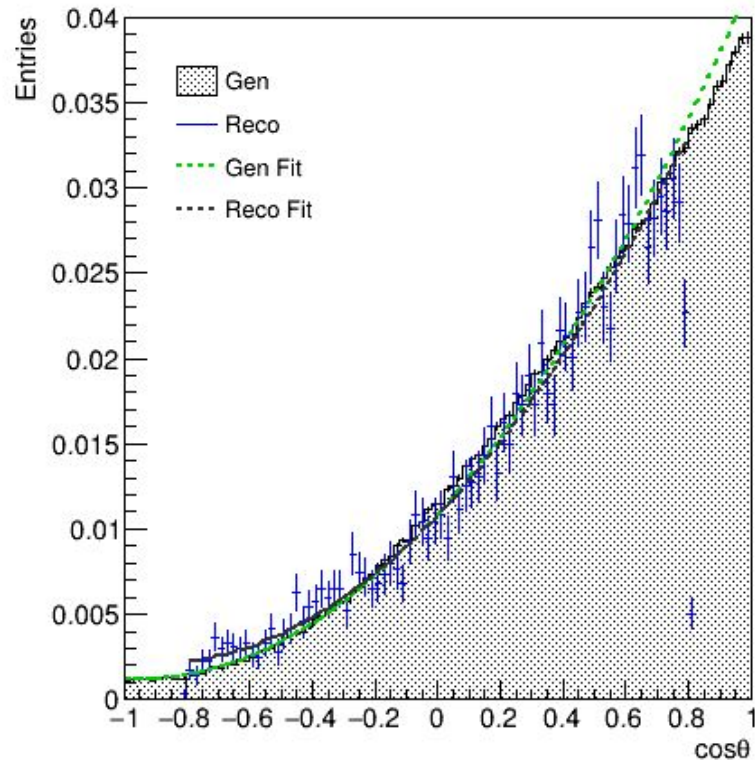
- Both PFO should have momentum window
 $20 < \text{Lead PFO mom} < 60$
- Lead PFO charge \pm or $-+$
- # TPC hits **$210 < \text{Lead PFO hits}$**
- Offset cut < 1.0
- $\text{kdEdx_dist} < (\text{pdEdx_dist} \ \& \ \text{pidEdx_dist})$

Secondary PFO Counting

- SPFO is not LPFO
- SPFO is Kaon (**determined from dEdx dist**)
- SPFO has opposite charge compared to LPFO
- SPFO should at least have 10 GeV momentum.
- Count number of such SPFO. (should be = 0)

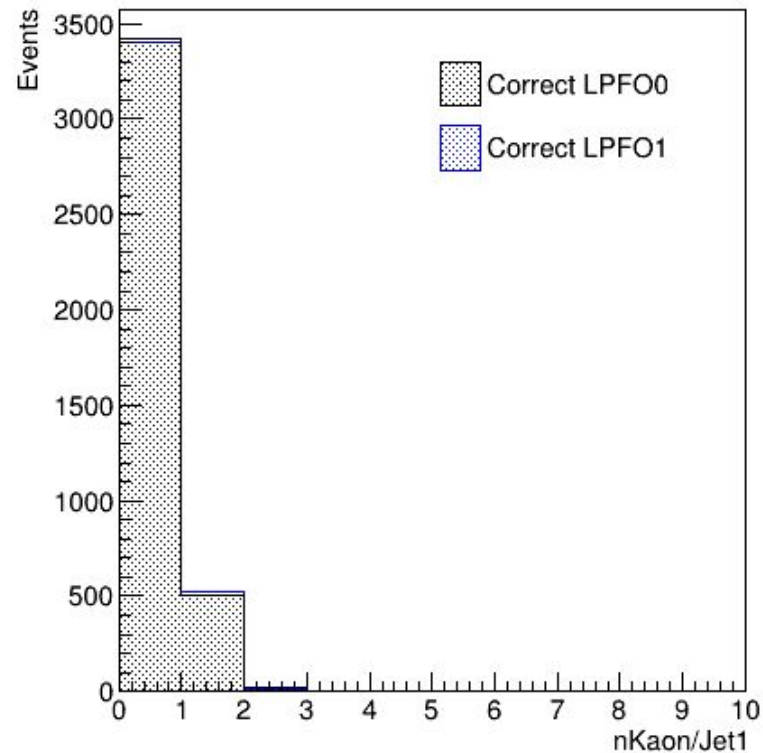
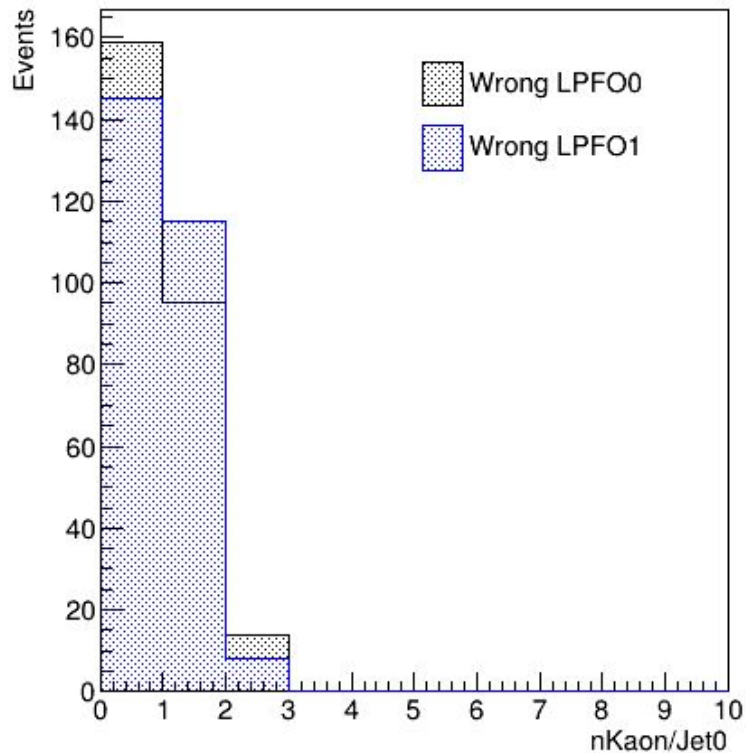
Notes

- TPC hits -> changed from base
- Normalization changed (integrate from $-0.8 < \cos < 0.8$) because of cut in # TPC hits
- Momentum window minimum changed from 10 -> 20 GeV



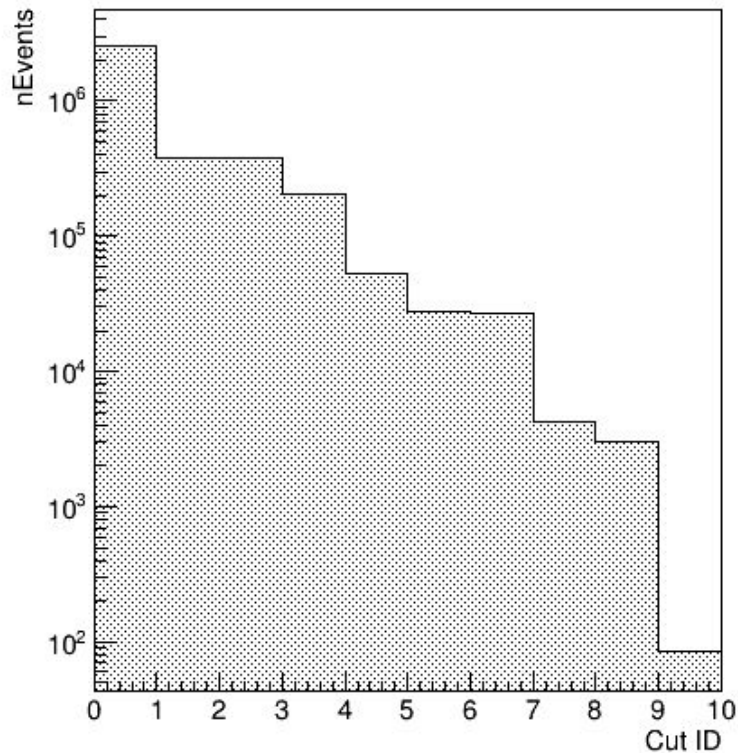
Migration

SPFO Kaon Opposite Charge Multiplicity



Number of Events

0	# Total Events (ss)	2,512,257
1	# after Gen sel	374,563
2	# after PFO sel	374,399
3	Charge check	201,967
4	Momentum check	53,227
5	TPC hit check	27,921
6	Offset check	26,848
7	dEdx dist min check	4,211
8	Opp K SPFO check	3,036
9	Migration	86 (2.8%)



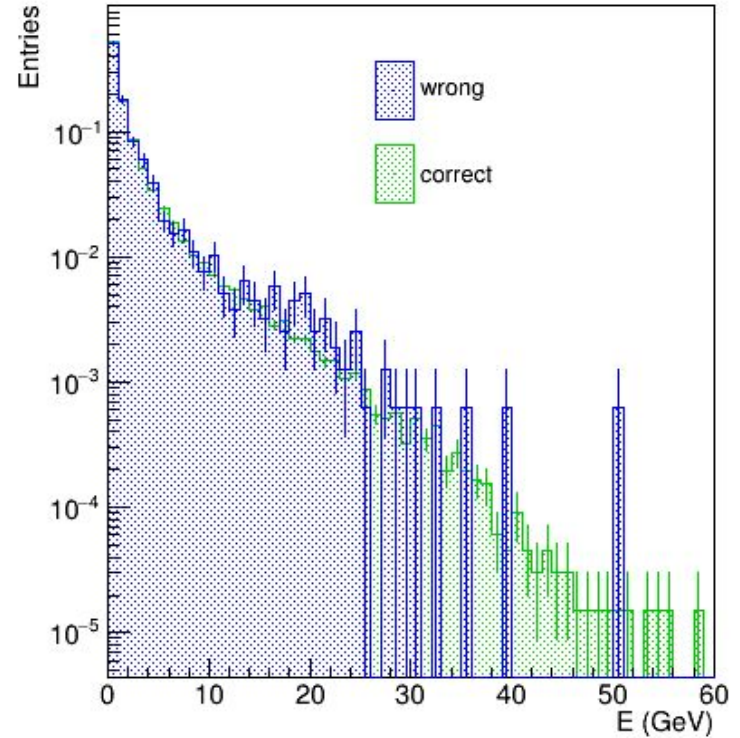
Neutral PFOs

Difference in Number

- LPFO Selection
 - Currently LPFO is selected among the charged PFOs.
 - This is done by 2 ways:
 - PFO should have 1 track.
 - LPFO should be charged.
 - The first selection was removed to take a look at neutral PFOs in selected events.
 - Events w/ Neutral PFOs \subset Events w/o Neutral PFOs
 - Thus, in current code, it will dump the event if the LPFO has charge 0.

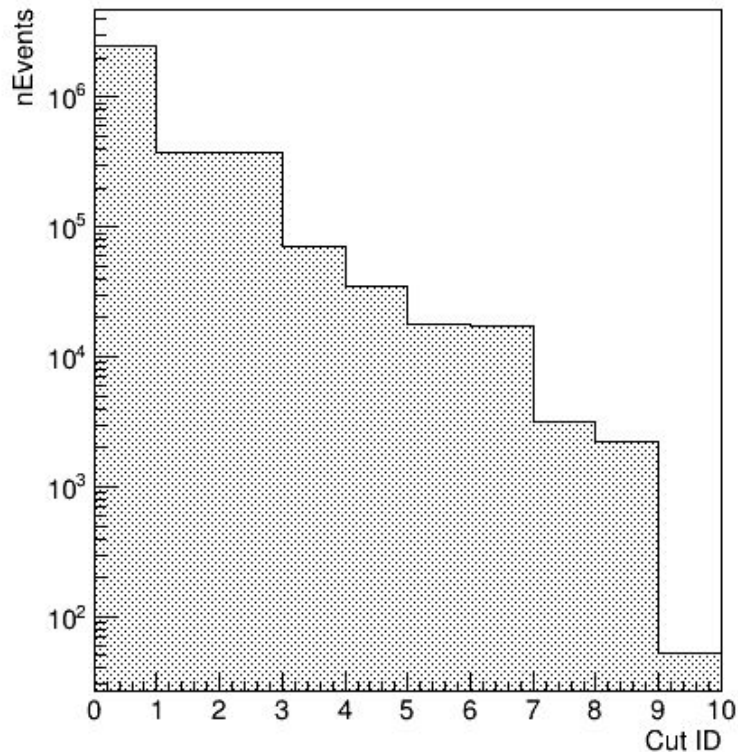
Energy Neutral PFO

Energy of Neutral PFOs



Number of Events

0	# Total Events (ss)	2,512,257
1	# after Gen sel	374,563
2	# after PFO sel	374,563
3	Charge check	70,516
4	Momentum check	35,222
5	TPC hit check	17,967
6	Offset check	17,306
7	dEdx dist min check	3,138
8	Opp K SPFO check	2,215
9	Migration	53 (2.4%)



Analysis Steps

- Reconstruct SSbar process using generator information
 - Summer 2021
 - PID was performed by checking with the Generator Information.
 - Done to **explore the maximum efficiency** that can be achieved by this analysis.
 - Understanding the characteristics of the process itself.
- Reconstruct SSbar process using dE/dx distance PID
 - Fall 2021 - Winter 2022
 - PID was performed using **dE/dx distance information**.
 - Still use **Gen Info for Signal Selection**
 - Tight selection was applied to **achieve high purity**.
- Analysis Refinement
 - Winter 2022
 - Counter migration
 - Increase selection efficiencies.
 - Start of use **Reco Info for ISR removal**.

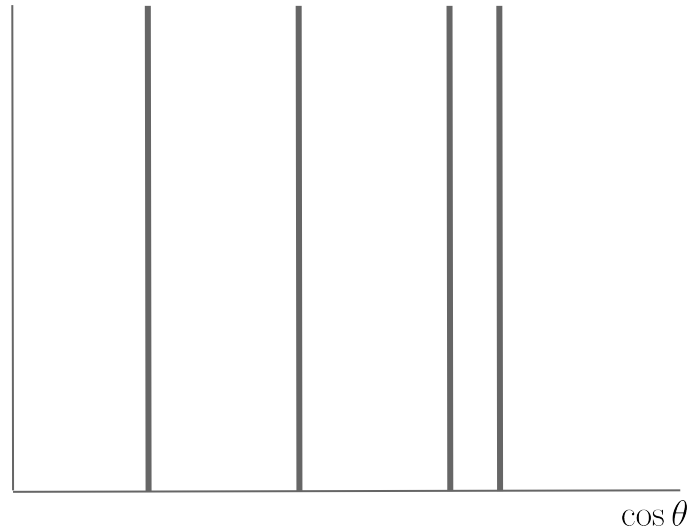
Definitions of Stability and Purity

Stability and Efficiency

$$\text{stability} = \frac{N_{rec} \cap N_{gen}}{N_{gen}}$$

$$\text{purity} = \frac{N_{rec} \cap N_{gen}}{N_{reco}}$$

$$\begin{array}{l} N_{gen} \\ N_{rec} \\ N_{rec} \cap N_{gen} \end{array} \quad \mathbf{4}$$

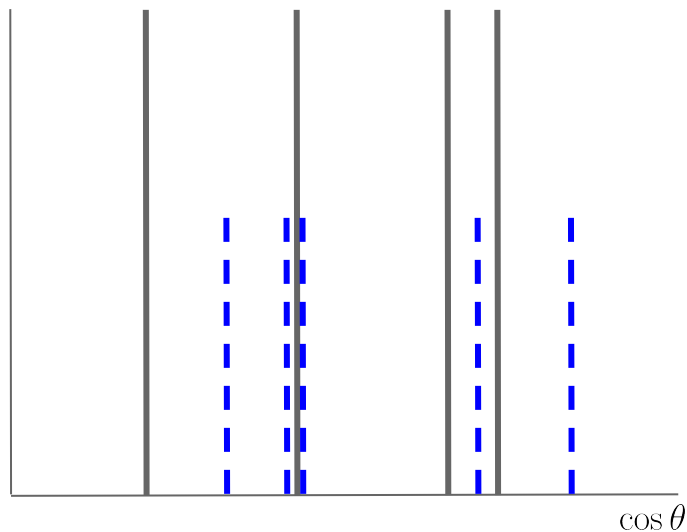


Stability and Efficiency

$$\text{stability} = \frac{N_{rec} \cap N_{gen}}{N_{gen}}$$

$$\text{purity} = \frac{N_{rec} \cap N_{gen}}{N_{reco}}$$

$$\begin{array}{r} N_{gen} \quad \mathbf{4} \\ N_{rec} \quad \mathbf{5} \\ N_{rec} \cap N_{gen} \end{array}$$



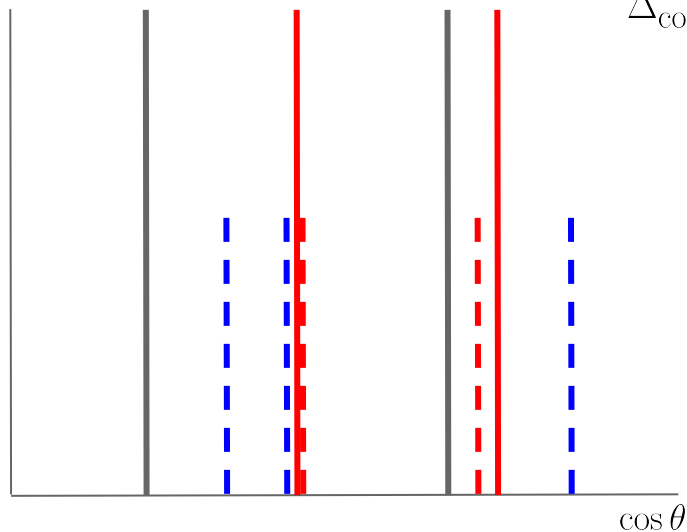
Stability and Efficiency

$$\text{stability} = \frac{N_{rec} \cap N_{gen}}{N_{gen}}$$

$$\text{purity} = \frac{N_{rec} \cap N_{gen}}{N_{reco}}$$

$$\begin{aligned} N_{gen} & \mathbf{4} \\ N_{rec} & \mathbf{5} \\ N_{rec} \cap N_{gen} & \mathbf{2} \end{aligned}$$

$$\Delta_{\cos\theta} < 0.02$$



Stability and Efficiency

$$\text{stability} = \frac{N_{rec} \cap N_{gen}}{N_{gen}}$$

$$\text{purity} = \frac{N_{rec} \cap N_{gen}}{N_{reco}}$$

