NLO Electroweak Corrections to Lepton Collider Processes

IDT-WG3-Phys Open Meeting

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July 13, 2023

Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

NLO EW corrections in an automated tool

$\Rightarrow$ Include $\mathcal{O}(\alpha)$ corrections in cross sections and differential distributions for arbitrary final states at hadron and lepton colliders

Major enhancement effects of $\mathcal{O}(\alpha)$ corrections **at lepton colliders**

- $\mathcal{O}(\alpha \log^2 p_{ij}^2/M_W^2)$ EW Sudakov suppressions large
  - for any high-energy lepton collider, $p_{ij}^2 \sim \hat{s} \sim s$
- $\mathcal{O}(\alpha \log s/m_l^2)$ enhancements for QED ISR
  - especially for high-energy e^+e^- colliders, $s \gg m_e^2$
- $\mathcal{O}(\alpha \log s/E_\gamma^2)$ enhancements due to soft photon radiation
  - especially for lepton-collider energies close to mass thresholds

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What do we do?

- Apply universal principles of NLO EW corrections in the Monte-Carlo generator `WHIZARD` [EPJ C71 (2011) 1742]

Overview

Parts:

I) Automation of NLO EW corrections in `WHIZARD`

II) Application of fixed $\mathcal{O}(\alpha)$ corrections to muon-collider processes

III) Status: NLL electron PDFs for lepton-collider processes at NLO EW

I) Automation of NLO EW corrections in WHIZARD

I) NLO framework in WHIZARD

What is WHIZARD?

Multi-purpose event generator for cross sections and differential distributions of **arbitrary processes** at HEP experiments (LHC, Belle II, ILC/CLIC/FCC/CEPC, MuCol, ...)

recent version: v3.1.2
team: **Wolfgang Kilian, Thorsten Ohl, Jürgen Reuter**
PB, Nils Kreher, Krzysztof Mękała, Tobias Striegl
webpage: https://whizard.hepforge.org/
support: https://launchpad.net/whizard
email contact: whizard@desy.de

Essential elements of WHIZARD at NLO

- physics models: SM (internal BSM and UFO models)
- phase-space integrator: VAMP (VEGAS AMPlified) [CPC 120 (1999) 13],
  VAMP2[EPJ C79 (2019) 4 344] incl. MPI parallelization
- matrix elements: tree-level ME generator O'Mega [LC-TOOL (2001) 040], interface to OLPs
  OpenLoops[1907.13071], RECOLA[1711.07388] and GoSam[1404.7096]
- infrared subtraction: FKS-scheme (POWHEG matching + PYTHIA-interface)

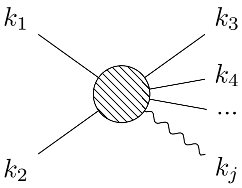
I) NLO framework in WHIZARD

(contributors: PB, B. C.-Nejad, W. Kilian, J. Reuter, V. Rothe, P. Stienemeier, C. Weiss)

$$\sigma_{\text{NLO}} = \underbrace{\int d\Phi_n \mathcal{B}}_{\text{Born}} + \underbrace{\int d\Phi_{n+1} \mathcal{R}}_{\text{div. real}} + \underbrace{\int d\Phi_n \mathcal{V}}_{\text{div. virtual}} = \text{finite}$$

Need observables **exclusive** in kinematic properties!

$$\sigma_{\text{NLO}} = \int d\Phi_n \mathcal{B} + \int \underbrace{d\Phi_{n+1} \left[\mathcal{R} - d\sigma_S\right]}_{\text{finite by construction}} + \underbrace{\int d\Phi_n \mathcal{V} + \int d\Phi_n d\sigma_{S,\text{int}}}_{\text{IR poles cancelled analyt.}}$$



'j' radiated with several different emitters
⇒ Subtract singularities related to IR splittings systematically!

Frixione-Kunszt-Signer (FKS) subtraction

Divide phase space into disjoint regions with **at most one** soft and/or collinear singularity.
⇒ kinematical weight factors related to pairs (i, j)

I) NLO framework in WHIZARD: NLO QCD

(contributors: PB, B. C.-Nejad, W. Kilian, J. Reuter, V. Rothe, P. Stienemeier, C. Weiss)

For pp and e^+e^- colliders

- Automation of fNLO simulation of cross sections and distributions
  - → Validated for about 50 processes with MG5_aMC@NLO[1405.0301] and SHERPA[1905.09127]
- Automation of POWHEG-matched event generation for NLO QCD corrections
  (thanks to Pascal Stienemeier)
  - → Validated for $pp \to e^+e^-$ and $e^+e^- \to t\bar{t}j$ with POWHEG-BOX[1002.2581]

I) NLO framework in WHIZARD: NLO EW

Extension to electroweak corrections – Basics

- QED FKS subtraction terms:

$$d\sigma_{S,\text{coll}} \sim \alpha \underbrace{\hat{P}^{\mu\nu}_{E\to(i,j),\text{QED}} \mathcal{B}^{(E)}_{\mu\nu}}_{\text{pol. AP kernel} \times \text{spin-corr.}}, \; d\sigma_{S,\text{soft}} \sim \alpha \sum_{k,l=1}^{n} \underbrace{\frac{\bar{k}_k \cdot \bar{k}_l}{(\bar{k}_k \cdot \hat{k}_j)(\bar{k}_l \cdot \hat{k}_j)} \mathcal{B}_{kl}}_{\text{eikonal} \times \text{charge-corr.}}$$

- EW loop contributions (interface to OpenLoops, RECOLA, GoSam)
- EW renorm. schemes & photons entering at Born level

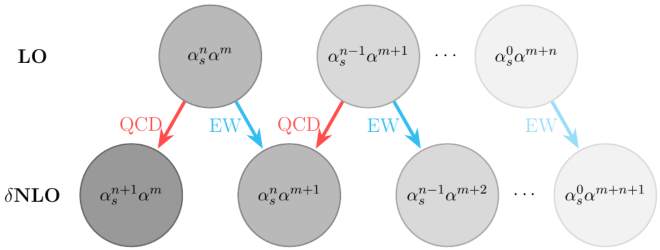
$Q^2_\gamma \to 0$	$Q^2_\gamma \sim$ EW scale
on-shell photons no γ splittings	*off-shell* photons $\gamma^* \to f\bar{f}$
$\alpha(0)$	$\alpha\|_{G_\mu}$, $\alpha(M_Z)$
$\left[\frac{\delta\alpha(0)}{\alpha(0)} + \delta Z_{AA}\right]_{\text{light}} = 0$	$\left[\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} + \delta Z_{AA}\right]_{\text{light}} + \delta Z_{\gamma,\text{PDF}}$ $\to$ finite overall photon factor $\neq 0$

with photon virtuality Q^2_γ

$\to$ α coupling constant, renormalization factors

I) NLO framework in WHIZARD: NLO EW and mixed

Interfering correction types (NLO QCD×EW): for processes with $\mathcal{O}(\alpha_s^n)$ contributions with $n \geq 1$:



Example: $pp \to Zj$ at $\mathcal{O}(\alpha\alpha_s)$:
Contributions from $q\bar{q} \to Zg\gamma$ at $\mathcal{O}(\alpha^2\alpha_s)$
$\Rightarrow$ Need cancellations from $[\mathcal{B}(q\bar{q} \to Zg)$ at $\mathcal{O}(\alpha\alpha_s)] \times$ [QED splitting]
and $[\mathcal{B}(q\bar{q} \to Z\gamma)$ at $\mathcal{O}(\alpha^2)] \times$ [QCD splitting]

For lepton collider processes relevant in case of at least 3 colored final states

I) NLO framework in WHIZARD: Lepton collisions at NLO EW

Fixed-order massive approximation for NLO cross sections and distributions:

- IS leptons considered as massive $\Rightarrow$ no collinear counterterms needed
- lepton mass dependencies kept explicit in matrix elements
- naive NLO phase-space construction with on-shell projection:
  - radiated momentum according to FKS parametrisation
  - keep IS momenta fixed
  - boost of Born FS into recoiling system of radiated photon

I) NLO framework in WHIZARD: NLO EW

pp processes: Validation for

- $pp \to VV(H)$, $VH(H)$, $t\bar{t}(V/H)$ for $V \in \{W^{\pm}, Z\}$ with MATRIX/MUNICH orig. ref. [1412.5157]
- $pp \to e^{+}\nu_e(H/j/\mu^{-}\bar{\nu}_\mu)$, $e^{+}e^{-}(H/j/\mu^{+}\mu^{-})$, Hjj, tj with MG5_aMC@NLO [1804.10017]

Lepton-collider processes: Checks with MCSANCee, e. g.

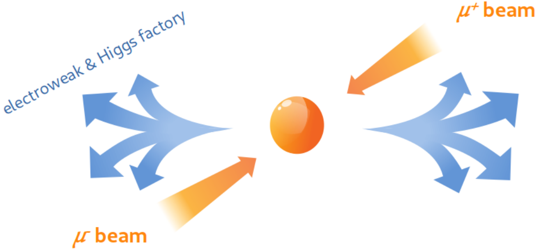
$e^{+}e^{-} \to HZ$	MCSANCee[Sadykov,2020]		WHIZARD+RECOLA			σ^{sig}
$\sqrt{s}$ [GeV]	$\sigma^{\mathrm{tot}}_{\mathrm{LO}}$ [fb]	$\sigma^{\mathrm{tot}}_{\mathrm{NLO}}$ [fb]	$\sigma^{\mathrm{tot}}_{\mathrm{LO}}$ [fb]	$\sigma^{\mathrm{tot}}_{\mathrm{NLO}}$ [fb]	δ_{EW} [%]	LO/NLO
250	225.59(1)	206.77(1)	225.60(1)	207.0(1)	−8.25	0.4/2.1
500	53.74(1)	62.42(1)	53.74(3)	62.41(2)	+16.14	0.2/0.3
1000	12.05(1)	14.56(1)	12.0549(6)	14.57(1)	+20.84	0.5/0.5

$e^{+}e^{-} \to \mu^{+}\mu^{-}$	MCSANCee[2206.09469]		WHIZARD+RECOLA			σ^{sig}
$\sqrt{s}$ [GeV]	$\sigma^{\mathrm{tot}}_{\mathrm{LO}}$ [pb]	$\sigma^{\mathrm{tot}}_{\mathrm{NLO}}$ [pb]	$\sigma^{\mathrm{tot}}_{\mathrm{LO}}$ [pb]	$\sigma^{\mathrm{tot}}_{\mathrm{NLO}}$ [pb]	δ_{EW} [%]	LO/NLO
5	2978.6(1)	3434.2(1)	2978.7(1)	3433.5(3)	+15.27	0.3/2.2
7	1519.6(1)	1773.8(1)	1519.605(4)	1773.1(2)	+16.68	0.05/3.0

II) Application of fixed $\mathcal{O}(\alpha)$ corrections to muon-collider processes

II) Multi-boson processes at a muon collider at NLO EW

[PB, W. Kilian, J. Reuter, P. Stienemeier; JHEP 12 (2022)]



large IS mass:

- large scales (multi-TeV)

→ high new physics discovery potential: Scanning for BSM theories related to $(g-2)_\mu$

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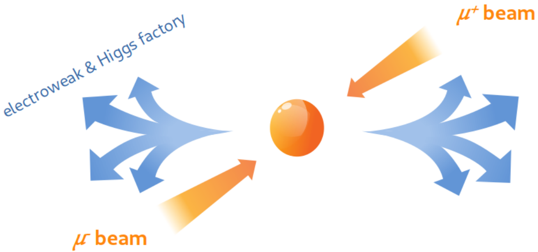

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- reduced Bremsstrahlung; 'leading log. term beyond NLO' $\sim (\alpha/\pi)^2 \log^2(Q^2/m^2) \sim 0.1\%$ sufficiently small

→ fixed $\mathcal{O}(\alpha)$ expansion viable

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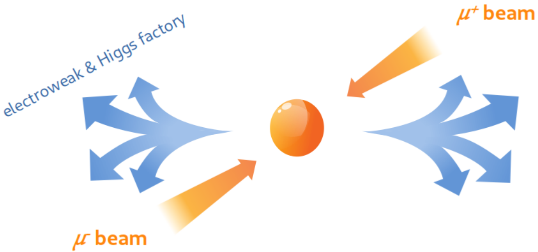

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⇒ Fixed-order massive approximation for $\mu^+\mu^- \to V^n H^m$ with $V \in \{W^\pm Z\}$ and $n + m \leq 4$ at NLO EW

II) Multi-boson processes at a muon collider at NLO EW

[PB, W. Kilian, J. Reuter, P. Stienemeier; JHEP 12 (2022)]

WHIZARD+RECOLA, G_μ scheme, $m_\mu = 0.1056...$ GeV

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\text{incl}}_{\text{LO}}$ [fb]	δ_{EW} [%]	δ_{ISR} [%]
W^+W^-	$4.6591(2) \cdot 10^2$	$+4.0(2)$	$+13.82(4)$
ZZ	$2.5988(1) \cdot 10^1$	$+2.19(6)$	$+15.71(4)$
HZ	$1.3719(1) \cdot 10^0$	$-1.51(4)$	$+30.24(3)$
W^+W^-Z	$3.330(2) \cdot 10^1$	$-22.9(2)$	$+2.90(9)$
W^+W^-H	$1.1253(5) \cdot 10^0$	$-20.5(2)$	$+7.10(8)$
ZZZ	$3.598(2) \cdot 10^{-1}$	$-25.5(3)$	$+5.24(8)$
HZZ	$8.199(4) \cdot 10^{-2}$	$-19.6(3)$	$+8.39(8)$
HHZ	$3.277(1) \cdot 10^{-2}$	$-25.2(1)$	$+7.58(7)$
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$-33.1(4)$	$-1.3(1)$
W^+W^-ZZ	$1.209(1) \cdot 10^0$	$-42.2(6)$	$-1.8(1)$
W^+W^-HZ	$8.754(8) \cdot 10^{-2}$	$-30.9(5)$	$-0.1(1)$
W^+W^-HH	$1.058(1) \cdot 10^{-2}$	$-38.1(4)$	$+1.7(1)$
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$-42.2(2)$	$+0.8(1)$
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$-34.4(2)$	$+1.4(1)$
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$-36.5(2)$	$+2.2(1)$
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$-25.7(2)$	$+5.7(1)$

with $\delta_{\text{EW}} = \sigma^{\text{incl}}_{\text{NLO}}/\sigma^{\text{incl}}_{\text{LO}} - 1$ and $\delta_{\text{ISR}} = \sigma^{\text{incl}}_{\text{LO,LL-ISR}}/\sigma^{\text{incl}}_{\text{LO}} - 1$

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$$\Lambda_{\text{EW,Sud}} \sim -\frac{\alpha}{8\pi} \sum_{k,l\neq k} \sum_{a=\gamma,Z,W} I^a(k) I^{\bar{a}}(l) \log^2 \frac{(p_k+p_l)^2}{M_W^2} \Rightarrow \text{virtual } \mathcal{V}$$

with $\delta_{\text{EW}} = \sigma^{\text{incl}}_{\text{NLO}}/\sigma^{\text{incl}}_{\text{LO}} - 1$ and $\delta_{\text{ISR}} = \sigma^{\text{incl}}_{\text{LO,LL-ISR}}/\sigma^{\text{incl}}_{\text{LO}} - 1$

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$$\text{LL PDF } \Gamma^{\text{LL}(1)}_{\mu/\mu} \sim \frac{\alpha}{2\pi} \log \frac{s}{m_\mu^2} \Rightarrow \text{real } \mathcal{R}$$

with $\delta_{\text{EW}} = \sigma^{\text{incl}}_{\text{NLO}}/\sigma^{\text{incl}}_{\text{LO}} - 1$ and $\delta_{\text{ISR}} = \sigma^{\text{incl}}_{\text{LO,LL-ISR}}/\sigma^{\text{incl}}_{\text{LO}} - 1$

II) Multi-boson processes at a muon collider at NLO EW

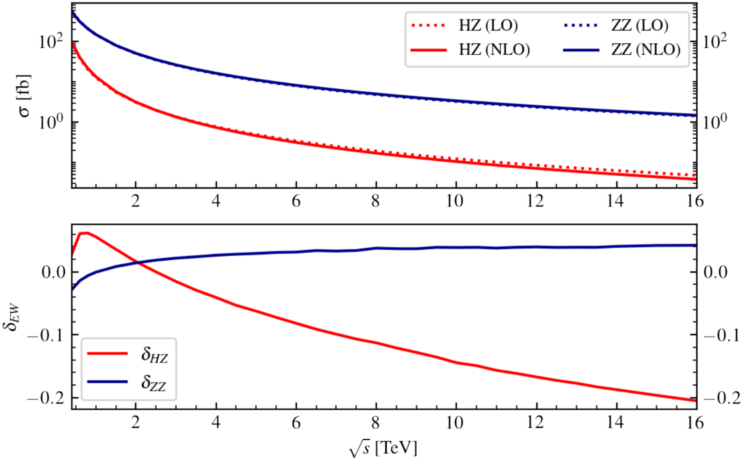
[PB, W. Kilian, J. Reuter, P. Stienemeier; JHEP 12 (2022)]

$\mu^+\mu^- \to X$	$\sqrt{s} = 10$ TeV		$\sqrt{s} = 14$ TeV	
	$\sigma_{\text{LO}}^{\text{incl}}$ [fb]	δ_{EW} [%]	$\sigma_{\text{LO}}^{\text{incl}}$ [fb]	δ_{EW} [%]
W^+W^-	$5.8820(2) \cdot 10^1$	$+3.9(2)$	$3.2423(1) \cdot 10^1$	$+3.6(2)$
ZZ	$3.2730(4) \cdot 10^0$	$+3.9(1)$	$1.80357(9) \cdot 10^0$	$+3.8(2)$
HZ	$1.22929(8) \cdot 10^{-1}$	$-14.12(7)$	$6.2702(4) \cdot 10^{-2}$	$-18.7(1)$
W^+W^-Z	$9.609(5) \cdot 10^0$	$-39.0(2)$	$6.369(3) \cdot 10^0$	$-45.0(4)$
W^+W^-H	$2.1263(9) \cdot 10^{-1}$	$-38.4(5)$	$1.2846(6) \cdot 10^{-1}$	$-43.3(9)$
ZZZ	$8.565(4) \cdot 10^{-2}$	$-38.5(9)$	$5.475(3) \cdot 10^{-2}$	$-44.2(6)$
HZZ	$1.4631(6) \cdot 10^{-2}$	$-34.9(4)$	$8.754(4) \cdot 10^{-3}$	$-39.7(4)$
HHZ	$6.083(2) \cdot 10^{-3}$	$-51.6(5)$	$3.668(1) \cdot 10^{-3}$	$-59.4(3)$

Suppression due to EW Sudakov logarithms at high energies pronounced for **(di-)Higgsstrahlung!**

II) Multi-boson processes at a muon collider at NLO EW

[PB, W. Kilian, J. Reuter, P. Stienemeier; JHEP 12 (2022)]



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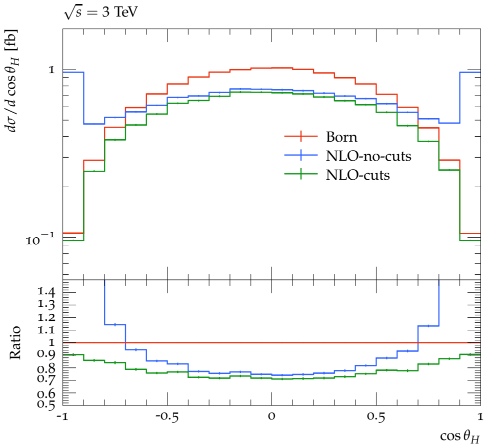
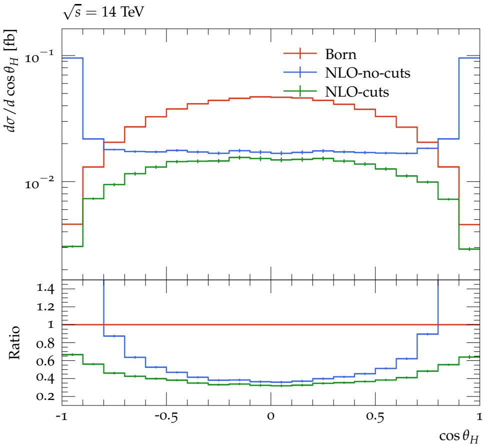
Fixed order differential distributions: $d\sigma(\mu^+\mu^- \to HZ)/d\cos\theta_H$





'NLO-cuts': phase-space cut on hard photons occuring at NLO: $E_\gamma < 0.7\sqrt{s}$

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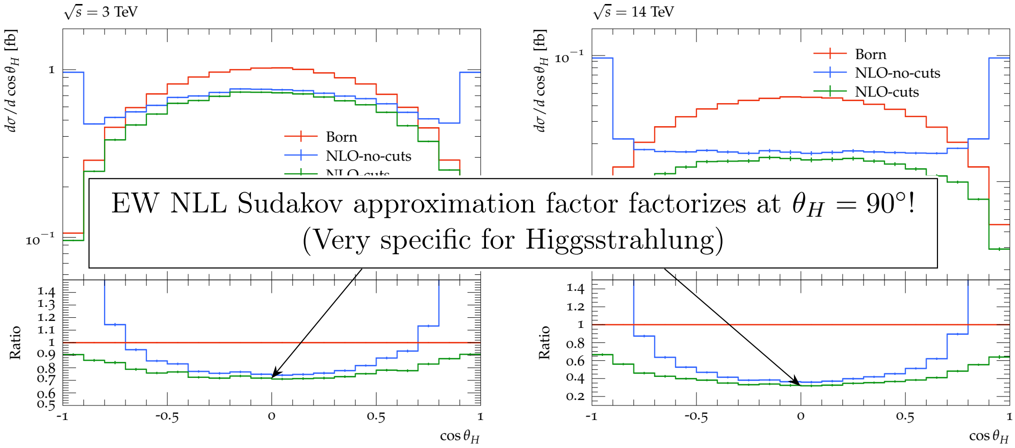

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II) Multi-boson processes at a muon collider at NLO EW

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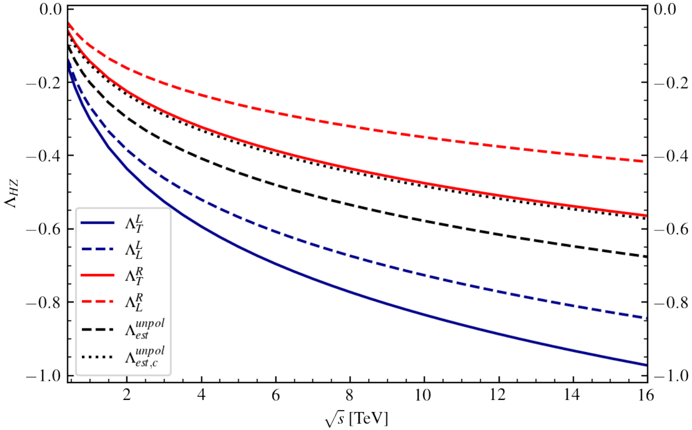
NLL EW $\mu^+\mu^- \to HZ$ Sudakov factor: $\to \Lambda_{\text{est}}^{\text{unpol}}(\theta_H = 90°)$ (black dashed line)



- Λ_λ^κ: Sudakov factors for muon chiralities $\kappa = L, R$ and Z polarisations $\lambda = T, L$
- $\Lambda_{\text{est}}^{\text{unpol}}$: estimated unpolarised correction factor at $\theta_H = 90°$
- $\Lambda_{\text{est,c}}^{\text{unpol}}$: $\Lambda_{\text{est}}^{\text{unpol}}$ without angular dependent terms

III) Status:
NLL electron PDFs for lepton-collider processes at NLO EW

III) Precision of lepton-collider observables

Soft photon radiation: $\sim (\alpha/\pi)^k \log^k \left(\frac{s}{E_\gamma^2}\right)$ ($\rightarrow$ YFS resummation)

Collinear ISR:

$$\sim \underbrace{\underbrace{(\alpha/\pi)^k \log^k \left(\frac{s}{m^2}\right)}_{\text{LL}} + (\alpha/\pi)^k \log^{k-1} \left(\frac{s}{m^2}\right)}_{\text{NLL}} + \ldots$$

$\Rightarrow$ Observables unreliable for $s \gg m^2$ unless resumming these logs!

III) Collinear factorisation

$$d\bar{\sigma}_{e^+e^-}(p_{e^+}, p_{e^-}, m^2) = \sum_{ij=e^\pm,\gamma} \int dz_+ dz_- \Gamma_{i/e^+}(z_+, \mu^2, m^2) \Gamma_{j/e^-}(z_-, \mu^2, m^2)$$
$$\times\, d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2)$$

- Absorb all resummed collinear logarithms (mass dependencies!) into $\Gamma_{i/e}$
  - $d\bar{\sigma}_{e^+e^-}$ at LO+LL accuracy:

$$d\bar{\sigma}^{\mathrm{LO+LL}}_{e^+e^-} = \Gamma^{\mathrm{LO+LL}}_{e^+/e^+} \star \Gamma^{\mathrm{LO+LL}}_{e^-/e^-} \star d\hat{\sigma}^{\mathrm{LO}}_{e^+e^-}$$

with $\Gamma^{\mathrm{LO+LL}}_{e/e}$ from [Cacciari, Deandrea, Montagna, Nicrosini, 1992; Skrypek, Jadach, 1991]

  - $d\bar{\sigma}_{e^+e^-}$ at NLO+NLL accuracy:

$$d\bar{\sigma}^{\mathrm{NLO+NLL}}_{e^+e^-} = \sum_{ij=e^\pm,\gamma} \Gamma^{\mathrm{NLO+NLL}}_{i/e^+} \star \Gamma^{\mathrm{NLO+NLL}}_{j/e^-} \star d\hat{\sigma}^{\mathrm{NLO}}_{ij}$$

with $\Gamma^{\mathrm{NLO+NLL}}_{j/e^-}$ from [Frixione, 1909.03886; Bertone, Cacciari, Frixione, Stagnitto, 1911.12040]

III) NLL electron PDFs for lepton-collider processes at NLO EW

$$d\bar{\sigma}_{e^+e^-}^{\text{NLO+NLL}} = \Gamma_{i/e^+}^{\text{NLO+NLL}} \star \Gamma_{j/e^-}^{\text{NLO+NLL}} \star d\hat{\sigma}_{ij}^{\text{NLO}}$$

- accurate enough for any IS lepton mass and any collider energy
- electron PDFs have **singular** behavior at $z \simeq 1$

$\Rightarrow$ requires appropriate MC integration methods to ensure numerical stability in $z \to 1$ regions

<u>In `WHIZARD`:</u>

- $d\bar{\sigma}_{e^+e^-}^{\text{LO+LL}}$ with $\Gamma_{i/e}^{\text{LO+LL}}$ in the form of (well-known) 'ISR structure functions'
- $d\bar{\sigma}_{e^+e^-}^{\text{NLO+NLL}}$ work-in-progress with $\Gamma_{i/e}^{\text{NLO+NLL}}$ implemented
  (bottle-neck: adequate mapping of random numbers to beam-energy fractions at NLO)

Summary

I) Automated computation of NLO EW corrections in `WHIZARD`
   - → well-tested for pp processes
   - → fixed-order EW corrections to lepton collider processes

II) Application of fixed $\mathcal{O}(\alpha)$ corrections to muon-collider processes
   - → EW corrections highly significant for multi-TeV scales and high boson multiplicities at a future muon collider

III) NLL electron PDFs for lepton-collider processes at NLO EW
   - → motivated for a fully generic usage of EW corrections for any lepton-collider setup
   - → work to be done in `WHIZARD` w.r.t. a decent treatment of the numerics

Back-Up

1. FKS subtraction scheme & POWHEG matching
2. Validations
3. Electron/photon PDFs for lepton collisions
4. Complex-mass scheme at NLO
5. NLL EW $\mu^+\mu^- \to HZ$ Sudakov factor
6. `WHIZARD` features

Back-Up: FKS subtraction scheme

FKS parametrisation:

For $2 \to n$ processes: integrands parametrised by

Φ_n for $\mathcal{B}$, $\mathcal{V}$, $d\sigma_{S,\text{int}}$ and $\Phi_{n+1} = (\Phi_n, \Phi_{\text{rad}})$ for $\mathcal{R}$, $d\sigma_S$

$$\text{FKS variables:} \quad \Phi_{\text{rad}} \to \{\xi, y, \phi\}$$

$$d\Phi_{n+1} = d\Phi_{\text{rad}} d\Phi_n = \underbrace{\mathcal{J}(\xi, y, \phi)}_{\text{Jacobian}} d\xi dy d\phi d\Phi_n$$

with $\xi \equiv 2E_{\text{rad}}/\sqrt{s}$, $y \equiv \cos\theta_{ij}$ and ϕ: angle difference in transversal plane

$$\text{collinear limit:} \quad y \to 1$$
$$\text{soft limit:} \quad \xi \to 0$$

Back-Up: FKS subtraction scheme

IR cancellation:

- Define:

$$\mathcal{R}_{(i,j)} = \mathcal{S}_{(i,j)}\mathcal{R}$$

with $\mathcal{S}_{(i,j)}$ depending on the kinematics of (i,j), $\sum_{i,j}\mathcal{S}_{(i,j)} = 1$ and $\lim_{y\to 1}\mathcal{S}_{(i,j)} = 1$, $\lim_{\xi\to 0}\mathcal{S}_{(i,j)} = \mathcal{S}^{\text{soft}}_{(i,j)}$

Subtraction:

$$\tilde{\mathcal{R}}(\xi,y) \equiv (1-y)\xi^2\mathcal{R}(\xi,y)$$

$$\frac{\hat{\tilde{\mathcal{R}}}_{(i,j)}(\xi,y)}{\xi^2(1-y)} = \frac{1}{\xi^2(1-y)}\left(\tilde{\mathcal{R}}_{(i,j)}(\xi,y) - \underbrace{\tilde{\mathcal{R}}_{(i,j)}(0,y)}_{\text{soft}} - \underbrace{\tilde{\mathcal{R}}_{(i,j)}(\xi,1)}_{\text{collinear}} + \underbrace{\tilde{\mathcal{R}}_{(i,j)}(0,1)}_{\text{soft-collinear}}\right)$$

- Subtraction "events" get Born phase-space configuration
  $\Rightarrow$ Mind IR-safe observables for event generation!

$$\lim_{p_i\|p_j} O_{n+1}(p_1,\ldots,p_i,\ldots,p_j,\ldots,p_{n+1}) = O_n(p_1,\ldots,p_{ij},\ldots,p_n)$$

$$\lim_{p_i\to 0} O_{n+1}(p_1,\ldots,p_j,\ldots,p_{n+1}) = O_n(p_1,\ldots,p_{j-1},p_{j+1},\ldots,p_n)$$

Back-Up: FKS subtraction scheme

Subtraction terms: For split. partons $\bar{\mathcal{I}}_i \to \mathcal{I}_i \mathcal{I}_j$ and $k_i^2 = 0$ for emitting parton $\mathcal{I}_i$ after splitting

- collinear limit: unreg. polarised splitting functions $\times$ spin-correlated ME^2

$$\lim_{y\to 1} \tilde{\mathcal{R}}_{(i,j)}(\xi, y) \simeq \tilde{\mathcal{R}}_{(i,j)}(\xi, 1) = \lim_{y\to 1} \frac{8\pi \alpha_s (1-y)\xi^2}{\bar{k}_i^2} \hat{P}^{\mu\nu}_{\bar{\mathcal{I}}_i \to \mathcal{I}_i \mathcal{I}_j, \mathrm{QCD}}(z, k_\perp) \mathcal{B}^{(i)}_{\mu\nu}$$

$$\text{For } \bar{\mathcal{I}}_i = g, \quad \mathcal{B}^{(i)}_{\mu\nu} = N_B \sum_{\{m\}, s_i, s_i'} \mathcal{M}_n(\{m\}, s_i) \mathcal{M}_n^\dagger(\{m\}, s_i')(\epsilon_{s_i})^*_\mu (\epsilon_{s_i'})_\nu$$

  with $\{m\}$ colour, spins of Born conf. and s_i the spin of emitting gluon

- soft limit: eikonal $\times$ color- or charge-correlated Born ME^2

$$\lim_{\xi\to 0} \tilde{\mathcal{R}}_{(i,j)}(\xi, y) \simeq \tilde{\mathcal{R}}_{(i,j)}(0, y) = 4\pi \alpha_s (1-y) \sum_{k,l=1}^{n} \frac{\bar{k}_k \cdot \bar{k}_l}{(\bar{k}_k \cdot \hat{k}_j)(\bar{k}_l \cdot \hat{k}_j)} \mathcal{B}_{kl}$$

$$\mathcal{B}_{kl} = -|\mathcal{M}^n_{kl}|^2 = \langle \mathcal{M}^n | \mathbf{T}_k \cdot \mathbf{T}_l | \mathcal{M}^n \rangle$$

  with $\mathcal{I}_j = g$ the radiated parton and $\mathbf{T}_k$ the colour charge operator

QCD $\to$ QED: $\{g, \alpha_s, \hat{P}^{\mu\nu}_{\bar{\mathcal{I}}_i \to \mathcal{I}_i \mathcal{I}_j, \mathrm{QCD}}, \mathbf{T}_k\} \longrightarrow \{\gamma, \alpha, \hat{P}^{\mu\nu}_{\bar{\mathcal{I}}_i \to \mathcal{I}_i \mathcal{I}_j, \mathrm{QED}}, \mathbf{Q}_k\}$

Back-Up: FKS subtraction scheme

Regularisation by integrated subtraction terms:

From dimensional regularisation with $d = 4 - 2\varepsilon$ and expansions in ε

$\xi^{-1-2\varepsilon} = -\frac{1}{2\varepsilon}\delta(\xi) + \left(\frac{1}{\xi}\right)_+ - 2\varepsilon\left(\frac{\log\xi}{\xi}\right)_+ = -\frac{1}{2\varepsilon}\delta(\xi) + \mathcal{P}_+(\xi)$ and

$(1-y)^{-1-\varepsilon} = -\frac{2^{-\varepsilon}}{\varepsilon}\delta(1-y) + \left(\frac{1}{1-y}\right)_+ - \varepsilon\left(\frac{\log(1-y)}{1-y}\right)_+$ we get

$$
\begin{aligned}
\int d\Phi_{\text{rad}}\mathcal{R} &= \int d\Phi_{\text{rad}}(\xi,y)\frac{\tilde{\mathcal{R}}(\xi,y)}{\xi^2(1-y)} \\
&= \frac{s^{1-\varepsilon}}{(4\pi)^{3-2\varepsilon}}\int d\Omega^{(2-2\varepsilon)}\int_{-1}^{1} dy(1-y)^{-1-\varepsilon}\int_0^{\xi_{\max}} d\xi\xi^{-1-2\varepsilon}\tilde{\mathcal{R}}(\xi,y) \\
&= \underbrace{\frac{I^{(2)}_{\text{soft-coll}}}{\varepsilon^2} + \frac{I^{(1)}_{\text{soft}}}{\varepsilon} + + I^{(0)}_{\text{soft}}}_{①} + \underbrace{\frac{I^{(1)}_{\text{coll}}}{\varepsilon} + I^{(0)}_{\text{coll}}}_{②} + \int \underbrace{d\Phi_{\text{rad}}\hat{\mathcal{R}}}_{③} + \mathcal{O}(\varepsilon)
\end{aligned}
$$

with plus-distributions $\int_{-1}^{1} dy\left(\frac{g(y)}{1-y}\right)_+ f(y) = \int_{-1}^{1} dy g(y)\frac{f(y)-f(1)}{1-y}$

- ① soft (and soft-collinear) limit: $\sim -\frac{1}{2\varepsilon}\int dy(1-y)^{-1-\varepsilon}\int d\xi\delta(\xi)\tilde{\mathcal{R}}(\xi,y)$
- ② collinear limit: $\sim -\frac{2^{-\varepsilon}}{\varepsilon}\int d\xi\mathcal{P}_+(\xi)\int dy\delta(1-y)\tilde{\mathcal{R}}(\xi,y)$
- ③ subtracted Real: $d\phi dy d\xi\frac{\mathcal{J}(\xi,y,\phi)}{\xi}\left(\frac{1}{\xi}\right)_+\left(\frac{1}{1-y}\right)_+\tilde{\mathcal{R}}(\xi,y)$

Backup: FKS & POWHEG matching

(contributors: PB, B. C.-Nejad, W. Kilian, J. Reuter, V. Rothe, P. Stienemeier, C. Weiss)

FKS subtraction per α_r region

$$\mathcal{R} = \sum_{\alpha_r} \mathcal{R}_{\alpha_r} = \sum_{\alpha_r} \mathcal{S}_{\alpha_r} \mathcal{R} \qquad \text{for } \mathcal{I}_{\alpha_r} = (i,j) \in P_{\text{FKS}}(f_r)$$

works in conjunction with POWHEG matching scheme

$$d\sigma_{\text{NLO}} = \bar{\mathcal{B}}(\Phi_n) \left(\Delta(p_{T,\min}) + \Delta(k_T(\Phi_{n+1})) \frac{\mathcal{R}(\Phi_{n+1})}{\mathcal{B}(\Phi_n)} d\Phi_{\text{rad}} \right) d\Phi_n$$

using a *modified* Sudakov form factor

$$\Delta(\Phi_n, p_T) = \exp\left[- \int \frac{\mathcal{R}(\Phi_{n+1})}{\mathcal{B}(\Phi_n)} \theta\left(k_T(\Phi_{n+1}) - p_T\right) d\Phi_{\text{rad}} \right]$$

$$\Delta^{f_\mathcal{B}}(\Phi_n, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_\mathcal{B}\}} \Delta^{f_\mathcal{B}}_{\alpha_r}(\Phi_n, p_T)$$

Validations: NLO EW

Cross-validation of WHIZARD and MUNICH/MATRIX orig. ref. [Kallweit *et. al.*, 1412.5157]

process $pp \to$	MUNICH$_{\text{(CS)}}$ +OpenLoops $\sigma^{\text{tot}}_{\text{NLO}}$ [fb]	WHIZARD +OpenLoops $\sigma^{\text{tot}}_{\text{NLO}}$ [fb]	δ [%]	dev [%]	σ^{sig}
ZZ	$1.05729(1) \cdot 10^4$	$1.05729(11) \cdot 10^4$	-4.20	0.0001	0.01
W^+Z	$1.71505(2) \cdot 10^4$	$1.71507(2) \cdot 10^4$	-0.15	0.001	0.88
W^-Z	$1.08576(1) \cdot 10^4$	$1.08574(1) \cdot 10^4$	$+0.07$	0.001	0.90
W^+W^-	$7.93106(7) \cdot 10^4$	$7.93087(21) \cdot 10^4$	$+4.55$	0.002	0.89
ZH	$6.18523(6) \cdot 10^2$	$6.18533(6) \cdot 10^2$	-5.29	0.002	1.17
W^+H	$7.18070(7) \cdot 10^2$	$7.18072(9) \cdot 10^2$	-2.31	0.0003	0.18
W^-H	$4.59289(4) \cdot 10^2$	$4.59299(5) \cdot 10^2$	-2.15	0.002	1.62
ZZZ	$9.7429(2) \cdot 10^0$	$9.7417(11) \cdot 10^0$	-9.47	0.012	1.01
W^+W^-Z	$1.08288(2) \cdot 10^2$	$1.08293(10) \cdot 10^2$	$+7.67$	0.004	0.45
W^+ZZ	$2.0188(4) \cdot 10^1$	$2.0188(23) \cdot 10^1$	$+1.58$	0.0001	0.01
W^-ZZ	$1.09844(2) \cdot 10^1$	$1.09838(12) \cdot 10^1$	$+3.09$	0.006	0.51
$W^+W^-W^+$	$8.7979(2) \cdot 10^1$	$8.7991(15) \cdot 10^1$	$+6.18$	0.014	0.79
$W^+W^-W^-$	$4.9447(1) \cdot 10^1$	$4.9441(2) \cdot 10^1$	$+7.13$	0.013	2.52
ZZH	$1.91607(2) \cdot 10^0$	$1.91614(18) \cdot 10^0$	-8.78	0.004	0.39
W^+ZH	$2.48068(2) \cdot 10^0$	$2.48095(28) \cdot 10^0$	$+1.64$	0.011	0.96
W^-ZH	$1.34001(1) \cdot 10^0$	$1.34016(15) \cdot 10^0$	$+2.51$	0.011	1.02
W^+W^-H	$9.7012(2) \cdot 10^0$	$9.700(2) \cdot 10^0$	$+9.83$	0.014	0.75
ZHH	$2.39350(2) \cdot 10^{-1}$	$2.39337(32) \cdot 10^{-1}$	-11.06	0.005	0.41
W^+HH	$2.44794(2) \cdot 10^{-1}$	$2.44776(24) \cdot 10^{-1}$	-12.04	0.007	0.74
W^-HH	$1.33525(1) \cdot 10^{-1}$	$1.33471(19) \cdot 10^{-1}$	-11.53	0.041	2.80

LHC setup (Run II), $\delta \equiv (\sigma^{\text{tot}}_{\text{NLO}} - \sigma^{\text{tot}}_{\text{LO}})/\sigma^{\text{tot}}_{\text{LO}}$, $\text{dev} \equiv |\sigma^{\text{tot}}_{\texttt{WHIZARD}} - \sigma^{\text{tot}}_{\texttt{MUNICH}}|/\sigma^{\text{tot}}_{\texttt{WHIZARD}}$

Validations: NLO EW

Pure electroweak *pp* processes with off-shell vector bosons

process $pp \to$	α^m	MG5_aMC@NLO[1804.10017] $\sigma^{\text{tot}}_{\text{NLO}}$ [pb]	WHIZARD+OpenLoops $\sigma^{\text{tot}}_{\text{NLO}}$ [pb]	δ [%]	$\sigma^{\text{sig}}_{\text{NLO}}$
$e^+\nu_e$	α^2	$5.2005(8)\cdot 10^3$	$5.1994(4)\cdot 10^3$	-0.73	1.24
e^+e^-	α^2	$7.498(1)\cdot 10^2$	$7.498(1)\cdot 10^2$	-0.50	0.004
$e^+\nu_e\mu^-\bar{\nu}_\mu$	α^4	$5.2794(9)\cdot 10^{-1}$	$5.2816(9)\cdot 10^{-1}$	$+3.69$	1.69
$e^+e^-\mu^+\mu^-$	α^4	$1.2083(3)\cdot 10^{-2}$	$1.2078(3)\cdot 10^{-2}$	-5.25	1.26
$He^+\nu_e$	α^3	$6.4740(17)\cdot 10^{-2}$	$6.4763(6)\cdot 10^{-2}$	-4.04	1.24
He^+e^-	α^3	$1.3699(2)\cdot 10^{-2}$	$1.3699(1)\cdot 10^{-2}$	-5.86	0.32
Hjj	α^3	$2.7058(4)\cdot 10^0$	$2.7056(6)\cdot 10^0$	-4.23	0.27
tj	α^2	$1.0540(1)\cdot 10^2$	$1.0538(1)\cdot 10^2$	-0.72	0.74

LHC setup (Run II): $\sqrt{s} = 13$ TeV $\mu_R = \mu_F = \frac{1}{2}\sum_i \sqrt{p^2_{T,i} + m^2_i}$ EW scheme: G_μ CMS

PDF set: `LUXqed_plus_PDF4LHC15_nnlo_100` cuts from ref. [1804.10017]

Validations: NLO EW and mixed

Cross-validation with `MUNICH/MATRIX` using `OpenLoops` for $pp \to t\bar{t}$ and $pp \to t\bar{t} + W^{\pm}/Z/H$ with complete NLO SM corrections, e. g.

$pp \to t\bar{t}W^+$	$\alpha_s^n \alpha^m$	σ^{tot} [fb] MUNICH$_{\text{(CS)}}$	σ^{tot} [fb] WHIZARD	σ^{sig} / *dev* MUNICH$_{\text{(CS)}}$-WHIZARD
LO_{21}	$\alpha_s^2\alpha$	$2.411403(1) \cdot 10^2$	$2.4114(1) \cdot 10^2$	0.72 / 0.003%
LO_{12}	$\alpha_s\alpha^2$	0.000	0.000	0.00 / 0.000%
LO_{03}	α^3	$2.31909(1) \cdot 10^0$	$2.3193(1) \cdot 10^0$	1.76 / 0.009%
δNLO_{31}	$\alpha_s^3\alpha$	$1.18993(2) \cdot 10^2$	$1.1905(5) \cdot 10^2$	1.06 / 0.048%
δNLO_{22}	$\alpha_s^2\alpha^2$	$-1.09511(9) \cdot 10^1$	$-1.0947(3) \cdot 10^1$	1.13 / 0.035%
δNLO_{13}	$\alpha_s\alpha^3$	$2.93251(3) \cdot 10^1$	$2.9334(8) \cdot 10^1$	1.14 / 0.030%
δNLO_{04}	α^4	$5.759(3) \cdot 10^{-2}$	$5.756(4) \cdot 10^{-2}$	0.58 / 0.049%

Non-negligible and even enhanced EW effects for α_s subleading contributions at NLO!

($pp \to b\bar{b}X$ in validation progress)

Validations: NLO EW and mixed

Comparison with MG5_aMC@NLO for $pp \to e^+\nu_e j$ and $pp \to e^+e^-j$ at NLO EW

process $pp \to Xj$	$\alpha_s^n\alpha^m$	MG5_aMC@NLO $\sigma^{\mathrm{tot}}_{\mathrm{LO}}$ [pb]	MG5_aMC@NLO $\sigma^{\mathrm{tot}}_{\mathrm{NLO}}$ [pb]	WHIZARD+OpenLoops $\sigma^{\mathrm{tot}}_{\mathrm{LO}}$ [pb]	WHIZARD+OpenLoops $\sigma^{\mathrm{tot}}_{\mathrm{NLO}}$ [pb]	WHIZARD+OpenLoops δ [%]	σ^{sig} LO/NLO
$e^+\nu_e j$	$\alpha_s\alpha^2$	914.81(6)	904.75(8)	914.74(7)	904.59(7)	-1.11	0.8/1.5
e^+e^-j	$\alpha_s\alpha^2$	150.59(1)	149.09(2)	150.59(1)	149.08(2)	-1.00	0.05/0.4

LHC-setup (Run II), cuts with photon recombination **and** jet clustering

Back-Up: Electron/photon PDFs for lepton collisions

- LL resummation[Cacciari, Deandrea, Montagna, Nicrosini, 1992; Skrypek, Jadach, 1991]:
  Non-singlet evolution equation

$$\Gamma_e(x,\mu^2) = \delta(1-x) + \int_{m^2}^{\mu^2} \frac{dq^2}{q^2} \frac{\alpha(q^2)}{2\pi} \int_x^1 dz P_{ee}(z) \Gamma_e\left(\frac{x}{z}, q^2\right)$$

One-loop accurate regularised (unpolarised) Altarelli-Parisi kernels

$$P_{ee}(z) = \langle \hat{P}_{ee} \rangle(z) - \delta(1-z) \int_0^1 dt \langle \hat{P}_{ee} \rangle(t), \qquad \langle \hat{P}_{ee} \rangle(z) = \frac{1+z^2}{1-z}$$

Recursive approach via auxiliary function $G(x,\mu^2)$

$$G(x,\mu^2) = \int_x^1 dt \Gamma_e(t,\mu^2) \qquad \Gamma_e(x,\mu^2) = -\frac{\partial}{\partial x} G(x,\mu^2)$$

Solution in asymptotic $x \simeq 1$ limit

$$\Gamma_e(x,\mu^2) = \frac{e^{\eta(\frac{3}{4}-\gamma_E)}}{\Gamma(1+\eta)} \eta (1-x)^{\eta-1} \qquad \eta = \frac{\alpha}{\pi} \log \frac{\mu^2}{m^2}$$

Back-Up: Electron/photon PDFs for lepton collisions

Alternative approach: Identically, transforming the integro-differential evolution equations into Mellin space by

$$M[f] \equiv f_N = \int_0^1 dz z^{N-1} f(z) \qquad\qquad M[g \star h] = M[g]M[h] \quad .$$

$\Rightarrow$ using the asymptotic limit $N \to \infty$ analogously to $z \to 1$ in z-space
$\Rightarrow$ analytical Mellin inversion of the resulting solution
<u>'All x' solution:</u>
$G(x, \mu^2)$ and $\Gamma_e(x, \mu^2)$ can be written as a perturbative series expressed as

$$G(x, \mu^2) = \sum_{n=0}^{\infty} \frac{\eta^n}{2^n n!} I_n(x), \qquad\qquad \Gamma_e(x, \mu^2) = \sum_{n=0}^{\infty} \frac{\eta^n}{2^n n!} \frac{\partial I_n(x)}{\partial x}$$

Find recurrence relation

$$I_n(x) = \int_x^1 dz P(z) I_{n-1}\left(\frac{x}{z}\right)$$

Boundary conditions $G^{(0)}(x, \mu^2) = G(x, m^2) = 1$ implicating $I_0(x) = 1$ and $I_1(x) = \int_x^1 dz P(z)$
$\Rightarrow$ 'all x' solution for G and Γ_e up to $\mathcal{O}(\alpha^3)$ by iterations up to I_3

Back-Up: Electron/photon PDFs for lepton collisions

- NLO initial conditions of electron and photon PDFs [Frixione, 1909.03886]:
  Approach:

$$d\bar{\sigma}_{e^+e^-}(p_{e^+}, p_{e^-}, m^2) = \sum_{ij=e^\pm,\gamma} \int dz_+ dz_- \Gamma_{i/e^+}(z_+, \mu^2, m^2) \Gamma_{i/e^-}(z_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) \quad (1)$$

  - explicit short-distance cross section computation for specific but arbitrary process $e^+e^- \to u\bar{u}(\gamma)$
  - parton-level cross section $d\hat{\sigma}_{ij}$ computed with massless electrons
  - particle-level cross section $d\bar{\sigma}_{kl} = d\sigma_{kl} + \mathcal{O}\left(\left(\frac{m^2}{s}\right)^p\right)$, $p \geq 1$

  → (7) solved for PDFs Γ_{i/e^-},Γ_{i/e^+}
- NLL resummation [Bertone, Cacciari, Frixione, Stagnitto, 1911.12040]:
  Recursive solutions valid for all z values computed up to $\mathcal{O}(\alpha^3)$ matched to the asymptotic large z solution (valid for $z \simeq 1$) retaining all orders in α

Back-Up: Complex-mass scheme at NLO

Renormalised self-energy:

$$\hat{\Sigma}^i(p^2) = \Sigma^i(p^2) - \delta M_i^2$$

Complex location of the pole $p^2 = \mu_i^2$ of propagator: $\mu_i^2 - M_{0,i}^2 + \Sigma(\mu_i^2) = 0 \quad \Rightarrow \hat{\Sigma}^i(\mu_i^2)$ vanishes

$\Rightarrow$ renormalised masses $M_i^2 = M_{0,i}^2 - \delta M_i^2$ fixed at this pole due to OS condition $\delta M_i^2 = \Sigma(p^2)|_{p^2=\mu_i^2}$

Complex-mass scheme requires calculating self-energies for complex squared momenta!

Solutions:

- analytic continuation of the self-energies in the complex momentum variable to the unphysical Riemann sheet (`MadLoop`)[Frederix *et. al.*, 1804.10017]
- expansion of self-energies around real arguments such that one-loop accuracy is retained (`OpenLoops`, `Recola`) [Denner *et. al.*, 0505042]
  - 2-point integrals with $p^2 = \mu_i^2 = M_i^2 - i\Gamma_i M_i$ can be obtained through first-order expansion in Γ_i/M_i around $p^2 = M_i^2$

Back-Up: NLL EW $\mu^+\mu^- \to HZ$ Sudakov factor

Using the abbreviations for double and single logarithmic factors

$$L(s, M_W^2) = \frac{\alpha}{4\pi} \log^2 \frac{s}{M_W^2} \qquad l(s, M_W^2) = \frac{\alpha}{4\pi} \log \frac{s}{M_W^2}$$

For $s \gg M_W$, leading logarithmic, angular-independent, terms (from exchange of soft-collinear gauge bosons between pairs of external legs)

$$\Lambda_{l,\lambda}^{\kappa} = A_\lambda^\kappa \, L(s, M_W^2) + B_\lambda^\kappa \, \log \frac{M_Z^2}{M_W^2} l(s, M_W^2) + C_\lambda$$

with $\lambda = T, L$ the transverse and longitudinal polarisation of the Z boson, and $\kappa = L, R$ the muon initial state chirality

$$\begin{aligned} A_T^\kappa &= -\frac{1}{2}\left[2C_{\mu^\kappa}^{\mathrm{EW}} + C_\Phi^{\mathrm{EW}} + C_{ZZ}^{\mathrm{EW}}\right] & A_L^\kappa &= -\left[C_{\mu^\kappa}^{\mathrm{EW}} + C_\Phi^{\mathrm{EW}}\right] \\ B_T^\kappa &= 2(I_{\mu_\kappa}^Z)^2 + (I_H^Z)^2 & B_L^\kappa &= 2\left[(I_{\mu_\kappa}^Z)^2 + (I_H^Z)^2\right] \\ C_T &= \delta_H^{LSC,h} & C_L &= \delta_H^{LSC,h} + \delta_\chi^{LSC,h} \end{aligned} .$$

Back-Up: NLL EW $\mu^+\mu^- \to HZ$ Sudakov factor

Subleading, angular-dependent, terms due to $W^\pm$ boson exchange between initial- and final-state legs

$$\Lambda^{\kappa}_{\theta,\lambda} = -\delta_{\kappa L} \frac{D_\lambda}{I^Z_{\mu_\kappa}}\, l(s, M_W^2) \left[\log\frac{|t|}{s} + \log\frac{|u|}{s}\right]$$

Mandelstam variables t and u approximated in the high-energy limit

$$t = \ (p_{\mu^+} - p_H)^2 \sim -\frac{s}{2}(1 - \cos\theta_H) \qquad u = \ (p_{\mu^+} - p_Z)^2 \sim -\frac{s}{2}(1 + \cos\theta_H)$$

Back-Up: NLL EW $\mu^+\mu^- \to HZ$ Sudakov factor

Estimation for the unpolarised approximation factor:

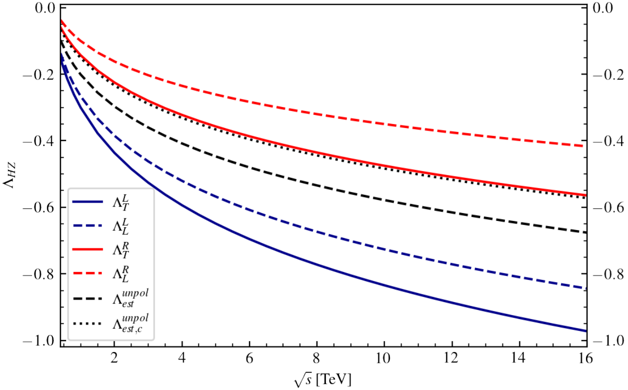
- Born amplitudes for transverse polarized Z bosons are suppressed by M_Z^2/s

$$\Lambda_\lambda^\kappa \mathcal{M}_0^{\mu_\kappa^+\mu_\kappa^- \to HZ_\lambda} \xrightarrow{s \gg M_W^2} \delta_{\lambda L} \Lambda_\lambda^\kappa \mathcal{M}_0^{\mu_\kappa^+\mu_\kappa^- \to HZ_\lambda} \tag{2}$$

- Chirality and helicity of the muon coincide in the ultrarelativistic limit (two helicity configurations $(+,-)$ and $(-,+)$ remaining, equivalent to chiralities $\kappa = L, R$). Spin-averaging yields

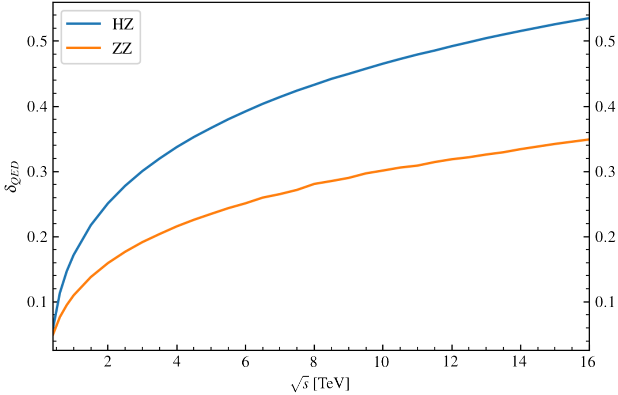
$$\Lambda_{\text{est}}^{\text{unpol}} = \frac{\sum_\kappa \Lambda_L^\kappa |\mathcal{M}_0^{\mu_\kappa^+\mu_\kappa^- \to HZ_L}|^2}{|\mathcal{M}_0^{\mu^+\mu^- \to HZ_L}|^2} \tag{3}$$

Back-Up: NLL EW $\mu^+\mu^- \to HZ$ Sudakov factor



- Λ_λ^κ: Sudakov factors for muon chiralities $\kappa = L, R$ and Z polarisations $\lambda = T, L$
- $\Lambda_{\text{est}}^{\text{unpol}}$: estimated unpolarised correction factor at $\theta_H = 90°$
- $\Lambda_{\text{est,c}}^{\text{unpol}}$: $\Lambda_{\text{est}}^{\text{unpol}}$ without angular dependent terms

Back-Up: NLO QED corrections to $\mu^+\mu^- \rightarrow HZ/ZZ$



Relative QED corrections $\delta_{QED} = \sigma^{\text{incl}}_{\text{NLO,QED}}/\sigma^{\text{incl}}_{\text{LO}} - 1$ to HZ and ZZ production at the muon collider as a function of the collider energy, $\sqrt{s}$

Back-Up: WHIZARD features

WHIZARD provides

- phase space evaluation with VAMP2 [Braß *et. al.*: 1811.09711]:
  - twofold self-adaptive multi-channel parametrization
  - OpenMP and MPI for parallelization ⇒ speedup of factor $\mathcal{O}(100)$
- matching to parton showers: POWHEG scheme
- showering and hadronization: PYTHIA6 shipped with WHIZARD, interface between WHIZARD and PYTHIA8
- event formats: LHE, HepMC2/3, Stdhep, LCIO, ...
- special support for lepton collider processes:

beamstrahlung	CIRCE1/CIRCE2 [CPC 101 (1997) 269]
bremsstrahlung	LL resummation via ISR and EPA functions
beam polarization	inclusion for a user-defineable setup