

Physics case for e+eat 500 GeV and above

Georg Weiglein, DESY & UHH Tokyo, 07 / 2024





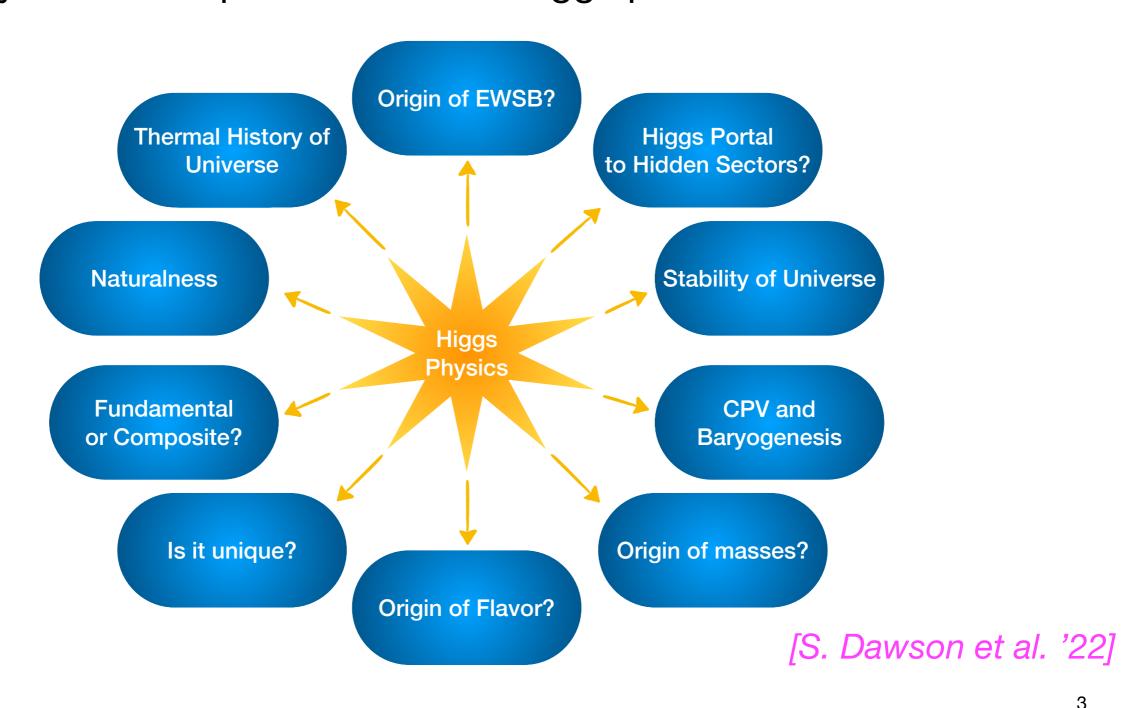
Outline

Introduction

- 500 GeV: guaranteed measurements
- Beyond 500 GeV: guaranteed "measurements"
- Sensitivity to new particles at 500 GeV and beyond
- Conclusions

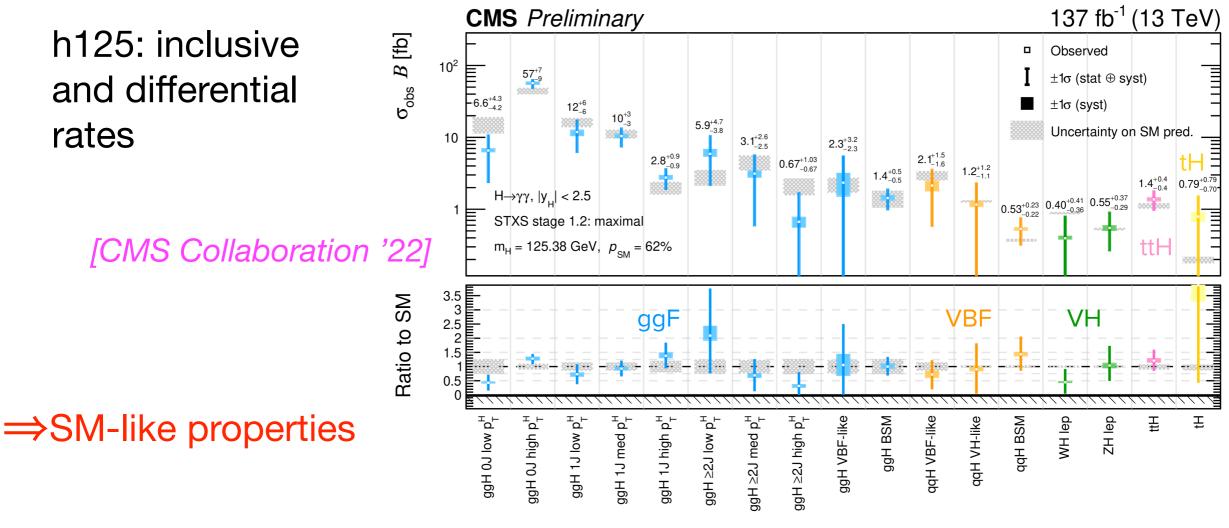
Introduction

Most of the open questions of particle physics are directly related to Higgs physics and in particular to the Higgs potential



Properties of the detected Higgs boson (h125)

The Standard Model of particle physics uses a "minimal" form of the Higgs potential with a single Higgs boson that is an elementary particle

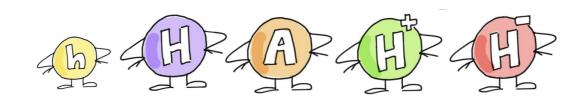


The LHC results on the discovered Higgs boson within the current uncertainties are compatible with the predictions of the Standard Model, but also with a wide variety of other possibilities, corresponding to very different underlying physics

Simple example of extended Higgs sector: 2HDM

Two Higgs doublet model (2HDM):

- **CP conserving** 2HDM with two complex doublets: $\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + \rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}$, $\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + \rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}$



[K. Radchenko '23]

- **Softly broken** \mathbb{Z}_2 **symmetry** $(\Phi_1 \to \Phi_1; \Phi_2 \to \Phi_2)$ entails 4 Yukawa types
- Potential:

$$V_{\text{2HDM}} = m_{11}^2 (\Phi_1^{\dagger} \Phi_1) + m_{22}^2 (\Phi_2^{\dagger} \Phi_2) - m_{12}^2 (\Phi_1^{\dagger} \Phi_2 + \Phi_2^{\dagger} \Phi_1) + \frac{\lambda_1}{2} (\Phi_1^{\dagger} \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^{\dagger} \Phi_2)^2 + \lambda_3 (\Phi_1^{\dagger} \Phi_1) (\Phi_2^{\dagger} \Phi_2) + \lambda_4 (\Phi_1^{\dagger} \Phi_2) (\Phi_2^{\dagger} \Phi_1) + \frac{\lambda_5}{2} ((\Phi_1^{\dagger} \Phi_2)^2 + (\Phi_2^{\dagger} \Phi_1)^2),$$

- Free parameters: m_h , m_H , m_A , $m_{H^{\pm}}$, m_{12}^2 , $\tan \beta$, $\cos(\beta - \alpha)$, v

$$\tan \beta = v_2/v_1 v^2 = v_1^2 + v_2^2 \sim (246 \text{ GeV})^2$$

In alignment limit, $cos(\beta - \alpha) = 0$: h couplings are as in the SM at tree level

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s eigenvalues m_h , m_H , m_A , $m_{H\pm}$ and angle α reaking mass scale

$$M^2 = \frac{2m_3^2}{s_{2\beta}}$$

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$$m_{\Phi}^2 = M^2 + \tilde{\lambda}_{\Phi} v^2, \quad \Phi \in \{H, A, H^{\pm}\}$$

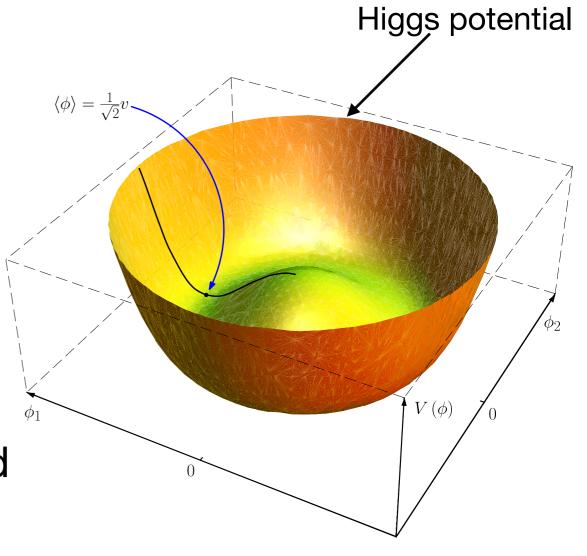
where $M^2 = 2 m_{12}^2 / \sin(2\beta)$

Sizeable splitting between m_{ϕ} and M induces large BSM contributions the Higgs self-couplings (see below)

What is the underlying dynamics of electroweak symmetry breaking?

The vacuum structure is caused by the Higgs field through the Higgs potential. We lack a deeper understanding of this!

We do not know where the Higgs potential that causes the structure of the vacuum actually comes from and which form of the potential is realised in nature. Experimental input is needed to clarify this!



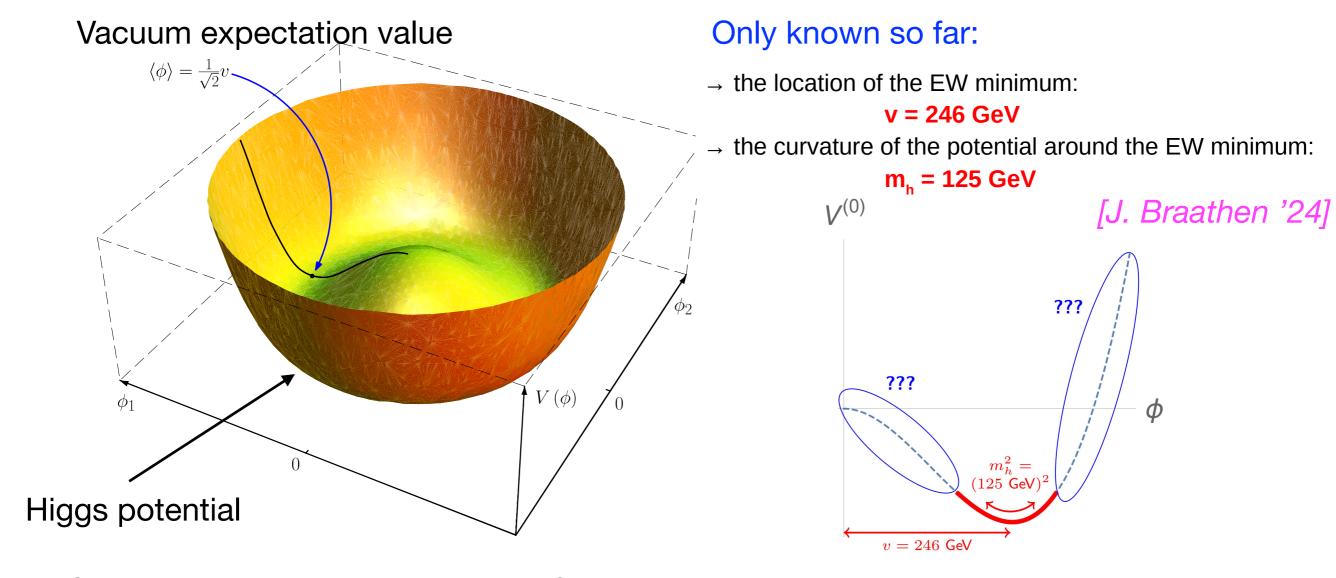
Single doublet or extended Higgs sector? (new symmetry?)

Fundamental scalar or compositeness? (new interaction?)

Higgs potential: the "holy grail" of particle physics



Crucial questions related to electroweak symmetry breaking: what is the form of the Higgs potential and how does it arise?

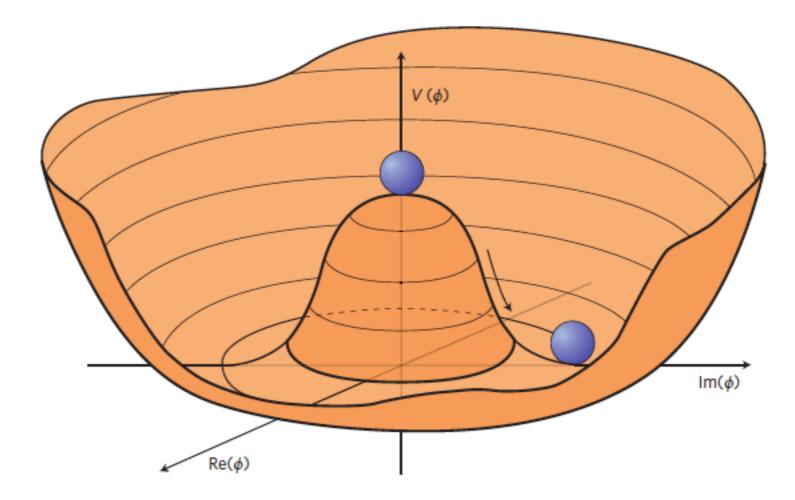


Information can be obtained from the trilinear and quartic Higgs self-couplings, which will be a main focus of the experimental and theoretical activities in particle physics during the coming years

Higgs potential: the "holy grail" of particle physics



The simple picture



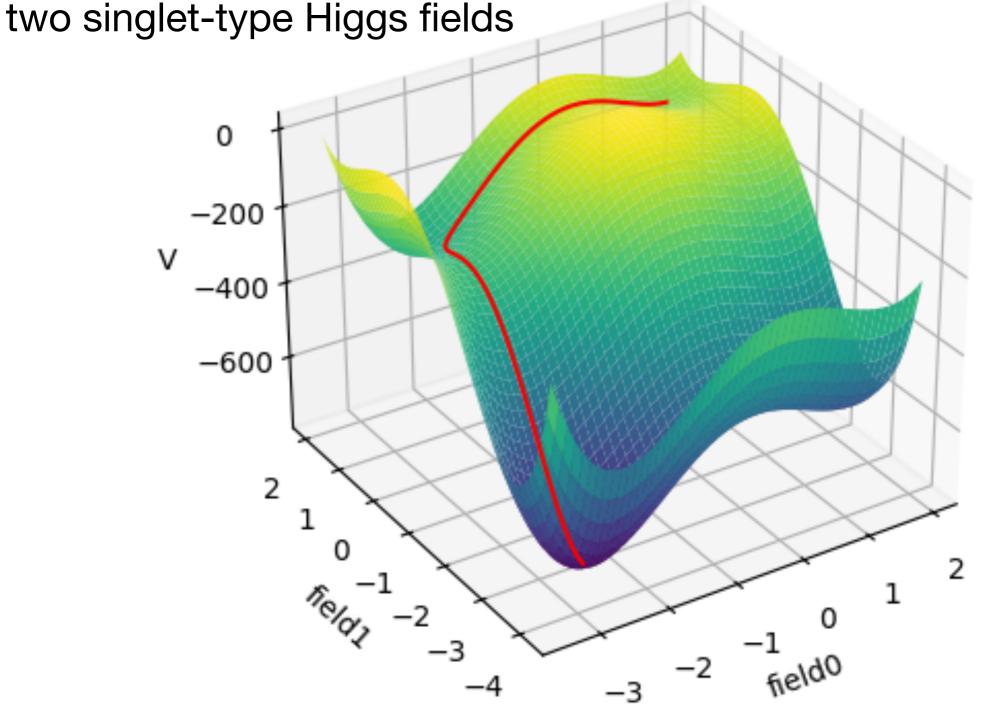
refers to the case of a single Higgs doublet field

If more than one scalar field is present, the Higgs potential is a multidimensional function of the components of the different scalar fields

The Higgs potential and vacuum stability

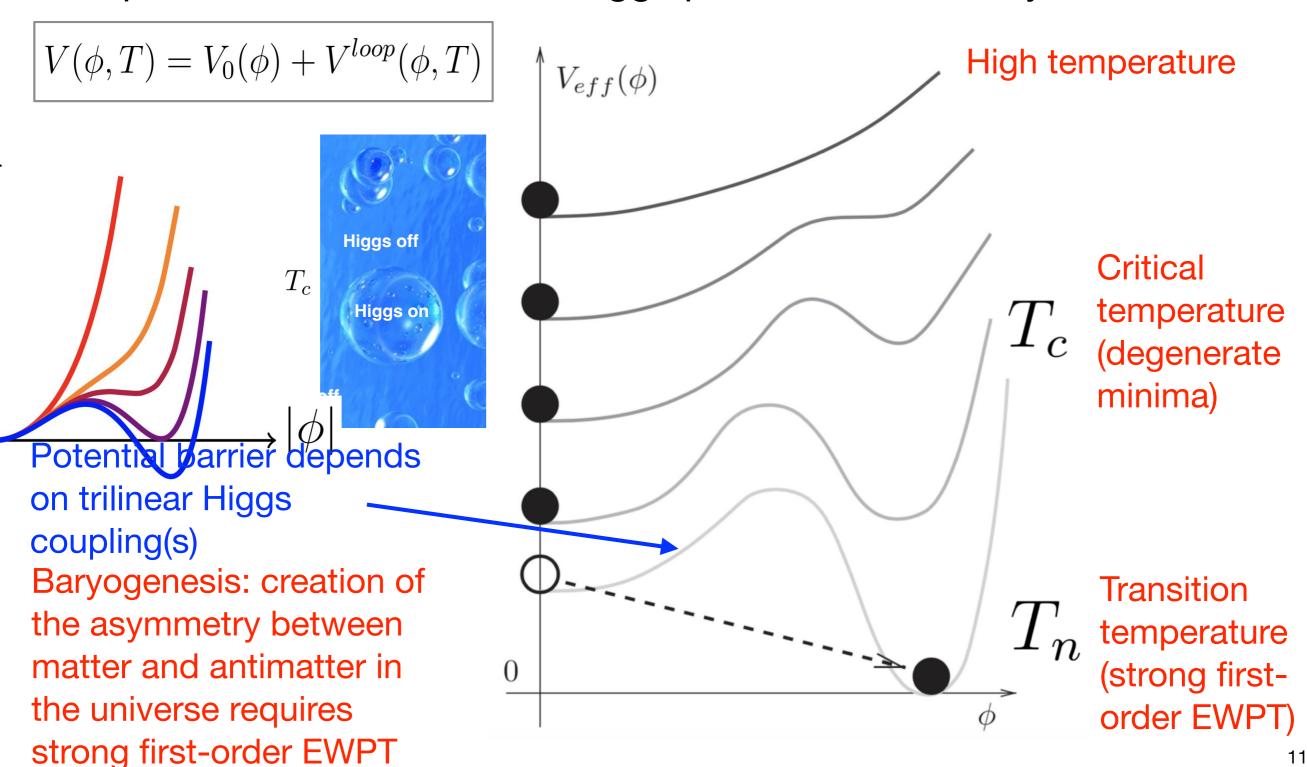
[T. Biekötter, F. Campello, G. W. '24]

Tunneling from a local minimum into the global minimum: toy example,



The Higgs potential and the electroweak phase transition (EWPT)

[D. Gorbunov, V. Rubakov]
Temperature evolution of the Higgs potential in the early universe:



Higgs couplings to fermions and gauge bosons: the quest for identifying the underlying physics

- Future Higgs factories: what can we learn from the enhanced precision (~factor 10 better than HL-LHC) in comparison to the direct searches at the HL-LHC (existing limits and future prospects)?
- How significant will possible patterns of deviations be? How stringent are indirect hints for additional particles (typically scale like coupling/mass²)?
- How well can one distinguish between different realisations of possible BSM physics?

Questions of this kind have hardly been touched upon, for instance, at the previous update of the European Strategy for Particle Physics, but they are crucial for making the case for a (low-energy) e+e- Higgs factory in the wider scientific community!

500 GeV: guaranteed measurements

- Higgs couplings to fermions and bosons: Z h125 + vv h125
- tt h125 (c.m. energy slightly above 500 GeV beneficial)
- Z h125 h125, trilinear Higgs self-coupling

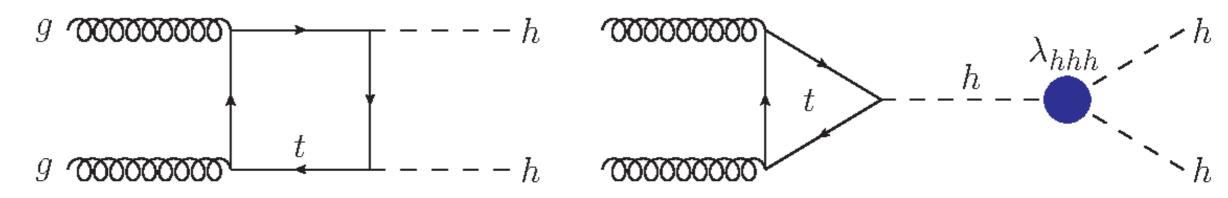
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I'll focus on accessing the trilinear Higgs self-coupling via Z h125 h125 production in the following

Trilinear Higgs self-coupling and the Higgs pair production process: LHC and e+e- collider

Sensitivity to the trilinear Higgs self-coupling from Higgs pair production:

→ Double-Higgs production → $λ_{hhh}$ enters at LO → most direct probe of $λ_{hhh}$



[Note: Single-Higgs production (EW precision observables) $\rightarrow \lambda_{hhh}$ enters at NLO (NNLO)]

Note: the "non-resonant" experimental limit on Higgs pair production

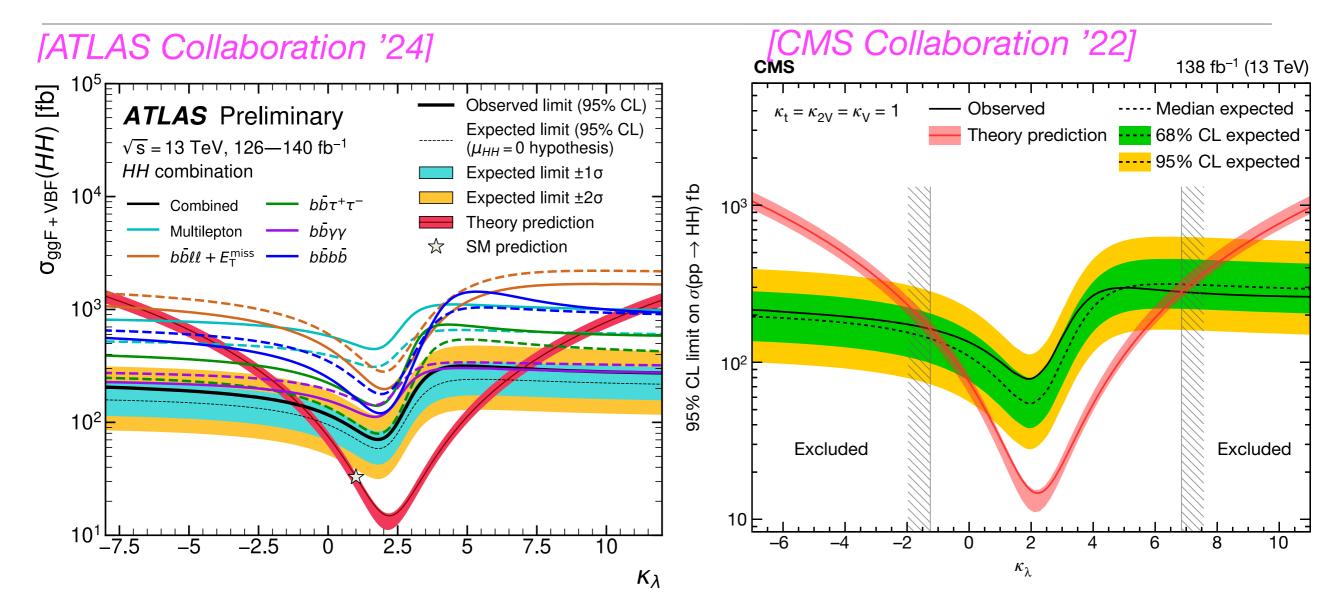
obtained by ATLAS and CMS depends on $\kappa_{\lambda} = \lambda_{hhh} / \lambda_{hhh}$

e+e- Higgs factory:

Indirect constraints from measurements of single Higgs production and electroweak precision observables at lower energies are not competitive

Direct measurement of trilinear Higgs self-coupling is possible a at lepton collider with at least 500 GeV c.m. energy

LHC, bound on the trilinear Higgs self-coupling: κ_{λ}



Using only information from di-Higgs production and assuming that new physics only affects the trilinear Higgs self-coupling, this limit on the cross section translates to:

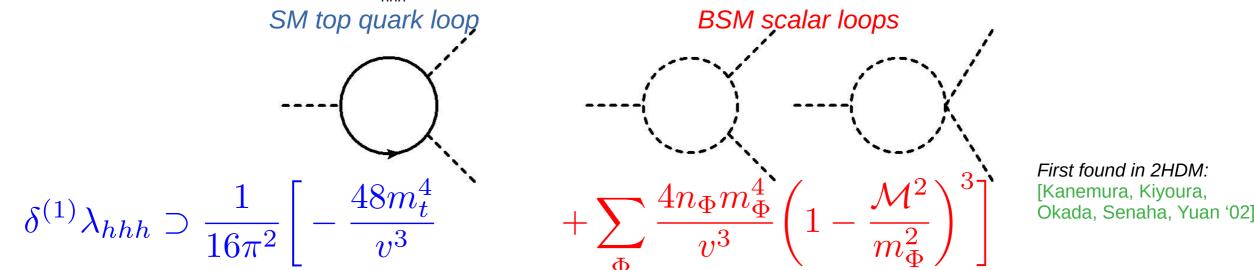
ATLAS: $-1.2 < \kappa_{\lambda} < 7.2$ at 95% C.L. [ATLAS Collaboration '24]

CMS: $-1.2 < \kappa_{\lambda} < 6.5$ at 95% C.L. [CMS Collaboration '22]

Effects of BSM particles on the trilinear Higgs coupling

Trilinear Higgs coupling in extended Higgs sectors: potentially large loop contributions

Leading one-loop corrections to λ_{hhh} in models with extended sectors (like 2HDM):



 \mathcal{M} : **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM

 n_Φ : # of d.o.f of field Φ

> Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda} v^2$

 \Rightarrow Large effects possible for sizeable splitting between m_Φ and ${\cal M}$

Two-loop predictions for the trilinear Higgs coupling in the 2HDM vs. current experimental bounds

[H. Bahl, J. Braathen, G. W. '22]

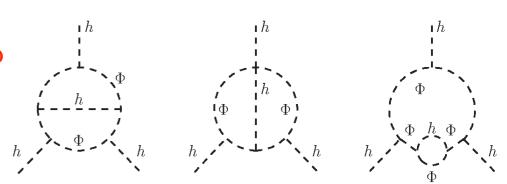
The largest loop corrections to λ_{hhh} in the 2HDM are induced by the quartic couplings between two SM-like Higgs bosons h (where one external Higgs is possibly replaced by its vacuum expectation value) and two BSM Higgs bosons ϕ of the form

$$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_{\Phi}^2)}{v^2} \qquad \Phi \in \{H, A, H^{\pm}\}$$

Leading two-loop corrections involving heavy BSM Higgses and the top quark in the effective potential approximation

[J. Braathen, S. Kanemura '19, '20]

 \Rightarrow Incorporation of the highest powers in $g_{hh\phi\phi}$

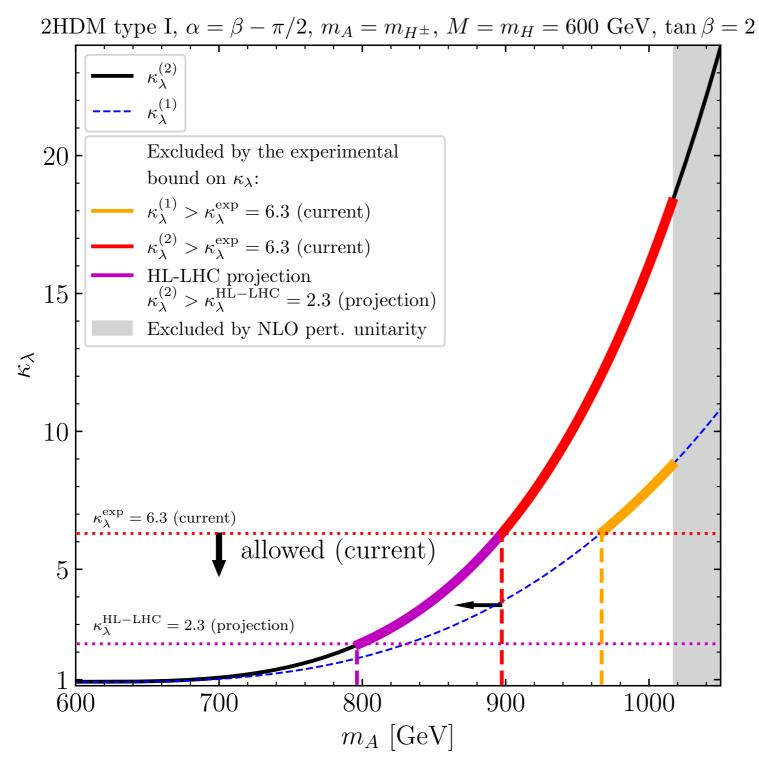


Analysis is carried out in the alignment limit of the 2HDM ($\alpha = \beta - \pi/2$) \Rightarrow h has SM-like tree-level couplings

Trilinear Higgs coupling: current experimental limit vs. prediction from extended Higgs sector (2HDM)

Prediction for x_{λ} up to the two-loop level:

[H. Bahl, J. Braathen, G. W. '22, Phys. Rev. Lett. 129 (2022) 23, 231802]

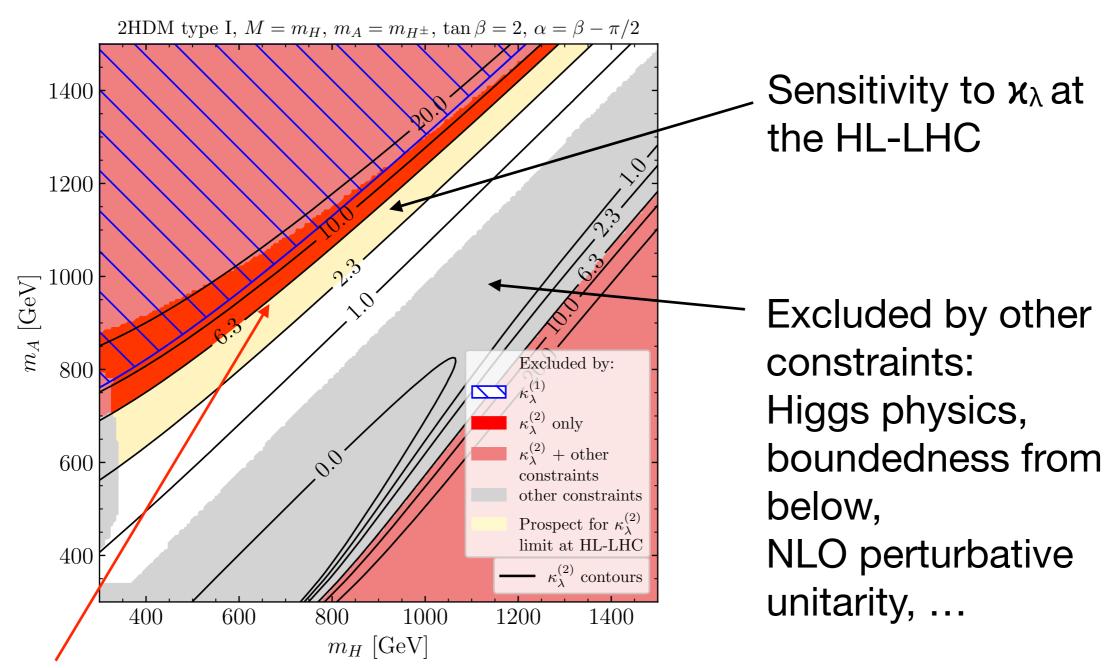


⇒ Current experimental limit excludes important parameter region that would be allowed by all other constraints!

Experimental limit on the trilinear Higgs coupling already has sensitivity to probe extended Higgs sectors!

Constraints in the mass plane of H and A

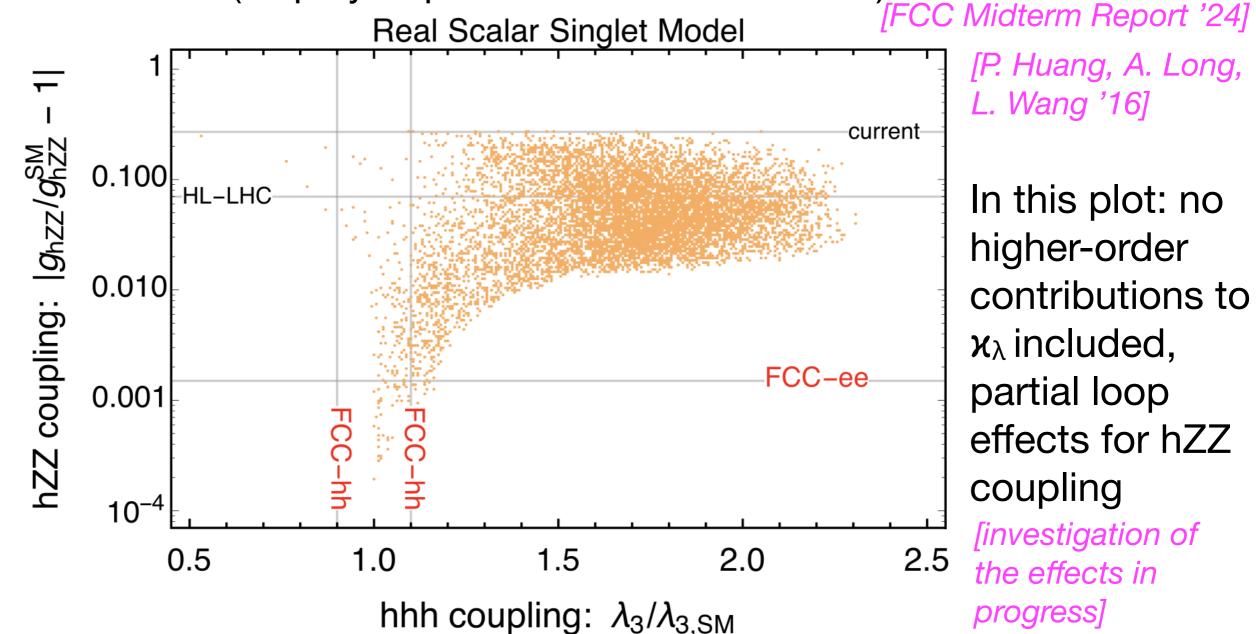
[H. Bahl, J. Braathen, G. W. '22]



⇒ LHC limits exclude parameter regions that would be allowed by all other constraints; high sensitivity of future limits / measurements!

Correlation of deviations in κ_{λ} with effects in other couplings? Real scalar singlet model

This plot caused some discussions in the context of strategies for future colliders (displayed points feature a FOEWPT):



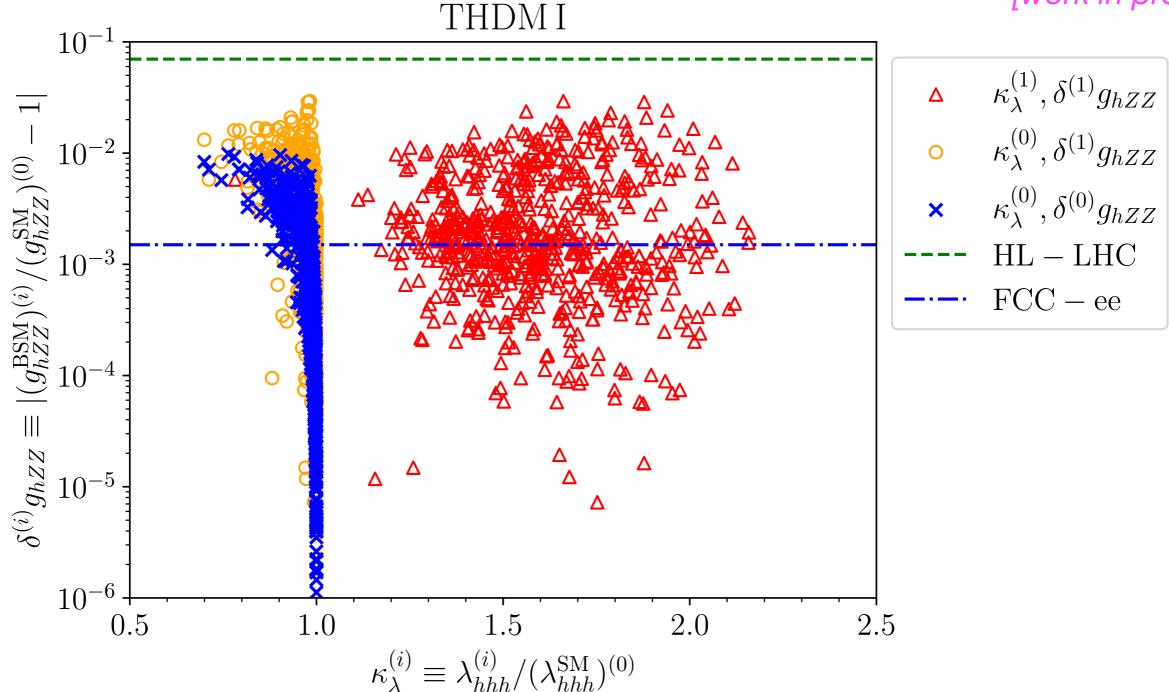
 \Rightarrow Do the deviations in x_{λ} have to be small if the FCC-ee does not find a deviation in the h125 coupling to ZZ?

Correlation of deviations in x_{λ} with effects in other couplings? Two Higgs Doublet model



sensitivity

[H. Bahl et al. '24] [work in progress]



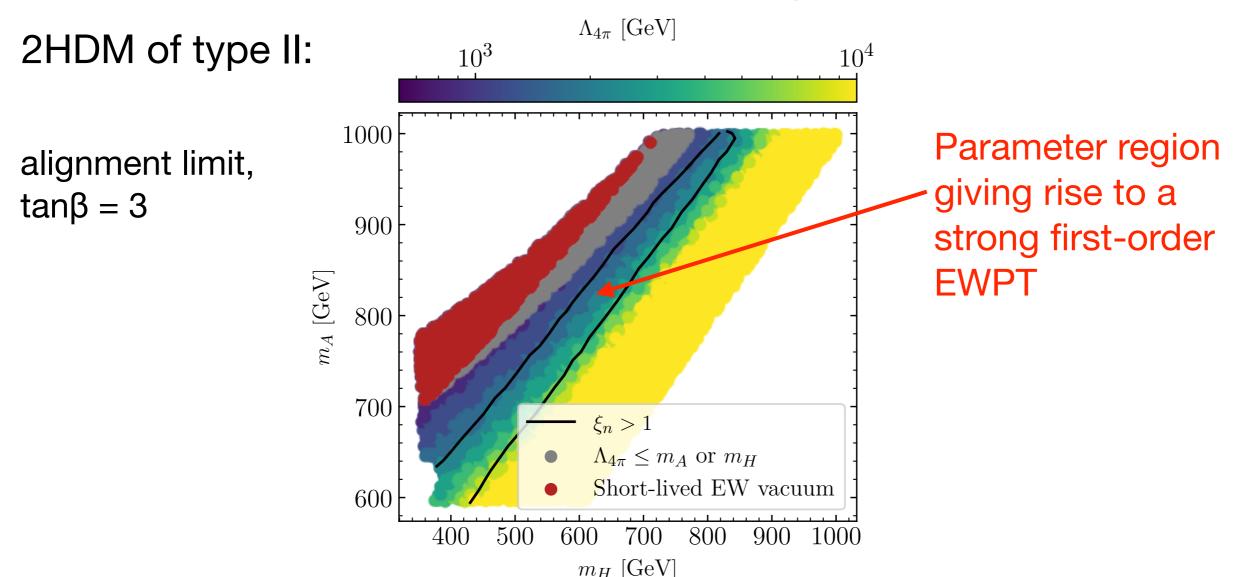
 \Rightarrow Large deviations in \varkappa_{λ} possible for effects in g_{hZZ} below the FCC

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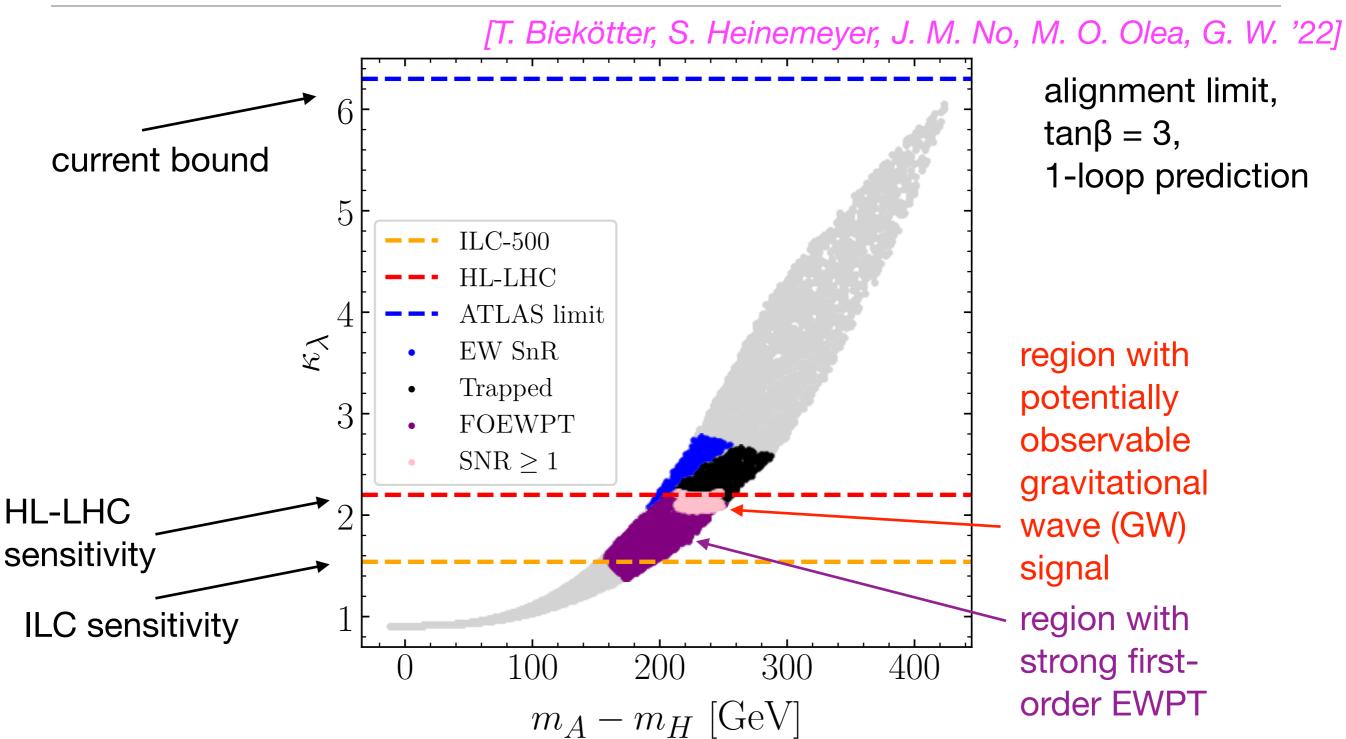
Connection between the trilinear Higgs coupling and the evolution of the early Universe

2HDM, N2HDM, ...: the parameter region giving rise to a strong first-order EWPT, which may cause a detectable gravitational wave signal, is correlated with an enhancement of the trilinear Higgs self-coupling and with "smoking gun" signatures at the LHC

[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '22]

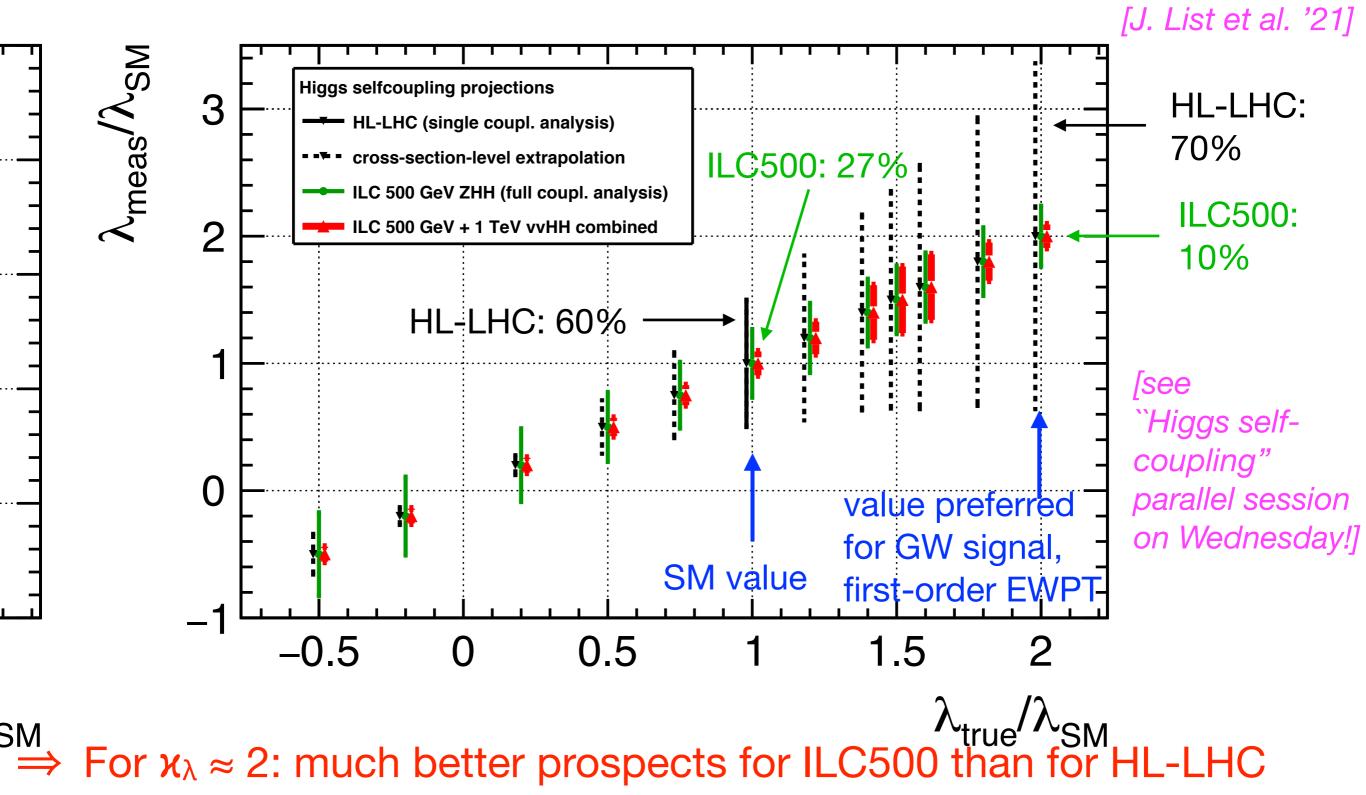


Relation between trilinear Higgs coupling and strong first-order EWPT with potentially observable GW signal



 \Rightarrow Region with strong first-order EWPT and potentially detectable GW signal is correlated with significant deviation of x_{λ} from SM value

Prospects for measuring the trilinear Higgs coupling: HL-LHC vs. ILC (500 GeV, Higgs pair production)



Reason: different interference contributions

Beyond 500 GeV: guaranteed "measurements"

- Higgs couplings to fermions and bosons, vv h125: high statistics
- Z h125 h125, vv h125 h125
- Z h125 h125 h125, vv h125 h125 h125, quartic Higgs self-coupling

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I'll focus on accessing the quartic Higgs self-coupling via Z h125 h125 h125 and $\nu\nu$ h125 h125 h125 production in the following

LHC: HHH production and Higgs self-couplings

Is it possible to obtain bounds from triple Higgs production on κ_3 and κ_4 that go beyond the existing theoretical bounds from perturbative unitarity? Potential for κ_3 constraints beyond the ones from di-Higgs production?

How big could the deviations in x_4 from the SM value (= 1) be in BSM scenarios?

Bounds from perturbative unitarity

- Process relevant for κ_3 , κ_4 is $HH \to HH$ scattering (see also [Liu et al `18])
- Jacob-Wick expansion allows to extract partial waves

$$\beta(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$$
 Wigner functions
$$a_{fi}^J = \frac{\beta^{1/4}(s,m_{f_1}^2,m_{f_1}^2)\beta^{1/4}(s,m_{i_1}^2,m_{i_1}^2)}{32\pi s} \int\limits_{-1}^1 \mathrm{d}\cos\theta\,\mathcal{D}_{\mu_i\mu_f}^J\,\mathcal{M}(s,\cos\theta)$$

Tree level unitarity:

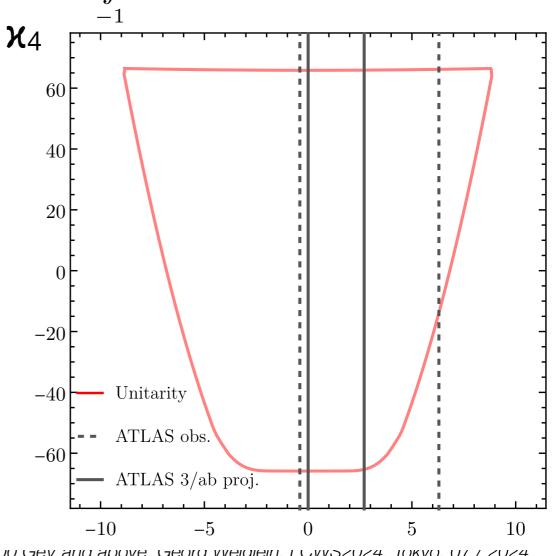
$$\operatorname{Im} a_{ii}^0 \ge |a_{ii}^0|^2 \implies |\operatorname{Re} a_{ii}^0| \le \frac{1}{2}$$

95 % CL

ATLAS current bounds: [-0.4, 6.3]

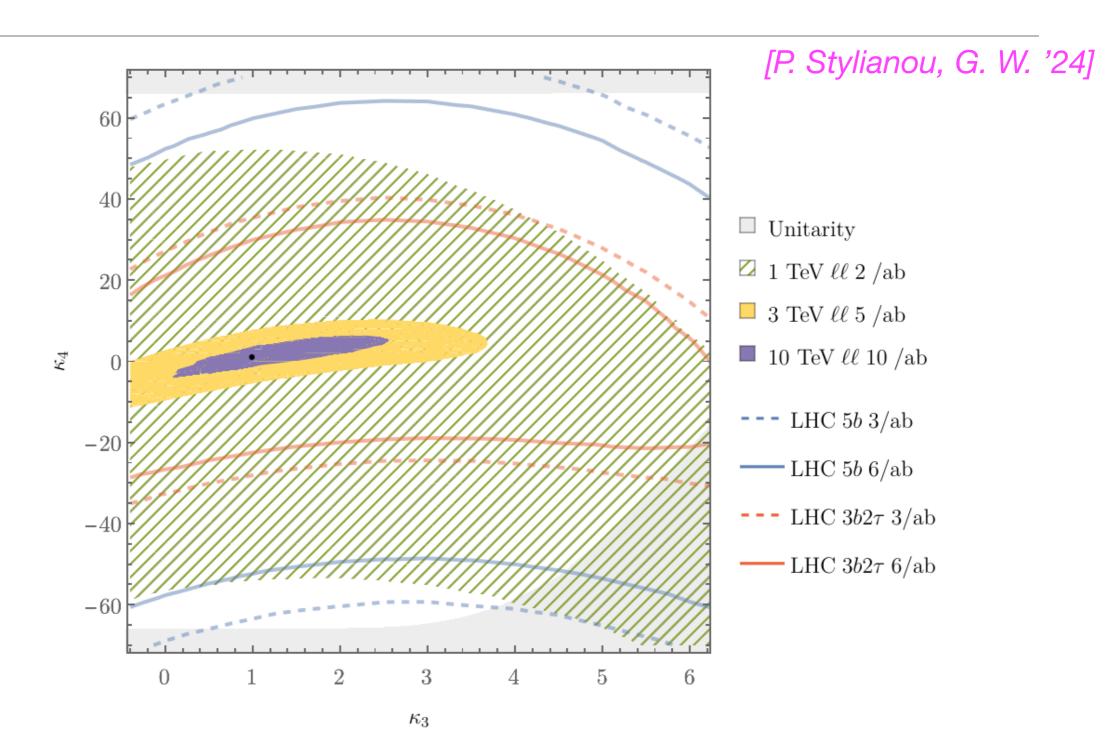
CMS & ATLAS HH projections: [0.1, 2.3]

[ATLAS 2211.01216] [CERN Yellow Rep. 1902.00134]



X

Triple Higgs production: HL-LHC vs. lepton colliders



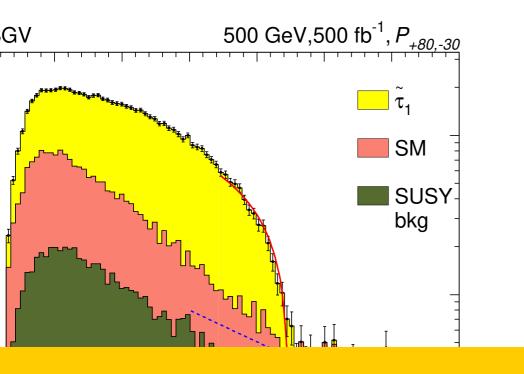
HL-LHC is competitive to 1 TeV lepton collider; higher-energetic lepton colliders have better sensitivity

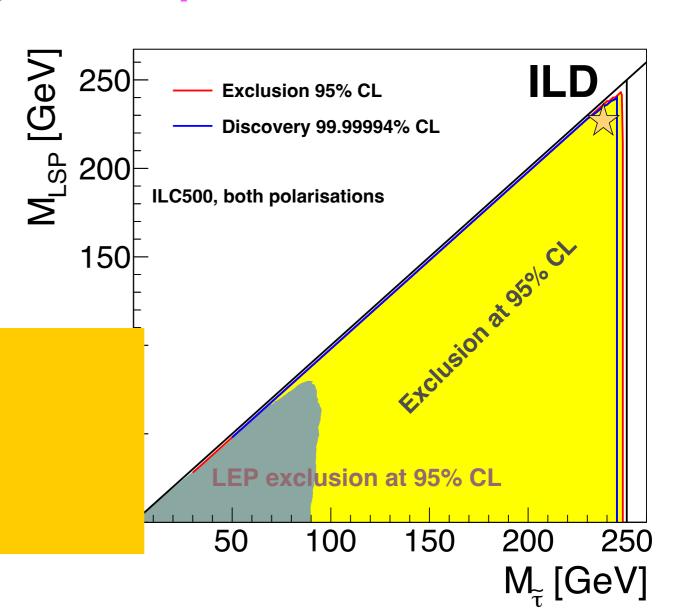
sitivity to new particles at 500 GeV and beyond



on and discovery reach for charged s, charginos, ...) via photon exchange

mgg resन et al. '16]

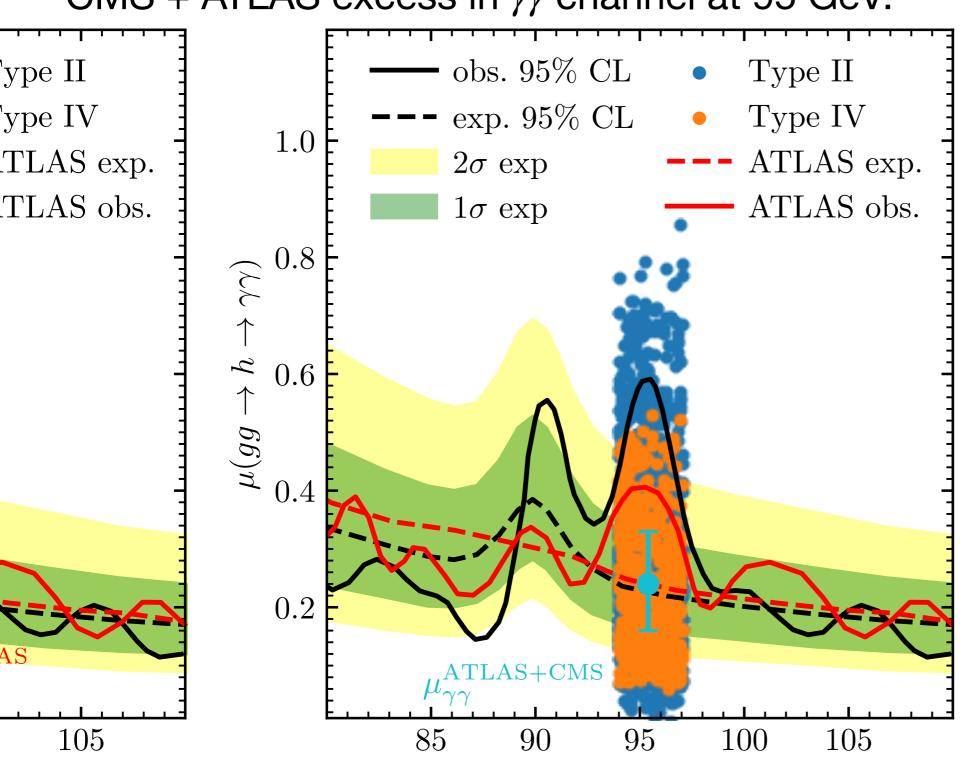




Different case: how about a light BSM Higgs boson?

 m_h

CMS + ATLAS excess in $\gamma\gamma$ channel at 95 GeV:



[T. Biekötter,

S. Heinemeyer,

G. W. '23]

Example interpretation: S2HDM, type II and IV

⇒Good description in extended Higgs sectors with an additional doublet and a singlet

Different case: how about a light BSM Higgs boson?

- Possible signal (h95) with significant ZZ h95 coupling (possibly explaining also the "LEP excess" near 95 GeV)
- ⇒ h95 can be studied in detail at 250 GeV e+e- Higgs factory

- Possible signal (h95) explaining only the LHC excess in the $\gamma\gamma$ channel E.g.: CP-odd Higgs boson at 95 GeV
- ⇒ Expect sizeable coupling of h95 to top quarks
 Prospects at e+e- colliders?
 e+e- → t t h95, e+e- → Z h95 h95 (via intermediate h125), ...
- ⇒ Need higher c.m. energy (about 500 GeV for t t h95 final state) and high luminosity

Conclusions

The possibility to measure the trilinear Higgs self-coupling in the Zhh production process at an e+e- Linear Collider with c.m. energy of at least 500 GeV is a qualitative game-changer compared to the capabilities of lower-energy Higgs factories

This, in combination with the significantly extended reach for BSM searches, is a strong motivation for designing a possible future e⁺e⁻ Higgs factory such that an upgrade to at least 500 GeV is possible

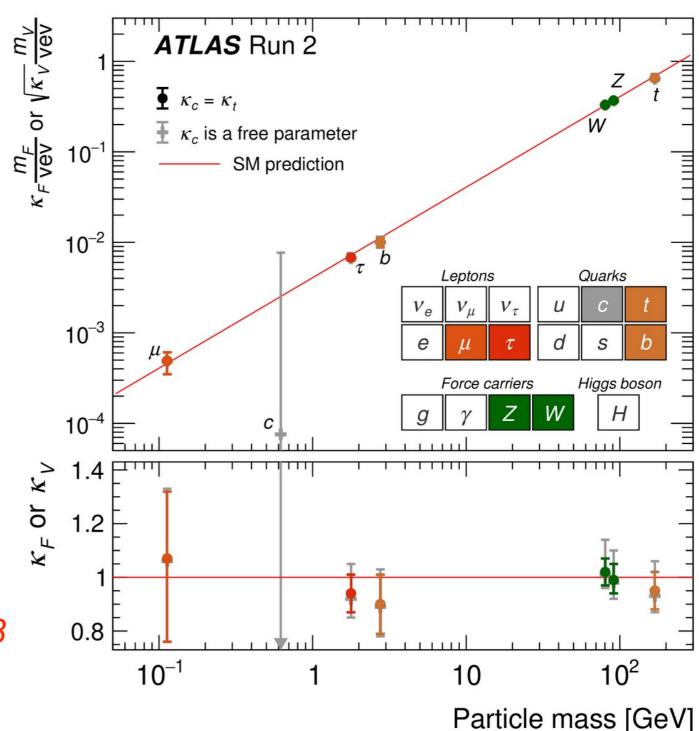
The highest-energetic lepton colliders provide sensitivity for constraining the quartic Higgs self-coupling

Backup

Properties of the detected Higgs boson (h125)

Couplings of the detected Higgs boson to other particles:

[ATLAS Collaboration '22]





Nobel Prize 2013

⇒Agrees with predictions of the Brout-Englert-Higgs (BEH) mechanism

Electroweak phase transition and baryon asymmetry

Observed Baryon Asymmetry of the Universe (BAU)

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$
 [Planck '18]

 n_b : baryon no. density $n_{\overline{b}}$: antibaryon no. density n,: photon no. density

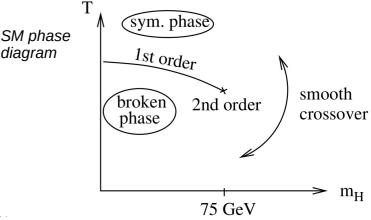


Sakharov Conditions

(for dynamical generation of baryon asymmetry)

[J. M. No '23]

- **B** Violation
- C/CP Violation × not enough in SM
- Departure from Thermal Equilibrium



SM CP Violation insufficient by ~ 10 orders of magnitude

via 3-family fermion mixing (CKM matrix)

Sakharov conditions:

- baryon (or lepton) number violation starting from symmetric state
- treat baryons and anti-baryons differently (to remove anti-matter)
- suppress inverse processes

diagram

Strongly first-order EWPT in the 2HDM

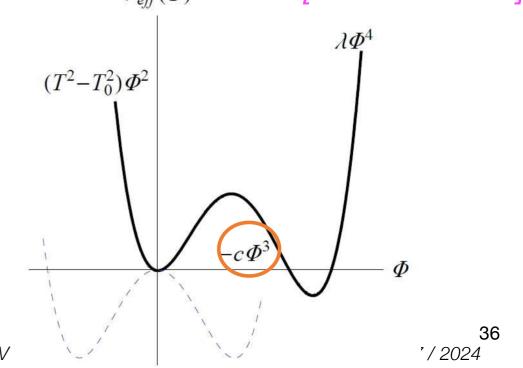
Barrier is re
$$m_i^2 = \mu_S^2 + \lambda_{HS} h^2$$
 effective potential

Arises from higher-order contributions and thermal corrections to the potential, in particular:

$$-\frac{T}{12\pi} \left[\mu_S^2 + \lambda_{HS} h^2 + \Pi_S \right]^{3/2}$$

 \Rightarrow For sizeable quartic couplings an effective cubic term in the Higgs potential is generated [M. O. Olea '23]

⇒ Yields mass splitting between the BSM Higgs bosons and sizeable corrections to the trilinear Higgs coupling



Physics case for e+e- at 500 GeV

EWPT: are there additional sources for CP violation in the Higgs sector?

Baryogenesis: creation of the asymmetry between matter and antimatter in the universe requires a strong first-order electroweak phase transition (EWPT)

First-order EWPT does not work in the SM The amount of CP violation in the SM (induced by the CKM phase) is not sufficient to explain the observed asymmetry between matter and anti-matter in the universe

First-order EWPT can be realised in extended Higgs sectors could give rise to detectable gravitational wave signal

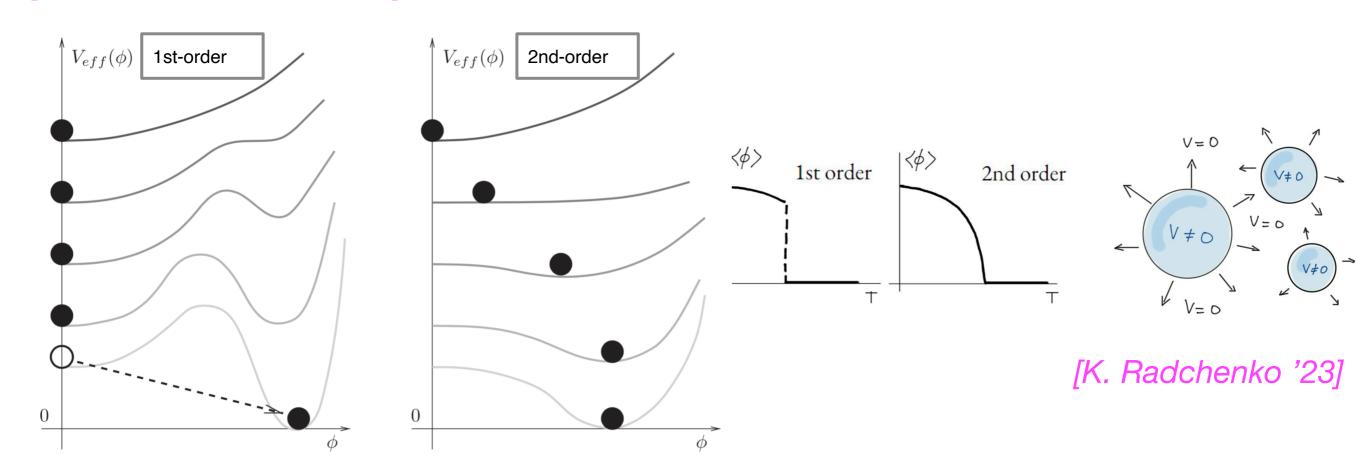
⇒ Search for additional sources of CP violation

 $\begin{array}{c} h \\ e \\ \hline \end{array} \begin{array}{c} V_{\mu} \\ e \\ \hline \end{array} \begin{array}{c} V_{\mu} \\ e \\ \hline \end{array}$ $\begin{array}{c} e \\ \hline \end{array} \begin{array}{c} V_{\mu} \\ e \\ \hline \end{array} \begin{array}{c} e \\ \end{array}$

But: strong experimental constraints from limits on electric dipole moments (EDMs)

First-order vs. second order EWPT

[D. Gorbunov, V. Rubakov]



Potential barrier needed for first-order EWPT, depends on trilinear Higgs coupling(s)

Deviation of trilinear Higgs coupling from SM value is a typical feature of a strong first-order EWPT

"x framework" and EFT approach for coupling analyses

Simplified framework for coupling analyses: deviations from SM parametrised by "scale factors" x_i , where $x_i = g_{Hii}/g^{SM, (0)}_{Hii}$

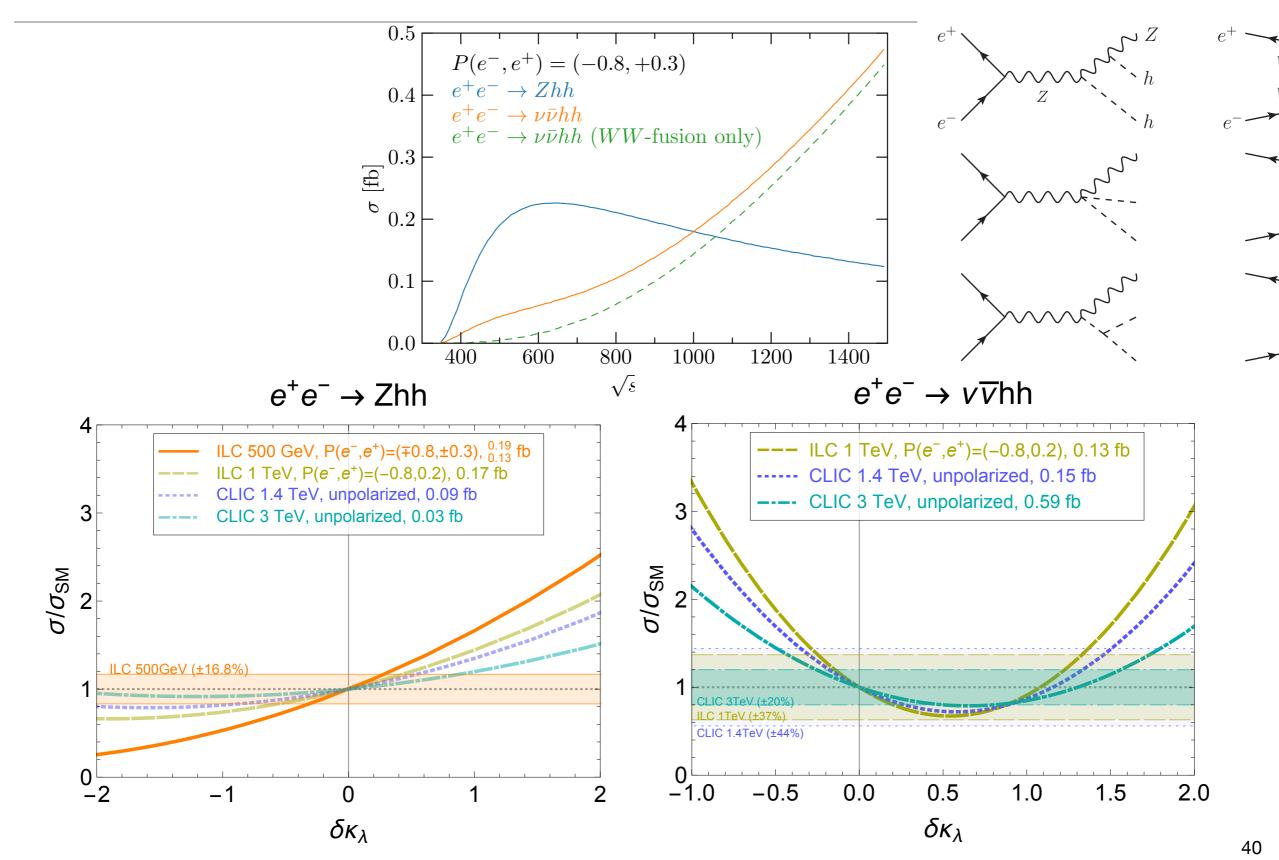
Assumptions inherent in the x framework: signal corresponds to only one state, no overlapping resonances, etc., zero-width approximation, only modifications of coupling strengths (absolute values of the couplings) are considered

→ Assume that the observed state is a CP-even scalar

Theoretical assumptions in determination of the x_i : $x_i \le 1$, no invisible / undetectable decay modes, ...

EFT: fits for Wilson coefficients of higher-dimensional operators in SMEFT Lagrangian, ...

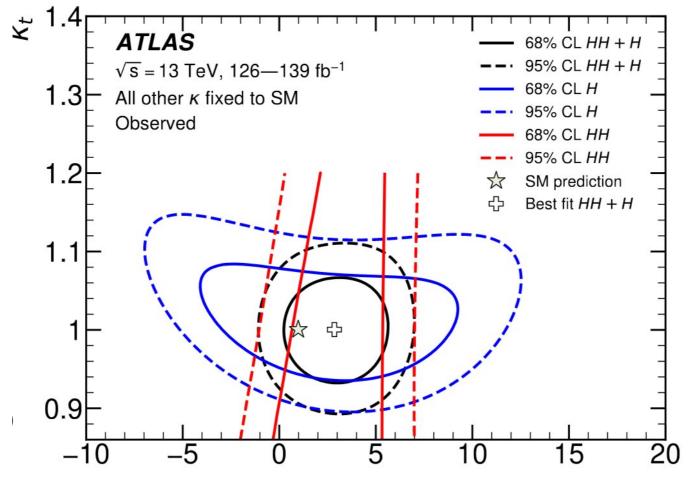
Higgs pair production at e+e- colliders



Experimental constraints on \mathbf{x}_{λ}

[ATLAS Collaboration '22]

Combination assumption	Obs. 95% CL	Exp. 95% CL	Obs. value $^{+1}_{-1}\sigma$
HH combination	$-0.6 < \kappa_{\lambda} < 6.6$	$-2.1 < \kappa_{\lambda} < 7.8$	$\kappa_{\lambda} = 3.1^{+1.9}_{-2.0}$
Single- <i>H</i> combination	$-4.0 < \kappa_{\lambda} < 10.3$	$-5.2 < \kappa_{\lambda} < 11.5$	$\kappa_{\lambda} = 2.5^{+4.6}_{-3.9}$
<i>HH</i> + <i>H</i> combination	$-0.4 < \kappa_{\lambda} < 6.3$	$-1.9 < \kappa_{\lambda} < 7.5$	$\kappa_{\lambda} = 3.0^{+1.8}_{-1.9}$
$HH+H$ combination, κ_t floating	$-0.4 < \kappa_{\lambda} < 6.3$	$-1.9 < \kappa_{\lambda} < 7.6$	$\kappa_{\lambda} = 3.0^{+1.8}_{-1.9}$
$HH+H$ combination, κ_t , κ_V , κ_b , κ_τ floating	$-1.3 < \kappa_{\lambda} < 6.1$	$-2.1 < \kappa_{\lambda} < 7.6$	$\kappa_{\lambda} = 2.3^{+2.1}_{-2.0}$



Check of applicability of the experimental limit on κ_{λ}

The assumption that new physics only affects the trilinear Higgs selfcoupling is expected to hold at most approximately in realistic models

BSM models can modify Higgs pair production via resonant and non-resonant contributions

The current experimental limit can only probe scenarios with large deviations from the SM

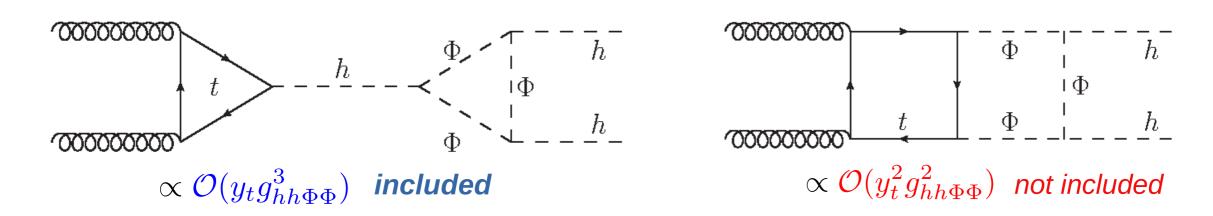
 \Rightarrow Direct application of the experimental limit on \varkappa_{λ} is possible if sub-leading effects are less relevant

Check of applicability of the experimental limit on κ_{λ}

Alignment limit: h has SM-like tree-level couplings

Resonant contribution to Higgs pair production with H or A in the s channel is absent in the alignment limit

The dominant new-physics contributions enter via trilinear coupling



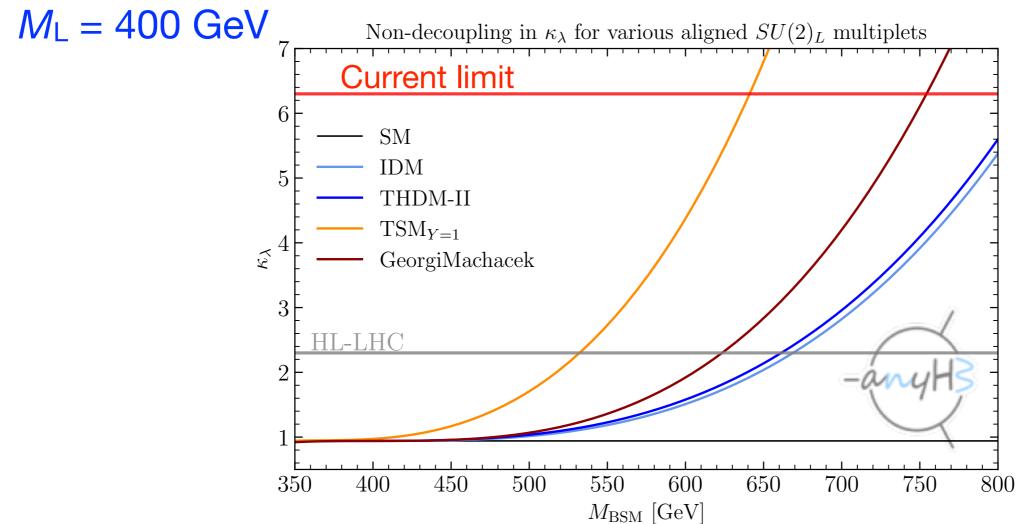
 \Rightarrow The leading effects in $g_{hh\phi\phi}$ to the Higgs pair production process are correctly incorporated at the 1- and 2-loop order via the corrections to the trilinear Higgs coupling!

Higgs self-couplings in extended Higgs sectors

Effect of splitting between BSM Higgs bosons:

Very large corrections to the Higgs self-couplings, while all couplings of h₁₂₅ to gauge bosons and fermions are SM-like (tree-level couplings agree with the SM in the alignment limit)

[H. Bahl, J. Braathen, M. Gabelmann, G. W. '23]



Single-Higgs processes: λ enters at loop level

[E. Petit '19]

How to measure deviations of λ_3

- ◆ The Higgs self-coupling can be assessed using di-Higgs production and single-Higgs production
- ♦ The sensitivity of the various future colliders can be obtained using four different methods:

di-Higgs single-H 1. di-H, excl. 3. single-H, excl. exclusive • Use of $\sigma(HH)$ • single Higgs processes at higher order • only deformation of κλ • only deformation of κλ 2. di-H, glob. 4. single-H, glob. • Use of σ(HH) • single Higgs processes at higher order • deformation of $\kappa\lambda$ + of the single-H couplings global (a) do not consider the effects at higher order • deformation of $\kappa\lambda$ + of the single Higgs of κλ to single H production and decays couplings (b) these higher order effects are included

Note: this is based on the assumption that there is a large shift in λ, but no change anywhere else!

Single-Higgs processes: λ enters at loop level

[B. Heinemann '19]

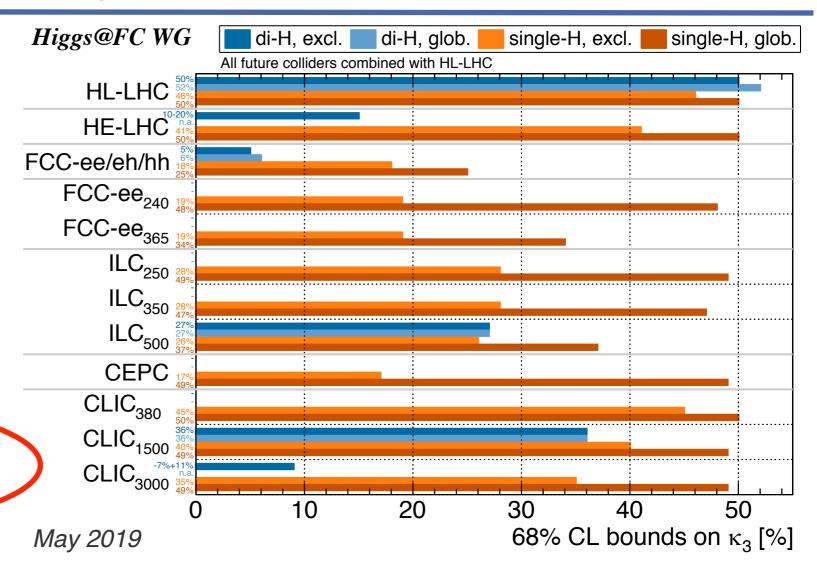
Sensitivity to λ: via single-H and di-H production

Di-Higgs:

- HL-LHC: ~50% or better?
- Improved by HE-LHC (~15%), ILC₅₀₀ (~27%), CLIC₁₅₀₀ (~36%)
- Precisely by CLIC₃₀₀₀ (~9%),
 FCC-hh (~5%),
- Robust w.r.t other operators

Single-Higgs:

- Global analysis: FCC-ee365 and ILC500 sensitive to ~35% when combined with HL-LHC
 - ~21% if FCC-ee has 4 detectors
- Exclusive analysis: too sensitive to other new physics to draw conclusion

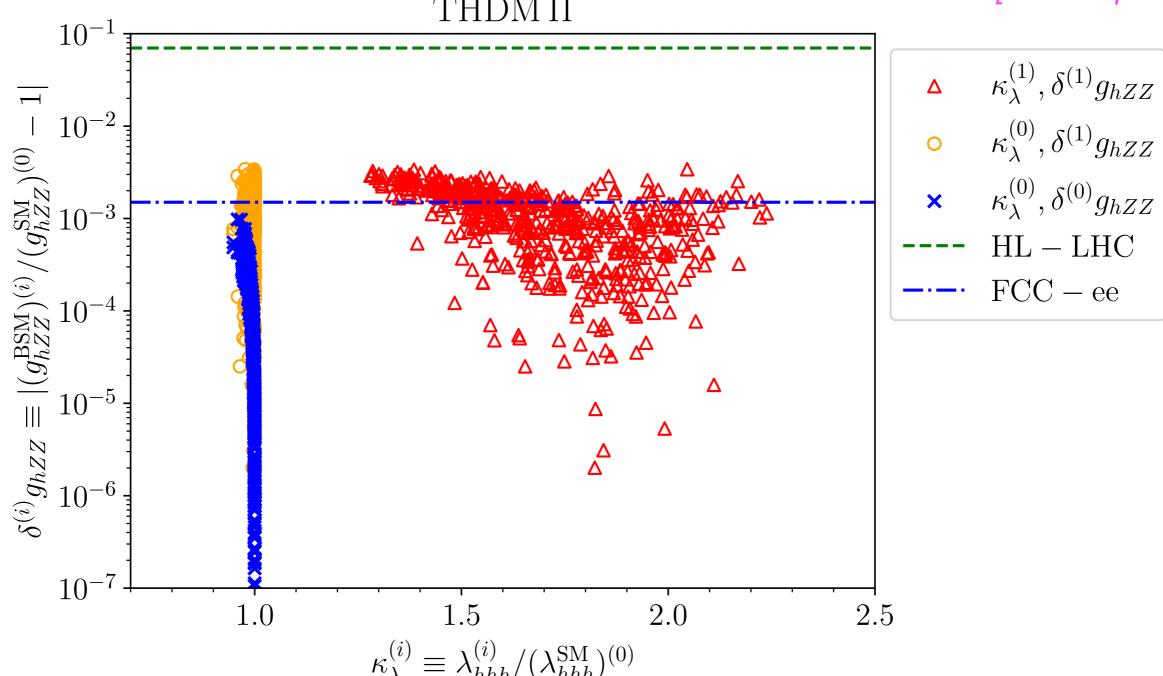




Correlation of deviations in x_{λ} with effects in other couplings? Two Higgs Doublet model



[H. Bahl et al. '24] [work in progress]

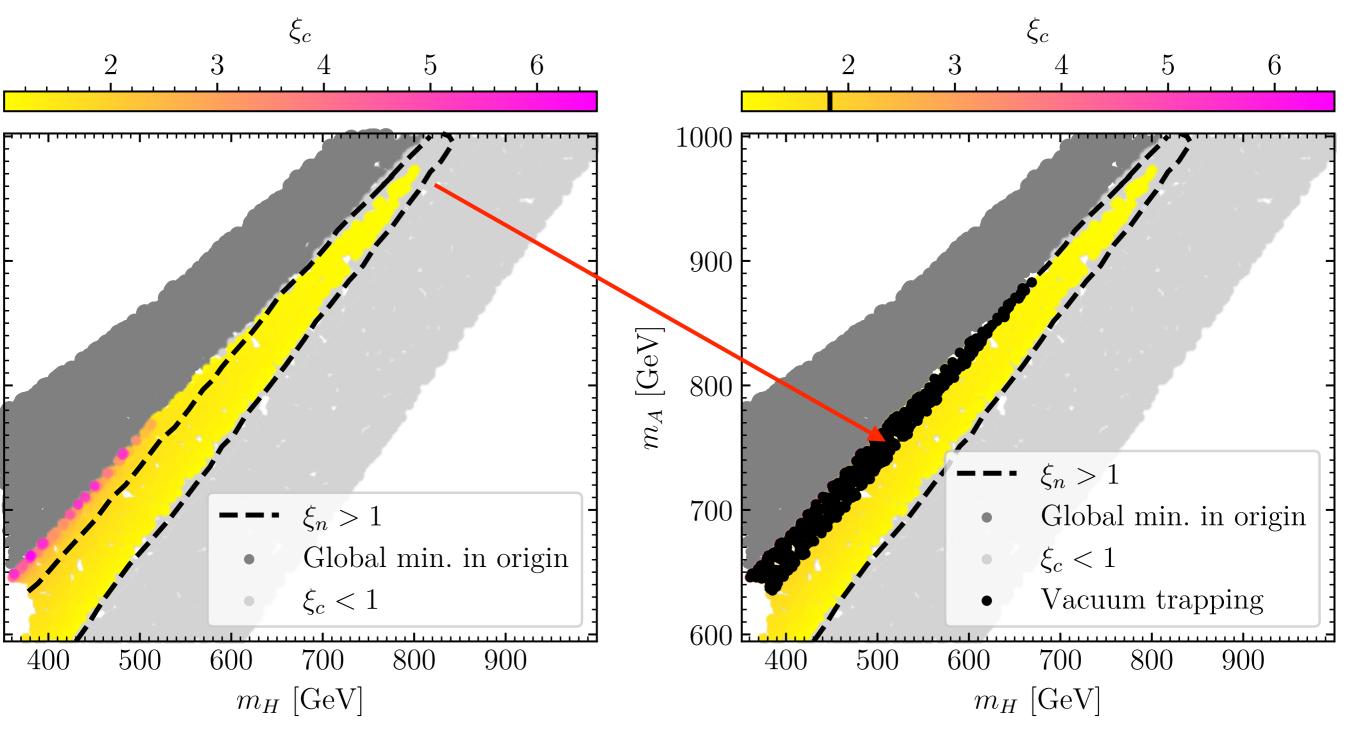


 \Rightarrow Large deviations in \varkappa_{λ} possible for effects in g_{hZZ} below the FCC

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2HDM of type II: region of strong first-order EWPT

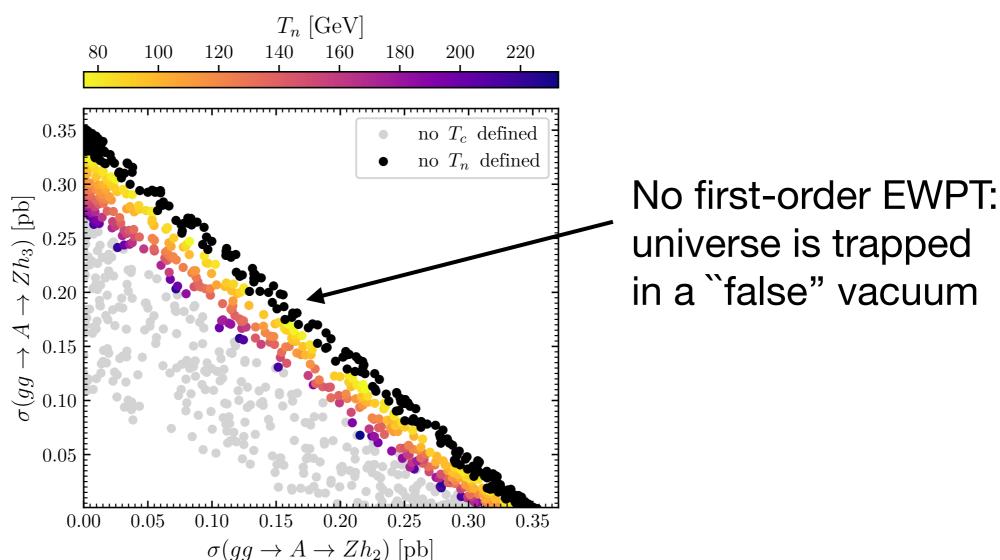
[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '22]



N2HDM (two doublets + real singlet) example

"Smoking gun" collider signatures: $A \rightarrow Z h_2$, $A \rightarrow Z h_3$ Nucleation temperature for the first-order EWPT, N2HDM scan:

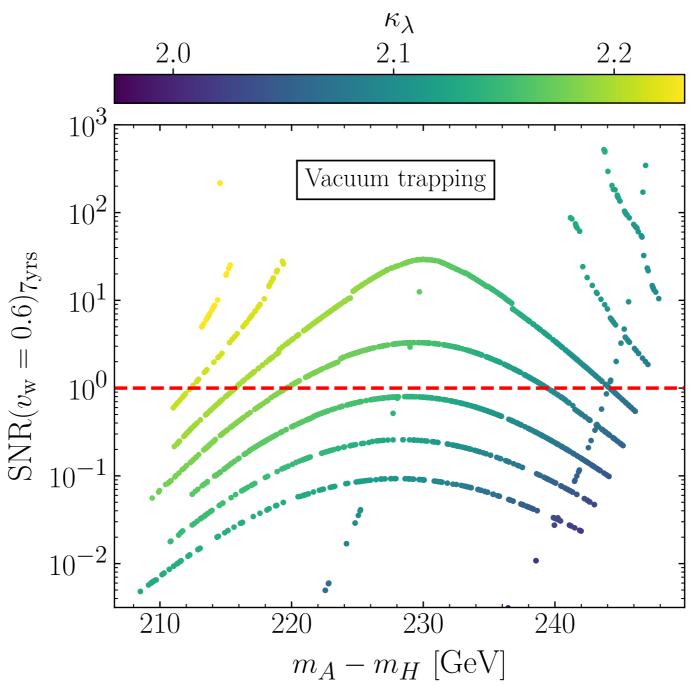
[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '21]



⇒ Lower nucleation temperatures, i.e. stronger first-order EWPTs, are correlated with larger signal rates at the LHC!

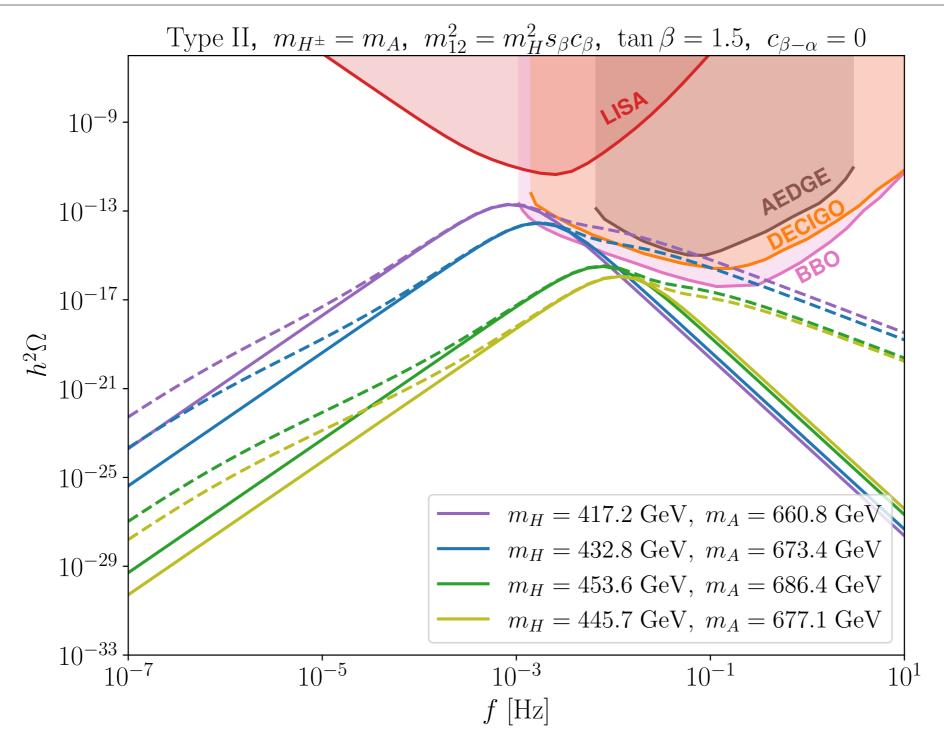
Correlation of x_{λ} with the signal-to-noise ratio (SNR) of a gravitational wave signal at LISA

[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '22]



 \Rightarrow Region with potentially detectable gravitational wave signal: significant enhancement of x_{λ} and non-vanishing mass splitting

GW spectra of scenarios fitting the excess

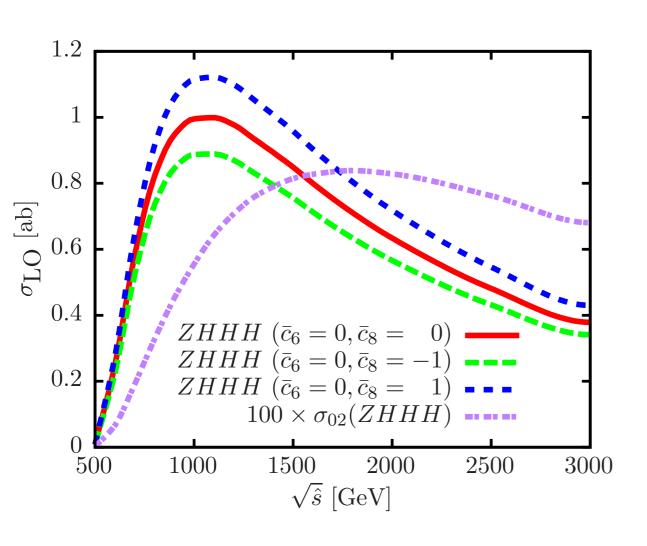


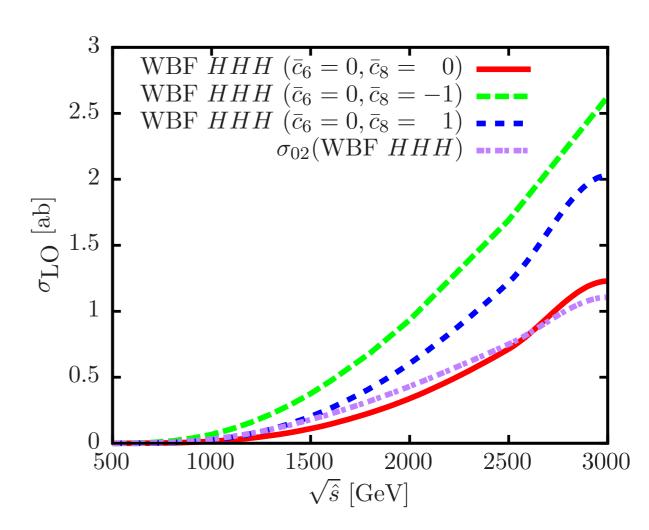
[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, K. Radchenko, G. W. '23]

⇒ Prospects for GW detection depend very sensitively on the precise details of the mass spectrum of the additional Higgs bosons

Triple Higgs production at e+e- colliders

[F. Maltoni et al. '18]





Prospects for the HL-LHC: 6b and 4b2T channels comb.

[P. Stylianou, G. W. '24]

Assumption: No correlations

Combination of further channels and improvements of tagging/reconstruction methods could enhance results further

