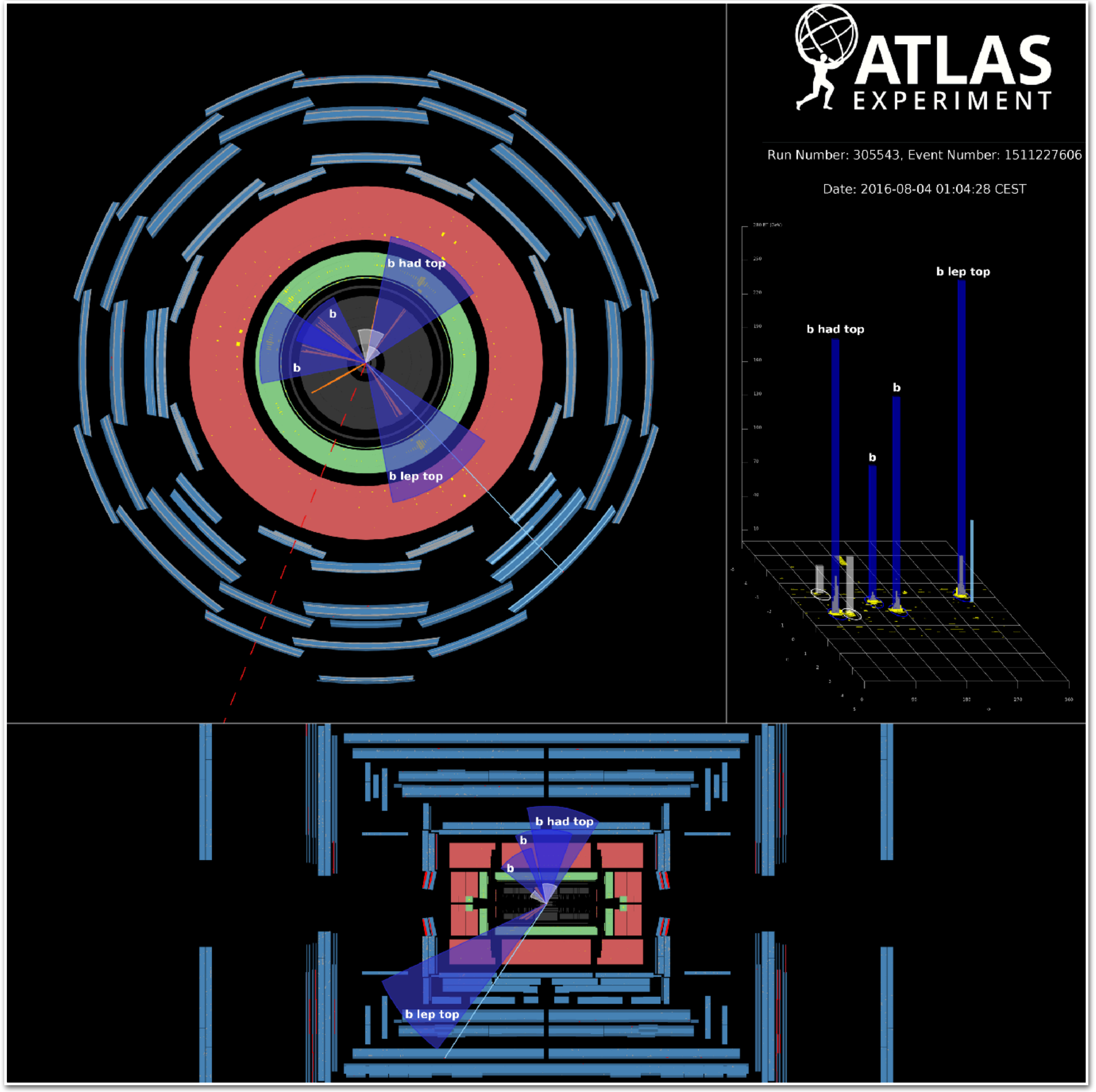
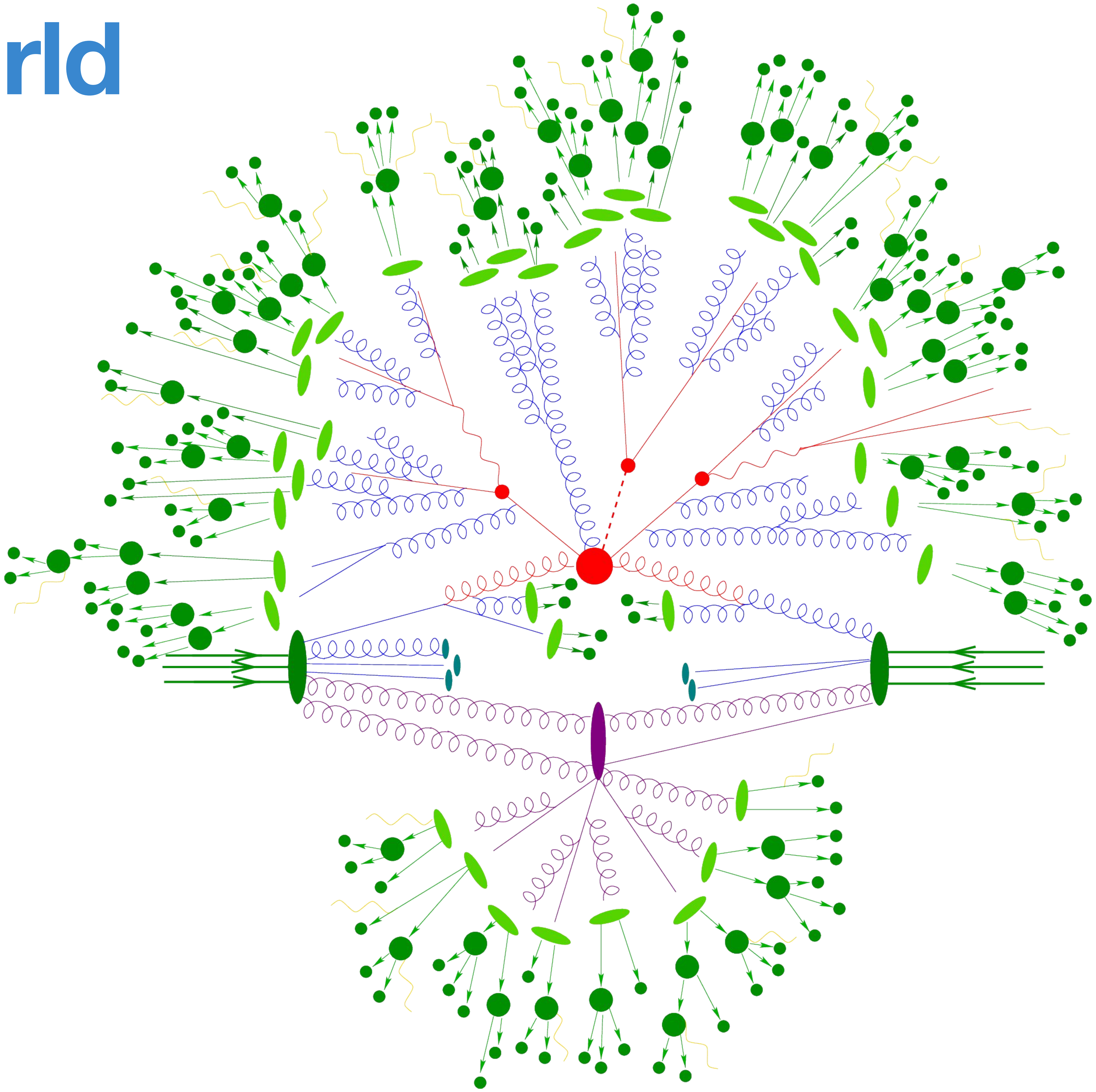
The Alaric parton shower algorithm

based on
Herren, Höche, Krauss, DR, Schönherr JHEP 10 (2023) 091 [arXiv:2208.06057]
Höche, Krauss, DR [arXiv:2404.14360]

Daniel Reichelt, 9 July 2024

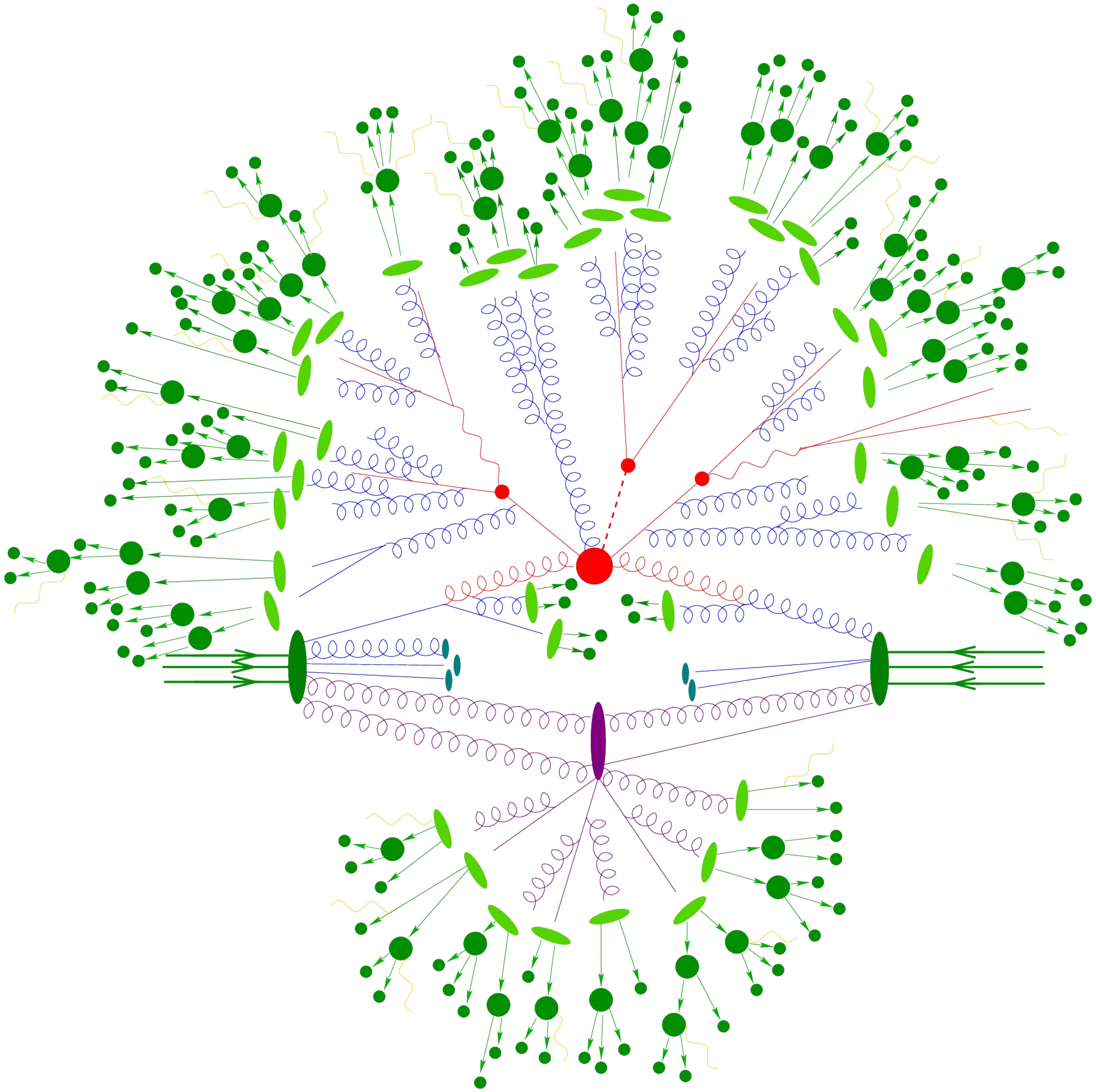
Colliders in the real world





Colliders for theorists

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - PDF/Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays



A Logarithmically Accurate Resummation In C++

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - Underlying event
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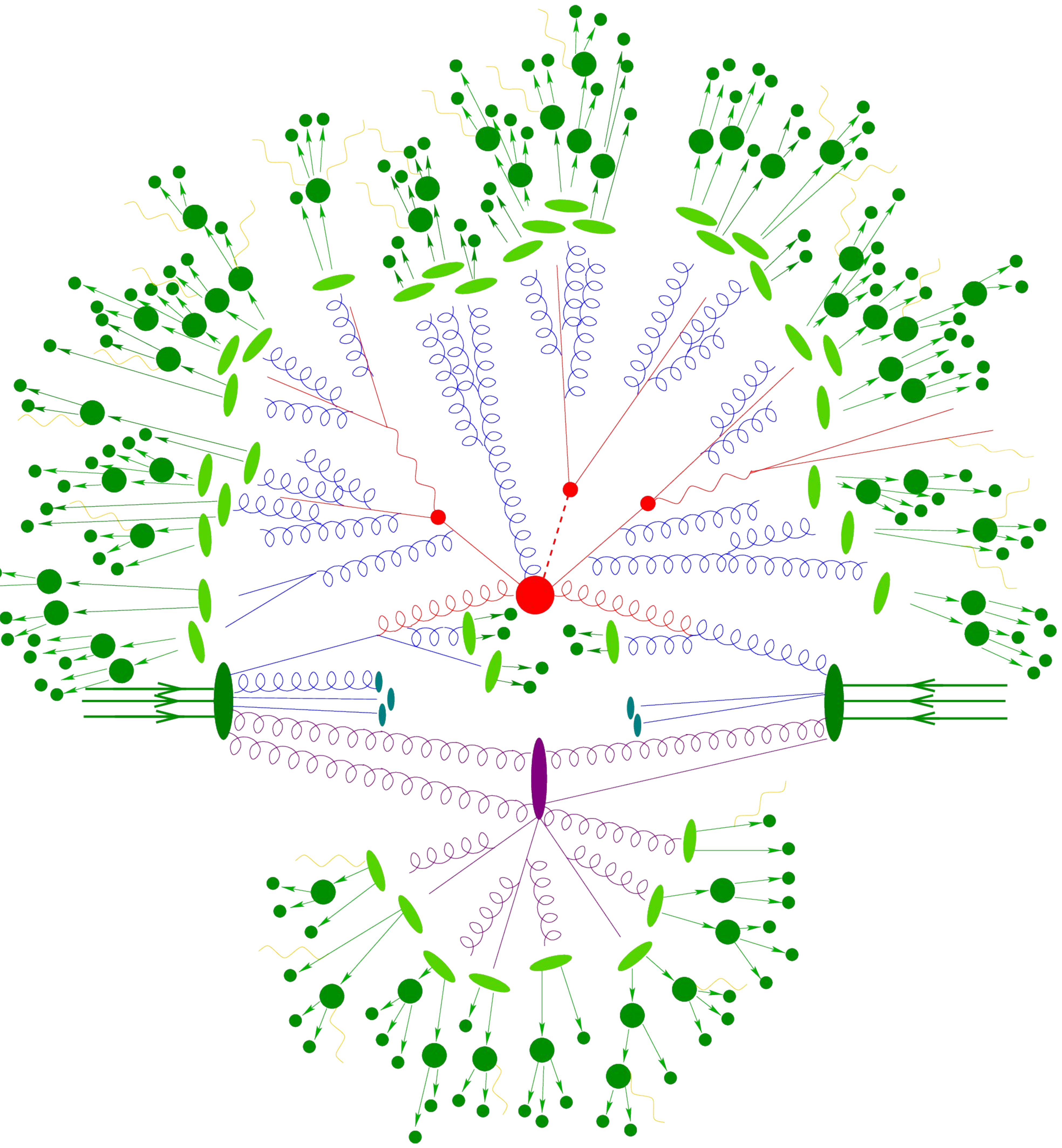
  - QED radiation
  - Hadron Decays

This Talk:

Why?

- parton showers resum large logs ~ NLL, but open questions on actual accuracy
- starting work towards NNLL/NLO evolution → probably better resolve this first
- recent formal discussion → current dipole showers need reworking
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]

Alaric in Sherpa



- Hard Process
  - ME from usual generators AMEGIC/Comix, including multi-jet merging [Höche, Krauss, DR '24] of higher final state multiplicities in CKKW scheme
- Parton Shower
  - Alaric (instead of CSS/Dire showers)
  - including quark masses [Höche, Assi '23]
- PDF/Underlying event
  - default Sherpa model (no dedicated tune yet)
- Hadronisation
  - plots here with interface to Pythia 6/8
- Hadron Decays/Beam Remnants etc.
  - default Sherpa modules

Parton showers - Cliff notes version

- no-emission probability (sudakov factor)

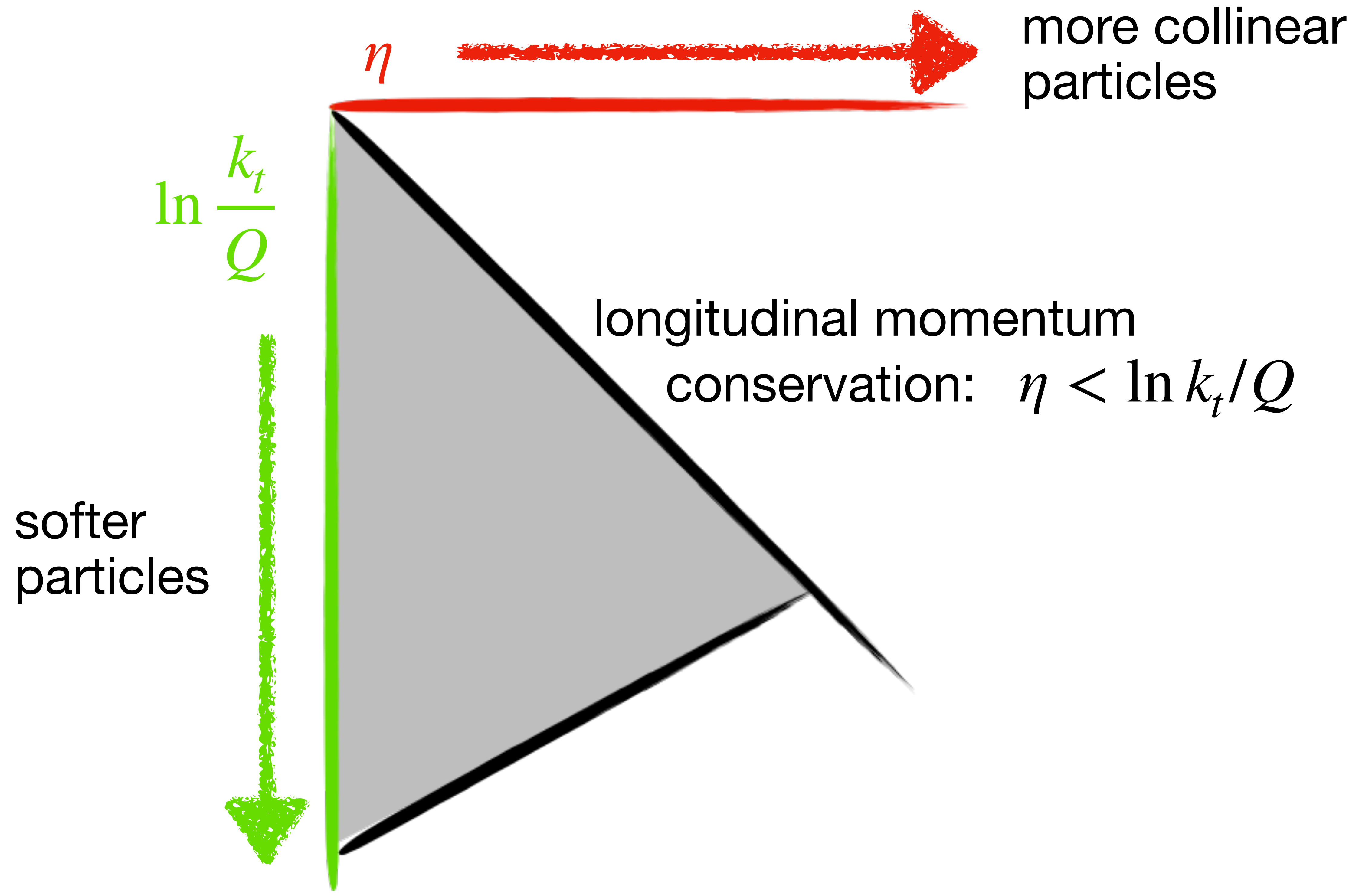
$$\sim \exp\left[-\int_{t_0}^{t_1} \frac{dk_t}{k_t} dz \frac{\alpha_S}{2\pi} P(z)\right]$$

- Main ingredients to a shower:

  1. splitting kernels $P(z)$ captures soft and collinear limits of matrix elements

  2. fill phase space ordered in evolution variable $(k_t, \theta, q^2, \ldots) \Rightarrow$ here k_t ordered shower

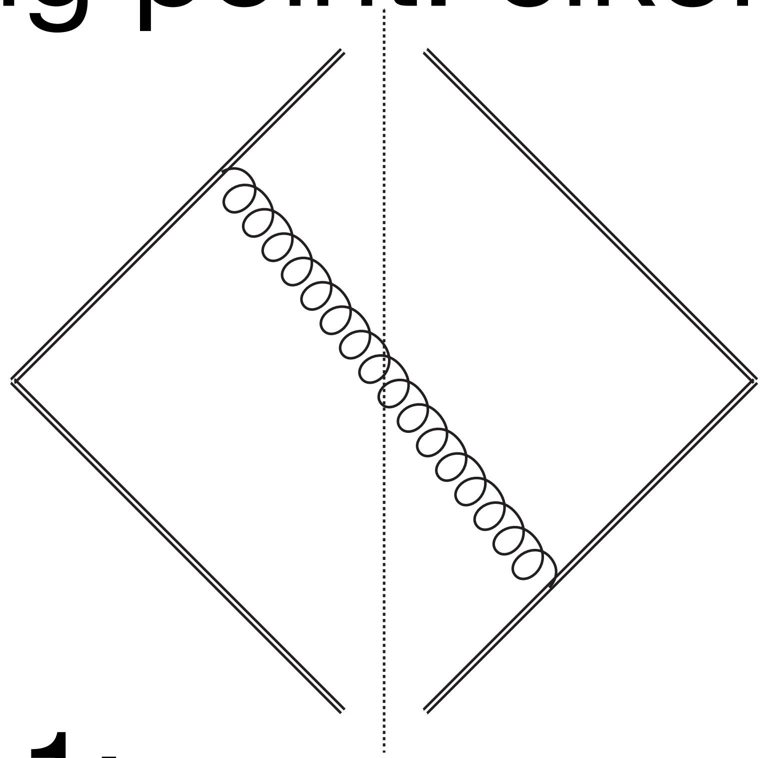
  3. generate new final state after emission according to recoil scheme



Splitting of Eikonal

Starting point: eikonal

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} \equiv \frac{W_{ik,j}}{E_j^2}$$



naive implementation leads to soft double counting need to split into ij and kj collinear terms [Marchesini, Webber '88]

Option 1:

$$W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j} \,, \qquad \text{where} \qquad \tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} + \frac{1}{1 - \cos\theta_{ij}} - \frac{1}{1 - \cos\theta_{jk}}\right)$$

- e.g. Angular ordered shower, downside: problems with NGLs

Option 2: follow [Catani, Seymour '97]

$$W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j} \,, \qquad \text{where} \qquad \bar{W}^i_{ik,j} = \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(2 - \cos\theta_{ij} - \cos\theta_{jk})}$$

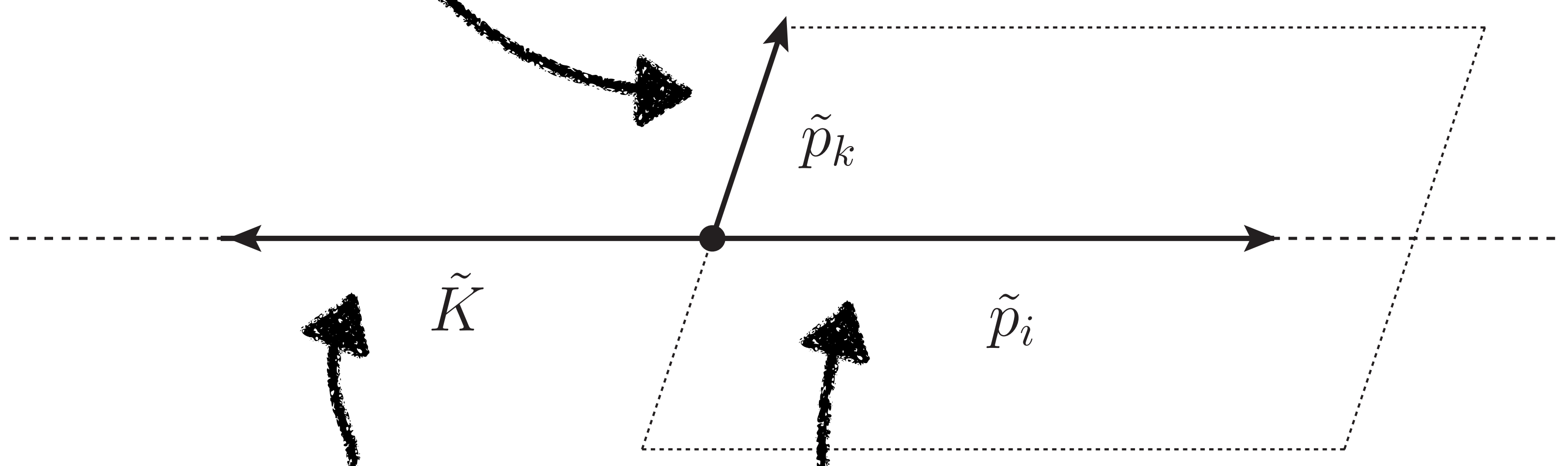
- full phase space coverage, splitting functions remain positive definite

Note related ideas in [Forshaw, Holguin, Plätzer '20]

Kinematics - global recoil scheme

- Before splitting:

colour spectator

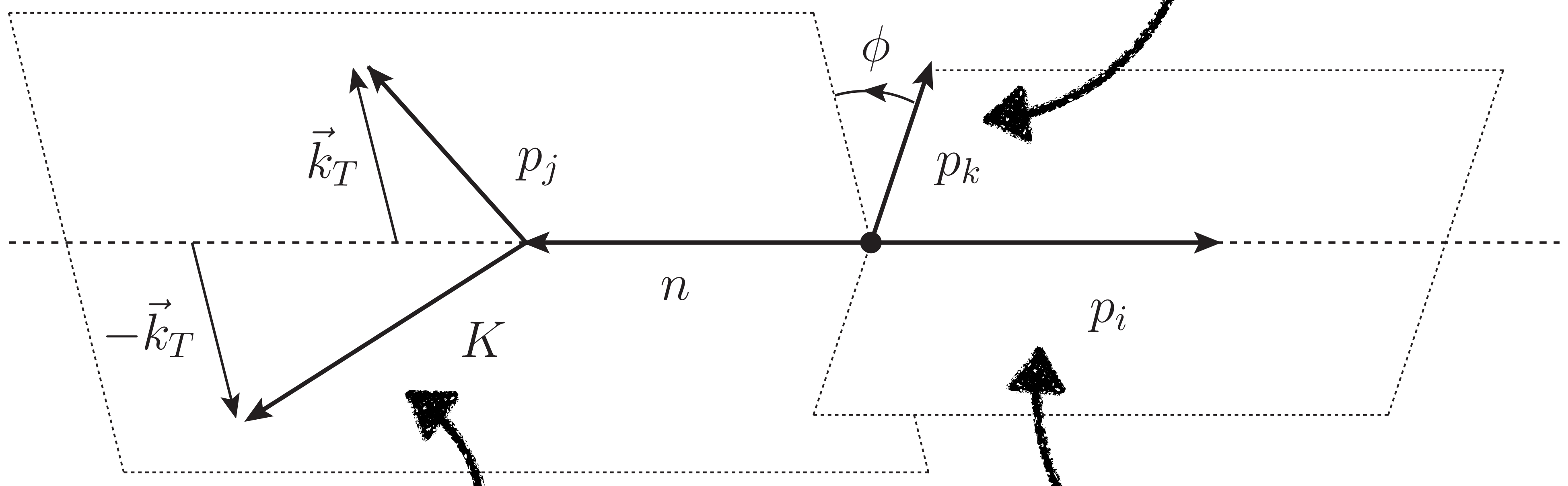


$\sum$ other momenta k_i

splitter

- After splitting:

$p_k = \tilde{p}_k$



$p_i = z\tilde{p}_i$

$K^2 = \tilde{K}^2$

$\tilde{K} + \tilde{p}_i = K + p_i + p_j$

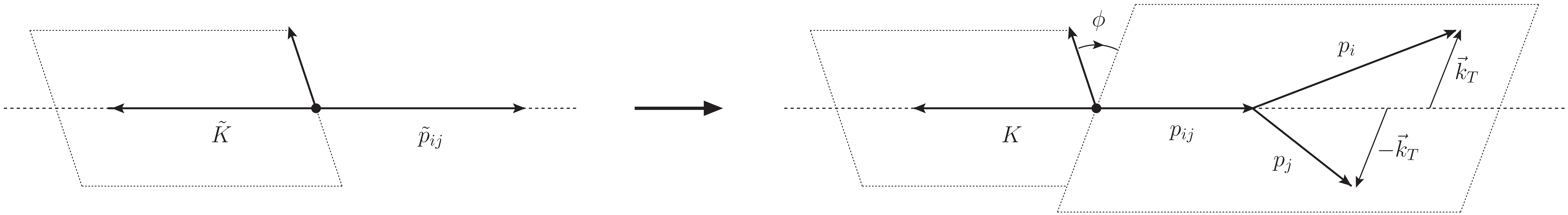
$$k_i^\mu \to \Lambda^\mu{}_\nu k_i^\nu$$

[Catani, Seymour '97]

$$\Lambda^\mu{}_\nu = g^\mu{}_\nu - \frac{(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{K\cdot\tilde{K} + \tilde{K}^2} + 2\frac{K^\mu \tilde{K}_\nu}{\tilde{K}^2} \quad \to \quad \Lambda^\mu{}_\nu \tilde{K}^\nu = K^\mu$$

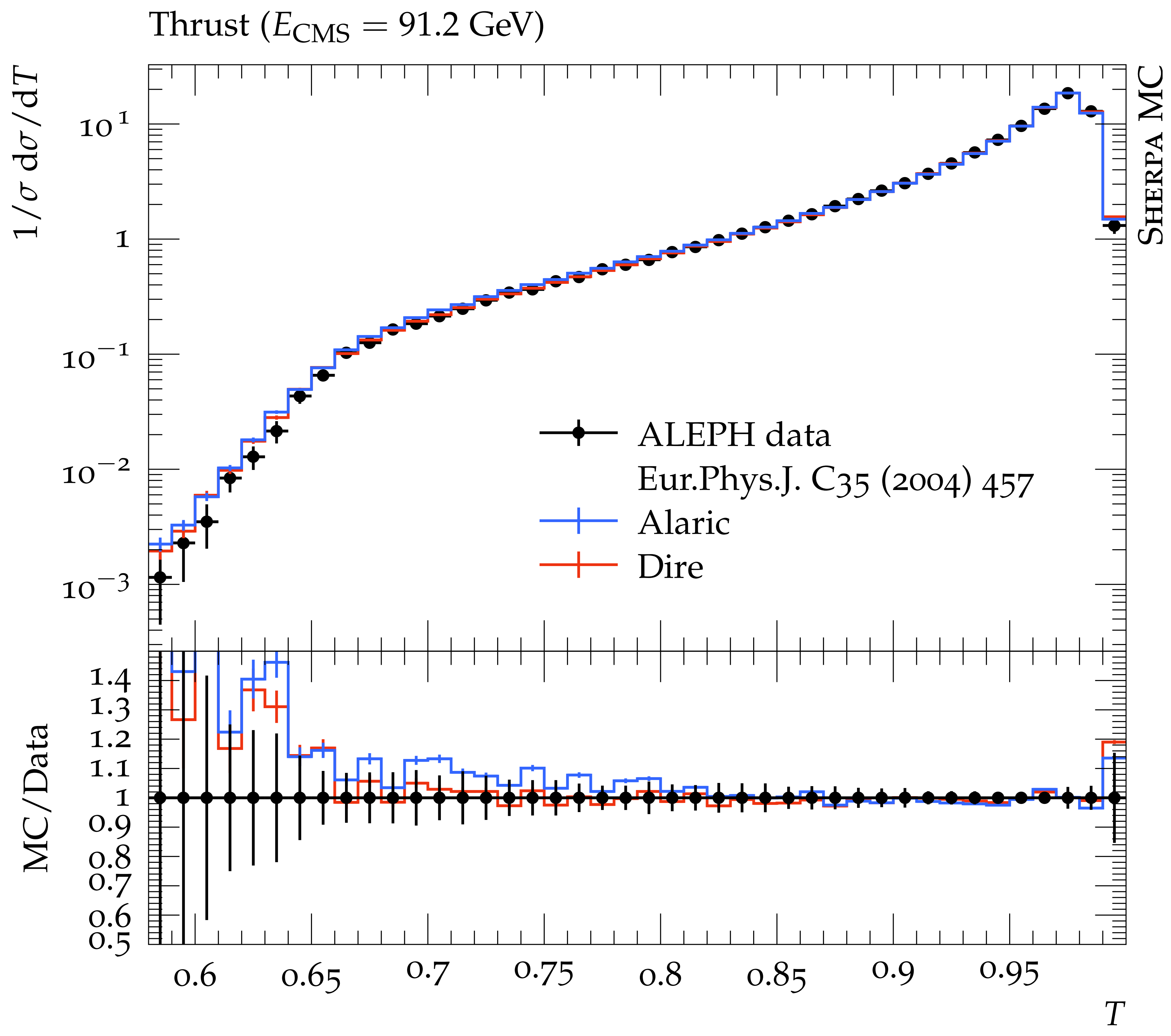
Kinematics - splitting vs. radiation kinematics

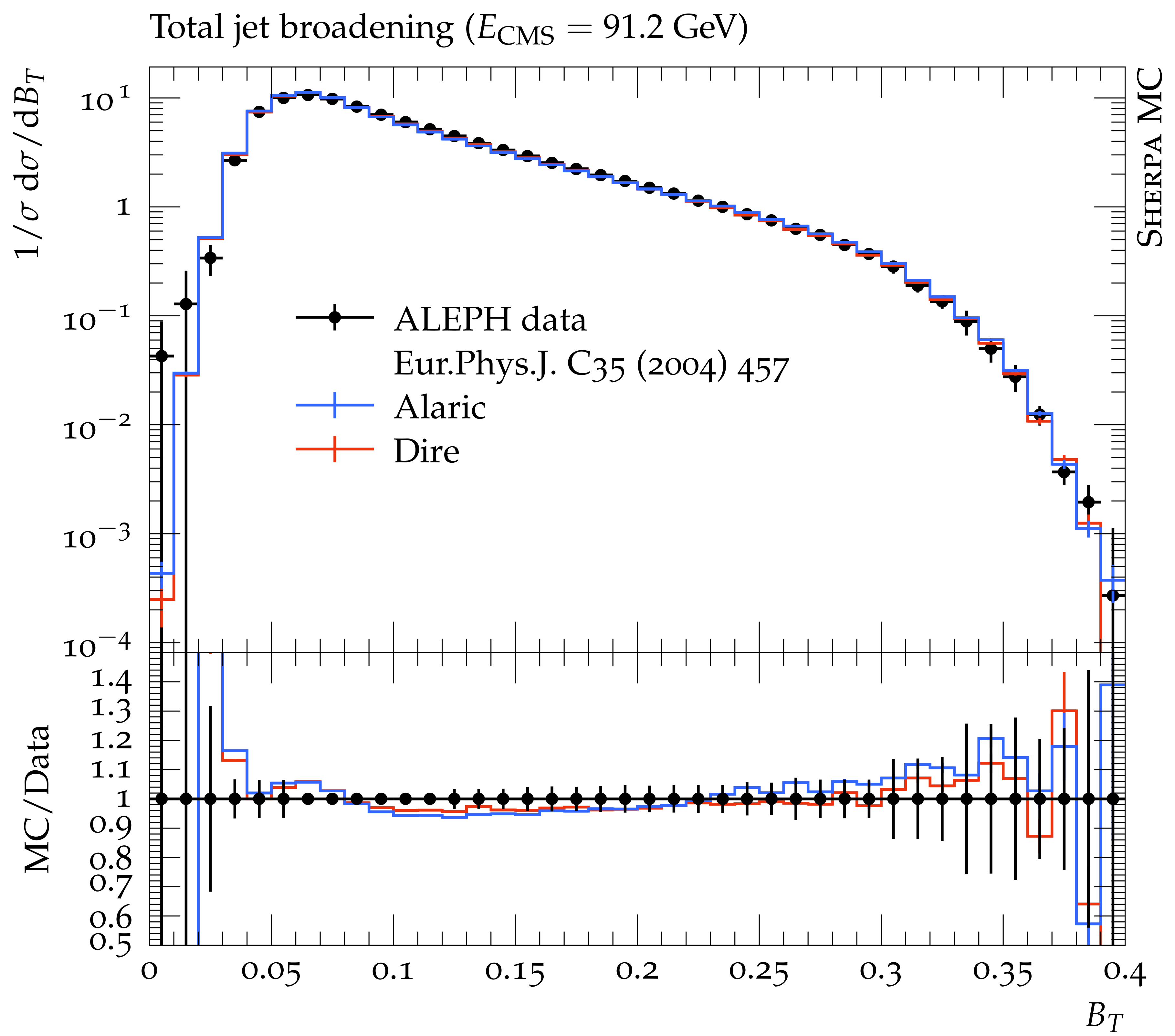
- Before splitting:
- After splitting:



- alternative: share transverse momentum recoil between splitter and spectator
- advantage: treat both particles symmetric, seems like a natural choice for e.g. $g \to q\bar{q}$ splitting (at least naively)
- disadvantage: significant impact on emitter kinematics possible, only applicable to purely collinear splitting functions (see later)

LEP observables





Thrust:

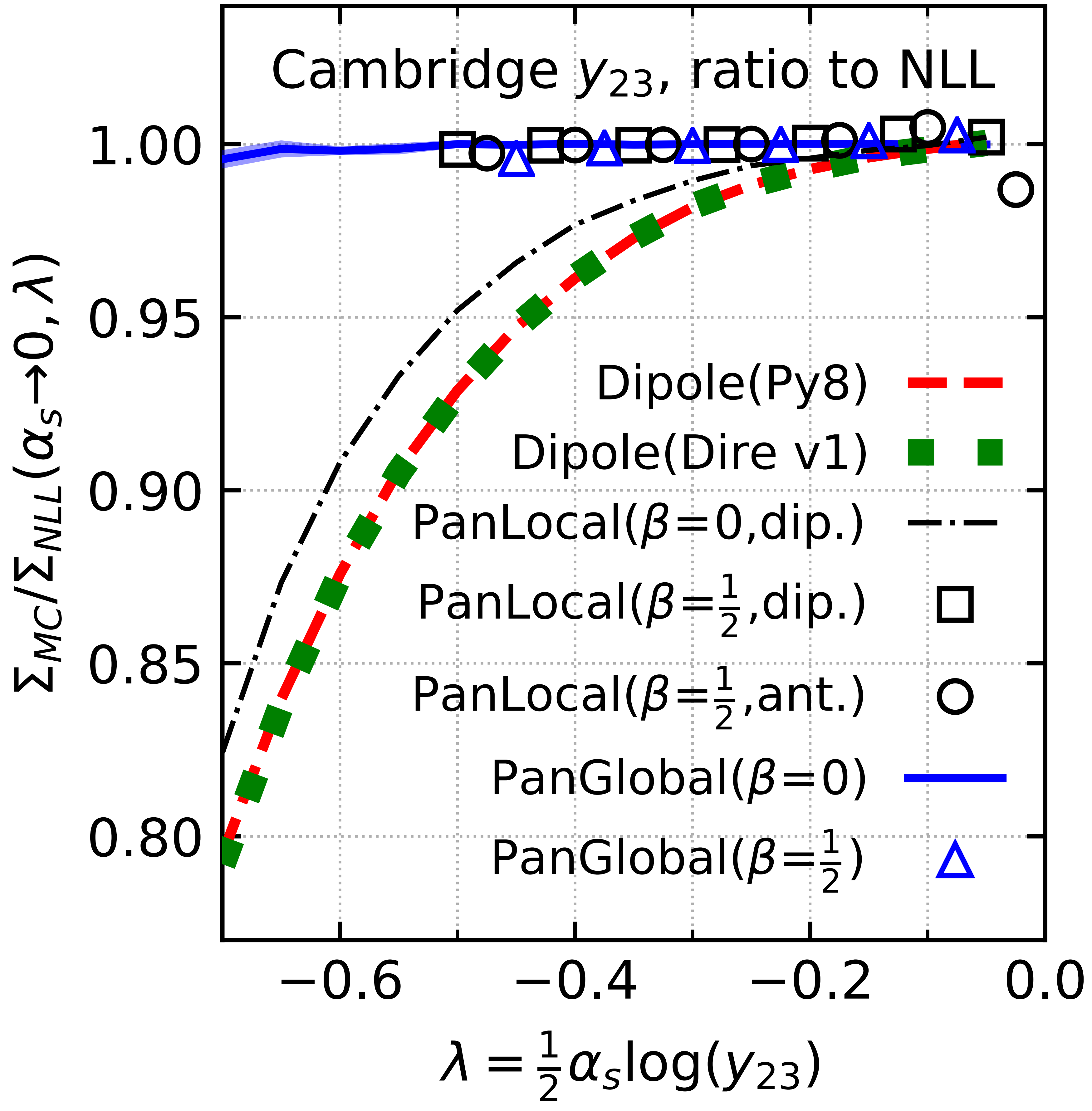
- Note this is T, not 1-T: soft physics is to the right
- Note there is no matching, relevant for small T

Total Broadening:

- soft physics is left hand side
- some deviations from data, but similar to Dire

New Parton Showers - NLL accuracy

- typical claim based on accuracy of splitting functions etc.
  - parton showers ~ NLL accurate if CMW scheme for strong coupling is used
- observation in [Dasgupta, Dreyer, Hamilton, Monni, Salam '18] (PanScales collaboration):
  - subtleties arise in distribution of recoil for subsequent emissions ⇒ phase space where accuracy is spoiled if soft gluon absorbs recoil
  - \+ in colour assignment
  - also: set of tests for shower accuracy [Dasgupta, Dreyer, Hamilton, Monni, Salam '20]

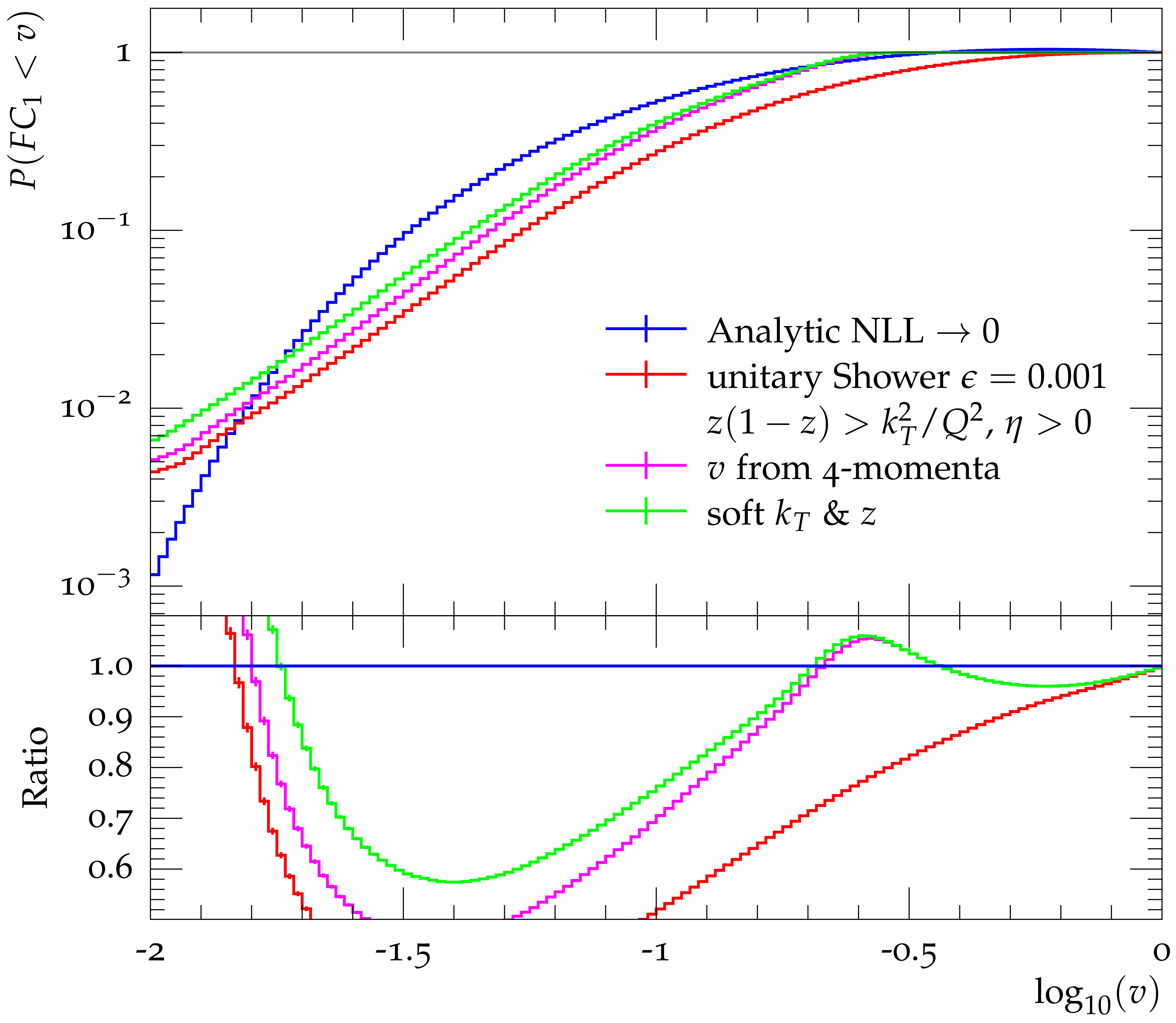


New Parton Showers - NLL accuracy

- Several solutions/re-evaluations of parton shower concepts:
- [Dasgupta, Dreyer, Hamilton,Monni, Salam, Soyez ’20], [vanBeekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso,Soyez ’22]
  - partitioning of splitting functions and appropriate choice of evolution variable can lead to NLL accurate shower for local and global recoil strategies
- [Forshaw, Holguin, Plätzer ’20]
  - Connections between angular ordered and dipole showers
- [Nagy, Soper ’11]
  - local transverse, global longitudinal recoil
- [Herren, Krauss, DR, Schönherr, Höche ’22]
  - global recoil, enables analytic comparison to resummation and proof of NLL accuracy
- [Preuss ’24]
  - global recoil in antenna shower Vinca

Beyond logarithmic accuracy

- Observations
  - LL and NLL accurate showers can be very similar (e.g. failing of NLL accuracy numerically undetectable for Dire in prominent observables like Thrust)
  - NLL accurate showers can differ significantly from NLL result away from strict limit
  - ⇒ subleading effect play a significant role in phenomenological successful parton showers, more systematic understanding desirable, see also [Höche, Siegert, DR ’17]



Alaric beyond NLL - subleading effects

assume Sudakov decompose like

$$p_i^\mu = z_i \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_i\, 2p_{ij}\bar{n}}\, \bar{n}^\mu + k_t^\mu \ ,$$

$$p_j^\mu = z_j \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_j\, 2p_{ij}\bar{n}}\, \bar{n}^\mu - k_t^\mu$$

derivation of splitting functions leads to:

$$P^{(\mathrm{F})}_{qq\,\|}(p_i, p_j, \bar{n}) = C_F\, (1-\varepsilon)(1-z_i)$$

$$P^{(\mathrm{F})}_{gg\,\|}(p_i, p_j, \bar{n}) = 2C_A\, z_i z_j \ ,$$

$$P^{(\mathrm{F})}_{gq\,\|}(p_i, p_j, \bar{n}) = T_R \left[\, 1 - \frac{2\, z_i z_j}{1-\varepsilon}\, \right] .$$

actual shower kinematics:

$$p_i = z\, \tilde{p}_i \ ,$$

$$p_j = (1-z)\, \tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\, \tilde{p}_i\big) - k_\perp \ ,$$

$$K = \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\, \tilde{p}_i\big) + k_\perp \ ,$$

$$p_i = \frac{z}{1-v(1-z+\kappa)}\, \hat{p}_{ij} + \frac{z}{1-v(1-z+\kappa)}\, k_\perp + \mathcal{O}\left(\frac{k_\perp^2}{2\tilde{p}_i\tilde{K}}\right) \ ,$$

$$p_j = \frac{(1-z)(1-v) - v\kappa}{1-v(1-z+\kappa)}\, \hat{p}_{ij} - \frac{z}{1-v(1-z+\kappa)}\, k_\perp + \mathcal{O}\left(\frac{k_\perp^2}{2\tilde{p}_i\tilde{K}}\right)$$

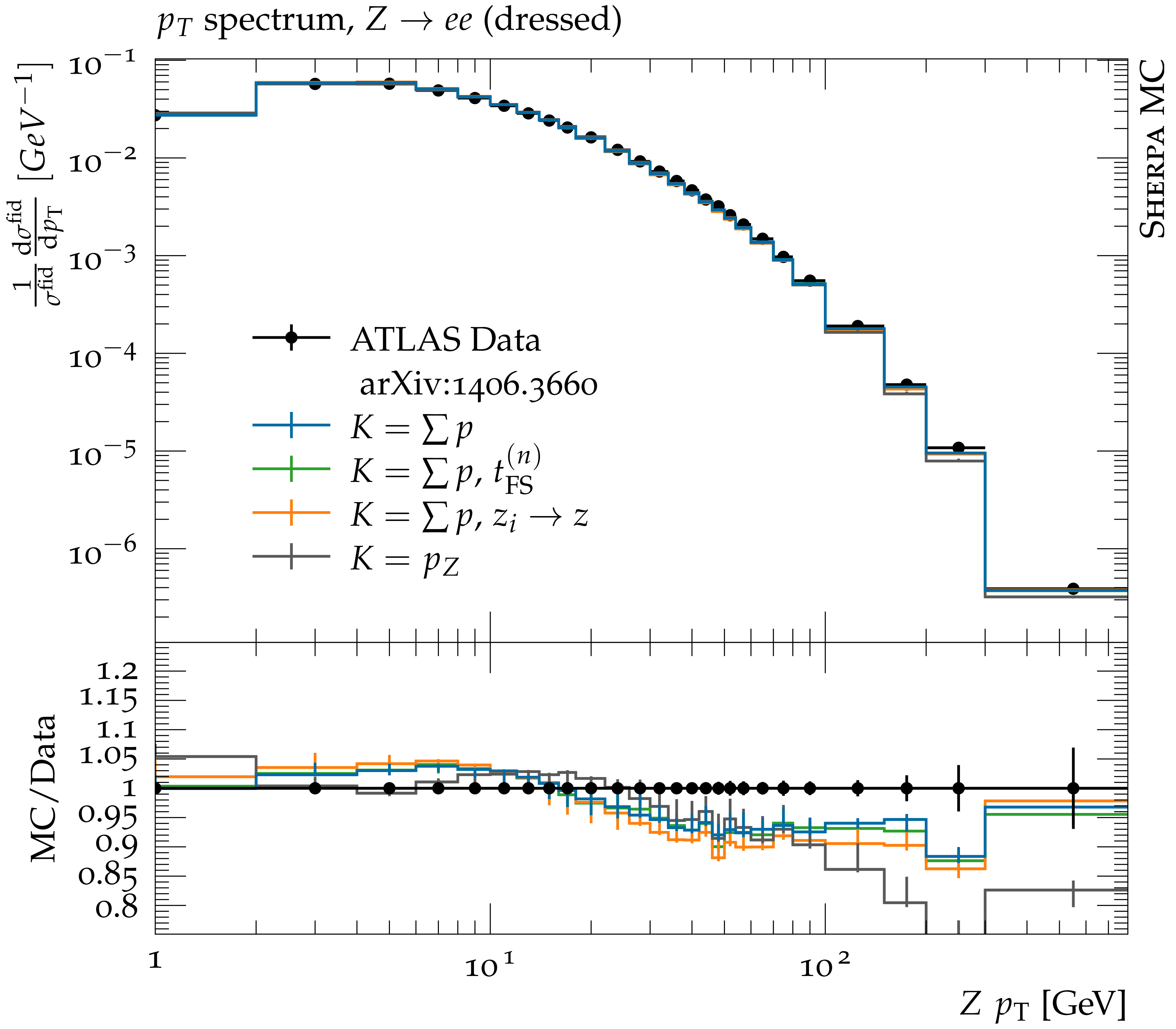
ultimately, “proper” splitting variables:

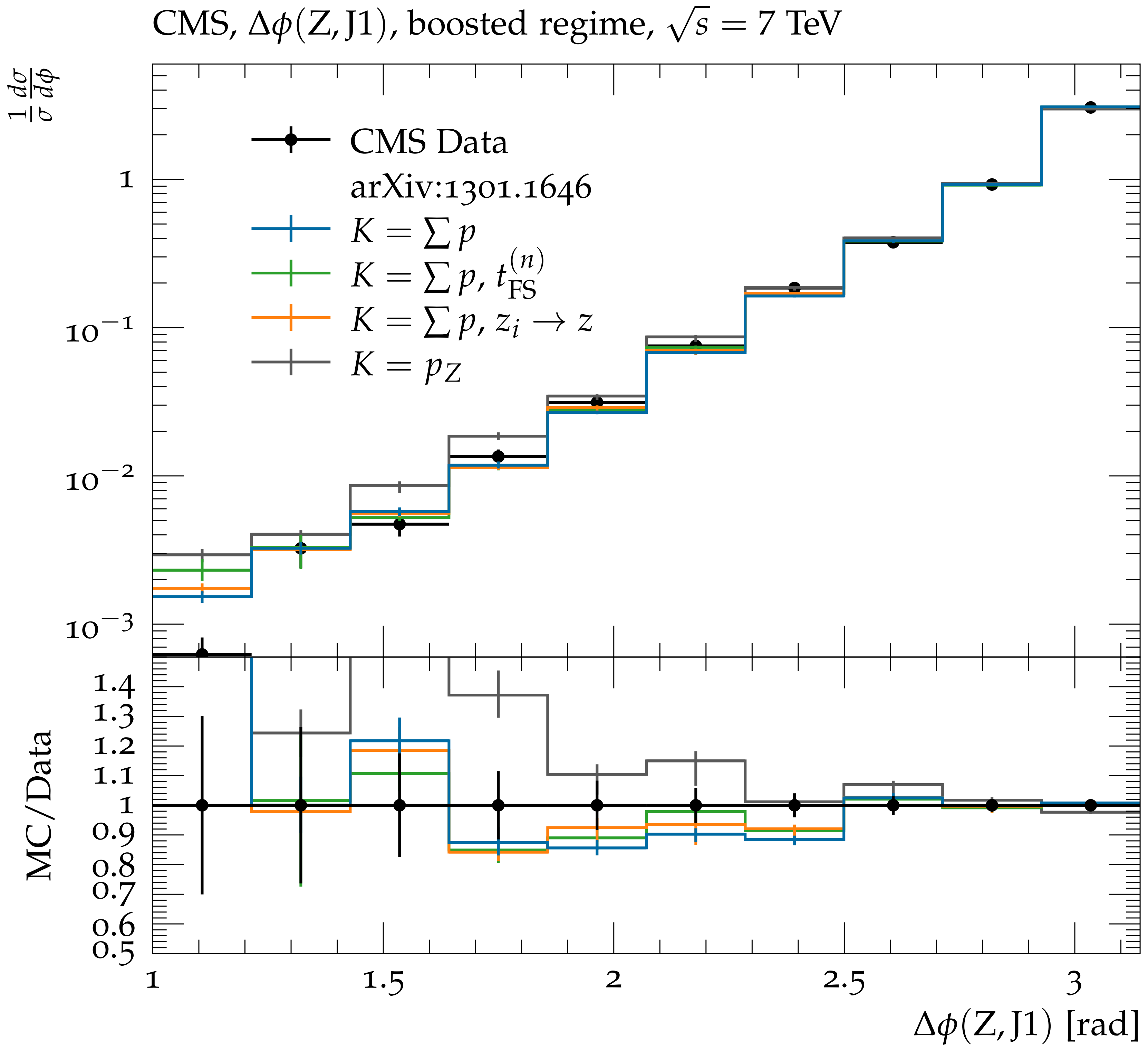
$$z_i = \frac{z}{1-v(1-z+\kappa)} \ ,$$

$$z_j = 1 - \frac{z}{1-v(1-z+\kappa)}$$

Alaric — subleading effects in Z+jets

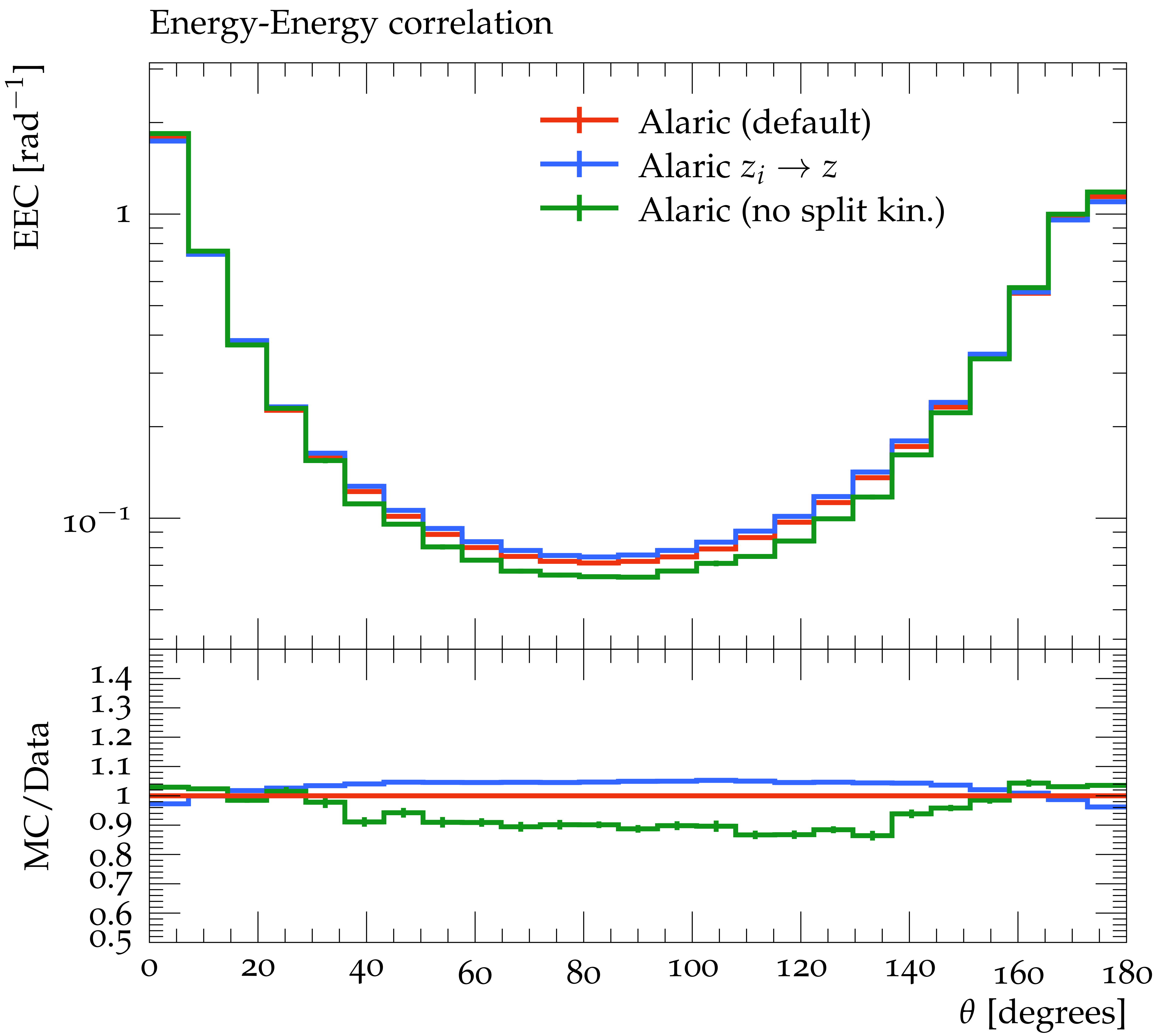
- effects/choices beyond NLL accuracy:
  - choice of evolution variable (up to factors of $z \sim 1$)
  - identify PS parameter z with z_i, z_j
  - choice of recoil momentum K (NLL accuracy needs "hard" K)





Towards subleading effects for lepton colliders

- same variations available for lepton colliders (as far as they are applicable)
- example here: EEC from $q\bar{q}$ final state at the Z pole
- systematic variations not captured by e.g. scale variations
  - additional uncertainty
  - kinematics enter splitting functions, hope for systematic reduction at higher orders



Summary

- Alaric parton shower
  - partial fractioning of eikonal leading to positive splitting functions filling full phase space
  - global kinematics for soft splitting functions, guarantees NLL and analytic tracking of accuracy
- new developments:
  - CKKW merging
  - systematic variations of NLL ambiguities
- future
  - MC@NLO matching on the way (enabling NLL' accuracy in the soft limit)
  - higher order splitting functions with Dire technique
  - spin correlations to complete radiation pattern

Compare: resummation e.g. in CAESAR

- factorisation of matrix elements in soft collinear limit well known
- how to extract NLL observable independent (i.e. without additional information)?
- method from [Banfi, Salam, Zanderighi '05]: need explicit implementation of soft-collinear limit*:

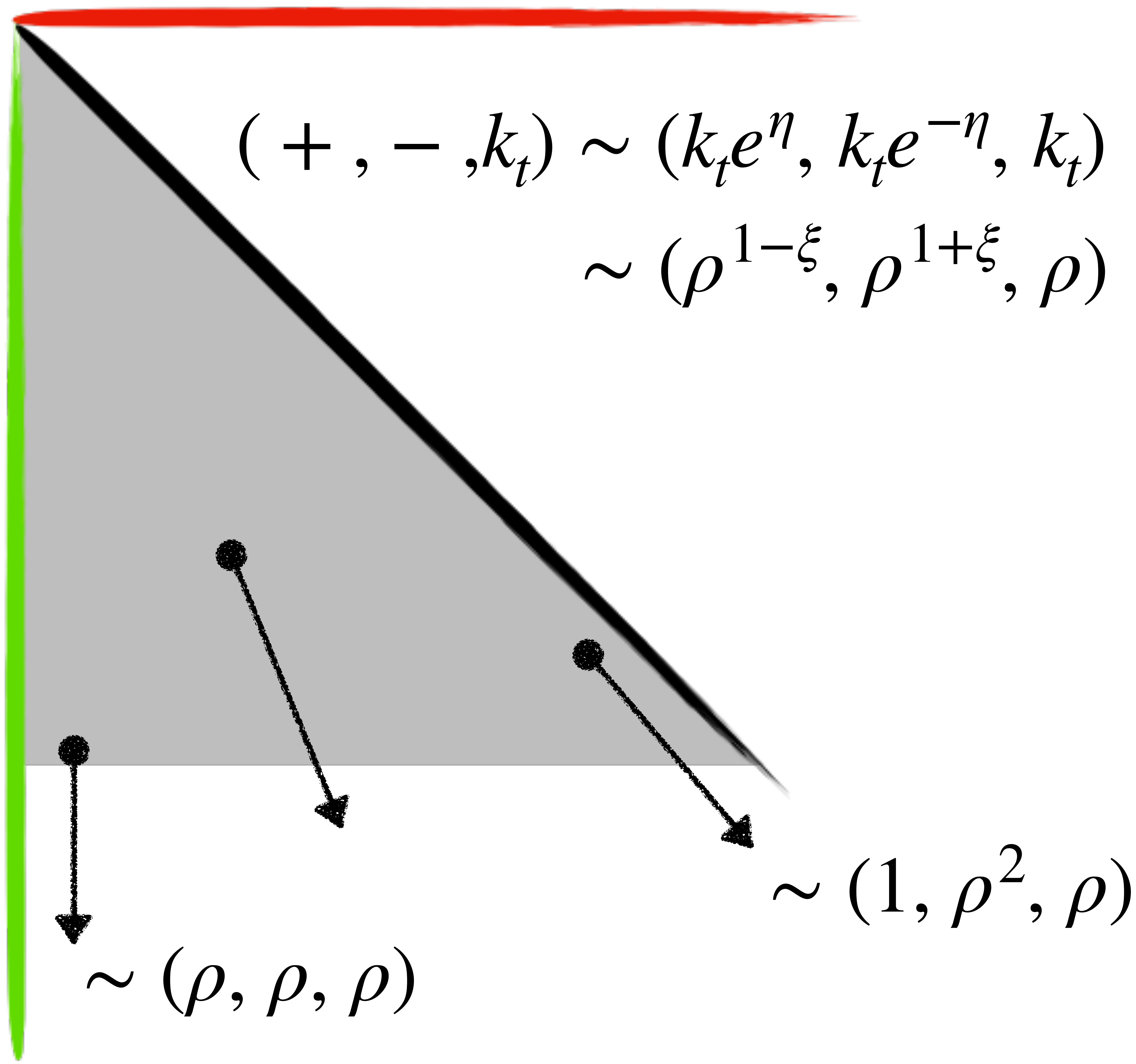
$$k_t^\rho = k_t \rho \qquad \xi = \frac{\eta}{\eta_{\max}}$$

$$\eta^\rho = \eta - \xi \ln \rho$$

and assume

$$V(k_i^\rho) = \rho V(k_i)$$

→ numerically evaluate phase space integrals in this limit



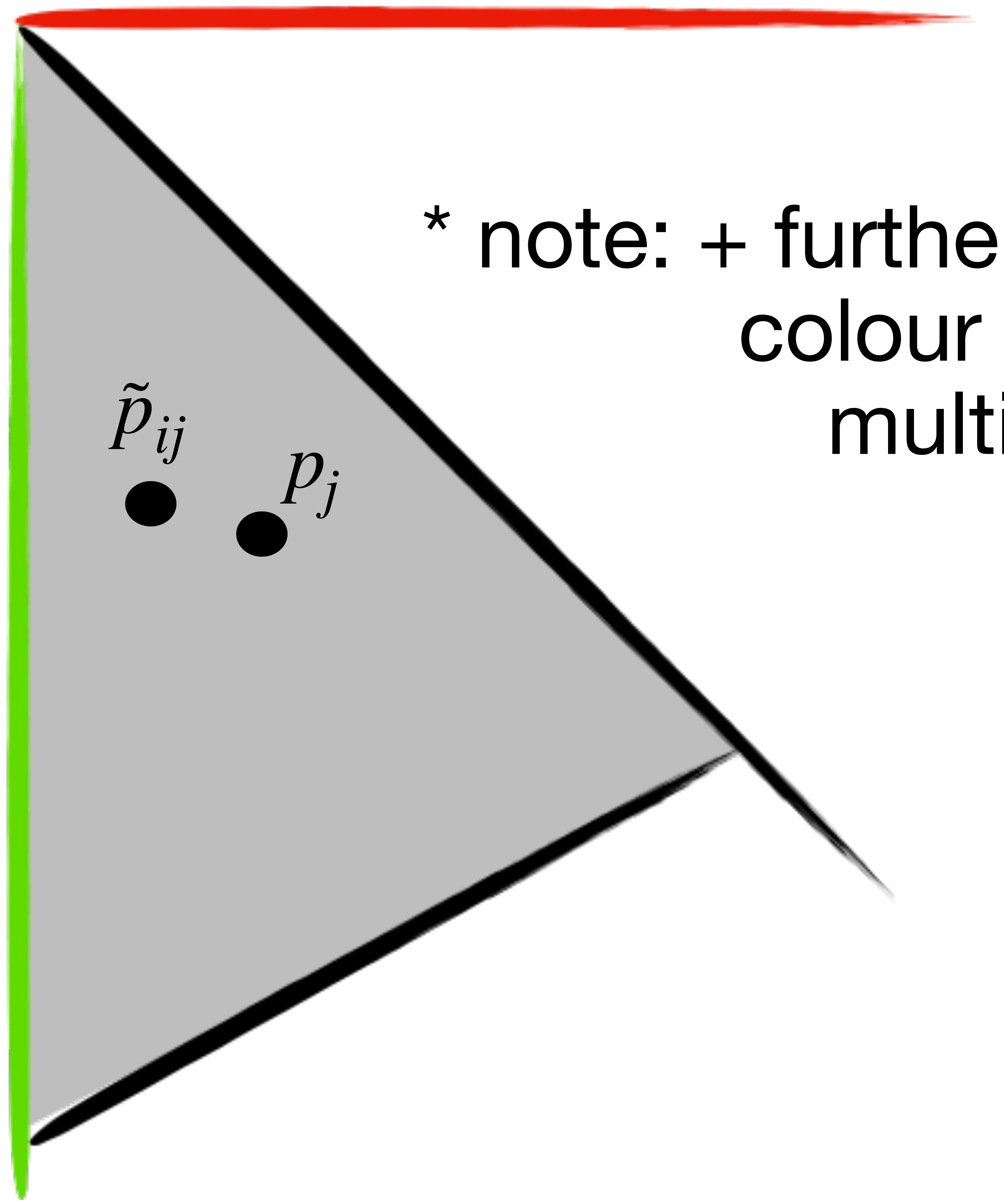
* example assuming $V(k_t, \eta) \sim k_t/Q$ for brevity

Effect of recoil on accuracy

- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \to 0$)?*
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]

$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$

- consider situation where we first emit $\tilde{p}_{ij}$ from p_a, p_b, then emit p_j, $\tilde{p}_{ij} \to p_i, p_j$



* note: + further problems for colour assignment in multiple emissions

- transverse momentum of p_i will be $k_t^i \sim k_t^{ij} + k_t^j \to k_t^{ij}$ as $\dfrac{k_t^j}{k_t^i} \to 0$

- but, relevant limit is $\dfrac{\Delta k_t^i}{k_t^i} \to \dfrac{\rho k_t^j}{\rho k_t^i} = \mathcal{O}(1)$

Alaric at the LHC — jets

- [Höche, Krauss, DR '24] extend Alaric method to IS evolution
- satisfactory description of inclusive and dijet events
- transverse momentum spectrum of leading jet and ratio 3-to-2 jet rate
- NLL accuracy shown numerically for FS, in addition to analytic proof