Fragmentation functions at future lepton colliders

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based on 2305.14620 with ChongYang Liu, XiaoMin Shen, Bin Zhou
2401.02781, 2407.04422 with ChongYang Liu, XiaoMin Shen, HongXi Xing, YuXiang Zhao
and work in progress with Bin Zhou

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Outline

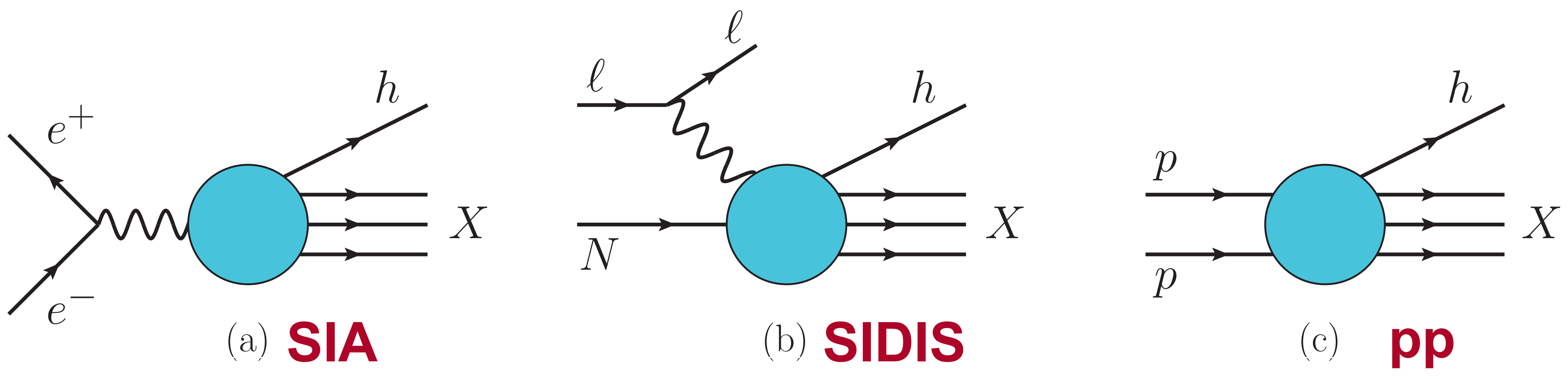
- 1. Introduction
- 2. NPC23: a global analysis of FFs to light charged hadrons
- 3. Projection on FFs at future lepton colliders
- 4. Summary

Single inclusive hadron production

- In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), semi-inclusive DIS (SIDIS), pp collisions

single inclusive hadron production/observable

[1607.02521]



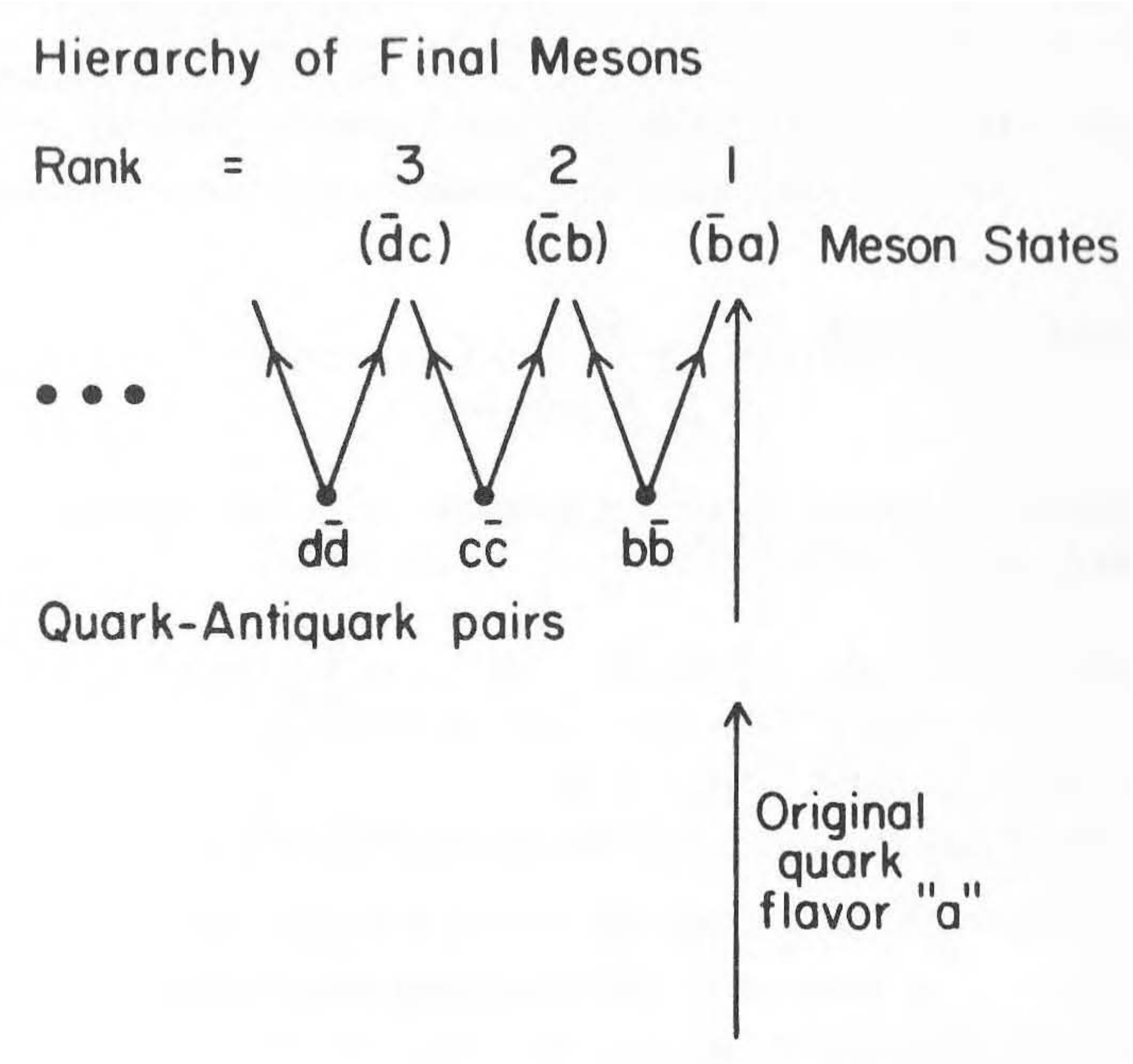
(a) **SIA** (b) **SIDIS** (c) **pp**

$$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{e^+e^-\to hX}}{dz} = F^h(z,Q^2)\,, \quad z = \frac{2\,E_h}{\sqrt{s}}$$

exp. definition of unpolarized collinear FFs

other forms: polarized FFs, TMD FFs, di-hadron FFs

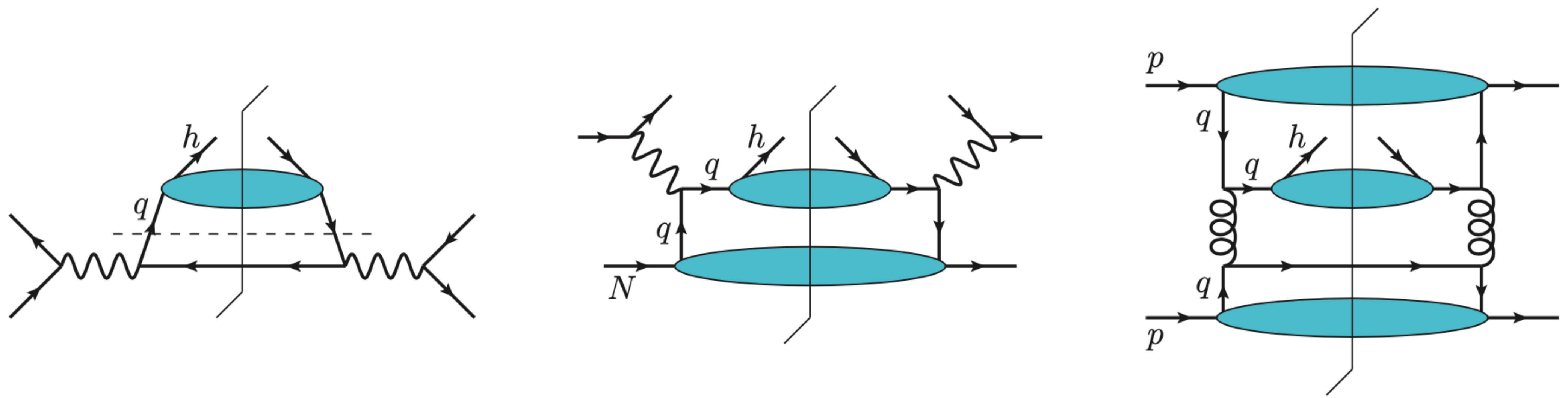
parton model **[Field&Feynman]**



distribution of momentum to mesons via creation of quark-antiquark pairs in cascade

QCD collinear factorization

- QCD collinear factorization ensures universal separation of long-distance and short-distance contributions in high energy scatterings involving initial/final state hadrons, and enables predictions on cross sections



(a) **SIA** (b) **SIDIS** (c) **pp**

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{e^+e^-\to hX}}{dz} = \frac{1}{\sum_q e_q^2}\left(2F_1^h(z,Q^2)+F_L^h(z,Q^2)\right)$$

$$2F_1^h(z,Q^2) \;=\; \sum_q e_q^2\left(D_1^{h/q}(z,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\left(C_1^q\otimes D_1^{h/q}+C_1^g\otimes D_1^{h/g}\right)(z,Q^2)\right)$$

$$\frac{d^3\sigma^{\ell p\to \ell hX}}{dx\,dy\,dz} = \frac{2\pi\alpha_{\rm em}^2}{Q^2}\left(\frac{1+(1-y)^2}{y}\,2F_1^h(x,z,Q^2)+\frac{2(1-y)}{y}\,F_L^h(x,z,Q^2)\right)$$

$$\begin{aligned}2F_1^h(x,z,Q^2) \;=\; \sum_q e_q^2\Bigg(f_1^{q/p}D_1^{h/q}+\frac{\alpha_s(Q^2)}{2\pi}\Big(&f_1^{q/p}\otimes C_1^{qq}\otimes D_1^{h/q}\\ &+f_1^{q/p}\otimes C_1^{gq}\otimes D_1^{h/g}+f_1^{g/p}\otimes C_1^{qg}\otimes D_1^{h/q}\Big)\Bigg]\,,\end{aligned}$$

- coefficient functions, hard scattering; infrared (IR) safe, calculable in pQCD, independent of the hadron
- FFs/PDFs, reveal inner structure of hadrons or parton-hadorn transition; NP origin, universal, e.g. DIS vs. pp collisions; fitted from data
- runnings of FFs/PDFs with μ_D/μ_f are governed by the DGLAP equation

unpolarized collinear FFs, operator definition

$$\begin{aligned}D_1^{h/q}(z) \;=\; &\frac{z}{4}\sum\!\!\!\!\!\!\int_X\int\frac{d\xi^+}{2\pi}\,e^{ik^-\xi^+}\,{\rm Tr}\Big[\langle 0|\,\mathcal{W}(\infty^+,\xi^+)\,\psi_q(\xi^+,0^-,\vec{0}_T)\,|P_h,S_h;X\rangle\\ &\times\langle P_h,S_h;X|\,\bar\psi_q(0^+,0^-,\vec{0}_T)\,\mathcal{W}(0^+,\infty^+)\,|0\rangle\,\gamma^-\Big]\,.\end{aligned}$$

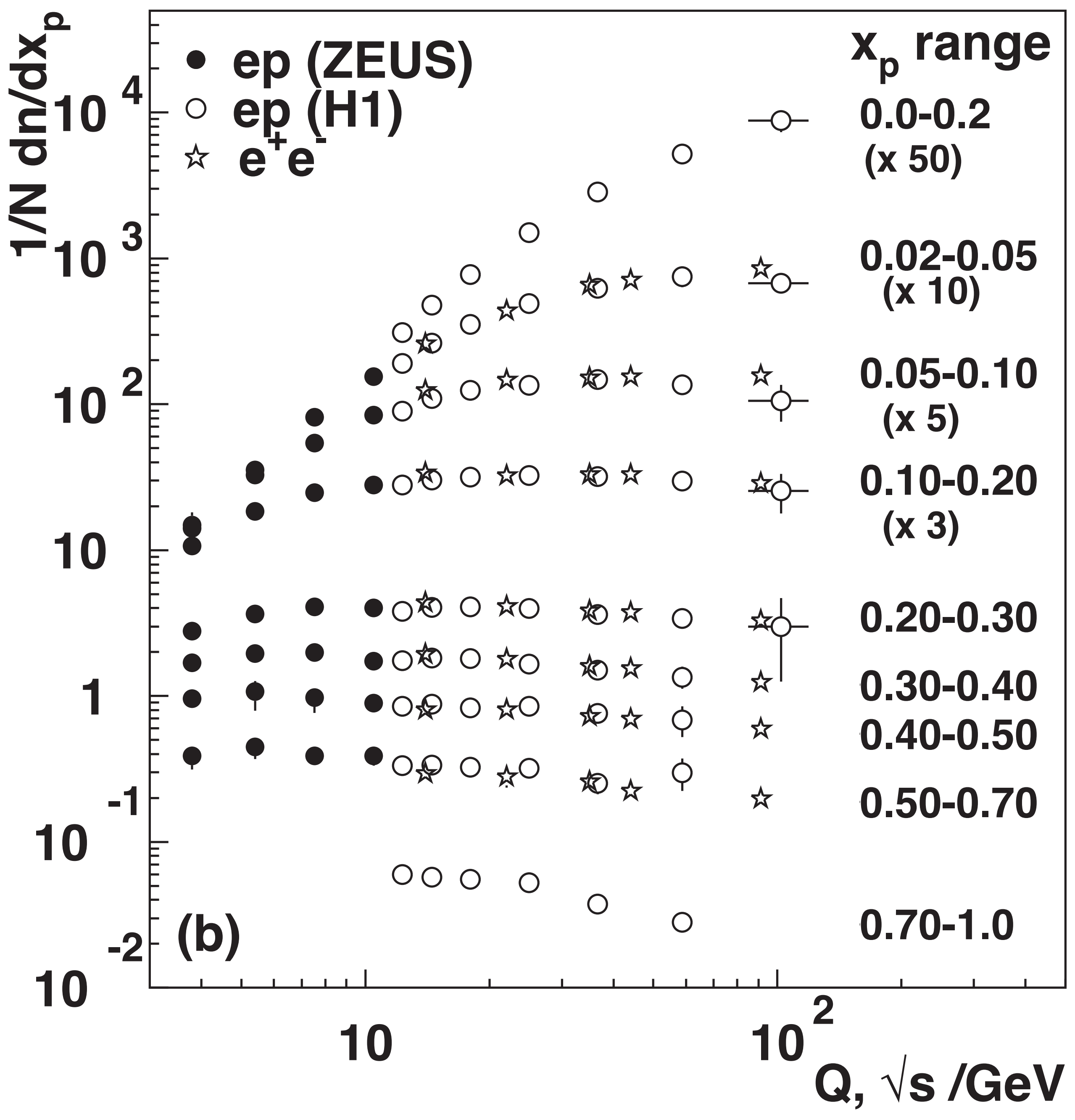
$$\frac{d}{d\ln\mu^2}D_1^{h/i}(z,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\sum_j\int_z^1\frac{du}{u}\,P_{ji}(u,\alpha_s(\mu^2))\,D_1^{h/j}\left(\frac{z}{u},\mu^2\right)$$

[Collins, Soper, Sterman]

Global data and phenomenological analysis

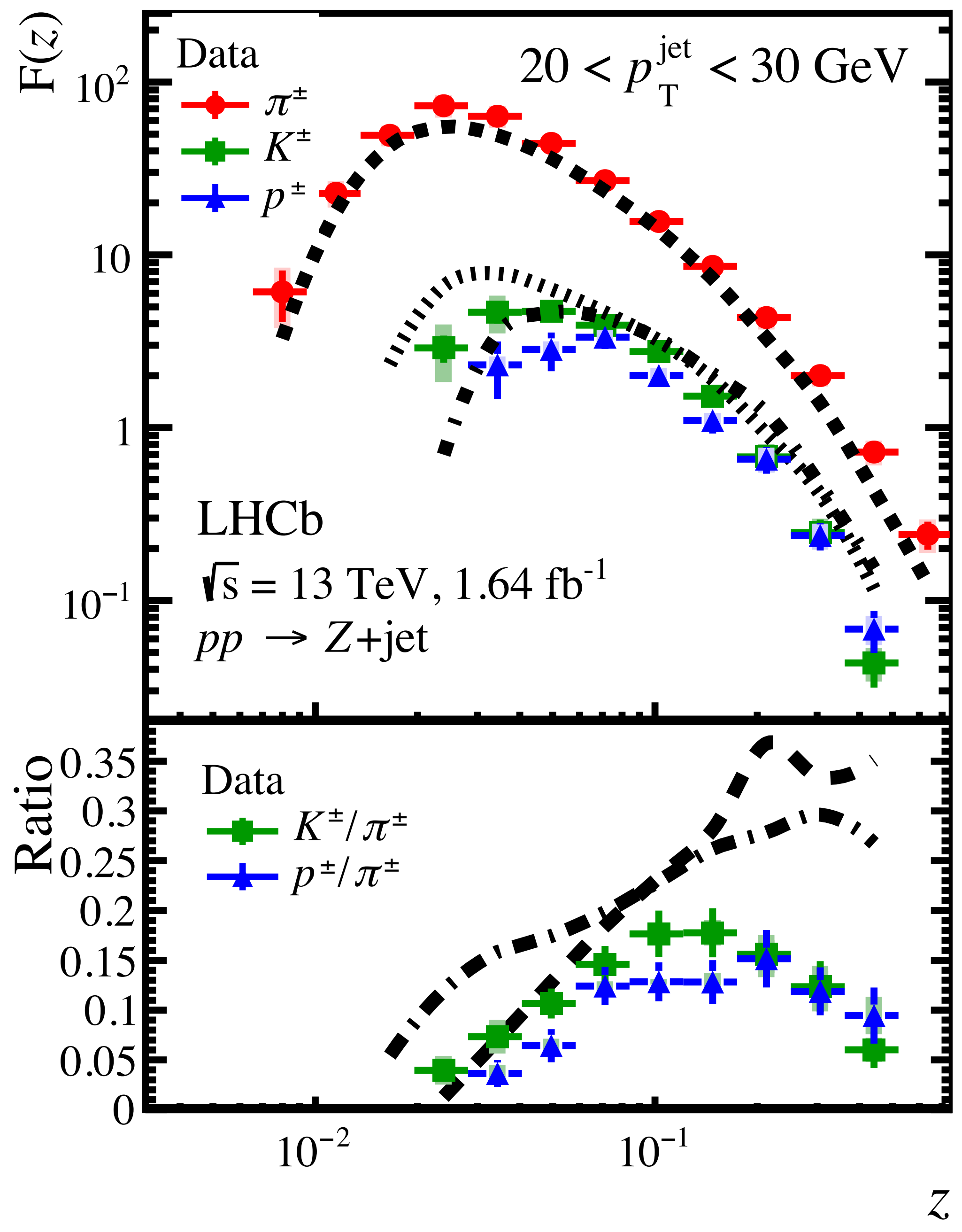
- Measurements are available from colliders SLAC, LEP, HERA, RHIC, LHC and fixed-target HERMES, COMPASS experiments for various charged hadrons as well as neutral hadrons; many groups provide phenomenological FFs from global analysis at NLO/NNLO in QCD

single incl. production of unidentified charged hadrons (SIA & SIDIS)



[Particle data group]

Jet fragmentation to light charged hadrons (LHCb)



[2208.11691]

$$z = \frac{\mathbf{p}_{\rm had} \cdot \mathbf{p}_{\rm jet}}{|\mathbf{p}_{\rm jet}|^2}, \quad F(z) = \frac{1}{N_{Z+\rm jet}} \frac{dN_{\rm had}(z)}{dz}$$

global analysis

- ❖ major groups/families include BKK, AKK, HKNS, DSS, NNFF, MAPFF, JAM etc.
- ❖ mostly done at NLO in QCD since exact NNLO coefficient functions only known recently for SIDIS
- ❖ different determination can be quite different due to selection of data sets as well as theory treatments, not converge as well as the case of PDF fits

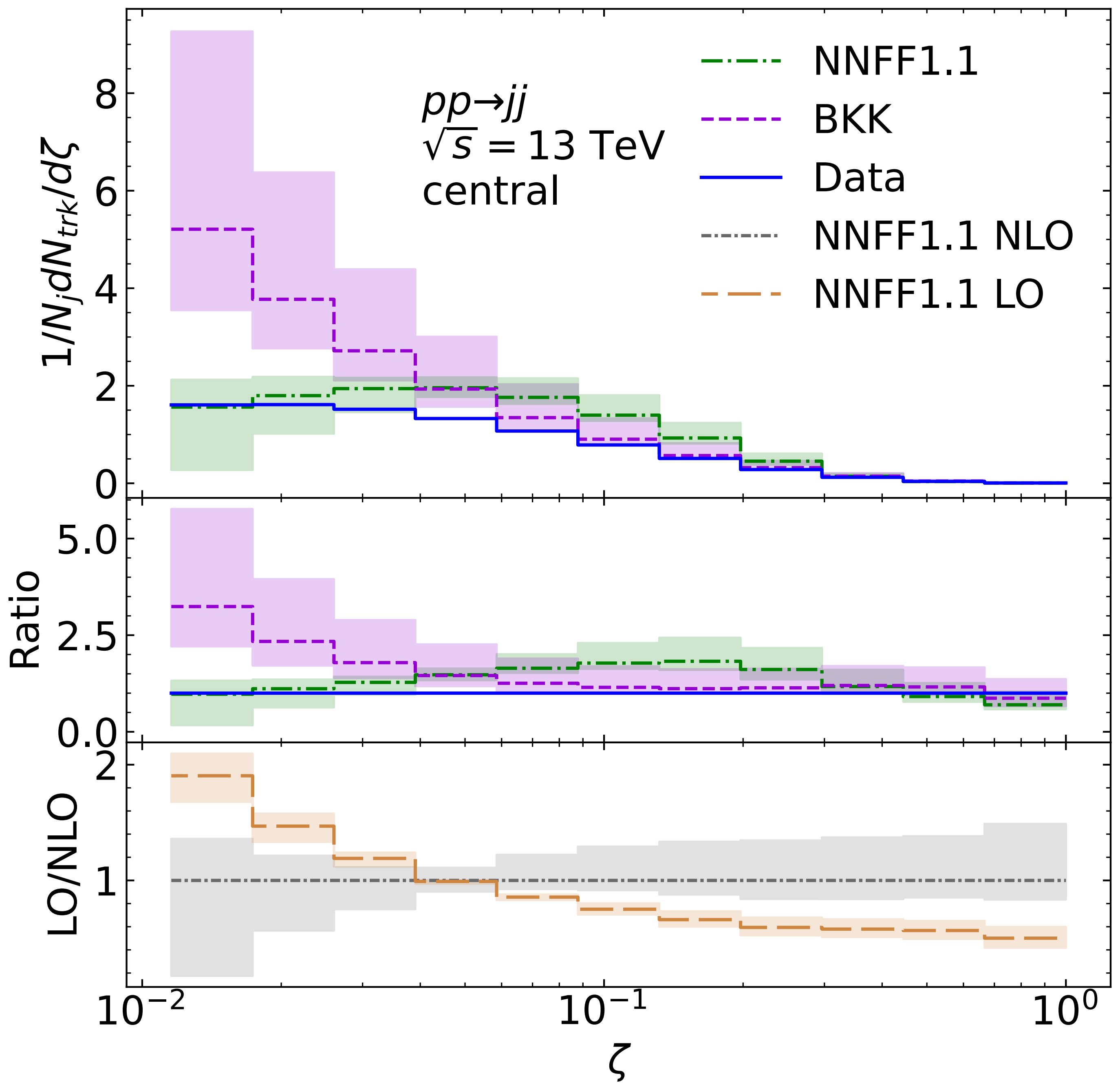
[1607.02521 for a review]

FMNLO (fragmentation at NLO in QCD)

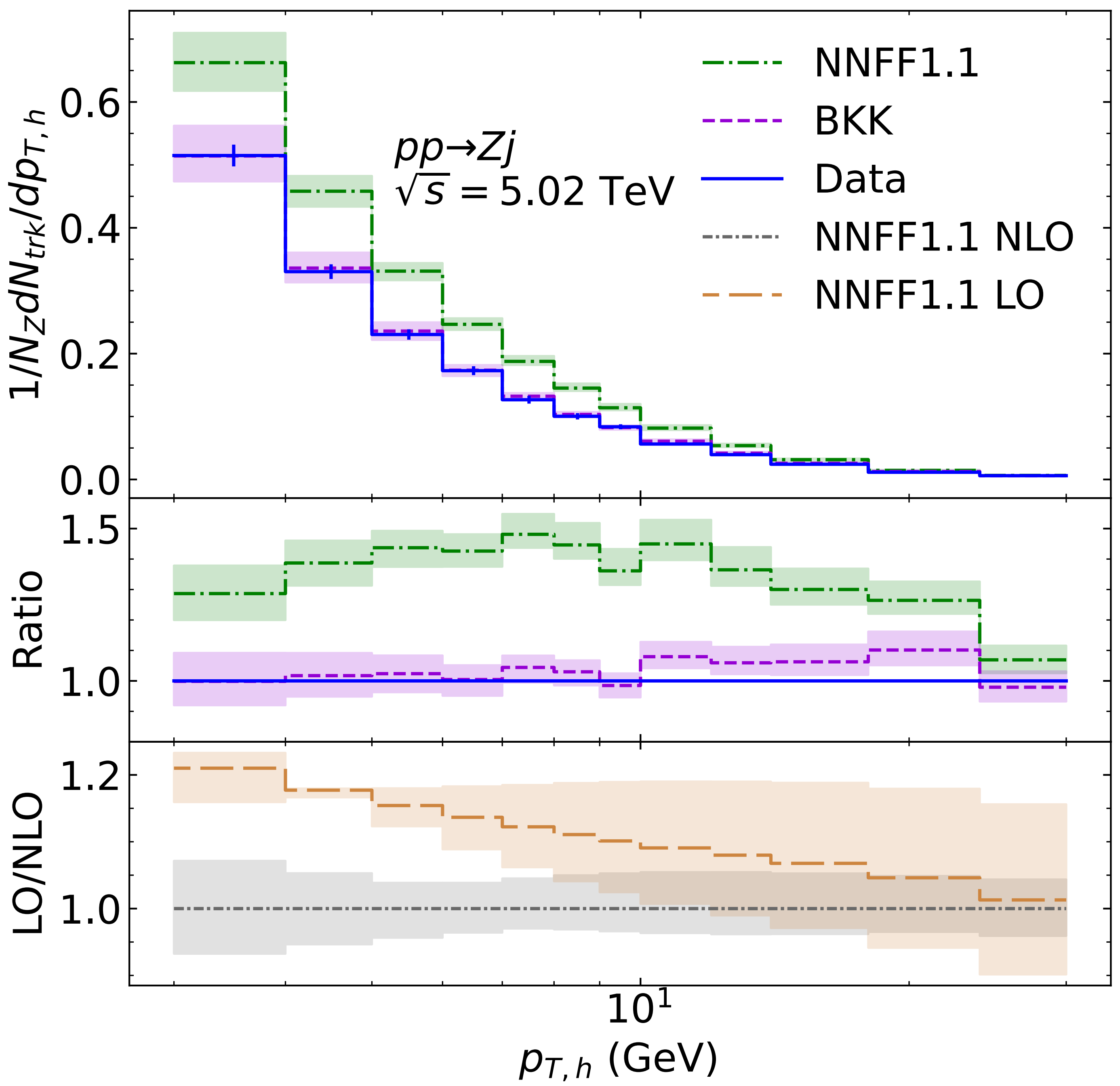
- FMNLO is a program for automated and fast calculations of fragmentation cross sections of arbitrary processes. It is based on a hybrid scheme of phase-space slicing method and local subtraction method, accurate to NLO in QCD

  - automation of fragmentation calculations for arbitrary hard processes at NLO, within SM and BSMs via MG5_aMC@NLO
  - fast convolution algorithms of partonic cross sections with FFs without repeating the time consuming MC integrations
  - future goal/generalizations: transverse observables, NNLO corrections

QCD inclusive dijets at LHC



Z-boson tagged jet




🔥 **News**

2023.05: 🎉🎉 FMNLOv1.0 first release of `FMNLO` interfaced with `MG5_aMC@NLO`.

https://fmnlo.sjtu.edu.cn/~fmnlo/

[JG, Liu, Shen, Zhou, 2305.14620]

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NPC23 analysis of FFs

- Establishing a new framework on global analysis of fragmentation functions to identified charged hadrons, including charged pion, kaon and proton, using most recent data from SIA, SIDIS, and pp collisions

parametrization of FFs to charged pion/kaon/proton at an initial scale (Q=5 GeV):

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to-π^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$\bar{d} \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F				4

parton-to-K^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y					x	4
$\bar{s} \simeq u$	Y	-	-		-	x	1
$\bar{u}=d=\bar{d}=s$	N					x	4
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

parton-to-p	favored	α	β	a_0	a_1	a_2	d.o.f.
$u = 2d$	Y					x	4
$\bar{u} = \bar{d} = s = \bar{s}$	N				x	x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

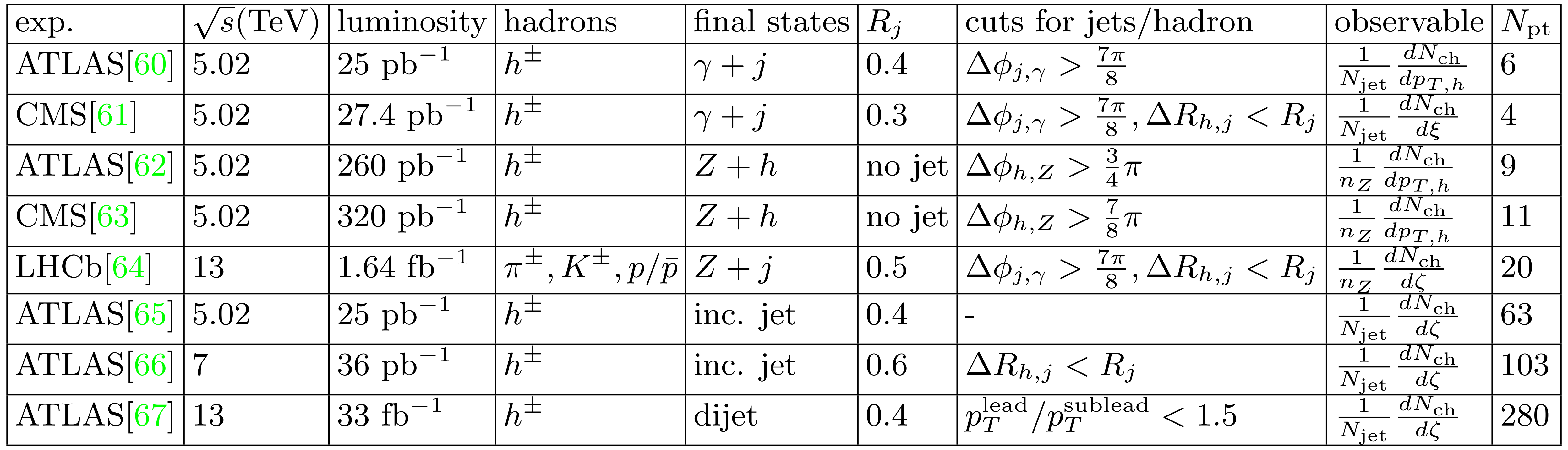
- a joint determination of FFs to charged pion, kaon and proton at NLO in QCD (63 parameters) including estimation of uncertainties with Hessian sets
- apply a strong selection criteria on the kinematics of fragmentation processes to ensure validity of LT factorization and perturbative calculations ($z>0.01$ and $E_h/p_{T,h}>4$ GeV)
- including theory uncertainties (residual scale variations) into the covariance matrix
- use fast interpolation techniques for calculations of cross sections which largely increase efficiency of the global fit

[JG, CY Liu, XM Shen, HX Xing, YX Zhao, 2401.02781]

Selection of data

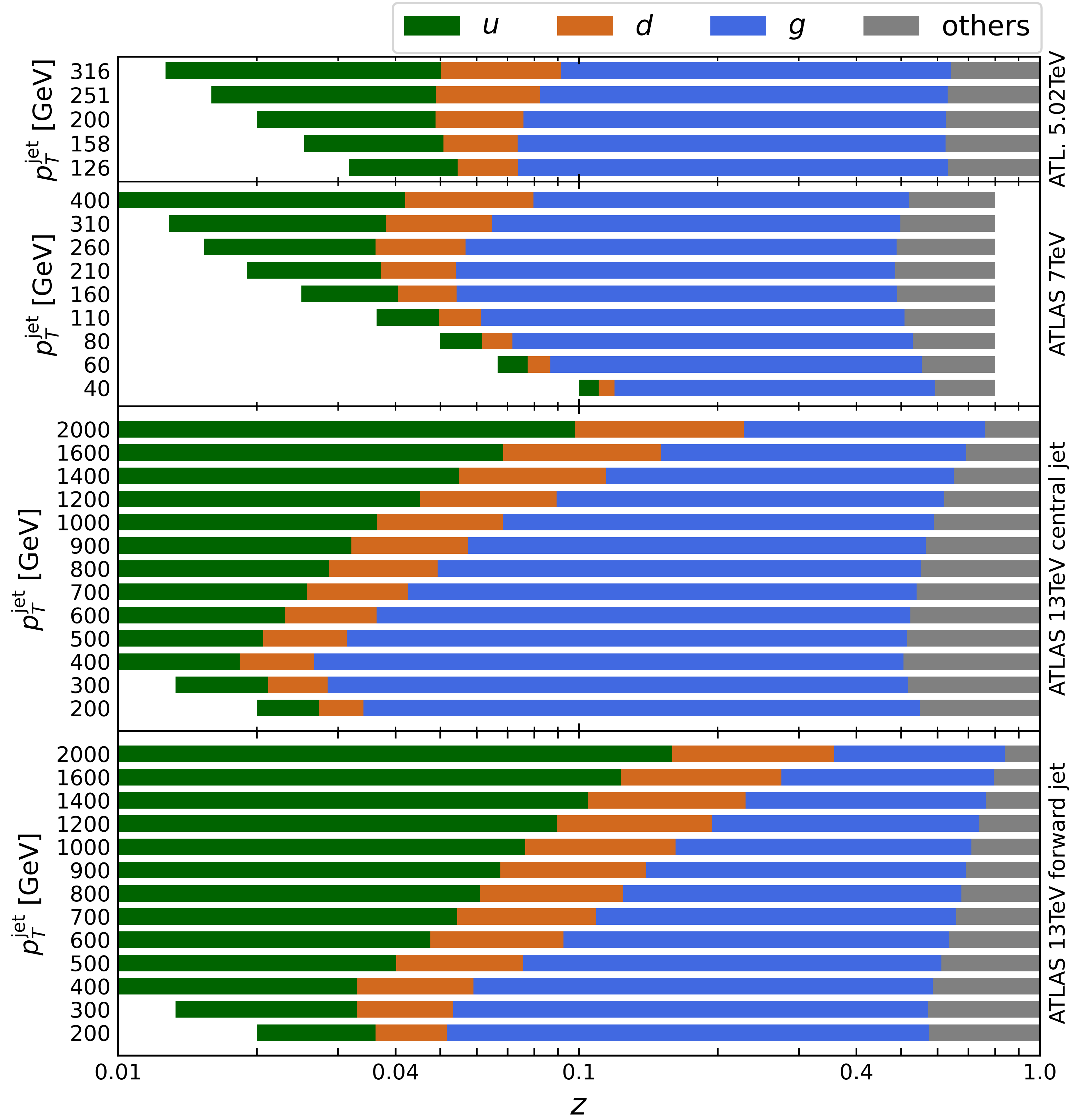
- For the first time the jet fragmentation data from LHC have been incorporated into the global analysis of FFs to light charged hadrons, including from processes of incl. jet, dijet, Z or photon tagged jet productions, due to the development of FMNLO

LHC measurements for hadron inside jet measurements (jet fragmentation)



exp.	$\sqrt{s}$(TeV)	luminosity	hadrons	final states	R_j	cuts for jets/hadron	observable	$N_{\rm pt}$
ATLAS[60]	5.02	25 $\rm pb^{-1}$	$h^\pm$	$\gamma+j$	0.4	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{dp_{T,h}}$	6
CMS[61]	5.02	27.4 $\rm pb^{-1}$	$h^\pm$	$\gamma+j$	0.3	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\xi}$	4
ATLAS[62]	5.02	260 $\rm pb^{-1}$	$h^\pm$	$Z+h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{4}\pi$	$\frac{1}{n_Z}\frac{dN_{\rm ch}}{dp_{T,h}}$	9
CMS[63]	5.02	320 $\rm pb^{-1}$	$h^\pm$	$Z+h$	no jet	$\Delta\phi_{h,Z} > \frac{7}{8}\pi$	$\frac{1}{n_Z}\frac{dN_{\rm ch}}{dp_{T,h}}$	11
LHCb[64]	13	1.64 $\rm fb^{-1}$	$\pi^\pm, K^\pm, p/\bar{p}$	$Z+j$	0.5	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{n_Z}\frac{dN_{\rm ch}}{d\zeta}$	20
ATLAS[65]	5.02	25 $\rm pb^{-1}$	$h^\pm$	inc. jet	0.4	-	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\zeta}$	63
ATLAS[66]	7	36 $\rm pb^{-1}$	$h^\pm$	inc. jet	0.6	$\Delta R_{h,j} < R_j$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\zeta}$	103
ATLAS[67]	13	33 $\rm fb^{-1}$	$h^\pm$	dijet	0.4	$p_T^{\rm lead}/p_T^{\rm sublead} < 1.5$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\zeta}$	280

kinematic/flavor coverage (LO) for ATLAS jet fragmentation



- ❖ LHC measurements on hadron inside jet provide essential inputs for u/d/g flavor separation with wide kinematic coverages, both in energy scale Q and in momentum fraction z
- ❖ In dijets or inclusive jets production, low p_T and central (high p_T and forward) jets are mostly initiated by g(u-quark); Z or photon tagged jets are more likely from u/d quarks

Selection of data

- Other data include ratios of inclusive production rates of different hadrons measured in pp collisions, single incl. hadron production from SIA (w/wo heavy-flavor tagging) mostly at Z-pole, and incl. hadron production in SIDIS from HERA and COMPASS, for identified or unidentified charged hadrons

incl. hadron production at RHIC and LHC (pp)

exp.	$\sqrt{s_{NN}}$(TeV)	# events (million)	$p_{T,h}$	hadrons	observable	$N_{\rm pt}$
ALICE[58]	13	40-60(pp)	$[2, 20]$ GeV	$\pi, K, p,\ K^0_S$	$K/\pi, p/\pi, K^0_S/\pi$	49
ALICE[58]	7	150(pp)	$[3, 20]$ GeV	π, K, p	13TeV/7TeV for π, K, p	37
ALICE[57]	5.02	120(pp)	$[2, 20]$ GeV	π, K, p	$K/\pi, p/\pi$	34
ALICE[56]	2.76	40(pp)	$[2, 20]$ GeV	π, K, p	$K/\pi, p/\pi$	27
STAR[68]	0.2	14(pp)	$[3, 15]$ GeV	π, K, p, K^0_S	$K/\pi,\ p/\pi^+,\ \bar{p}/\pi^-,\ K^0_S/\pi,\ \pi^-/\pi^+,\ K^-/K^+$	60

incl. hadron production mostly at Z-pole (SIA)

exp.	$\sqrt{s}$	lum.(n_Z)	final states	hadrons	$N_{\rm pt}$
OPAL[51]	m_Z	780 000	$Z \to q\bar{q}$	$\pi^\pm, K^\pm$	20
ALEPH[52]	m_Z	520 000	$Z \to q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	42
DELPHI[53]	m_Z	1 400 000	$Z \to q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	39
			$Z \to b\bar{b}$	$\pi^\pm, K^\pm, p(\bar{p})$	39
SLD[77]	m_Z	400 000	$Z \to q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
			$Z \to b\bar{b}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
			$Z \to c\bar{c}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
TASSO[75]	34GeV	77 pb^{-1}	inc. had.	$\pi^\pm, K^\pm, p(\bar{p})$	3
TASSO[75]	44GeV	34 pb^{-1}	inc. had.	$\pi^\pm, \pi^0$	5
TPC[76]	29GeV	70 pb^{-1}	inc. had.	$\pi^\pm, K^\pm$	12
OPAL[54]	201.7GeV	433 pb^{-1}	inc. had.	$h^\pm$	17
DELPHI[55]	189GeV	157.7 pb^{-1}	inc. had.	$\pi^\pm, K^\pm, p(\bar{p})$	9

incl. hadron production at HERA and COMPASS (SIDIS)

exp.	$\sqrt{s}$(GeV)	luminosity	kinematic cuts	hadrons	obs	$N_{\rm pt}$
H1[69]	318	44 pb^{-1}	$Q^2 \in$[175,20000] GeV^2	$h^\pm$	$D \equiv \frac{1}{N}\frac{dn_{h^\pm}}{dx_p}$	16
H1[70]	318	44 pb^{-1}	$Q^2 \in$[175,8000] GeV^2	$h^\pm$	$A \equiv \frac{D^+ - D^-}{D^+ + D^-}$	14
ZEUS[71]	300,318	440 pb^{-1}	$Q^2 \in$[160,40960] GeV^2	$h^\pm$	D	32
COMPASS06[72, 73]	17.3	540 pb^{-1}	$x \in [0.14, 0.4], y \in [0.3, 0, 5]$	π, K, h	$\frac{dM^h}{dz}$	124
COMPASS16[74]	17.3	-	$x \in [0.14, 0.4], y \in [0.3, 0, 5]$	π, K, p	$\frac{dM^h}{dz}$	97

Quality of the fit

- A best-fit with good agreements to the global data sets (1370 points in total) are found, χ^2/N well below 1; individual agreements to the 138 sub-datasets are also tested, motivating usage of a tolerance $\Delta\chi^2\sim2$ in determination of Hessian uncertainties

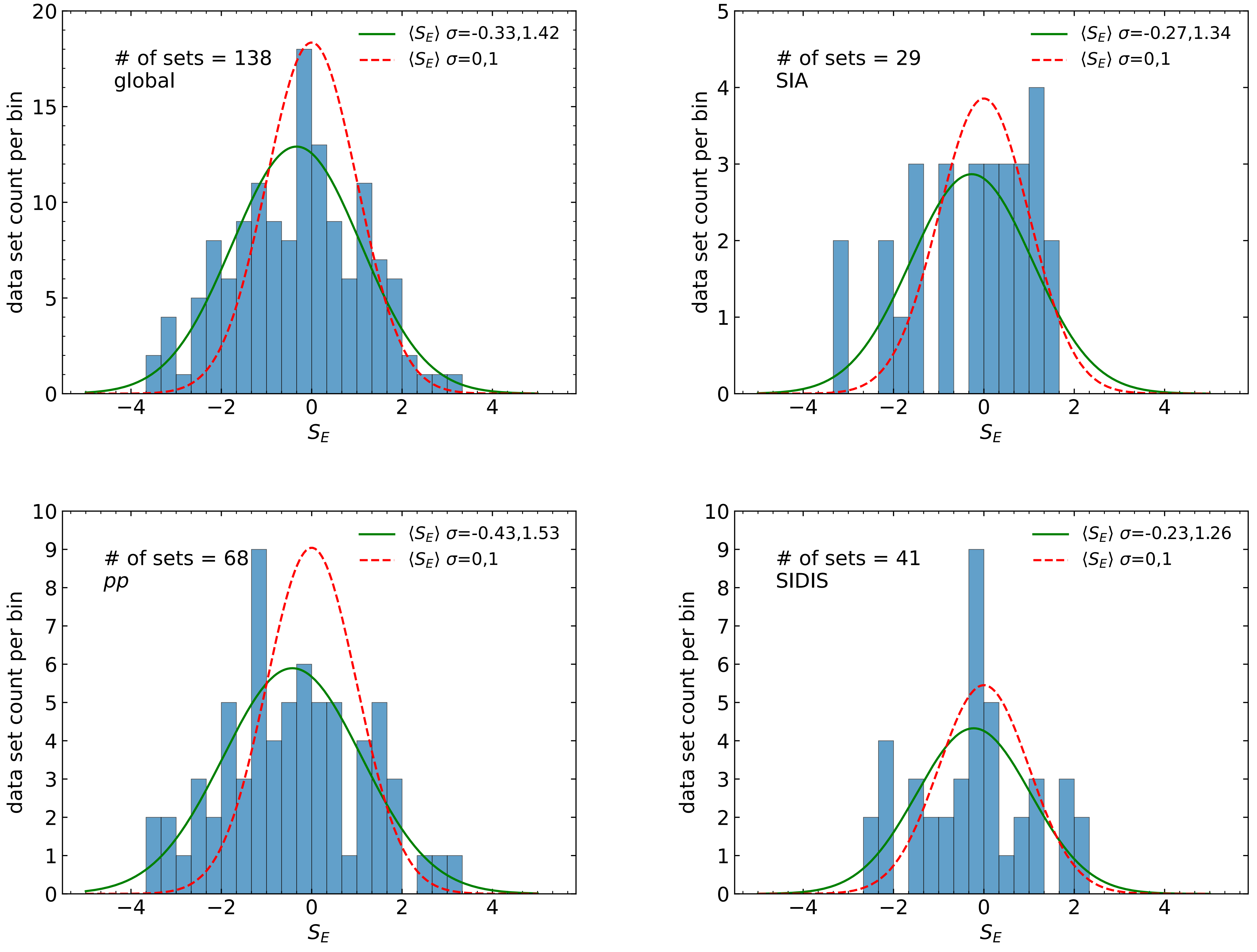
[CTEQ-TEA]

overall agreement: χ^2 breakdown to sub-groups for the best-fit

Experiments	N_{pt}	χ^2	χ^2/N_{pt}
ATLAS jets †	446	350.8	0.79
ATLAS Z/γ+jet †	15	31.8	2.12
CMS Z/γ+jet †	15	17.3	1.15
LHCb Z+jet	20	30.6	1.53
ALICE inc. hadron	147	150.6	1.02
STAR inc. hadron	60	42.2	0.70
pp sum	703	623.3	0.89
TASSO	8	7.0	0.88
TPC	12	11.6	0.97
OPAL	20	16.3	0.81
OPAL (202 GeV) †	17	24.2	1.42
ALEPH	42	31.4	0.75
DELPHI	78	36.4	0.47
DELPHI (189 GeV)	9	15.3	1.70
SLD	198	211.6	1.07
SIA sum	384	353.8	0.92
H1 †	16	12.5	0.78
H1 (asy.) †	14	12.2	0.87
ZEUS †	32	65.5	2.05
COMPASS (06I)	124	107.3	0.87
COMPASS (16p)	97	56.8	0.59
SIDIS sum	283	254.4	0.90
Global total	1370	1231.5	0.90

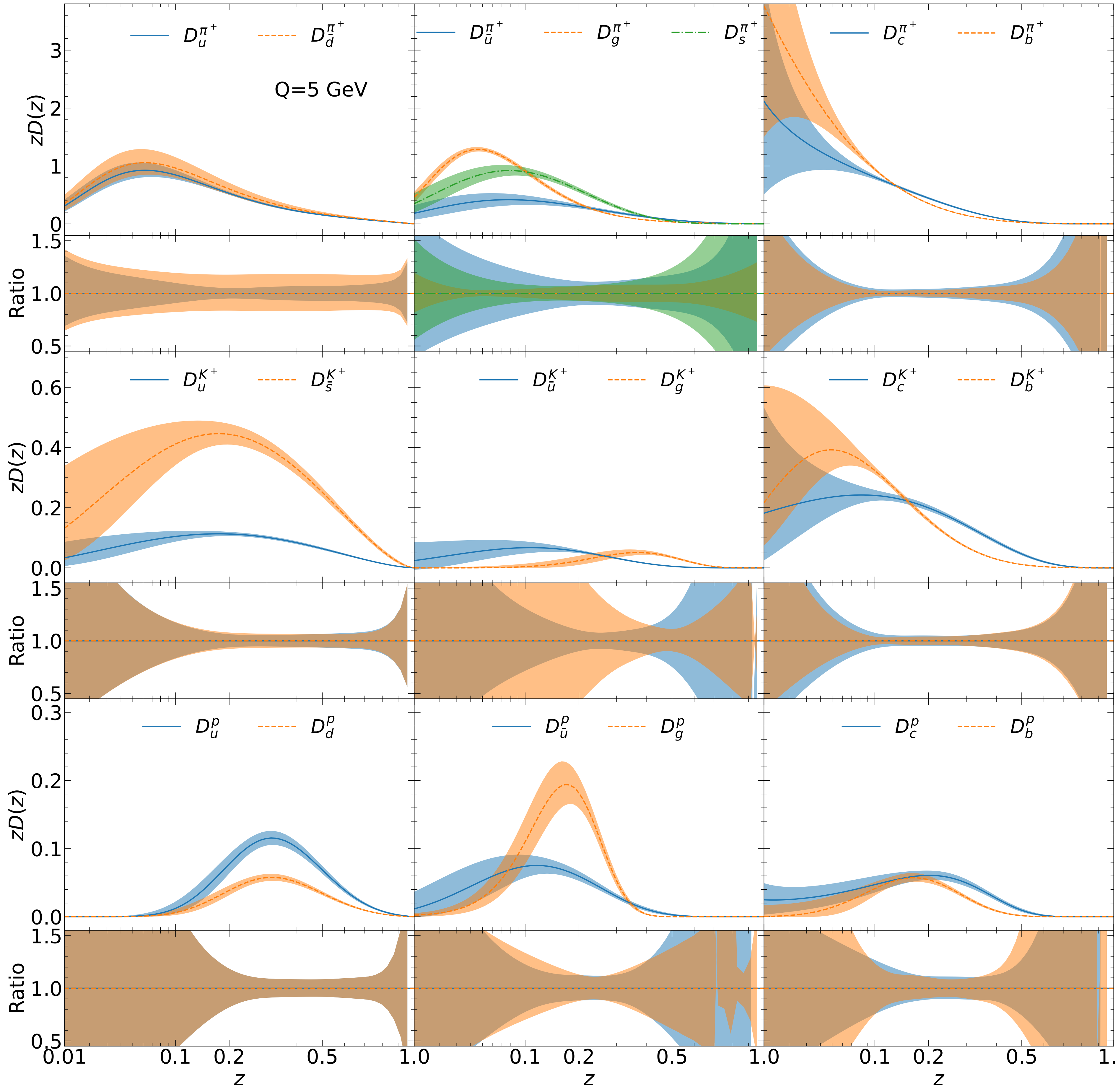
individual agreement: distributions of the effective Gaussian variable

$$S_E=\frac{(18N_{pt})^{3/2}}{18N_{pt}+1}\left\{\frac{6}{6-\ln(\chi^2/N_{pt})}-\frac{9N_{pt}-1}{9N_{pt}}\right\}$$



FFs to light charged hadrons

- We arrive at a best-fit of the charged pion, kaon and proton FFs together with 126 Hessian error FFs, two for each of the eigenvector direction; FFs are generally well constrained in the region with z~0.1-0.7



FFs (positively charged) vs. momentum fraction

- our results show an uncertainty of 3%, 4% and 8% for FFs of gluon to pion at z=0.05, 0.1 and 0.3, respectively
- similarly an uncertainty of 4%, 4% and 7% for FFs of u-quark to pion, kaon and proton at z=0.3, respectively
- FFs of heavy-quarks are well constrained for z between 0.1~0.5 due to the tagged SIA events of Z-pole measurements
- a preference for larger FFs of s quark to pion due to pulls from SIA data
- high precision of gluon FFs is mostly due to the data of jet fragmentation from the LHC

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Opportunities at future lepton colliders

- High luminosity and high energies of future lepton colliders open new opportunities for precision determination of FFs, especially with production of the W boson pairs and the Higgs boson with hadronic decays **[work in progress, Bin Zhou, JG]**

proposed hadron multiplicity measurements from annihilation to quarks

e^+e^- annihilation							
$\sqrt{s}$ (GeV)	luminosity (ab^{-1})			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC-*ee*	ILC				
91.2	60	150	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	132
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	65
160	4.2	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	168
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	83
161	-	10	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	168
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	83
240	13	5	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	186
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	92
250	-	-	2	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	186
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	92
350	-	0.2	0.2	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	98
360	0.65	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	98
365	-	1.5	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	98
500	-	-	4	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^{\pm}$	98

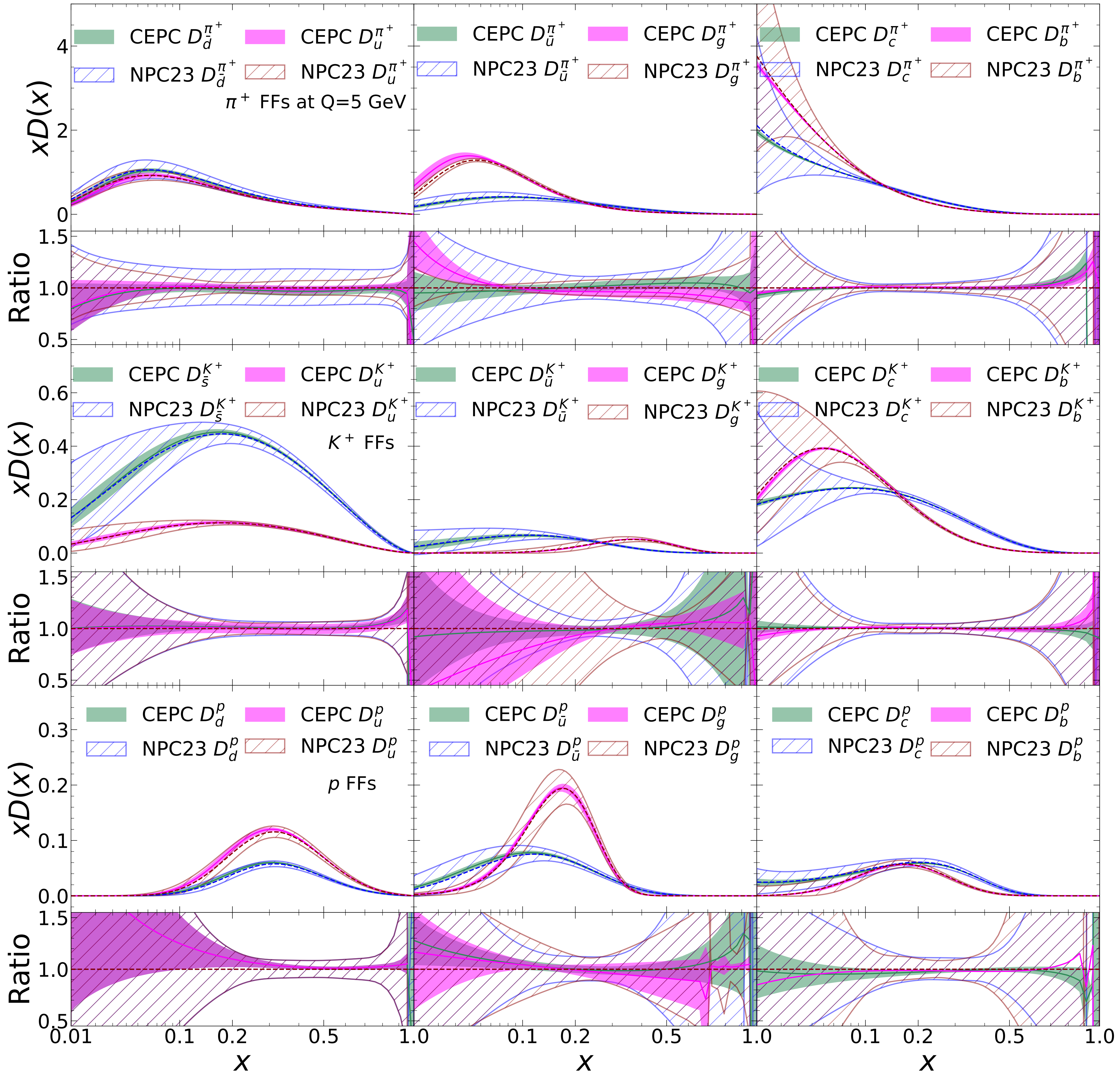
proposed hadron multiplicity measurements from decays of W or Higgs bosons

W boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC-*ee*	ILC				
80.419	116	68	62	$W^-W^{+*} \to W^-q\bar{q}$	-	$h^{+,-}$	120
	58	34	31	$W^-W^{+*} \to W^-c\bar{s}$			
Higgs boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC-*ee*	ILC				
125	0.23	0.09	0.07	gg	-	$h^{\pm}$	77
	0.08	0.03	0.02	$c\bar{c}$			
	1.53	0.59	0.47	$b\bar{b}$			

- (anti-)quark flavor separation from measurements at different energies, on angular distributions, and with heavy-flavor tagging
- d/s quark separation from W boson decays; direct probe of gluon fragmentation from Higgs boson decays

Projection for constraints on FFs

- ✦ Pseudo-data on the proposed measurements are constructed using NPC23 FFs as the truth theory; fits to FFs at NLO in QCD are carried out with data solely from future electron-positron colliders



FFs (positively charged) vs. momentum fraction

- ❖ assuming same (un-)correlated systematic uncertainties as SLD measurements; statistical errors calculated based on prescribed luminosities
- ❖ fits using the same fitting framework as NPC23; theoretical uncertainties from scale variations of the NLO calculations are included
- ❖ best-fit agrees well with the truth FF; uncertainties are greatly reduced taking the CEPC as an example

[work in progress, Bin Zhou, JG]

Projection for constraints on FFs

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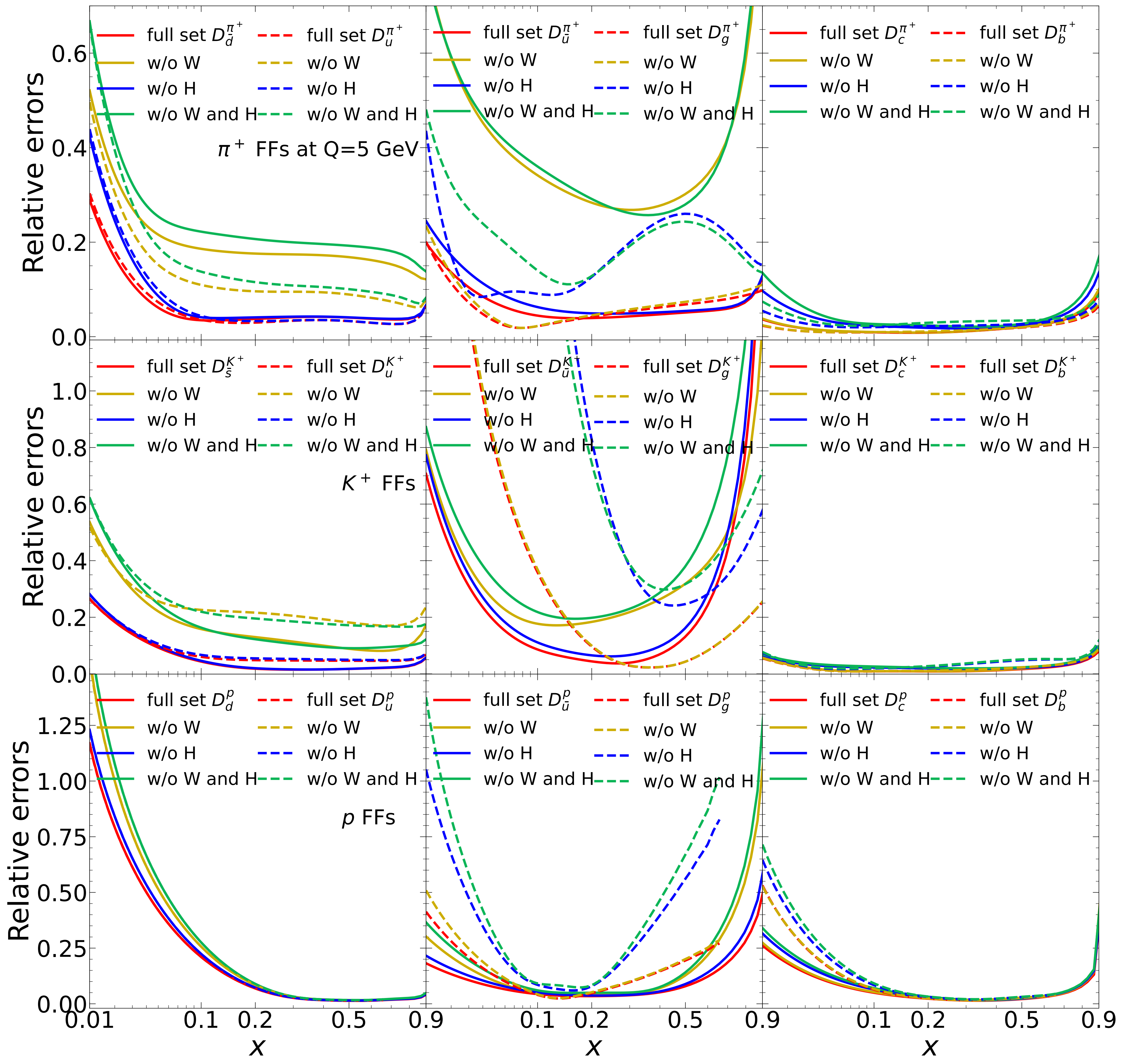

FFs (positively charged) vs. momentum fraction

- assuming same (un-)correlated systematic uncertainties as SLD measurements; statistical errors calculated based on prescribed luminosities
- fits using the same fitting framework as NPC23; theoretical uncertainties from scale variations of the NLO calculations are included
- best-fit agrees well with the truth FF; uncertainties are greatly reduced taking the CEPC as an example
- W boson data are essential for quark flavor separation; similarly Higgs boson data for constraining gluon FF

[work in progress, Bin Zhou, JG]

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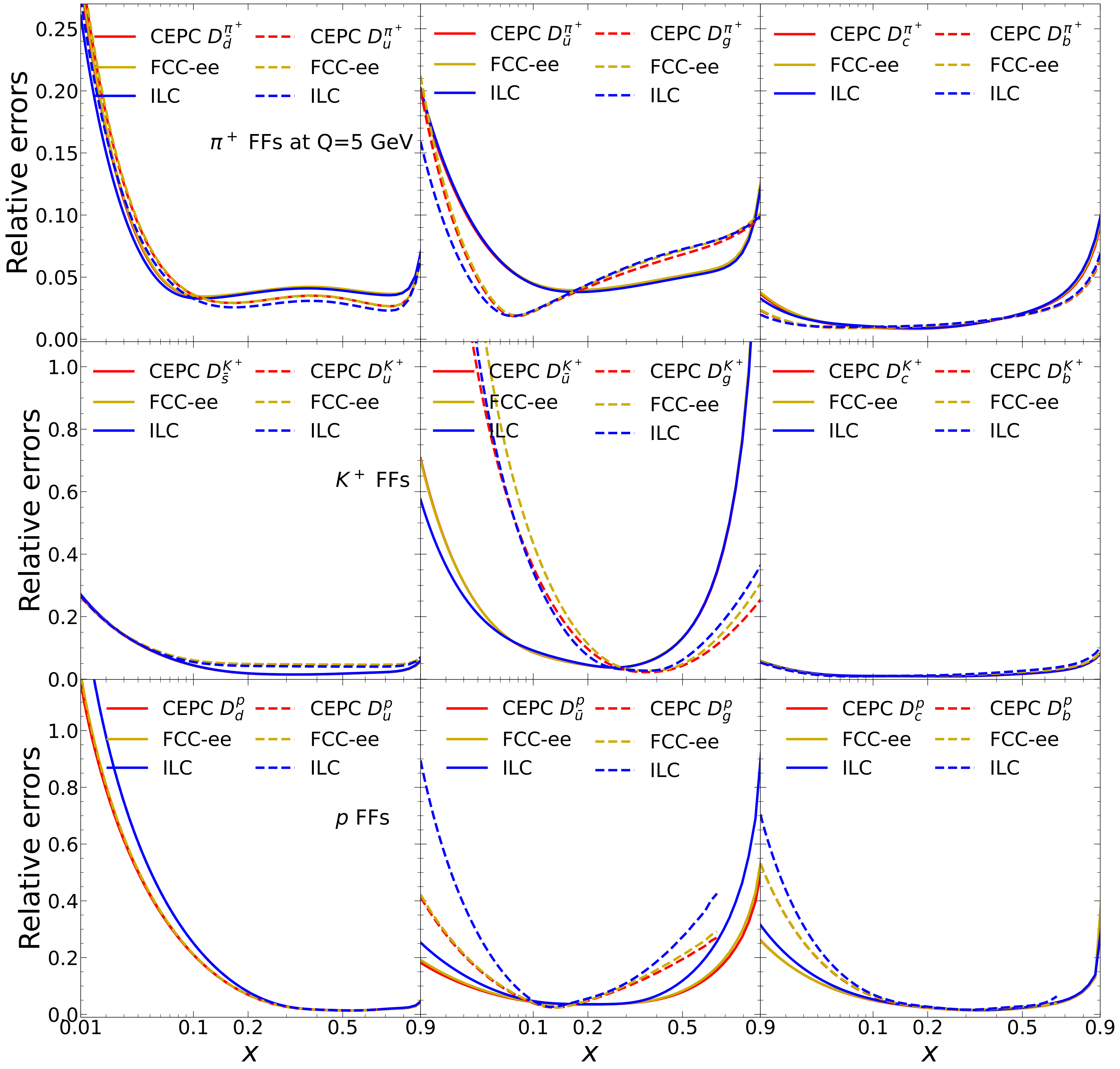

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- ILC, FCC-ee and CEPC give quite similar results except in regions statistics are limited

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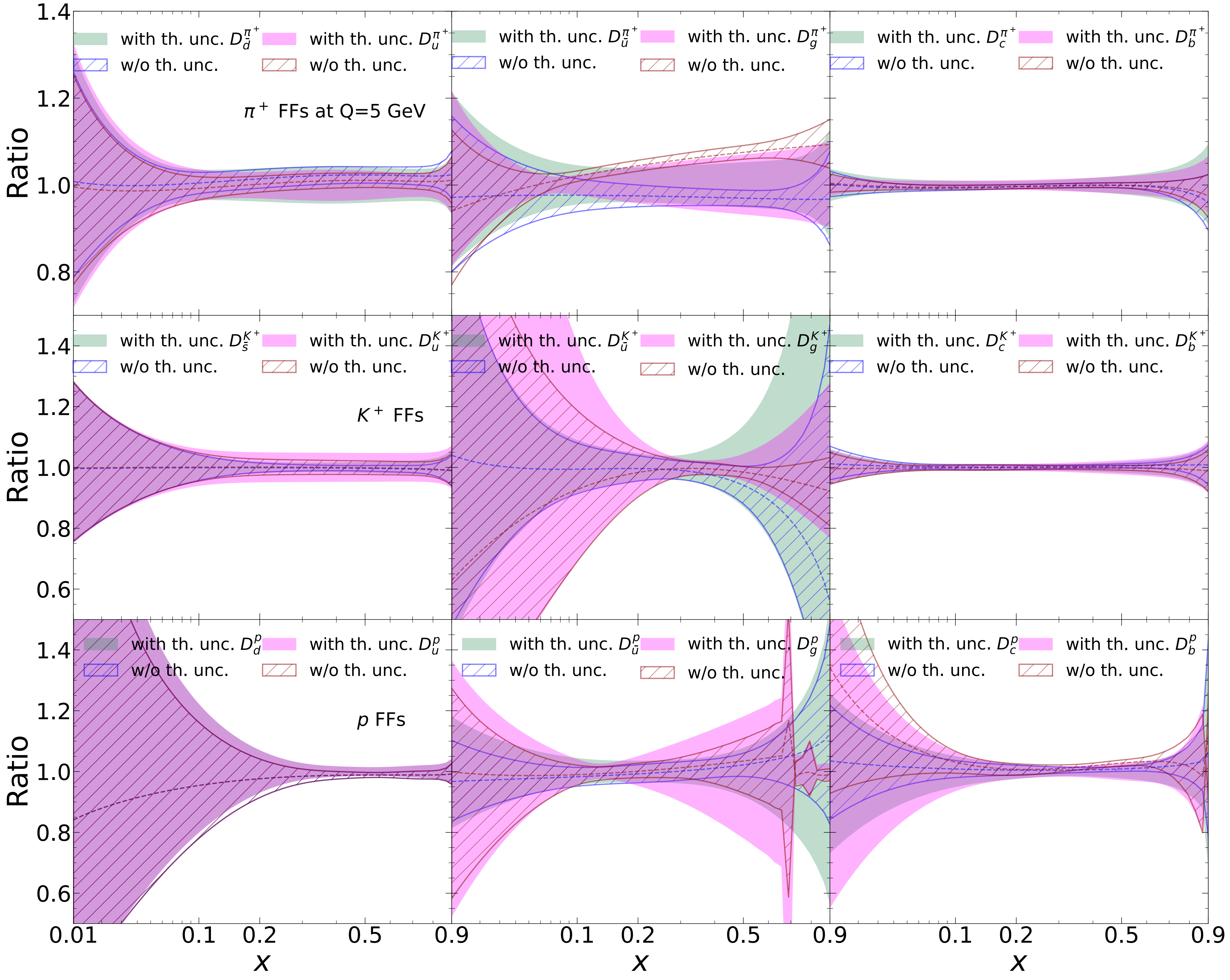
Impact of theoretical uncertainties

- Impact of theoretical uncertainties (scale variations at NLO in QCD) are large considering high precision of data; NNLO corrections to most of the selected processes of hadron production are known and implemented in FMNLO **[e.g., NNLO corrections for Higgs decays to gg in 0912.0369]**



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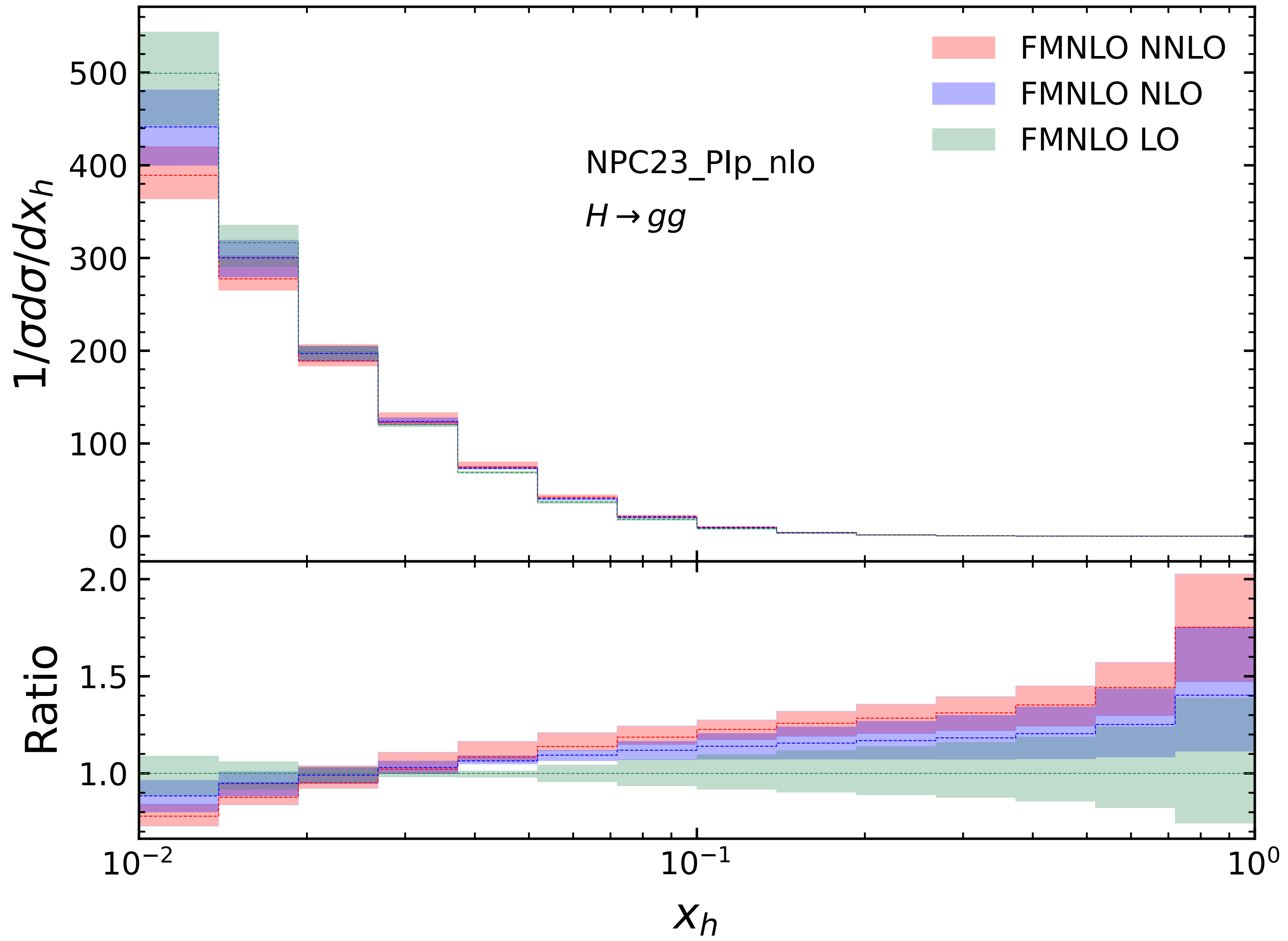

- removal of theoretical uncertainties (NLO) leads to reduction of FF uncertainties by more than a factor of two in many cases, e.g., FFs for favored quarks, heavy quarks and gluons
- scale variations at NNLO are smaller but not negligible; further theoretical calculations are needed, e.g., NNLO for 3-jet production, N3LO for others

Summary

- Fragmentation functions (FFs) are essential non-perturbative inputs for precision calculations of hadron production cross sections in high energy scattering from first principle of QCD

- FMNLO is a program for automated and fast calculations of fragmentation processes at NLO in QCD is now publicly available, which is desirable for global analysis of FFs providing much improved efficiency and capability for arbitrary hard processes

- NPC23 is an up-to-date global analysis of FFs to identified charged hadrons, including charged pion, kaon and proton, at NLO in QCD, using most recent data from SIA, SIDIS, and pp collisions; constraints on gluon FFs are greatly improved

- Future high-energy lepton colliders offer great opportunities on further understanding and much precise determinations of FFs, especially with measurements on hadronic decays of the abundantly produced Higgs boson and the W boson, independent of inputs from SIDIS and pp collisions

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Thank you for your attention!