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Third family quark mass hierarchy and FCNC in the Universal Seesaw Model

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Outline

- Introduction
- The model
- Quark Sector
- Higgs Sector
- Hierarchy in the Top and Bottom Sector
- Higgs and Z boson FCNCs
- Summary

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- USM introduces:
  - An additional gauge group, $\mathrm{SU}(2)_{\mathrm{R}}$ with the corresponding Higgs doublet
  - Vector-like quark (VLQ) whose mass parameter plays role in the seesaw-like mechanism

Introduction

Table 1. Quark mass and their corresponding Yukawa coupling [PDG]

Quark mass	Yukawa coupling
$m_u = 2.16\ \text{MeV}$	$y_u^{\text{SM}} \simeq 1.24 \times 10^{-5}$
$m_d = 4.70\ \text{MeV}$	$y_d^{\text{SM}} \simeq 2.7 \times 10^{-5}$
$m_s = 93.5\ \text{MeV}$	$y_s^{\text{SM}} \simeq 5.37 \times 10^{-4}$
$m_c = 1.273\ \text{GeV}$	$y_c^{\text{SM}} \simeq 7.31 \times 10^{-3}$
$m_b = 4.183\ \text{GeV}$	$y_b^{\text{SM}} \simeq 2.4 \times 10^{-2}$
$m_t = 172.57\ \text{GeV}$	$y_t \simeq 0.99$

m_u, m_d, m_s from $\overline{\text{MS}}$ at $\mu = 2\ \text{GeV}$

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m_t from direct measurement

$v \approx 246.22\ \text{GeV}$

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$$y_u^{\text{SM}} \simeq \frac{y_{u_L} v_R y_{u_R}}{\sqrt{2} M_U} \simeq \frac{v_R}{\sqrt{2} M_U} \simeq 10^{-5}$$

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- How about the top quark?

Introduction

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- Our research:
  - Study the third family quark mass hierarchy (top and bottom quarks) in the massless case of the first and second generation
  - Find the VLQ mass parameters (MT and MB) using the current experimental data and using the exact mass eigenvalues
  - Investigate the phenomenology implications → FCNCs process within this model

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The model

- Universal seesaw model is based on $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{Y}'}$

Table 2. Quark and Higgs fields with their quantum numbers under the model gauge group

Quark and Higgs Fields	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{SU}(2)_\mathrm{R}$	$\mathrm{U}(1)_{\mathrm{Y}'}$
$q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}$	$1/6$
$q_R^i = \begin{pmatrix} u_R^i \\ d_R^i \end{pmatrix}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$1/6$
$T_{L,R}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$2/3$
$B_{L,R}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$-1/3$
$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/2$
$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$

$$Q = I_L^3 + \underbrace{I_R^3 + Y'}_{Y = I_R^3 + Y'}$$

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$q_R^i = \begin{pmatrix} u_R^i \\ d_R^i \end{pmatrix}$	**3**	**1**	**2**	$1/6$
$T_{L,R}$	**3**	**1**	**1**	$2/3$
$B_{L,R}$	**3**	**1**	**1**	$-1/3$
$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}$	**1**	**2**	**1**	$1/2$
$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$	**1**	**1**	**2**	$1/2$

$$Q = I_L^3 + I_R^3 + Y'$$

$$Y = I_R^3 + Y'$$

$$\langle \phi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}$$

$$\langle \phi_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix}$$

The model

- The Lagrangian of the model (excluding the QCD part)

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_q + \mathcal{L}_H + \mathcal{L}_{\text{gauge}}, \\
\mathcal{L}_q &= \overline{q_L^i} i\gamma^\mu D_{L\mu} q_L^i + \overline{q_R^i} i\gamma^\mu D_{R\mu} q_R^i + \overline{T} i\gamma^\mu D_{T\mu} T + \overline{B} i\gamma^\mu D_{B\mu} B \\
&\quad - Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \overline{T_L} \tilde{\phi}_R^\dagger q_R^3 - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \phi_R^\dagger q_R^i - h.c. \\
&\quad - \overline{T_L} M_T T_R - \overline{B_L} M_B B_R - h.c., \\
\mathcal{L}_H &= (D_L^\mu \phi_L)^\dagger (D_{L\mu}\phi_L) + (D_R^\mu \phi_R)^\dagger (D_{R\mu}\phi_R) - V(\phi_L, \phi_R), \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} F_{L\mu\nu}^a F_L^{a\mu\nu} - \frac{1}{4} F_{R\mu\nu}^a F_R^{a\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}
\end{aligned}
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where,

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\begin{aligned}
D_{L(R)\mu} q_{L(R)}^i &= \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g'_1 Y'_q B'_\mu\right) q_{L(R)}^i, \\
D_{L(R)\mu} \phi_{L(R)} &= \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g'_1 Y'_\phi B'_\mu\right) \phi_{L(R)}, \\
D_{T\mu} T &= \left(\partial_\mu + i g'_1 Y'_T B'_\mu\right) T, \\
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\mathcal{L}_H &= (D_L^\mu \phi_L)^\dagger (D_{L\mu}\phi_L) + (D_R^\mu \phi_R)^\dagger (D_{R\mu}\phi_R) - V(\phi_L, \phi_R), \\
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Kinetic terms of quark doublet and VLQs

where,

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Yukawa interactions and VLQs mass terms

$$- Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \overline{T_L} \tilde{\phi}_R^\dagger q_R^3 - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \phi_R^\dagger q_R^i - h.c.$$

$$- \overline{T_L} M_T T_R - \overline{B_L} M_B B_R - h.c.,$$

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\mathcal{L} &= \mathcal{L}_q + \mathcal{L}_H + \mathcal{L}_{\text{gauge}}, \\
\mathcal{L}_q &= \overline{q_L^i} i\gamma^\mu D_{L\mu} q_L^i + \overline{q_R^i} i\gamma^\mu D_{R\mu} q_R^i + \overline{T} i\gamma^\mu D_{T\mu} T + \overline{B} i\gamma^\mu D_{B\mu} B \\
&\quad - Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \overline{T_L} \tilde{\phi}_R^\dagger q_R^3 - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \phi_R^\dagger q_R^i - h.c. \\
&\quad - \overline{T_L} M_T T_R - \overline{B_L} M_B B_R - h.c., \\
\mathcal{L}_H &= (D_L^\mu \phi_L)^\dagger (D_{L\mu} \phi_L) + (D_R^\mu \phi_R)^\dagger (D_{R\mu} \phi_R) - V(\phi_L, \phi_R), \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} F_{L\mu\nu}^a F_L^{a\mu\nu} - \frac{1}{4} F_{R\mu\nu}^a F_R^{a\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}
\end{aligned}
$$

where,

$$
\begin{aligned}
D_{L(R)\mu} q_{L(R)}^i &= \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g_1' Y_q' B_\mu' \right) q_{L(R)}^i, \\
D_{L(R)\mu} \phi_{L(R)} &= \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g_1' Y_\phi' B_\mu' \right) \phi_{L(R)}, \\
D_{T\mu} T &= \left(\partial_\mu + i g_1' Y_T' B_\mu' \right) T, \\
D_{B\mu} B &= \left(\partial_\mu + i g_1' Y_B' B_\mu' \right) B,
\end{aligned}
$$

The model

- The Lagrangian of the model (excluding the QCD part)

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_q + \mathcal{L}_H + \mathcal{L}_{\text{gauge}}, \\
\mathcal{L}_q &= \overline{q_L^i} i\gamma^\mu D_{L\mu} q_L^i + \overline{q_R^i} i\gamma^\mu D_{R\mu} q_R^i + \overline{T} i\gamma^\mu D_{T\mu} T + \overline{B} i\gamma^\mu D_{B\mu} B \\
&\quad - Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \overline{T_L} \tilde{\phi}_R^\dagger q_R^3 - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \phi_R^\dagger q_R^i - h.c. \\
&\quad - \overline{T_L} M_T T_R - \overline{B_L} M_B B_R - h.c., \\
\mathcal{L}_H &= (D_L^\mu \phi_L)^\dagger (D_{L\mu}\phi_L) + (D_R^\mu \phi_R)^\dagger (D_{R\mu}\phi_R) - V(\phi_L, \phi_R), \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} F_{L\mu\nu}^a F_L^{a\mu\nu} - \frac{1}{4} F_{R\mu\nu}^a F_R^{a\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu},
\end{aligned}
$$

where,

$$
\begin{aligned}
V(\phi_L, \phi_R) &= \mu_L^2 \phi_L^\dagger \phi_L + \mu_R^2 \phi_R^\dagger \phi_R + \lambda_L (\phi_L^\dagger \phi_L)^2 + \lambda_R (\phi_R^\dagger \phi_R)^2 + 2\lambda_{LR} (\phi_L^\dagger \phi_L)(\phi_R^\dagger \phi_R), \\
F_{L\mu\nu}^a &= \partial_\mu W_{L\nu}^a - \partial_\nu W_{L\mu}^a - g_L \epsilon^{abc} W_{L\mu}^b W_{L\nu}^c, \\
F_{R\mu\nu}^a &= \partial_\mu W_{R\nu}^a - \partial_\nu W_{R\mu}^a - g_R \epsilon^{abc} W_{R\mu}^b W_{R\nu}^c, \\
B'_{\mu\nu} &= \partial_\mu B'_\nu - \partial_\nu B'_\mu.
\end{aligned}
$$

$$
\boxed{\begin{matrix} \mu_L^2 < 0 \\ \mu_R^2 < 0 \end{matrix}}
$$

The model

- We derive the Lagrangian starting from:

$$\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$$

The model

- We derive the Lagrangian starting from:

$$\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$$

$$\downarrow$$

$$\mathrm{SU(2)_L \times U(1)_Y}$$

$$\begin{pmatrix} B'_\mu \\ W^3_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} B_\mu \\ Z_{R\mu} \end{pmatrix}$$

$$\sin\theta_R = \frac{g'_1}{\sqrt{g_R^2 + g_1'^2}}, \qquad \cos\theta_R = \frac{g_R}{\sqrt{g_R^2 + g_1'^2}}$$

$$g' = g'_1 \cos\theta_R = g_R \sin\theta_R$$

$$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi_R^+ \\ v_R + h_R + i\chi_R^3 \end{pmatrix}$$

The model

- We derive the Lagrangian starting from:

$$\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$$

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$$\mathrm{SU(2)_L \times U(1)_Y}$$

$$\begin{pmatrix} B_\mu \\ W^3_{L\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_{L\mu} \end{pmatrix}$$

$$\cos\theta_W = \frac{g_L}{\sqrt{g_L^2 + g'^2}}, \qquad \sin\theta_W = \frac{g'}{\sqrt{g_L^2 + g'^2}}$$

$$e = g' \cos\theta_W = g_L \sin\theta_W$$

$$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi_L^+ \\ v_L + h_L + i\chi_L^3 \end{pmatrix}$$

$$\mathrm{U(1)_{em}}$$

Outline

- Introduction
- The model
- Quark Sector
- Higgs Sector
- Hierarchy in the Top and Bottom Sector
- Higgs and Z boson FCNCs
- Summary

Quark Sector

- In this talk, we skip the details of deriving the Lagrangian from:

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{Y}'} \longrightarrow \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \longrightarrow \mathrm{U}(1)_\mathrm{em}$$ [2407.00432]

Quark Sector

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- The final expression of the mass terms in the flavor basis (before diagonalization):

Quark Sector

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- The final expression of the mass terms in the flavor basis (before diagonalization):

$$\begin{aligned}\mathcal{L}_\mathrm{mass} = &-\begin{pmatrix} \overline{(\tilde{u}_L)^3} & \overline{(\tilde{u}_L)^4} \end{pmatrix} \begin{pmatrix} Y^3_{u_L}\frac{v_L}{\sqrt{2}}(W_{T_R})^{43} & Y^3_{u_L}\frac{v_L}{\sqrt{2}}(W_{T_R})^{44} \\ 0 & m_{u_4} \end{pmatrix} \begin{pmatrix} (\tilde{u}'_R)^3 \\ (\tilde{u}'_R)^4 \end{pmatrix} - h.c. \\ &- \begin{pmatrix} \overline{(\tilde{d}'_L)^3} & \overline{(\tilde{d}'_L)^4} \end{pmatrix} \begin{pmatrix} Y^3_{d_L}\frac{v_L}{\sqrt{2}}(W_{B_R})^{43} & Y^3_{d_L}\frac{v_L}{\sqrt{2}}(W_{B_R})^{44} \\ 0 & m_{d_4} \end{pmatrix} \begin{pmatrix} (\tilde{d}''_R)^3 \\ (\tilde{d}''_R)^4 \end{pmatrix} - h.c..\end{aligned}$$

Quark Sector

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- The final expression of the mass terms in the flavor basis (before diagonalization):

$$\begin{aligned}\mathcal{L}_\mathrm{mass} = &- \begin{pmatrix} \overline{(\tilde{u}_L)^3} & \overline{(\tilde{u}_L)^4} \end{pmatrix} \overbrace{\begin{pmatrix} Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{43} & Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{44} \\ 0 & m_{u_4} \end{pmatrix}}^{\mathbb{M}_t} \begin{pmatrix} (\tilde{u}'_R)^3 \\ (\tilde{u}'_R)^4 \end{pmatrix} - h.c. \\ &- \begin{pmatrix} \overline{(\tilde{d}'_L)^3} & \overline{(\tilde{d}'_L)^4} \end{pmatrix} \underbrace{\begin{pmatrix} Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{43} & Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{44} \\ 0 & m_{d_4} \end{pmatrix}}_{\mathbb{M}_b} \begin{pmatrix} (\tilde{d}''_R)^3 \\ (\tilde{d}''_R)^4 \end{pmatrix} - h.c..\end{aligned}$$

Quark Sector

- In this talk, we skip the details of deriving the Lagrangian from:

$$\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}} \longrightarrow \mathrm{SU(2)_L \times U(1)_Y} \longrightarrow \mathrm{U(1)_{em}}$$ [2407.00432]

- The final expression of the mass terms in the flavor basis (before diagonalization):

$$\mathcal{L}_{\rm mass} = -\begin{pmatrix} \overline{(\tilde{u}_L)^3} & \overline{(\tilde{u}_L)^4} \end{pmatrix} \overbrace{\begin{pmatrix} Y^3_{u_L}\frac{v_L}{\sqrt{2}}(W_{T_R})^{43} & Y^3_{u_L}\frac{v_L}{\sqrt{2}}(W_{T_R})^{44} \\ 0 & m_{u_4} \end{pmatrix}}^{\mathbb{M}_t} \begin{pmatrix} (\tilde{u}'_R)^3 \\ (\tilde{u}'_R)^4 \end{pmatrix} - h.c.$$
$$- \begin{pmatrix} \overline{(\tilde{d}'_L)^3} & \overline{(\tilde{d}'_L)^4} \end{pmatrix} \underbrace{\begin{pmatrix} Y^3_{d_L}\frac{v_L}{\sqrt{2}}(W_{B_R})^{43} & Y^3_{d_L}\frac{v_L}{\sqrt{2}}(W_{B_R})^{44} \\ 0 & m_{d_4} \end{pmatrix}}_{\mathbb{M}_b} \begin{pmatrix} (\tilde{d}''_R)^3 \\ (\tilde{d}''_R)^4 \end{pmatrix} - h.c..$$

where,

$$W_{T_R} = \begin{pmatrix} \cos\theta_{T_R} & \sin\theta_{T_R} \\ -\sin\theta_{T_R} & \cos\theta_{T_R} \end{pmatrix} \qquad \cos\theta_{T_R} = \frac{M_T}{m_{u_4}}, \quad \sin\theta_{T_R} = \frac{Y^3_{u_R}}{m_{u_4}}\frac{v_R}{\sqrt{2}}, \quad \cos\theta_{B_R} = \frac{M_B}{m_{d_4}}, \quad \sin\theta_{B_R} = \frac{Y^3_{d_R}}{m_{d_4}}\frac{v_R}{\sqrt{2}},$$

$$W_{B_R} = \begin{pmatrix} \cos\theta_{B_R} & \sin\theta_{B_R} \\ -\sin\theta_{B_R} & \cos\theta_{B_R} \end{pmatrix} \qquad m_{u_4} = \sqrt{\frac{(Y^3_{u_R})^2 v_R^2}{2} + M_T^2}, \qquad m_{d_4} = \sqrt{\frac{(Y^3_{d_R})^2 v_R^2}{2} + M_B^2}.$$

Quark Sector

- Changing from flavor basis into mass basis transformations:

$$(\tilde{u}_L)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{T_L})^{\alpha\beta} (u_L^m)^\beta, \qquad (\tilde{d}'_L)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{B_L})^{\alpha\beta} (d_L^m)^\beta,$$

$$(\tilde{u}'_R)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{T_R})^{\alpha\beta} (u_R^m)^\beta \qquad (\tilde{d}''_R)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{B_R})^{\alpha\beta} (d_R^m)^\beta$$

where

$$\widetilde{K}_{T_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{T_L} \end{pmatrix}, \quad \widetilde{K}_{T_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{T_R} \end{pmatrix}, \quad \widetilde{K}_{B_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{B_L} \end{pmatrix}, \quad \widetilde{K}_{B_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{B_R} \end{pmatrix}$$

Quark Sector

- Changing from flavor basis into mass basis transformations:

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where

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- Diagonalizing the mass matrix by the bi-unitary transformations:

$$K_{T_L}^\dagger \mathbb{M}_t K_{T_R} = (m_t^{\text{diag}}) = \text{diag}(m_t, m_{t'}),$$
$$K_{B_L}^\dagger \mathbb{M}_b K_{B_R} = (m_b^{\text{diag}}) = \text{diag}(m_b, m_{b'}).$$

Quark Sector

- Changing from flavor basis into mass basis transformations:

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$$(\tilde{u}'_R)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{T_R})^{\alpha\beta} (u_R^m)^\beta \qquad (\tilde{d}''_R)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{B_R})^{\alpha\beta} (d_R^m)^\beta$$

where

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$$K_{B_L}^\dagger \mathbb{M}_b K_{B_R} = (m_b^{\text{diag}}) = \text{diag}(m_b, m_{b'}).$$

- The mixing angles:

$$K_{T_L}^\dagger = \begin{pmatrix} \cos\phi_{T_L} & \sin\phi_{T_L} \\ -\sin\phi_{T_L} & \cos\phi_{T_L} \end{pmatrix}, \qquad K_{B_L}^\dagger = \begin{pmatrix} \cos\phi_{B_L} & \sin\phi_{B_L} \\ -\sin\phi_{B_L} & \cos\phi_{B_L} \end{pmatrix},$$

$$K_{T_R} = \begin{pmatrix} \cos\phi_{T_R} & -\sin\phi_{T_R} \\ \sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad K_{B_R} = \begin{pmatrix} \cos\phi_{B_R} & -\sin\phi_{B_R} \\ \sin\phi_{B_R} & \cos\phi_{B_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Quark Sector

- The exact mass eigenvalues:

Top quark and heavy top quark: TM, A.S.Adam, Y.Kawamura, AHP, Y.Shimizu, and K.Yamamoto, J.Phys.Conf.Ser.**2446** 012046 (2023)

$$m_t = -\frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

$$m_{t'} = \frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

Bottom quark and heavy bottom quark:

$$m_b = -\frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

$$m_{b'} = \frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

where,

$$m_{u_R} = Y_{u_R}^3 \frac{v_R}{\sqrt{2}}, \qquad m_{u_L} = Y_{u_L}^3 \frac{v_L}{\sqrt{2}}., \qquad m_{d_R} = Y_{d_R}^3 \frac{v_R}{\sqrt{2}}, \qquad m_{d_L} = Y_{d_L}^3 \frac{v_L}{\sqrt{2}}$$

Quark Sector

- The right-handed weak isospin current:

$$j^{\mu}_{3R} = \sum_{i=1}^{3} \left(\overline{u^i_R} \gamma^{\mu} u^i_R - \overline{d^i_R} \gamma^{\mu} d^i_R \right)$$

Quark Sector

- The right-handed weak isospin current:

$$j_{3R}^{\mu} = \sum_{i=1}^{3} \left(\overline{u_R^i} \gamma^\mu u_R^i - \overline{d_R^i} \gamma^\mu d_R^i \right) \xrightarrow{\mathrm{SU(2)_R} \otimes \mathrm{U(1)_{Y'}}}$$

Quark Sector

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$$- \sum_{i=1}^{2} \overline{(\tilde{d}''_R)^i} \gamma^{\mu} (\tilde{d}''_R)^i - \sum_{j,k=3}^{4} \overline{(\tilde{d}''_R)^j} \gamma^{\mu} (Z_{B_R})^{jk} (\tilde{d}''_R)^k,$$

Quark Sector

- The right-handed weak isospin current:

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$$- \sum_{i=1}^{2} \overline{(\tilde{d}''_R)^i} \gamma^{\mu} (\tilde{d}''_R)^i - \sum_{j,k=3}^{4} \overline{(\tilde{d}''_R)^j} \gamma^{\mu} (Z_{B_R})^{jk} (\tilde{d}''_R)^k,$$

$\xrightarrow{\mathrm{SU(2)_L \otimes U(1)_Y}}$

Quark Sector

- The right-handed weak isospin current:

$$j^{\mu}_{3R} = \sum_{i=1}^{3} \left(\overline{u^i_R} \gamma^\mu u^i_R - \overline{d^i_R} \gamma^\mu d^i_R \right) \xrightarrow{\mathrm{SU(2)_R \otimes U(1)_{Y'}}} j^{\mu}_{3R} = \sum_{i=1}^{2} \overline{(\tilde{u}'_R)^i} \gamma^\mu (\tilde{u}'_R)^i + \sum_{j,k=3}^{4} \overline{(\tilde{u}'_R)^j} \gamma^\mu (Z_{T_R})^{jk} (\tilde{u}'_R)^k$$

$$- \sum_{i=1}^{2} \overline{(\tilde{d}''_R)^i} \gamma^\mu (\tilde{d}''_R)^i - \sum_{j,k=3}^{4} \overline{(\tilde{d}''_R)^j} \gamma^\mu (Z_{B_R})^{jk} (\tilde{d}''_R)^k,$$

$\mathrm{SU(2)_L \otimes U(1)_Y}$

$$j^{\mu}_{3R} = \sum_{i=1}^{2} \overline{(\hat{u}^m_R)^i} \gamma^\mu (\hat{u}^m_R)^i + \sum_{j,k=3}^{4} \overline{(\hat{u}^m_R)^j} \gamma^\mu (\mathcal{Z}_{T_R})^{jk} (\hat{u}^m_R)^k$$

$$- \sum_{i=1}^{2} \overline{(\hat{d}^m_R)^i} \gamma^\mu (\hat{d}^m_R)^i + \sum_{j,k=3}^{4} \overline{(\hat{d}^m_R)^j} \gamma^\mu (\mathcal{Z}_{B_R})^{jk} (\hat{d}^m_R)^j k$$

Quark Sector

- The right-handed weak isospin current:

$$j_{3R}^{\mu} = \sum_{i=1}^{3}\left(\overline{u_R^i}\gamma^\mu u_R^i - \overline{d_R^i}\gamma^\mu d_R^i\right) \xrightarrow{\mathrm{SU(2)_R \otimes U(1)_{Y'}}} j_{3R}^{\mu} = \sum_{i=1}^{2}\overline{(\tilde{u}'_R)^i}\gamma^\mu(\tilde{u}'_R)^i + \sum_{j,k=3}^{4}\overline{(\tilde{u}'_R)^j}\gamma^\mu(Z_{T_R})^{jk}(\tilde{u}'_R)^k$$

$$- \sum_{i=1}^{2}\overline{(\tilde{d}''_R)^i}\gamma^\mu(\tilde{d}''_R)^i - \sum_{j,k=3}^{4}\overline{(\tilde{d}''_R)^j}\gamma^\mu(Z_{B_R})^{jk}(\tilde{d}''_R)^k,$$

$\xrightarrow{\mathrm{SU(2)_L \otimes U(1)_Y}}$

$$j_{3R}^{\mu} = \sum_{i=1}^{2}\overline{(\hat{u}_R^m)^i}\gamma^\mu(\hat{u}_R^m)^i + \sum_{j,k=3}^{4}\overline{(\hat{u}_R^m)^j}\gamma^\mu(\mathcal{Z}_{T_R})^{jk}(\hat{u}_R^m)^k$$

$$- \sum_{i=1}^{2}\overline{(\hat{d}_R^m)^i}\gamma^\mu(\hat{d}_R^m)^i + \sum_{j,k=3}^{4}\overline{(\hat{d}_R^m)^j}\gamma^\mu(\mathcal{Z}_{B_R})^{jk}(\hat{d}_R^m)^j k$$

$$(\mathcal{Z}_{T_R})^{jk} \equiv \sum_{i,l=3}^{4}(K_{T_R}^\dagger)^{ji}(Z_{T_R})^{il}(K_{T_R})^{lk},$$

$$j,k \in \{3,4\}$$

$$(\mathcal{Z}_{B_R})^{jk} \equiv \sum_{i,l=3}^{4}(K_{B_R}^\dagger)^{ji}(Z_{B_R})^{il}(K_{B_R})^{lk}$$

$$\mathcal{Z}_{T_R} = \begin{pmatrix} \cos^2\beta_{T_R} & -\sin\beta_{T_R}\cos\beta_{T_R} \\ -\sin\beta_{T_R}\cos\beta_{T_R} & \sin^2\beta_{T_R} \end{pmatrix}$$

$$\mathcal{Z}_{B_R} = \begin{pmatrix} \cos^2\beta_{B_R} & -\sin\beta_{B_R}\cos\beta_{B_R} \\ -\sin\beta_{B_R}\cos\beta_{B_R} & \sin^2\beta_{B_R} \end{pmatrix}$$

Quark Sector

- The left-handed weak isospin current:

$$j_{3L}^{\mu} = \sum_{i=1}^{3} \left(\overline{u_L^i} \gamma^{\mu} u_L^i - \overline{d_L^i} \gamma^{\mu} d_L^i \right)$$

Quark Sector

- The left-handed weak isospin current:

$$j_{3L}^{\mu} = \sum_{i=1}^{3} \left(\overline{u_L^i} \gamma^\mu u_L^i - \overline{d_L^i} \gamma^\mu d_L^i \right) \xrightarrow{\mathrm{SU(2)_L \otimes U(1)_Y}}$$

Quark Sector

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$$j^{\mu}_{3L} = \sum_{i=1}^{3} \left(\overline{u^i_L} \gamma^\mu u^i_L - \overline{d^i_L} \gamma^\mu d^i_L \right) \xrightarrow{\mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}} j^{\mu}_{3L} = \sum_{i=1}^{2} \overline{(\hat{u}^m_L)^i} \gamma^\mu (\hat{u}^m_L)^i + \sum_{j,k=3}^{4} \overline{(\hat{u}^m_L)^j} \gamma^\mu (\mathcal{Z}_{T_L})^{jk} (\hat{u}^m_L)^k$$

$$- \sum_{i=1}^{2} \overline{(\hat{d}^m_L)^i} \gamma^\mu (\hat{d}^m_L)^i - \sum_{j,k=3}^{4} \overline{(\hat{d}^m_L)^j l} \gamma^\mu (\mathcal{Z}_{B_L})^{jk} (\hat{d}^m_L)^k$$

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where

$$(\mathcal{Z}_{T_L})^{jk} = (K^\dagger_{T_L})^{j3} (K_{T_L})^{3k}, \qquad j,k \in \{3,4\}$$
$$(\mathcal{Z}_{B_L})^{jk} = (K^\dagger_{B_L})^{j3} (K_{B_L})^{3k}$$

$$\mathcal{Z}_{T_L} = \begin{pmatrix} \cos^2\phi_{T_L} & -\sin\phi_{T_L}\cos\phi_{T_L} \\ -\sin\phi_{T_L}\cos\phi_{T_L} & \sin^2\phi_{T_L} \end{pmatrix}$$

$$\mathcal{Z}_{B_L} = \begin{pmatrix} \cos^2\phi_{B_L} & -\sin\phi_{B_L}\cos\phi_{B_L} \\ -\sin\phi_{B_L}\cos\phi_{B_L} & \sin^2\phi_{B_L} \end{pmatrix}$$

Quark Sector

- The interaction between Higgs (h_L and h_R) with quarks:

$$
\begin{aligned}
\mathcal{L}_q \supset \mathcal{L}_{hH} = &-\frac{1}{v_L}\sum_{k,i=3}^{4}\Big[(\mathcal{Z}_{T_L} m_t^{\rm diag})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i + (m_t^{\rm diag}\mathcal{Z}_{T_L})^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i \\
&+(\mathcal{Z}_{B_L} m_b^{\rm diag})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i + (m_b^{\rm diag}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\, h_L \\
&-\frac{1}{v_R}\sum_{k,i=3}^{4}\Big[((1-\mathcal{Z}_{T_L})m_t^{\rm diag}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i \\
&+(\mathcal{Z}_{T_R} m_t^{\rm diag}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i + ((1-\mathcal{Z}_{B_L})m_b^{\rm diag}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i \\
&+(\mathcal{Z}_{B_R} m_b^{\rm diag}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\, h_R
\end{aligned}
$$

Quark Sector

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$$+(\mathcal{Z}_{B_L} m_b^{\rm diag})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i + (m_b^{\rm diag}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\, h_L$$

$$-\frac{1}{v_R}\sum_{k,i=3}^{4}\Big[((1-\mathcal{Z}_{T_L})m_t^{\rm diag}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i$$

$$+(\mathcal{Z}_{T_R} m_t^{\rm diag}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i + ((1-\mathcal{Z}_{B_L})m_b^{\rm diag}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i$$

$$+(\mathcal{Z}_{B_R} m_b^{\rm diag}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\, h_R$$

- Transformation from h_L and h_R basis into the diagonal mass eigenstate h and H

Quark Sector

- The interaction between Higgs (h_L and h_R) with quarks:

$$\mathcal{L}_q \supset \mathcal{L}_{hH} = -\frac{1}{v_L}\sum_{k,i=3}^{4}\Big[(\mathcal{Z}_{T_L} m_t^{\text{diag}})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i + (m_t^{\text{diag}}\mathcal{Z}_{T_L})^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i$$
$$+(\mathcal{Z}_{B_L} m_b^{\text{diag}})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i + (m_b^{\text{diag}}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\, h_L$$
$$-\frac{1}{v_R}\sum_{k,i=3}^{4}\Big[((1-\mathcal{Z}_{T_L})m_t^{\text{diag}}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i$$
$$+(\mathcal{Z}_{T_R} m_t^{\text{diag}}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i + ((1-\mathcal{Z}_{B_L})m_b^{\text{diag}}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i$$
$$+(\mathcal{Z}_{B_R} m_b^{\text{diag}}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\, h_R$$

- Transformation from h_L and h_R basis into the diagonal mass eigenstate h and H

$$\begin{pmatrix} h_L \\ h_R \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} h \\ H \end{pmatrix}$$

Quark Sector

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$$\tan 2\phi = \frac{2\lambda_{LR} v_R v_L}{\lambda_R v_R^2 - \lambda_L v_L^2} \qquad 0 \le |\phi| \le \frac{\pi}{4}$$

$$\tan\phi \simeq \frac{\lambda_{LR}}{\lambda_R}\frac{v_L}{v_R}$$

Quark Sector

- The interaction between Z_L and Z_R with quarks:

$$\mathcal{L}_q \supset \mathcal{L}_{ZZ'} = -\left[\frac{g_L}{2\cos\theta_W}(j^{\mu}_{3L}) - e\tan\theta_W(j^{\mu}_{\text{em}})\right] Z_{L\mu} - \left[\frac{g_R}{2\cos\theta_R}(j^{\mu}_{3R}) - g'\tan\theta_R\left(j^{\mu}_{\text{em}} - \frac{1}{2}(j^{\mu}_{3L})\right)\right] Z_{R\mu}$$

where,

$$j^{\mu}_{\text{em}} = \frac{2}{3}\sum_{\alpha=1}^{4}\overline{(\hat{u}^m)^{\alpha}}\gamma^{\mu}(\hat{u}^m)^{\alpha} - \frac{1}{3}\sum_{\alpha=1}^{4}\overline{(\hat{d}^m)^{\alpha}}\gamma^{\mu}(\hat{d}^m)^{\alpha}$$

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$$-\left[\frac{g_R}{2\cos\theta_R}(j_{3R}^{\mu}) - g'\tan\theta_R\left(j_{\rm em}^{\mu} - \frac{1}{2}(j_{3L}^{\mu})\right)\right] Z_{R\mu}$$

where,

$$j_{\rm em}^{\mu} = \frac{2}{3}\sum_{\alpha=1}^{4}\overline{(\hat{u}^m)^\alpha}\gamma^\mu(\hat{u}^m)^\alpha - \frac{1}{3}\sum_{\alpha=1}^{4}\overline{(\hat{d}^m)^\alpha}\gamma^\mu(\hat{d}^m)^\alpha$$

- Transformation from Z_L and Z_R basis into the diagonal mass eigenstate Z and Z'

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where,

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$$\begin{pmatrix} Z_{L\mu} \\ Z_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix}$$

Quark Sector

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$$-\left[\frac{g_R}{2\cos\theta_R}(j^\mu_{3R}) - g'\tan\theta_R\left(j^\mu_{\text{em}} - \frac{1}{2}(j^\mu_{3L})\right)\right] Z_{R\mu}$$

where,

$$j^\mu_{\text{em}} = \frac{2}{3}\sum_{\alpha=1}^{4}\overline{(\hat{u}^m)^\alpha}\gamma^\mu(\hat{u}^m)^\alpha - \frac{1}{3}\sum_{\alpha=1}^{4}\overline{(\hat{d}^m)^\alpha}\gamma^\mu(\hat{d}^m)^\alpha$$

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$$\tan 2\theta = \frac{2c_R s_R^3 s_W \frac{v_L^2}{v_R^2}}{s_W^2 - s_R^2\left(s_W^2\cos 2\theta_R + c_W^2 c_R^2\right)\frac{v_L^2}{v_R^2}}, \quad 0 \le \theta \le \frac{\pi}{4}$$

$$\tan\theta \simeq \mathcal{O}(v_L^2/v_R^2)$$

Outline

- Introduction
- The model
- Quark Sector
- Higgs Sector
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Higgs Sector

- After $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}} \longrightarrow \mathrm{SU(2)_L \times U(1)_Y} \longrightarrow \mathrm{U(1)_{em}}$, we collect the qudratic terms:

$$
\begin{aligned}
\mathcal{L}_H \supset \mathcal{L}_{\mathrm{quad}} = {} & (D^\mu_{\mathrm{em}}\chi_L^-)(D_{\mathrm{em}\mu}\chi_L^+) + (D^\mu_{\mathrm{em}}\chi_R^-)(D_{\mathrm{em}\mu}\chi_R^+) \\
& + i\frac{g_L v_L}{2}\{W_L^{+\mu}(D_{\mathrm{em}\mu}\chi_L^-) - W_L^{-\mu}(D_{\mathrm{em}\mu}\chi_L^+)\} + \frac{g_L^2 v_L^2}{4} W_L^{-\mu} W_{L\mu}^+ \\
& + i\frac{g_R v_R}{2}\{W_R^{+\mu}(D_{\mathrm{em}\mu}\chi_R^-) - W_R^{-\mu}(D_{\mathrm{em}\mu}\chi_R^+)\} + \frac{g_R^2 v_R^2}{4} W_R^{-\mu} W_{R\mu}^+ \\
& + \frac{1}{2}\left(\frac{g_L}{2}\frac{v_L}{\cos\theta_W}\right)^2 Z_L^\mu Z_{L\mu} + \frac{1}{2}\left\{\left(\frac{g_R}{2}\frac{v_R}{\cos\theta_R}\right)^2 + \left(\frac{g'}{2} v_L \tan\theta_R\right)^2\right\} Z_R^\mu Z_{R\mu} \\
& + \frac{g' v_L}{2}\tan\theta_R \frac{g_L}{2}\frac{v_L}{\cos\theta_W} Z_L^\mu Z_{R\mu} \\
& + \frac{1}{2}(\partial_\mu \chi_L^3)^2 + \frac{1}{2}(\partial_\mu \chi_R^3)^2 \\
& - \frac{1}{2}\frac{g_L v_L}{\cos\theta_W} Z_{L\mu}(\partial^\mu \chi_L^3) - \frac{1}{2}\frac{g_R v_R}{\cos\theta_R} Z_{R\mu}(\partial^\mu \chi_R^3) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}(\partial^\mu \chi_L^3) \\
& + \frac{1}{2}(\partial_\mu h_L)^2 + \frac{1}{2}(\partial_\mu h_R)^2 \\
& - h_L(2\lambda_{LR} v_R v_L) h_R - \frac{h_L^2}{2}(2\lambda_L v_L^2) - \frac{h_R^2}{2}(2\lambda_R v_R^2).
\end{aligned}
$$

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$$\mathcal{L}_H \supset \mathcal{L}_{\text{quad}} = (D^\mu_{\text{em}}\chi_L^-)(D_{\text{em}\mu}\chi_L^+) + (D^\mu_{\text{em}}\chi_R^-)(D_{\text{em}\mu}\chi_R^+)$$

$$+ i\frac{g_L v_L}{2}\{W_L^{+\mu}(D_{\text{em}\mu}\chi_L^-) - W_L^{-\mu}(D_{\text{em}\mu}\chi_L^+)\} + \frac{g_L^2 v_L^2}{4} W_L^{-\mu} W_{L\mu}^+$$

WL boson

$$+ i\frac{g_R v_R}{2}\{W_R^{+\mu}(D_{\text{em}\mu}\chi_R^-) - W_R^{-\mu}(D_{\text{em}\mu}\chi_R^+)\} + \frac{g_R^2 v_R^2}{4} W_R^{-\mu} W_{R\mu}^+$$

WR boson

Neutral gauge bosons

$$+ \frac{1}{2}\left(\frac{g_L}{2}\frac{v_L}{\cos\theta_W}\right)^2 Z_L^\mu Z_{L\mu} + \frac{1}{2}\left\{\left(\frac{g_R}{2}\frac{v_R}{\cos\theta_R}\right)^2 + \left(\frac{g'}{2} v_L \tan\theta_R\right)^2\right\} Z_R^\mu Z_{R\mu}$$

$$+ \frac{g' v_L}{2}\tan\theta_R \frac{g_L}{2}\frac{v_L}{\cos\theta_W} Z_L^\mu Z_{R\mu}$$

Nambu-Goldstone bosons

$$+ \frac{1}{2}(\partial_\mu \chi_L^3)^2 + \frac{1}{2}(\partial_\mu \chi_R^3)^2$$

$$- \frac{1}{2}\frac{g_L v_L}{\cos\theta_W} Z_{L\mu}(\partial^\mu \chi_L^3) - \frac{1}{2}\frac{g_R v_R}{\cos\theta_R} Z_{R\mu}(\partial^\mu \chi_R^3) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}(\partial^\mu \chi_L^3)$$

Higgs bosons

$$+ \frac{1}{2}(\partial_\mu h_L)^2 + \frac{1}{2}(\partial_\mu h_R)^2$$

$$- h_L(2\lambda_{LR} v_R v_L) h_R - \frac{h_L^2}{2}(2\lambda_L v_L^2) - \frac{h_R^2}{2}(2\lambda_R v_R^2).$$

Higgs Sector

- Mass of gauge bosons:

Higgs Sector

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- After expanding and taking $M_{W_R} \gg M_{W_L}$, the Z and Z' bosons mass can be expressed as,

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- After expanding and taking $M_{W_R} \gg M_{W_L}$, the Z and Z' bosons mass can be expressed as,

$$M^2_Z \simeq \frac{M^2_{W_L}}{c^2_W} \left(1 - \frac{M^2_{W_L}}{M^2_{W_R}} s^2_R t^2_W \right)$$

$$M^2_{Z'} \simeq \frac{M^2_{W_R}}{c^2_R} \left(1 + \frac{M^2_{W_L}}{M^2_{W_R}} s^2_R t^2_W \right)$$

Higgs Sector

- Higgs and heavy Higss masses:

Higgs Sector

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$$m^2_{h(H)} = \lambda_L v_L^2 + \lambda_R v_R^2 \overset{(+)}{-} \sqrt{(\lambda_L v_L^2 - \lambda_R v_R^2)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}$$

Higgs Sector

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$$m_{h(H)}^2 = \lambda_L v_L^2 + \lambda_R v_R^2 \overset{(+)}{-} \sqrt{(\lambda_L v_L^2 - \lambda_R v_R^2)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}$$

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- If we ignore $\mathcal{O}(v_L^2/v_R^2)$, the mass eigenvalues become,

$$m_h^2 \simeq 2\lambda_L \left(1 - \frac{\lambda_{LR}^2}{\lambda_L \lambda_R}\right) v_L^2$$

$$m_H^2 \simeq 2\lambda_R v_R^2$$

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Hierarchy in the Top and Bottom Sector

- How to explain the hierarchy of top and bottom quarks mass in a seesaw model
- Using exact mass eigenvalues formula:

Hierarchy in the Top and Bottom Sector

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Top sector

$$m_t = -\frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

$$m_{t'} = \frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

Hierarchy in the Top and Bottom Sector

- How to explain the hierarchy of top and bottom quarks mass in a seesaw model

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Top sector

$$m_t = -\frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

$$m_{t'} = \frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

Hierarchy in the Top and Bottom Sector

- How to explain the hierarchy of top and bottom quarks mass in a seesaw model
- Using exact mass eigenvalues formula:

Top sector

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Bottom sector

$$m_b = -\frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

$$m_{b'} = \frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

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where,

$$m_{u_R} = Y_{u_R}^3 \frac{v_R}{\sqrt{2}}, \qquad m_{u_L} = Y_{u_L}^3 \frac{v_L}{\sqrt{2}}., \qquad m_{d_R} = Y_{d_R}^3 \frac{v_R}{\sqrt{2}}, \qquad m_{d_L} = Y_{d_L}^3 \frac{v_L}{\sqrt{2}}$$

Hierarchy in the Top and Bottom Sector

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Bottom sector

$$m_b = -\frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

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- Assumptions and constraints (from PDG):

$$g_R \simeq 1, \quad Y_{u_R}^3 = Y_{u_L}^3 = Y_{d_R}^3 = Y_{d_L}^3 \simeq 1$$

$$m_{t'} > 1310\,\mathrm{GeV}; m_{b'} > 1390\,\mathrm{GeV}; M_{Z'} > 5150\,\mathrm{GeV}$$

Hierarchy in the Top and Bottom Sector

- How to explain the hierarchy of top and bottom quarks mass in a seesaw model

- Using exact mass eigenvalues formula:

Top sector

$$m_t = -\frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

$$m_{t'} = \frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

Bottom sector

$$m_b = -\frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

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- Assumptions and constraints (from PDG):

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$$m_{t'} > 1310\,\text{GeV}; m_{b'} > 1390\,\text{GeV}; M_{Z'} > 5150\,\text{GeV}$$

- We obtain: $M_{W_R} \gtrsim 5\,\text{TeV}; v_R \gtrsim 10\,\text{TeV}$

Hierarchy in the Top and Bottom Sector

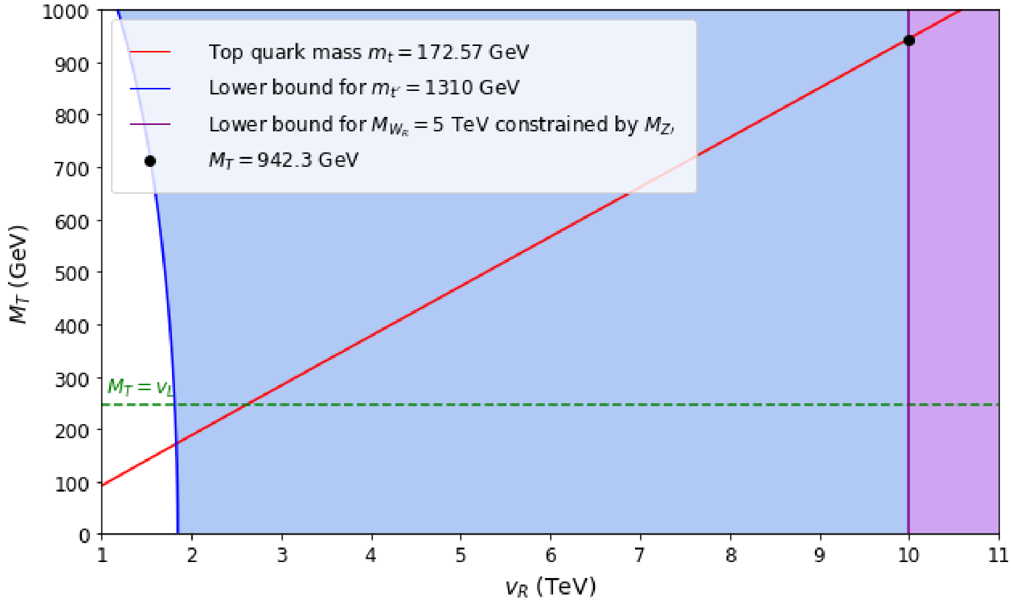

Fig 1. Constraints on v_R and M_T (Fig source: [2407.00732])

- At $v_R = 10\,\text{TeV} \longrightarrow M_T = 942.3\,\text{GeV}$
- Using the exact mass eigenvalues equation, we obtain $m_{t'} = 7.13\,\text{TeV}$
- The hierarchy: $v_L < M_T < v_R$

Hierarchy in the Top and Bottom Sector

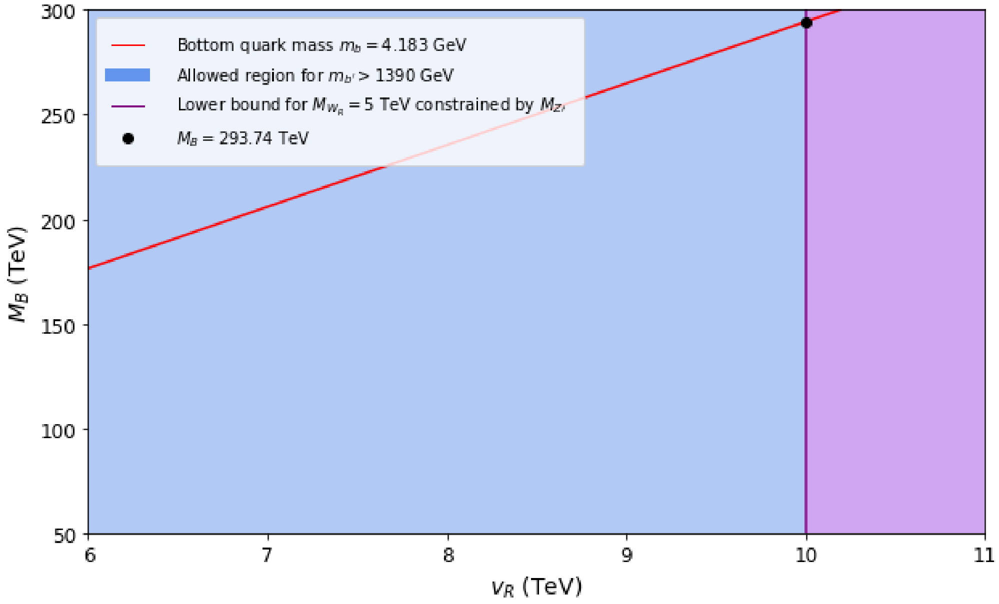

Fig 1. Constraints on v_R and M_B (Fig source: [2407.00732])

- At $v_R = 10\,\mathrm{TeV} \longrightarrow M_B = 293.74\,\mathrm{TeV}$
- Using the exact mass eigenvalues equation, we obtain $m_{b'} = 293.82\,\mathrm{TeV}$
- The hierarchy: $v_L < v_R \ll M_B$

Hierarchy in the Top and Bottom Sector

- The hierarchy: $v_L < M_T < v_R \ll M_B$

Hierarchy in the Top and Bottom Sector

- The hierarchy: $v_L < M_T < v_R \ll M_B$
- Using this hierarchy, we compute the approximation of the mass eigenvalues:

Hierarchy in the Top and Bottom Sector

- The hierarchy: $v_L < M_T < v_R \ll M_B$
- Using this hierarchy, we compute the approximation of the mass eigenvalues:

$$
\begin{aligned}
m_t^{\text{approx}} &\simeq \frac{v_R Y^3_{u_R} Y^3_{u_L} v_L}{2\sqrt{\frac{v_R^2}{2}(Y^3_{u_R})^2 + M_T^2}}, \\
m_{t'}^{\text{approx}} &\simeq \sqrt{\frac{v_R^2}{2}(Y^3_{u_R})^2 + M_T^2}, \\
m_b^{\text{approx}} &\simeq \frac{v_R Y^3_{d_R} Y^3_{d_L} v_L}{2M_B}, \\
m_{b'}^{\text{approx}} &\simeq M_B.
\end{aligned}
$$

Hierarchy in the Top and Bottom Sector

- The hierarchy: $v_L < M_T < v_R \ll M_B$
- Using this hierarchy, we compute the approximation of the mass eigenvalues:

$$
\begin{aligned}
m_t^{\text{approx}} &\simeq \frac{v_R Y_{u_R}^3 Y_{u_L}^3 v_L}{2\sqrt{\frac{v_R^2}{2}(Y_{u_R}^3)^2 + M_T^2}}, \\
m_{t'}^{\text{approx}} &\simeq \sqrt{\frac{v_R^2}{2}(Y_{u_R}^3)^2 + M_T^2}, \\
m_b^{\text{approx}} &\simeq \frac{v_R Y_{d_R}^3 Y_{d_L}^3 v_L}{2M_B}, \\
m_{b'}^{\text{approx}} &\simeq M_B.
\end{aligned}
$$

- Using the same numerical inputs, we obtain:

$$m_t^{\text{approx}} = 172.58\,\text{GeV}, m_{t'}^{\text{approx}} = 7.13\,\text{TeV}, m_b^{\text{approx}} = 4.19\,\text{GeV}, m_{b'}^{\text{approx}} = 293.74\,\text{TeV}$$

Outline

- Introduction
- The model
- Quark Sector
- Higgs Sector
- Hierarchy in the Top and Bottom Sector
- Higgs and Z boson FCNCs
- Summary

Higgs

Higgs and Z bosons FCNCs

- The interaction between Higgs and top-sector quarks:

$$\mathcal{L}_{ht} \simeq -\cos\phi \frac{m_t}{v_L}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi \frac{M_T}{m_{u_R}}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)(\bar{t}_L t'_R + \bar{t}'_R t_L)h$$
$$- \cos\phi \frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\right)(\bar{t}'_L t_R + \bar{t}_R t'_L)h - \cos\phi \frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2} - \frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h.$$

Higgs and Z bosons FCNCs

- The interaction between Higgs and top-sector quarks:

$$\mathcal{L}_{ht} \simeq -\cos\phi\frac{m_t}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi\frac{M_T}{m_{u_R}}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)(\bar{t}_L t'_R + \bar{t}'_R t_L)h$$
$$-\cos\phi\frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1+\frac{\lambda_{LR}}{\lambda_R}\right)(\bar{t}'_L t_R + \bar{t}_R t'_L)h - \cos\phi\frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2}-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h.$$

- Our findings:

Higgs and Z bosons FCNCs

- The interaction between Higgs and top-sector quarks:

correction

$$\mathcal{L}_{ht} \simeq -\cos\phi \frac{m_t}{v_L}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi \frac{M_T}{m_{u_R}}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)(\bar{t}_L t'_R + \bar{t}'_R t_L)h$$
$$- \cos\phi \frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\right)(\bar{t}'_L t_R + \bar{t}_R t'_L)h - \cos\phi \frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2} - \frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h.$$

- Our findings:
  - The Higgs-top coupling receives a small correction

Higgs and Z bosons FCNCs

- The interaction between Higgs and top-sector quarks:

$$\mathcal{L}_{ht} \simeq -\cos\phi \frac{m_t}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi\frac{M_T}{m_{u_R}}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)(\bar{t}_L t'_R + \bar{t}'_R t_L)h$$
$$-\cos\phi\frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1+\frac{\lambda_{LR}}{\lambda_R}\right)(\bar{t}'_L t_R + \bar{t}_R t'_L)h - \cos\phi\frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2}-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h.$$

suppression

- Our findings:
  - The Higgs-top coupling receives a small correction
  - The Higgs-heavy top coupling receives an overall suppression of $\mathcal{O}\left(v_L/v_R\right) \sim \mathcal{O}(10^{-2})$

Higgs and Z bosons FCNCs

- The interaction between Higgs and top-sector quarks:

$$\mathcal{L}_{ht} \simeq -\cos\phi \frac{m_t}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi \underbrace{\frac{M_T}{m_{u_R}}}_{\mathcal{O}\left(10^{-1}\right)}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)(\bar{t}_L t'_R + \bar{t}'_R t_L)h$$
$$-\cos\phi \frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1+\frac{\lambda_{LR}}{\lambda_R}\right)(\bar{t}'_L t_R + \bar{t}_R t'_L)h - \cos\phi \frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2}-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h.$$

- Our findings:
  - The Higgs-top coupling receives a small correction
  - The Higgs-heavy top coupling receives an overall suppression of $\mathcal{O}\left(v_L/v_R\right) \sim \mathcal{O}(10^{-2})$
  - The Higgs FCNC of $\bar{t}'_L t_R, \bar{t}_R t'_L$ type is more suppressed by a factor $\mathcal{O}\left(v_L/v_R\right) \sim \mathcal{O}(10^{-2})$ compare to $\bar{t}_L t'_R, \bar{t}'_R t_L$ type

Higgs and Z bosons FCNCs

- The interaction between Higgs and bottom-sector quarks:

$$\mathcal{L}_{hb} \simeq -\cos\phi \frac{m_b}{v_L}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\bar{b}bh - \cos\phi \frac{m_b m_{b'}}{m_{d_L} m_{d_R}}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{M_B^2}\right)(\bar{b}_L b'_R + \bar{b}'_R b_L)h$$
$$- \frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R} + \frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi \frac{m_{d_L}}{m_{b'}}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}'b'h.$$

Higgs and Z bosons FCNCs

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$$-\frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R}+\frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi\frac{m_{d_L}}{m_{b'}}\left(1-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}' b' h.$$

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Higgs and Z bosons FCNCs

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$$-\frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R} + \frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi \frac{m_{d_L}}{m_{b'}}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}'b'h.$$

- Our findings:
  - The Higgs-bottom coupling receives a small correction

Higgs and Z bosons FCNCs

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$$-\frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R}+\frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi \frac{m_{d_L}}{m_{b'}}\left(1-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}'b'h.$$

suppression

- Our findings:
  - The Higgs-bottom coupling receives a small correction
  - The Higgs-heavy bottom coupling receives a suppression of $\mathcal{O}\left(v_L/M_B\right) \sim \mathcal{O}(10^{-4})$

Higgs and Z bosons FCNCs

- The interaction between Higgs and bottom-sector quarks:

$$\mathcal{L}_{hb} \simeq -\cos\phi \frac{m_b}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\bar{b}bh - \cos\phi \frac{m_b m_{b'}}{m_{d_L} m_{d_R}}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{M_B^2}\right)(\bar{b}_L b'_R + \bar{b}'_R b_L)h$$

$$-\frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R}+\frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi \frac{m_{d_L}}{m_{b'}}\left(1-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}' b' h.$$

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  - The Higgs FCNC of $\bar{b}'_L b_R, \bar{b}_R b'_L$ type is suppressed by a factor $\mathcal{O}\left(v_L/v_R\right) \sim \mathcal{O}(10^{-2})$

Higgs and Z bosons FCNCs

- The interaction between Higgs and bottom-sector quarks:

≈1

$$\mathcal{L}_{hb} \simeq -\cos\phi \frac{m_b}{v_L}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\bar{b}bh - \cos\phi \frac{m_b m_{b'}}{m_{d_L} m_{d_R}}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{M_B^2}\right)(\bar{b}_L b'_R + \bar{b}'_R b_L)h$$
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  - The Higgs FCNC of $\bar{b}_L b'_R, \bar{b}'_R b_L$ type is not suppressed

Higgs and Z bosons FCNCs

- The interaction between Higgs and bottom-sector quarks:

$$\mathcal{L}_{hb} \simeq -\cos\phi \frac{m_b}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\bar{b}bh - \cos\phi \underbrace{\frac{m_b m_{b'}}{m_{d_L} m_{d_R}}}_{\approx 1}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{M_B^2}\right)\underline{(\bar{b}_L b'_R + \bar{b}'_R b_L)}h$$

$$-\frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R}+\frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi \frac{m_{d_L}}{m_{b'}}\left(1-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}'b'h.$$

- Our findings:
  - The Higgs-bottom coupling receives a small correction
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  - The Higgs FCNC of $\bar{b}'_L b_R, \bar{b}_R b'_L$ type is suppressed by a factor $\mathcal{O}\left(v_L/v_R\right) \sim \mathcal{O}(10^{-2})$
  - The Higgs FCNC of $\bar{b}_L b'_R, \bar{b}'_R b_L$ type is not suppressed

    because $Y_{d_L}^3 \simeq 1$

Z-boson

Higgs and Z bosons FCNCs

- The interaction between Z-boson and up-sector quarks:

$$\mathcal{L}^Z_{\bar{q}q} \supset \mathcal{L}^Z_t = -\frac{g_L}{2\cos\theta_W}\left\{\overline{(\hat{u}^m)^1}\gamma^\mu\left[(g_V)^{11}_u - (g_A)^{11}_u\gamma^5\right](\hat{u}^m)^1\right.$$

$$+\overline{(\hat{u}^m)^2}\gamma^\mu\left[(g_V)^{22}_u - (g_A)^{22}_u\gamma^5\right](\hat{u}^m)^2 + \bar{t}\gamma^\mu\left[(g_V)^{33}_u - (g_A)^{33}_u\gamma^5\right]t$$

$$+\bar{t}\gamma^\mu\left[(g_V)^{34}_u - (g_A)^{34}_u\gamma^5\right]t' + \overline{t'}\gamma^\mu\left[(g_V)^{43}_u - (g_A)^{43}_u\gamma^5\right]t$$

$$\left.+\overline{t'}\gamma^\mu\left[(g_V)^{44}_u - (g_A)^{44}_u\gamma^5\right]t'\right\}Z_\mu$$

Higgs and Z bosons FCNCs

- The interaction between Z-boson and up-sector quarks:

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$$+\overline{(\hat{u}^m)^2}\gamma^\mu\Big[(g_V)^{22}_u - (g_A)^{22}_u\gamma^5\Big](\hat{u}^m)^2 + \bar{t}\gamma^\mu\Big[(g_V)^{33}_u - (g_A)^{33}_u\gamma^5\Big]t$$
$$+\bar{t}\gamma^\mu\Big[(g_V)^{34}_u - (g_A)^{34}_u\gamma^5\Big]t' + \overline{t'}\gamma^\mu\Big[(g_V)^{43}_u - (g_A)^{43}_u\gamma^5\Big]t$$
$$+\overline{t'}\gamma^\mu\Big[(g_V)^{44}_u - (g_A)^{44}_u\gamma^5\Big]t'\Big\}Z_\mu$$

where

$$(g_V)^{\alpha\beta}_u = \frac{1}{2}\left((\kappa_{T_L})^{\alpha\beta} - (\kappa_{T_R})^{\alpha\beta}\right) - 2\kappa Q_u\delta^{\alpha\beta}$$
$$(g_A)^{\alpha\beta}_u = \frac{1}{2}\left((\kappa_{T_L})^{\alpha\beta} + (\kappa_{T_R})^{\alpha\beta}\right)$$
$$\kappa = \sin^2\theta_W\cos\theta - \sin\theta_W\tan\theta_R\sin\theta$$

Higgs and Z bosons FCNCs

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where

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FCNC

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Higgs and Z bosons FCNCs

- Our results:
  - The Z-boson FCNC process with the top-quark and heavy-top quark Ztt'

$$(\kappa_{T_L})^{34} = (\kappa_{T_L})^{43} = \cos\theta\left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m_{u_L} M_T}{m_{u_R}^2}$$

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Higgs and Z bosons FCNCs

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Outline

- Introduction
- The model
- Quark Sector Lagrangian
- Higgs Sector and Gauge Kinetic Terms Lagrangian
- Hierarchy in Top and Bottom Sector
- Higgs and Z boson FCNCs
- Summary

Summary

- We have presented the quark sector of universal seesaw model in the massless case of two lightest quark families

- We confirmed that the hierarchy of VLQ's mass parameters, v_L, and v_R

$$v_L < M_T < v_R \ll M_B$$

- We have shown that the Z-boson mediated FCNC process is suppressed for both (up and down) sectors

- On the other hand, the Higgs mediated FCNC of $\bar{b}_L b'_R, \bar{b}'_R b_L$ is not suppressed when $Y^3_{d_L} \simeq 1$

THANK YOU

BACKUP

Higgs Sector

Z and Z' bosons mass

- Mass matrix in the Z_L and Z_R basis:

$$\mathbb{M}_Z^2 = \begin{pmatrix} (\frac{g_L v_L}{2\cos\theta_W})^2 & \frac{1}{2}g' v_L \tan\theta_R \frac{g_L v_L}{2\cos\theta_W} \\ \frac{1}{2}g' v_L \tan\theta_R \frac{g_L v_L}{2\cos\theta_W} & (\frac{g_R v_R}{2\cos\theta_R})^2 + \left(\frac{1}{2}g' v_L \tan\theta_R\right)^2 \end{pmatrix}$$

- Define following transformation:

$$\begin{pmatrix} Z_{L\mu} \\ Z_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix}$$

- After diagonalizing the mass matrix, the mass eigenvalues and mixing angles are,

$$M^2_{Z(Z')} = \frac{M^2_{W_R}}{2c_R^2} \left\{ 1 + (c_R^2 + t_W^2)\frac{M^2_{W_L}}{M^2_{W_R}} \overset{(+)}{-} \sqrt{1 - \frac{2M^2_{W_L}}{M^2_{W_R}} \left(\frac{c_R^2 - s_W^2 s_R^2}{c_W^2} \right) + (c_R^2 + t_W^2)^2 \left(\frac{M^2_{W_L}}{M^2_{W_R}} \right)^2} \right\}$$

$$\tan 2\theta = \frac{2 c_R s_R^3 s_W \frac{v_L^2}{v_R^2}}{s_W^2 - s_R^2 \left(s_W^2 \cos 2\theta_R + c_W^2 c_R^2 \right) \frac{v_L^2}{v_R^2}}, \quad 0 \le \theta \le \frac{\pi}{4}$$

Higgs Sector Lagrangian

$$M^2_{Z(Z')} = \frac{M^2_{W_R}}{2c^2_R}\left\{1+(c^2_R+t^2_W)\frac{M^2_{W_L}}{M^2_{W_R}} \overset{(+)}{-} \sqrt{1-\frac{2M^2_{W_L}}{M^2_{W_R}}\left(\frac{c^2_R-s^2_W s^2_R}{c^2_W}\right)+(c^2_R+t^2_W)^2\left(\frac{M^2_{W_L}}{M^2_{W_R}}\right)^2}\right\}$$

- After expanding and taking $M_{W_R} \gg M_{W_L}$, the Z and Z' bosons mass can be approximated,

$$M^2_Z \simeq \frac{M^2_{W_L}}{c^2_W}\left(1-\frac{M^2_{W_L}}{M^2_{W_R}}s^2_R t^2_W\right),$$

$$M^2_{Z'} \simeq \frac{M^2_{W_R}}{c^2_R}\left(1+\frac{M^2_{W_L}}{M^2_{W_R}}s^2_R t^2_W\right).$$

Higgs Sector Lagrangian

Higgs boson mass

- Mass matrix in the h_L and h_R basis:

$$\mathbb{M}_h^2 = \begin{pmatrix} 2\lambda_L v_L^2 & 2\lambda_{LR} v_R v_L \\ 2\lambda_{LR} v_R v_L & 2\lambda_R v_R^2 \end{pmatrix}$$

- Define following transformation:

$$\begin{pmatrix} h_L \\ h_R \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

- After diagonalizing the mass matrix, the mass eigenvalues and mixing angles are,

$$m_{h(H)}^2 = \lambda_L v_L^2 + \lambda_R v_R^2 \overset{(+)}{-} \sqrt{(\lambda_L v_L^2 - \lambda_R v_R^2)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}$$

$$\tan 2\phi = \frac{2\lambda_{LR} v_R v_L}{\lambda_R v_R^2 - \lambda_L v_L^2}, \quad 0 \leq |\phi| \leq \frac{\pi}{4}$$

Higgs Sector Lagrangian

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$$\tan 2\phi = \frac{2\lambda_{LR} v_R v_L}{\lambda_R v_R^2 - \lambda_L v_L^2}$$

- If we ignore $\mathcal{O}(v_L^2/v_R^2)$, the mass eigenvalues and mixing angle become,

$$m_h^2 \simeq 2\lambda_L \left(1 - \frac{\lambda_{LR}^2}{\lambda_L \lambda_R}\right) v_L^2,$$

$$m_H^2 \simeq 2\lambda_R v_R^2,$$

$$\tan 2\phi \simeq \frac{2\lambda_{LR}}{\lambda_R} \frac{v_L}{v_R}$$

Higgs Sector Lagrangian

χ_L^3 and χ_R^3 mixing

$$\mathcal{L}_{\rm quad} \supset \mathcal{L}_\chi = \frac{1}{2}(\partial_\mu \chi_L^3)^2 + \frac{1}{2}(\partial_\mu \chi_R^3)^2$$
$$- \frac{1}{2}\frac{g_L v_L}{\cos\theta_W} Z_{L\mu}(\partial^\mu \chi_L^3) - \frac{1}{2}\frac{g_R v_R}{\cos\theta_R} Z_{R\mu}(\partial^\mu \chi_R^3) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}(\partial^\mu \chi_L^3).$$

- By changing into mass eigenstate Z and Z', also in terms of M_Z and $M_{Z'}$

$$\mathcal{L}_{\rm quad} \supset \mathcal{L}_\chi = \frac{1}{2}(\partial_\mu \chi_Z)^2 + \frac{1}{2}(\partial_\mu \chi_{Z'})^2$$
$$- M_Z\ (\partial^\mu \chi_Z) Z_\mu - M_{Z'}(\partial^\mu \chi_{Z'}) Z'_\mu,$$

where,

$$\begin{pmatrix} \chi_L^3 \\ \chi_R^3 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \chi_Z \\ \chi_{Z'} \end{pmatrix},$$
$$\cos\alpha = \frac{M_Z \cos\theta}{\sqrt{M_Z^2\cos^2\theta + M_{Z'}^2\sin^2\theta}},$$
$$\sin\alpha = \frac{M_{Z'} \sin\theta}{\sqrt{M_Z^2\cos^2\theta + M_{Z'}^2\sin^2\theta}}.$$

Higgs Sector Lagrangian

- Therefore, the quadratic terms of Higgs sector Lagrangian written in terms of the mass basis of the Z bosons, Higgs bosons and Nambu-Goldstone bosons,

$$
\begin{aligned}
\mathcal{L}_H \supset \mathcal{L}_{\text{quad}} = &\left(D^{\mu}_{\text{em}}\chi^-_L - iM_{W_L}W^{\mu-}_L\right)\left(D_{\text{em}\mu}\chi^+_L + iM_{W_L}W^+_{L\mu}\right) \\
&+ \left(D^{\mu}_{\text{em}}\chi^-_R - iM_{W_R}W^{\mu-}_R\right)\left(D_{\text{em}\mu}\chi^+_R + iM_{W_R}W^+_{R\mu}\right) \\
&+ \frac{1}{2}\left(\partial_\mu\chi_Z - M_Z Z_\mu\right)^2 + \frac{1}{2}\left(\partial_\mu\chi_{Z'} - M_{Z'}Z'_\mu\right)^2 \\
&+ \frac{1}{2}\left(\partial_\mu h\right)^2 - \frac{1}{2}m_h^2 h^2 + \frac{1}{2}\left(\partial_\mu H\right)^2 - \frac{1}{2}m_H^2 H^2,
\end{aligned}
$$

where,

$$
D_{\text{em}\mu}\chi^+_{L(R)} = (\partial_\mu + ieA_\mu)\chi^+_{L(R)}
$$

Kinetic terms of gauge fields

- Recall the kinetic terms of gauge fields which invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{L\mu\nu}F_L^{a\mu\nu} - \frac{1}{4}F^a_{R\mu\nu}F_R^{a\mu\nu} - \frac{1}{4}B'_{\mu\nu}B'^{\mu\nu}$$

where,

$$F^a_{L\mu\nu} = \partial_\mu W^a_{L\nu} - \partial_\nu W^a_{L\mu} - g_L\epsilon^{abc}W^b_{L\mu}W^c_{L\nu},$$
$$F^a_{R\mu\nu} = \partial_\mu W^a_{R\nu} - \partial_\nu W^a_{R\mu} - g_R\epsilon^{abc}W^b_{R\mu}W^c_{R\nu},$$
$$B'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu$$

- Starting from $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}} \longrightarrow \mathrm{SU(2)_L \times U(1)_Y}$

$$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\chi_R^+ \\ v_R + h_R + i\chi_R^3 \end{pmatrix}$$

$$\begin{pmatrix} B'_\mu \\ W^3_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix}\begin{pmatrix} B_\mu \\ Z_{R\mu} \end{pmatrix}$$

Kinetic terms of gauge fields

$\mathrm{SU(2)_L \times U(1)_Y}$

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} =& -\frac{1}{4}F^a_{L\mu\nu}F^{a\mu\nu}_L - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& -\frac{1}{2}(\partial_\mu W^+_{R\nu} - \partial_\nu W^+_{R\mu})(\partial^\mu W_R^{-\nu} - \partial^\nu W_R^{-\mu}) \\
& - i(\partial_\mu W^+_{R\nu} - \partial_\nu W^+_{R\mu})(g_R \cos\theta_R Z^\nu_R + g' B^\nu)W_R^{-\mu} \\
& + i(\partial^\mu W_R^{-\nu} - \partial^\nu W_R^{-\mu})(g_R \cos\theta_R Z_{R\nu} + g' B_\nu)W^+_{R\mu} \\
& - \Big\{(g_R \cos\theta_R Z_{R\nu} + g' B_\nu)W^+_{R\mu}(g_R \cos\theta_R Z^\nu_R + g' B^\nu)W_R^{-\mu} \\
& \quad -(g_R \cos\theta_R Z_{R\mu} + g' B_\mu)W^+_{R\nu}(g_R \cos\theta_R Z^\nu_R + g' B^\nu)W_R^{-\mu}\Big\} \\
& - \frac{1}{4}F^0_{Z_R\mu\nu}F^{0\mu\nu}_{Z_R} + iW^-_{R\mu}W^+_{R\nu}(g_R \cos\theta_R F^{0\mu\nu}_{Z_R} + g' B^{\mu\nu}) \\
& + \frac{1}{2}g_R^2(W^-_{R\mu}W^+_{R\nu} - W^+_{R\mu}W^-_{R\nu})(W_R^{-\mu}W_R^{+\nu}),
\end{aligned}
$$

where

$$
\begin{aligned}
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
F^a_{L\mu\nu} &= \partial_\mu W^a_{L\nu} - \partial_\nu W^a_{L\mu} - g_L \epsilon^{abc} W^b_{L\mu} W^c_{L\nu}, \\
F^0_{Z_R\mu\nu} &= \partial_\mu Z_{R\nu} - \partial_\nu Z_{R\mu}.
\end{aligned}
$$

Kinetic terms of gauge fields

$$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \longrightarrow \mathrm{U}(1)_{\mathrm{em}}$$

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} = & -\frac{1}{4} F^0_{Z\mu\nu} F^{0\mu\nu}_{Z} - \frac{1}{4} F^0_{Z'\mu\nu} F^{0\mu\nu}_{Z'} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& - \frac{1}{2}\left(\mathcal{D}_\mu W^+_{L\nu} - \mathcal{D}_\nu W^+_{L\mu}\right)\left(\mathcal{D}^\mu W_L^{-\nu} - \mathcal{D}^\nu W_L^{-\mu}\right) \\
& - \frac{1}{2}\left(\mathcal{D}_\mu W^+_{R\nu} - \mathcal{D}_\nu W^+_{R\mu}\right)\left(\mathcal{D}^\mu W_R^{-\nu} - \mathcal{D}^\nu W_R^{-\mu}\right) \\
& + \frac{g_L^2}{2}\left((W_L^- \cdot W_L^-)(W_L^+ \cdot W_L^+) - \left(W_L^- \cdot W_L^+\right)^2\right) \\
& + \frac{g_R^2}{2}\left((W_R^- \cdot W_R^-)(W_R^+ \cdot W_R^+) - \left(W_R^- \cdot W_R^+\right)^2\right) \\
& + i\left\{g_L \cos\theta_W \cos\theta F_Z^{0\mu\nu} + g_L \cos\theta_W \sin\theta F_{Z'}^{0\mu\nu} + eF^{\mu\nu}\right\}\left(W^-_{L\mu} W^+_{L\nu}\right) \\
& + i\left\{-(g_R \cos\theta_R \sin\theta + e\tan\theta_W \cos\theta) F_Z^{0\mu\nu} + (g_R \cos\theta_R \cos\theta - e\tan\theta_W \sin\theta) F_{Z'}^{0\mu\nu} + eF^{\mu\nu}\right\}\left(W^-_{R\mu} W^+_{R\nu}\right),
\end{aligned}$$

where

$$\begin{aligned}
F^0_{Z\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu, & \mathcal{D}_\mu W^+_{R\nu} &= (D_{\mathrm{em}\mu} W^+_{R\nu}) - i(e\tan\theta_W Z_{L\mu} - g_R \cos\theta_R Z_{R\mu}) W^+_{R\nu}, \\
F^0_{Z'\mu\nu} &= \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, & \mathcal{D}_\mu W^+_{L\nu} &= (D_{\mathrm{em}\mu} W^+_{L\nu}) + i g_L \cos\theta_W Z_{L\mu} W^+_{L\nu}, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, & D_{\mathrm{em}\mu} G_\nu &= (\partial_\mu + ieA_\mu) G_\nu,
\end{aligned}$$

Higgs and Z bosons FCNCs

- Recall the FCNCs couplings:

$$\mathcal{Z}_{T_L} = \begin{pmatrix} \cos^2\phi_{T_L} & -\sin\phi_{T_L}\cos\phi_{T_L} \\ -\sin\phi_{T_L}\cos\phi_{T_L} & \sin^2\phi_{T_L} \end{pmatrix} \qquad \mathcal{Z}_{T_R} = \begin{pmatrix} \cos^2\beta_{T_R} & -\sin\beta_{T_R}\cos\beta_{T_R} \\ -\sin\beta_{T_R}\cos\beta_{T_R} & \sin^2\beta_{T_R} \end{pmatrix}$$

$$\mathcal{Z}_{B_L} = \begin{pmatrix} \cos^2\phi_{B_L} & -\sin\phi_{B_L}\cos\phi_{B_L} \\ -\sin\phi_{B_L}\cos\phi_{B_L} & \sin^2\phi_{B_L} \end{pmatrix} \qquad \mathcal{Z}_{B_R} = \begin{pmatrix} \cos^2\beta_{B_R} & -\sin\beta_{B_R}\cos\beta_{B_R} \\ -\sin\beta_{B_R}\cos\beta_{B_R} & \sin^2\beta_{B_R} \end{pmatrix}$$

- The mixing angles in the approximation forms:

$$\sin\phi_{T_L} \simeq -\frac{m_{u_L}M_T}{M_T^2+m_{u_R}^2}, \quad \cos\phi_{T_L}\simeq 1, \quad \sin\phi_{T_R}\simeq \frac{m_{u_L}^2 m_{u_R} M_T}{(M_T^2+m_{u_R}^2)^2}, \quad \cos\phi_{T_R}\simeq 1$$

$$\sin\phi_{B_L} \simeq -\frac{m_{d_L}}{M_B}, \quad \cos\phi_{B_L}\simeq 1, \quad \sin\phi_{B_R}\simeq \frac{m_{d_L}^2 m_{d_R}}{M_B^3}, \quad \cos\phi_{B_R}\simeq 1$$

$$\sin\beta_{T_R} \simeq \frac{m_{u_R}}{\sqrt{M_T^2+m_{u_R}^2}}, \quad \cos\beta_{T_R}\simeq \frac{M_T}{\sqrt{M_T^2+m_{u_R}^2}}, \quad \sin\beta_{B_R}\simeq\frac{m_{d_R}}{M_B}, \quad \cos\beta_{B_R}\simeq 1.$$

where,

$$m_{u_R} = Y_{u_R}^3\frac{v_R}{\sqrt{2}}, \qquad m_{u_L} = Y_{u_L}^3\frac{v_L}{\sqrt{2}}., \qquad m_{d_R} = Y_{d_R}^3\frac{v_R}{\sqrt{2}}, \qquad m_{d_L} = Y_{d_L}^3\frac{v_L}{\sqrt{2}}$$