A Wakefield Resilient, High Shunt Impedance Accelerating Structure for Cold Copper Collider (C^3)

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SLAC NATIONAL ACCELERATOR LABORATORY

What is New in the Cavity Design

The Initial Proposed Design

- Based on a **distributed coupling accelerating (DCA)** rf structure where the phase advance between the accelerating standing wave cavities is **π**.
- **Aperture radius of 2.624 mm** for a shunt impedance of 300 MΩ/m with 77K copper walls.

Proposed Design

- A novel DCA rf structure with **3π/4** phase advance between the individually fed cavities.
- **Aperture radius of 3.55 mm** for the same shunt impedance and the peak field constraints.
- Because of this **35% larger aperture**, this rf structure is much more resilient to both short-range and long-range wakefield effects.

What is New in the RF Manifold

Current Implementation of 3π/4 DCA

The researchers at SLAC have already proposed and designed a 3π/4 DCA with **four feeding waveguide manifolds**.

The implementation of four waveguide manifolds is, however, **mechanically challenging**.

Proposed Implementation of 3π/4 DCA

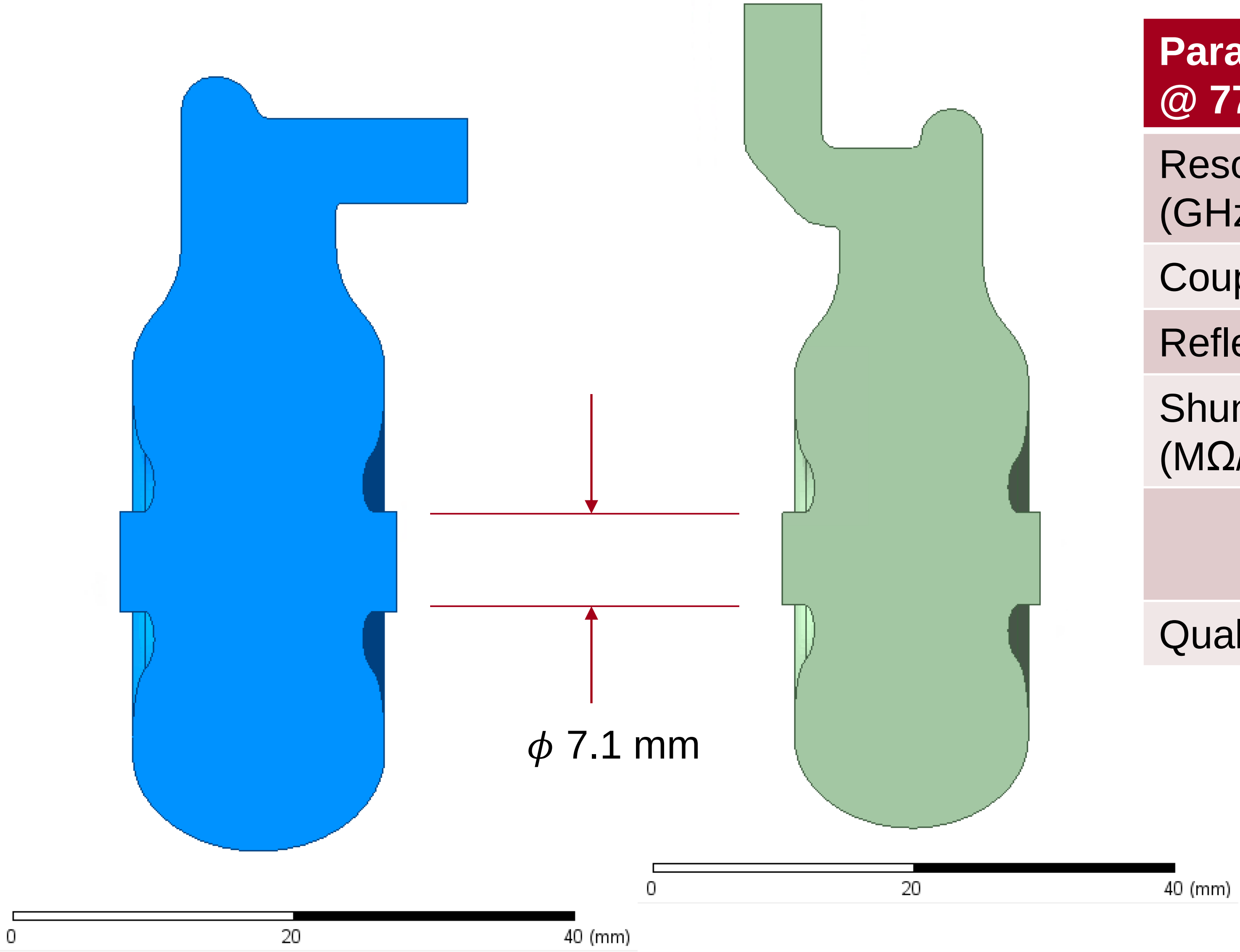
We present a novel 3π/4 DCA for C^3 which is based on only **two waveguide manifolds**.

This rf structure comprises of 56 cavities where cavities are fed in pairs through a standard π phase advance rf manifold. This is achieved by pairing the cavities as, first and third, second and forth, and so on. With such pairing, the phase advance between the two cavities in a pair is π/2 and the phase advance between successive pairs is π.

C^3 Parameters

Parameter	Symbol	$C^3 250$	$C^3 550$
frequency (GHz)	f	5.712	5.712
bunch spacing (rf cycles)	n_{bs}	30	20
bunch charge (nC)	q_b	1	1
bunch length (mm)	σ_z	0.1	0.1
cold shunt impedance (MΩ/m)	r_s	317	317
gradient (V/m)	G	70	120
rf wavelength (mm)	$\lambda = \frac{c}{f}$	52.4847	52.4847
acc. structure length (m)	$L = 56 \times \frac{3}{8} \times \lambda$	1.10218	1.10218
bunch spacing (ns)	$t_{bs} = \frac{n_{bs}}{f}$	5.2521	3.5014
beam current (A)	$I_b = \frac{q_b}{t_{bs}}$	0.1904	0.2856
bunch form factor	$F = e^{-\frac{1}{2}\left(2\pi\frac{\sigma_z}{\lambda}\right)^2}$	0.999928	0.999928
rf power lost in walls (MW)	$P_{walls} = \frac{G^2 L}{r_s}$	17.0368	50.0674
rf power delivered to beam (MW)	$P_b = F I_b G L$	14.6888	37.7711
optimal coupling coefficient	$\beta^* = 1 + \frac{P_b}{P_{rf}} = 1 + \frac{F I_b r_s}{G}$	1.86218	1.75441
designed coupling coefficient	β	1.86218	1.86218
coupling coeff. at room temp.	$\beta_{RT} = 0.38\beta$	0.707628	0.707628
rf power reflected (MW)	$P_{ref} = P_{walls} \frac{\beta^* \left((\beta-1)\sqrt{\beta^*} - (\beta^*-1)\sqrt{\beta}\right)^2}{(\beta+1)^2 \beta^* - \left((\beta-1)\sqrt{\beta^*} - (\beta^*-1)\sqrt{\beta}\right)^2}$	0.	0.0774395
total rf power required (MW)	$P_{tot} = P_{walls} + P_b + P_{ref}$	31.7256	87.916
rf power per unit length (MW)/m	P_{tot}/L	28.7845	79.7657

Cavity (with two different coupler implementations) Parameters Determined Through HFSS® Simulations



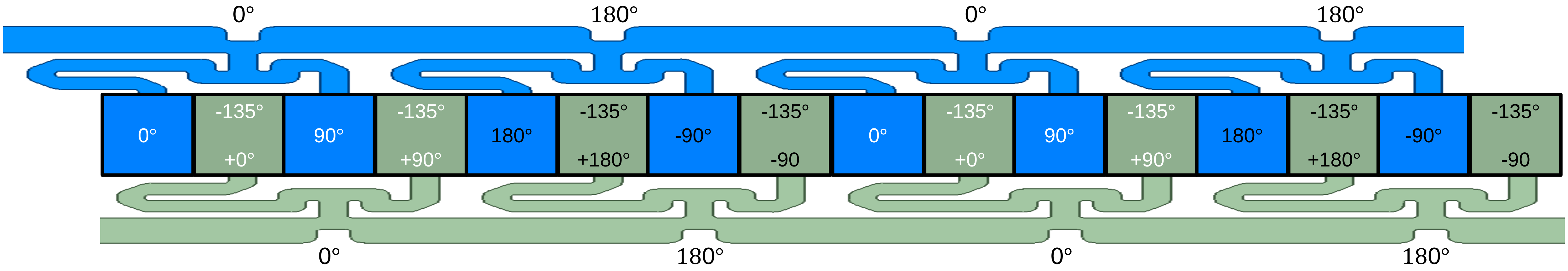
Parameters @ 77 K	Cavity Left	Cavity Right
Resonant Frequency, f_0 (GHz)	5.712	5.712
Coupling Coefficient, β	1.8625	1.8634
Reflection Coefficient, Γ	0.3013	0.3015
Shunt Impedance, r_s (MΩ/m)	317.22	317.22
$\frac{\eta_0 H_{surf\,peak}}{Gradient}$	1.14	1.14
Quality Factor, Q_0	30124	30122

Cavity Phases in an Accelerator Made of 135° Phase Advance Cavities ($e^{i\omega t}$ convention)

Beam Direction

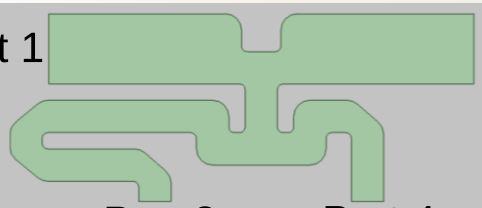
0°	-135°	-270°	-405°	-540°	-675°	-810°	-945°	-1080°	-1215°	-1350°	-1485°	-1620°	-1755°	-1890°	-2025°

Equivalent phases and Feeding Scheme:



There has to be a phase difference of 135° between the upper and lower rf feeding manifold.

The General S-Parameter for Four-Port Network That Makes a Unit of This Distributed Feed (n_0: Number of Networks to be Cascaded in Each Manifold)



Port 3 feeds n^{th} cavity at ϕ_{c_n} phase.

Port 4 feeds $(n+1)^{th}$ cavity at $\phi_{c_{n+1}} = \phi_{c_n} - 135° \times 2 = \phi_{c_n} + 90°$ phase.

$$S = -\frac{1}{2n_0+1}\begin{pmatrix} 1 & 2n_0 & -\sqrt{2n_0} & -i\sqrt{2n_0} \\ 2n_0 & 1 & \sqrt{2n_0} & i\sqrt{2n_0} \\ -\sqrt{2n_0} & \sqrt{2n_0} & \frac{1}{2}\left(2n_0 - 1 + e^{i\phi}(2n_0+1)\right) & \frac{i}{2}\left(2n_0 - 1 - e^{i\phi}(2n_0+1)\right) \\ -i\sqrt{2n_0} & i\sqrt{2n_0} & \frac{i}{2}\left(2n_0 - 1 - e^{i\phi}(2n_0+1)\right) & -\frac{1}{2}\left(2n_0 - 1 + e^{i\phi}(2n_0+1)\right) \end{pmatrix}.$$

Here, ϕ is an arbitrary phase factor.

- A compact four-port network is designed to achieve S_{11}, S_{12}, S_{13} and S_{14}. Then ϕ is determined as,

$$\phi = Arg[-(S_{33} + iS_{34})] = Arg[-iS_{43} - S_{44}].$$

- For tuning purpose, the reflection coefficient at ports 3 or 4 of a detuned cavity should be $e^{i\theta}$ where $\theta = -(\phi + k\pi)/2$ and k is an odd integer.

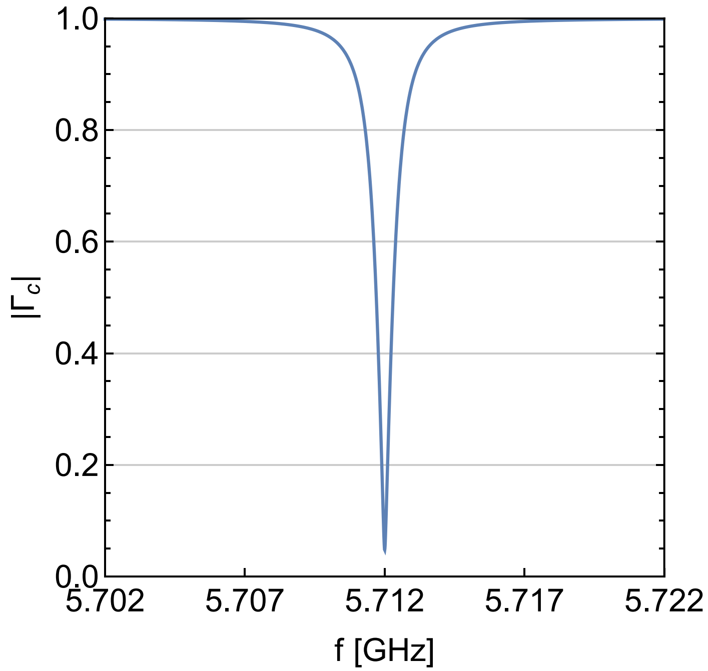
- The guide length l_c (for ports 3 and 4) to the plane where cavity presents a reflection coefficient of -1 when detuned, is determined as:

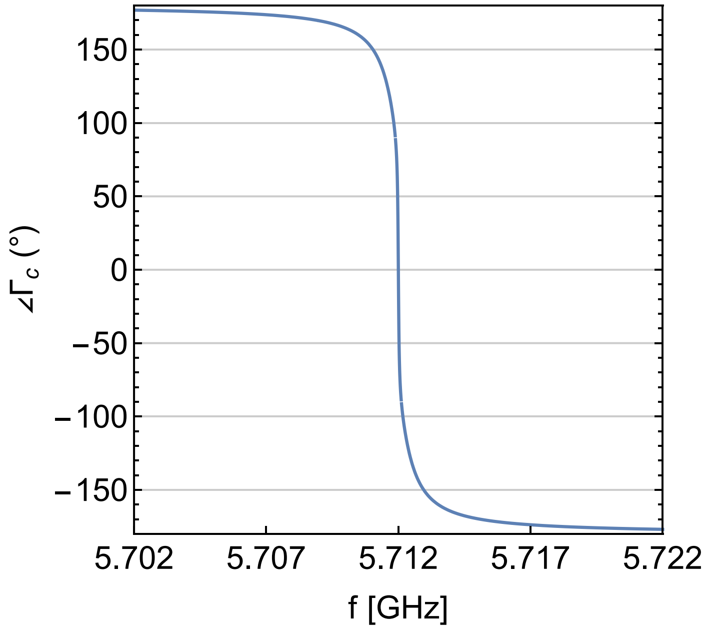
$$l_c = \frac{m\pi - \theta}{4\pi}\lambda_g.$$

Here, m is an odd integer and λ_g is guide wavelength of port 3 and 4.

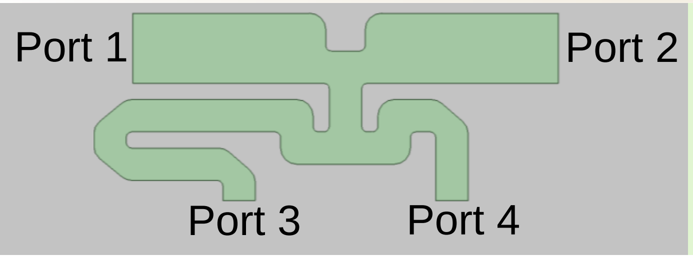
A Potential CCC Cavity Response

Here is the reflection coefficient from a cavity resonant at 5.712 GHz with intrinsic quality factor of 11,500 and coupling coefficient of 1.1.



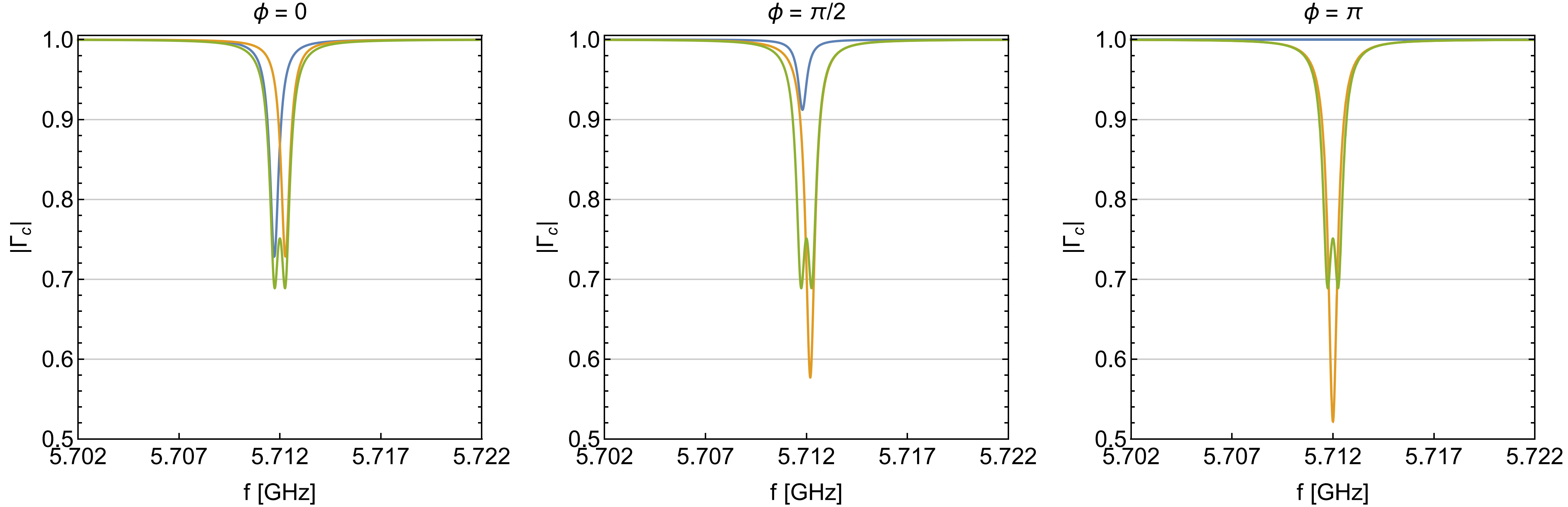


Response of Cavities Through the Manifold (for Tuning Purpose)



When all cavities in the manifold are detuned except, possibly, those connected to port 3 and/or 4 of one of the four-port unit networks then the reflection coefficient at Port 2 of that network is 1.

The graphs below show the response of the cavities as seen through Port 1 when one or none of them are detuned for various values of ϕ.

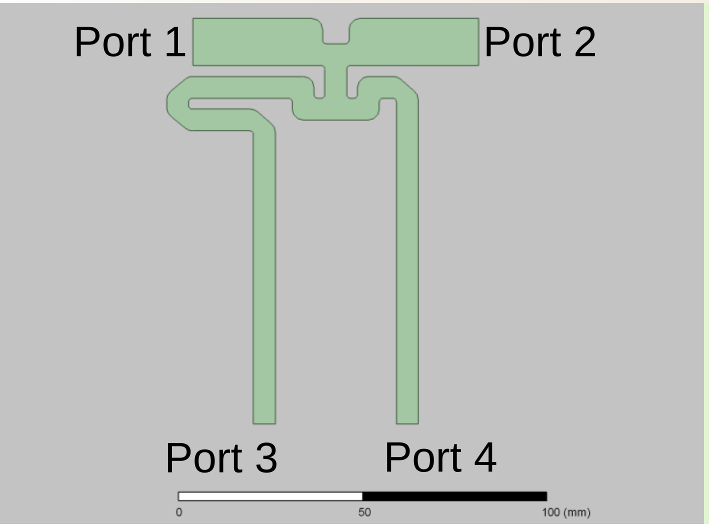


-- Blue: Response of cavity at port 3 when cavity at port 4 is detuned

-- Orange: Response of cavity at port 4 when cavity at port 3 is detuned

-- Green: Response of both cavities at port 3 and port 4

Design of Four-Port Network



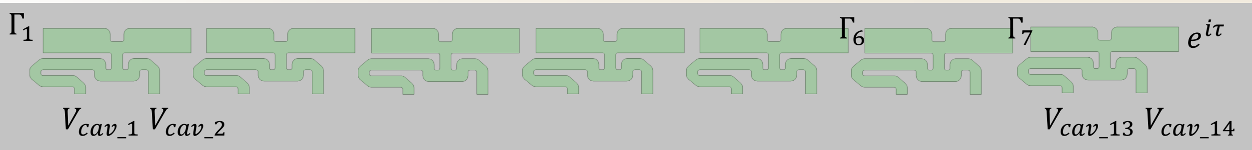
$$S = -\frac{1}{2n_0+1}\begin{pmatrix} 1 & 2n_0 & -\sqrt{2n_0} & -i\sqrt{2n_0} \\ 2n_0 & 1 & \sqrt{2n_0} & i\sqrt{2n_0} \\ -\sqrt{2n_0} & \sqrt{2n_0} & \frac{1}{2}\left(2n_0 - 1 + e^{i\phi}(2n_0+1)\right) & \frac{i}{2}\left(2n_0 - 1 - e^{i\phi}(2n_0+1)\right) \\ -i\sqrt{2n_0} & i\sqrt{2n_0} & \frac{i}{2}\left(2n_0 - 1 - e^{i\phi}(2n_0+1)\right) & -\frac{1}{2}\left(2n_0 - 1 + e^{i\phi}(2n_0+1)\right) \end{pmatrix}.$$

Here, ϕ is an arbitrary phase factor.

- The cavity length for 135° phase advance structure is 19.68 mm. We intend to make a 1.1 m structure using 56 of these cavities. Using a four-way splitter, we will use four arms of distributed feed with $n_0 = 7$, where each four-port network unit feeds two cavities. Thus, 4 x 7 x 2 = 56.
- The above figure shows view of the symmetry plan of a network that was designed in HFSS ® with $n_0 = 7$. We achieved $\phi = -107.8°$. The magnitudes and phases of the ideal (top entry) and achieved (bottom entry) S-matrix are given below.

$$|S| = \begin{pmatrix} \binom{0.0667}{0.0667} & \binom{0.9333}{0.9335} & \binom{0.2494}{0.2503} & \binom{0.2494}{0.2481} \\ \binom{0.9333}{0.9335} & \binom{0.0667}{0.0664} & \binom{0.2494}{0.2482} & \binom{0.2494}{0.2502} \\ \binom{0.2494}{0.2503} & \binom{0.2494}{0.2482} & \binom{0.5527}{0.5512} & \binom{0.755}{0.7563} \\ \binom{0.2494}{0.2481} & \binom{0.2494}{0.2502} & \binom{0.755}{0.7563} & \binom{0.5527}{0.5513} \end{pmatrix}, \quad \angle S(°) = \begin{pmatrix} \binom{180.}{-178.4} & \binom{180.}{179.9} & \binom{0.}{0.} & \binom{90.}{90.} \\ \binom{180.}{179.9} & \binom{180.}{-178.3} & \binom{180.}{179.7} & \binom{-90.}{-89.6} \\ \binom{0.}{0.} & \binom{180.}{179.7} & \binom{120.5}{120.3} & \binom{-50.9}{-50.8} \\ \binom{90.}{90.} & \binom{-90.}{-89.6} & \binom{-50.9}{-50.8} & \binom{-59.5}{-59.} \end{pmatrix}.$$

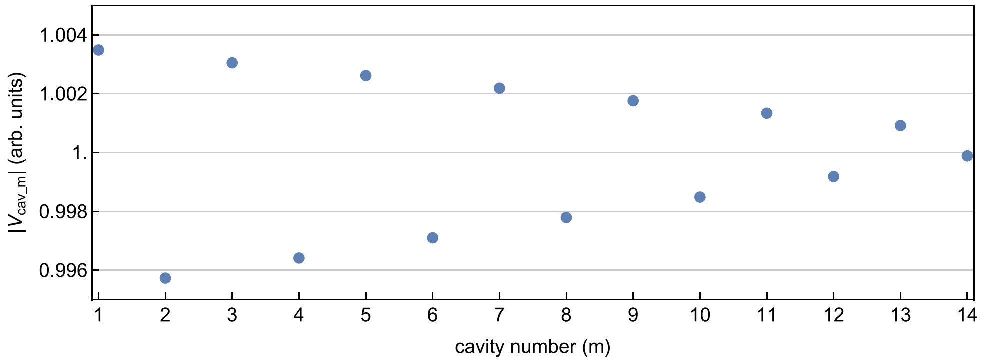
The Results of Analytical Cascading of 7 of These Designed Four-Port Networks

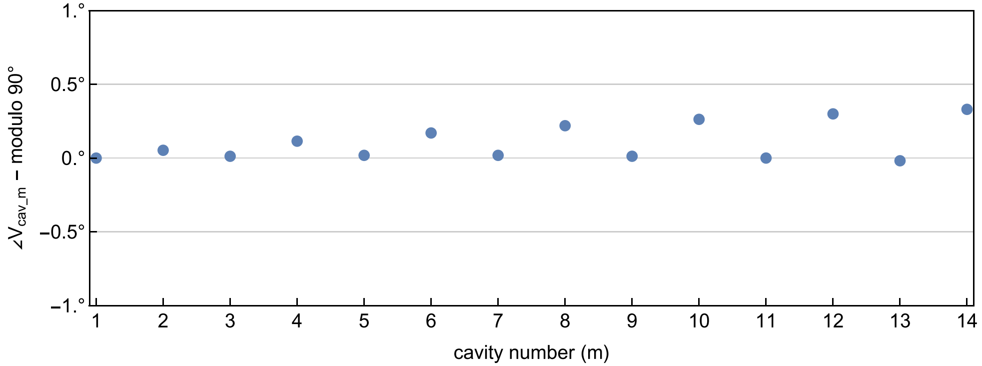


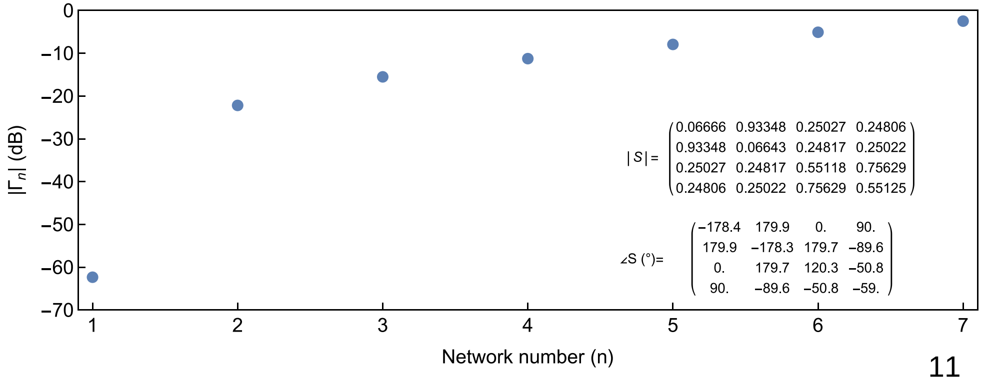
The above figure is a depiction of analytical cascading where the last one is truncated with a reflection coefficient of $e^{i\tau}$, where $\tau = 6.3355$ for minimum overall reflection Γ_1. Here we assume no reflection from cavity ports.

The top two figures on the right show the stability of the relative cavity voltages and phases.

The bottom figure on the right shows the reflection coefficient at the entrance of each network. Note that this 7 x 2 distributed feed is well matched with overall reflection coefficient of less than -60 dB.

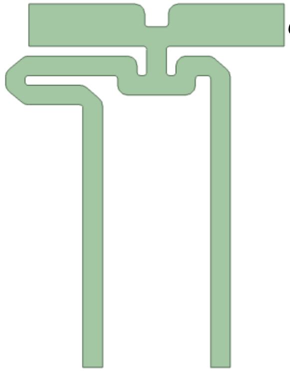




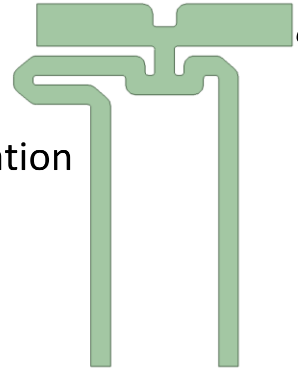


Implementation of a Compact Termination for the Last Network in a Right Going 7x2 Manifold

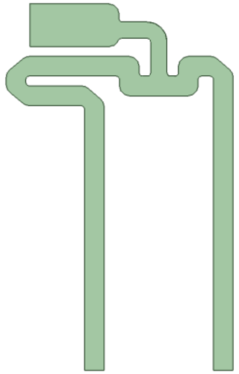
I: Ideal network with termination of reflection coefficient 1.



II: Implemented network with termination for minimum overall reflection.



III: Compact Implementation

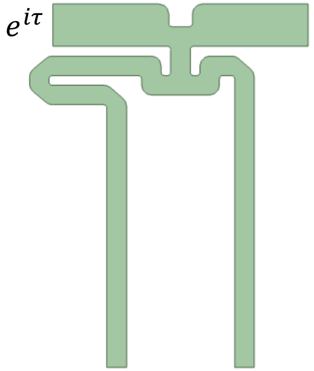


The magnitudes and phases of the resultant 3x3 S-matrix corresponding to I (top entry), II (middle entry) and III (bottom entry) are given below.

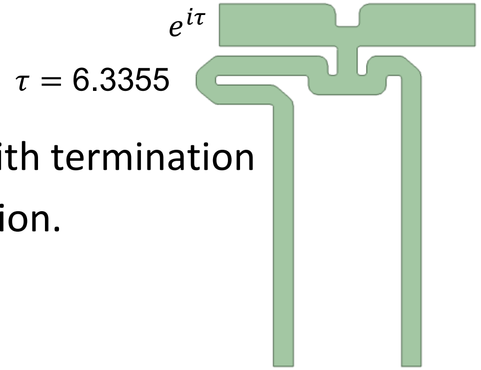
$$|S| = \begin{pmatrix} \begin{pmatrix}0.7500\\0.7506\\0.7504\end{pmatrix} & \begin{pmatrix}0.4677\\0.4675\\0.4673\end{pmatrix} & \begin{pmatrix}0.4677\\0.4670\\0.4674\end{pmatrix} \\ \begin{pmatrix}0.4677\\0.4675\\0.4673\end{pmatrix} & \begin{pmatrix}0.5255\\0.5263\\0.5392\end{pmatrix} & \begin{pmatrix}0.7107\\0.7103\\0.7007\end{pmatrix} \\ \begin{pmatrix}0.4677\\0.4670\\0.4674\end{pmatrix} & \begin{pmatrix}0.7107\\0.7103\\0.7007\end{pmatrix} & \begin{pmatrix}0.5255\\0.5267\\0.5391\end{pmatrix} \end{pmatrix}.$$

$$\angle S(^\circ) = \begin{pmatrix} \begin{pmatrix}0.0\\2.6\\2.7\end{pmatrix} & \begin{pmatrix}0.0\\1.1\\1.0\end{pmatrix} & \begin{pmatrix}90.0\\91.4\\91.4\end{pmatrix} \\ \begin{pmatrix}0.0\\1.1\\1.0\end{pmatrix} & \begin{pmatrix}115.0\\114.7\\116.0\end{pmatrix} & \begin{pmatrix}-47.9\\-48.0\\-46.9\end{pmatrix} \\ \begin{pmatrix}90.0\\91.4\\91.4\end{pmatrix} & \begin{pmatrix}-47.9\\-48.0\\-46.9\end{pmatrix} & \begin{pmatrix}-65.\\-64.7\\-63.3\end{pmatrix} \end{pmatrix}.$$

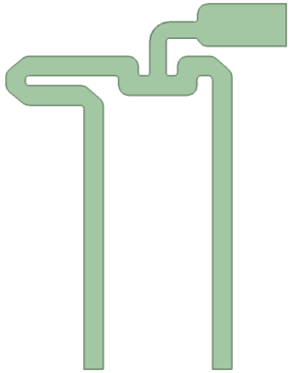
Implementation of a Compact Termination for the Last Network in a Left Going 7x2 Manifold



I: Ideal network with termination of reflection coefficient 1.



II: Implemented network with termination for minimum overall reflection.



III: Compact Implementation

The magnitudes and phases of the resultant 3x3 S-matrix corresponding to I (top entry), II (middle entry) and III (bottom entry) are given below.

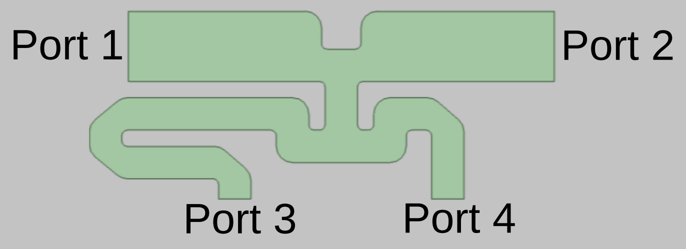
$$|S| = \begin{pmatrix} \begin{pmatrix}0.7500\\0.7507\\0.7505\end{pmatrix} & \begin{pmatrix}0.4677\\0.4671\\0.4673\end{pmatrix} & \begin{pmatrix}0.4677\\0.4673\\0.4673\end{pmatrix} \\ \begin{pmatrix}0.4677\\0.4671\\0.4673\end{pmatrix} & \begin{pmatrix}0.5255\\0.5266\\0.5390\end{pmatrix} & \begin{pmatrix}0.7107\\0.7103\\0.7008\end{pmatrix} \\ \begin{pmatrix}0.4677\\0.4673\\0.4673\end{pmatrix} & \begin{pmatrix}0.7107\\0.7103\\0.7008\end{pmatrix} & \begin{pmatrix}0.5255\\0.5264\\0.5390\end{pmatrix} \end{pmatrix}.$$

$$\angle S(°) = \begin{pmatrix} \begin{pmatrix}0.0\\2.6\\2.8\end{pmatrix} & \begin{pmatrix}180\\-178.9\\-179.0\end{pmatrix} & \begin{pmatrix}-90.0\\-88.6\\-88.6\end{pmatrix} \\ \begin{pmatrix}180\\-178.9\\-179.0\end{pmatrix} & \begin{pmatrix}115.0\\114.6\\116.0\end{pmatrix} & \begin{pmatrix}-47.9\\-48.0\\-46.9\end{pmatrix} \\ \begin{pmatrix}-90.0\\-88.6\\-88.6\end{pmatrix} & \begin{pmatrix}-47.9\\-48.0\\-46.9\end{pmatrix} & \begin{pmatrix}-65.\\-64.6\\-63.3\end{pmatrix} \end{pmatrix}.$$

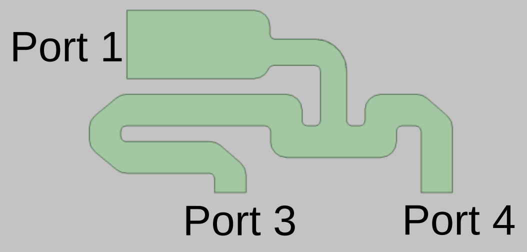
Fixing the lengths of Port 3 and Port 4

For tuning purpose, the guide length l_c (for ports 3 and 4) to the plane where cavity presents a reflection coefficient of -1 when detuned, is determined to be -77.05 mm. As seen here, we have reduced the length of ports 3 and 4 accordingly. The cavities with response equivalent to the one shown in slide # 3 will be attached to ports 3 and 4 as shown here.

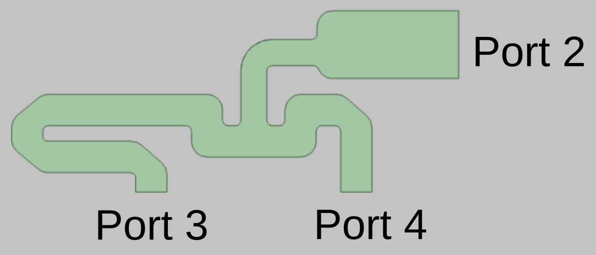
Four-Port Network



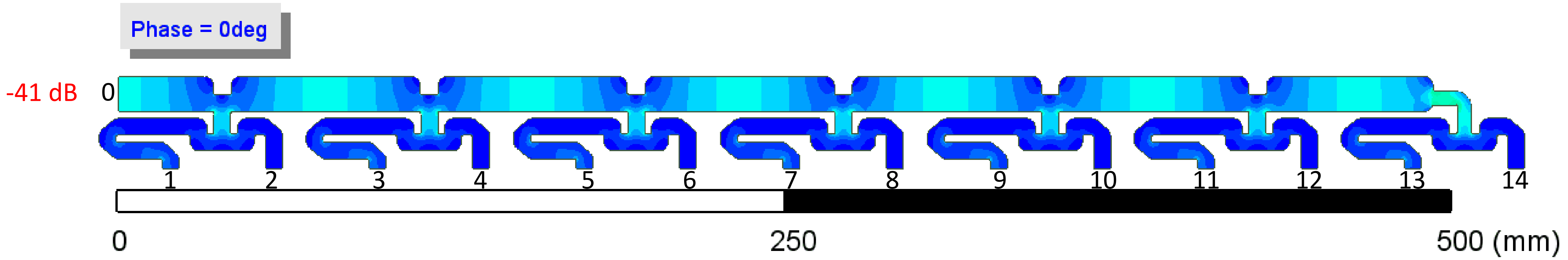
Three-Port Network at the End of Right Going Manifold

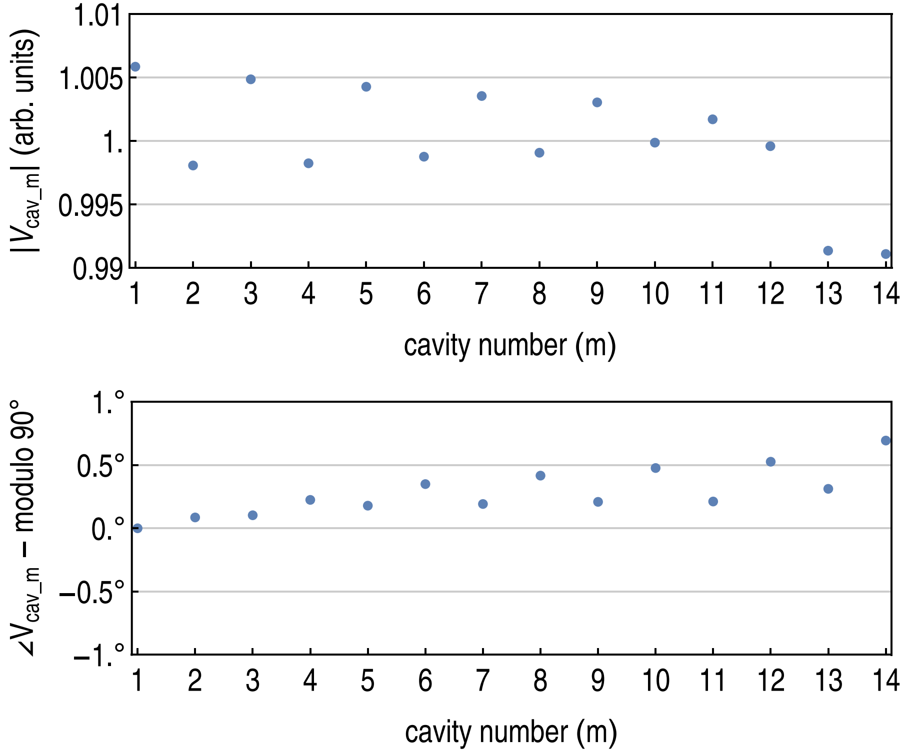


Three-Port Network at the End of Left Going Manifold

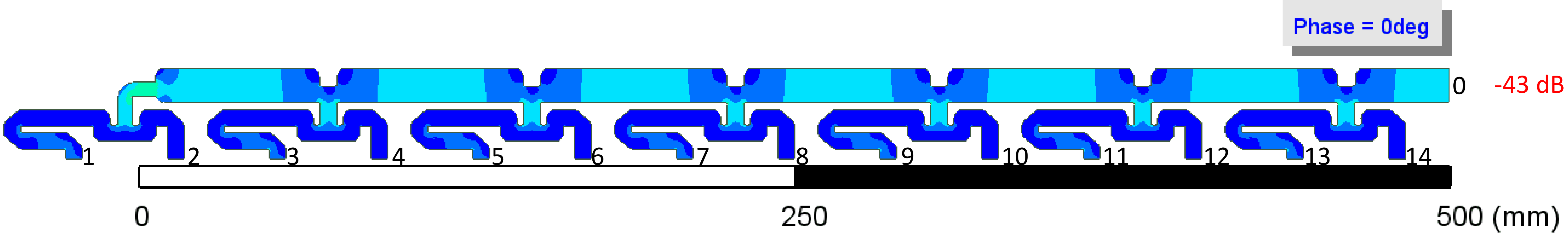


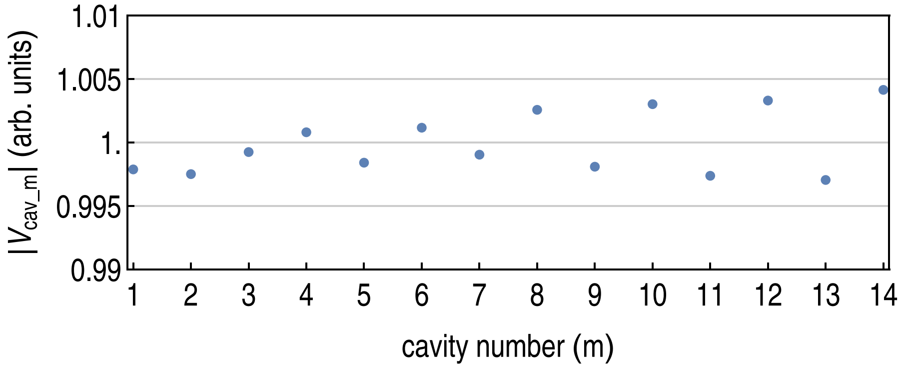
Right Going 7x2 Manifold

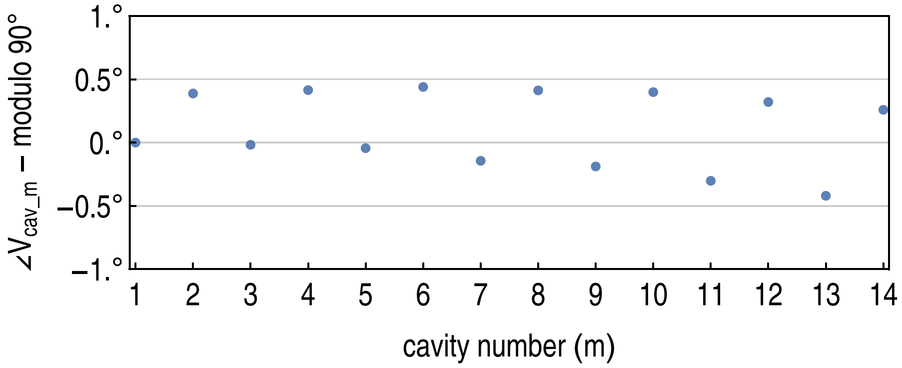




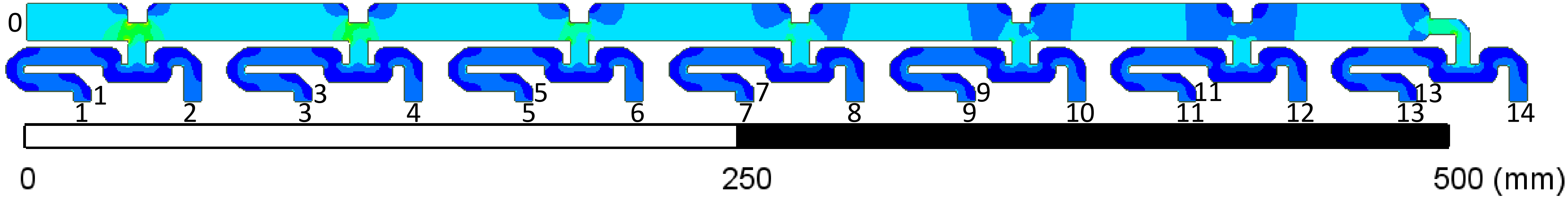
Left Going 7x2 Manifold







2x2 S-Matrix of Right Going 7x2 Manifold When All, Except an Odd Cavity Port # n, are Shorted (Detuned Cavities)



The magnitudes of the resultant 2x2 S-matrix are calculated to be as follows when n is odd.

$$|S| = \begin{pmatrix} 0.9576 & 0.2880 \\ 0.2880 & 0.9576 \end{pmatrix}.$$

The HFSS simulation of a particular such scenario yielded following magnitudes of the 2x2 S-matrix. Here the upper to lower values correspond to $n = 1,3,5,7,9,11,13$, respectively.

$$|S| = \begin{pmatrix} \begin{pmatrix} 0.9579 \\ 0.9578 \\ 0.9578 \\ 0.9577 \\ 0.9576 \\ 0.9577 \\ 0.9555 \end{pmatrix} & \begin{pmatrix} 0.2870 \\ 0.2873 \\ 0.2875 \\ 0.2877 \\ 0.2880 \\ 0.2878 \\ 0.2949 \end{pmatrix} \\ \begin{pmatrix} 0.2870 \\ 0.2873 \\ 0.2875 \\ 0.2877 \\ 0.2880 \\ 0.2878 \\ 0.2949 \end{pmatrix} & \begin{pmatrix} 0.9579 \\ 0.9578 \\ 0.9578 \\ 0.9577 \\ 0.9576 \\ 0.9577 \\ 0.9555 \end{pmatrix} \end{pmatrix}.$$

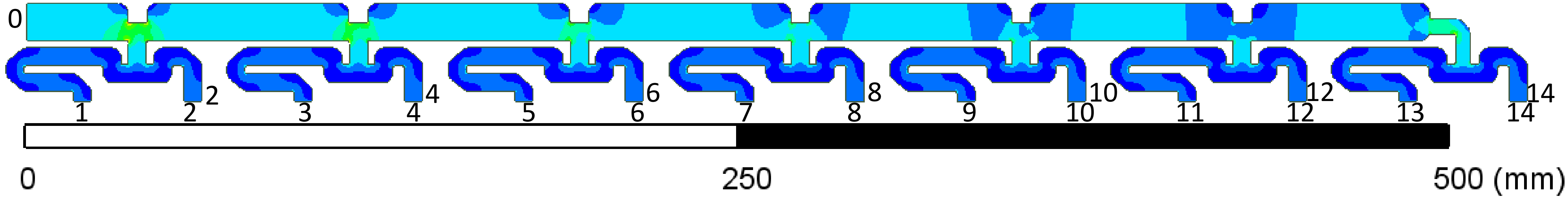
The phases of the resultant 2x2 S-matrix are calculated to be as follows when n is odd. Here, the upper values correspond to $n = 1,5,9,13$ while the lower values correspond to $n = 3,7,11$.

$$\angle S(°) = \begin{pmatrix} 1.6 & \begin{pmatrix} 122.2 \\ -57.8 \end{pmatrix} \\ \begin{pmatrix} 122.2 \\ -57.8 \end{pmatrix} & 62.8 \end{pmatrix}.$$

The HFSS simulation of a particular such scenario yielded following phases of the 2x2 S-matrix. Here the upper to lower values correspond to $n = 1,3,5,7,9,11,13$, respectively.

$$\angle S(°) = \begin{pmatrix} \begin{pmatrix} 4.1 \\ 4.2 \\ 4.1 \\ 4.2 \\ 4.2 \\ 4.1 \\ 4.0 \end{pmatrix} & \begin{pmatrix} 123.4 \\ -56.5 \\ 123.5 \\ -56.5 \\ 123.5 \\ -56.6 \\ 124.7 \end{pmatrix} \\ \begin{pmatrix} 123.4 \\ -56.5 \\ 123.5 \\ -56.5 \\ 123.5 \\ -56.6 \\ 124.7 \end{pmatrix} & \begin{pmatrix} 62.8 \\ 62.8 \\ 62.8 \\ 62.8 \\ 62.8 \\ 62.8 \\ 65.4 \end{pmatrix} \end{pmatrix}.$$

2x2 S-Matrix of Right Going 7x2 Manifold When All, Except an Even Cavity Port # n, are Shorted (Detuned Cavities)



The magnitudes of the resultant 2x2 S-matrix are calculated to be as follows when n is even.

$$|S| = \begin{pmatrix} 0.6807 & 0.7326 \\ 0.7326 & 0.6807 \end{pmatrix}.$$

The HFSS simulation of a particular such scenario yielded following magnitudes of the 2x2 S-matrix. Here the upper to lower values correspond to $n = 1,3,5,7,9,11,13$, respectively.

$$|S| = \begin{pmatrix} \begin{pmatrix} 0.6834 \\ 0.6830 \\ 0.6826 \\ 0.6819 \\ 0.6814 \\ 0.6807 \\ 0.6869 \end{pmatrix} & \begin{pmatrix} 0.7300 \\ 0.7305 \\ 0.7308 \\ 0.7314 \\ 0.7319 \\ 0.7326 \\ 0.7267 \end{pmatrix} \\ \begin{pmatrix} 0.7300 \\ 0.7305 \\ 0.7308 \\ 0.7314 \\ 0.7319 \\ 0.7326 \\ 0.7267 \end{pmatrix} & \begin{pmatrix} 0.6834 \\ 0.6830 \\ 0.6826 \\ 0.6819 \\ 0.6814 \\ 0.6807 \\ 0.6869 \end{pmatrix} \end{pmatrix}.$$

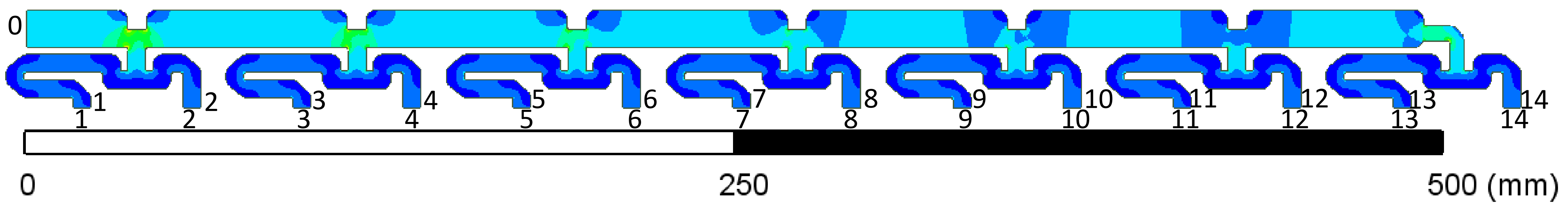
The phases of the resultant 2x2 S-matrix are calculated to be as follows when n is odd. Here, the upper values correspond to $n = 2,6,10,14$ while the lower values correspond to $n = 4,8,12$.

$$\angle S(°) = \begin{pmatrix} 16.0 & \begin{pmatrix} -112.8 \\ 67.2 \end{pmatrix} \\ \begin{pmatrix} -112.8 \\ 67.2 \end{pmatrix} & -61.7 \end{pmatrix}.$$

The HFSS simulation of a particular such scenario yielded following phases of the 2x2 S-matrix. Here the upper to lower values correspond to $n = 2,4,6,8,10,12,14$, respectively.

$$\angle S(°) = \begin{pmatrix} \begin{pmatrix} 18.4 \\ 18.4 \\ 18.5 \\ 18.6 \\ 18.5 \\ 18.6 \\ 18. \end{pmatrix} & \begin{pmatrix} -111.7 \\ 68.3 \\ -111.6 \\ 68.4 \\ -111.6 \\ 68.5 \\ -111.4 \end{pmatrix} \\ \begin{pmatrix} -111.7 \\ 68.3 \\ -111.6 \\ 68.4 \\ -111.6 \\ 68.5 \\ -111.4 \end{pmatrix} & \begin{pmatrix} -61.9 \\ -61.8 \\ -61.8 \\ -61.7 \\ -61.7 \\ -61.6 \\ -60.8 \end{pmatrix} \end{pmatrix}.$$

3x3 S-Matrix of Right Going 7x2 Manifold When All, Except Two Consecutive Cavity Ports # n & $n+1$, are Shorted (Detuned Cavities) 1/2



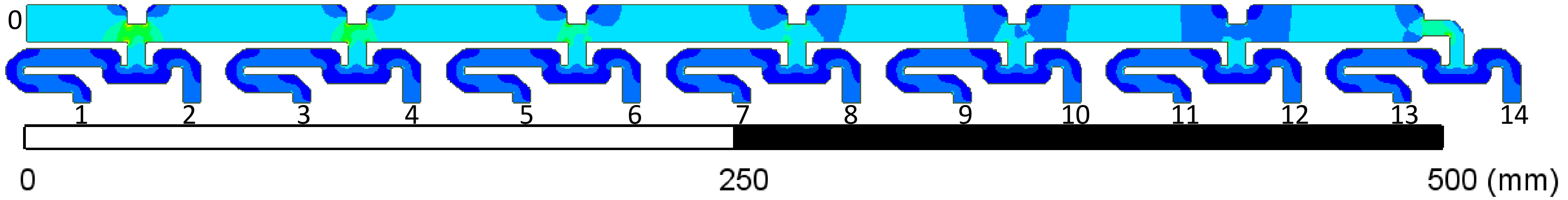
The magnitudes of the resultant 3x3 S-matrix are calculated to be as follows.

$$|S| = \begin{pmatrix} 0.7506 & 0.4675 & 0.4670 \\ 0.4675 & 0.5263 & 0.7103 \\ 0.4670 & 0.7103 & 0.5267 \end{pmatrix}.$$

The HFSS simulation of a particular such scenario yielded following magnitudes of the 3x3 S-matrix. Here the upper to lower values correspond to $n = 1,3,5,7,9,11,13$, respectively.

$$|S| = \begin{pmatrix} \begin{pmatrix} 0.7525 \\ 0.7522 \\ 0.7520 \\ 0.7513 \\ 0.7510 \\ 0.7508 \\ 0.7530 \end{pmatrix} & \begin{pmatrix} 0.4660 \\ 0.4662 \\ 0.4664 \\ 0.4669 \\ 0.4672 \\ 0.4674 \\ 0.4659 \end{pmatrix} & \begin{pmatrix} 0.4654 \\ 0.4657 \\ 0.4658 \\ 0.4664 \\ 0.4667 \\ 0.4668 \\ 0.4648 \end{pmatrix} \\ \begin{pmatrix} 0.4660 \\ 0.4662 \\ 0.4664 \\ 0.4669 \\ 0.4672 \\ 0.4674 \\ 0.4659 \end{pmatrix} & \begin{pmatrix} 0.5278 \\ 0.5273 \\ 0.5272 \\ 0.527 \\ 0.5266 \\ 0.5263 \\ 0.5385 \end{pmatrix} & \begin{pmatrix} 0.7101 \\ 0.7103 \\ 0.7103 \\ 0.7101 \\ 0.7102 \\ 0.7104 \\ 0.7021 \end{pmatrix} \\ \begin{pmatrix} 0.4654 \\ 0.4657 \\ 0.4658 \\ 0.4664 \\ 0.4667 \\ 0.4668 \\ 0.4648 \end{pmatrix} & \begin{pmatrix} 0.7101 \\ 0.7103 \\ 0.7103 \\ 0.7101 \\ 0.7102 \\ 0.7104 \\ 0.7021 \end{pmatrix} & \begin{pmatrix} 0.5283 \\ 0.5278 \\ 0.5277 \\ 0.5275 \\ 0.5271 \\ 0.5267 \\ 0.5395 \end{pmatrix} \end{pmatrix}.$$

3x3 S-Matrix of Right Going 7x2 Manifold When All, Except Two Consecutive Cavity Ports # n & $n+1$, are Shorted (Detuned Cavities) 2/2



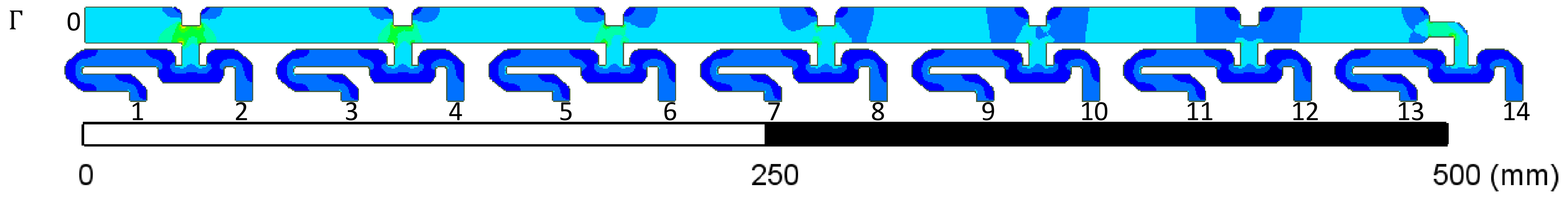
The phases of the resultant 3x3 S-matrix are calculated to be as follows. Here, the upper values correspond to $n = 1,5,9,13$ while the lower values correspond to $n = 3,7,11$.

$$\angle S(°) = \begin{pmatrix} 2.6 & \begin{pmatrix} 163.7 \\ -16.3 \end{pmatrix} & \begin{pmatrix} -106.0 \\ 74.0 \end{pmatrix} \\ \begin{pmatrix} 163.7 \\ -16.3 \end{pmatrix} & 79.9 & -82.8 \\ \begin{pmatrix} -106.0 \\ 74.0 \end{pmatrix} & -82.8 & -99.5 \end{pmatrix}.$$

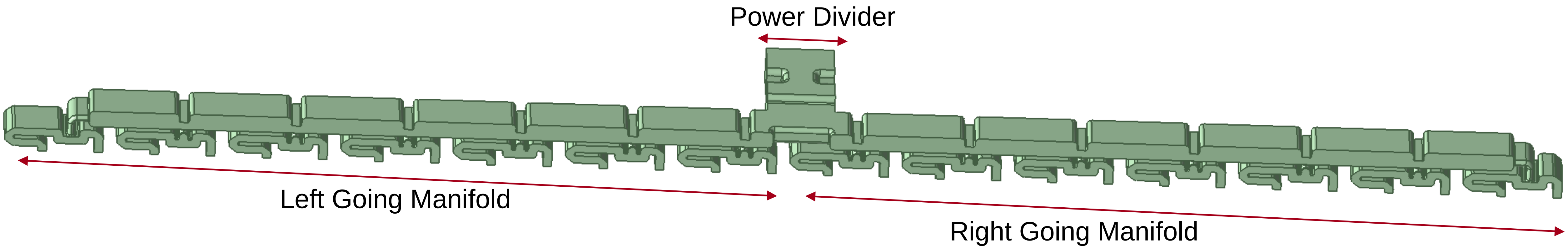
The HFSS simulation of a particular such scenario yielded following phases of the 3x3 S-matrix. Here the upper to lower values correspond to $n = 1,3,5,7,9,11,13$, respectively.

$$\angle S(°) = \begin{pmatrix} \begin{pmatrix} 5.1 \\ 5.2 \\ 5.2 \\ 5.2 \\ 5.2 \\ 5.2 \\ 4.2 \end{pmatrix} & \begin{pmatrix} 164.8 \\ -15.1 \\ 164.9 \\ -15.1 \\ 164.9 \\ -15.1 \\ 164.2 \end{pmatrix} & \begin{pmatrix} -104.8 \\ 75.2 \\ -104.7 \\ 75.2 \\ -104.7 \\ 75.3 \\ -105.5 \end{pmatrix} \\ \begin{pmatrix} 164.8 \\ -15.1 \\ 164.9 \\ -15.1 \\ 164.9 \\ -15.1 \\ 164.2 \end{pmatrix} & \begin{pmatrix} 80. \\ 80. \\ 80. \\ 79.9 \\ 79.9 \\ 79.9 \\ 81.1 \end{pmatrix} & \begin{pmatrix} -82.9 \\ -82.9 \\ -82.9 \\ -82.9 \\ -82.8 \\ -82.8 \\ -82.1 \end{pmatrix} \\ \begin{pmatrix} -104.8 \\ 75.2 \\ -104.7 \\ 75.2 \\ -104.7 \\ 75.3 \\ -105.5 \end{pmatrix} & \begin{pmatrix} -82.9 \\ -82.9 \\ -82.9 \\ -82.9 \\ -82.8 \\ -82.8 \\ -82.1 \end{pmatrix} & \begin{pmatrix} -99.5 \\ -99.4 \\ -99.5 \\ -99.5 \\ -99.5 \\ -99.5 \\ -98.7 \end{pmatrix} \end{pmatrix}.$$

Reflection (Γ) Through the Right Going Manifold (for Tuning Purpose)

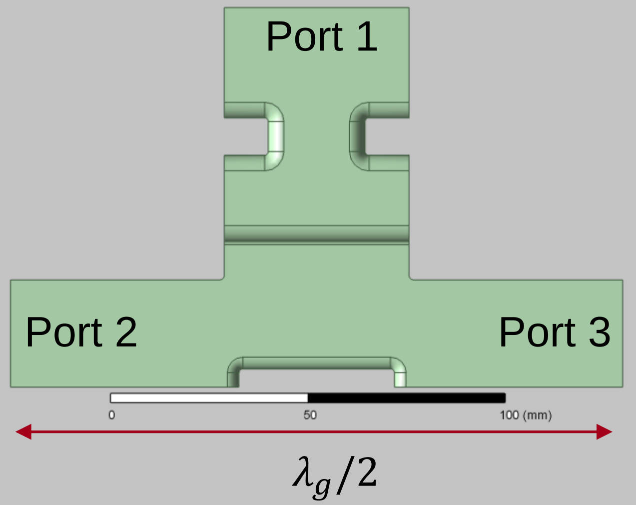


Power Divider Between the Right Going and the Left Going Manifold



Requirements for the Power Divider:

- It has to be implemented as an E-bend divider to naturally give 180° phase difference between the right going and left going manifold.
- The input waveguide must have WR187 cross-section
- For tuning purpose, the S-matrix of the power divider should be of the form shown here on the right (up to an arbitrary phase factor for port 1).



$$S = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}}0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

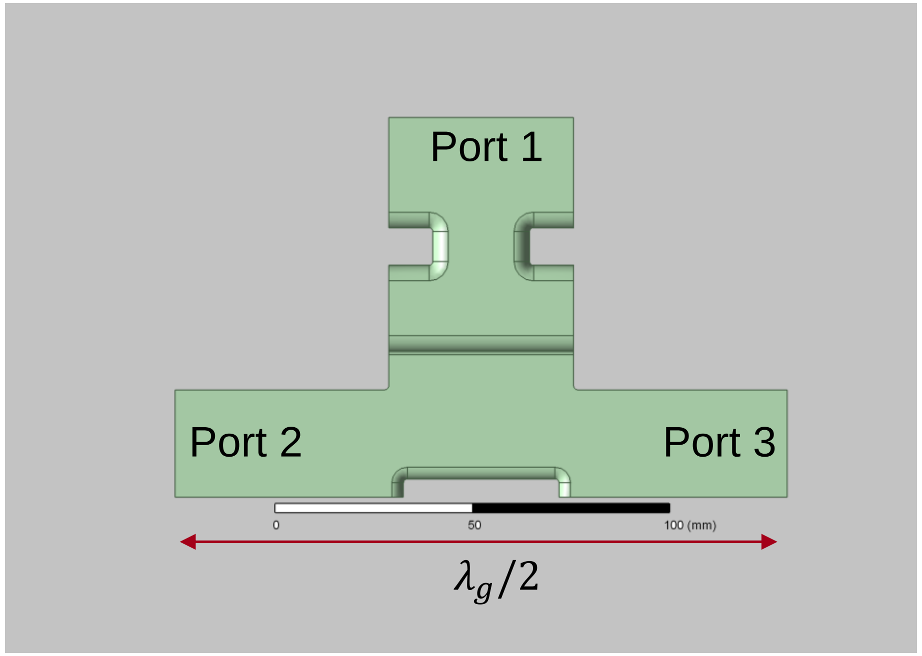
Design of the Power Divider

The magnitudes and phases of the ideal (top entry) and achieved (bottom entry) S-matrix are given below.

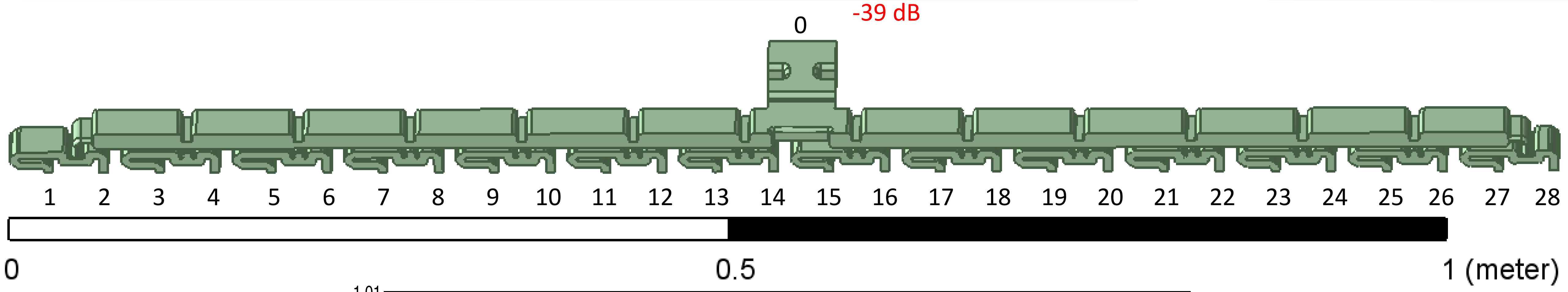
$$|S| = \begin{pmatrix} \binom{0.00000}{0.00082} & \binom{0.70711}{0.70710} & \binom{0.70711}{0.70711} \\ \binom{0.70711}{0.70710} & \binom{0.50000}{0.49972} & \binom{0.50000}{0.50029} \\ \binom{0.70711}{0.70711} & \binom{0.50000}{0.50029} & \binom{0.50000}{0.49971} \end{pmatrix}.$$

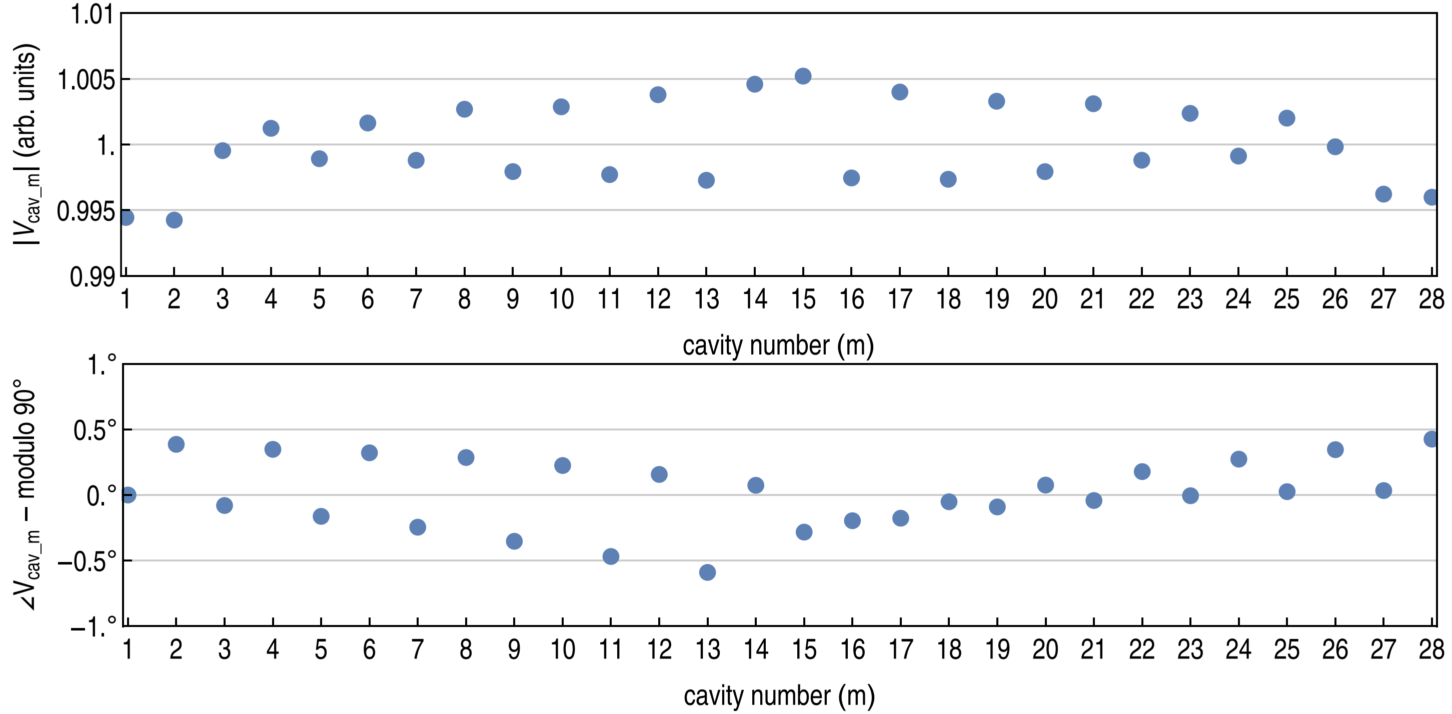
$$\angle S\ (°) = \begin{pmatrix} \binom{0}{-135} & \binom{0}{0} & \binom{0}{0} \\ \binom{0}{0} & \binom{180.0}{179.9} & \binom{0.0}{-0.2} \\ \binom{0}{0} & \binom{0.0}{-0.2} & \binom{180.0}{179.9} \end{pmatrix}.$$

.

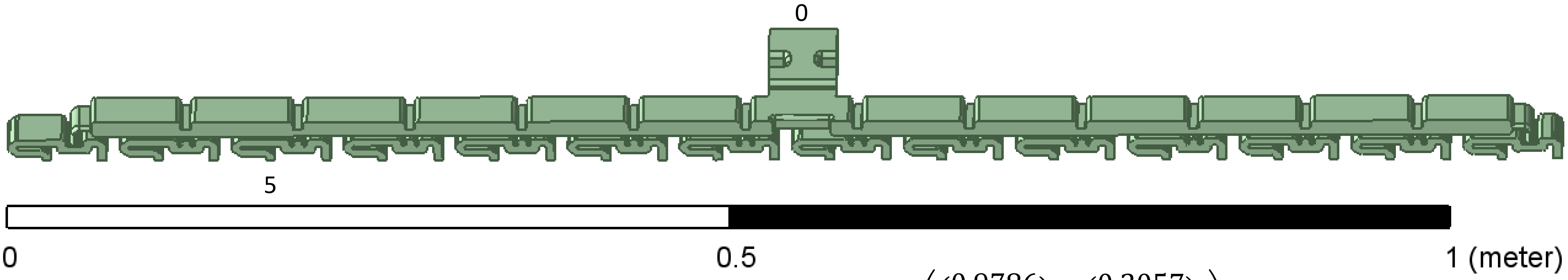


The 2x7x2 Manifold





2x2 S-Matrix of 2x7x2 Manifold
When All, Except Cavity Port # 5, are Shorted (Detuned Cavities)

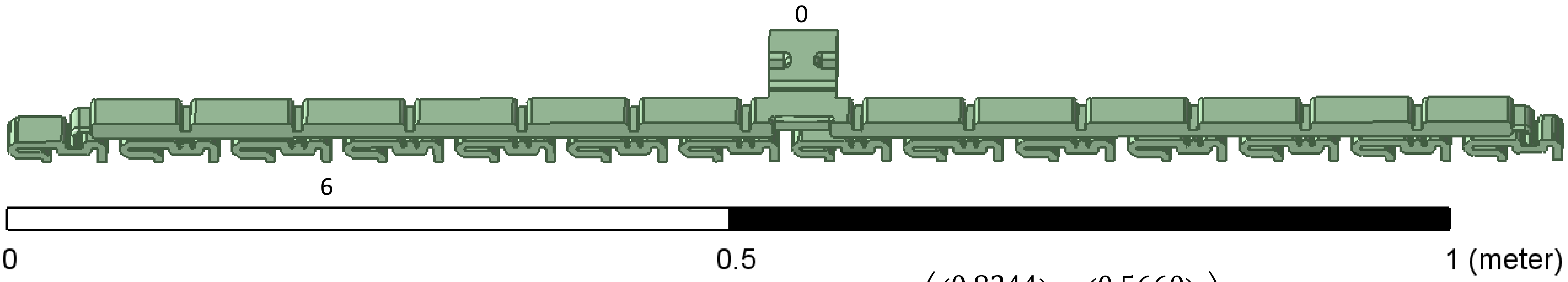


The magnitudes and phases of the calculated (top entry) and simulated (bottom entry) S-matrix are given as follows.

$$|S| = \begin{pmatrix} \begin{pmatrix} 0.9786 \\ 0.9790 \end{pmatrix} & \begin{pmatrix} 0.2057 \\ 0.2037 \end{pmatrix} \\ \begin{pmatrix} 0.2057 \\ 0.2037 \end{pmatrix} & \begin{pmatrix} 0.9786 \\ 0.9790 \end{pmatrix} \end{pmatrix}.$$

$$\angle S\ (^\circ) = \begin{pmatrix} \begin{pmatrix} 42.4 \\ 45.5 \end{pmatrix} & \begin{pmatrix} -37.4 \\ -35.8 \end{pmatrix} \\ \begin{pmatrix} -37.4 \\ -35.8 \end{pmatrix} & \begin{pmatrix} 62.7 \\ 62.8 \end{pmatrix} \end{pmatrix}.$$

2x2 S-Matrix of 2x7x2 Manifold
When All, Except Cavity Port # 6, are Shorted (Detuned Cavities)

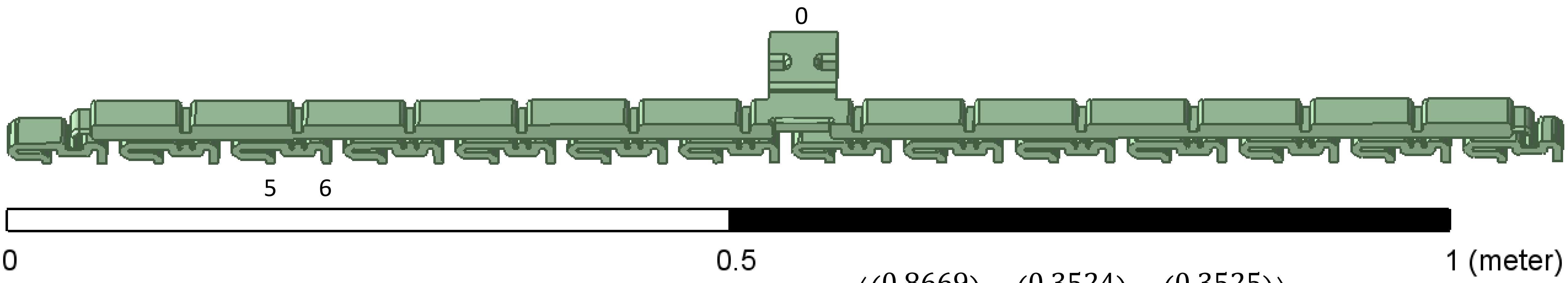


$$|S| = \begin{pmatrix} \begin{pmatrix} 0.8244 \\ 0.8271 \end{pmatrix} & \begin{pmatrix} 0.5660 \\ 0.5621 \end{pmatrix} \\ \begin{pmatrix} 0.5660 \\ 0.5621 \end{pmatrix} & \begin{pmatrix} 0.8244 \\ 0.8271 \end{pmatrix} \end{pmatrix}.$$

The magnitudes and phases of the calculated (top entry) and simulated (bottom entry) S-matrix are given as follows.

$$\angle S\ (°) = \begin{pmatrix} \begin{pmatrix} 49.5 \\ 52.4 \end{pmatrix} & \begin{pmatrix} 85.0 \\ 86.6 \end{pmatrix} \\ \begin{pmatrix} 85 \\ 86.6 \end{pmatrix} & \begin{pmatrix} -59.5 \\ -59.1 \end{pmatrix} \end{pmatrix}$$

2x2 S-Matrix of 2x7x2 Manifold
When All, Except Cavity Port # 5 and 6, are Shorted (Detuned Cavities)

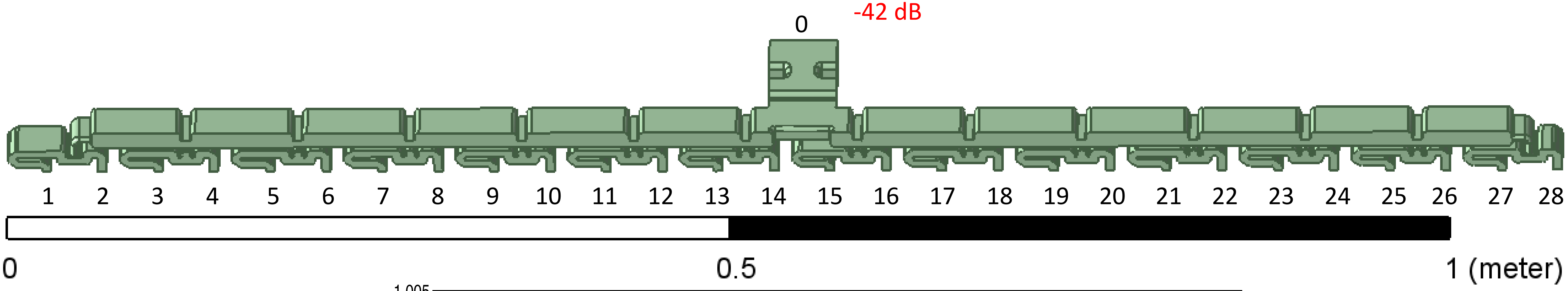


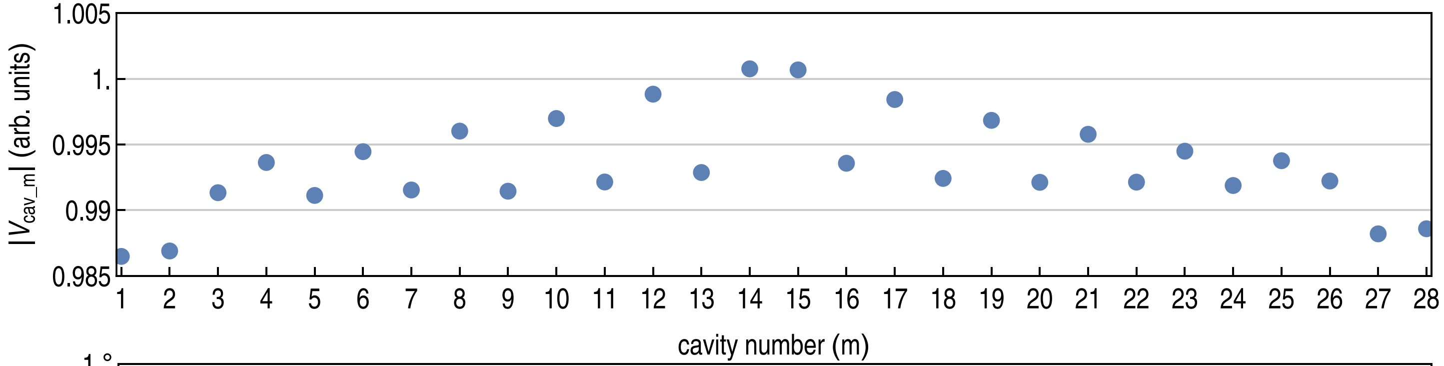
The magnitudes and phases of the calculated (top entry) and simulated (bottom entry) S-matrix are given as follows.

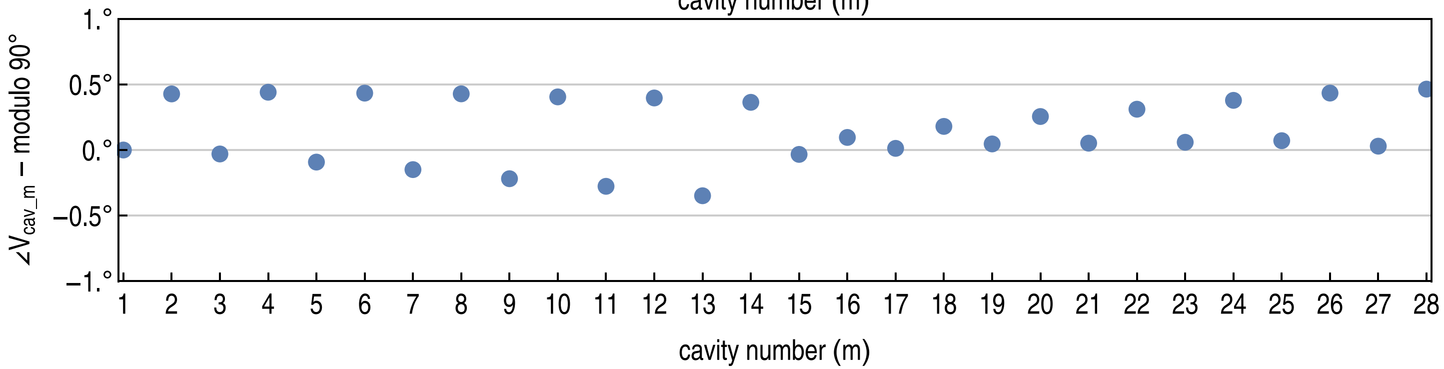
$$|S| = \begin{pmatrix} \begin{pmatrix}0.8669\\0.8694\end{pmatrix} & \begin{pmatrix}0.3524\\0.3493\end{pmatrix} & \begin{pmatrix}0.3525\\0.3495\end{pmatrix} \\ \begin{pmatrix}0.3524\\0.3493\end{pmatrix} & \begin{pmatrix}0.5526\\0.552\end{pmatrix} & \begin{pmatrix}0.7553\\0.7571\end{pmatrix} \\ \begin{pmatrix}0.3525\\0.3495\end{pmatrix} & \begin{pmatrix}0.7553\\0.7571\end{pmatrix} & \begin{pmatrix}0.5525\\0.5519\end{pmatrix} \end{pmatrix}.$$

$$\angle S\ (°) = \begin{pmatrix} \begin{pmatrix}42.9\\46.0\end{pmatrix} & \begin{pmatrix}3.9\\5.5\end{pmatrix} & \begin{pmatrix}94.3\\95.9\end{pmatrix} \\ \begin{pmatrix}3.9\\5.5\end{pmatrix} & \begin{pmatrix}85.3\\85.5\end{pmatrix} & \begin{pmatrix}-85.7\\-85.6\end{pmatrix} \\ \begin{pmatrix}94.3\\95.9\end{pmatrix} & \begin{pmatrix}-85.7\\-85.6\end{pmatrix} & \begin{pmatrix}-93.9\\-93.6\end{pmatrix} \end{pmatrix}.$$

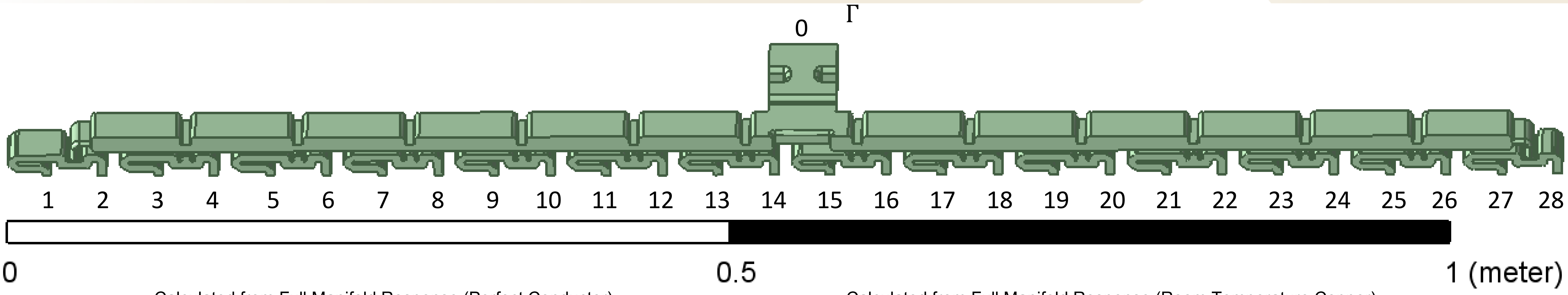
The 2x7x2 Manifold (Cold Copper $\sigma = 2.5^2 \times 5.8 \times 10^7\ S/m$)

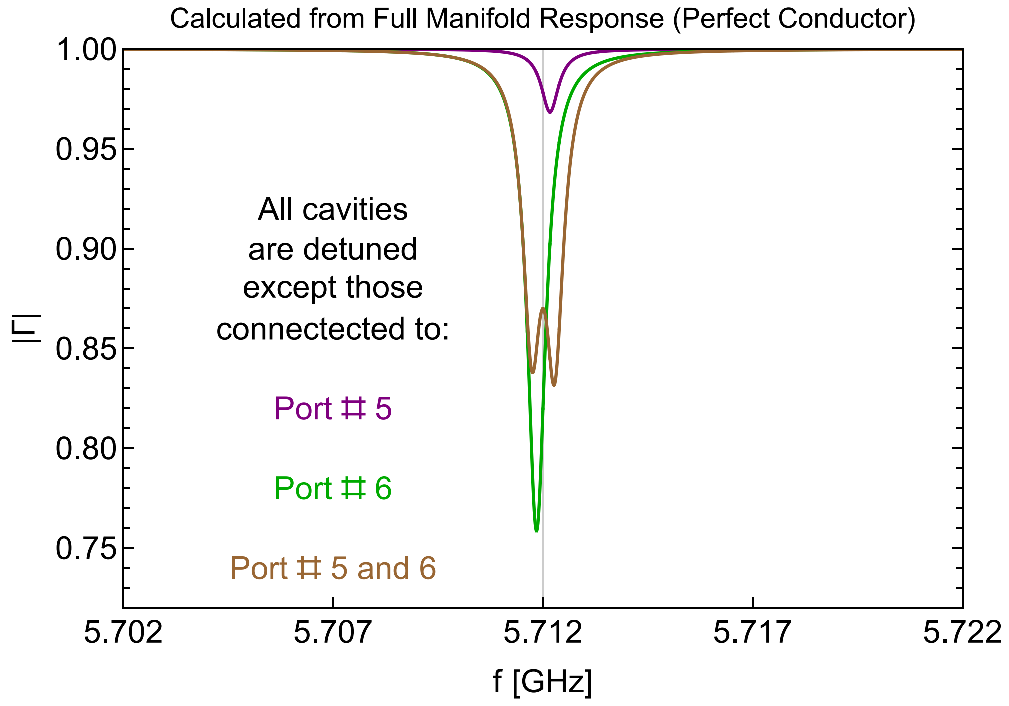


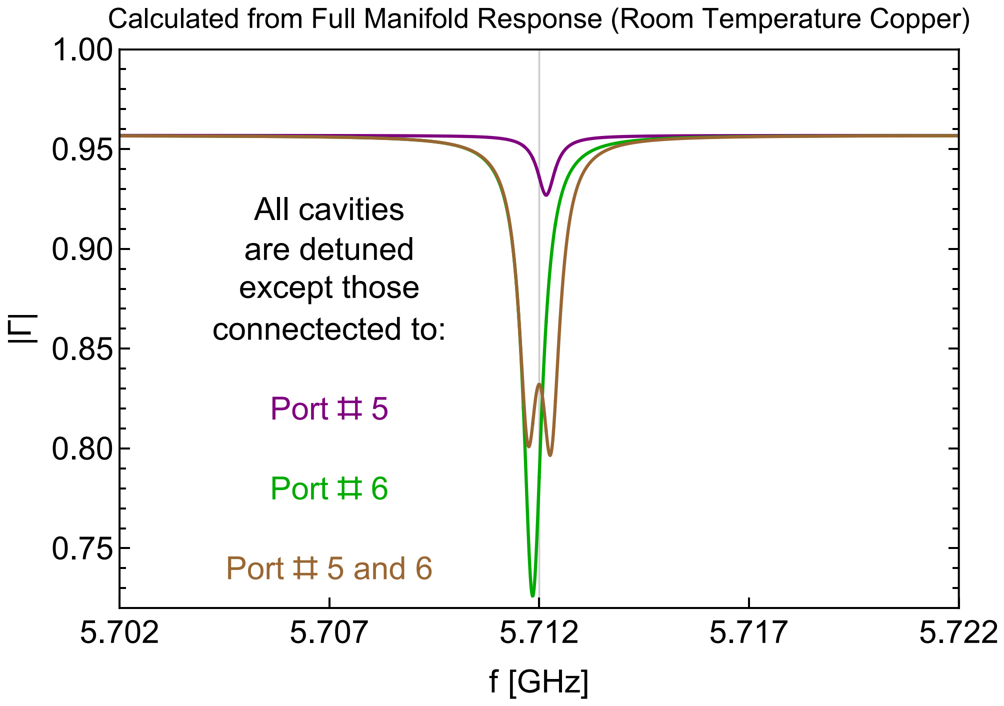




Reflection (Γ) Through the 2x7x2 Manifold (Room Temperature Copper, $\sigma = 5.8 \times 10^7\ S/m$, for Tuning Purpose)



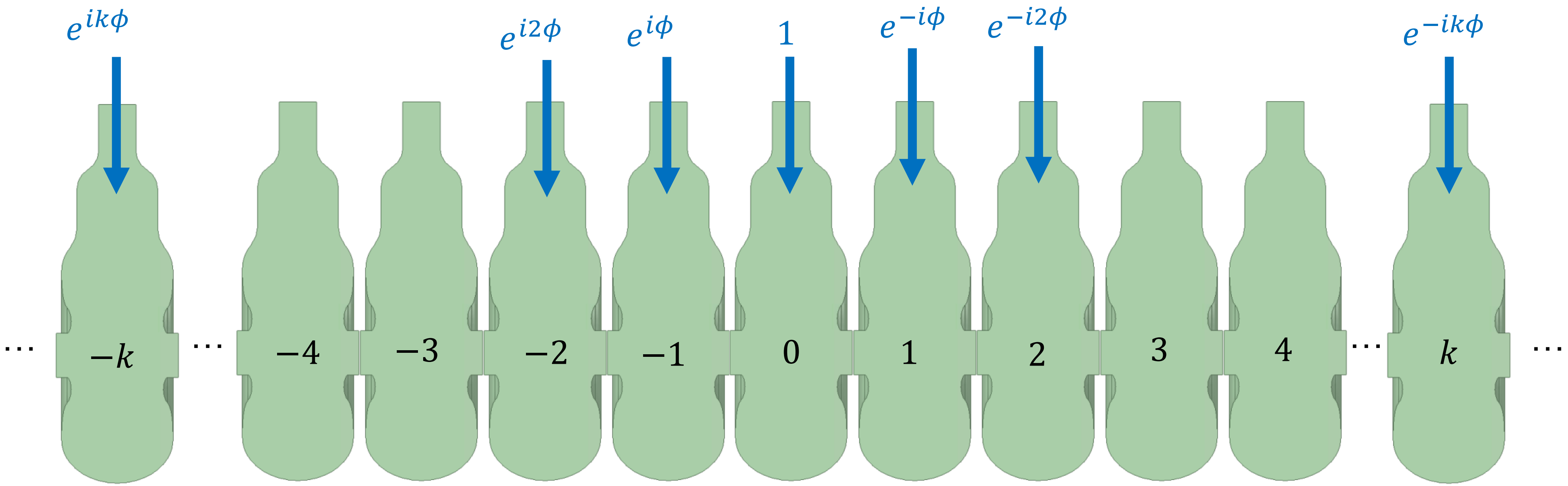




Next Work: Theory of Periodic Coupled Microwave Cavities

Each cavity is fed with rf signals of equal amplitude but phased advanced by $-\phi$ with respect to the previous.

We have derived the inter-cavity coupling coefficients from the measurements of the reflected signal from only a single cavity corresponding to only three values of ϕ: 0, $\pi/2$, and π. We are working on this theory to guide the design of the end cavities for a uniform amplitude and periodic phase excitation even in the case of strong inter-cavity coupling.



Thanks

Questions are welcome!