W-boson pair production at lepton colliders in the Feynman-Diagram gauge

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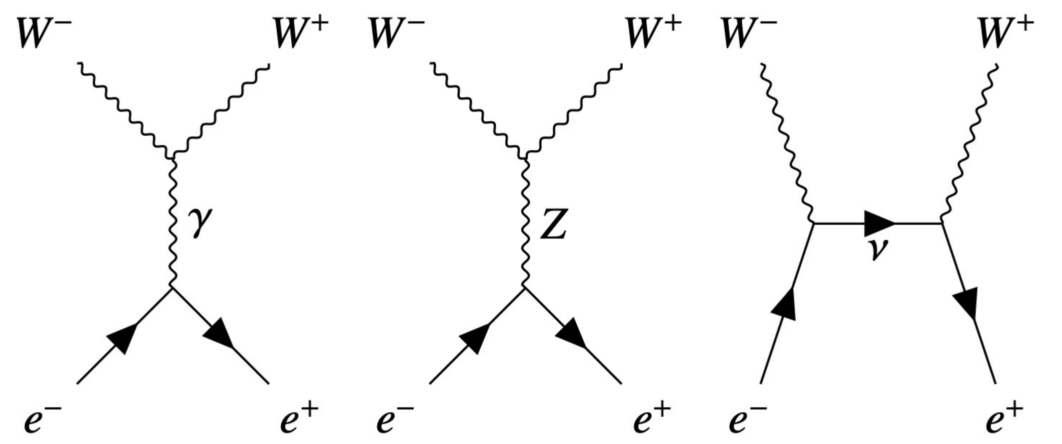
1. Introduction



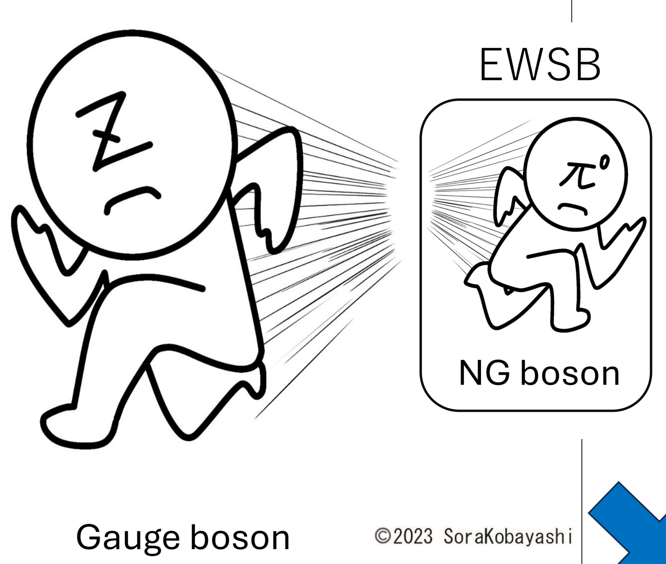
$$e^-(k,\sigma) + e^+(\bar{k},\bar{\sigma}) \to W^-(p,\lambda) + W^+(\bar{p},\bar{\lambda})$$

- We revisit $e^-e^+ \to W^-W^+$ in the SM.
- This process has been thoroughly studied theoretically and experimentally in the LEP era.
- This process is still very important for precision test of the electroweak (EW) theory in future high-energy lepton collider, such as the ILC, CEPC and FCC-ee.
- The gauge cancellation among amplitudes is an obstacle to numerical evaluations, especially at high energy.
  ➡ causes cancellation of significant digits.
- Recently a new gauge fixing was proposed and it indicates gauge cancellation among the interfering amplitudes can be avoided!!

"Feynman-Diagram Gauge" [1],[2],[3]

2. Gauge choices

- The treatment of Nambu-Goldstone (NG) bosons depend on gauge choices.
- In the early universe (before EW symmetry breaking), gauge bosons run at the speed of light (mass 0).



When EW symmetry breaks, NG bosons appear.

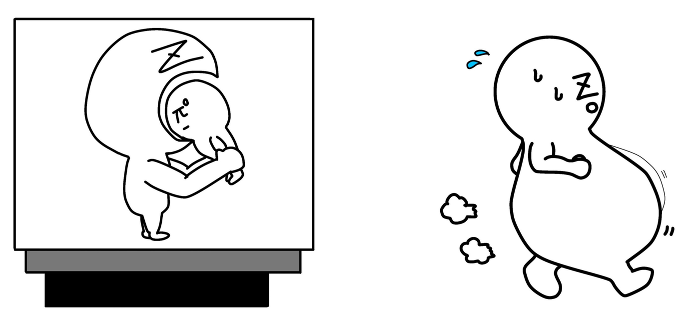
$$q^\mu = (E,0,0,q),$$

$$\beta = \frac{q}{E} = \sqrt{1-\frac{m^2}{E^2}},$$

$$\gamma = \frac{E}{m} = \frac{1}{\sqrt{1-\beta^2}}.$$

Unitary (U) gauge

Gauge bosons eat NG bosons and become fat (run slower = get the mass).



$\mu, \nu = 0$ to 3,

$$G_{\mu\nu}(q) = \frac{i}{q^2 - m^2 + i\epsilon}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}\right)$$

$$\epsilon^\mu(q,0) = \frac{1}{m}(q,0,0,E) = \gamma(\beta,0,0,1)$$

Feynman-Diagram (FD) gauge

Gauge bosons and NG bosons run together (run slower = get the mass).

$M, N = 0$ to 4,

$$n^\mu = (\mathrm{sgn}(q^0), -\vec{q}/|\vec{q}|)$$

$$G_{MN}(q) = \frac{i}{q^2 - m^2 + i\epsilon}\begin{pmatrix} -g_{\mu\nu} + \frac{q_\mu n_\nu + n_\mu q_\nu}{n\cdot q} & i\frac{m n_\mu}{n\cdot q} \\ -i\frac{m n_\nu}{n\cdot q} & 1 \end{pmatrix}$$

$$\epsilon^M(q,0) = (\tilde{\epsilon}^\mu(q,0),\, i)$$

$$\tilde{\epsilon}^\mu(q,0) = \epsilon^\mu(q,0) - \frac{q^\mu}{Q} = \frac{1}{\gamma(1+\beta)}(-1,0,0,1)$$

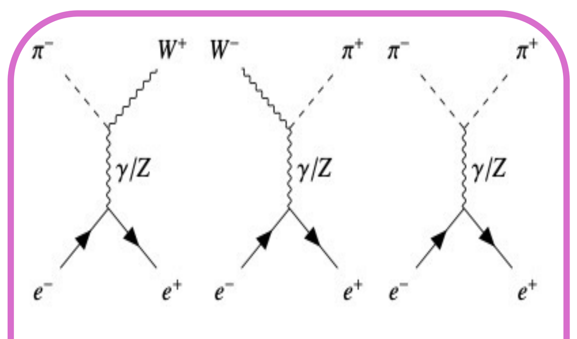
3. Helicity amplitudes

$$\mathcal{M}_{i\sigma}^{\lambda\bar{\lambda}} = \sqrt{2}\, e^2\, c_i\, \tilde{\mathcal{M}}_i^{\lambda\bar{\lambda}}(\theta)\, \varepsilon\, d^{J_0}_{\Delta\sigma,\Delta\lambda}(\theta)\, P_i(\theta)$$

U gauge

$\Delta\lambda$	$(\lambda,\bar{\lambda})$	$\tilde{\mathcal{M}}_\gamma^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_Z^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_\nu^{\lambda\bar{\lambda}}(\theta)$
0	$(0,0)$	$-2\gamma^2\beta+\beta$	$2\gamma^2\beta-\beta$	$2\gamma^2(\beta-\cos\theta)-\beta$
+1	$(+,0),(0,-)$	$-2\gamma\beta$	$2\gamma\beta$	$2\gamma(\beta-\cos\theta)-\frac{1}{\gamma}$
−1	$(0,+),(-,0)$	$-2\gamma\beta$	$2\gamma\beta$	$2\gamma(\beta-\cos\theta)+\frac{1}{\gamma}$
0	$(\pm,\pm)$	$-\beta$	β	$\beta-\cos\theta$
±2	$(\pm,\mp)$	0	0	$-\sqrt{2}$

i	γ	Z	ν
c_i	1	$s_W^{-2}\left(-\frac{1}{2}\delta_{\sigma,-1}+s_W^2\right)$	$s_W^{-2}\delta_{\sigma,-1}$
$P_i(\theta)^{-1}$	1	$1-m_Z^2/s$	$1+\beta^2-2\beta\cos\theta$

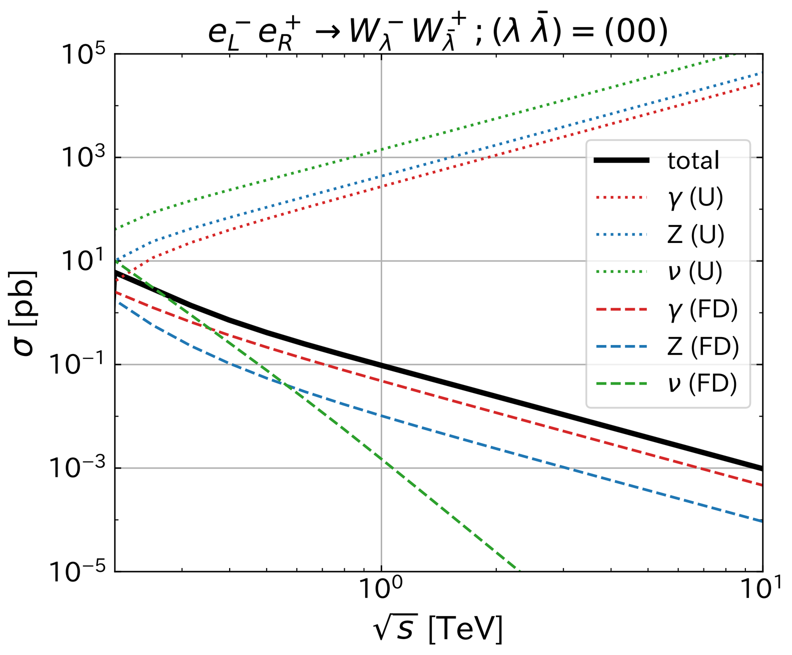


FD gauge

$\Delta\lambda$	$(\lambda,\bar{\lambda})$	$\tilde{\mathcal{M}}_\gamma^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_Z^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_\nu^{\lambda\bar{\lambda}}(\theta)$
0	$(0,0)$	$\frac{1}{\gamma^2}\frac{3+\beta}{(1+\beta)^2}+1$	$-\frac{1}{\gamma^2}\frac{3+\beta}{(1+\beta)^2}-\frac{s_W^2}{c_W^2}\left(\frac{\beta}{2s_W^2}-1\right)$	$-\frac{1}{\gamma^2}\frac{2}{(1+\beta)^2}(1+\cos\theta)$
+1	$(+,0),(0,-)$	$\frac{1}{2\gamma}\left(\frac{3-\beta}{1+\beta}+1\right)$	$-\frac{1}{2\gamma}\left(\frac{3-\beta}{1+\beta}-\frac{s_W^2}{c_W^2}\right)$	$-\frac{1}{\gamma}\frac{2}{1+\beta}(1+\cos\theta)$
−1	$(0,+),(-,0)$	$\frac{1}{2\gamma}\left(\frac{3-\beta}{1+\beta}+1\right)$	$-\frac{1}{2\gamma}\left(\frac{3-\beta}{1+\beta}-\frac{s_W^2}{c_W^2}\right)$	$\frac{1}{\gamma}\frac{2}{1+\beta}(\beta-\cos\theta)$
0	$(\pm,\pm)$	$-\beta$	β	$\beta-\cos\theta$
±2	$(\pm,\mp)$	0	0	$-\sqrt{2}$

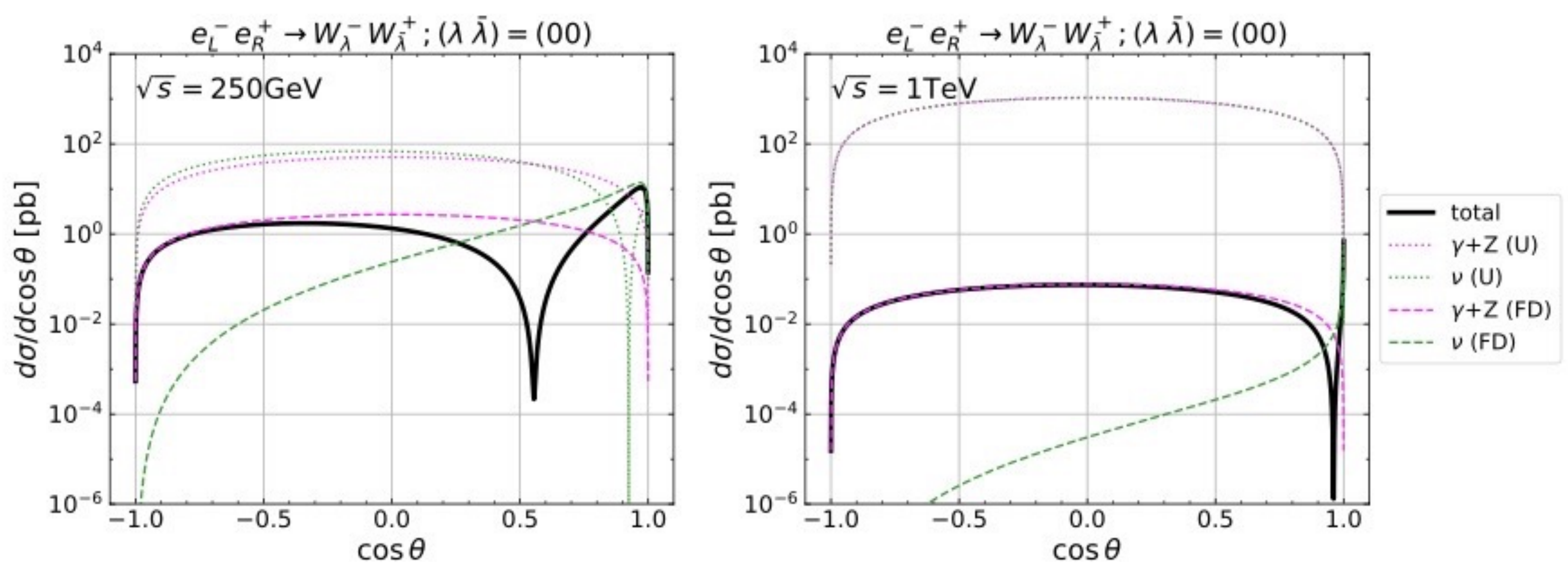
4. Cross sections

Total cross section



- **The solid black line** is the physical observable.
  $total \propto |M_\gamma + M_Z + M_\nu|^2$
- Dotted lines (U gauge) and dashed lines (FD gauge) are unphysical quantities.
  $\gamma \propto |M_\gamma|^2, Z \propto |M_Z|^2, \nu \propto |M_\nu|^2$

Differential cross section



・U gauge :
- Each amplitude has energy growth.
  →Large cancellation among the three amplitudes (γ, Z, ν) at high energies.
- No clue for the physical distribution from each contribution.

・FD gauge :
- No energy growth of individual amplitudes at all.
  →No unphysical cancellation.
- The contribution from the associated Goldstone bosons is manifest.
- Clear indication to the physical distribution from each contribution.

Our analytic results in the FD gauge provide a new insight into gauge theories !!

[1] J. Chen, K. Hagiwara, J. Kanzaki and K. Mawatari, "Helicity amplitudes without gauge cancellation for electroweak processes", arXiv:2203.10440, Eur.Phys.J.C 83 (2023) 922
[2] J. Chen, K. Hagiwara, J. Kanzaki, K. Mawatari and Y. Zheng , "Helicity amplitudes in light-cone and Feynman-diagram gauges", arXiv:2211.14562, (2022), Eur.Phys.J.Plus 139 (2024) 332
[3] K. Hagiwara, J. Kanzaki, O. Mattelaer, K. Mawatari and Y. Zheng, "Automatic generation of helicity amplitudes in Feynman-Diagram gauge", arXiv:2405.01256