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HIGGS PRODUCTION AT $\mu^+\mu^+$ COLLIDERS

AND A NEW CALCULATION SCHEME FOR TOTAL CROSS SECTIONS WITH INTERMEDIATE PHOTONS

Will appear on the arXiv very soon!

International Workshop on Future Linear Colliders (LCWS) 2024
July 9th 2024

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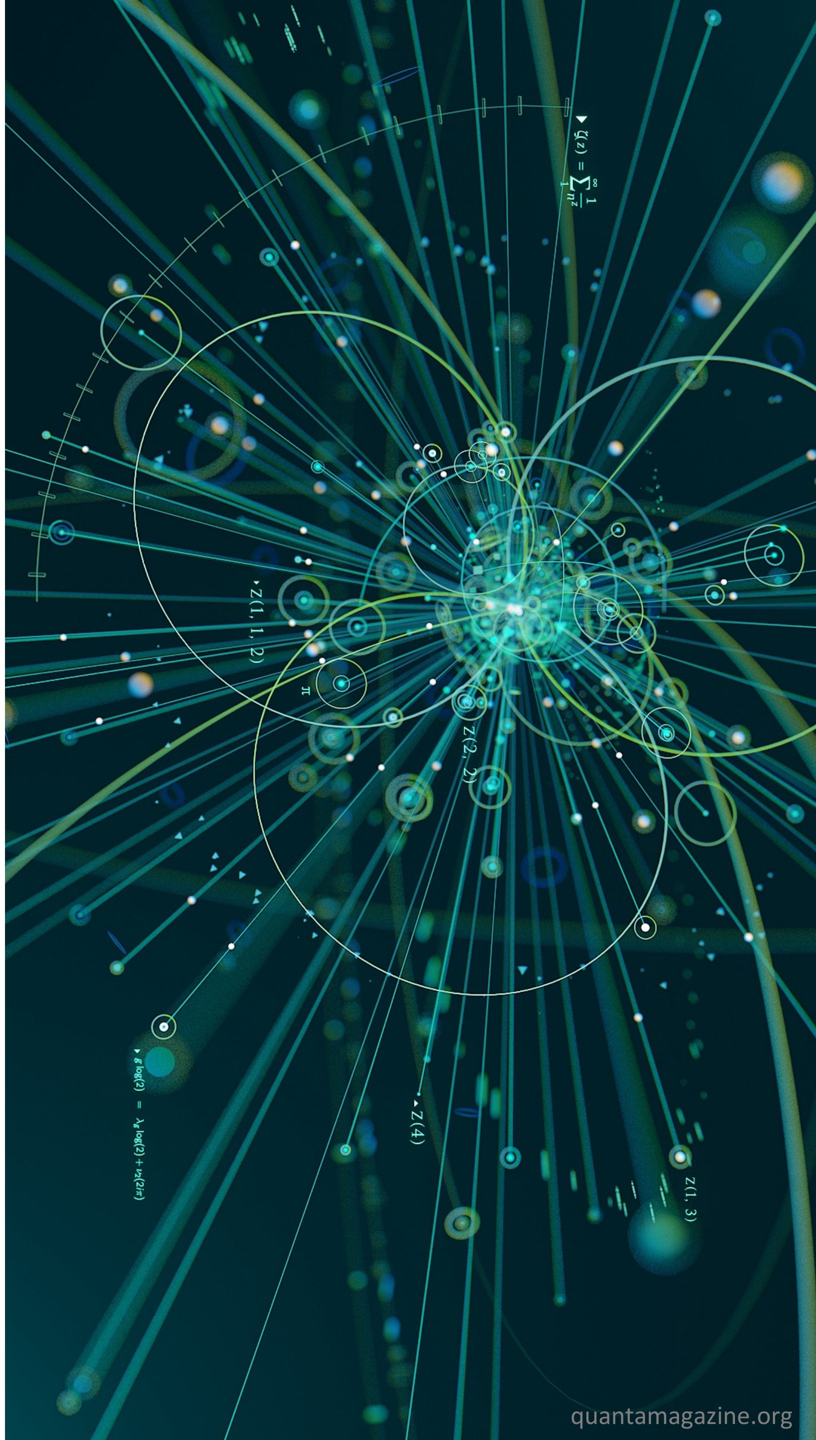


OUTLINE

- WHAT KIND OF MUON COLLIDERS?
- THE TOTAL CROSS SECTION OF $\mu^+\mu^+ \longrightarrow \mu^+ W^+ h \overline{\nu}_\mu$
  - THE EQUIVALENT PHOTON APPROXIMATION
  - THE NEW CALCULATION SCHEME
  - RESULTS FOR HIGGS PRODUCTION
- CONCLUSION

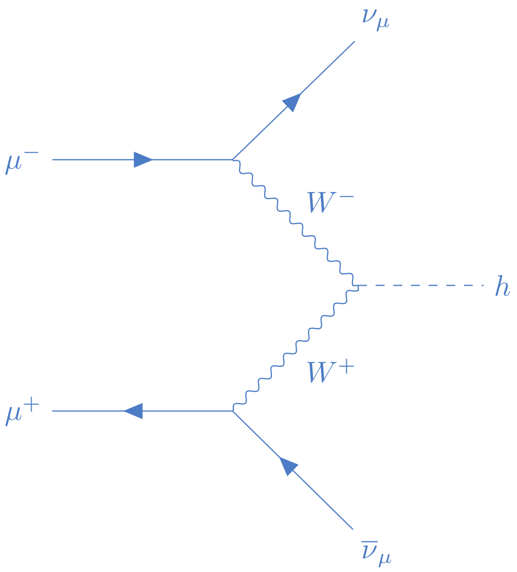
WHAT KIND OF MUON COLLIDERS?

	$\mu^+\mu^-$	μ^+e^-	$\mu^+\mu^+$
Electrically Neutral	✓	✓	✗
Vector Boson Fusion (VBF) w/o s-Channel Background (e.g., W^+W^-, $q\bar{q}$)	✗	✓	✓
Special Advantage	Resonant s-Channel Production	Flavor Physics	μ^+ Cooling Technology Already Exsists!
Disadvantage	μ^- Cooling Technology Needed	Lower Energy	Suppression From Extra Vertex…?

[See, e.g.: The muon Smasher's guide; Al Ali et al.; 2021;
μTRISTAN; Hamada, Kitano, Matsudo, Takaura, Yoshida; 2022]

THE TOTAL CROSS SECTION OF $\mu^+\mu^+ \longrightarrow \mu^+ W^+ h \overline{\nu}_\mu$

- VBF generally enhanced as $\sim \log\left(s/M_V^2\right)$, but need extra coupling!
- $\mu^+\mu^+$: Collinear emission of photon $\longrightarrow$ Collinear divergence (regulated by muon mass)
- Num. instabilities in num. phase-space integral of event generator MadGraph
- Equivalent Photon (Weizsäcker-Williams) Approximation for collinear photons



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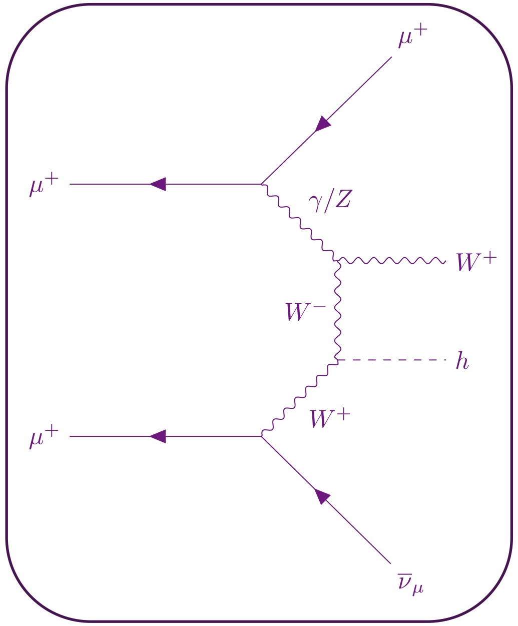

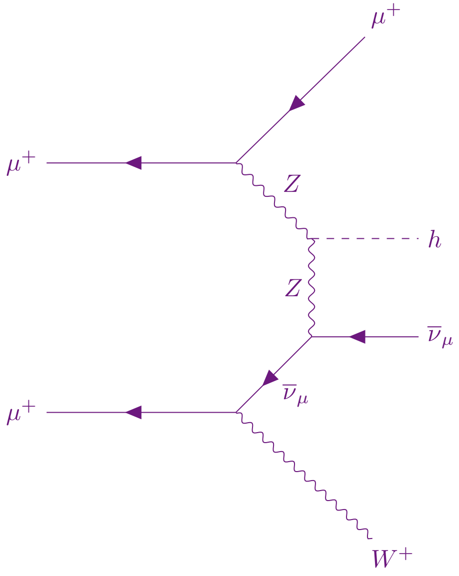

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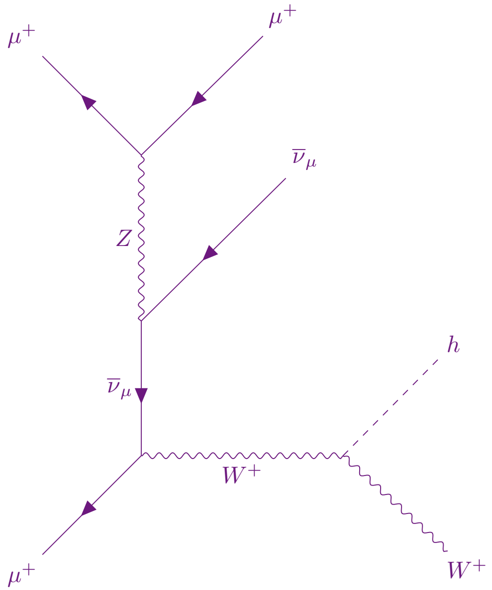


THE EQUIVALENT PHOTON APPROXIMATION

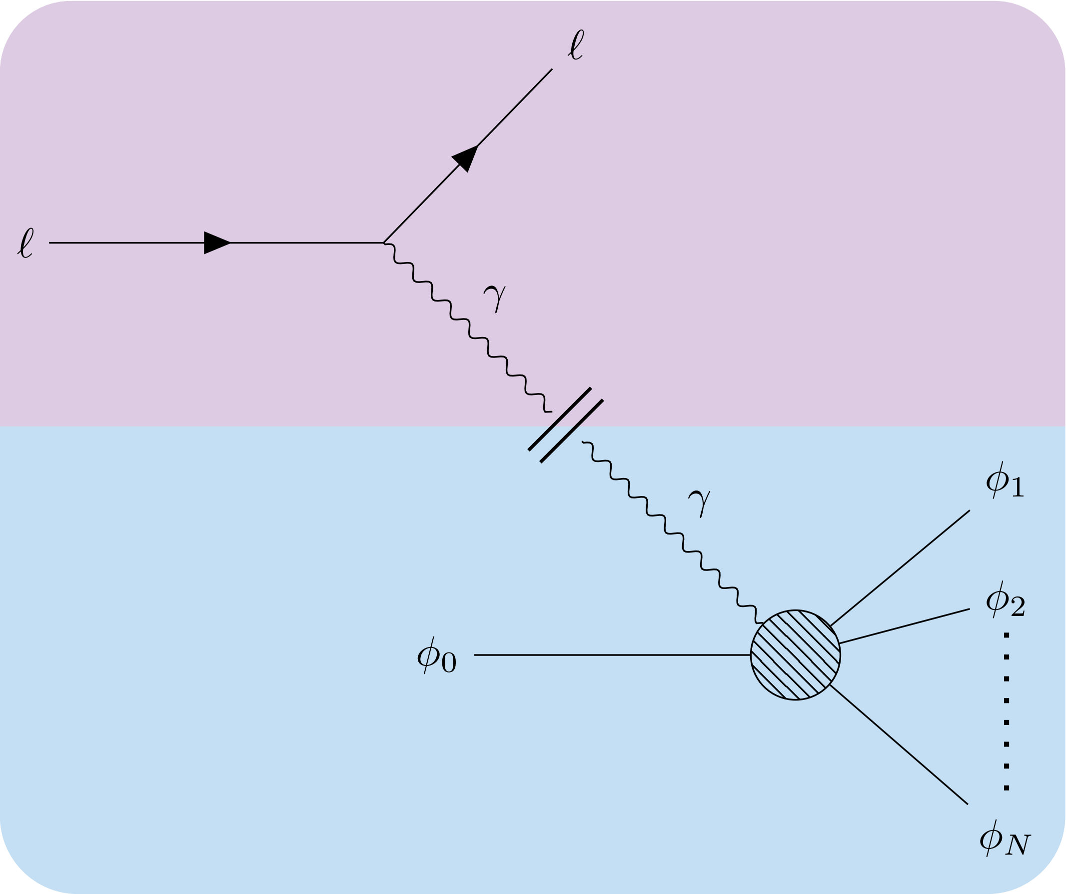
Intermediate photon quasi on-shell $\longrightarrow$ factorization!

$$\sigma = \sigma_{\mathrm{EPA}} + \Delta\sigma_{\mathrm{non-factorizable}}(q^2)$$

$$= \int \mathrm{d}x \left(\sigma_\gamma(xs)\, f_{\gamma/\ell}(x) + \mathcal{O}(1/x) \right)$$

Partonic cross section

Parton Distribution Function (PDF)



[Improving the Weizsacker-Williams approximation in electron-proton collisions; Frixione, Mangano, Nason, Ridolfi; 1993]

THE NEW CALCULATION SCHEME

Phase-space of lepton emitting photon

$$\sigma = \int \frac{\mathrm{d}^3 p_\ell}{E_\ell} \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 p_\ell / E_\ell} = \int \mathrm{d}p_\mathrm{T} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\mathrm{T}}$$

$$= \int\limits_{p_\mathrm{T} < p_\mathrm{T}^{(\mathrm{cut})}} \mathrm{d}p_\mathrm{T} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\mathrm{T}} + \int\limits_{p_\mathrm{T} > p_\mathrm{T}^{(\mathrm{cut})}} \mathrm{d}p_\mathrm{T} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\mathrm{T}}$$

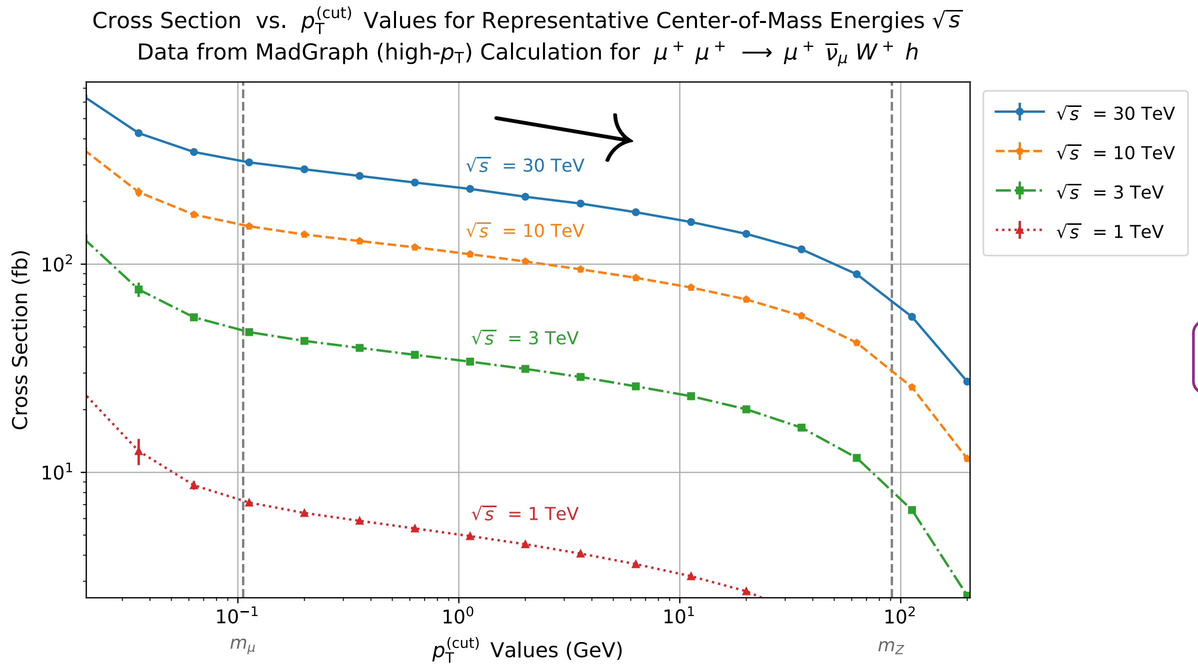
$$= \sigma \,\Big|_{p_\mathrm{T} < p_\mathrm{T}^{(\mathrm{cut})}} + \sigma \,\Big|_{p_\mathrm{T} > p_\mathrm{T}^{(\mathrm{cut})}}$$

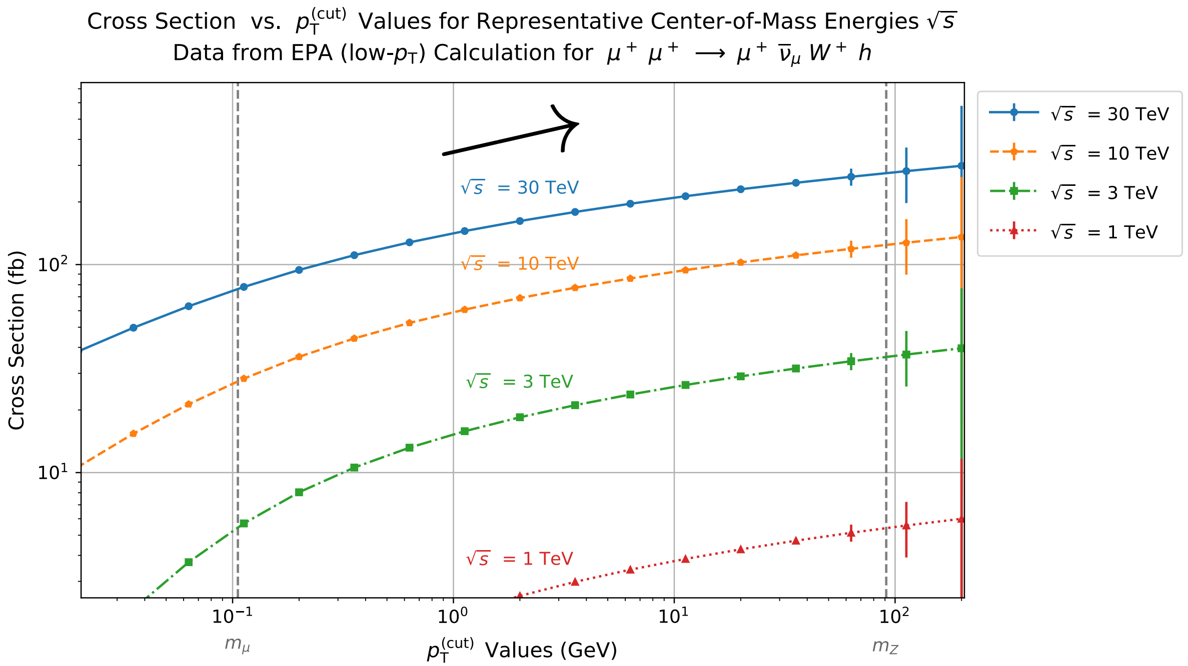
EPA

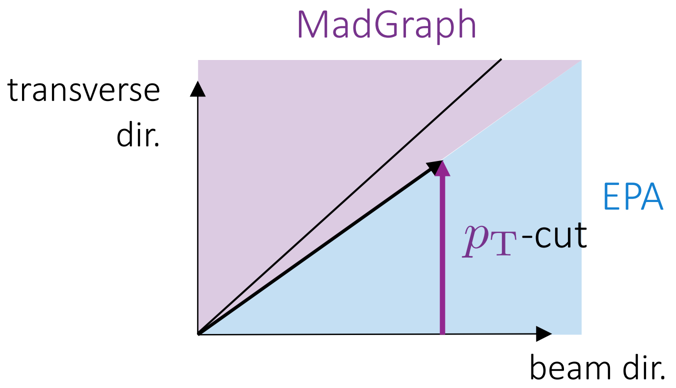
MadGraph

[Similar procedure "matrix element matching" in: The Effective Vector Boson approximation in High-Energy Muon Collisions; Ruiz, Costantini, Maltoni, Mattelaer; 2022]

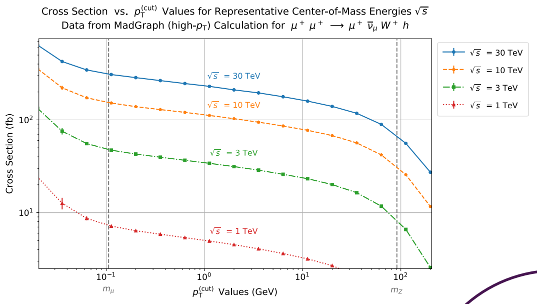
RESULTS FOR HIGGS PRODUCTION – 1 / 2

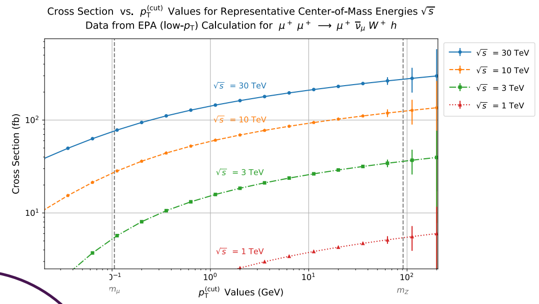


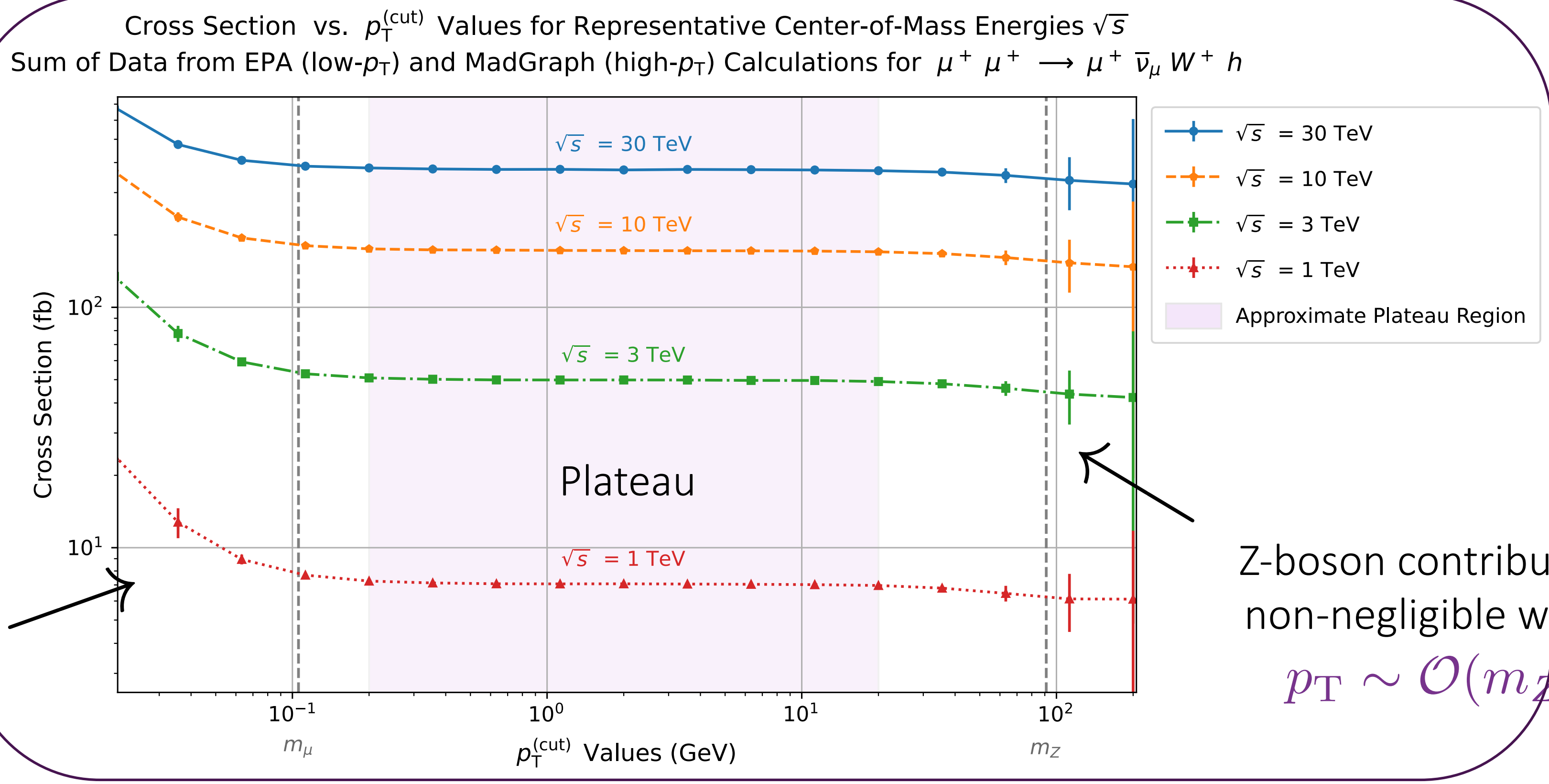




RESULTS FOR HIGGS PRODUCTION – 1 / 2



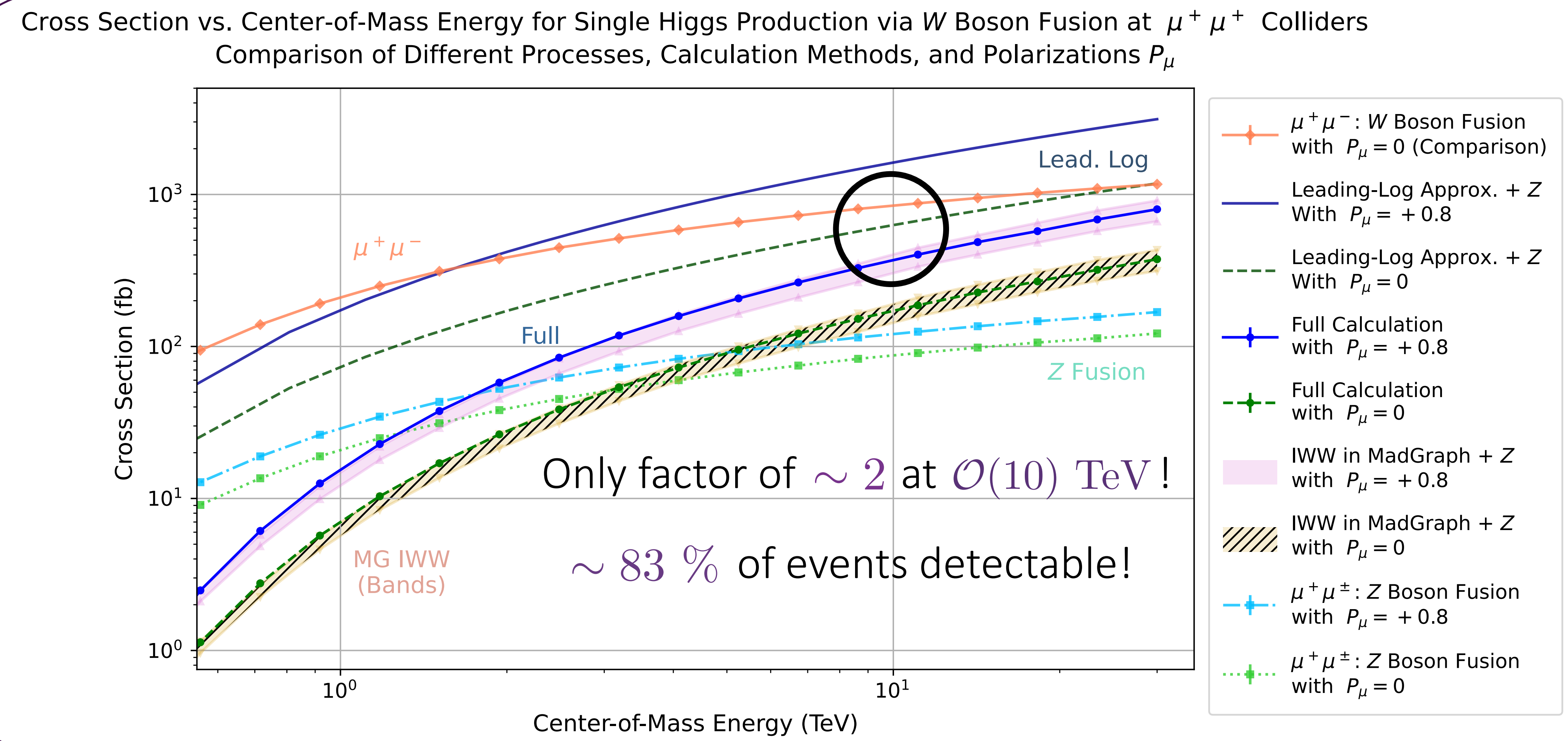




Numerical instability when $p_T \lesssim m_\mu$

Z-boson contribution non-negligible when $p_T \sim \mathcal{O}(m_Z)$

RESULTS FOR HIGGS PRODUCTION – 2 / 2



EVOLUTION WITH TRIPLE LOGARTHIMS

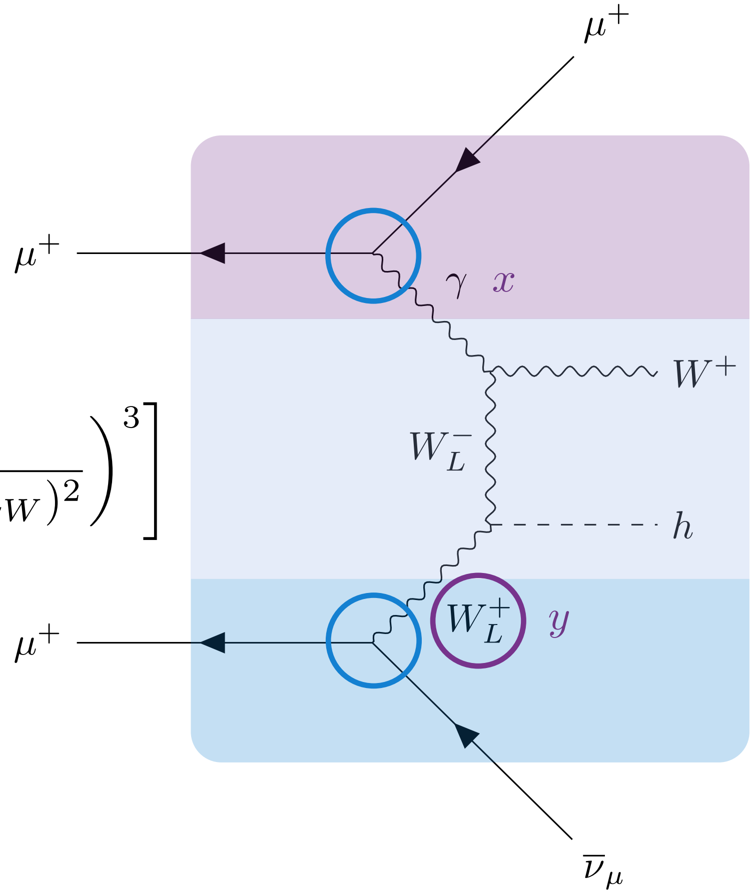
- Approximations
  - Photon and W boson collinear
  - Fusion of longitudinal W-boson polarizations dominates

Photon-W Cross Section — W PDF — Photon PDF

$$\sigma_{\mu^+\mu^+\to\mu^+\overline{\nu}_\mu W^+ h}(s) = \int \mathrm{d}x\,\mathrm{d}y\; \sigma_{\gamma W_\mathrm{L}^+\to\overline{\nu}_\mu W^+ h}(xy\,s)\, f_{W_\mathrm{L}^+/\mu^+}(y)\, f_{\gamma/\mu^+}(x)$$

$$\sim \frac{(1+P_{\mu^+})\,\alpha^4}{4\pi m_W^2 \sin^4\theta_\mathrm{W}} \left[\log\frac{s}{m_\mu^2} \left(\log\frac{s}{(m_h+m_W)^2} \right)^2 - \frac{1}{3} \left(\log\frac{s}{(m_h+m_W)^2} \right)^3 \right]$$

Three logarithms! Only one for $\mu^+\mu^-$!



[See, e.g.: Improving the Weizsacker-Williams approximation in electron-proton collisions; Frixione, Mangano, Nason, Ridolfi; 1993;
The Effective Vector Boson Approximation in High-Energy Muon Collisions; Ruiz, Costantini, Maltoni, Mattelaer; 2022;
Electroweak Splitting Function and High Energy Showering; Chen, Han, Tweedie; 2018;
Theory of e+e- Collisions at Very High Energy; Peskin; 1988
Higgs boson production in eγ collisions; Hagiwara, Watanabe, Zerwas; 1992]

CONCLUSION

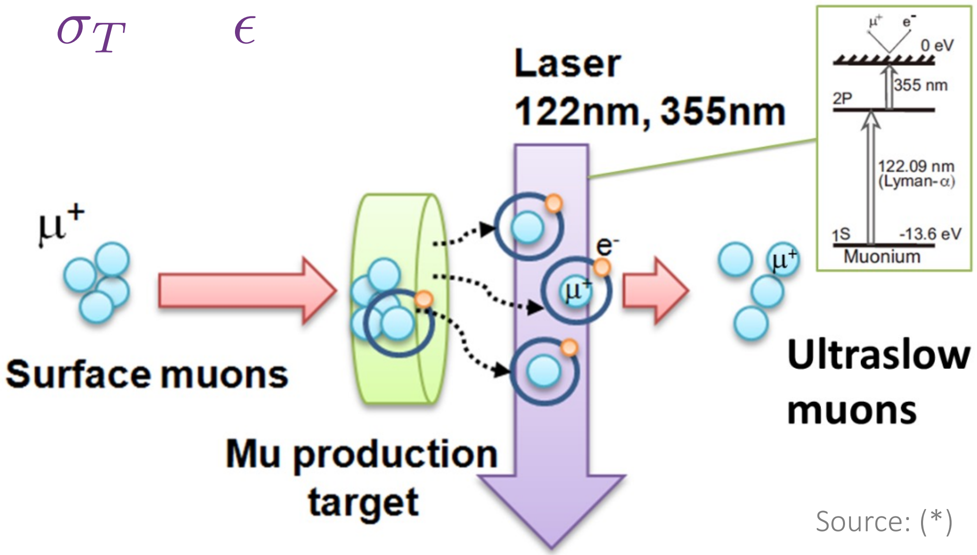
- Calculation of $\mu^+\mu^+ \longrightarrow \mu^+ W^+ h \bar{\nu}_\mu$ not straightforward due to collinear divergences of intermediate photon
- p_T-separated calculation scheme works well!
- Total cross section only factor of ~ 2 smaller than for $\mu^+\mu^-$, despite naïve suppression! $\longrightarrow$ Ennhancement from three collinear/soft logarithms
- Also important for other high-energy lepton colliders
- Comparable cross section + well-detectable final state + available cooling technology …

$\Longrightarrow$ $\mu^+\mu^+$ collider is a great Higgs factory!

BACKUP

MUON COLLIDER TECHNOLOGY

- Why is cooling so important? $\frac{\mathrm{d}N}{\mathrm{d}t} = \mathcal{L} \cdot \sigma\,, \quad \mathcal{L} \sim \frac{1}{\sigma_T} \sim \frac{1}{\epsilon}$
- Cooling
  - μ^- : Ionization cooling
  - μ^+ : Ultra-cold muons via muonium
- Polarization
  - μ^- : Possible, but big luminosity loss (energy selection, cooling)
  - μ^+ : Achievable without luminosity loss



Source: (*)

[See, e.g.: Re-Acceleration of Ultra Cold Muon in J-PARC Muon Facility; Kondo et al.; 2018 (*);
Polarized Beams in a Muon Collider; Cline, Norum, Rossmanith; 1996;
Demonstration of colling by the Muon Ionization Cooling Experiment; 2020]

DERIVATION OF EPA PDF

- Consider $e(p) + p(k) \to e(p') + X$

$$\mathrm{d}\sigma_{ep} = \frac{1}{8k\cdot p}\,\frac{\overbrace{W^{\mu\nu}}^{\text{Hard process}}\,\overbrace{T_{\mu\nu}}^{\text{Electron}}}{q^4}\,\overbrace{\frac{\mathrm{d}^3p'}{(2\pi)^3\,2E'}}^{\pi\mathrm{d}q^2\mathrm{d}y}$$

- Real photon $\longrightarrow$ current conservation $\implies q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = q^\mu T_{\mu\nu} = q^\nu T_{\mu\nu} = 0$

- $T^{\mu\nu}$ fully determined, $W_{\mu\nu}$ structure determined up to coefficients

- Assume factorization, integrate over $q^2 \implies \mathrm{d}\sigma_{ep} \overset{!}{=} \sigma_{\gamma p}(q,k) f_\gamma^{(e)}(y)\mathrm{d}y$

$$\implies f_\gamma^{(e)} = \frac{\alpha_{\mathrm{EM}}}{2\pi}\left[2m_e^2 y\left(\frac{1}{q^2_{\mathrm{max}}} - \frac{1}{q^2_{\mathrm{min}}}\right) + \frac{1+(1-y)^2}{y}\log\frac{q^2_{\mathrm{min}}}{q^2_{\mathrm{max}}}\right]$$

[Improving the Weizsacker-Williams approximation in electron-proton collisions; Frixione, Mangano, Nason, Ridolfi; 1993]

p_T-CUT SCHEME: EPA PART

Note

$$q^2 = (p' - p)^2$$

$$q^2_{\mathrm{min}} \leq q^2 \leq q^2_{\mathrm{max}} \leq 0$$

$$\left|q^2_{\mathrm{min}}\right| \geq \left|q^2_{\mathrm{max}}\right|$$

From kinematics:

$$q^2_{\mathrm{max}} = -\frac{m_e^2 y^2}{1-y}$$

$$q^2_{\mathrm{min}} = -\frac{1}{1-y}\left[\left(p_\mathrm{T}^{(\mathrm{cut})}\right)^2 + m_\ell^2\, y^2\right]$$

$$\Longrightarrow \quad f_\gamma^{(\ell),p_\mathrm{T}^{(\mathrm{cut})}}(y) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left[-2\,\frac{1-y}{y}\,\frac{\left(p_\mathrm{T}^{(\mathrm{cut})}\right)^2}{\left(p_\mathrm{T}^{(\mathrm{cut})}\right)^2 + m_\ell^2\, y^2} + \frac{1+(1-y)^2}{y}\,\log\frac{\left(p_\mathrm{T}^{(\mathrm{cut})}\right)^2 + m_\ell^2\, y^2}{m_\ell^2\, y^2}\right]$$