# Determination of $\mathcal{CP}$-violating *HZZ* interaction with polarised beams at the ILC

[arXiv: 2405.08494]

**Cheng Li**[1], Gudrid Moortgat-Pick[2]

[1]School of Science
Sun Yat-sen University

[2]II. Institut für Theoretische Physik
Universität Hamburg

LCWS 2024, Tokyo
July 9, 2024

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

中山大學
SUN YAT-SEN UNIVERSITY

# Motivation

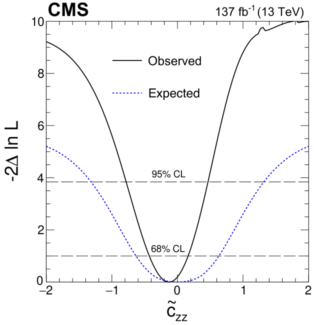
1. The CP violation in *HVV* interaction can be a possible source of the baryogenesis
2. Achieving highest precision for determination the CP properties of *HZZ* coupling via Z decay at the future $e^+e^-$ collider.
3. Polarised $e^+e^-$ beams can be used to improve the sensitivity to the CP properties of *HZZ* coupling, particularly for the transversely polarised beams

# CP violation in Higgs to gauge bosons interaction

$$\mathcal{L}_{\text{EFF}} = c_{\text{SM}}\, Z_\mu Z^\mu H - \frac{c_{HZZ}}{v} Z_{\mu\nu} Z^{\mu\nu} H - \frac{\widetilde{c}_{HZZ}}{v} Z_{\mu\nu} \tilde{Z}^{\mu\nu} H \qquad (1)$$

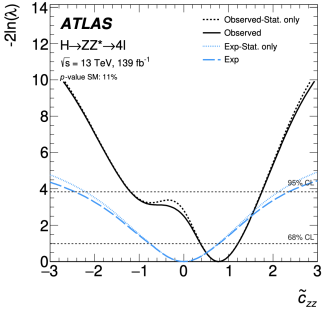
At LHC: $H \to 4\ell$ measurement:

[CERN-EP-2021-054]




$$(\tilde{c}_{ZZ})_{\text{CMS}} \sim [-0.66, 0.51]$$

[CERN-EP-2023-030]

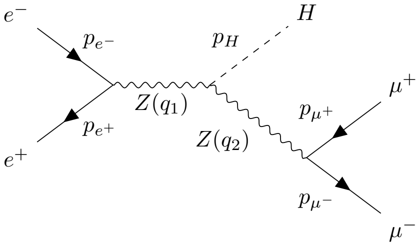



$$(\tilde{c}_{ZZ})_{\text{ATLAS}} \sim [-1.2, 1.75]$$

# Probing the CP violation at $e^+e^-$ collider

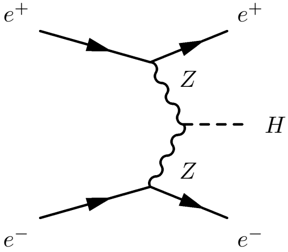
- Probe the CP-violation of $HZZ$ at $e^+e^-$ collider via $Z$ decay from Higgs strahlung process or $Z$-fusion process

## Higgs Strahlung




- Unpolarised study at CEPC [Q. Sha et al. 22']
- The effect of the initial polarized electrons is carried by the $Z$ boson and transferred to the $\mu^+\mu^-$ pair by the $Z$ decay

## $Z$ fusion




- $Z$-fusion study at 1 TeV [I. Bozovic et al. 24']
- $Z$-fusion process **cannot** carry the spin information of initial transversely polarised beams, since the final state electron and positron are unpolarised

# Initial beam polarisation and spin density matrix

## Spin formalism [H. E. Haber, 94']

polarisation matix for the initial beams:

$$\frac{1}{2}(1-\sigma\cdot P)_{\lambda\lambda'} = \frac{1}{2}\begin{pmatrix} 1-P^3 & P^1-iP^2 \\ P^1+iP^2 & 1+P^3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-f\cos\theta_P & f\sin\theta_P e^{-i\phi_P} \\ f\sin\theta_P e^{i\phi_P} & 1+f\cos\theta_P \end{pmatrix} \quad (2)$$

Bouchiat-Michel formula:

$$u(p,\lambda')\bar{u}(p,\lambda) = \frac{1}{2}(1+2\gamma_5)\not{p}\delta_{\lambda\lambda'} + \frac{1}{2}\gamma_5(\not{s}^1_-\sigma^1_{\lambda\lambda'} + \not{s}^2_-\sigma^2_{\lambda\lambda'})\not{p} \quad (3)$$

$$v(p,\lambda')\bar{v}(p,\lambda) = \frac{1}{2}(1-2\gamma_5)\not{p}\delta_{\lambda\lambda'} + \frac{1}{2}\gamma_5(\not{s}^1_+\sigma^1_{\lambda\lambda'} + \not{s}^2_+\sigma^2_{\lambda\lambda'})\not{p} \quad (4)$$

Spin density matrix for Higgs strahlung:

$$\begin{aligned}\rho^{ii'}(e^+e^-\to ZH) =& \frac{1}{2}(\delta_{\lambda_r\lambda'_r} + P^m_-\sigma^m_{\lambda_r\lambda'_r})\frac{1}{2}(\delta_{\lambda_u\lambda'_u} + P^n_+\sigma^n_{\lambda_u\lambda'_u})M^i_{\lambda_r\lambda_u}M^{*i'}_{\lambda'_r\lambda'_u} \\ =& (1-P^3_-P^3_+)A^{ii'} + (P^3_- - P^3_+)B^{ii'} + \sum_{mn}^{1,2} P^m_- P^n_+ C^{ii'}_{mn}\end{aligned} \quad (5)$$

where $C_{mn}$ is the part with transversely polarised beams.

- Note that, one would not see any transverse polarisation effect when only one beams transversely polarised

# Amplitude and CP-violation contribution

In order to simplify the analysis and get the idea of CP-violation effect, we only consider the additional contribution from the CP-odd term $\widetilde{c}_{HZZ}$

$$\begin{aligned}|\mathcal{M}|^2 =& |c_{\rm SM}\mathcal{M}_{\rm SM} + \widetilde{c}_{HZZ}\widetilde{\mathcal{M}}_{HZZ}|^2 \\ =& |c_{\rm SM}\mathcal{M}_{\rm SM}|^2 + |c_{\rm SM}\widetilde{c}_{HZZ}\mathcal{M}_{\rm SM}\widetilde{\mathcal{M}}_{HZZ}| + |\widetilde{c}_{HZZ}\widetilde{\mathcal{M}}_{HZZ}|^2\end{aligned} \tag{6}$$

where

$$c_{\rm SM} \propto \cos\xi_{CP}, \qquad \widetilde{c}_{HZZ} \propto \sin\xi_{CP} \tag{7}$$

Concerning the beam polarisation

$$\begin{aligned}|\mathcal{M}|^2 =& (1 - P_-^3 P_+^3)(\cos^2\xi_{CP}\,\mathcal{A}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{A}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{A}}_{\text{CP-even}}) \\ &+ (P_-^3 - P_+^3)(\cos^2\xi_{CP}\,\mathcal{B}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{B}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{B}}_{\text{CP-even}}) \\ &+ \sum_{mn}^{1,2} P_-^m P_+^n \left(\cos^2\xi_{CP}\,\mathcal{C}^{mn}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{C}^{mn}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{C}}^{mn}_{\text{CP-even}}\right)\end{aligned} \tag{8}$$

Only the interference term is CP-odd, which yield the CP-violation via triple-product correlations

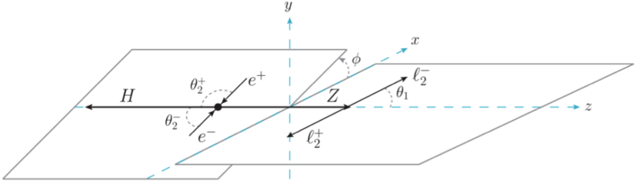
$$\mathcal{A}_{\text{CP-odd}}, \mathcal{B}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\alpha\beta}[p^\mu_{e-} p^\nu_{e+} p^\alpha_{\mu+} p^\beta_{\mu-}] \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{p}_{e^-} \tag{9}$$

$$\mathcal{C}^{mn}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\rho\sigma}[(p_{e^-} + p_{e^+})^\mu p^\nu_{\mu^+} p^\rho_{\mu^-} s^\sigma_{e^-}] \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{s}_{e^-} \tag{10}$$

- The idea of using transverse polarisation to probe the CP properties of $HZZ$ coupling see also [*S. Biswal et al. '09*]

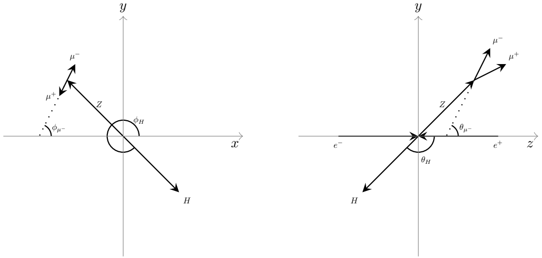
# CP-sensitive observables

## Coordinate systems with unpolarised or longitudinal polarised beams




- The $\phi$ is the azimuthal angle difference between the $\mu^-$-$\mu^+$ plane and the $Z$-$H$ plane

## Coordinate systems with transversely polarised beams ($\vec{n}_y \propto \vec{s}_{e^-}$, $\vec{n}_x \propto \vec{s}_{e^-} \times \vec{p}_{e^-}$, $\vec{n}_z \propto \vec{p}_{e^-}$)




- The $\phi_{\mu^-}$ is the azimuthal angle of the $\mu^-$-$\mu^+$ plane with fixing the $y$-axis orientation to $\vec{s}_{e^-}$

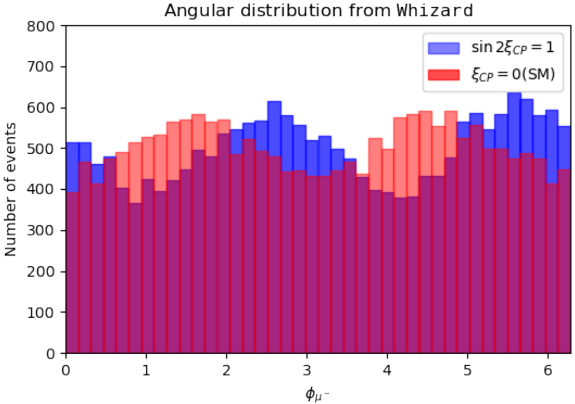
# Angular distribution

## Monte Carlo simulation by Whizard [1]

- We fix the total cross-section to the SM tree-level cross-section, and use 100% parallel transverse polarisation beams

$$\sigma_{\text{tot}} = \cos^2 \xi_{CP}\, \sigma_{\text{SM}} + \sin^2 \xi_{CP} \tilde{\kappa}^2_{HZZ}\, \tilde{\sigma}_{\text{HZZ}} = \sigma_{\text{SM}}, \tag{11}$$

$$P_-^2 = P_+^2 = 100\% \tag{12}$$




- The angular distribution of muon azimuthal angle is sensitive to the CP-violation

[1] http://whizard.hepforge.org

# Azimuthal asymmetry

Construct the observables sensitive to CP-violation:

$$\mathcal{O}^{T}_{CP} \propto \cos\theta_H \sin 2\phi_{\mu^-}, \qquad \mathcal{O}^{UL}_{CP} \propto \cos\theta_\mu \sin\phi \tag{13}$$

We can define the following asymmetries:

$$\mathcal{A}^{T}_{CP} = \frac{N(\mathcal{O}^{T}_{CP} < 0) - N(\mathcal{O}^{T}_{CP} > 0)}{N_{\text{tot}}} \tag{14}$$

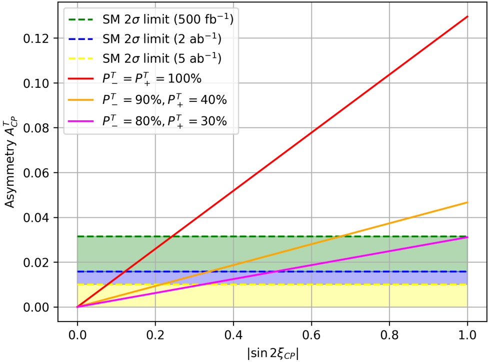
$$\mathcal{A}^{UL}_{CP} = \frac{N(\mathcal{O}^{UL}_{CP} < 0) - N(\mathcal{O}^{UL}_{CP} > 0)}{N_{\text{tot}}} \tag{15}$$

Statistical uncertainty (based on binomial distribution) of the Asymmetry:

$$\Delta\mathcal{A} = \sqrt{\frac{1 - \mathcal{A}^2}{N_{\text{tot}}}} \tag{16}$$

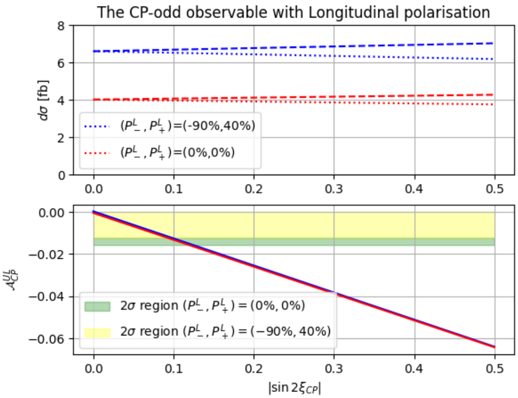
# Variation of CP-mixing angle

We fix the total cross-section, and vary the CP-mixing angle $\xi_{CP}$




- This $\mathcal{A}_{CP}^T$ is linearly depending on the CP-mixing angle $\sin 2\xi_{CP}$
- The stronger transverse polarisation leads to larger $\mathcal{A}_{CP}^T$.
- For $(P_{e^-}^T, P_{e^+}^T) = (80\%, 30\%)$ and $L = 500\ \mathrm{fb}^{-1}$, one cannot distinguish the CP-violating case from CP-conserving case for any CP-mixing angle $\xi_{CP}$ with only using $\mathcal{A}_{CP}^T$ observable.

# Variation of CP-mixing angle

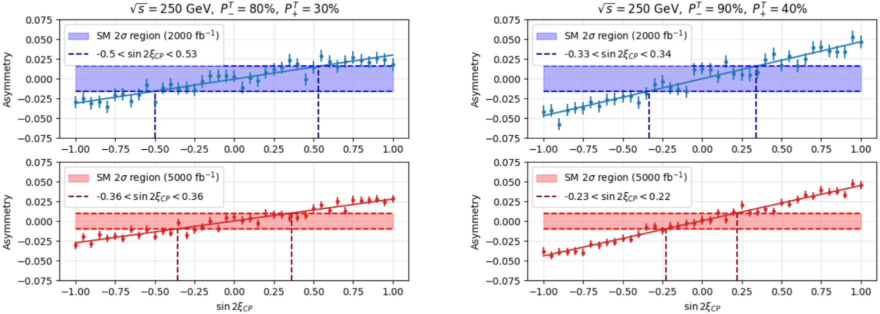



- The $\mathcal{A}^{UL}_{CP}$ linearly depends on the $\sin 2\xi_{CP}$ as well, while the beams polarisation cannot change the $\mathcal{A}^{UL}_{CP}$.
- One can also simultaneously measure the $\mathcal{A}^{UL}_{CP}$ when initial beams are transversely polarised.

# Determination of the CP-mixing angle

We made a linear fit for the asymmetries with respect to the $\sin 2\xi_{CP}$

$$\mathcal{A}_i = a \sin 2\xi_{CP} + b \tag{17}$$




- The fitting results for Monte-Carlo simulation data are basically match to the analytical calculation.

# Determination of the CP-mixing angle

- Simply combine the two asymmetries

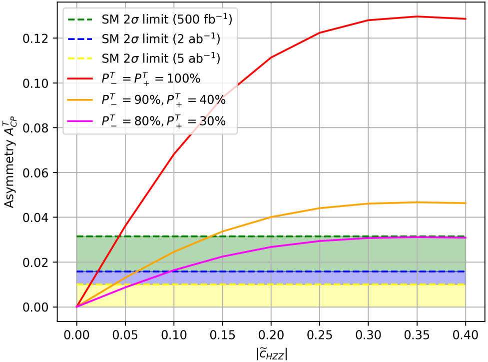
$$\chi^2_{\mathcal{A}_{\mathcal{CP}}} = \left(\frac{\mathcal{A}^{T}_{CP}}{\Delta\mathcal{A}^{T}_{CP}}\right)^2 + \left(\frac{\mathcal{A}^{UL}_{CP}}{\Delta\mathcal{A}^{UL}_{CP}}\right)^2 < 3.81 \tag{18}$$

| $(P_-, P_+)$ Observables | $\mathcal{L}$ [$ab^{-1}$] | sin $2\xi_{CP}$ limit (95% C.L.) $\mathcal{A}^{T}_{CP}$ | Combine $\mathcal{A}^{T}_{CP}$ & $\mathcal{A}^{UL}_{CP}$ | $\mathcal{A}^{UL}_{CP}$ |
|---|---|---|---|---|
| Transverse polarisation | | | | |
| (80%, 30%) | 2.0 | [-0.50, 0.53] | [-0.113, 0.125] | |
| (80%, 30%) | 5.0 | [-0.36, 0.36] | [-0.068, 0.079] | |
| (90%, 40%) | 2.0 | [-0.33, 0.34] | [-0.118, 0.110] | |
| (90%, 40%) | 5.0 | [-0.23, 0.22] | [-0.066, 0.077] | |
| (100%, 100%) | 5.0 | [-0.082, 0.069] | [-0.056, 0.051] | |
| Longitudinal polarisation | | | | |
| (−80%, 30%) | 2.0 | | | [-0.119,0.082] |
| (−80%, 30%) | 5.0 | | | [-0.066,0.063] |
| (−90%, 40%) | 2.0 | | | [-0.085,0.106] |
| (−90%, 40%) | 5.0 | | | [-0.059,0.062] |
| (−100%, 100%) | 5.0 | | | [-0.047,0.053] |

* The systematic uncertainties can be cancelled out by the CP-odd asymmetry, since the background contribution is basically CP-even.

# Variation of the CP-odd coupling

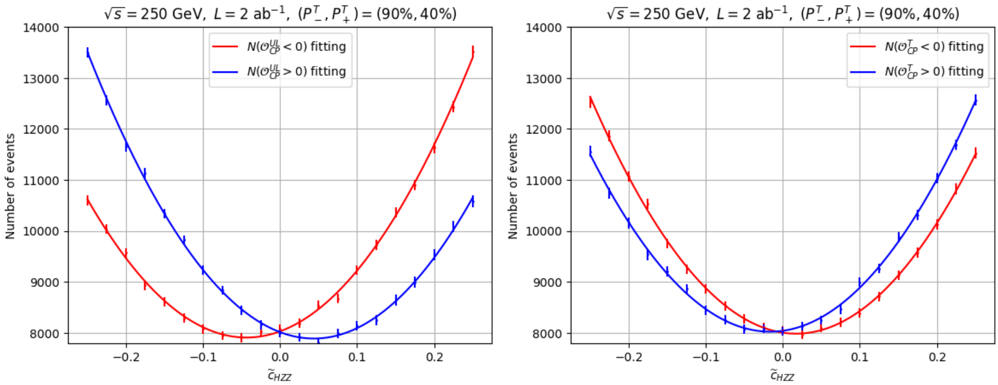
We fix $c_{\rm SM} = 1$ and vary $\widetilde{c}_{HZZ}$, in this case $\sigma_{\rm tot}$ would be increased by $\widetilde{c}_{HZZ}$




- The $\mathcal{A}_{CP}^T$ can reach to maximal when $\widetilde{c}_{HZZ} \sim 0.35$, and asymmetry $\mathcal{A}_{CP}^T$ would decrease for much higher $\widetilde{c}_{HZZ}$.
- For $(P_{e^-}^T, P_{e^+}^T) = (80\%, 30\%)$ and $L = 500~\mathrm{fb}^{-1}$, one still cannot determine any CP-odd coupling $\widetilde{c}_{HZZ}$.

# Determination of the CP-odd coupling

## Monte Carlo simulation by `Whizard`




- We made the quadratic function fit for the signal regions with varying $\widetilde{c}_{HZZ}$

$$N_i = a\widetilde{c}^2_{HZZ} + b\widetilde{c}_{HZZ} + c \tag{19}$$

# Determination of the CP-odd coupling

- One can combine the signal regions

$$\chi^2_N = \sum_i \left( \frac{(N(\mathcal{O}_i < 0) - N^{\rm SM}(\mathcal{O}_i < 0))^2}{N(\mathcal{O}_i < 0)} + \frac{(N(\mathcal{O}_i > 0) - N^{\rm SM}(\mathcal{O}_i > 0))^2}{N(\mathcal{O}_i > 0)} \right) \tag{20}$$

| $(P_-, P_+)$ | Luminosity [$\text{ab}^{-1}$] | $\tilde{c}_{HZZ}$ ($\times 10^{-2}$) limit (95% C.L.) | | |
|---|---|---|---|---|
| Observables | | $\mathcal{O}^T_{CP}$ | Combine $\mathcal{O}^{UL}_{CP}$ & $\mathcal{O}^T_{CP}$ | $\mathcal{O}^{UL}_{CP}$ |
| Transverse polarisation | | | | |
| (80%, 30%) | 2.0 | [-4.45,4.65] | [-2.26, 1.93] | |
| (80%, 30%) | 5.0 | [-3.55,3.85] | [-1.29, 1.06] | |
| (90%, 40%) | 2.0 | [-4.55,4.15] | [-2.24, 1.69] | |
| (90%, 40%) | 5.0 | [-2.65,3.75] | [-1.12, 0.98] | |
| Longitudinal polarisation | | | | |
| $(-80\%, 30\%)$ | 2.0 | | | [-1.55,1.96] |
| $(-80\%, 30\%)$ | 5.0 | | | [-1.01,1.16] |
| $(-90\%, 40\%)$ | 2.0 | | | [-1.73,1.53] |
| $(-90\%, 40\%)$ | 5.0 | | | [-0.93,1.18] |

* The explicit combined results can be obtained by the background simulation and log-likelihood estimation

# Comparison

## Determination of the CP-odd coupling

| | 95% C.L. ($2\sigma$)limit | | | | | | |
|---|---|---|---|---|---|---|---|
| Experiments | ATLAS | CMS | HL-LHC | CEPC | CLIC | CLIC | ILC |
| Processes | $H \to 4\ell$ | $H \to 4\ell$ | $H \to 4\ell$ | $HZ$ | $W$-fusion | $Z$-fusion | $HZ,\ Z \to \mu^+\mu^-$ |
| $\sqrt{s}$ [GeV] | 13000 | 13000 | 14000 | 240 | 3000 | 1000 | 250 |
| Luminosity [fb$^{-1}$] | 139 | 137 | 3000 | 5600 | 5000 | 8000 | 5000 |
| $(\|P_-\|, \|P_+\|)$ | | | | | | | $(90\%, 40\%)$ |
| $\tilde{c}_{HZZ}$ $(\times 10^{-2})$ | [-16.4, 24.0] | [-9.0, 7.0] | [-9.1, 9.1] | [-1.6, 1.6] | [-3.3, 3.3] | [-1.1, 1.1] | [-1.1, 1.0] |
| $f_{CP}^{HZZ}(\times 10^{-5})$ | [-409.82, 873.58] | [-123.78, 74.91] | [-126.54, 126.54] | [-3.92, 3.92] | [-16.66, 16.66] | [-1.85, 1.85] | [-1.85, 1.53] |
| $\tilde{c}_{ZZ}$ | [-1.2, 1.75] | [-0.66, 0.51] | [-0.66, 0.66] | [-0.12, 0.12] | [-0.24, 0.24] | [-0.08, 0.08] | [-0.08, 0.07] |

- The $e^+e^-$ colliders can significantly improve the sensitivity to CP-odd $HZZ$ coupling compared to the LHC or HL-LHC.
- The sensitivity with polarised beams is better than the analysis with unpolarised beams, where the center-of-mass energy and luminosity are similar.
- The $Z$-fusion process can have similar sensitivity but with much higher center-of-mass energy.

# Summary

## Conclusions

- The $e^+e^-$ collider can achieve high precision to CP properties of $HZZ$ interaction.
- The initial transversely polarised beams introduce additional CP-odd observables, which can be combined and improve the sensitivity to CP-odd structure.
- The longitudinally polarised beams enhance the total cross-section and suppress the statistical uncertainty, which can improve the CP-odd structure sensitivity as well.
- Both transverse and longitudinal polarisation improve compared to unpolarised case, where the transverse polarisation offers more observables

Thank you!

# Back up

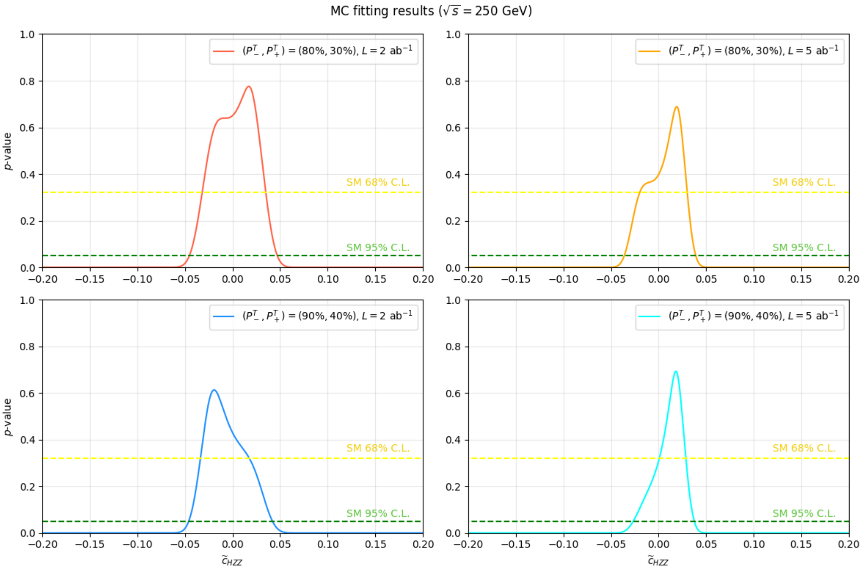
Matching conditions between different interpretations

$$f_{CP}^{HZZ} = \frac{\Gamma_{H\to ZZ}^{CP-\mathrm{odd}}}{\Gamma_{H\to ZZ}^{CP-\mathrm{even}} + \Gamma_{H\to ZZ}^{CP-\mathrm{odd}}}, \tag{21}$$

$$\frac{\Gamma_{H\to ZZ}^{CP-\mathrm{odd}}}{\Gamma_{H\to ZZ}^{CP-\mathrm{even}}} \sim \frac{\sigma_3}{\sigma_{\mathrm{SM}}}[pp \to H \to 4\ell(13\ \mathrm{TeV})] \sim 0.153. \tag{22}$$

$$\widetilde{c}_{HZZ} = \frac{g_1^2 + g_2^2}{4}\tilde{c}_{ZZ} = \frac{m_Z^2}{v^2}\tilde{c}_{ZZ}. \tag{23}$$

# Back up



MC fitting results ($\sqrt{s} = 250$ GeV)
$(P_-^T, P_+^T) = (80\%, 30\%), L = 2$ ab$^{-1}$
$(P_-^T, P_+^T) = (80\%, 30\%), L = 5$ ab$^{-1}$
$(P_-^T, P_+^T) = (90\%, 40\%), L = 2$ ab$^{-1}$
$(P_-^T, P_+^T) = (90\%, 40\%), L = 5$ ab$^{-1}$
SM 68% C.L.
SM 95% C.L.
p-value
$\tilde{c}_{HZZ}$