Prospects of measuring quantum entanglement in $\tau\tau$ final states at a future e^+e^- Higgs factory

Philip Bechtle, Cedric Breuning, Klaus Desch, Christian Grefe

LCWS 2024 – University of Tokyo

09.07.2024

INTRODUCTION

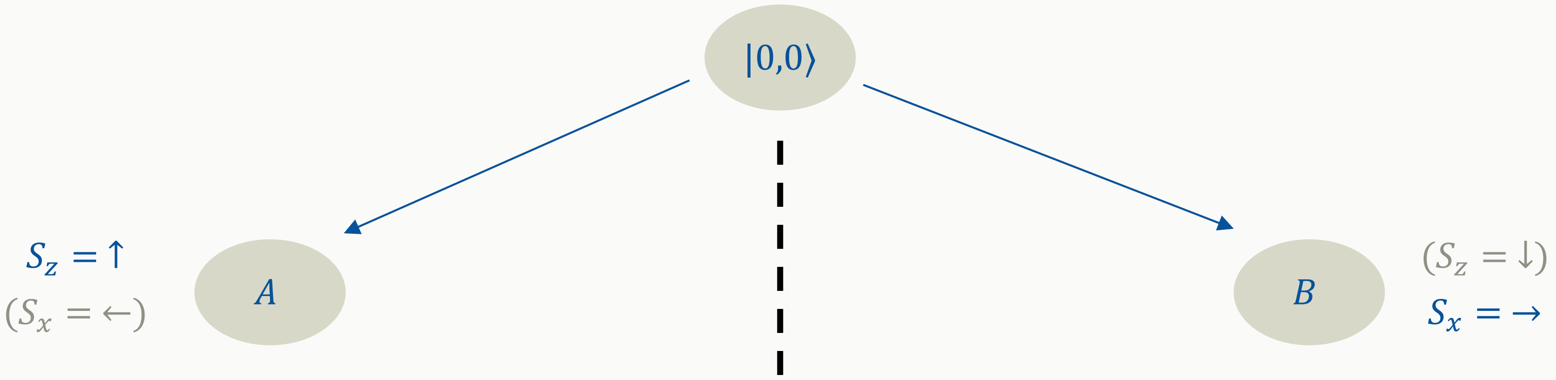
“Can Quantum-Mechanical Description of Physical Reality

Be Considered Complete?”

Einstein, Podolsky, Rosen: Phys. Rev 47.777

→ **Einstein-Podolsky-Rosen (EPR) Paradox**

SIMPLIFIED EPR PARADOX *(Bohm, Aharonov: Phys. Rev. 108.1070)*



But: $[S_z, S_x] \neq 0$

- *Einstein*: Quantum mechanics (QM) is incomplete
- Postulation of additional “hidden variables” (local hidden variable theories, LHVT)
- *Bell*: Inequality showing differences between LHVT and QM

UNIVERSITÄT BONN

BELL TESTS

- Testing the *Bell* inequality to test LHVT vs. entangled states in QM
- Many *Bell* tests over the last decades: rule out LHVT
  - Photons, low energies

  →Measure entanglement at collider energies
- Can use *Clauser-Horne-Shimony-Holt* (CHSH) inequality instead of *Bell* inequality

CHSH INEQUALITY

- CHSH Operator:

$$\mathfrak{B}_{CHSH} = \vec{A} \cdot \vec{\sigma} \otimes \left(\vec{B} + \vec{B}'\right) \cdot \vec{\sigma} + \vec{A}' \cdot \vec{\sigma} \otimes \left(\vec{B} - \vec{B}'\right) \cdot \vec{\sigma}$$

with unit vectors $\vec{A}, \vec{A}', \vec{B}, \vec{B}' \in \mathbb{R}^3$

- Assuming local realism: $|\langle \mathfrak{B}_{CHSH} \rangle| \leq 2$ **CHSH inequality**
- QM: construct system/state that violates this inequality: $|\langle \mathfrak{B}_{CHSH} \rangle| \leq 2\sqrt{2}$

CHSH inequality violated → QM, entangled state

CHSH INEQUALITY IN 2 SPIN ½ PARTICLES SYSTEM

- Spin density matrix (I: Identity operator):

$$\rho = \frac{1}{4}\left[I \otimes I + \sum_i B_i^+(\sigma_i \otimes I) + \sum_j B_j^-(I \otimes \sigma_j) + \sum_{ij} C_{ij}(\sigma_i \otimes \sigma_j)\right]$$

Polarization A — Polarization B — Correlations

- CHSH inequality:

$$|\langle \mathfrak{B}_{CHSH} \rangle| = |\mathrm{Tr}(\rho \mathfrak{B}_{CHSH})| \leq 2$$

UNIVERSITÄT BONN

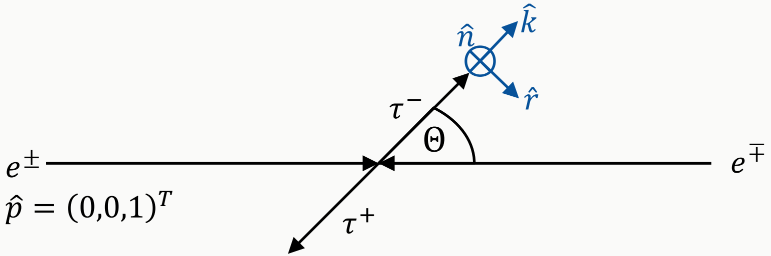
A SIMPLER CONDITION *(PHYS. LET. A 200 (1995) 340-344)*

- Focus on correlation part in ρ: $C_{ij} = \mathrm{Tr}\left(\rho(\sigma_i \otimes \sigma_j)\right)$
- Calculate $M = C^T C$ with eigenvalues $m_1 \geq m_2 \geq m_3$
- CHSH inequality violated if

$$\mathfrak{m}_{12} = m_1 + m_2 > 1$$

HOW TO CALCULATE C_{ij}? *(arXiv:2211.10513v2)*

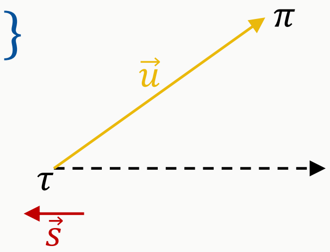
First: need coordinate system in which spins are measured



- $\hat{k}$ is the flight direction of the τ^- in the $\tau^-\tau^+$ rest frame
- $\hat{r} = (\hat{p} - \cos\Theta \cdot \hat{k})/\sin\Theta$
- $\hat{n} = \hat{k} \times \hat{r}$

HOW TO CALCULATE C_{ij}? *(arXiv:2211.10513v2)*

- We know $C_{ij} = \mathrm{Tr}\left(\rho(\sigma_i \otimes \sigma_j)\right)$
  $\rightarrow$ $\boxed{C_{ij} = \langle s_i^A s_j^B \rangle}$ with $s_z^A|\pm, m_B\rangle = \pm|\pm, m_B\rangle$ and $i, j \in \{r, n, k\}$



- **In $\boldsymbol{\tau \rightarrow \pi\nu}$ (1p0n) decay** (τ rest frame): $P(\vec{u}|\vec{s}) = 1 + \alpha \vec{s} \cdot \vec{u}$

- Can show: $\left\langle u_i^{\pi^-} u_j^{\pi^+} \right\rangle = -\frac{1}{9}\left\langle s_i^{\tau^-} s_j^{\tau^+} \right\rangle$ with $u_i^{\pi} = \vec{u}^{\pi} \cdot \vec{i} = \cos\theta_i^{\pi}$

MEASURING C_{ij}

Want to measure $C_{ij} = -9 \left\langle u_i^{\pi^-} u_j^{\pi^+} \right\rangle = -9 \left\langle \cos\theta_i^{\pi^-} \cos\theta_j^{\pi^+} \right\rangle$

$$C_{ij} = -9 \int \mathrm{d}\cos\theta_i^{\pi^-} \, \mathrm{d}\cos\theta_j^{\pi^+} \frac{\mathrm{d}\sigma \cdot \sigma^{-1}}{\mathrm{d}\cos\theta_i^{\pi^-} \, \mathrm{d}\cos\theta_j^{\pi^+}} \cos\theta_i^{\pi^-} \cos\theta_j^{\pi^+}$$

3. Sum bins

1. Normalized 2D Histogram

2. Multiply with bin centers

EXPECTATION FOR $H \rightarrow \tau\tau$ *(arXiv:2211.10513v2)*

- Spin density matrix for $H \rightarrow \tau\tau$ can be calculated
- Leads to

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

with Higgs CP phase δ → $\delta = 0$ in SM

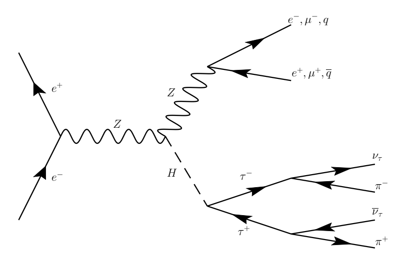
$$C_{ij} = \mathrm{diag}(1,1,-1) \rightarrow \mathfrak{m}_{12} = 2$$

EVENT GENERATION

- Basic process: MadGraph5_aMC@NLO (v.3.5.3) *(arXiv:1405.0301v2)*
- Showering/Hadronization/Decay: Pythia8 (v. 8.306) *(arXiv:1410.3012v1)*
- (Fast) Detector simulation: Delphes (v.3.5.0) with IDEA card *(arXiv:1307.6346v3)*

```
import model sm-full
define x = e- mu- u d s c b
define x~ = e+ mu+ u~ d~ s~ c~ b~
generate e+ e- > h z, h > ta+ ta-, z > x x~
```

- Beam energy 120 GeV
- FCC-ee Pythia8 configuration: [p8_ee_ZH_ecm240](p8_ee_ZH_ecm240) (without vertex smearing)



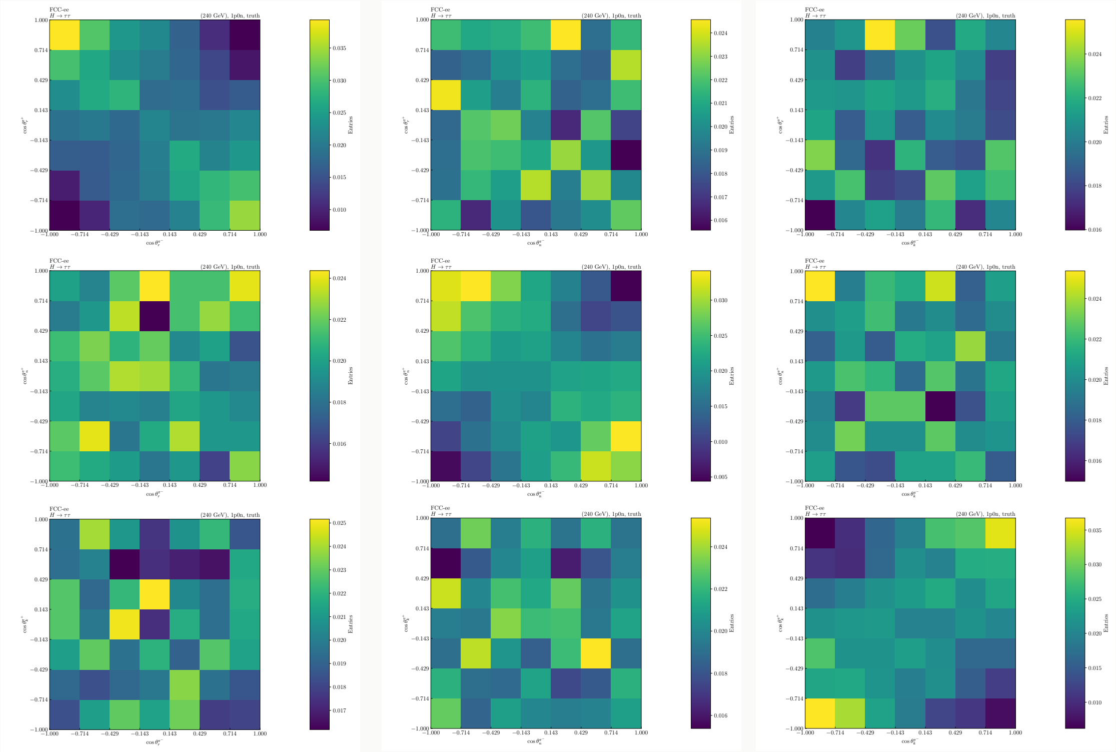
TRUTH LEVEL RESULTS

Normalized 2D histograms for C_{ij}

$$\frac{\mathrm{d}\sigma \cdot \sigma^{-1}}{\underbrace{\mathrm{d}\cos\theta_i^{\pi^-}}_{x\text{-axis}}\ \underbrace{\mathrm{d}\cos\theta_j^{\pi^+}}_{y\text{-axis}}}$$

Columns: $i = r$, $i = n$, $i = k$

Rows: $j = r$, $j = n$, $j = k$



TRUTH LEVEL RESULTS

Resulting correlation matrix

$$C = \begin{pmatrix} 0.932 & -0.067 & 0.008 \\ 0.01 & 0.926 & -0.009 \\ 0.047 & 0.034 & -0.866 \end{pmatrix}$$

which results in

$$\mathfrak{m}_{12} = 1.76$$

RECONSTRUCTION: p_τ (arXiv:2211.10513v2)

- Need $p_{\tau^\pm}$ for boosts and $\hat{k}$ (8 unknowns)
- Known: $p_{\pi^\pm},\ p_H$ (from measured Z decay products and known initial state)
- Have a number of conditions that constrain the unknown momenta

$p_{\tau^+} + p_{\tau^-} = p_H$ → 4 constraints

$p_{\tau^+}^2 = p_{\tau^-}^2 = m_\tau^2$ → 2 constraints

$(p_{\tau^\pm} - p_{\pi^\pm})^2 = m_\nu^2 = 0$ → 2 constraints

} 8 constraints for 8 unknowns

- Problem: gives 2 solutions

SELECTING THE CORRECT SOLUTION

- For selection solve
  $$\vec{x}_\pi + t_\pi \cdot \vec{p}_\pi + t_c \cdot (\vec{p}_\pi \times \vec{p}_{\tau,i}) = \vec{x}_\tau + t_\tau \cdot \vec{p}_{\tau,i}$$

$d = |t_c \cdot (\vec{p}_\pi \times \vec{p}_{\tau,i})|$

$\sigma_d = 5\mu\text{m}$ guessed to well working value

$l_i = |t_\tau \cdot \vec{p}_{\tau,i}|$

$l_\tau = c\tau_\tau \beta\gamma$

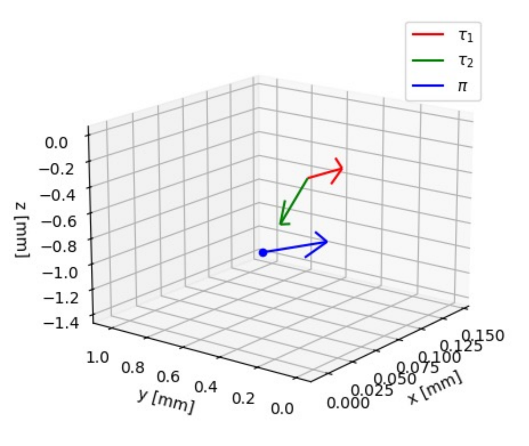
- Minimize $\log \mathcal{L} = \frac{d_i^2}{\sigma_d^2} + \log l_\tau + \frac{|l_i|}{l_\tau}$ and $l_i > 0$

τ track should hit π track

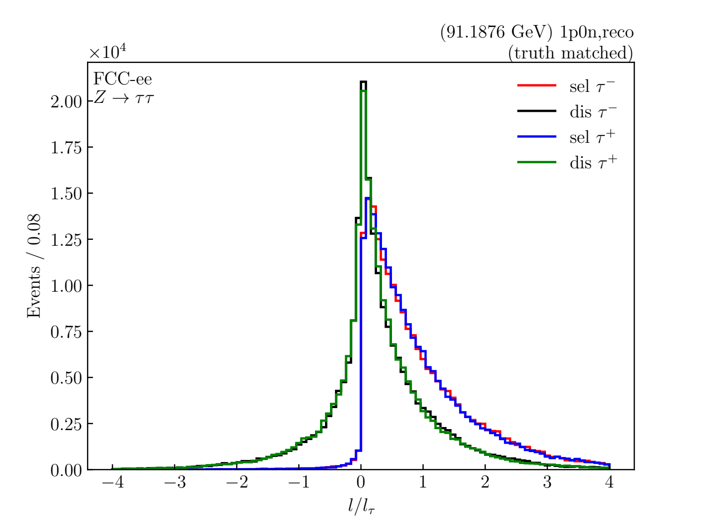
→ Closer distance

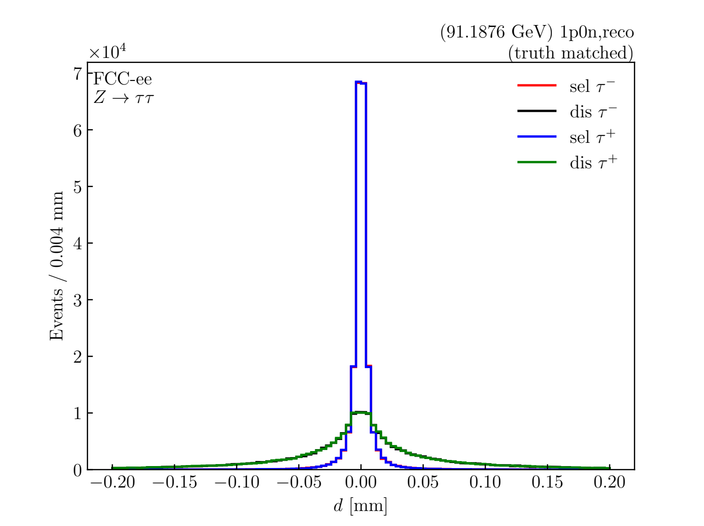
Probablilty τ decayed:

$$P(l) = l_\tau^{-1} \cdot \exp\left(-\frac{|l|}{l_\tau}\right)$$



SELECTION RESULTS

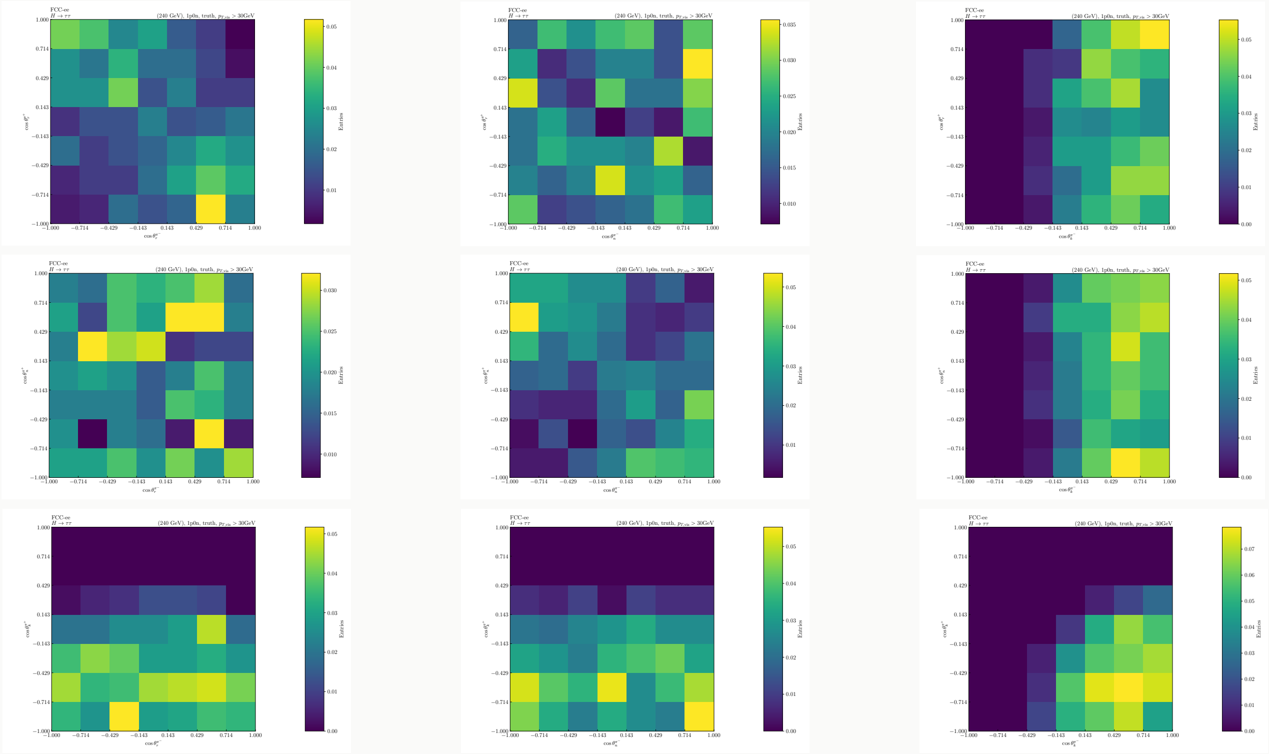




WHY e^+e^- COLLIDER?

- In theory also possible at pp collision
  - Harder reconstruction, background handling
  - No ”simple” way to reconstruct $\tau\tau$ rest frame
- Another “problem“ at the LHC:
  $p_{T,vis} > 40\ (30)\ \text{GeV}$ acceptance cut on (sub)leading τ (ATLAS)
  - Hopefully easier to counteract at e^+e^- collisions than at pp collisions

TRUTH LEVEL RESULTS WITH $p_{T,vis} > 30$ GeV



TRUTH LEVEL RESULTS WITH $p_{T,vis} > 30$ GeV

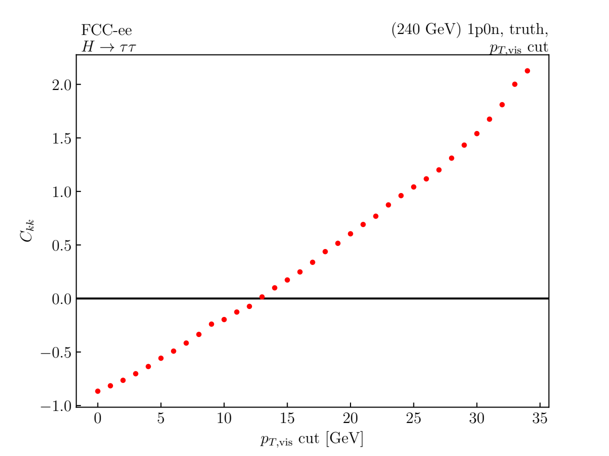
Resulting correlation matrix

$$C = \begin{pmatrix} 1.206 & 0.028 & -0.017 \\ -0.086 & 1.299 & 0.022 \\ -0.13 & -0.051 & \boxed{1.540} \end{pmatrix}$$

which results in

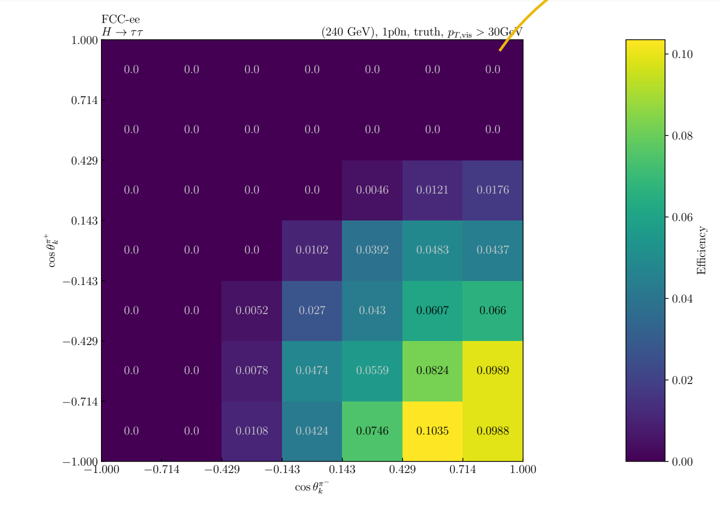
$$\mathfrak{m}_{12} = 4.1$$

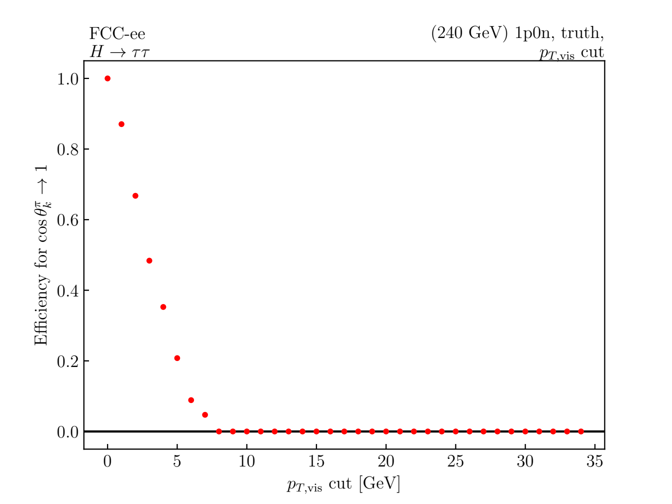
(values > 1 in correlation matrix:
probably caused by too low statistics)



EFFICIENCY OF THE $p_{T,vis}$ CUT

For C_{kk} look at the fraction that survives the $p_{T,vis}$ cut:





SUMMARY

- Violation of CHSH inequality → entangled states, rule out LHVT
- Show violation by measuring the correlation matrix C_{ij} → $\mathfrak{m}_{12} > 1$
- Observable at colliders: π momenta in $\tau \to \pi\nu$
  - Enough to reconstruct p_τ
  - Need to select correct solution
- Truth level results match expectations
- Problem: acceptance cuts

UNIVERSITÄT BONN

OUTLOOK

- Improve reconstruction
- Simulate and include background processes (arXiv:2211.10513v2)
  (Main background: $e^+e^- \to Z\tau\tau$ with $\tau\tau$ from γ^*/Z^* exchange)
  - Suppress by adding constraint $\sqrt{(p_{in} - p_Z)^2} \approx m_H$
- Analyse $Z \to \tau\tau$

UNIVERSITÄT BONN

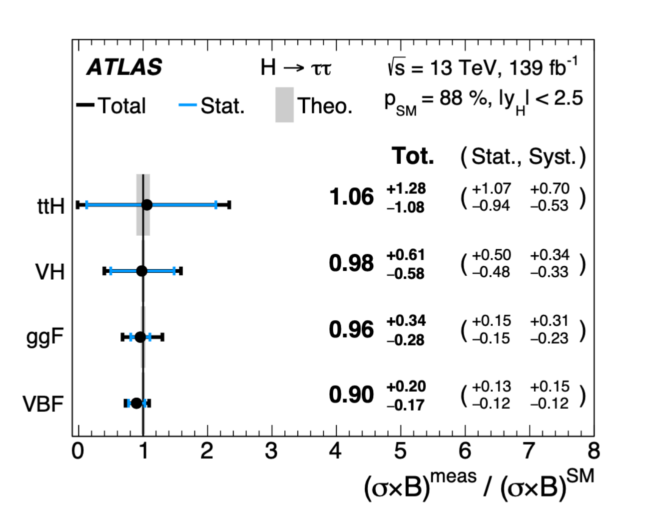
BACKUP

UNIVERSITÄT BONN

RECONSTRUCTING p_τ (arXiv:2211.10513v2)

- Basisvectors $p_H^\mu, p_{\pi^+}^\mu, p_{\pi^-}^\mu, q^\mu = m_H^{-2}\varepsilon^{\mu\nu\rho\sigma} p_{H\nu} p_{\pi^+\rho} p_{\pi^-\sigma}$
- Can write $p_{\tau^\pm}^\mu = \frac{1\mp a}{2} p_H^\mu \pm \frac{b}{2} p_{\pi^+}^\mu \mp \frac{c}{2} p_{\pi^-}^\mu \pm dq^2$ which fulfils $p_{\tau^+} + p_{\tau^-} = p_H$
- 2 equations (I, II) from $\left(p_{\tau^\pm} - p_{\pi^\pm}\right)^2 = m_\nu^2 = 0$ only dependent on a, b, c
- 2 equations (III, IV) from $p_{\tau^\pm}^2 = m_\tau^2$
  - Difference between III and IV delivers equation (V) only dependent on a, b, c
- Solve equation system I, II, V (only dependent on a, b, c), e.g. as matrix equation
- Sum of III and IV delivers equation for d^2
  - Discard negative solutions of d^2, try to select correct solution for positive d^2

TRIGGERS AND HIGH $p_{T,vis}$ CUT



arXiv:2201.08269: H production cross section in $H \rightarrow \tau^+\tau^-$ decay channel