Karlsruhe Institute of Technology
Institute for Theoretical Physics

BSM Triple Higgs couplings (at one-loop) at e^+e^- Colliders

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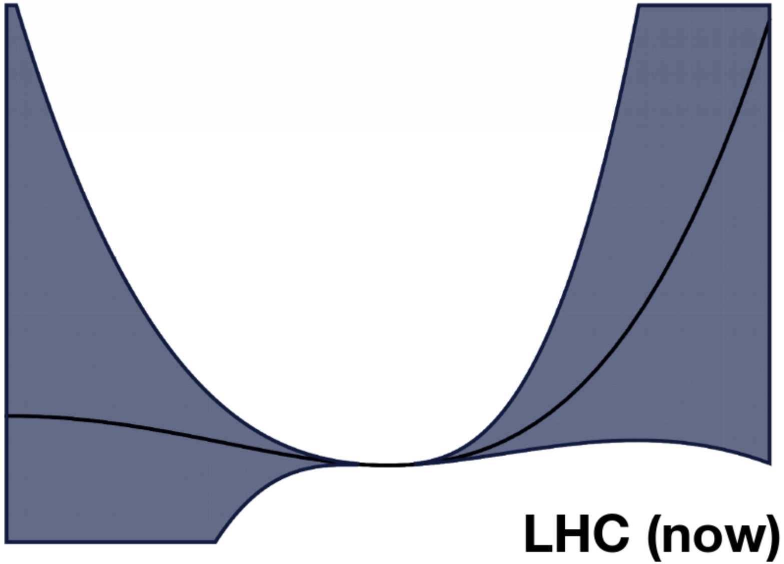
Higgs, Electroweak Parallel Session
University of Tokyo, Japan – July 10, 2024

Ongoing work with S. Heinemeyer and M. Mühlleitner

Motivation: BSM in the Higgs Sector

- The Higgs boson potential is essentially ***untested***
- Extended Higgs sectors can solve (at least some) of the SM problems
  - Dark matter, baryon asymmetry…
- Framework: **Two Higgs doublet model (2HDM)**
  - 5 Higgs bosons $h, H, A, H^{\pm}$ + new scalar interactions
  - **Large scalar couplings** are still allowed! *[FA, Heinemeyer, Herrero, 21, 22]*



Sketch of the current uncertainty in the (SM) Higgs potential, by Nathaniel Craig

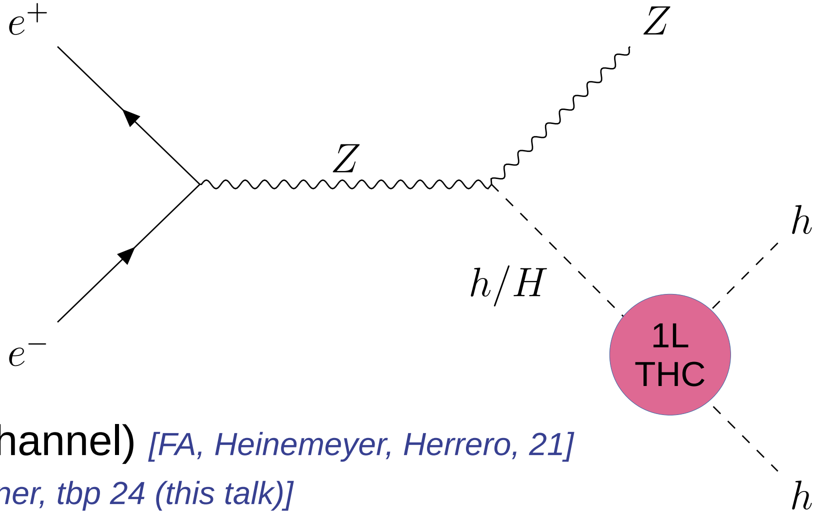
Where to look? At e^+e^- colliders!

- Large scalar couplings can give large 1L corrections to triple Higgs couplings (THCs) $\rightarrow$ potential signal at high energy e^+e^- colliders
  - e.g. 1L $\lambda^{(1)}_{hhh}$ correction can be well above 100% w.r.t the tree level *[Kanemura, Kiyoura, Okada, Senaha, Yuan, 02]*
- Our computation for **ILC**:

Tree level $e^+e^- \rightarrow hhZ$

+

1L corrected $\lambda^{(1)}_{hhh}$ and $\lambda^{(1)}_{hhH}$



- Bibliography in the 2HDM:
  - Tree-level THCs @ e^+e^- colliders (also VBF channel) *[FA, Heinemeyer, Herrero, 21]*
  - 1L THCs @ e^+e^- colliders *[FA, Heinemeyer, Mühlleitner, tbp 24 (this talk)]*
  - Tree, 1 and 2L THCs @(HL-)LHC *[FA, Heinemeyer, Radchenko, Mühlleitner, 22][Bahl, Braathen, Weiglein, 22] [Heinemeyer, Mühlleitner, Radchenko, Wieglein, 23]*

Two Higgs Doublet Model (2HDM)

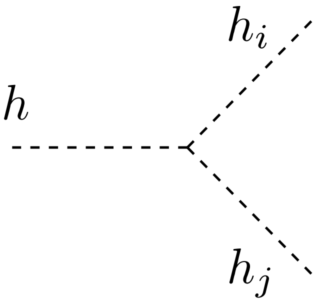
- SM + **second Higgs doublet** (CP-conserving + Z_2 symmetry)

5 physical Higgs bosons: h, H: (CP-even) A: (CP-odd) and $H^{\pm}$

- Input parameters:

$$m_h\ (\sim 125\ \mathrm{GeV}),\ m_H, m_A, m_{H^\pm},\ \tan\beta, \cos(\beta-\alpha) \equiv c_{\beta-\alpha},\ m_{12}^2 \equiv \bar{m}^2 s_\beta c_\beta$$

- ***Alignment limit***: for $c_{\beta-\alpha} = 0$ the SM interactions for h are recovered
- Notation for **THCs**:



$$= -ivn!\lambda^{(0)}_{hh_ih_j}$$

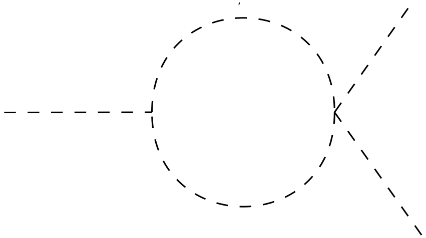
$$\kappa_\lambda^{(0,1)} \equiv \frac{\lambda^{(0,1)}_{hhh}}{\lambda^{(0)}_{\mathrm{SM}}}$$

with $\lambda^{(0)}_{\mathrm{SM}} = \frac{m_h^2}{2v^2} \simeq 0.13$

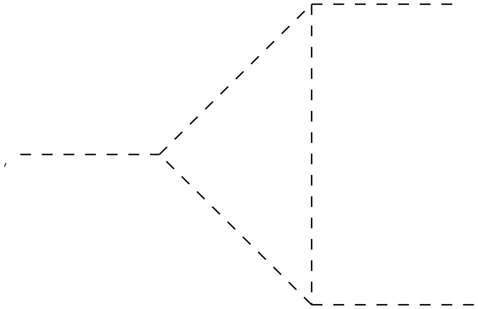
($n =$ # identical bosons)

Triple Higgs Couplings at 1 Loop

- **Effective potential**
  - ‘On-shell’ renormalization: 1L parameters are set equal to their tree-level values
  - `BSMPT` *[Basler, Biermann, Mühlleitner, Müller, Santos, 24]*



- **Full diagramatic** approach for $\lambda^{(1)}_{hhh}$
  - On-shell conditions for masses, angles, and WFRs MS-bar for m^2_{12} (small scale dependence)
  - The *finite momentum* effects are included!
  - `anyH3` *[Bahl, Braathen, Gabelmann, Weiglein, 23]*



- They will capture the pure scalar 1L corrections to $e^+e^- \to hhZ$ *(expected to be the main ones)*

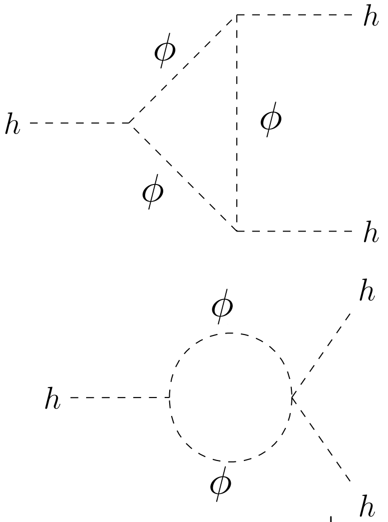
THCs: tree vs 1loop with constraints

Type	$\kappa_\lambda^{(0)}$	$\kappa_\lambda^{(1)}$	$\lambda_{hhH}^{(0)}$	$\lambda_{hhH}^{(1)}$
I	[-0.2, 1.2]	[0.2, 6.8]	[-1.6, 1.5]	[-2.1, 1.9]
II	[0.6, 1.0]	[0.7, 5.6]	[-1.5, 1.6]	[-1.7, 2.0]
LS	[0.5, 1.0]	[0.6, 5.6]	[-1.7, 1.7]	[-2.0, 2.1]
FL	[0.7, 1.0]	[0.8, 5.6]	[-1.6, 1.3]	[-1.9, 1.5]

- Scan of the parameter space
- Applied ***constraints*** to the 2HDM
    - EWPO
    - Tree-level unitarity + potential stability
    - BSM Higgs boson searches
    - Properties of the SM-like Higgs boson
        - *Close to the alignment!*
    - Flavor Observables

[ScannerS + HiggsTools + HDECAY]

κ_λ: tree level vs 1 loop



$\phi = H, A, H^\pm$

Type	$\kappa_\lambda^{(0)}$	$\kappa_\lambda^{(1)}$	$\lambda_{hhH}^{(0)}$	$\lambda_{hhH}^{(1)}$
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FL	[0.7, 1.0]	[0.8, 5.6]	[-1.6, 1.3]	[-1.9, 1.5]

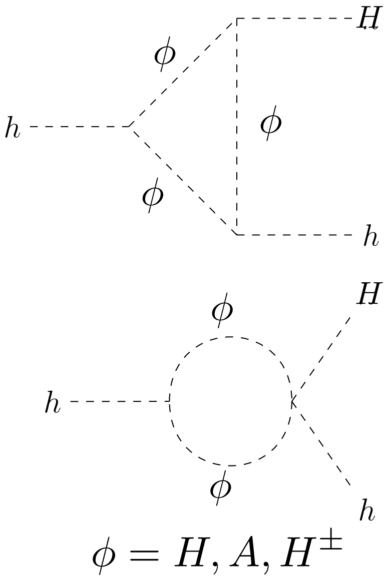
(results from the effective potential)

- Very large corrections are possible! $\lambda_{hhh}^{(1)} >> \lambda_{hhh}^{(0)}$
- h couplings to heavy Higgs bosons can be large ($\lambda_{h\phi\phi} \sim 15$)
  - Even at the ***alignment limit*** !!!

[FA, Heinemeyer, Herrero, 21, 22]

(In the SM, top-loops are ~ -8%)

λ_{hhH} : tree level vs 1 loop



$\phi = H, A, H^\pm$

Type	$\kappa_\lambda^{(0)}$	$\kappa_\lambda^{(1)}$	$\lambda_{hhH}^{(0)}$	$\lambda_{hhH}^{(1)}$
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(results from the effective potential)

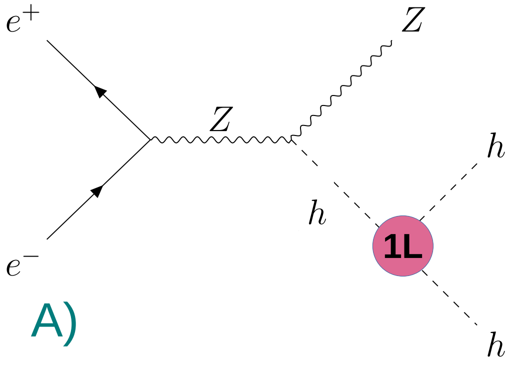
- 1L corrections for λ_{hhH} are *not as significant* as for λ_{hhh}
- Still interesting results: $\lambda_{hhH}^{(1)} \gtrsim \lambda_{hhH}^{(0)} \sim 0$ or change of sign in λ_{hhH}

Effects from THCs at $e^+e^- \to hhZ$

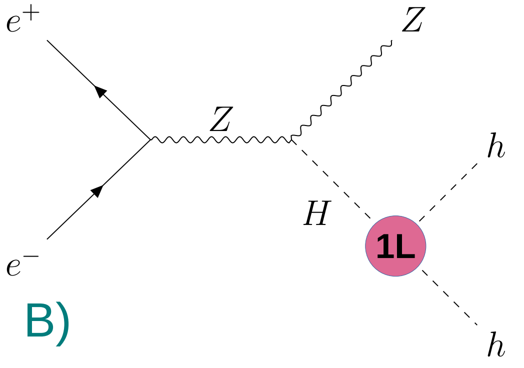
A) **Non-resonant** diagram with $\boldsymbol{\kappa_\lambda}$ $\Rightarrow$ at low m_{hh}

B) **Resonant $\boldsymbol{H}$** diagram with $\boldsymbol{\lambda_{hhH}}$ $\Rightarrow$ at $m_{hh} \simeq m_H$

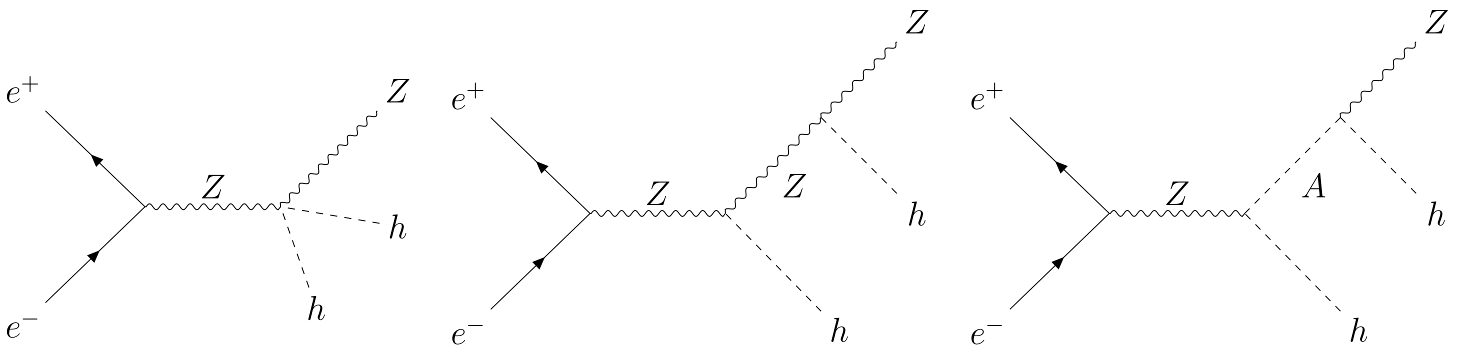
C) Resonant A diagram (no THC)



A)



B)



C)

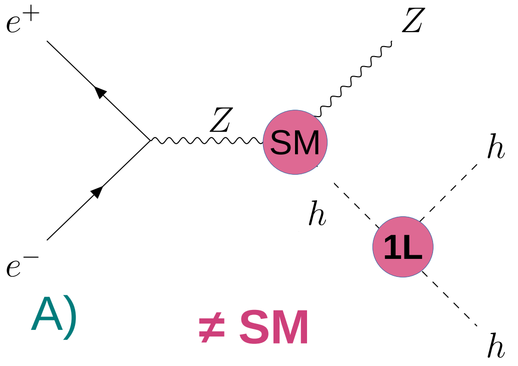
In the alignment limit ($c_{\beta-\alpha} = 0$)

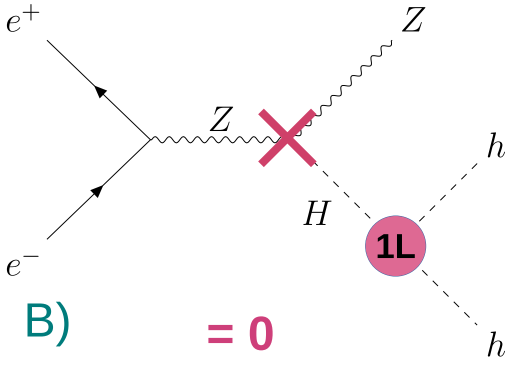
A) **Non-resonant** diagram
wit h

B) **Re** am
wit $\simeq m_H$

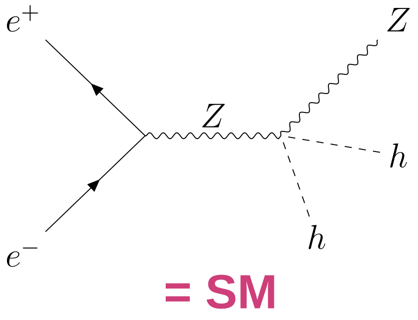
C) Resonant A diagram
(no THC)

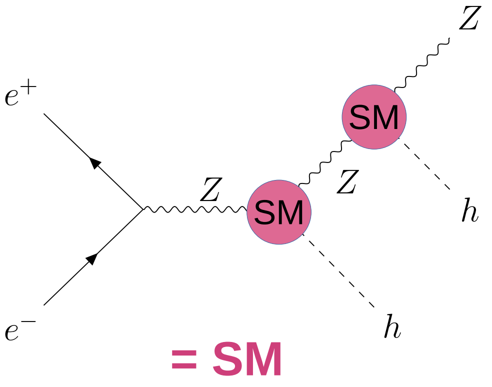
Only BSM effects in $\kappa_\lambda^{(1)}$

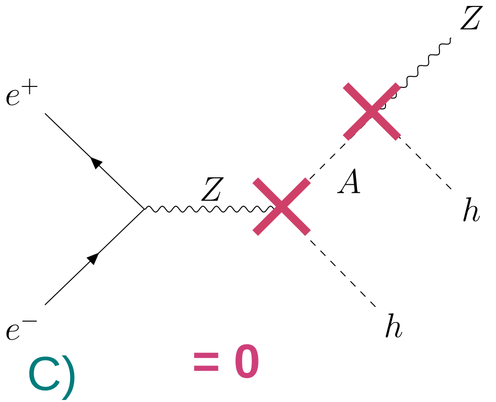




$$\kappa_\lambda^{(0)} = 1,$$
$$\lambda_{hhH}^{(0)} = 0$$





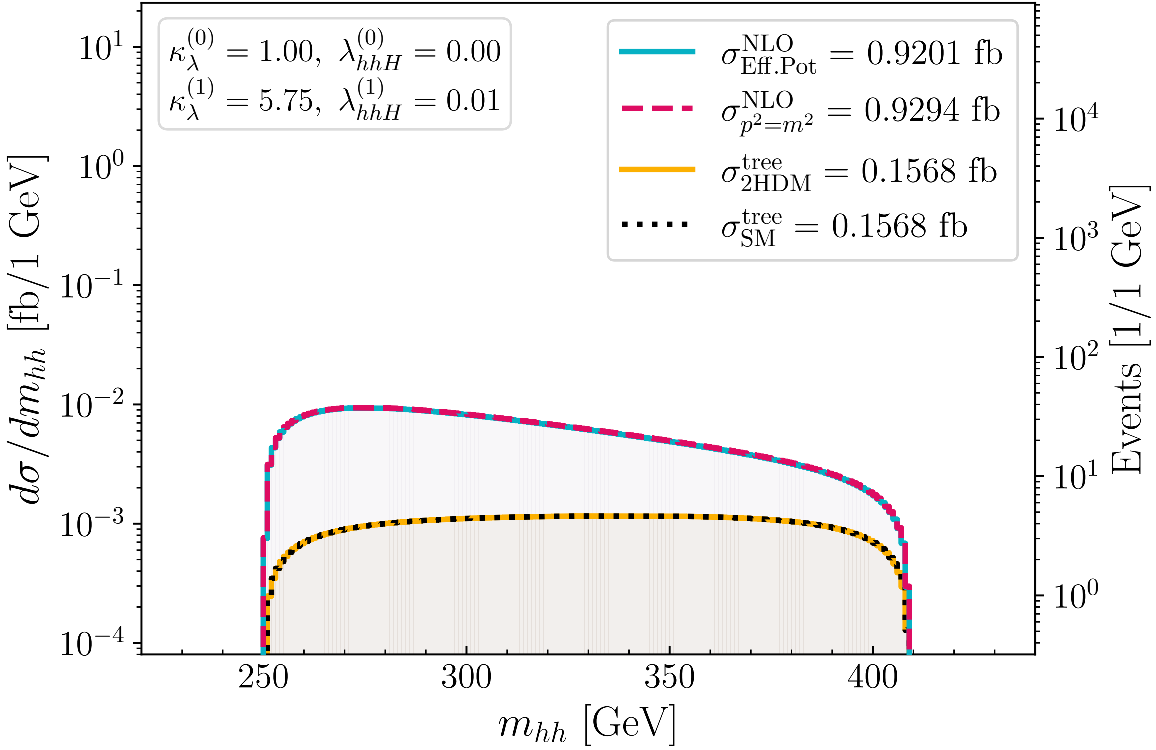


Access to $\kappa_\lambda^{(1)}$

Large 1L κ_λ @ILC500GeV

$$\text{BPal, all types!}$$
$$m_H = \bar{m} = 400\ \text{GeV},$$
$$m_A = m_{H^\pm} = 800\ \text{GeV},$$
$$\tan\beta = 3,\ \cos(\beta-\alpha) = 0$$

- XS 6 times larger than the tree-level !!!
- Momentum effects on $\kappa_\lambda^{(1)}(m_{hh})$ around 1-2%
- Better access, and sensitivity, to $\kappa_\lambda^{(1)}$ than in the SM!



Large 1L κ_λ @ILC1TeV

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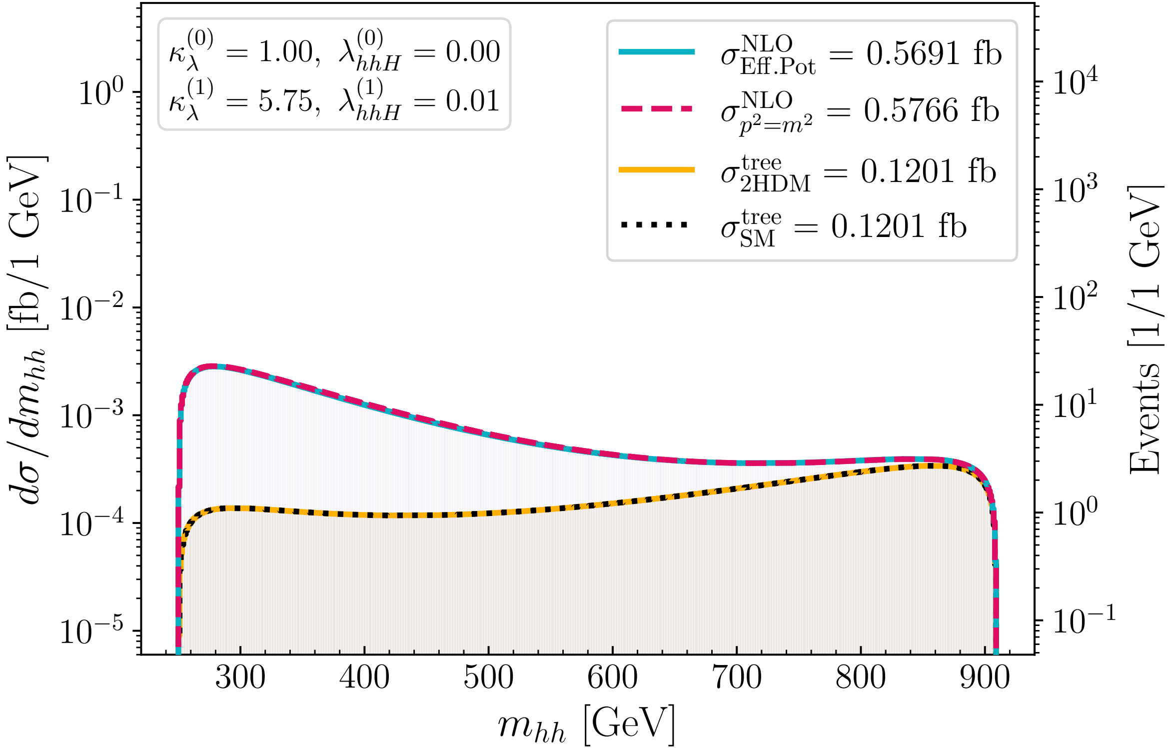
$$\text{BPal, all types!}$$

$$m_H = \bar{m} = 400\ \text{GeV},$$

$$m_A = m_{H^\pm} = 800\ \text{GeV},$$

$$\tan\beta = 3,\ \cos(\beta - \alpha) = 0$$

- Similar to the 500 GeV case
- All points with large scalar couplings can potentially lead to large κ_λ at 1L !!!



Access to $\lambda^{(1)}_{hhH}$

Expected events at the H peak

- We look at the expected **final 4b-jet events**:
  - b-tagging efficiency: ϵ_b = 80%
  - Acceptance $\mathcal{A}$ after the detection cuts:

$$\bar{N}_{4bZ} = N_{4bZ} \times \mathcal{A} \times \epsilon_b$$

$$\mathrm{p}_T^Z > 20\ \mathrm{GeV},\ p_T^b > 20\ \mathrm{GeV}, |\eta_b| < 2,\ \Delta R_{bb} > 0.4$$

- **Smearing** of the theoretical prediction for the m_{hh} distributions due to finite detector resolution
  - We consider 2% and 5% smearing
- **Size of the bin**:
  - Bins with at least 2 events inside the kinematically allowed region

Similar (but improved) analysis to
[FA, Heinemeyer, Herrero,21] [FA, Heinemeyer, Radchenko, Mühlleitner, 22]

‘Sensitivity’ to the H resonance

- **Theoretical ‘estimator’** to the possible access to the H resonance (and to λ_{hhH}) from the resonance (R) and the ‘continuum’ (C) (i.e. $\lambda_{hhH}=0$) events:

$$R=\sqrt{2\left(\left(s+b\right)\log\left(1+\frac{s}{b}\right)-s\right)}$$

$$s=\sum_i\left|\bar{N}^R_{i,4bZ}-\bar{N}^C_{i,4bZ}\right|$$

$$b=\sum_i\bar{N}^C_{i,4bZ}$$

- Three methods to compute the estimator R:
  - ‘Signal region’: consider bins where C and R separated by 2 or 3 std
  - Compute R for all bins and sum in quadrature

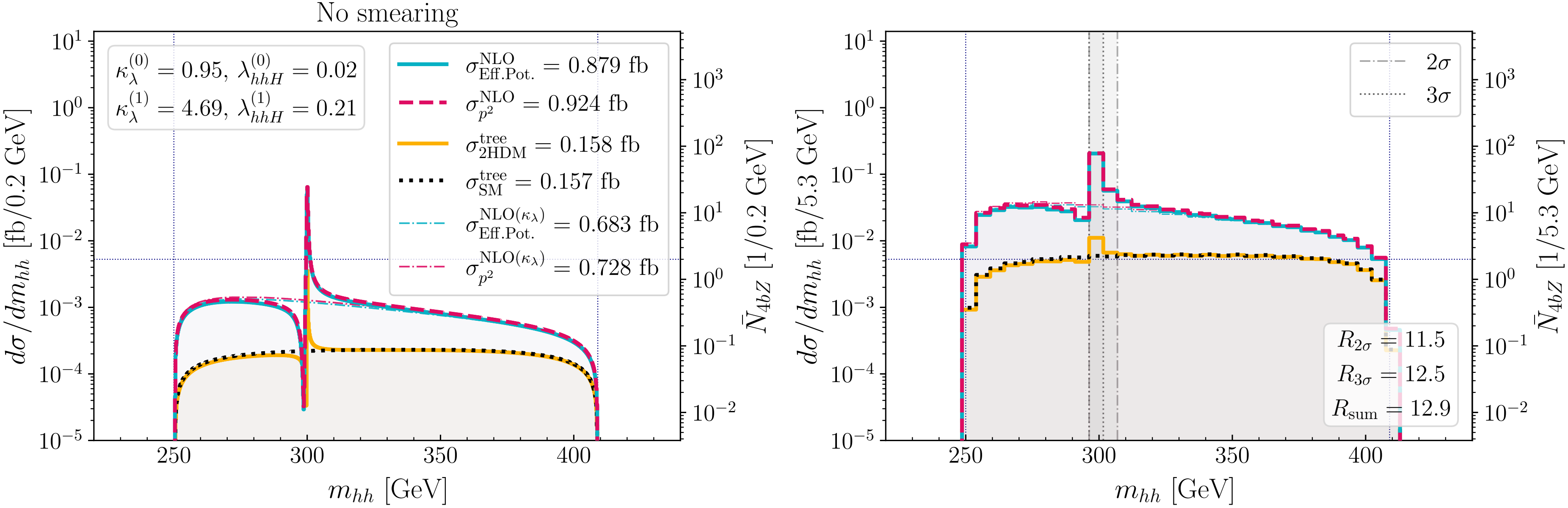
Similar (but improved) analysis to
[FA, Heinemeyer, Herrero,21] [FA, Heinemeyer, Radchenko, Mühlleitner, 22]

Large 1L λ_{hhH} @ILC500GeV (no smear)

BPlahhH-1, type I
$m_H = \bar{m} = 300$ GeV,
$m_A = m_{H^\pm} = 650$ GeV,
$\tan\beta = 12,\ \cos(\beta-\alpha) = 0.12$

- Large effect from $\kappa_\lambda^{(1)}$
- For this point $\lambda_{hhH}^{(0)} \sim 0 \ll \lambda_{hhH}^{(1)}$

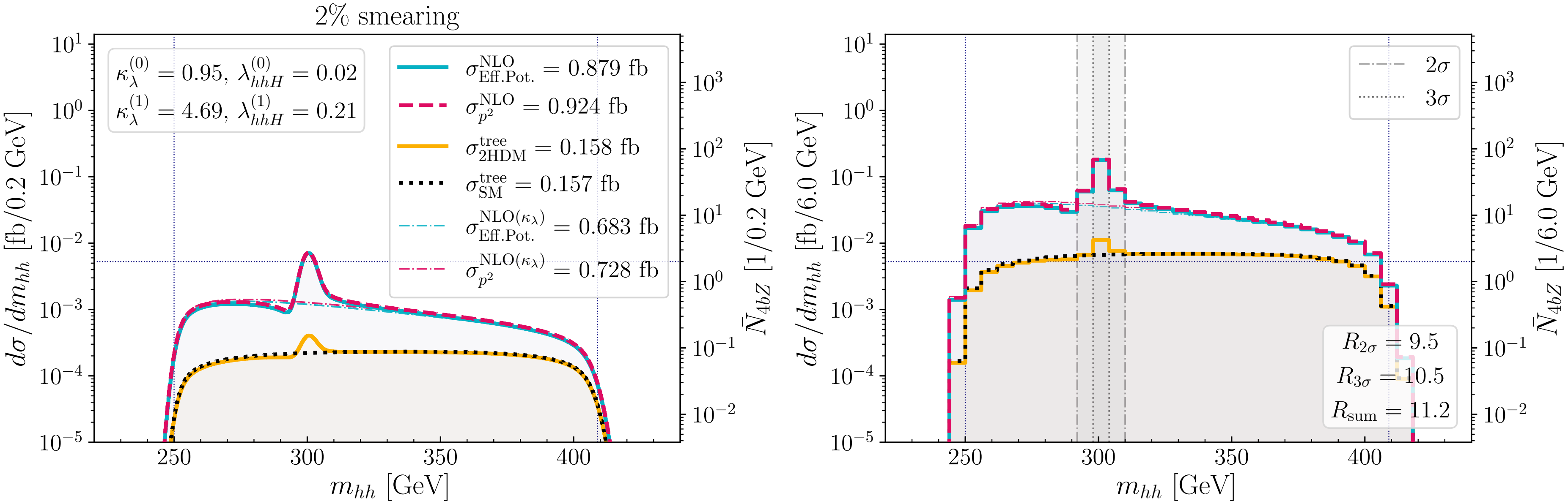
$\Rightarrow$ the H resonance is more prominent



Large 1L λ_{hhH} @ILC500 + 2% smear

BPlahhH-1, type I
$m_H = \bar{m} = 300$ GeV,
$m_A = m_{H^\pm} = 650$ GeV,
$\tan\beta = 12,\ \cos(\beta - \alpha) = 0.12$

- The resonance gets ‘smeared’
- Values for R get worst



Large 1L λ_{hhH} @ILC500 + 5% smear

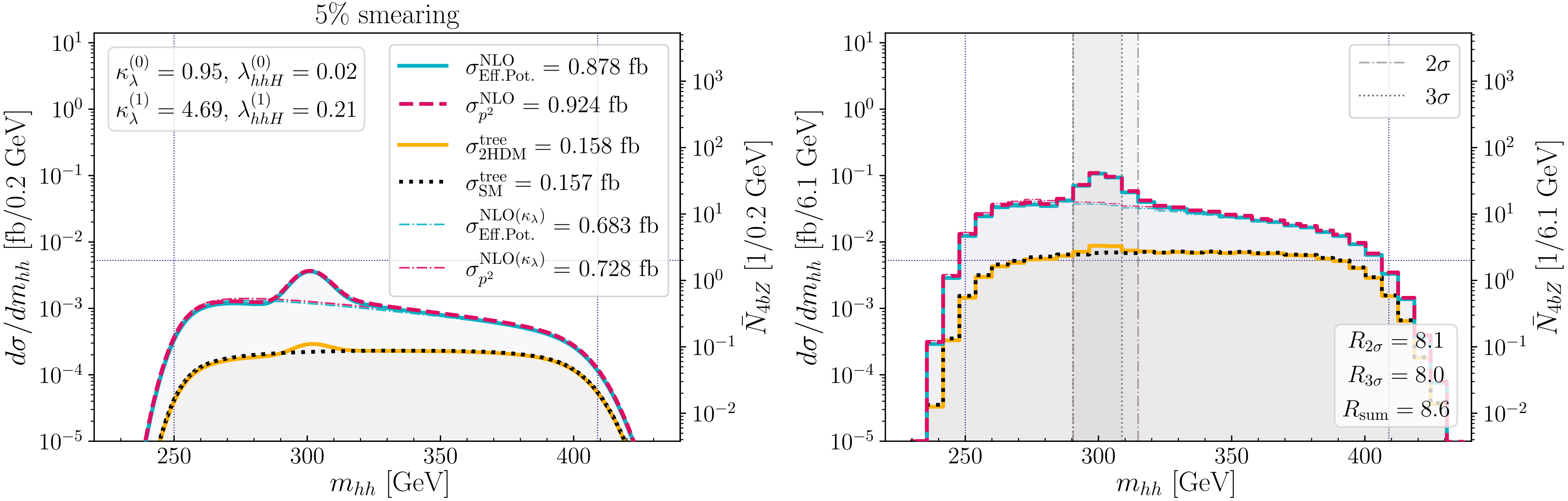
BPlahhH-1, type I

$m_H = \bar{m} = 300$ GeV,

$m_A = m_{H^\pm} = 650$ GeV,

$\tan\beta = 12,\ \cos(\beta - \alpha) = 0.12$

- The resonance gets even more ‘smeared’
- Values for R get even worst

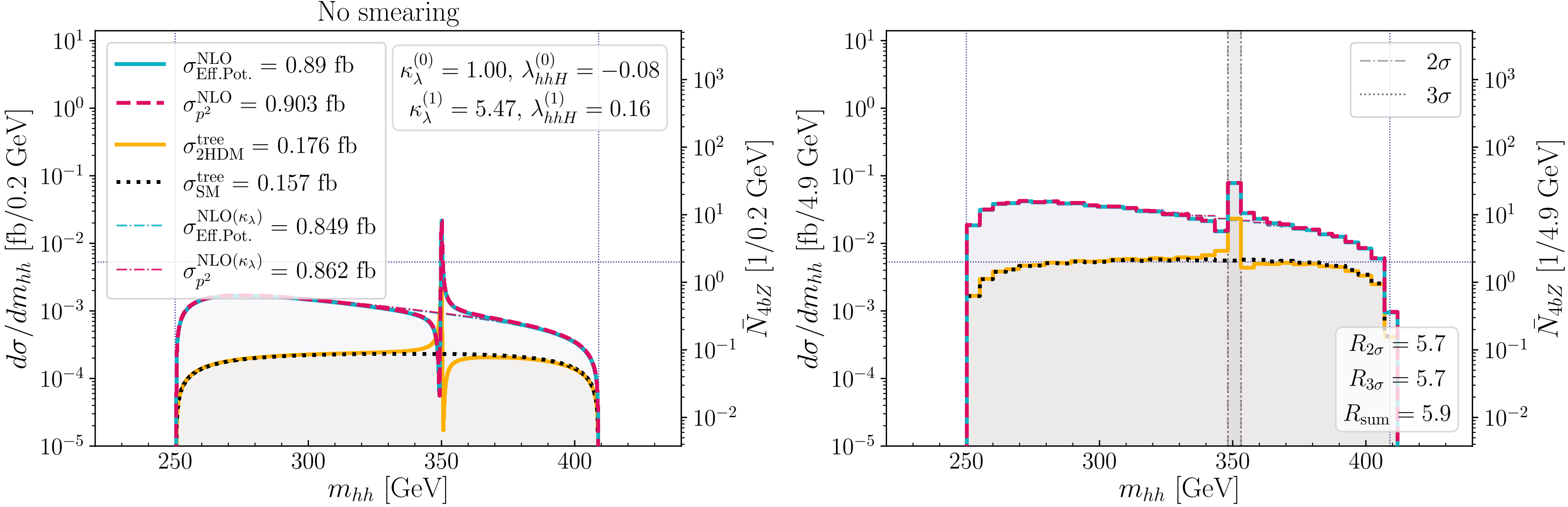


1L λ_{hhH} with different sign @ILC500GeV (no smear)

BPsign, type I

$$m_H = \bar{m} = 350 \text{ GeV},$$
$$m_A = m_{H^\pm} = 650 \text{ GeV},$$
$$\tan\beta = 20,\ \cos(\beta - \alpha) = 0.1$$

- In this point: $\text{sign}\left(\lambda_{hhH}^{(1)}\right) \neq \text{sign}\left(\lambda_{hhH}^{(0)}\right)$
  $\Rightarrow$ changes the dip-peak structure of the peak!
- Large effect from $\kappa_\lambda^{(1)}$



1L λ_{hhH} with different sign @ILC500GeV + 2% smear

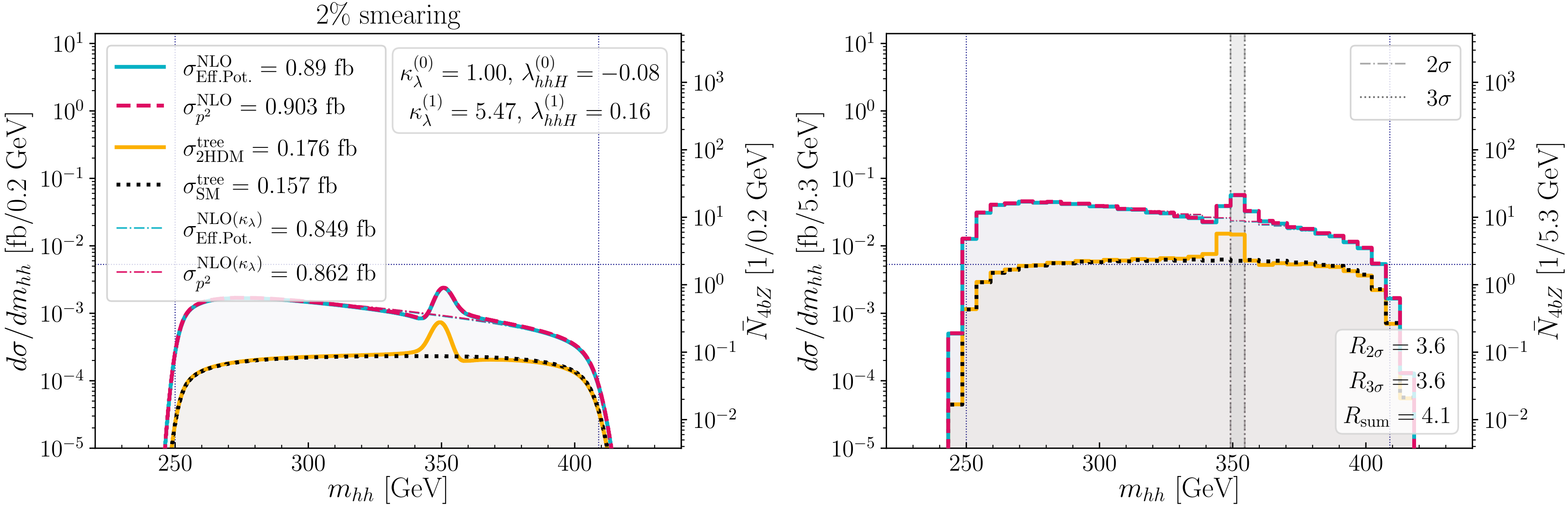
BPsign, type I

$m_H = \bar{m} = 350$ GeV,

$m_A = m_{H^\pm} = 650$ GeV,

$\tan\beta = 20, \ \cos(\beta - \alpha) = 0.1$

- The dip-peak structure gets washed out
  - No apparent sensitivity to the coupling sign

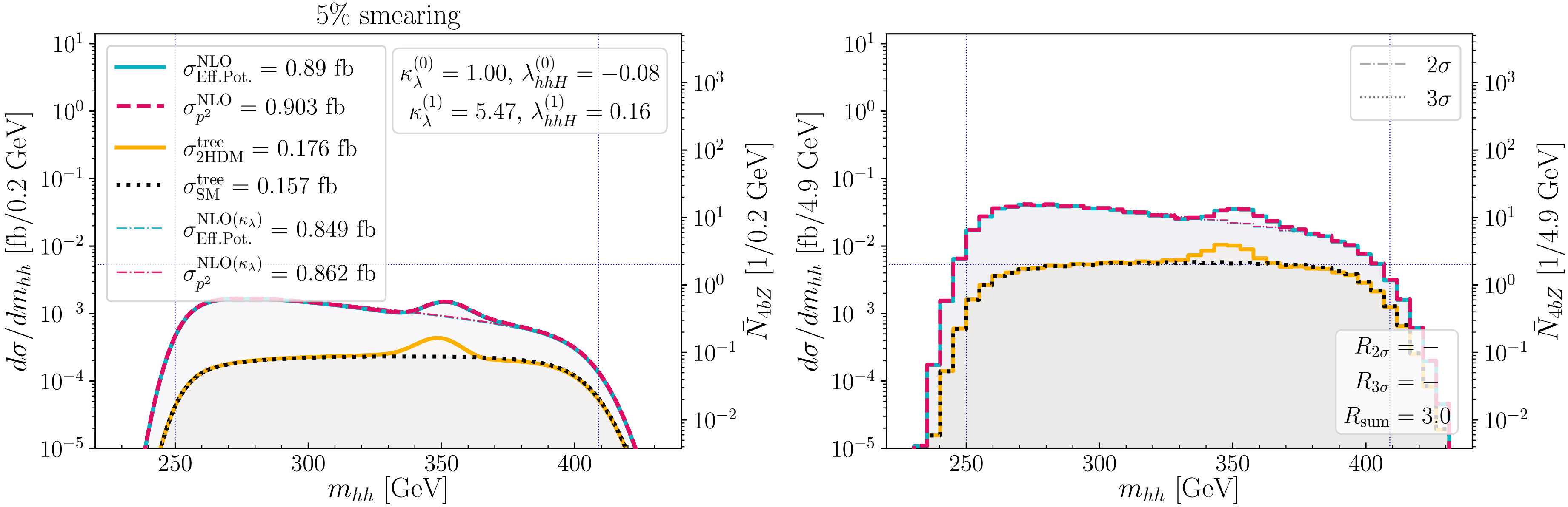


1L λ_{hhH} with different sign @ILC500GeV + 5% smear

BPsign, type I

$$m_H = \bar{m} = 350\ \text{GeV},$$
$$m_A = m_{H^\pm} = 650\ \text{GeV},$$
$$\tan\beta = 20,\ \cos(\beta-\alpha) = 0.1$$

- The resonance peak gets very difficult to detect :(



Results for R :

- Smearing decreases the value of R
    - Still optimistic results: the κ_λ enhancement helps
- Challenging access resonance H peaks and dip-peak/peak-dip structures
- Still a full experimental analysis is needed!

Point	$\sqrt{s}$	Smearing	Bin size	#bins$_{2\sigma}$	#bins$_{3\sigma}$	$R_{2\sigma}$	$R_{3\sigma}$	R_{sum}
BPlahhH-1	500	0%	5.3	2	1	11.5	12.5	12.9
BPlahhH-1	500	2%	6.0	3	1	9.5	10.5	11.2
BPlahhH-1	500	5%	6.1	4	3	8.1	8.0	8.6
BPlahhH-1	500	10%	5.3	5	3	6.0	5.1	6.6
BPlahhH-1	500	15%	5.1	4	0	4.1	-	5.7
BPlahhH-2	500	0%	8.1	2	2	10.5	10.5	10.6
BPlahhH-2	500	2%	7.6	2	2	10.6	10.6	11.0
BPlahhH-2	500	5%	9.0	4	2	8.2	8.3	8.9
BPlahhH-2	500	10%	9.0	5	4	6.9	6.7	7.2
BPlahhH-2	500	15%	8.7	6	3	5.9	4.7	6.4
BPlahhH-3	500	0%	9.0	2	1	3.4	2.8	4.5
BPlahhH-3	500	2%	9.2	1	1	2.6	2.6	4.4
BPlahhH-3	500	5%	9.4	2	0	2.9	-	4.1
BPlahhH-3	500	10%	10.4	1	0	1.8	-	3.5
BPlahhH-3	500	15%	10.7	0	0	-	-	3.1
BPsign	500	0%	4.9	1	1	5.7	5.7	5.9
BPsign	500	2%	5.3	1	1	3.6	3.6	4.1
BPsign	500	5%	4.9	0	0	-	-	3.0
BPsign	500	10%	4.8	0	0	-	-	2.2
BPsign	500	15%	3.9	0	0	-	-	1.8
BPext	500	0%	4.9	2	2	12.0	12.0	12.6
BPext	500	2%	4.9	2	2	11.7	11.7	11.9
BPext	500	5%	4.9	4	4	9.6	9.6	10.0
BPext	500	10%	4.8	7	5	8.2	7.6	8.8
BPext	500	15%	3.9	10	1	7.1	2.6	8.1

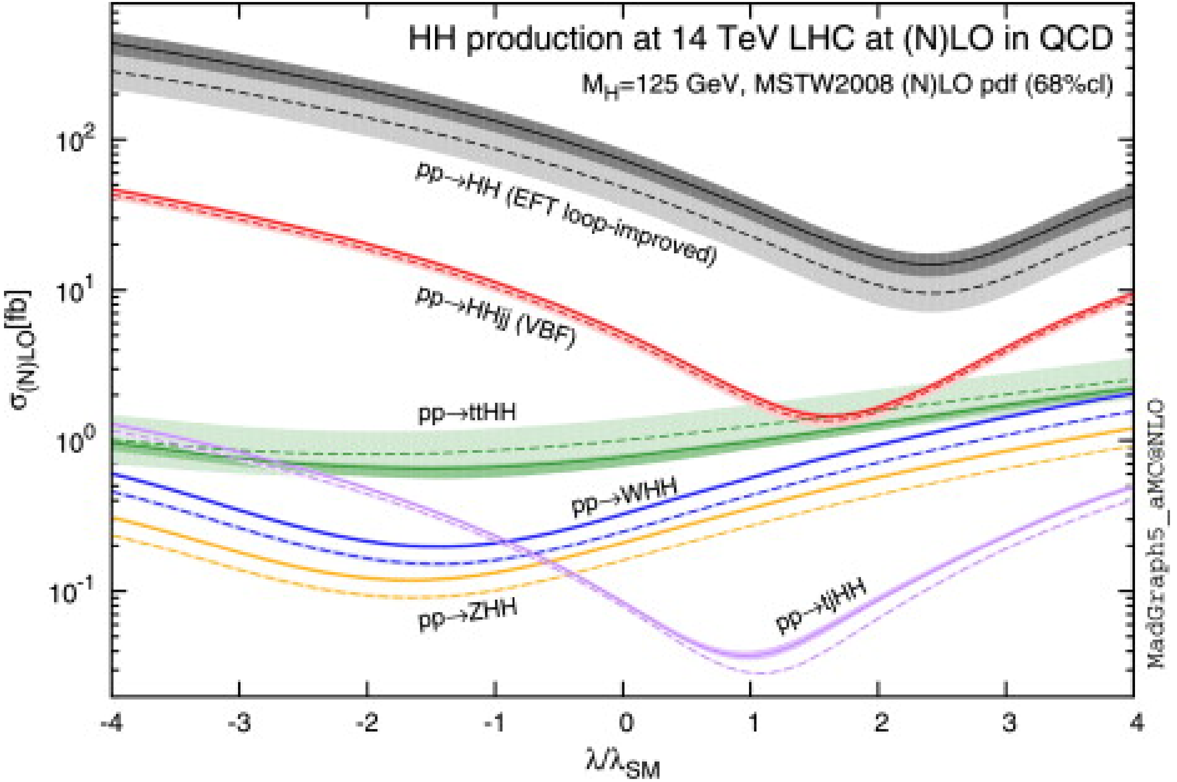
Summary & Conclusions

- Analysis of the **1L corrected triple Higgs couplings κ_λ and λ_{hhH}**, and their impact in **double Higgs production** at e^+e^- colliders in the 2HDM, specifically $e^+e^- \to hhZ$ at ILC
- **1L corrections to κ_λ can be very large**, *even in the alignment limit !!!*
  - Very distinct prediction even for a very SM-like Higgs boson!
  - No relevant effects from finite momentum
- **1L corrected λ_{hhH} can lead to interesting pheno!** Access via the H resonance peak
  - Analysis of the **final 4b-jet events + smearing + bin size**: access to the resonance peak may be challenging *(but an experimental analysis is needed)*
  - Resolution in the m_{hh} distributions will be crucial

Thanks for your attention! :)

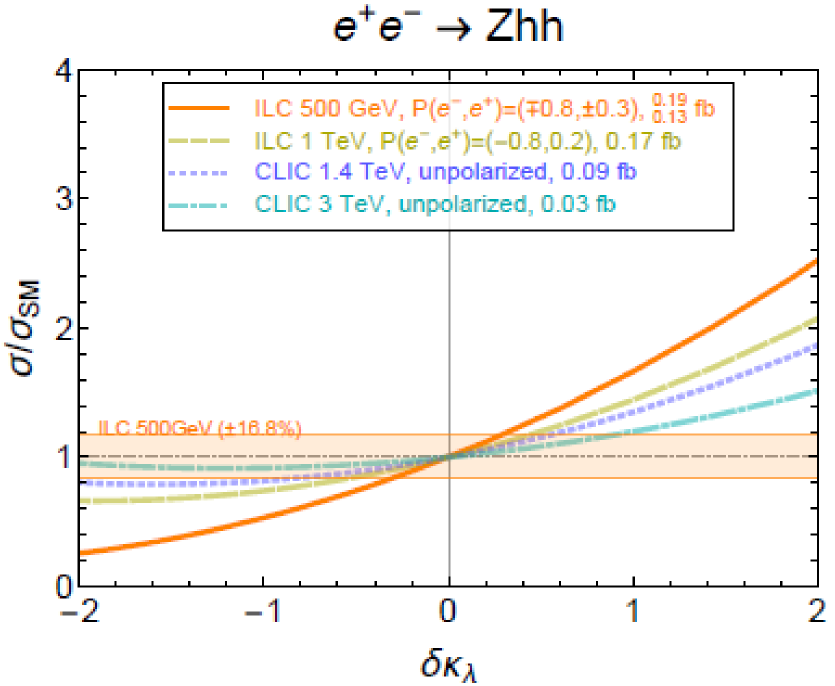
Back up

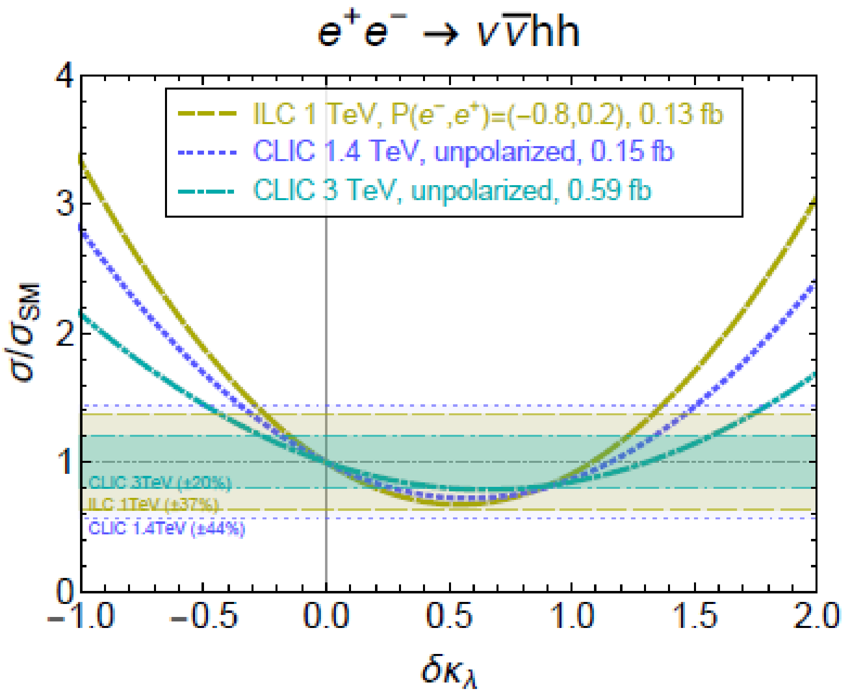
XS vs κ_λ in the SM at LHC



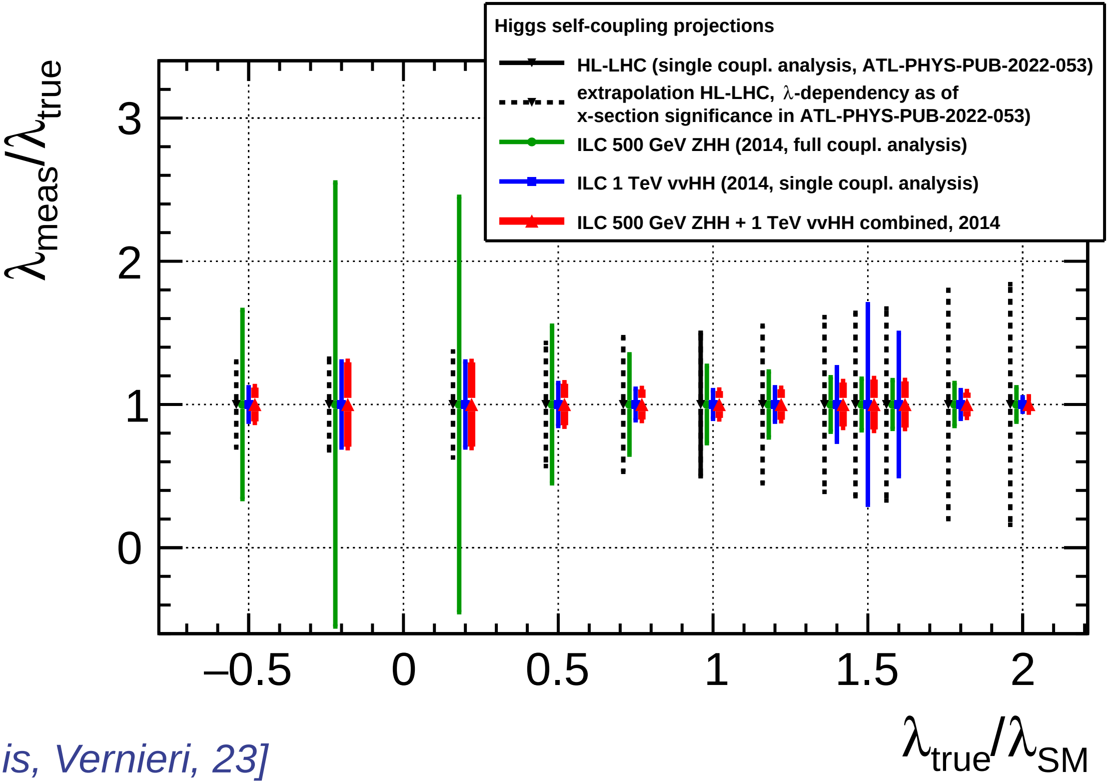
XS vs κ_λ in the SM at e^+e^- colliders

[Di Vita, Durieux, Grojean, Gu, Liu, Panico, Riembau, Vantalon, 18]





$\kappa_\lambda \neq 1$ at HL-LHC and e^+e^- colliders



[Torndal, List, Ntounis, Vernieri, 23]

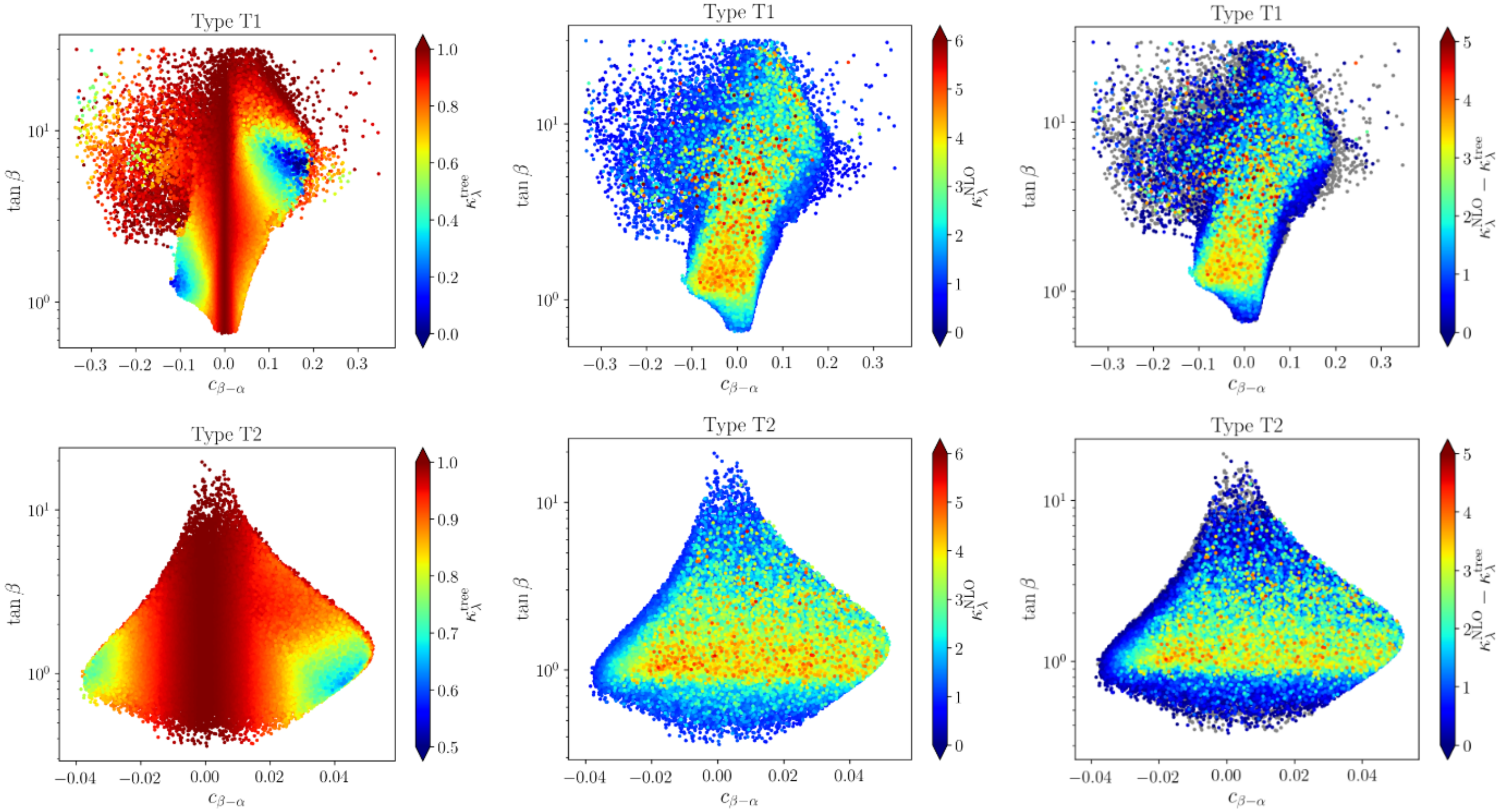
Main corrections to κ_λ

[Kanemura,Kiyoura,Okada,Senaha,Yuan,02]

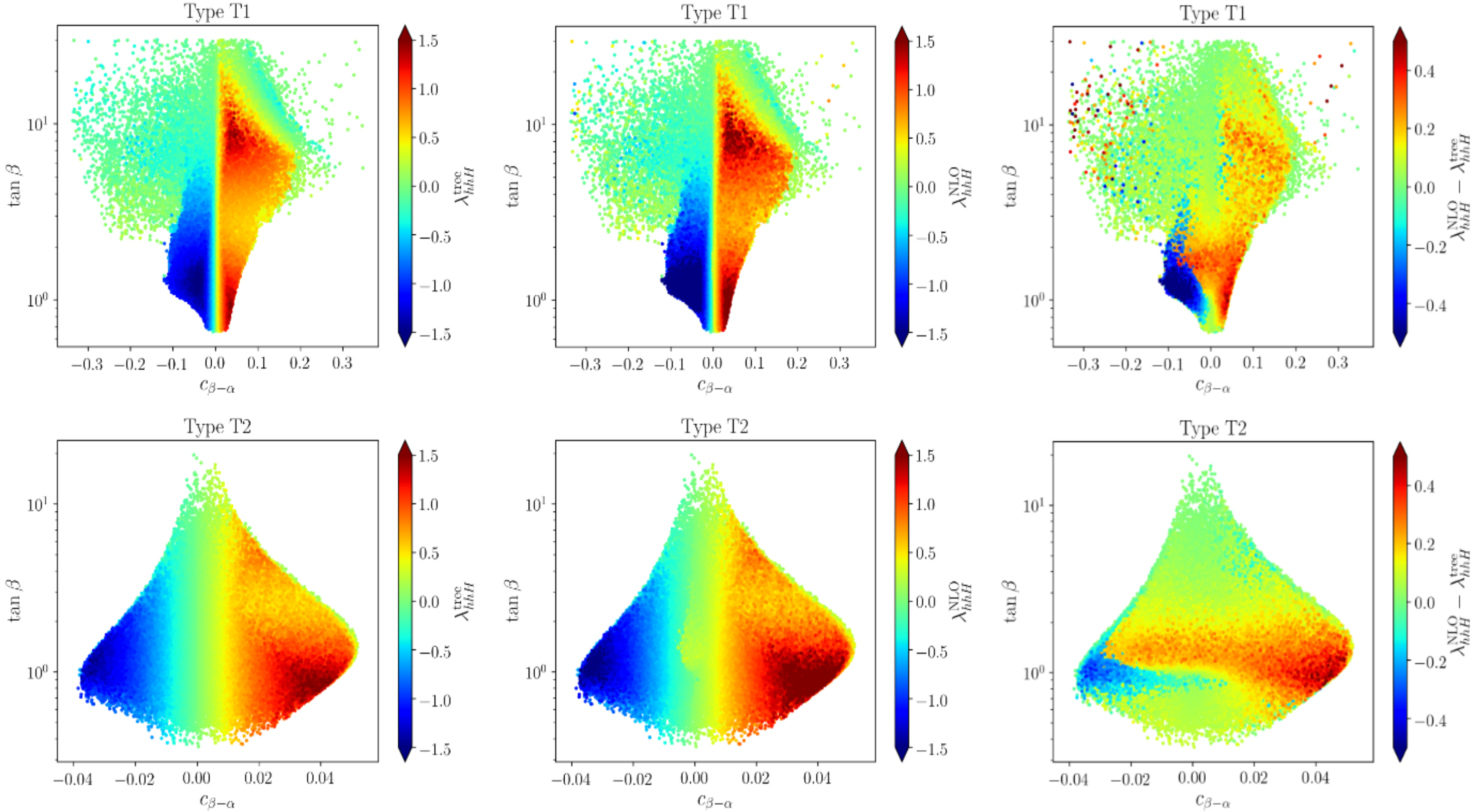
$$\kappa_\lambda^{(1)} \equiv \frac{\lambda_{hhh}^{(1)}}{\lambda_{\rm SM}^{(0)}} \simeq 1 + \sum_{\phi=H,A,H^\pm} \frac{m_\phi^4}{12\pi^2 m_h^2 v^2}\left(1-\frac{\bar{m}^2}{m_\phi^2}\right)^3$$

$$\lambda_{\rm SM}^{(1)} \simeq \lambda_{\rm SM}^{(0)}\left(1-\frac{m_t^4}{\pi^2 m_h^2 v^2}\right) \qquad \lambda_{\rm SM}^{(0)} = \frac{2m_h^2}{v^2} \simeq 0.13$$

Results for κ_λ



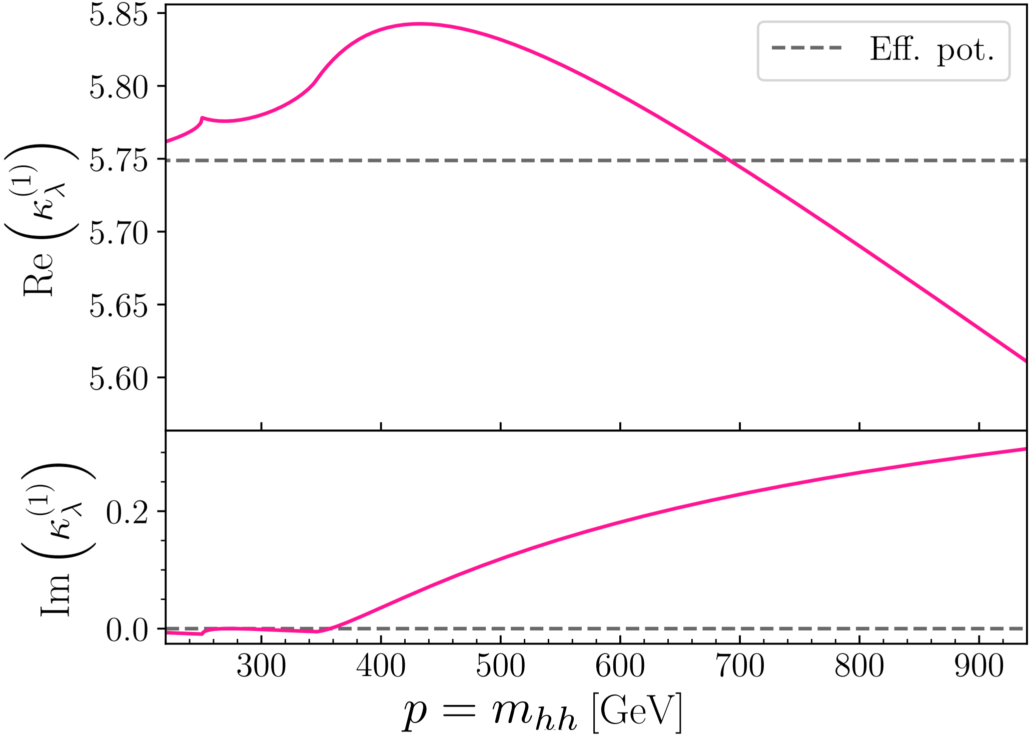
Results for λ_{hhH}



Example for large κ_λ at 1 loop

$$\text{BPal, all types!}$$
$$m_H = \bar{m} = 400 \text{ GeV},$$
$$m_A = m_{H^\pm} = 800 \text{ GeV},$$
$$\tan\beta = 3,\ \cos(\beta - \alpha) = 0$$

- Large $\kappa_\lambda^{(1)}$ due to large $\lambda_{hAA}^{(0)}$ and $\lambda_{hH^+H^-}^{(0)}$
- Good agreement between effective potential and diagramatic computation
  - Momentum dependence more important for large momentum



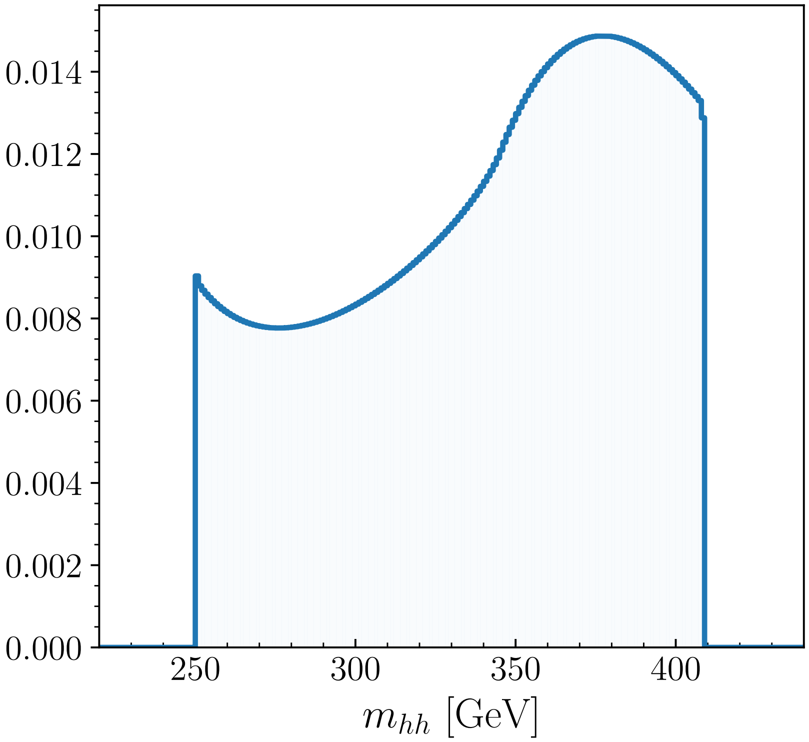
Relative difference w/ and wo/ p

BPal, all types!

$$m_H = \bar{m} = 400 \text{ GeV},$$

$$m_A = m_{H^\pm} = 800 \text{ GeV},$$

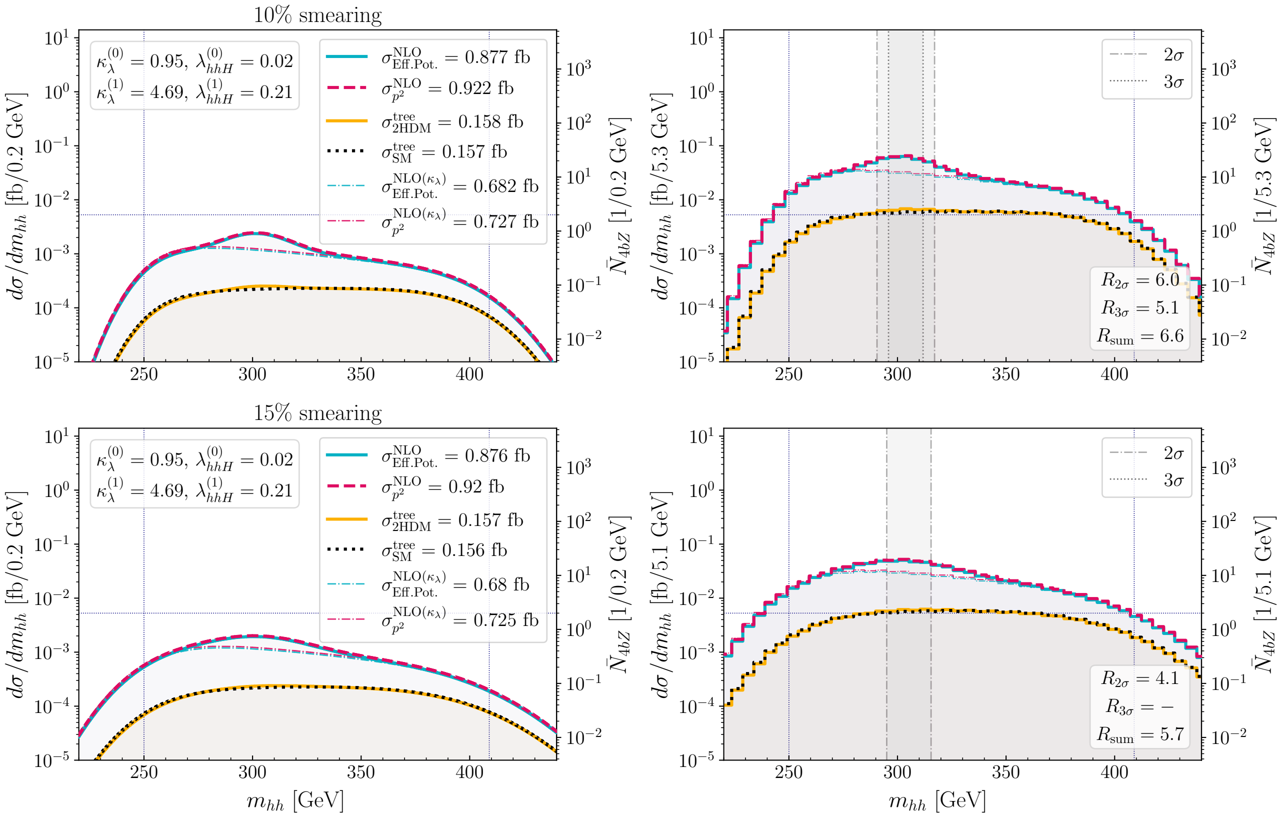
$$\tan\beta = 3,\ \cos(\beta - \alpha) = 0$$



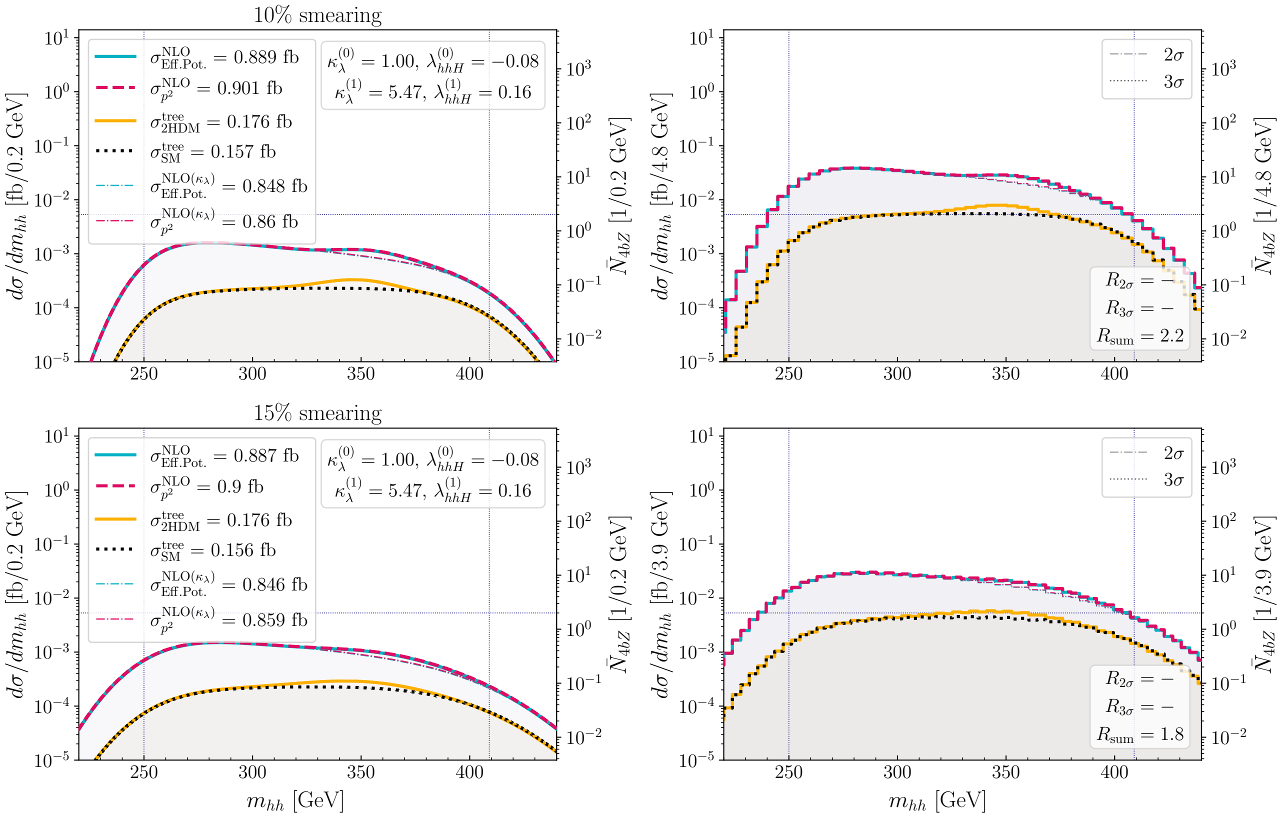
Results for R @1TeV:

Point	$\sqrt{s}$	Smearing	Bin size	$\#\text{bins}_{2\sigma}$	$\#\text{bins}_{3\sigma}$	$R_{2\sigma}$	$R_{3\sigma}$	R_{sum}
BPlahhH-1	1000	0%	12.4	1	1	11.5	11.5	11.6
BPlahhH-1	1000	2%	14.2	1	1	11.0	11.0	11.1
BPlahhH-1	1000	5%	14.4	2	2	8.2	8.2	8.8
BPlahhH-1	1000	10%	15.4	2	2	6.9	6.9	7.3
BPlahhH-1	1000	15%	15.4	4	2	6.1	5.4	6.4
BPlahhH-2	1000	0%	22.6	1	1	17.7	17.7	17.8
BPlahhH-2	1000	2%	22.3	1	1	17.7	17.7	17.8
BPlahhH-2	1000	5%	19.0	2	2	16.0	16.0	16.1
BPlahhH-2	1000	10%	19.2	4	4	13.6	13.6	14.5
BPlahhH-2	1000	15%	20.8	5	4	12.4	12.6	13.2
BPlahhH-3	1000	0%	32.1	1	1	3.8	3.8	4.2
BPlahhH-3	1000	2%	32.1	1	1	3.7	3.7	4.1
BPlahhH-3	1000	5%	29.9	1	1	3.7	3.7	4.4
BPlahhH-3	1000	10%	29.9	2	1	3.5	3.0	3.8
BPlahhH-3	1000	15%	26.7	2	0	3.0	-	3.5
BPsign	1000	0%	11.6	1	1	6.7	6.7	6.8
BPsign	1000	2%	12.4	1	1	6.1	6.1	6.2
BPsign	1000	5%	14.2	2	1	4.3	3.5	4.5
BPsign	1000	10%	14.9	2	0	3.2	-	3.5
BPsign	1000	15%	14.4	0	0	-	-	3.0

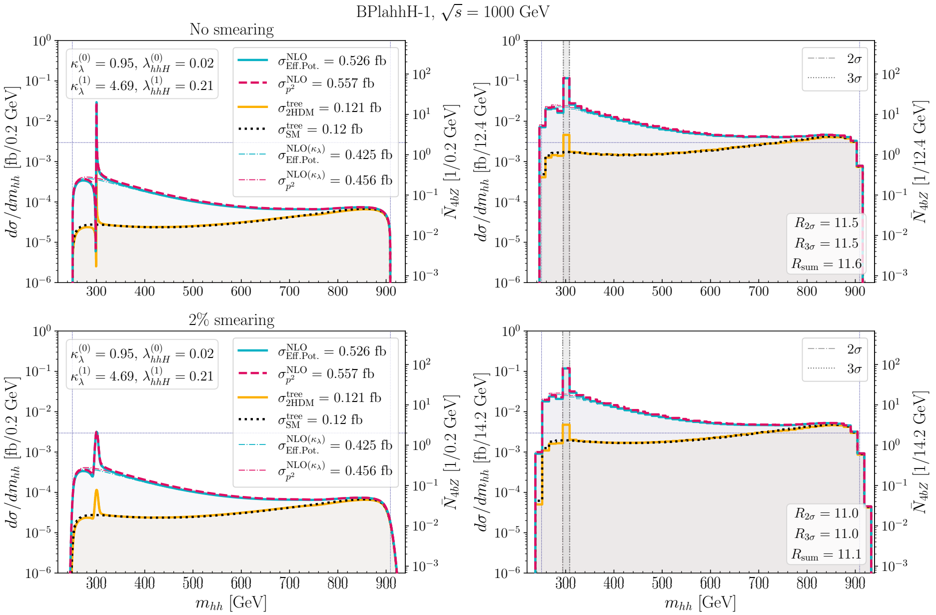
BPlahhH-1 @ILC 500GeV

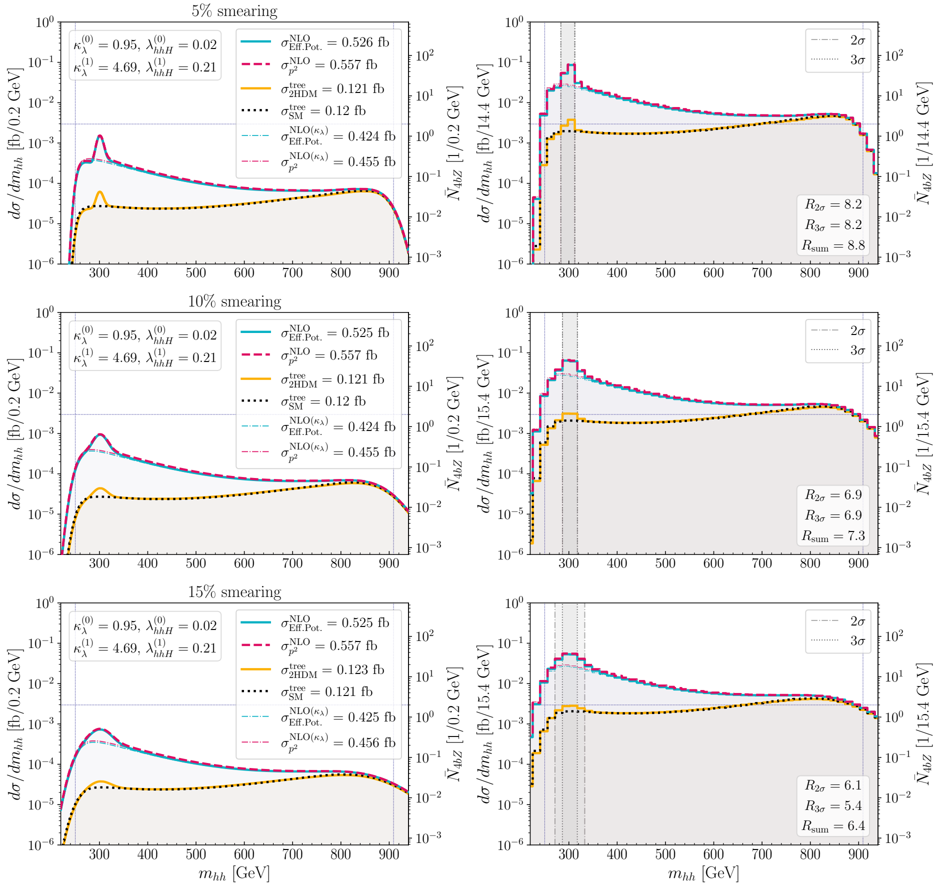


BPsign @ILC 500GeV

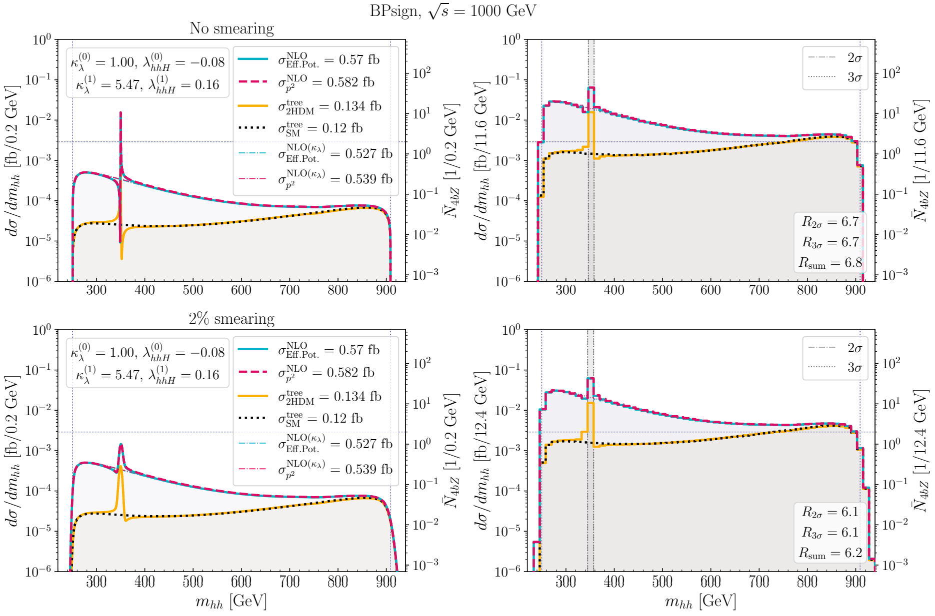


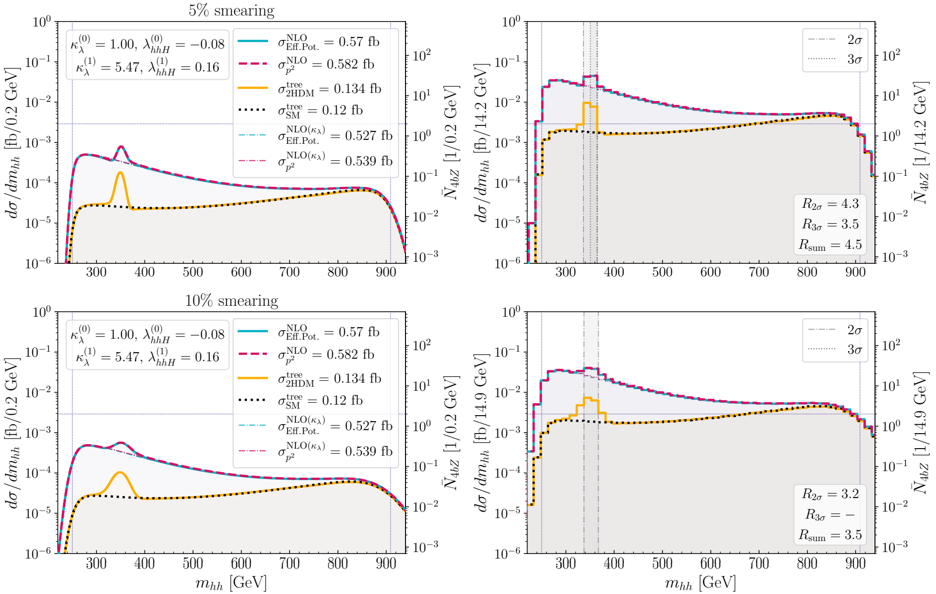
BPlahhH-1 @ILC 1TeV





BPsign @ILC 1TeV





2HDM Yukawa couplings

$$\mathrm{L}_{\mathrm{Yukawa}} \supset -\sum_{f=u,d,l} \frac{m_f}{v} \left[\xi_f^h \bar{f} f h + \xi_f^H \bar{f} f H + \xi_f^A \bar{f} \gamma_5 f A\right]$$

$$-\frac{\sqrt{2}}{v}\left[\bar{u}\left(\xi_d V_{\mathrm{CKM}} m_d P_R - \xi_u m_u V_{\mathrm{CKM}} P_L\right) d H^+ + \xi_l \bar{\nu} m_l P_R l H^+ + \mathrm{h.c.}\right]$$

	Type I	Type II	Type III	Type IV
ξ_u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ξ_d	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
ξ_l	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$

with $\xi_f^h = s_{\beta-\alpha} + \xi_f c_{\beta-\alpha}\,, \xi_f^H = c_{\beta-\alpha} - \xi_f s_{\beta-\alpha}\,, \xi_u^A = -i\xi_u\,, \xi_{d,l}^A = i\xi_{d,l}$

Great access to λ_{hhH} @CLIC 3TeV !

- Estimated “sensitivity” to λ_{hhH} from the expected **final 4b jet events** at the H peak:
  - Overall, larger “sensitivity” to λ_{hhH} at VBF channel at @3TeV
  - Also good “sensitivity” if m_H is very low

[FA, Heinemeyer, Herrero, 21]