Higgs Self-Coupling: Theory Status

LCWS2024

July 10th 2024

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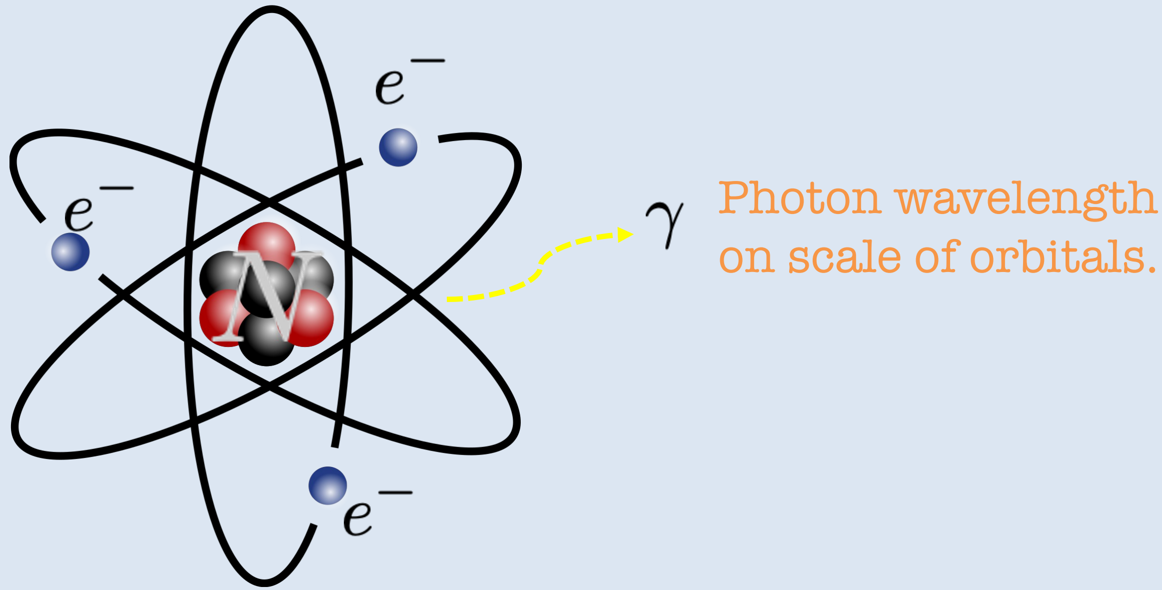
CERN

Zooming in on the Higgs

Effective Field Theory Basics

Consider exploring a neutral atom at eV energies:

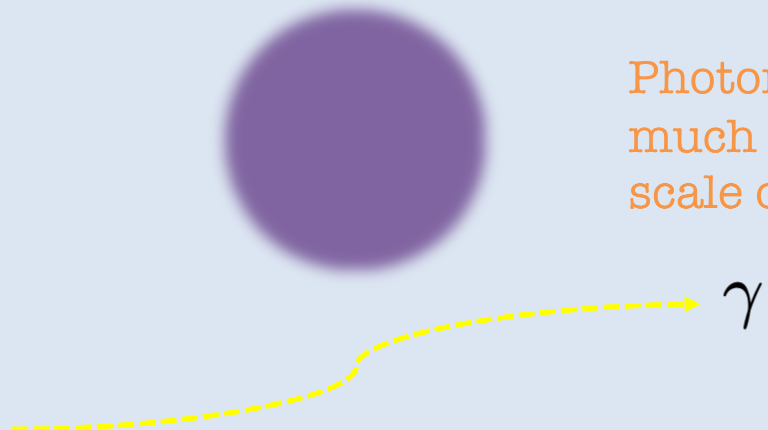


The appropriate theory at this length scale contains the photon, electrons and nucleus:

$$\mathcal{L} = \mathcal{L}(\gamma, e^-, N)$$

Effective Field Theory Basics

Consider exploring a neutral atom at much lower energies:



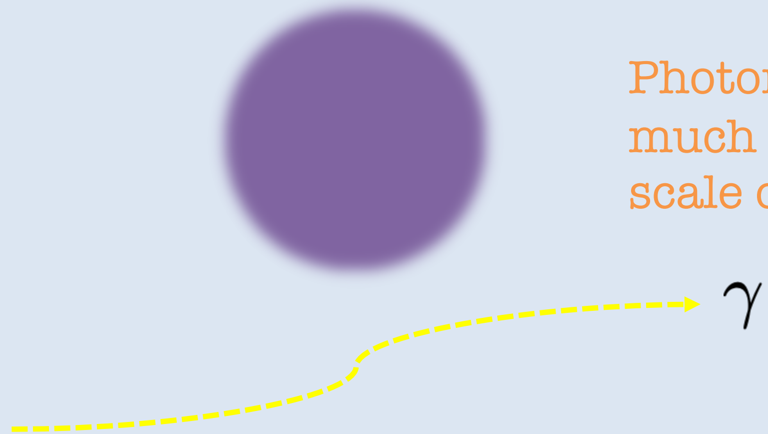
Photon wavelength much greater than scale of orbitals.

The appropriate theory at this length scale contains the photon and neutral atom...

$$\mathcal{L} = \mathcal{L}(\gamma, \chi)$$

Effective Field Theory Basics

Consider exploring a neutral atom at much lower energies:



Photon wavelength much greater than scale of orbitals.

Crucially, the substructure is encoded in "higher dimension operators", like dipoles or Rayleigh...

$$\mathcal{L} = ... + \frac{\chi^2}{\Lambda^2} F^{\mu\nu} F_{\mu\nu} + ...$$

Effective Field Theory Basics

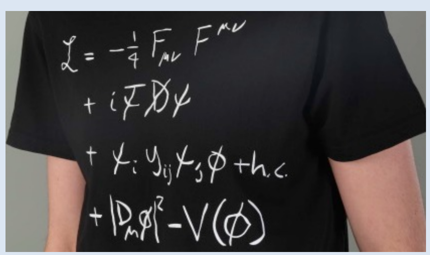
The same is true for the Higgs boson!

Collider wavelength greater than scale of microscopic new physics...

$\gamma, W, Z, g, \dots$

The Standard Model is an "Effective Field Theory". Unknown smaller distance physics in extra "operators":

$\mathcal{L} =$



$$+ \sum_{jk} \frac{c_j}{\Lambda^k} \mathcal{O}_{jk}$$

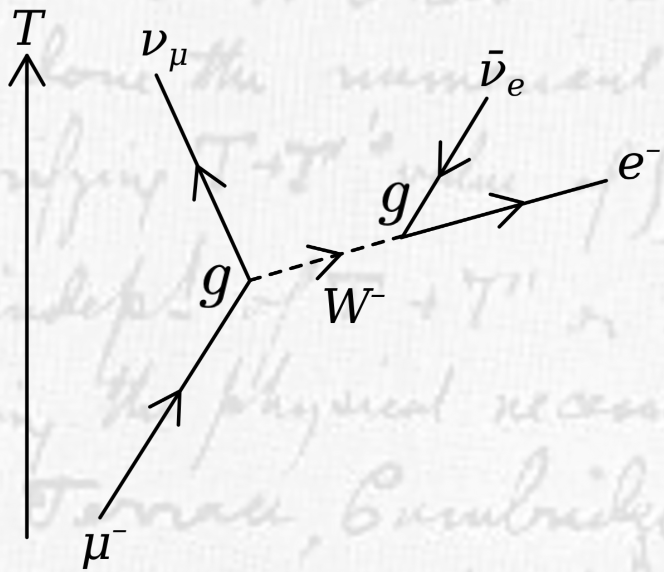
Organizing the Unknown

Naïve dimensional analysis:

$$[H] = [A_\mu] = \frac{1}{LC} \quad , \quad [\psi] = \frac{1}{L^{3/2}C}$$

Fields carry not only dimension of inverse length, but also inverse coupling.

Example: Muon Decay



Fermi Scale

Interaction: $\mathcal{L} \sim \frac{\psi^4}{\Lambda^2}$

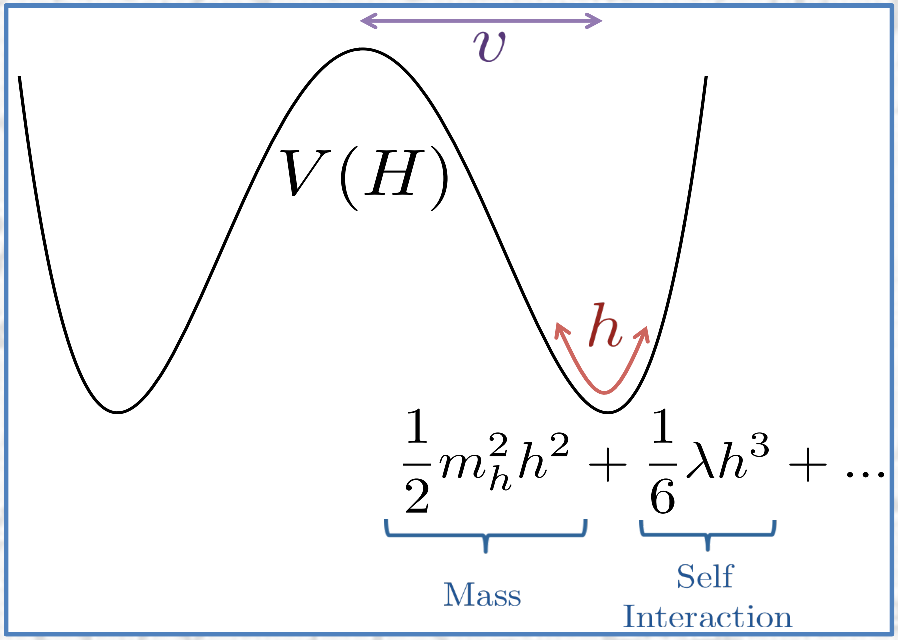
Dimension: $[\Lambda] = [G_F^{-1/2}] = \frac{[M_W]}{[g]}$

UV-completion

Coupling

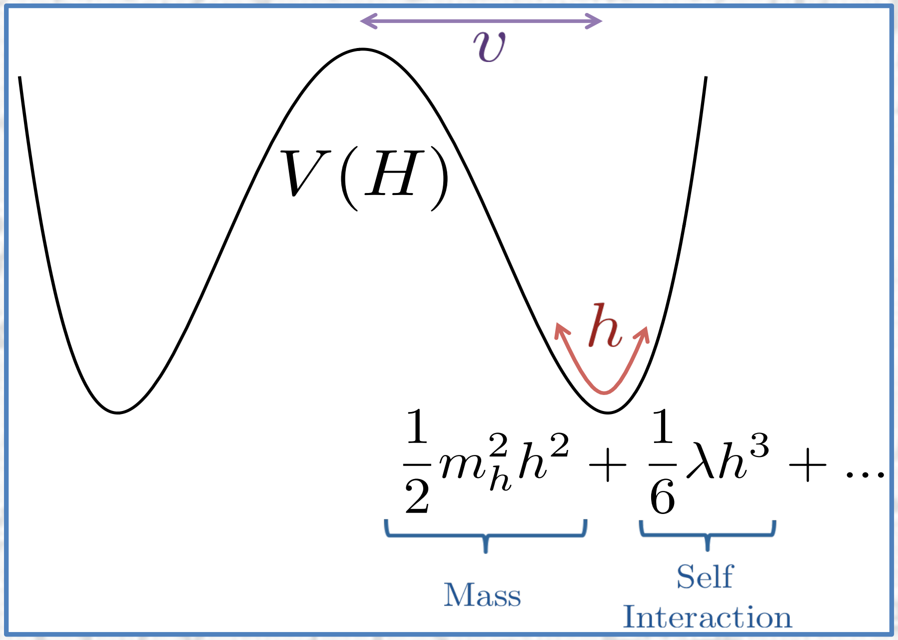
$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

What is the Higgs Field Potential?



Important because it determines how the Universe froze in the EW sector, giving mass to gauge bosons, fermions, the Higgs…

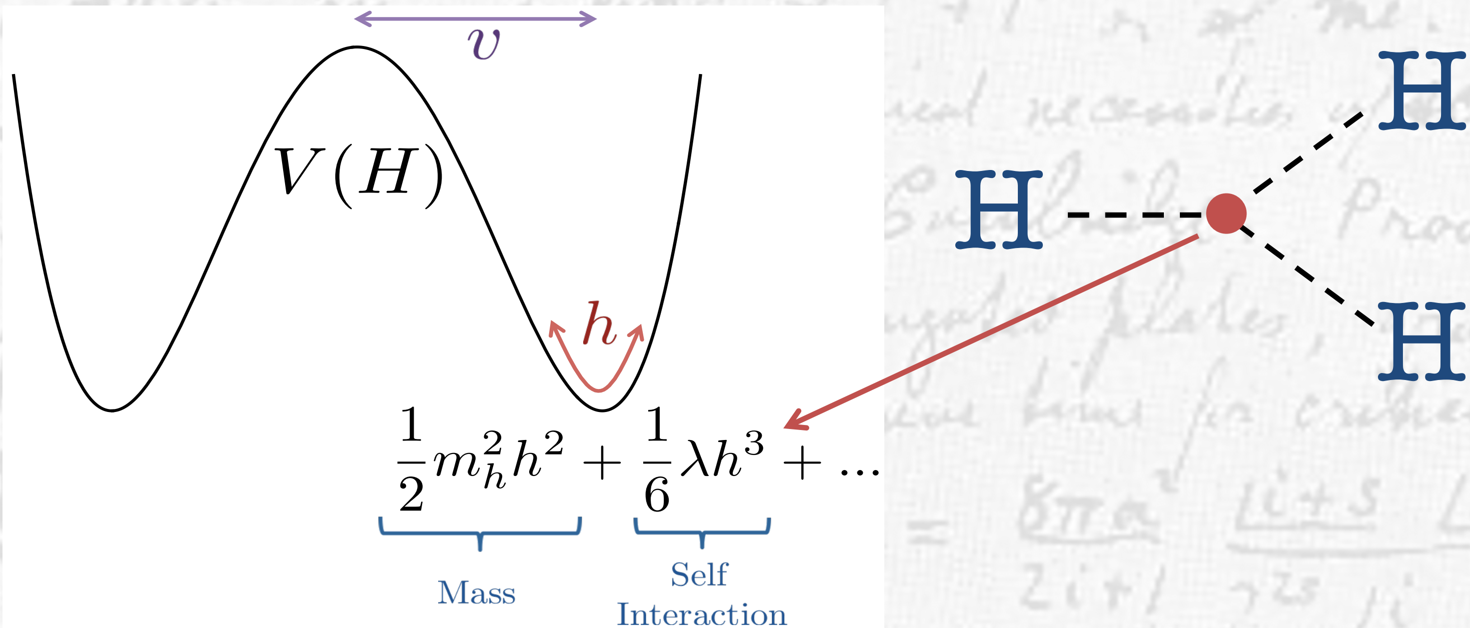
What is the Higgs Field Potential?



...because it determines how the Universe will end...

Naïve Dimensional Analysis

It's known that O_6 contributes to Higgs self-interaction, how it gives mass to itself, etc.



But less-well appreciated are the NDA aspects underlying it…

Naïve Dimensional Analysis

The fact that

$$[c_6] = [g^4]$$

and all other operator coefficients have

$$[c_j] \leq [g^2]$$

makes the self-coupling special, with one important implication I'll highlight today.

Self-Coupling Dominance

Suppose in fundamental theory leading interaction with microscopic physics is through parameter of coupling dimension

$$[y] = [g^2]$$

arising from a lower-dimension coupling with rule:

$$\kappa \propto y^2 \quad , \quad y \to -y$$

Then the only operator at $\hbar^0$ you can have is

$$\frac{\kappa |H|^6}{M^2}$$

all other dim-6 at least quantum-loop suppressed!

Self-Coupling Dominance

In other words, no obstruction to having Higgs self-coupling modifications a "loop factor" greater than **all** other couplings. Could have

$$\left|\frac{\delta_{h^3}}{\delta_{VV}}\right| \lesssim \min\left[\left(\frac{4\pi v}{m_h}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

without fine-tuning any parameters, as big as,

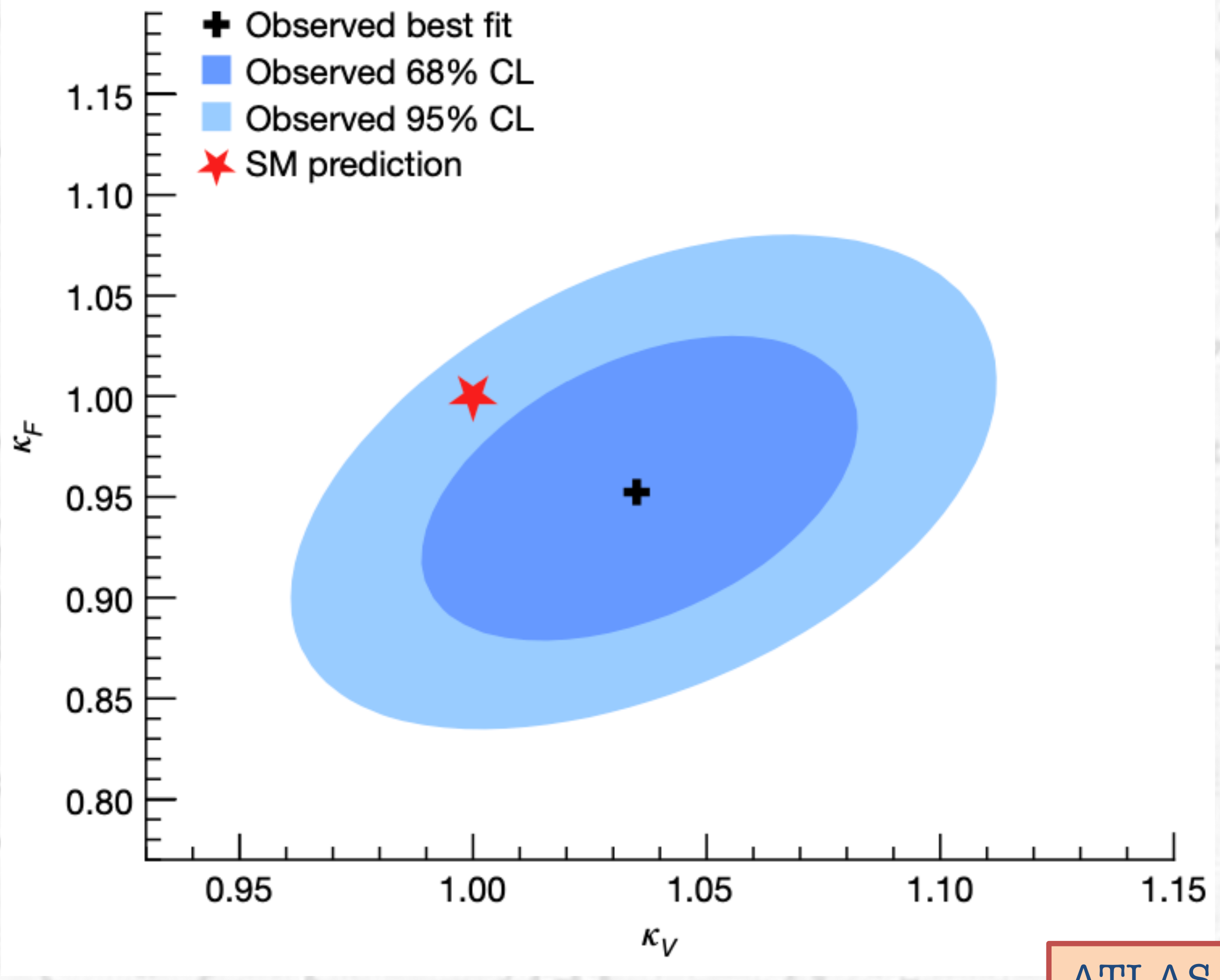
$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?



ATLAS, Nature, 2022

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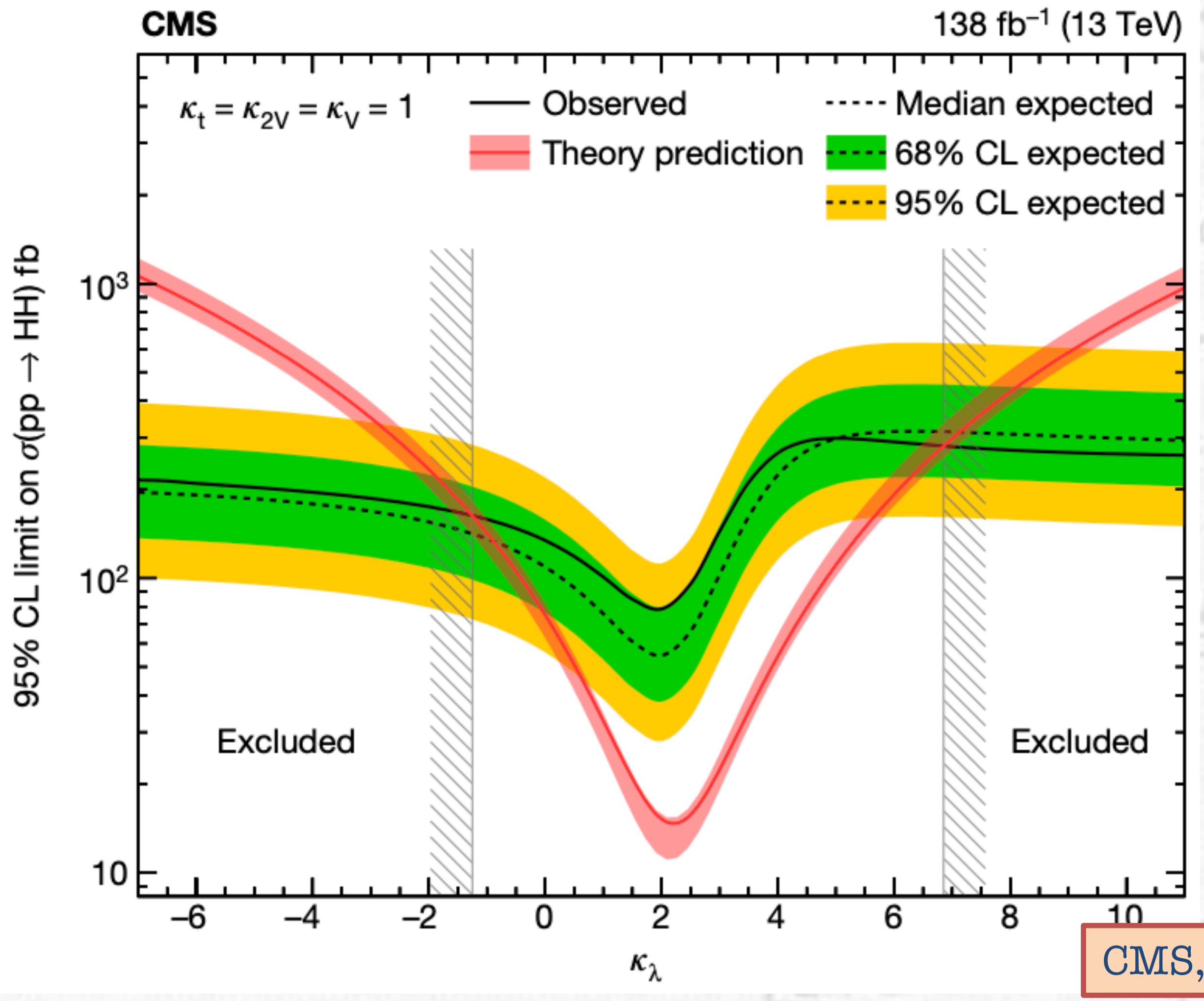

CMS, Nature, 2022

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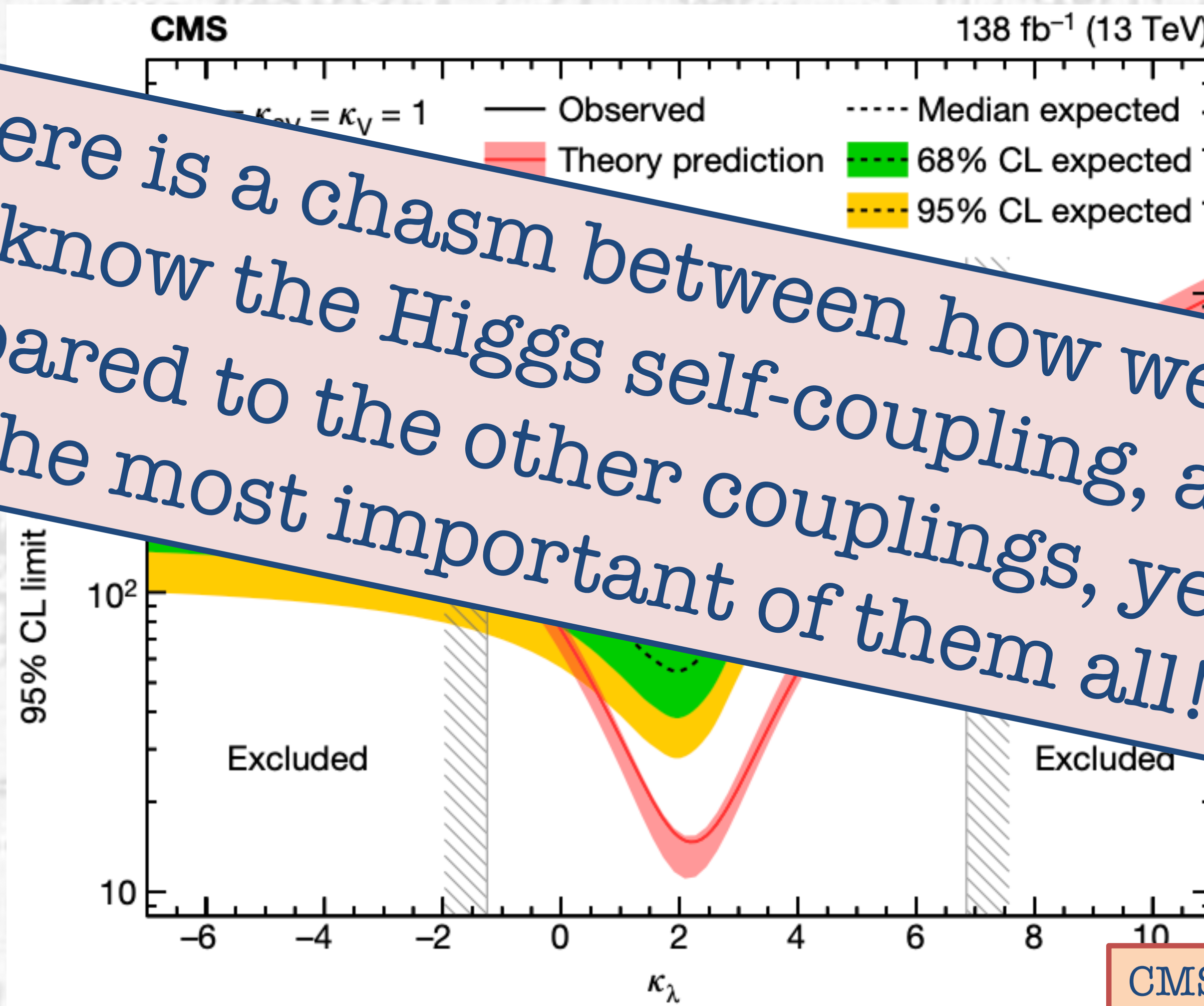


There is a chasm between how well we know the Higgs self-coupling, as compared to the other couplings, yet it is the most important of them all!

CMS, Nature, 2022

Self-Coupling Dominance

In other words, no obstruction from to having Higgs self-coupling modifications a loop factor greater than **all** other couplings. Could have

$$\left|\frac{\quad}{\delta_{VV}}\right| \quad \left[\left(\frac{4\pi v}{\quad}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

But can such a theory exist in practise?

without fine-tuning any parameters, as

$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Custodial Quadruplet

This is all well and good, but does such a theory exist? Yes: The custodial quadruplet scalar. Projecting the $(\mathbf{4},\mathbf{4})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ onto EW group we have

$$(\mathbf{4},\mathbf{4}) \quad \rightarrow \quad \mathbf{4}_{1/2} + \mathbf{4}_{3/2}$$

and including all couplings to the Higgs we have for scalar quadruplet

$$\mathcal{L}_{\mathrm{SO}(4)} = -\lambda\Big(H^*H^*(\epsilon H)\Phi + \frac{1}{\sqrt{3}}H^*H^*H^*\widetilde{\Phi}\Big) + \mathrm{h.c.}$$

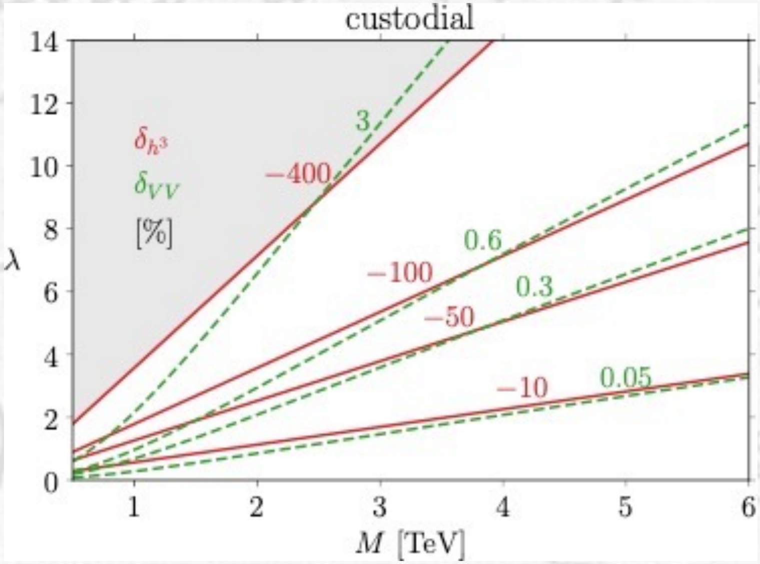
which has exactly the pattern described.

Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

$$-\frac{\delta_{VV}}{\delta_{h^3}} = 3\left(\frac{m_h}{4\pi v}\right)^2 + \left(\frac{m_h}{M}\right)^2 \approx \frac{1}{200} + \frac{1}{580}\left(\frac{3\ \text{TeV}}{M}\right)^2$$

Remarkably close to NDA estimate!



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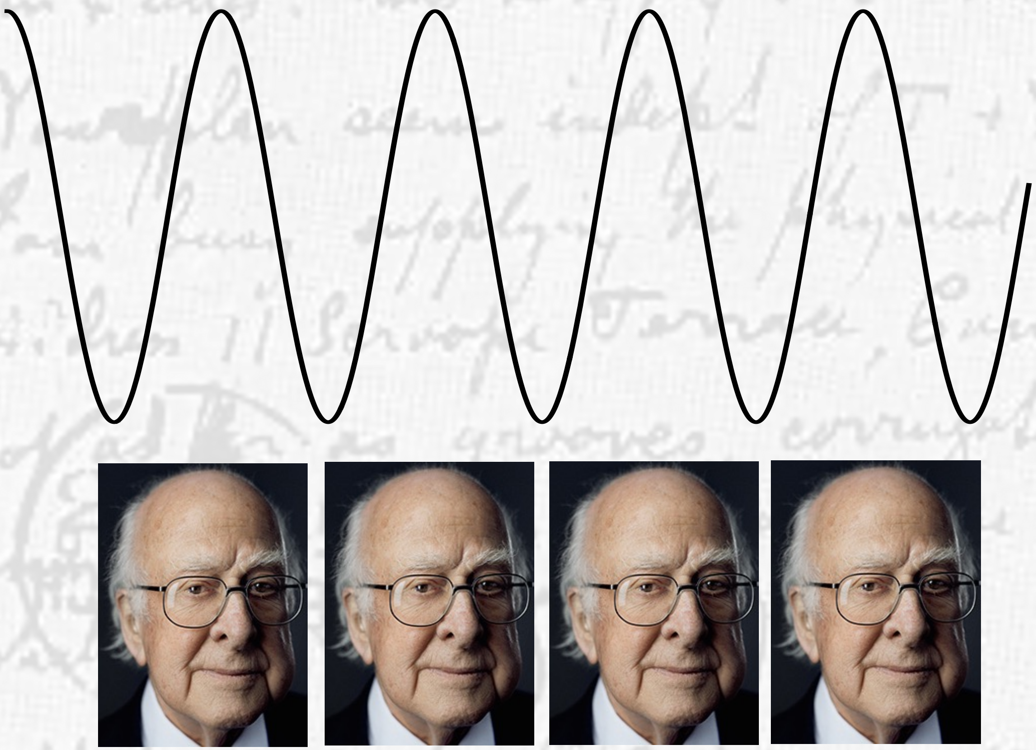
Is the Higgs Fundamental?

The Higgs boson has a size/wavelength. What's inside?



Precision measurements are different ways of probing the "compositeness of the Higgs".

$\lambda_h \approx 10^{-17}$ m

$\lambda_{10\text{ TeV}} \approx 10^{-19}$ m

Backdrop

This is exactly what happened with the pions…

$$\pi$$

$$m_\pi^2 \ll m_p^2$$

Why not the Higgs boson then?

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

"Compositeness" Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. Not so difficult to have a light Higgs

$$m_h^2 \sim \epsilon \Lambda^2$$

If one has $\epsilon \ll 1$. This is not fully possible in concrete models, since this is controlled by a symmetry which is already broken in SM.

However...

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

"Compositeness" Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. The position of the minimum of the potential doesn't care about this parameter:

$$V'(h) = 0 \Leftrightarrow F'(h/f) = 0$$

So, if this is to occur at $h = v \ll f$ then one has to fine-tune the contributions to the potential from the composite physics.

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

Compositeness Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. However, it is generic, like for pions, that the operator

$$\mathcal{O}_H \sim \frac{1}{f^2} \left(\partial^\mu |H|^2\right)^2$$

is generated. This modifies all Higgs couplings, including self-coupling, by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Let's scrutinize the assumptions...

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

How much symmetry breaking

How the symmetry is broken...

Assumption until now has been that the symmetry is broken in the most minimal ways.

Technically: Breaking "spurion" is in a low-index irrep of the global symmetry.

Beyond Minimality

Consider a simple scenario that could apply to the Higgs boson.

Example SO(N+1):

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - \frac{f^2}{2}\right)^2$$

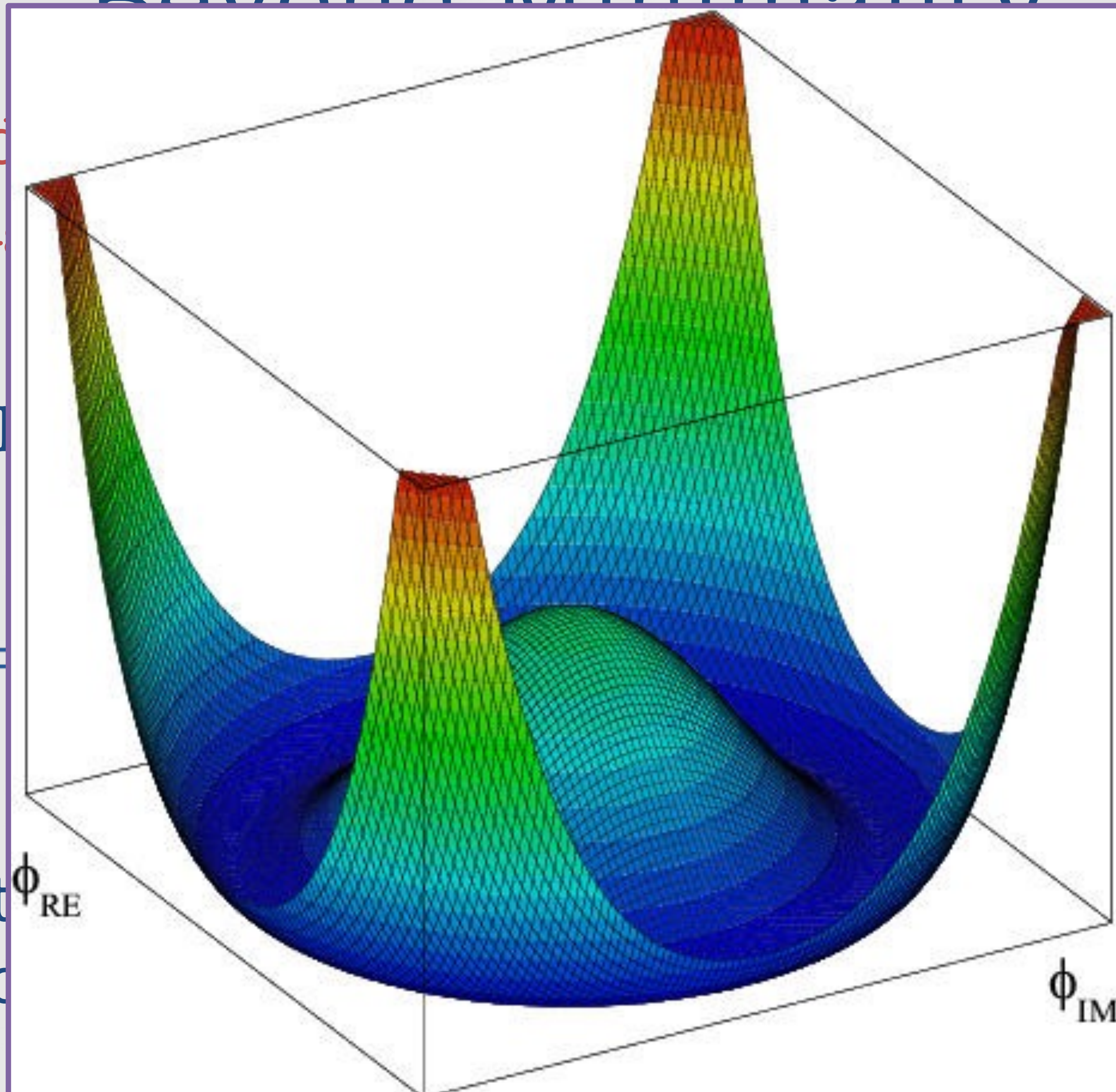
We get N massless pNGBs with decay constant "f" and unbroken SO(N).

Beyond Minimality

Consid ... ply to
the Hi...

Examp...

$\mathcal{L} = \ldots \Big)^2$



We get ... stant
"f" and ...

Beyond Minimality

Now assume some small explicit breaking "spurion" in a symmetric irrep with "n" indices:

$$V_\epsilon = \frac{\lambda}{f^{n-4}} \epsilon_{a_1, a_2, \ldots, a_n} \phi^{a_1} \phi^{a_2} \ldots \phi^{a_n}$$

How the symmetry is broken...

For the pNGB fields this generates a potential:

$$V = \epsilon m_\rho^2 f^2 \, G_n^{(N-1)/2}(\cos \Pi / f)$$

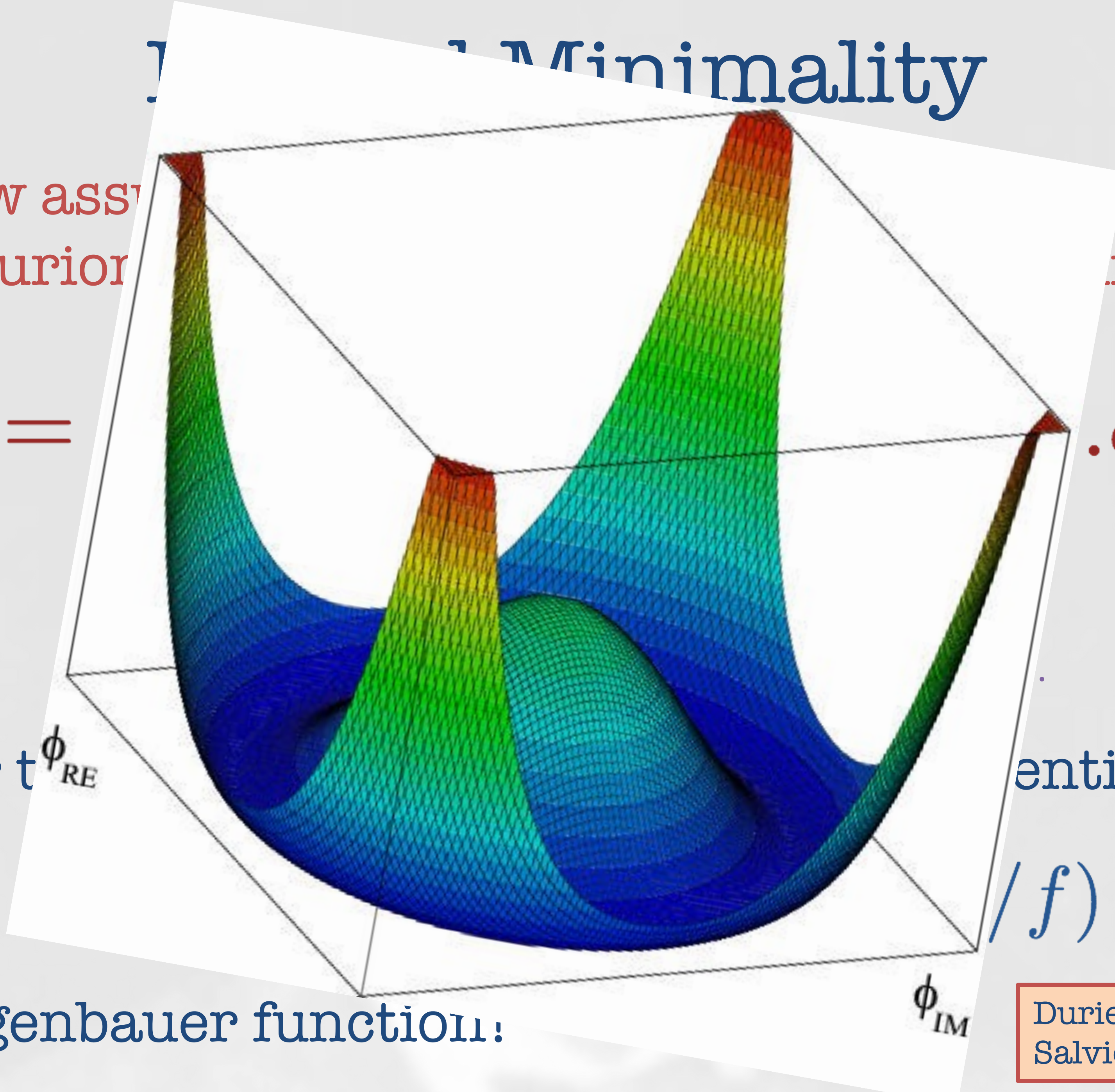
Gegenbauer function!

Durieux, MM, Salvioni. 2021

... Minimality

Now assu...
"spurion... ...ices:

$$V_\epsilon = \ldots .\phi^{a_n}$$



For t... ...ential:

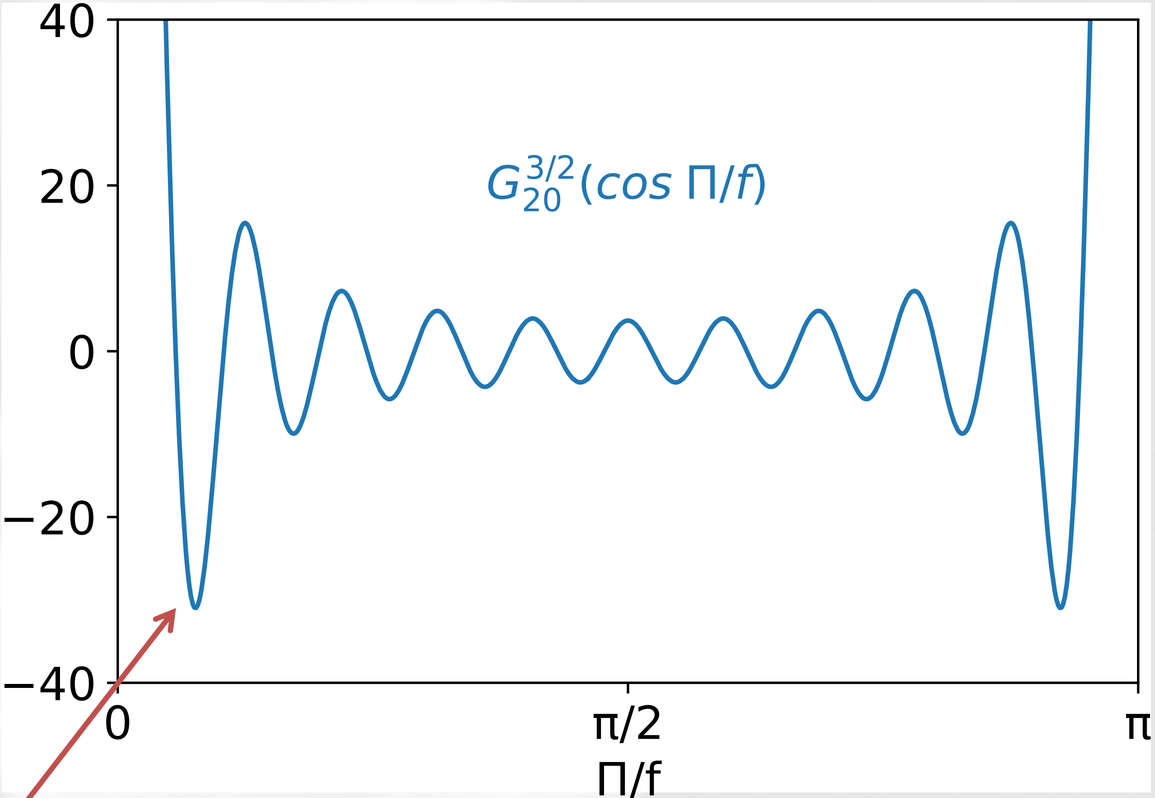
$$\ldots / f)$$

Gegenbauer functio...

Durieux, MM, Salvioni. 2021

Getting to know Gegenbauer

The Gegenbauer potential looks like:



Global minimum at naturally small field values:

$$\frac{\langle\Pi\rangle}{f} \approx \frac{j_{\lambda+1/2,1}}{n+\lambda} \approx \frac{5.1}{n}$$

Gegenbauer's Twin

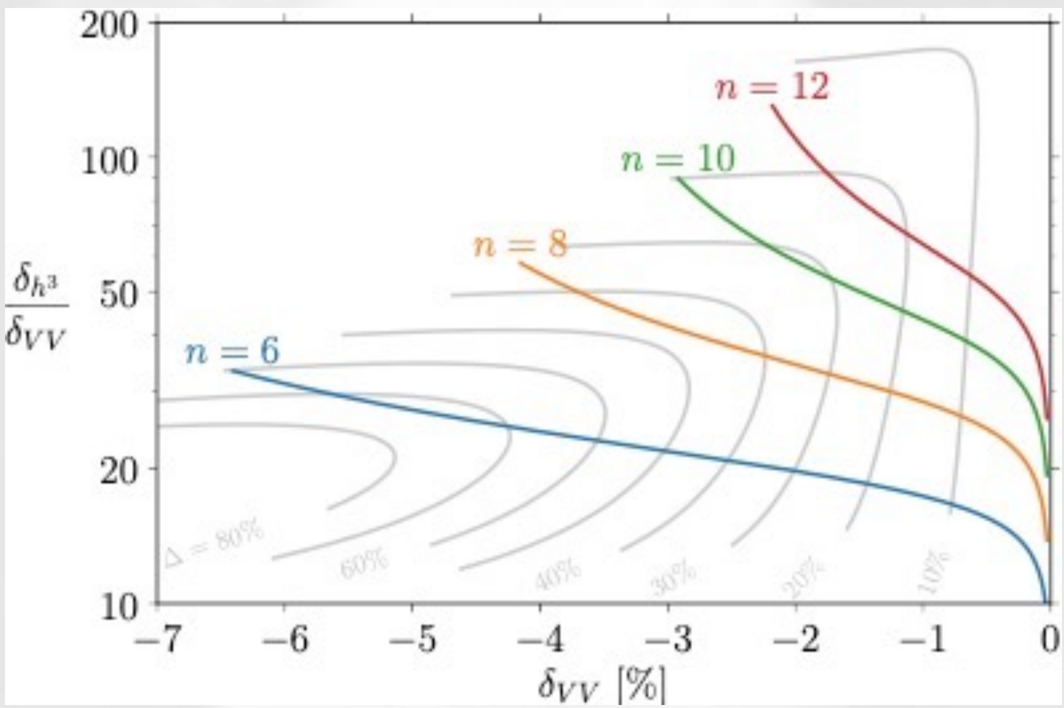
Durieux, MM, Salvioni. 2022

Gegenbauer contribution allows to naturally realise v<<f. On the other hand, for a standard composite Higgs model the top sector doesn't allow ϵ to be arbitrarily small…

Twin Higgs models, however, address that particular aspect. Could "Gegenbauer's Twin" allow both $\epsilon \ll 1$ and $v \ll f$?

Gegenbauer's Twin

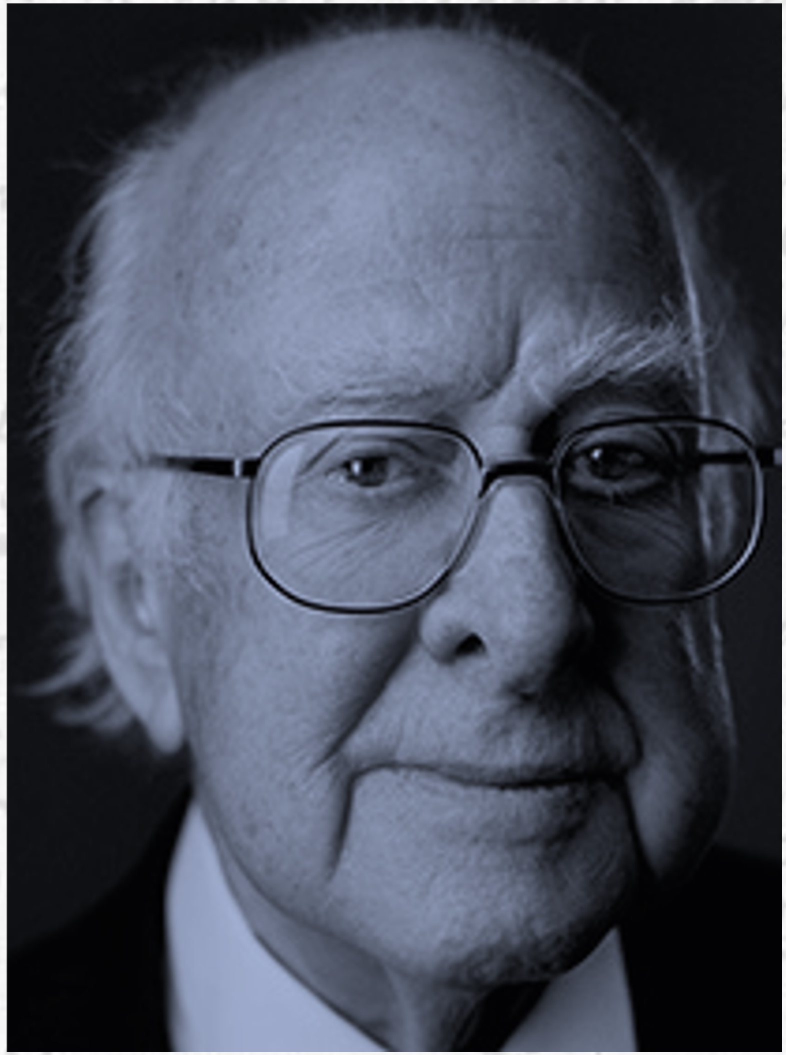
Modifications to self-interaction relative to other couplings are huge:



Naturalness could show up in self-interaction!

Fine-tuning is small. Huge corrections to Higgs self-coupling!

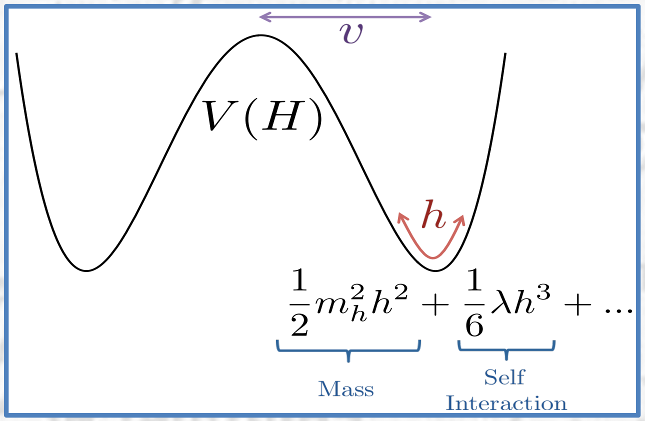
How well do we know the Higgs?

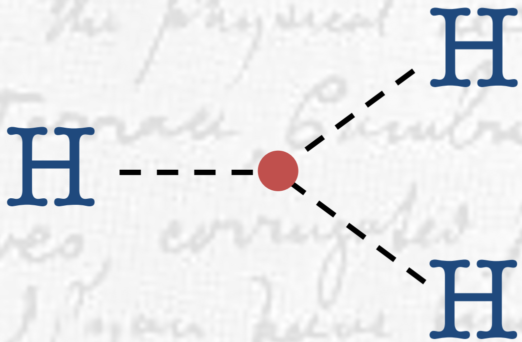


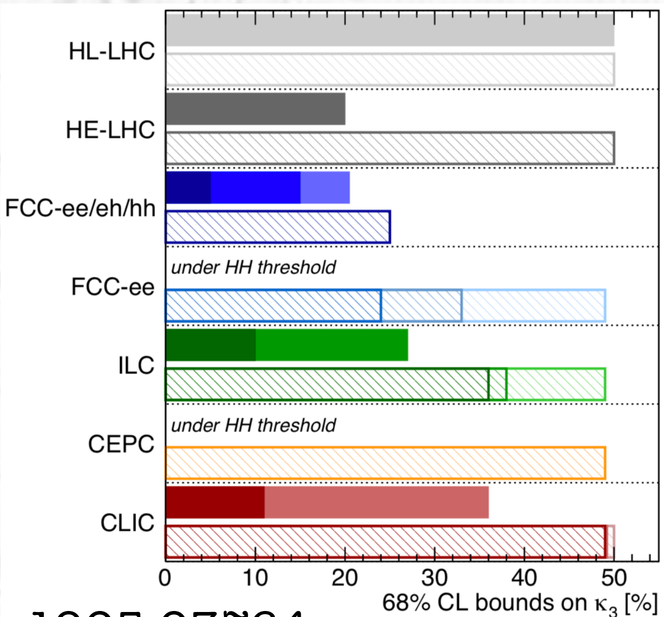
Barely.

Future of Higgs Self-Coupling

Future facilities can give us valuable new insights into the nature of the Higgs potential.







1905.03764

Rich interplay between direct/indirect, HL-LHC, Higgs factory, future High energy machines.

Conclusions

Higgs physics is still in its nascence. Pions were discovered in the early 1940's. Their fundamental origin, QCD, was developed theoretically in the early 1970's and only experimentally established in the late 1970's.

It has been eleven years since the discovery of the Higgs boson.

As it stands, we don't know how it interacts with itself; a property with far-reaching implications.

We must be patient and determined to uncover its origins.

O.T.' R.U. ATOME? $\iint \text{Sphaer}^2\, dS$ was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(s)})^2 dS$ in 4 lines, thus verifying T+T''s value of $\iint (\mathfrak{D}_i^{(s)})^2 dS$

Your plan seems indep^t of T + T' or of me. Publish!

I am busy supplying the physical necessities of scientific life at Chem 11 Scroope Terrace, Cambridge. Prooves have got as far as grooves, corrugated plates, gratings & rings. If you have time for criticism than [illegible] [illegible]

$$\iint \left(Y_i^{(s)}\right)^2 dS = \frac{8\pi a^2}{2i+1} \frac{\lfloor i+s \;\lfloor i-s}{2^{2s} \lfloor i \;\lfloor i}$$

except when $s=0$ when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

Hence

$$\int_{-1}^{+1} \left(\mathfrak{D}_i^{(s)}\right)^2 d\mu = \frac{2}{2i+1} \frac{2^{2s} \lfloor i-s \;\lfloor s \;\lfloor s}{\lfloor i+s}$$

without exception

$\frac{d^s}{d\mu^s}$