On the PMNS Matrix: Patterns and Non-Unitarity

A frequentist approach to probe certain properties of the PMNS matrix

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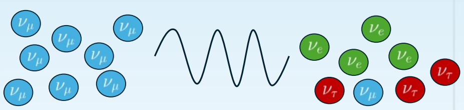
PMNS NEUTRINO OSCILLATIONS

Neutrino Oscillation:



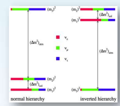
Neutrinos change flavor as they propagate.

The PMNS Paradigm:

$$\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} \xrightarrow{U_{PMNS}} \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

Mass states — Flavour states

$$P(\nu_\alpha \to \nu_\beta)(L,E) = \sum_{i,j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{ij} L}{2E}\right)$$



Two possible mass hierarchies

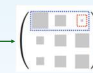
Oscillation is determined by 7 parameters:

- ❑ **Three mass differences** squared of the mass states
- ❑ **3 mixing angles** and **1 complex CP-violating phase**.
- ❑ The hierarchy of masses is not yet known.
- ❑ The value of the complex CP-violating phase is very uncertain.

PDG, Neutrino Masses, Mixing, and Oscillations

Ordering	Param	bfp ± 1σ
NO	$\theta_{12}/10^\circ$	33.41 ± 0.72
	$\theta_{23}/^\circ$	42.1 ± 1.0
	$\theta_{13}/^\circ$	8.58 ± 0.11
	$\delta_{CP}/^\circ$	$232 + 36 - 26$
	$\Delta m^2_{21}/10^{-5}\,\mathrm{eV}^2$	7.41 ± 0.21
	$\Delta m^2_{32}/10^{-3}\,\mathrm{eV}^2$	2.433 ± 0.026
IO	$\theta_{12}/10^\circ$	33.41 ± 0.72
	$\theta_{23}/^\circ$	49.0 ± 1.2
	$\theta_{13}/^\circ$	8.57 ± 0.11
	$\delta_{CP}/^\circ$	$276 + 22 - 29$
	$\Delta m^2_{21}/10^{-5}\,\mathrm{eV}^2$	7.41 ± 0.20
	$\Delta m^2_{32}/10^{-3}\,\mathrm{eV}^2$	-2.486 ± 0.028

$$U \equiv \begin{pmatrix}1&0&0\\0&\cos\theta_{23}&\sin\theta_{23}\\0&-\sin\theta_{23}&\cos\theta_{23}\end{pmatrix}\begin{pmatrix}\cos\theta_{13}&0&\sin\theta_{13}e^{-i\delta}\\0&1&0\\-\sin\theta_{13}e^{i\delta}&0&\cos\theta_{13}\end{pmatrix}\begin{pmatrix}\cos\theta_{12}&\sin\theta_{12}&0\\-\sin\theta_{12}&\cos\theta_{12}&0\\0&0&1\end{pmatrix}$$



Standard Parametrization of the PMNS matrix

BEYOND PMNS

Non-unitarity PMNS

Are there any other neutrino flavor or mass states?

$$U_{PMNS}: \begin{pmatrix}U_{e1}&U_{e2}&U_{e3}&\cdots&U_{en}\\U_{\mu1}&U_{\mu2}&U_{\mu3}&\cdots&U_{\mu n}\\U_{\tau1}&U_{\tau2}&U_{\tau3}&\cdots&U_{\tau n}\\U_{s_1 1}&U_{s_1 2}&U_{s_1 3}&\cdots&U_{s_1 n}\\\vdots&\vdots&\vdots&\ddots&\vdots\\U_{s_{n-3}1}&U_{s_{n-3}2}&U_{s_{n-3}3}&\cdots&U_{s_{n-3}n}\end{pmatrix}$$

Extra mass states

Extra flavor states (sterile neutrinos)

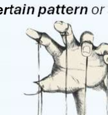
A minimalist approach: only considering a non-unitary 3x3 PMNS without making any additional assumption. Parametrizing a general mixing matrix by its QR decomposition components

$$U \equiv \begin{pmatrix}1&0&0\\0&\cos\theta_{23}&\sin\theta_{23}\\0&-\sin\theta_{23}&\cos\theta_{23}\end{pmatrix}\begin{pmatrix}\cos\theta_{13}&0&\sin\theta_{13}e^{-i\delta}\\0&1&0\\-\sin\theta_{13}e^{i\delta}&0&\cos\theta_{13}\end{pmatrix}\begin{pmatrix}\cos\theta_{12}&\sin\theta_{12}&0\\-\sin\theta_{12}&\cos\theta_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}r_{11}&0&0\\r_{21}e^{i\phi_{12}}&r_{22}&0\\r_{31}e^{i\phi_{13}}&r_{32}e^{i\phi_{23}}&r_{33}\end{pmatrix}$$

δ θ_{ij} r_{ij} ϕ_{ij}

Patterns of PMNS

*Do the parameters follow a **certain pattern** or are they completely arbitrary?*

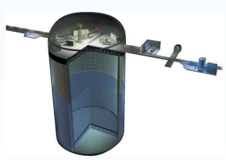


$$U \equiv \begin{pmatrix}1&0&0\\0&\cos\theta_{23}&\sin\theta_{23}\\0&-\sin\theta_{23}&\cos\theta_{23}\end{pmatrix}\begin{pmatrix}\cos\theta_{13}&0&\sin\theta_{13}e^{-i\delta}\\0&1&0\\-\sin\theta_{13}e^{i\delta}&0&\cos\theta_{13}\end{pmatrix}\begin{pmatrix}\cos\theta_{12}&\sin\theta_{12}&0\\-\sin\theta_{12}&\cos\theta_{12}&0\\0&0&1\end{pmatrix}$$

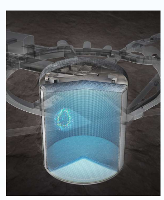
Discrete Non-Abelian Groups for flavor symmetries

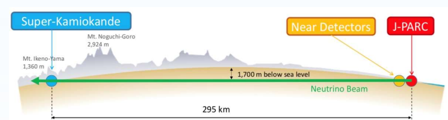
S_4 — Symmetry group — A_4 → Leptonic subgroup, Neutrino subgroup → Unitary Mixing Matrix → Formulas of PMNS matrix oscillation parameters

OSCILLATION EXPERIMENTS



Super-Kamiokande		Hyper-Kamiokande
Mozumi	Site	Tochibora
11,129	Number of ID PMTs	20,000
40%	Photo-coverage	20% (x2 efficiency)
50 kton/ 22.5 kton	Mass/Fiducial Mass	258 kton/ 186 kton
41m/ 39m	Height/ Width	71m/ 68m
500 kW to 1MW	Beam power	1.3 MW





OSCILLATION FRAMEWORK

P-theta

A Bayesian-Frequentist framework to perform far detector fit and make inference on neutrino oscillations in T2K and HK experiments.

The likelihood function in this framework

$$\ln \mathcal{L}(\mathbf{o},\mathbf{f} \mid \text{Data}) = \sum_{s,i}\left[N^{\text{exp}}_{s,i}(\mathbf{o},\mathbf{f}) - N^{\text{obs}}_{s,i} + N^{\text{obs}}_{s,i}\ln\frac{N^{\text{obs}}_{s,i}}{N^{\text{exp}}_{s,i}(\mathbf{o},\mathbf{f})}\right] + \frac{1}{2}(\mathbf{f}-\mathbf{f_0})^T\mathbf{V}^{-1}(\mathbf{f}-\mathbf{f_0})$$

Constrained by near detector data

$$N^{\nu_\alpha}_{\text{obs}}(E^{\text{rec}}_\nu) = \Phi(E^{\text{true}}_\nu) \otimes \sigma(E^{\text{true}}_\nu) \otimes S(E^{\text{true}}_\nu, E^{\text{rec}}_\nu) \otimes D(E^{\text{true}}_\nu) \otimes P_{\nu_\mu\to\nu_\alpha}(E^{\text{true}}_\nu;\mathbf{o})$$

Number of observed neutrinos = Neutrino Flux ⊗ Neutrino cross-section ⊗ Energy smearing ⊗ Detector effects ⊗ Oscillation probability

Systematic model

This part is our goal

CURRENT RESULTS

Non-unitarity PMNS

- ❑ Non-Unitarity adds a total of 9 free parameters to the unitary model.
- ❑ The amplitudes are constrained by conservation of probability and Cauchy-Schwarz inequality
- ❑ We modified Ptheta and ModProb to be compatible with non-unitarity
- ❑ We generate two sets of non-unitary matrices verifying all constraints, we look at the effect on apparent angles if fitted into a Unitary model.

Non-Unitary Set 1: small perturbation

$r_{11} = 0.9616$
$r_{21} = 0.02419$
$r_{22} = 0.9749$
$r_{31} = 0.03429$
$r_{32} = 0.01996$
$r_{33} = 0.9658$
$\phi_{12} = 2.714$
$\phi_{13} = -0.4088$
$\phi_{23} = 0.5466$

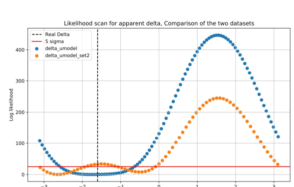
$$\begin{pmatrix}r_{11}&0&0\\r_{21}e^{i\phi_{12}}&r_{22}&0\\r_{31}e^{i\phi_{13}}&r_{32}e^{i\phi_{23}}&r_{33}\end{pmatrix}$$

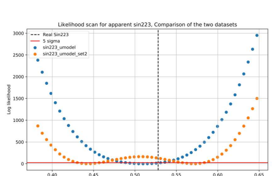
Non-Unitary Set 2: big perturbation

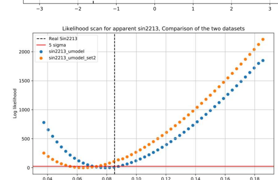
$r_{11} = 0.8049$
$r_{21} = 0.1868$
$r_{22} = 0.9199$
$r_{31} = 0.1427$
$r_{32} = 0.1979$
$r_{33} = 0.8242$
$\phi_{12} = -2.337$
$\phi_{13} = 1.126$
$\phi_{23} = -0.4941$

Apparent angles for the two sets.

Non-unitarity generates anomalies if fitted into a unitary model such as the appearance of a degeneracy for the CP-violating phase.







Patterns of PMNS

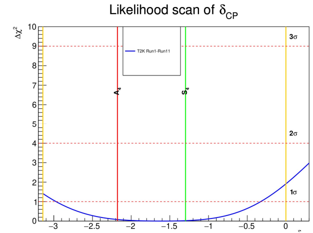
Models studied (for specified subgroups)

The model is true ⟷ $T_{\text{model}} = 0$

S_4 → $T_{S_4}(\delta,\theta_{23},\theta_{13}) = 2\sqrt{2}\sin(2\theta_{23})\sin(\theta_{13})\sqrt{1-3\sin^2\theta_{13}}\cos(\delta) - ((1+5\sin^2\theta_{13})\cos 2\theta_{23})$

A_4 → $T_{A_4}(\delta,\theta_{23},\theta_{13}) = \cos\delta\sin(2\theta_{23})\sin(\theta_{13})\sqrt{2-3\sin^2(\theta_{13})} - \cos(2\theta_{23})\cos(2\theta_{13})$

Likelihood scan of δ_{CP}



SK

HK

Likelihood scan of δ_{CP}