Possible effects of the composite dark matter at the linear collider

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Abstract. The existence of the dark matter is currently one of the strong motivations for beyond the standard model. We consider the model of the composite dark matter. Our model assumes that meson-like dark matter (call it dark mesons) is a bounded state of dark quark and anti-dark quark pairs, where they have a confining force at work. The confining force is based on the QCD-like hidden colour gauge theory. At the future linear collider, we can study the missing energy for the final state of our model.

1 Introduction

We consider a composite dark matter model where the dark matter is pion of $SU(N_c)$ dynamical chiral symmetry breaking. We have presented a model scheme for dark mesons as bound states of two kinds of Dirac spinor dark quarks ψ_i $(i = u, d)$. The Lagrangian of the dark matter sector in the $SU(N_c)$ dark colour model is given by [1–4],

$$\mathcal{L}_{DM} = Tr\,\overline{\psi_i}(i\gamma^\mu\partial_\mu + g_{DM}\gamma^\mu G_\mu + g' Q\gamma^\mu B_\mu - y\phi)\psi_i, \tag{1}$$

where G_μ is the gauge field, and g_{DM} is the gauge coupling for dark colour $SU(3)_{HC}$ sector, analogous to QCD. The Q is hypercharge, y is the Yukawa couplings. The notation "Tr" in Eq. (1) represents summing over the dark flavour and hidden colour indices. The B_μ is the $U(1)_Y$ gauge field consisting of A_μ and Z_μ boson. The ϕ is a singlet real scalar particle that connects the standard model sector and the dark matter sector. The parameters g' and Q are essential for linking the hidden sector to the Standard Model, thereby explaining the dark matter's characteristics and the cosmic phase transitions observed in the universe. The dark mesons are built from a pair of dark quarks ψ_i and $\bar{\psi}_i$ as,

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$$\tilde{\pi}^+ = \bar{\psi}_d\gamma_5\psi_u, \quad \tilde{\pi}^0 = \frac{\bar{\psi}_u\gamma_5\psi_u - \bar{\psi}_d\gamma_5\psi_d}{\sqrt{2}}, \quad \tilde{\pi}^- = \bar{\psi}_u\gamma_5\psi_d, \tag{2}$$

$$\tilde{\rho}^+_\mu = \bar{\psi}_d\gamma_\mu\psi_u, \quad \tilde{\rho}^0_\mu = \frac{\bar{\psi}_u\gamma_\mu\psi_u - \bar{\psi}_d\gamma_\mu\psi_d}{\sqrt{2}}, \quad \tilde{\rho}^-_\mu = \bar{\psi}_u\gamma_\mu\psi_d, \tag{3}$$

$$\tilde{a}^+_{1\mu} = \bar{\psi}_d\gamma_\mu\gamma_5\psi_u, \quad \tilde{a}^0_{1\mu} = \frac{\bar{\psi}_u\gamma_\mu\gamma_5\psi_u - \bar{\psi}_d\gamma_\mu\gamma_5\psi_d}{\sqrt{2}}, \quad \tilde{a}^-_{1\mu} = \bar{\psi}_u\gamma_\mu\gamma_5\psi_d, \tag{4}$$

$$\tilde{\sigma} = \frac{\bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d}{\sqrt{2}}. \tag{5}$$

2 Mass spectra of the dark matter

In the previous work for hadron physics by our group, it has been shown that the following relation holds between the mass of the π meson and the masses of the ρ and σ mesons in lattice full QCD using Wilson fermion [5].

$$m_\rho a = 0.3767(m_\pi a)^2 + 0.8099, \tag{6}$$

$$m_\sigma a = 1.6818(m_\pi a)^2 + 0.3219, \tag{7}$$

where a is a lattice spacing. The lattice calculation is employed by a lattice size $8^3 \times 16$ and a lattice coupling $\beta = 4.8$, corresponding to a lattice spacing $a = 0.207(9)$ fm. In addition, we calculated the mass ratio of the π meson to the ρ and a_1 mesons by the quenched lattice QCD simulations using Truncated Overlap Fermion (TOF) with the lattice chiral symmetry [6]:

$$m_\rho a = 0.466(m_\pi a)^2 + 0.743, \tag{8}$$

$$m_{a_1} a = 0.561(m_\pi a)^2 + 1.146. \tag{9}$$

The lattice calculation was employed by a lattice size $8^3 \times 24$ with plaquette gauge action and a lattice coupling $\beta = 5.7$. The fermion parameters for the TOF were set to the 5th-dimensional lattice size $N_5 = 32$ and the height of the domain wall $M_5 = 1.65$. Taking the limit in these expressions where the mass of the π meson is zero, we obtain the following ratios of meson masses:

$$\left(\frac{m_\sigma}{m_\rho}\right)_{lattice} = 0.3975, \quad \left(\frac{m_{a_1}}{m_\rho}\right)_{lattice} = 1.542. \tag{10}$$

We use an effective theory for the dark matter interactions based on technicolour theory [7]. By setting the mass scale of the hadron v=f_π where f_π is the pion decay constant, and the mass scale of the dark matter v_{DM}, we scale up the dark meson mass $\tilde{m}_{DM}$ from the mass of the meson m [8]. The mass of the dark meson is given by

$$\tilde{m}_{DM} = m\frac{v_{DM}}{v}. \tag{11}$$

The meson mass m can be used for the experimental value.

In addition, when using the results of lattice QCD calculations, Eq. (12) shows the cases of σ meson and a_1 meson. Here, m is the experimental value of the ρ meson.

$$\tilde{m}_\sigma = m_\rho^{exp}\left(\frac{m_\sigma}{m_\rho}\right)_{lattice}\frac{v_{DM}}{v}, \quad \tilde{m}_{a_1} = m_\rho^{exp}\left(\frac{m_{a_1}}{m_\rho}\right)_{lattice}\frac{v_{DM}}{v}. \tag{12}$$

3 Decay modes and width for the composite dark matter

The scalar potential for mass mixing between the Higgs boson h of the Standard Model and a scalar ϕ is given by [1, 4, 9],

$$V(h,\phi) = -\frac{1}{2}\mu_H^2 h^2 - \frac{1}{2}\mu_S\phi^2 - \frac{\lambda_H}{4}h^4 - \frac{\lambda_{HS}}{4}h^2\phi^2 - \frac{\lambda_S}{4}\phi^4, \tag{13}$$

After the spontaneous electroweak symmetry breaking, the vacuum expectation values of the fields h and ϕ are

$$v_h^2 \equiv \frac{2\lambda_{HS}\mu_S^2 - 4\lambda_S\mu_H^2}{4\lambda_H\lambda_S - \lambda_{HS}^2}, \omega^2 \equiv \frac{2\lambda_{HS}\mu_H^2 - 4\lambda_H\mu_S^2}{4\lambda_H\lambda_S - \lambda_{HS}^2}. \tag{14}$$

Furthermore, the mass matrix for scalar state $(h,\phi)^T$ is derived by [4],

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v_h^2 & \lambda_{HS} v_h \omega \\ \lambda_{HS} v_h \omega & 2\lambda_S\omega^2 \end{pmatrix}. \tag{15}$$

The matrix $\mathcal{M}^2$ can be diagonalized through an orthogonal transformation

$$\mathcal{O}^T\mathcal{M}^2\mathcal{O} = \begin{pmatrix} M_{H1}^2 & 0 \\ 0 & M_{H2}^2 \end{pmatrix}, \tag{16}$$

where

$$\mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \text{ with } \tan 2\theta = \frac{\lambda_{HS} v_h \omega}{\lambda_S\omega^2 - \lambda_H v_h^2}, \tag{17}$$

and the mass eigenvalues are

$$M_{H_1,H_2}^2 = \lambda_H v_h^2 + \lambda_S\omega^2 \mp \frac{\lambda_S\omega^2 - \lambda_H v_h^2}{\cos 2\theta}. \tag{18}$$

The extended Higgs sector will act as a double–portal for the dark matter particles.

In this paper, as the first stage of the research, we consider the case where $\theta \simeq 0$, $H_1 \simeq h$, and $H_2 \simeq \phi$. We examined the decay modes of ϕ. A real singlet scalar particle provides the current mass of dark quarks, which explicitly breaks chiral symmetry. This gives mass to the Nambu-Goldstone bosons. In the case of $SU(2)_V$, $\tilde{\pi}^+, \tilde{\pi}^-, \tilde{\pi}^0$ are candidates for the dark matter. Moreover, $\tilde{\sigma}$ meson is also the dark matter candidate because this is the chiral partner of the $\tilde{\pi}$ meson. A part of the effective Lagrangian depending on ϕ is written following,

$$\mathcal{L}_{int} = g_\pi\phi(\tilde{\pi}^+\tilde{\pi}^- + \tilde{\pi}^-\tilde{\pi}^+ + \tilde{\pi}^0\tilde{\pi}^0) + g_\sigma\phi\tilde{\sigma}\tilde{\sigma}, \tag{19}$$

and corresponding diagram is shown in Fig. 1.

Now we assume that the value of the coupling g_π is same as g_σ; $g_\pi = g_\sigma \equiv g$. The decay widths are described as follows;

$$\Gamma_{\phi\to\tilde{\pi}^+\tilde{\pi}^-} = 2\frac{g^2}{4\pi}\frac{1}{M_\phi^2}\sqrt{(\frac{M_\phi}{2})^2 - (\tilde{m}_\pi)^2}, \tag{20}$$

$$\Gamma_{\phi\to\tilde{\pi}^0\tilde{\pi}^0} = \frac{g^2}{4\pi}\frac{1}{M_\phi^2}\sqrt{(\frac{M_\phi}{2})^2 - (\tilde{m}_\pi)^2}, \tag{21}$$

$$\Gamma_{\phi\to\tilde{\sigma}\tilde{\sigma}} = \frac{g^2}{4\pi}\frac{1}{M_\phi^2}\sqrt{(\frac{M_\phi}{2})^2 - (\tilde{m}_\sigma)^2}. \tag{22}$$

The total decay width of ϕ is

$$\Gamma_{total}^\phi = \Gamma_{\phi\to\tilde{\pi}^+\tilde{\pi}^-} + \Gamma_{\phi\to\tilde{\pi}^0\tilde{\pi}^0} + \Gamma_{\phi\to\tilde{\sigma}\tilde{\sigma}}. \tag{23}$$

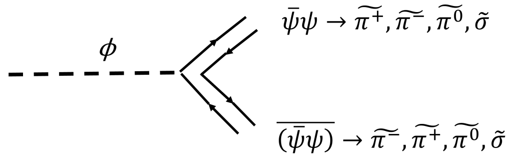

Figure 1. A diagram for the decay of a singlet scalar particle ϕ.

4 Results

In hadron physics, the experimental value of σ is not fixed, so our lattice calculations are used. The median experimental value from the PDG [10] was used for the other mesons. We also use $m_\pi = 140$ MeV and $v = f_\pi = 130$ MeV [10]. We chose the dark pion masses 1.08 and 1.8 TeV. A recent analysis shows that the dark pion has a mass of 1.8 TeV [11].

Fig. 2 shows the decay widths for coupling constant $g^2/4\pi$. Table 1 represents the branching ratio for ϕ. The mass of the dark σ, $\tilde{m}_\sigma$, was derived from Eq. (11) and the σ mass obtained in two different ways, as described below: (a) the median of the experimental values [10], (b) the Lattice results of our previous study in Eq. (10) and the experimental values from Eq. (12).

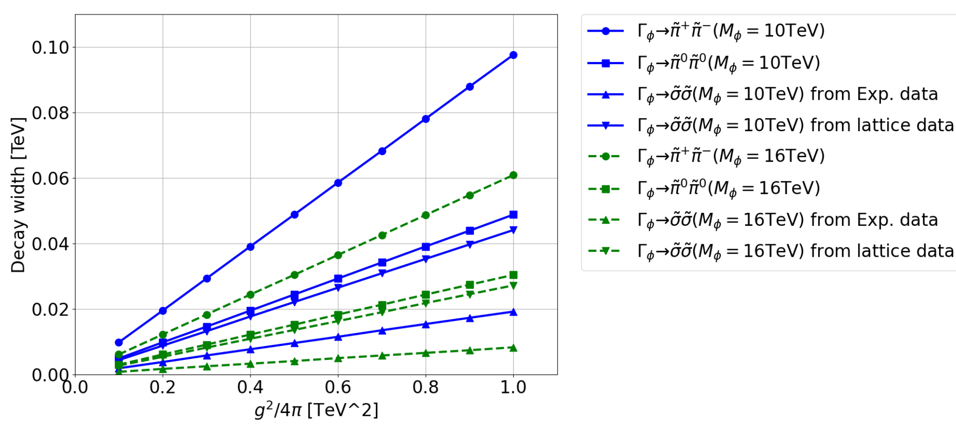

Figure 2. Decay width for coupling constant $g^2/4\pi$.

Table 1. Brunching ratio from scalar field ϕ to various dark meson pairs.

	v_{DM} [TeV]	M_ϕ [TeV]	$\tilde{m_\pi}$ [TeV]	$\tilde{m_\sigma}$ [TeV]	$\frac{\Gamma_{\phi\to m_{\pi^+}^{\sim} m_{\pi^-}^{\sim}}}{\Gamma_{total}}$	$\frac{\Gamma_{\phi\to m_{\pi^0}^{\sim} m_{\pi^0}^{\sim}}}{\Gamma_{total}}$	$\frac{\Gamma_{\phi\to \tilde{m_\sigma}\tilde{m_\sigma}}}{\Gamma_{total}}$
(a)	1.00	10.0	1.08	4.62	0.589	0.295	0.116
(b)	1.00	10.0	1.08	2.35	0.512	0.256	0.232
(a)	1.67	16.0	1.80	7.72	0.611	0.306	0.083
(b)	1.67	16.0	1.80	3.94	0.514	0.257	0.229

5 Summary and Discussion

The decay mode $\phi \to \tilde{\pi}^0\tilde{\pi}^0$ is a visible process because $\tilde{\pi}^0$ breaks into 2γ. The other decay modes are invisible processes. We assume that $\tilde{\sigma}$ decay into $\tilde{\pi}^+\tilde{\pi}^-$ and $\tilde{\pi}^0\tilde{\pi}^0$, do not decay into 2γ (In hadron physics, it is not yet known whether σ meson decay into photons or not). $\tilde{\pi}^+$ and $\tilde{\pi}^-$ could be the stable dark matters. We are supposed to identify the dark matter through an invisible process at the future linear collider.

Our current research focused on the decay of ϕ. However, it is important to consider ϕ and Higgs mixing. The Higgs sectors have two states, heavy and light, and it is thought that the states are mixed. We plan to incorporate these decay modes in our future studies [12]. We anticipate that the effects of the dark matter will be detectable at the future linear collider through an invisible process.

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References

[1] T. Hur and P. Ko, Scale invariant extension of the standard model with strongly interacting hidden sector. Phys. Rev. Lett. **106**, 141802 (2011). https://doi.org/10.1103/PhysRevLett.106.141802

[2] Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky and J. G. Wacker, Model for Thermal Relic Dark Matter of Strongly Interacting Massive Particles. Phys. Rev. Lett. **115**, 021301 (2015). https://doi.org/10.1103/PhysRevLett.115.021301

[3] Y. Ametani, M. Aoki, H. Goto and J. Kubo, Nambu-Goldstone Dark Matter in a Scale Invariant Bright Hidden Sector. Phys. Rev. D **91**, 115007 (2015). https://doi.org/10.1103/PhysRevD.91.115007

[4] G. Arcadi, A. Djouadi and M. Kado, The Higgs-portal for dark matter: effective field theories versus concrete realizations. Eur. Phys. J. C **81**, 653 (2021). https://doi.org/10.1140/epjc/s10052-021-09411-2

[5] T. Kunihiro, S. Muroya, A. Nakamura, C. Nonaka, M. Sekiguchi and H. Wada, The Low-lying Scalar Mesons and Related Topics. Nucl. Phys. B Proc. Suppl. **186**, 294 (2009). https://doi.org/10.1016/j.nuclphysbps.2008.12.066

[6] Y. Murakami, M. Sekiguchi, H. Wada and M. Wakayama, Mass spectroscopy of excited light mesons using truncated overlap fermions. J. Phys. Comm. **6**, 105009 (2022). https://doi.org/10.1088/2399-6528/ac9f00

[7] L. Susskind, Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory. Phys. Rev. D **20**, 2619 (1979). https://doi.org/10.1103/PhysRevD.20.2619

[8] Y. Murakami, M. Sekiguchi, H. Wada, M. Wakayama, Mass spectroscopy of dark matter in SU(3) hidden colour. PoS **441**, 336 (2023). https://doi.org/10.22323/1.441.0336

[9] P. Ko, Electroweak symmetry breaking and cold dark matter from strongly interacting dark sector, PoS **ICHEP2010**, 436 (2010). https://doi:10.22323/1.120.0436

[10] S. Navas *et al.* (Particle Data Group Collaboration), Review of Particle Physics. Phys. Rev. D **110**, 030001 (2024). https://doi.org/10.1103/PhysRevD.110.030001

[11] T. Abe, R. Sato and T. Yamanaka, Composite Dark Matter with Forbidden Annihilation. (2024). https://doi.org/10.48550/arXiv.2404.03963

[12] Y. Murakami, M. Sekiguchi, H. Wada and M. Wakayama, we now processing.