The ALARIC parton shower

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Abstract. Parton showers are important tools in the event generation chain for present and future colliders. Recently, their formally achieved accuracy has been under extended scrutiny. This contribution will present a novel take on dipole parton showers, resulting in the design of a new parton shower called ALARIC that is implemented in the SHERPA framework. Its resummation properties, including analytic and numerical proofs of its NLL accuracy, will be discussed alongside the latest developments.

1 Introduction

Parton showers serve as crucial steps in modern Monte Carlo event generators [1]. They connect the hard scale at which fixed order perturbation theory is adequate to calculate scattering amplitudes with the low scale objects that are observed in particle physics experiments like the current LHC or future lepton colliders. While parton showers have been phenomenologically very successful for decades now, their further development in several directions, such as more accurate treatment of colour [2–15], spin correlations between subsequent emissions [16–18] or the inclusion of higher order splitting functions [19–25], remains an active field of research of high relevance particularly to precision experiments at future lepton colliders. The logarithmic correctness and sub-leading effects of recoil and the imposed unitarity have seen particular interest over the last years [26–34]. This has resulted in the construction of several new parton showers designed to be logarithmically accurate [31]. This contribution reports on the ALARIC parton shower [31] which has been shown to be very successful in describing LEP data already at NLL and has recently been extended to include initial state radiation [34] and multijet merging, thereby enabling a comprehensive exploration of data from the LHC.

2 Basics of the ALARIC parton shower algorithm

This section will briefly review the basic theory principles of the ALARIC parton shower. The focus will be on the crucial steps of partial fractioning the eikonal to define positive splitting functions covering the full phase space and on the definition of the soft recoil scheme used in the parton shower. For a more detailed treatment, see the original publications [31, 34, 35]

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and references therein. Let us start by analysing a general QCD matrix element in the limit that gluon j becomes soft. The squared amplitude factorizes as [36]

$$ {}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n = -8\pi\alpha_s \sum_{i,k\neq j} {}_{n-1}\langle 1,\ldots,\not{j},\ldots,n\big|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}\big|1,\ldots,\not{j},\ldots,n\rangle_{n-1} \;, \tag{1} $$

where $\mathbf{T}_i$ and $\mathbf{T}_k$ are the color insertion operators defined in [37] and the soft eikonal is given by

$$ w_{ik,j} = \frac{p_i p_k}{(p_i p_j)(p_j p_k)} \;. \tag{2} $$

The eikonal exhibits singularities in the limits where the gluon j becomes either soft, $E_j \to 0$, or collinear to either i or k, $\theta_{ij} \to 0$ or $\theta_{jk} \to 0$. If one were to implement the soft eikonal for each of the radiators i and k in the collinear limit, the soft-collinear contribution to the emission probability [38] would therefore be double-counted. This can be solved by following the technique of [39], which leads to angular ordered parton showers that are very successful and implement the correct radiation pattern for example for event shapes, but have problems in filling the relevant phase space for non-global observables [40]. In Alaric, the eikonal is rather separated into two parts by partial fractioning. Specifically,

$$ w_{ik,j} = \bar{w}^{i}_{ik,j} + \bar{w}^{k}_{ki,j} \quad \text{with} \quad \bar{w}^{i}_{ik,j} = \frac{1}{p_i p_j}\frac{l_{ik} p_i}{l_{ik} p_j} \;, \tag{3} $$

and where

$$ l^{\mu}_{ik} = \frac{p^{\mu}_i}{p_i n} + \frac{p^{\mu}_k}{p_k n} \;. \tag{4} $$

with an for now arbitrary reference vector n. The soft eikonal can now be matched to the collinear splitting function, see [31].
Overall four-momentum conservation is satisfied by selecting a general set of momenta $\mathcal{K}$ in the event to define the recoil momentum

$$ \tilde{K} = \sum_{k\in\mathcal{K}} k \;. \tag{5} $$

For the purpose of this discussion lets take all momenta in the amplitude as outgoing, such that overall momentum conservation is written as

$$ \sum_{k\in\mathcal{K}} k^{\mu} + \sum_{k\in\overline{\mathcal{K}}} k^{\mu} = 0 \tag{6} $$

where $\overline{\mathcal{K}}$ is the sum over the remaining momenta in the event but not in $\mathcal{K}$.
The emitter is shifted as

$$ p_i = z\,\tilde{p}_i \;, \qquad n = \tilde{K} + (1-z)\,\tilde{p}_i \;, \tag{7} $$

which implies $p_i + n = \tilde{p}_i + \tilde{K}$. A light-like variant of n can be defined as

$$ \bar{n} = n - \frac{n^2}{2\tilde{p}_i n}\,\tilde{p}_i = \tilde{K} - \kappa\,\tilde{p}_i \;, \qquad \text{where} \qquad \kappa = \frac{\tilde{K}^2}{2\tilde{p}_i\tilde{K}} \;. \tag{8} $$

and new momenta are constructed as

$$ p_j = v\,\bar{n} + \frac{1}{v}\frac{\mathrm{k}_\perp^2}{2\tilde{p}_i\tilde{K}}\,\tilde{p}_i - k_\perp \;, \qquad\qquad \text{where} \qquad v = \frac{p_i p_j}{p_i\tilde{K}} \tag{9} $$

$$ K = (1-v)\,\bar{n} + \frac{1}{1-v}\frac{\mathrm{k}_\perp^2 + \tilde{K}^2}{2\tilde{p}_i\tilde{K}}\,\tilde{p}_i + k_\perp \;. \tag{10} $$

The construction is consistent if

$$\mathrm{k}_\perp^2 = v(1-v)\,2p_jK - v^2K^2 = v(1-v)(1-z)\,2\tilde{p}_i\tilde{K} - v^2\tilde{K}^2\ . \tag{11}$$

Inserting this relation into Eq. (9) produces the final mapping

$$\begin{aligned} p_j &= (1-z)\,\tilde{p}_i + v(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i) + k_\perp\ ,\\ K &= \qquad\quad \tilde{K} - v(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i) - k_\perp\ . \end{aligned} \tag{12}$$

Distributing the recoil amongst the particles making up $\tilde{K}$ implies a Lorentz transformation [37]

$$p_l^\mu \to \Lambda^\mu_{\ \nu}(\tilde{K},K)\,p_l^\nu\ , \qquad \text{where} \qquad \Lambda^\mu_{\ \nu}(\tilde{K},K) = g^\mu_{\ \nu} - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2K^\mu\tilde{K}_\nu}{\tilde{K}^2}\ . \tag{13}$$

If one would simply replace $\tilde{p}_i$ with p_i and p_j, the momenta in $\overline{\mathcal{K}}$ would sum to

$$\sum_{k\in\overline{\mathcal{K}}} k \equiv -K = -\tilde{K} - \tilde{p}_i + p_i + p_j\ . \tag{14}$$

This clearly breaks momentum conservation, *i.e.* Eq. (6) does no longer hold. The necessary change in a collection of momenta can be expressed as a Lorentz transformation that transforms $\tilde{K}$ to K, or its inverse transforming K to $\tilde{K}$,

$$\Lambda^\mu_{\ \nu}(\tilde{K},K)\tilde{K}^\nu = K \tag{15}$$

$$\Lambda^\mu_{\ \nu}(K,\tilde{K})K^\nu = \tilde{K}\ . \tag{16}$$

Note this relies on the momentum mapping ensuring that the momentum squared of K, $\tilde{K}$ does not change. Now overall momentum conservation can be satisfied by making either of the two following replacements:

$$k^\mu \to \Lambda^\mu_{\ \nu}(\tilde{K},K)k^\nu \quad \forall\, k \in \mathcal{K} \tag{17}$$

or

$$k^\mu \to \Lambda^\mu_{\ \nu}(K,\tilde{K})k^\nu \quad \forall\, k \in \overline{\mathcal{K}}\ . \tag{18}$$

3 Proof of accuracy preserving recoil scheme

An idealised parton shower works by iterative procedure generating higher multiplicity final states from lower multiplicity ones by adding additional particles according to the splitting function usually derived in the soft and collinear limits of QCD matrix elements. In practice, away from the exact limits, it is necessary to distribute final recoil to particles in the event either coming from the hard process or from previous parton shower emissions. Importantly, there is a discrepancy between the final particle momenta produced by the shower and the momenta entering into the calculation of emission probabilities at each step of the shower. It is hence necessary that the recoil scheme of the parton shower only produces small changes of the momenta of previous emissions, that should be vanishing in the limit taken also in a resummed calculation. In the ALARIC recoil scheme these changes can be traced analytically. The relevant method is for example used in the CAESAR formalism [41], see [42, 43] for a practical implementation in SHERPA [44, 45], to extract a pure NLL contribution from a general multiple emission integral of the form

$$\mathcal{F}(v) = \int \mathrm{d}^3k_1|M(k_1)|^2\, e^{-R'\ln\frac{v}{\epsilon v_1}} \sum_{m=0}^{\infty}\frac{1}{m!}\Big(\prod_{i=2}^{m+1}\int_{\epsilon v_1}^{v_1}\mathrm{d}^3k_i|M(k_i)|^2\Big)\Theta(v - V(\{p\},k_1,\ldots,k_n))\ . \tag{19}$$

where v_1 is the value of the observable in the leading (in v) emission, and k_1 is the corresponding momentum. This can be achieved by taking the limit

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)} \,, \qquad \eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b} \,, \qquad \text{where} \qquad \xi = \frac{\eta}{\eta_{\max}} \,. \tag{20}$$

where a, b_l parameterise a generic observable in the soft limit as

$$V(k) = \left(\frac{k_{T,l}}{Q}\right)^a e^{-b_l\eta_l} \,, \tag{21}$$

and $k_{T,l}, \eta_l$ are the transverse momentum and rapidity of an emission relative to leg l.
As discussed in the previous section, in the ALARIC shower mapping the emitter is just rescaled but does not absorb any transverse recoil,

$$p_i = z\tilde{p}_i \,. \tag{22}$$

The newly emitted parton momentum p_j on the other hand will have a finite transverse momentum relative to that direction that is to be compensated. This happens by either of the replacements from Eq. (18)
It has been shown in [31] that this does not impose a significant change in the momenta of already existing soft gluons. The general idea is to rewrite the momentum K as $\tilde{K}$ plus a vanishing correction

$$K^\mu = \tilde{K}^\mu - X^\mu \,, \qquad \text{where} \qquad X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu \,. \tag{23}$$

For this it is important that $\mathcal{K}$ contains some momenta that are not scaling with ρ, which is the only explicit condition on the choice of recoil momentum at this level. Following [31], the Lorentz transformation can now be written as

$$\Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_{\ \nu} + \tilde{K}^\mu A_\nu + X^\mu B_\nu \,, \tag{24}$$

where

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right], \qquad \text{and} \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2} \,. \tag{25}$$

The leading contributions to the coefficient in Eq. (24) in the $\rho \to 0$ limit are given by

$$A^\nu \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2} \,, \qquad \text{and} \qquad B^\nu \xrightarrow{\rho\to 0} \frac{\tilde{K}^\nu}{\tilde{K}^2} \,. \tag{26}$$

The relevant quantity to consider now is the difference of a generic momentum before and after applying the respective transformation:

$$\Delta p_l^\mu = p_l^\mu - \Lambda^\mu_{\ \nu} p_l^\nu = \pm\left[\tilde{K}^\mu A_\nu + X^\mu B_\nu\right] p_l^\nu \,, \tag{27}$$

where the plus or minus in the last expression correspond to performing one or the other transformation. As has been argued in [31], this difference indeed corresponds to a vanishing change in each of the momentum components. For completeness, the result from [31] are:

$$\begin{aligned}
&\frac{\Delta p_l^{0,3}}{p_l^{0,3}} \sim \rho^{(1-\max(\xi_i,\xi_j))/a} \,, \\
&\frac{\Delta p_l^{1,2}}{p_l^{1,2}} \sim \rho^{(1-\xi_l-b/(a+b)(\max(\xi_i,\xi_j)-\xi_l))/a} < \rho^{(1-b/(a+b))(1-\xi_l)/a} \,.
\end{aligned} \tag{28}$$

It is evident that for $\xi_l < 1$ and $\max(\xi_i,\xi_j) < 1$, the relative changes vanish in the $\rho \to 0$ limit. The only potential issue is at $\xi_l = 1$ and/or $\max(\xi_i,\xi_j) = 1$, which however correspond to a phase-space region of measure zero.
Corresponding numerical checks have been performed and documented in [31].

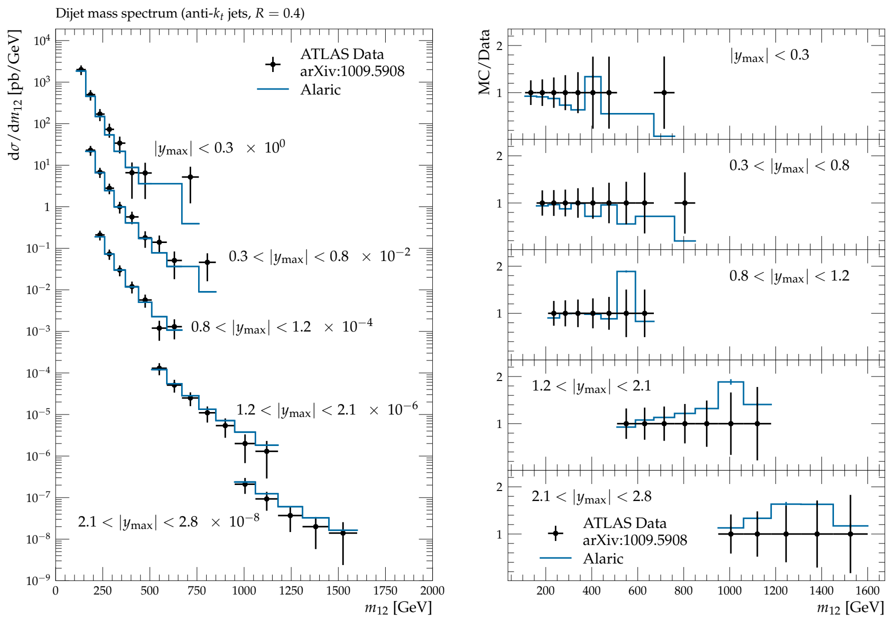

Figure 1. Dijet mass spectrum measured by ATLAS [46] for anti-k_t jets with transverse momentum $p_T > 60$ GeV within a rapidity range of $|y_{max}| < 2.8$, compared to predictions from ALARIC.

4 ALARIC for collider phenomenology

The ALARIC parton shower has been compared to a variety of LEP data in [31], finding overall good agreement in line with the quality of description achieved with previous dipole showers like DIRE. Fig. 1 illustrates the description of jet data measured at the LHC, complementing the study performed in [34]. Predictions are compared with dijet data measured by the ATLAS collaboration [46]. Jets are reconstructed with the anti-k_t algorithm [47] and required to have a transverse momentum of $p_T > 60$ GeV. The data are presented in several rapidity slices in the central region $|y_{max}| < 2.8$. ALARIC describes the mass spectrum well over the full range, within the experimentally quoted uncertainty. Some potentially systematic trends are visible, which however do not appear significant and would need a closer examination including potential correlations between errors both on the experimental as well as theoretical side. Additionally, Fig. 2 compares ALARIC predictions to data for jet angularities measured in [48], which have been studied within the SHERPA framework previously in [49–51]. The description of the data is generally good, in line with the observations made in the above references on the data agreement of both established dipole parton showers as well as NLL resummation.

5 Conclusion and Outlook

This contribution discussed the recently introduced ALARIC parton shower, in particular its main underlying construction principles, its analytically tractable resummation properties and the succesful description of collider data with an NLL parton shower. Going forward towards the precision era both at the LHC as well as in preparing for future lepton colliders will see

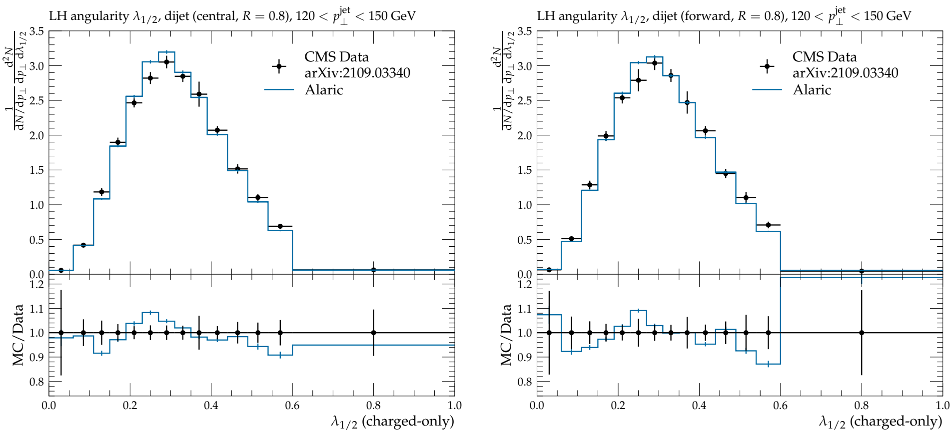

Figure 2. Les Houches angularity $\lambda_{1/2}$ measured by CMS [48] in dijet events on central jets in the transverse momentum range $120 < p_T^{\text{jet}} < 150$ GeV compared to predictions from ALARIC

the development of parton showers including NLO splitting functions [19–25] and aiming for NNLL accuracy. Simultaneously, the standard of fixed order calculations is becoming NNLO. Despite some progress in mitigating non-perturbative corrections with the help of jet substructure techniques such as soft drop grooming, see for example [52–67], a better control of these corrections becomes increasingly important with further progress in perturbative accuracy achieved in general purpose Monte Carlo event generators.

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