Adiabatic self-matching of witness bunches in a plasma wakefield accelerator

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Abstract. Plasma wakefield acceleration offers high accelerating gradients, making it an interesting candidate for future colliders. However, the large accelerating fields are coupled with large focusing fields, leading to accelerated bunches with extremely high charge density. In these regimes, the electron witness bunch can significantly perturb the plasma ion density. The resulting nonlinear focusing field can lead to emittance growth, reducing the beam quality. In this work, we show that by accelerating in a single stage, as would be possible with a high-energy proton driver, the witness bunch can adiabatically match to the focusing fields, greatly reducing the emittance growth. This suggests that many of the constraints for electron acceleration in plasma can be avoided.

1 Introduction

Plasma wakefield acceleration makes use of plasma to transfer energy from a driver to a witness bunch [1, 2]. The accelerating gradients can be extremely high, essentially limited by the length of the driver, which should be on the order of the plasma skin depth $1/k_p$, with $k_p = \sqrt{ne^2/\varepsilon_0 mc^2}$ the plasma wavenumber and n the plasma density. Wakefield accelerators generally operate near the wavebreaking amplitude $E_c = (mc^2/e)(1/k_p)$, which is typically on the order of 1-10 GV/m [3, 4].

The energy gain is then limited by the driver energy. Electron bunches and laser pulses do not have sufficient energy to accelerate a witness bunch to collider-relevant energies, and so staging would be required, with multiple acceleration sections each using a fresh driver [5, 6]. Proton bunches, however, can readily be accelerated to high energies in circular accelerators [7] and so would make ideal drive bunches. A recent study has investigated the main challenges to the development of a Higgs factory based on proton-driven wakefield acceleration (PDPWFA) [8]. Chief amongst these is that the short proton bunches required to excite high accelerating gradients are not currently available, but the study has motivated further research in this area.

In this work, we consider another potential barrier to the acceleration of collider-relevant bunches in plasma, namely the response of the plasma ions. In order to reach high luminosity, the accelerated bunches need to have low emittance and high charge. The plasma response to such a bunch is shown in Fig. 1, with parameters chosen to match those detailed in the recently proposed PDPWFA scheme for a Higgs factory [8]. A short proton drive bunch

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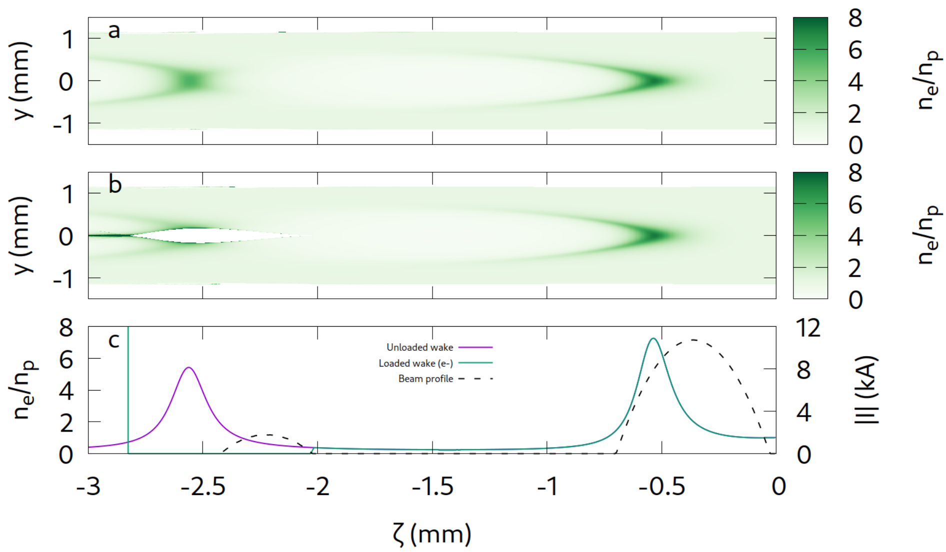

Figure 1. The plasma response to the drive and witness beams in a PDPWFA. a) The proton driver (propagating to the right) excites a density perturbation of the plasma electrons. b) A trailing electron witness trailing the driver is positioned to extract energy from the wakefields, and drives its own cavitated wake. c) Lineouts show the profiles of the driver (right) and witness (left), and the on-axis plasma electron density for the unloaded (a) and loaded (b) cases.

excites a quasilinear wakefield, and the witness bunch trails the drive bunch such that it gains energy from the wake. Although the driver only creates a quasilinear wake, the witness electron bunch has a sufficiently high charge density to drive a plasma blowout, a cavitated area free of plasma electrons [9]. In this case, the focussing fields acting on the witness bunch are entirely due to the plasma ions, which move relatively little due to their larger mass. In the absence of ion motion, the resulting force on the witness bunch increases linearly with transverse displacement, allowing the emittance of the accelerated bunch to be preserved [10].

The matched radius for the witness bunch σ_{ic}, such that the focussing force from the plasma ion column is equal to the emittance pressure from the witness, then satisfies:

$$\sigma_{ic}^2 = \sqrt{\frac{2}{\gamma}}\frac{\epsilon_N}{k_p}, \tag{1}$$

where γ is the Lorentz factor and ϵ_N the normalised emittance of the witness bunch. The scheme considered here uses $\epsilon_N = 0.1$ µm and $1/k_p = 300$ µm. The energy at injection is 1 GeV, which gives a matched radius of 1 µm. For the ideal case with no secondary ionization and no ion motion, an initially matched bunch remains matched during acceleration and is adiabatically focussed, with the radius reaching 300 nm at 125 GeV.

However, simultaneously avoiding ion motion and secondary ionization is challenging, especially for the extremely compact bunches considered here. Our PDPWFA makes use of an electron witness bunch with a population of 1×10^{10} particles and an RMS length of 105 µm. In order to avoid secondary ionization, we make use of a singly-ionized lithium. However, in this case, the motion of plasma ions cannot be neglected.

In general, the proton drive bunch moves more slowly than the witness bunch it accelerates, and so the witness gradually catches up with the driver. This "dephasing" can be avoided

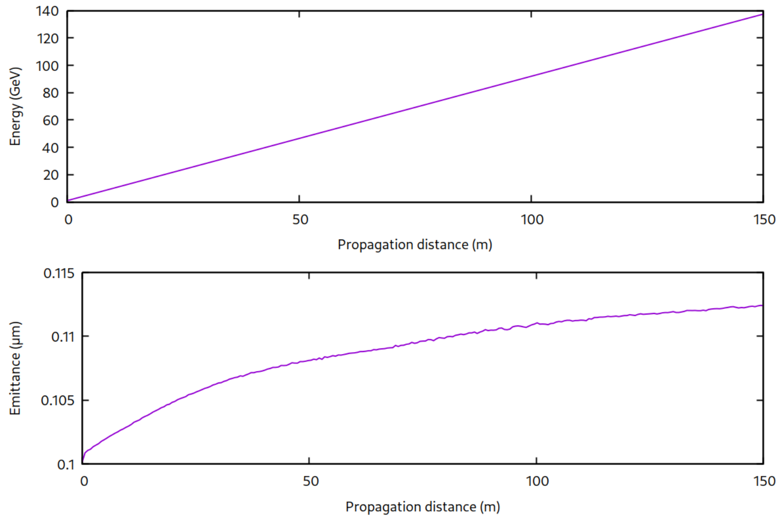

Figure 2. Evolution of the witness a) energy and b) normalised emittance for the case of a non-evolving drive bunch. A lithium plasma is used, with the emittance growth arising due to the motion of ions.

by increasing the plasma density along the accelerating length, decreasing the plasma period and maintaining the witness bunch in the accelerating phase of the wake [11, 12]. To simplify the simulations presented here, a non-evolving proton driver moving at the vacuum speed of light is used, allowing the evolution of the witness bunch and the ion motion it induces to be isolated. Although the proton driver has a population of 1×10^{11} particles, it has a RMS radius of 240 µm and an RMS length of 150 µm, resulting in a charge density which is too low to perturb the plasma ion density.

Simulations were carried out with the quasistatic PIC code LCODE [13, 14]. Figure 2 shows the witness energy and emittance as a function of propagation distance. As expected for a non-evolving driver, the energy gain is linear with distance, reaching the required energy of 125 GeV within 150 m. The emittance slowly increases due to the development of nonlinear focussing fields caused by ion motion. Figure 3a shows 1D projections of the witness bunch along a Cartesian transverse axis after different acceleration distances. Adiabatic focussing causes the beam radius to decrease as its energy increases. Figure 3 shows the corresponding focussing field, taken at the midpoint of the witness bunch. The focussing field at injection is essentially linear, as the charge density of the witness bunch is insufficient to drive significant ion motion. However, as the bunch radius decreases the charge density increases, scaling as $\sqrt{1/\gamma}$ for the case of no ion motion. At later propagation distances, the focussing field becomes increasingly nonlinear. However, we note that despite the significant nonlinearity in the focussing field, the witness emittance is well controlled, increasing by only $\sim 12\%$ as it is accelerated to 125 GeV.

To better understand how the emittance growth remains so well controlled, we consider slices of the witness bunch after 150 m acceleration, as shown in Fig 4a. Each lineout corresponds to the projection of one quartile of the bunch charge, from front to back. The sum of the four lines therefore gives the projection shown in Fig 3. The projection of the first quartile closely matches the expected Gaussian distribution in the absence of ion motion, shown by the black dashed line, calculated by using the initial emittance of 100 nm and the energy after 150 m in Eq. 1. However, towards the back of the bunch, the profile becomes more tightly

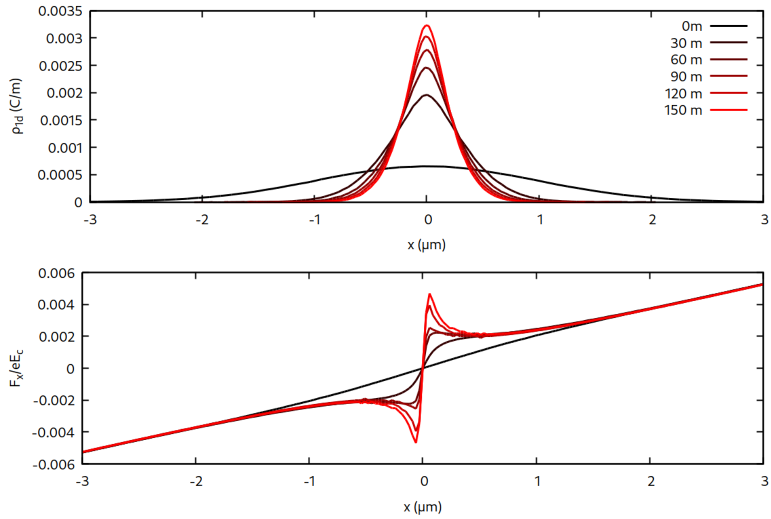

Figure 3. a) 1D transverse projections of the witness charge after different acceleration lengths, showing adiabatic focussing. b) The corresponding transverse focussing fields acting on the centre of the bunch, which become increasingly nonlinear due to the influence of ion motion driven by the witness.

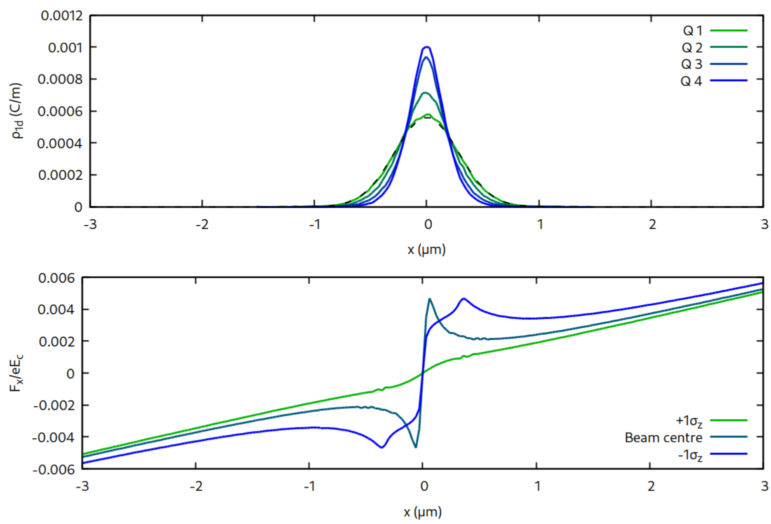

Figure 4. a) 1D projections of the witness charge after 150 m acceleration, showing different sections of the beam, from first (front) to fourth (rear) quartile. The black dashed line shows the expected profile in the absence of ion motion. b) The focussing field one sigma ahead of, at, and one sigma behind the bunch centre.

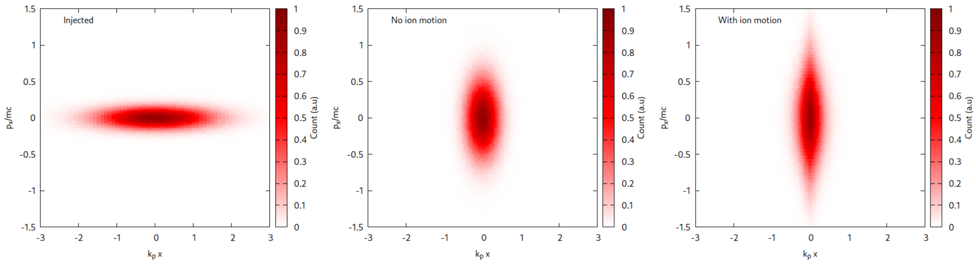

Figure 5. Phase-space projections of the witness bunch at injection (left), and after 150 m acceleration, for the case with no ion motion (centre) and with ion motion (right).

focussed, and more sharply peaked than a Gaussian distribution. Lineouts of the focussing field taken at different positions along the bunch show the focussing field is essentially linear near the start of the bunch, sharply peaked at the bunch centre, and gradually relaxing towards the bunch tail as the witness charge density decreases.

This sharp peaking of the distribution suggests that witness particles become trapped in smaller orbits as the focussing fields near the axis adiabatically increase over the acceleration length. This is indeed verified when looking at the evolution of the transverse phase-space, shown if Fig. 5. For no ion motion, the area of the phase-space ellipse is preserved and it remains a Gaussian ellipsoid, becoming narrower in x and wider in p_x. When ion motion is included, the phase-space area increases slightly, and shows regions near the axis where particles with large transverse momenta are trapped by the strongly nonlinear focussing fields.

To conclude, these simulation results show the impact of ion motion on the witness bunch quality in a plasma wakefield accelerator. By injecting at relatively low energy, the initial focussing fields are essentially linear and matching can be achieved for a witness bunch with a Gaussian transverse profile. As the witness bunch is accelerated, its radius decreases and the focussing fields become increasingly nonlinear due to the impact of ion motion. However, since the nonlinearity develops slowly, witness particles with a high transverse momentum can become trapped in the sharply peaked focussing fields near the axis, resulting in only a small increase in the beam emittance. This "adiabatic self-matching" is especially beneficial to proton-driven wakefield schemes as the witness bunch can be accelerated to high energies in a single plasma stage. Further work should be carried out to investigate the impact of the resulting distribution on the final focus.

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