Luminosity Studies for the Cool Copper Collider

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Abstract. Achieving high instantaneous luminosity while managing the beam-induced background (BIB) is critical for the successful operation of any future electron-positron (e^+e^-) collider. In this work, we summarize the first extensive luminosity studies for a proposed linear collider, the Cool Copper Collider (C^3), which are originally presented in [1]. We evaluate the impact of key beam parameters on the luminosity of C^3, and tune these parameters with the objective of optimizing the luminosity, without a commensurate increase in the accompanying BIB. Additionally, using a common simulation framework, we perform a comparative analysis of the luminosity and BIB characteristics of C^3 with those for other linear colliders. Finally, we present preliminary results towards an automatized luminosity optimization scheme.

1 Introduction

The Particle Physics community has agreed to pursue a high-energy e^+e^- collider as the next step beyond the LHC. Such a collider will enable precision measurements of the Higgs boson, the top quark, and electroweak observables, with the potential to uncover signatures of physics beyond the Standard Model [2–4]. Several such e^+e^- colliders have been proposed over the past few decades, both linear in design, such as the Compact Linear Collider (CLIC) [5], the International Linear Collider (ILC) [6] and the Cool Copper Collider (C^3) [7], as well as circular, the Future Circular Collider (FCC-ee) [8] and the Circular Electron Positron Collider (CEPC) [9].

For all these machines, the rate at which hard scatter events take place are determined by their instantaneous luminosity. Unlike the LHC, where beams are µm-sized, beams focused down to the nm scale are necessary to reach similar luminosities. This leads to intense electromagnetic fields in the bunches, giving rise to beam-beam interactions that affect the instantaneous luminosity, through self-focusing of the opposing bunches and emission of beamstrahlung photons, while, at the same time, leading to the creation of beam-induced background (BIB) particles.

In this work, we present extensive simulations of these beam-beam effects for one of these colliders, C^3, with the purpose of optimizing the beam parameters to maximize the instantaneous luminosity, without a commensurate increase in the BIB. We also perform a

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comparison between various proposed colliders in terms of the achievable luminosity and the magnitude of the expected BIB. Finally, we present preliminary results towards an automatized luminosity optimization framework that can be utilized without the need to run additional beam-beam simulations.

2 Luminosity Optimization

At a linear e^+e^- collider, the instantaneous luminosity is given by:

$$\mathcal{L} = H_D \frac{N_e^2 n_b f_r}{4\pi\sigma_x^*\sigma_y^*} = H_D \mathcal{L}_{\text{geom}} \tag{1}$$

where N_e is the number of particles per bunch, n_b is the number of bunches per bunch train, f_r is the train repetition rate and $\sigma_{x,y}^*$ are the horizontal and vertical, respectively, root-mean-square (RMS) beam sizes at the Interaction Point (IP), calculated as:

$$\sigma_{x,y}^* = \sqrt{\frac{\epsilon_{x,y}^*\beta_{x,y}^*}{\gamma}} \tag{2}$$

with $\epsilon_{x,y}^*, \beta_{x,y}^*$ denoting, respectively, the normalized emittances and beta functions at the IP in the horizontal (x) and vertical (y) directions and γ being the relativistic Lorentz factor of the beam particles. Finally, H_D is a dimensionless enhancement factor, which accounts for the increase in luminosity due to beam-beam interactions leading to the oppositely charged beams attracting and focusing each other down at the IP. The geometric luminosity $\mathcal{L}_{\text{geom}}$ is simply the luminosity obtained when these interactions are neglected.

The enhancement factor H_D has typical values between $\sim 1.5 - 2.5$ for proposed e^+e^- colliders and has been measured experimentally at the Stanford Linear Collider (SLC) [10]. The effect of H_D can therefore not be neglected in any luminosity optimization considerations. However, an analytical formula for H_D as a function of various beam parameters is not known, and the extraction of H_D has long been carried out through Particle-In-Cell (PIC) simulations of the interactions of the colliding bunches. We follow the same approach in this study and rely on simulation obtained with the GUINEA-PIG generator [11].

Using the beam parameters introduced above, we can define the so-called average Beamstrahlung parameter [12]

$$\langle\Upsilon\rangle = \frac{5}{6}\frac{N_e r_e^2 \gamma}{\alpha(\sigma_x^* + \sigma_y^*)\sigma_z^*} \tag{3}$$

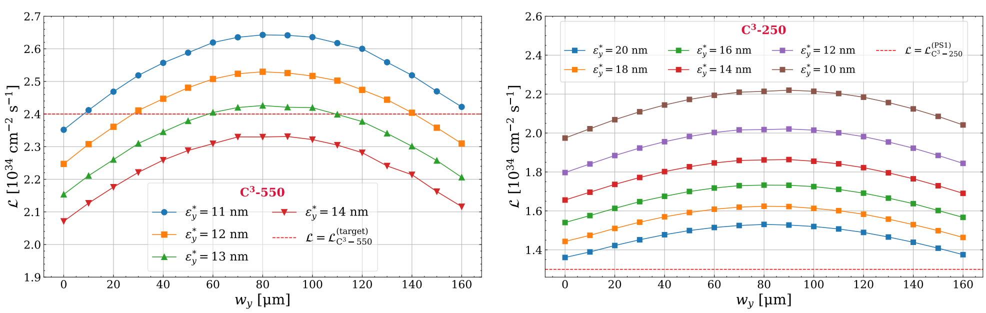
which expresses the strength of Beamstrahlung emission for Gaussian beams, where α is the fine structure constant and σ_z^* the bunch length, i.e. the RMS beam size in the beam direction (z). Larger values of $\langle\Upsilon\rangle$ correspond to more intense Beamstrahlung photon emission and hence to larger number of BIB particles produced. For the proposed future e^+e^- colliders, $\langle\Upsilon\rangle$ is typically much less than one.

Since the rate of collisions is directly proportional to the instantaneous luminosity $\mathcal{L}$, the goal is generally to achieve as high $\mathcal{L}$ as possible. However, maximizing $\mathcal{L}$ is a challenging task for several reasons. First, $\mathcal{L}$ is a multivariate function of various beam parameters and its exact dependence on said parameters is not given by an analytical formula, since the enhancement factor H_D in (1) is obtained from simulations, as explained earlier. Second, the luminosity maximization process is subject to several constraints, coming both from accelerator requirements (e.g. maximum achievable emittance reduction in damping rings, emittance growth in main linac, lattice performance, breakdown rates requirements etc.) and detector requirements (e.g. maximum allowed occupancy from BIB particles). Correctly specifying all these constraints requires deep understanding of the performance of the machine and the detectors and is a challenging task.

3 Beam parameter optimization for C^3

Nevertheless, a conventional luminosity optimization process relies on running simulations for various combinations of the input beam parameters and performing scans of the resulting luminosity in order to find the combinations of beam parameters for which it is maximized. In [1] we present a detailed analysis of the dependence of $\mathcal{L}$ for C^3 on the emittances $\epsilon^*_{x,y}$, the bunch length σ^*_z, and the vertical waist shift w_y, which will be introduced in the following. Here, we present a summary of the process and highlight the key results.

We start with C^3 at 550 GeV, since this center-of-mass energy sets the most stringent requirements, and evaluate how $\mathcal{L}$ varies as a function of the vertical waist shift w_y, which is the displacement along the beam direction (z) of the focal point of the bunches in the vertical (y) direction. This has been found to lead to $\mathcal{O}(10\%)$ luminosity enhancement [13] owing to beam-beam interactions actually shifting the focal point of the bunches to the IP. This is observed in Figure 1a, where a maximization of $\mathcal{L}$ for $w_y = 80$ µm can be seen. The same holds true for C^3-250 as well, as can be seen in Figure 1b. We also notice that the target luminosity for C^3-550 of $2.4 \cdot 10^{34}$ cm^{-2}s^{-1} is achieved for this waist shift value and ϵ^*_y values below $\sim$ 13 nm. Additionally, in Figures 2a, 2b we investigate how $\mathcal{L}$ varies as a function of ϵ^*_y for different values of σ^*_z and ϵ^*_x. We observe that $\mathcal{L}$ increases as either of these parameters is decreased, however, at the same time, Beamstrahlung, and the resulting BIB are also enhanced, as can be seen from Eq. (3).

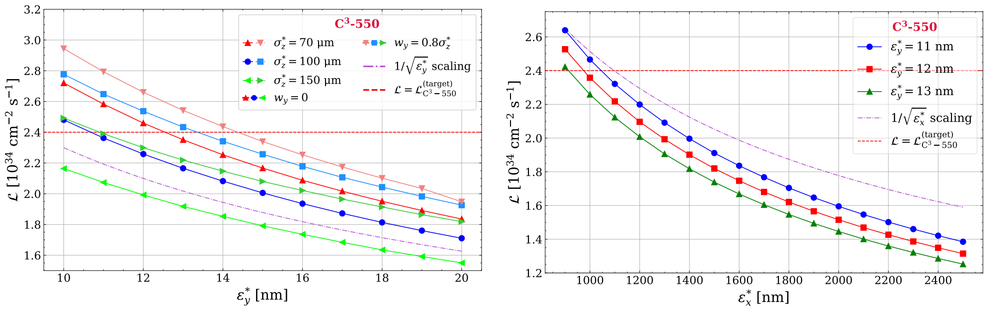


(a) (b)

Figure 1: Instantaneous luminosity as a function of the vertical waist shift w_y and for different values of the vertical emittance ϵ^*_y (a) for C^3-550 and (b) for C^3-250. In both plots, all other parameters are kept at their nominal values, while the horizontal red dashed lines indicate the target luminosity.

Based on these observations, and after evaluating the impact of these parameters on C^3-250 as well, we propose in [1] a new parameter set for C^3, which we call Parameter Set 2 (PS2) and which can be summarized in the following changes:

- A reduction of ϵ^*_y from 20 nm to 12 nm in order to increase $\mathcal{L}$.
- An introduction of $w_y = 80$ µm in order to increase $\mathcal{L}$.



(a) (b)

Figure 2: Instantaneous luminosity for C^3-550 as a function of (a) the vertical ϵ_y^* and (b) the horizontal ϵ_x^* emittance and further modified beam parameters as shown in each figure. All other beam parameters are kept to their nominal values. The scaling of the luminosity in the absence of beam-beam interactions $1/\sqrt{\epsilon^*}$ is also given in each case. In both plots, the horizontal red dashed line indicates the target luminosity for C^3-550.

- A slight increase of ϵ_x^* from 900 nm to 1000 nm in order to mitigate the effects of Beamstrahlung.

These changes are summarized in Table 1 and ensure that the target luminosity of $1.3\ (2.4) \cdot 10^{34}\ \text{cm}^{-2}\text{s}^{-1}$ for C^3 at 250 (550) GeV can be achieved or surpassed while keeping the BIB at tolerable levels.

Table 1: Baseline (PS1) scenario and new proposed set of beam parameters (PS2) for C^3 at 250 and 550 GeV.

Parameter	Symbol [unit]	C^3-250 (PS1)	C^3-250 (PS2)	C^3-550 (PS1)	C^3-550 (PS2)
Center-of-mass Energy	$\sqrt{s_0}$ [GeV]	250		550	
RMS bunch length	σ_z^* [µm]	100		100	
Horizontal beta function at IP	β_x^* [mm]	12		12	
Vertical beta function at IP	β_y^* [mm]	0.12		0.12	
Normalized horizontal emittance at IP	ϵ_x^* [nm]	900	1000	900	1000
Normalized vertical emittance at IP	ϵ_y^* [nm]	20	12	20	12
RMS horizontal beam size at IP	σ_x^* [nm]	210	221	142	149
RMS vertical beam size at IP	σ_y^* [nm]	3.1	2.4	2.1	1.6
Vertical waist shift	w_y [µm]	0	80	0	80
Geometric Luminosity	$\mathcal{L}_{geom}\ \left[10^{34}\ \text{cm}^{-2}\ \text{s}^{-1}\right]$	0.75	0.92	0.93	1.14
Average Beamstrahlung Parameter	$\langle\Upsilon\rangle$	0.065	0.062	0.21	0.20
Total Luminosity	$\mathcal{L}\ \left[10^{34}\ \text{cm}^{-2}\ \text{s}^{-1}\right]$	1.35	1.90	1.70	2.40
Peak luminosity fraction	$\mathcal{L}_{0.01}/\mathcal{L}$ [%]	73	74	52	54
Enhancement Factor	H_D	1.8	2.1	1.8	2.1

4 Comparison with other linear colliders

After optimizing the beam parameter set for C^3, we used the same GUINEA-PIG simulation framework to perform a comparison of the luminosity spectrum of C^3 with two other linear colliders, CLIC and ILC. These spectra are obtained from the distributions of beam particles' energy after beam-beam interactions and account for the effects of both natural energy spread of the beam, as well as Beamstrahlung-induced spread. The input beam parameters for CLIC and ILC used in the simulations are shown in Table 2, and the corresponding luminosity spectra are compared in Figure 3. As can be seen, the spectra for both C^3 and ILC at 250 GeV have comparable spread, with C^3 achieving higher luminosity at the peak. On the contrary, C^3 at 550 GeV has a significantly larger spread than ILC at 500 GeV and additional beam parameter modifications would be required to mitigate that. Finally, comparing the C^3 spectra for the two parameter sets – PS1 and PS2 –, we see that PS2 achieves its goal of luminosity gain at the peak without significant broadening of the overall spectrum.

Table 2: Beam parameters for various linear collider proposals. For C^3, the baseline beam parameters are given, as found in [7], which we refer to as Parameter Set 1 (PS1) in this work.

Parameter	Symbol [unit]	CLIC [14]	ILC-250 [15]	ILC-500 [15]	C^3-250 (PS1) [7]	C^3-550 (PS1) [7]
Center-of-mass Energy	$\sqrt{s_0}$ [GeV]	380	250	500	250	550
Bunch Population ($\boldsymbol{x_1}$)	N_e [10^9 particles]	5.18	20.0	20.0	6.24	6.24
Horizontal emittance ($\boldsymbol{x_2}$)	ϵ_x^* [nm]	950	5000	5000	900	900
Vertical emittance ($\boldsymbol{x_3}$)	ϵ_y^* [nm]	30	35	35	20	20
Horizontal beta function ($\boldsymbol{x_4}$)	β_x^* [mm]	8.2	13	22	12	12
Vertical beta function ($\boldsymbol{x_5}$)	β_y^* [mm]	0.1	0.41	0.49	0.12	0.12
RMS bunch length ($\boldsymbol{x_6}$)	σ_z^* [µm]	70	300	300	100	100
RMS horizontal beam size	σ_x^* [nm]	145	516	474	210	142
RMS vertical beam size	σ_y^* [nm]	2.8	7.7	5.9	3.1	2.1
Num. Bunches per Train	n_b	352	1312	1312	133	75
Train Rep. Rate	f_r [Hz]	50	5	5	120	120
Geometric Luminosity	$\mathcal{L}_{\text{geom}}$ $\left[10^{34}\text{ cm}^{-2}\text{ s}^{-1}\right]$	0.91	0.53	0.74	0.75	0.93
Average Beamstrahlung	$\langle\Upsilon\rangle$	0.17	0.028	0.062	0.065	0.21
Total Luminosity	$\mathcal{L}$ $\left[10^{34}\text{ cm}^{-2}\text{ s}^{-1}\right]$	1.67	1.35	1.80	1.35	1.70
Enhancement Factor	H_D	1.8	2.6	2.4	1.8	1.8

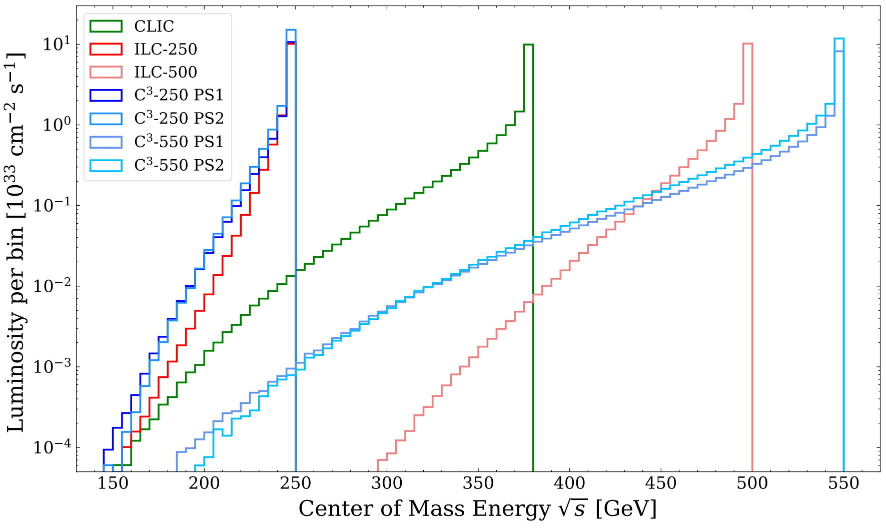


Figure 3: Luminosity spectra for the linear collider proposals under consideration here, as obtained from GUINEA-PIG simulations using the beam parameters from Tables 1, 2.

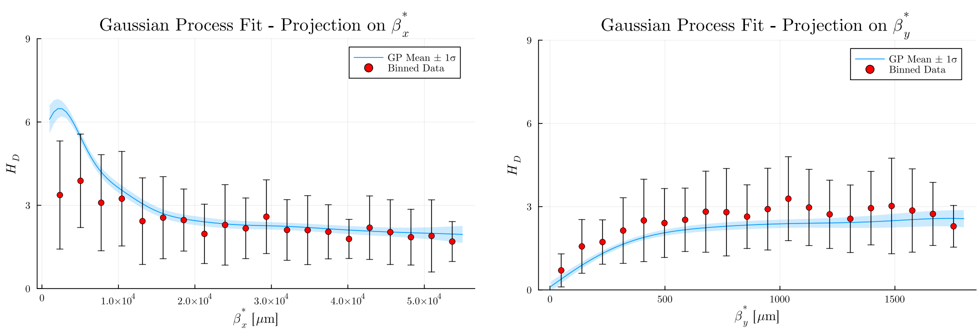
5 Surrogate model optimization

In the previous two sections, we presented optimization results obtained from discrete sampling on a grid of beam-parameter combinations. However, this approach has several limitations, as it relies on extensive simulation runs, which can be computationally intensive, does not inherently account for the correlations between the various beam parameters, and does not provide a concrete way to impose constraints on the beam parameters.

To overcome this, we train a Gaussian Process (GP) [16] probabilistic surrogate model on a large number ($\sim \mathcal{O}(10^4)$) of GUINEA-PIG simulation results obtained from a uniform sampling of the following beam parameters: $(N_e, \epsilon_x^*, \epsilon_y^*, \beta_x^*, \beta_y^*, \sigma_z^*)$ to predict the enhancement factor H_D for any given combination of the input parameters. We then use this to define a surrogate model for the luminosity, which, rearranging Eq. (1), is written as:

$$\hat{\mathcal{L}}(\boldsymbol{x}) = \frac{n_b f_\gamma \gamma}{4\pi}\left(\hat{H}_D(\boldsymbol{x})\frac{N_e^2}{\sqrt{\epsilon_x^*\beta_x^*\epsilon_y^*\beta_y^*}}\right), \quad \boldsymbol{x} = \left(N_e, \epsilon_x^*, \epsilon_y^*, \beta_x^*, \beta_y^*, \sigma_z^*\right) \tag{4}$$

where $\hat{H}_D$ is the surrogate model for the enhancement factor obtained from the GP, and $\boldsymbol{x}$ is the 6-dimensional feature vector of the model. An example of the prediction of the surrogate model when projected on a single dimension (i.e. one beam parameter) is shown in Figure 4.



(a) (b)

Figure 4: Gaussian Process Fit for $\mathrm{H_D}$ as a function of the (a) horizontal β_x^* and (b) vertical β_y^* beta functions. The blue line indicates the mean prediction of the GP model, and the blue-shaded region represents the 68% confidence region. The data points have been binned in equal-width intervals and the average value of H_D within each bin is plotted. The error bars indicate the standard deviation within each bin. For the prediction, all other components of the design vector $\boldsymbol{x}$ were set to their average value over the sample space.

Using this surrogate model we can then perform efficient luminosity optimization, without the need for additional GUINEA-PIG grid sampling, and using highly performant black-box optimizers, with the added benefit of being able to impose constraints on the optimization problem in a straightforward way. A test run was performed where the optimization problem solved was formulated as:

$$\text{maximize } \hat{\mathcal{L}} = \frac{n_b f_r \gamma}{4\pi}\left(\hat{H}_D(\boldsymbol{x})\frac{N_e^2}{\sqrt{\epsilon_x^*\beta_x^*\epsilon_y^*\beta_y^*}}\right), \boldsymbol{x} = \left(N_e, \epsilon_x^*, \epsilon_y^*, \beta_x^*, \beta_y^*, \sigma_z^*\right)$$
$$\text{subject to } \boldsymbol{x}^{(\min)} \leq \boldsymbol{x} \leq \boldsymbol{x}^{(\max)}, \langle\Upsilon\rangle = \frac{5r_e^2\gamma}{6\alpha}\frac{N_e}{\left(\sigma_x^* + \sigma_y^*\right)\sigma_z^*} \leq \Upsilon^{\max} \tag{5}$$

where the rectangular constraints $\boldsymbol{x}^{(\min)}, \boldsymbol{x}^{(\max)}$ on $\boldsymbol{x}$ where chosen such that $\boldsymbol{x}$ is allowed to vary within ±10% of the nominal values of Table 2 for each collider and the upper imposed limit $\Upsilon^{\max}$ on the Beamstrahlung parameter was set equal to the respective $\langle\Upsilon\rangle$ value of Table 2. Conceptually, this optimization problem answers the question: *What's the highest achievable luminosity if we allow the beam parameters to change within 10% of their chosen values while keeping the Beamstrahlung at the same or smaller levels?* The optimization problem was solved using the `JuMP` [17] domain-specific modeling language, while the `AbstractGPs` package [18] was used for the surrogate model, all within the `Julia` [19] programming language. The results are presented in Table 3 and indicate that with ±10% variations in beam parameters, in some cases up to ∼ 35% luminosity gain can be achieved. Although this is a first and preliminary result, it indicates the potential of this method.

Table 3: Luminosity gain across various colliders obtained through the surrogate model optimization scheme. The values with the "literature" subscript correspond to those found in Table 2, the ones with the subscript "GP" refer to the estimation from the Gaussian process surrogate model, and the ones with the subscript "sim" refer to the results of GUINEA-PIG simulations.

Quantity	CLIC	ILC-250	ILC-500	C^3-250	C^3-550
$\frac{4\pi}{n_b f_r \gamma}\mathcal{L}_{\text{literature}}$ [$\times 10^{29}$ m^{-2}]	3.2069	10.572	7.0479	4.3453	4.4107
$\langle\Upsilon\rangle_{\text{literature}}$	0.17	0.028	0.062	0.065	0.21
$\frac{4\pi}{n_b f_r \gamma}\mathcal{L}_{\text{GP}}$ [$\times 10^{29}$ m^{-2}]	5.0557	12.65	9.993	5.9109	6.6508
$\frac{4\pi}{n_b f_r \gamma}\mathcal{L}_{\text{sim}}$ [$\times 10^{29}$ m^{-2}]	4.9582	10.862	9.447	5.6218	5.7022
$\langle\Upsilon\rangle_{\text{GP}}$	0.12266	0.02801	0.02961	0.06501	0.08825
Overall $\mathcal{L}$ gain (%)	**35.3**	**2.7**	**25.4**	**22.7**	**22.6**

6 Conclusions

We presented the results of the first extensive luminosity optimization analysis for the C^3 collider concept and proposed a new parameter set, which achieves significant luminosity gains, reaching or exceeding the C^3 luminosity goals while keeping beamstrahlung and the resulting beam-induced background at similar levels. Further studies on the impact of these beam parameter modifications on the accelerator and detector design are necessary to inform whether these luminosity gains are indeed achievable. Finally, we presented preliminary yet promising results on a surrogate model for automatized luminosity optimization and are working on extending and improving its performance.

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