Optimizing the Higgs self-coupling measurement at ILC and C^3

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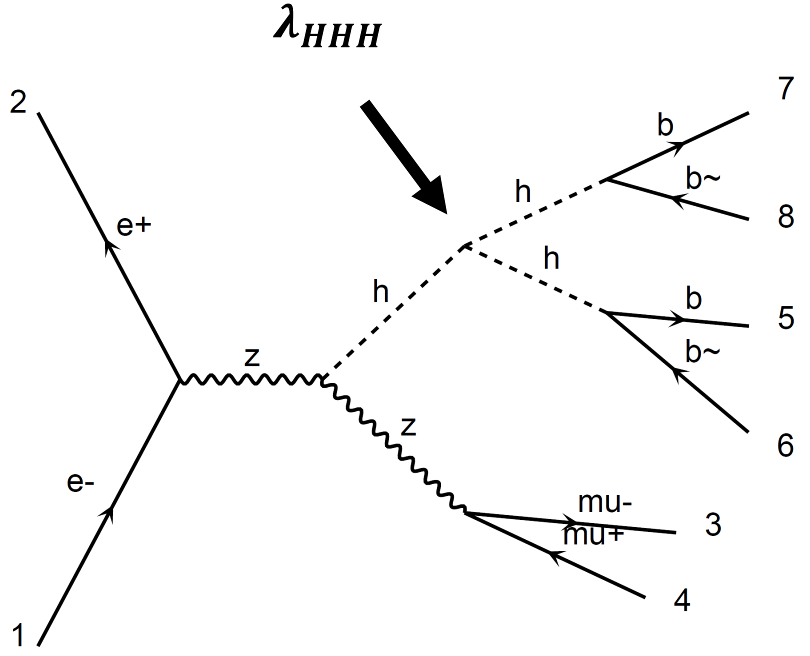
The Matrix Element Method (MEM)

- method for calculating event-likelihoods, use cases:
  - process discrimination (Neyman-Pearsson lemma)
  - parameter estimation
- Goal here: separate ZHH vs. ZZH → $\mu^-\mu^+b\bar{b}b\bar{b}$
- for each event $\boldsymbol{y}$ and process i (ZHH, ZZH), solve

$$P_i\,(\boldsymbol{y} \mid \boldsymbol{a}) = \frac{1}{\sigma_i(\boldsymbol{a}) \cdot A_i(\boldsymbol{a})} \int |M_i(\boldsymbol{x},\boldsymbol{a})|^2 W_i(\boldsymbol{y} \mid \boldsymbol{x}) \epsilon_i(\boldsymbol{x}) d\boldsymbol{\Phi}_n(\boldsymbol{x})$$

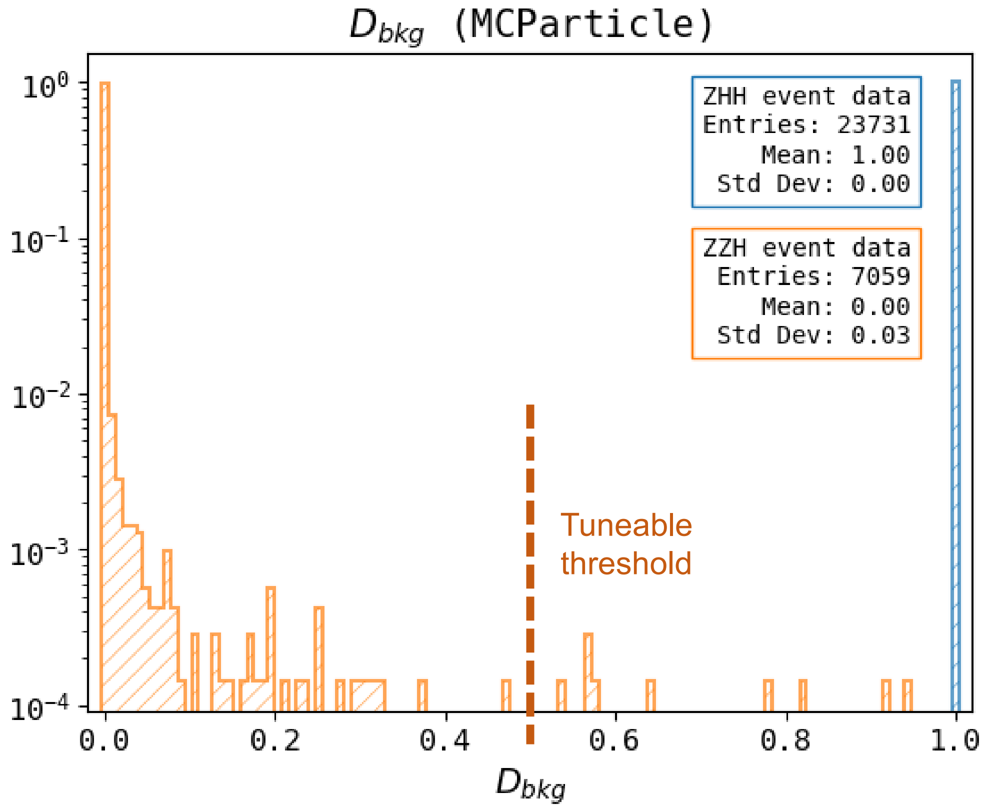
  - $M_i(\boldsymbol{x},\boldsymbol{a})$ LO matrix element (HELAS-based Physsim, J. Tian)
  - $W_i(\boldsymbol{y}|\boldsymbol{x})$ detector transfer functions: PDF for measuring $\boldsymbol{y}$ given $\boldsymbol{x}$; fitted from ILD full-simulation



$\boldsymbol{a}$: theory parameters; e.g. λ_{HHH}
$A_i(\boldsymbol{a})$: signal acceptance
$\epsilon_i(\boldsymbol{x})$: detector efficiency

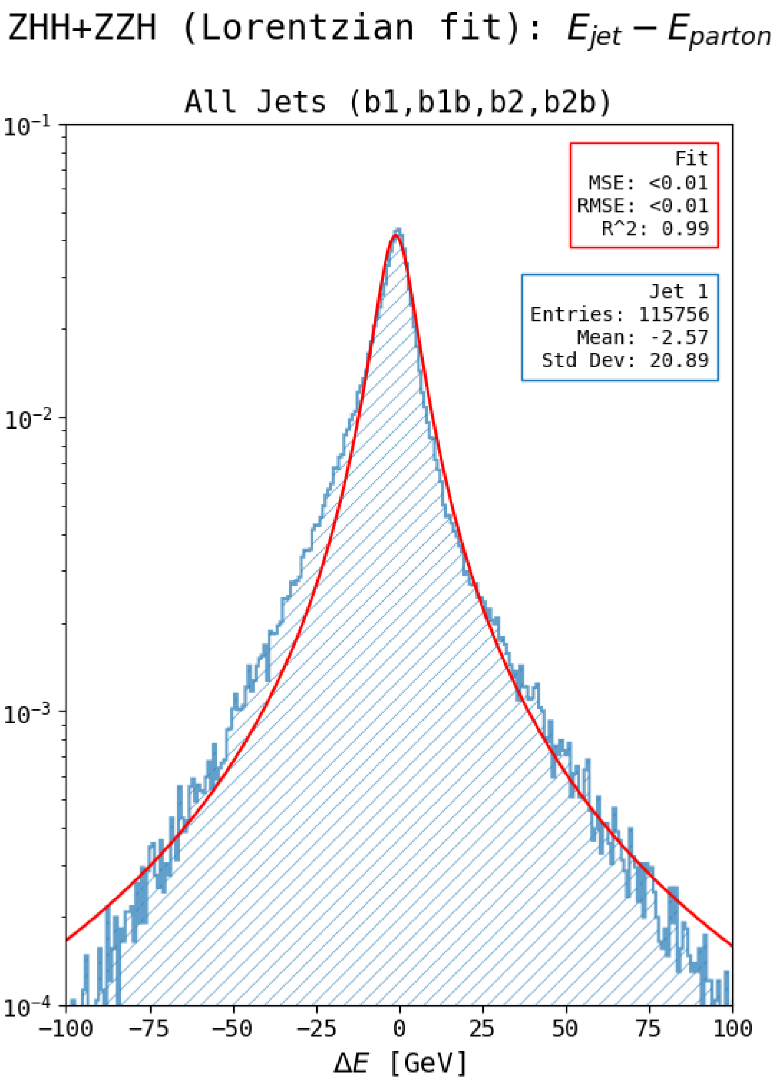
MEM generator level check

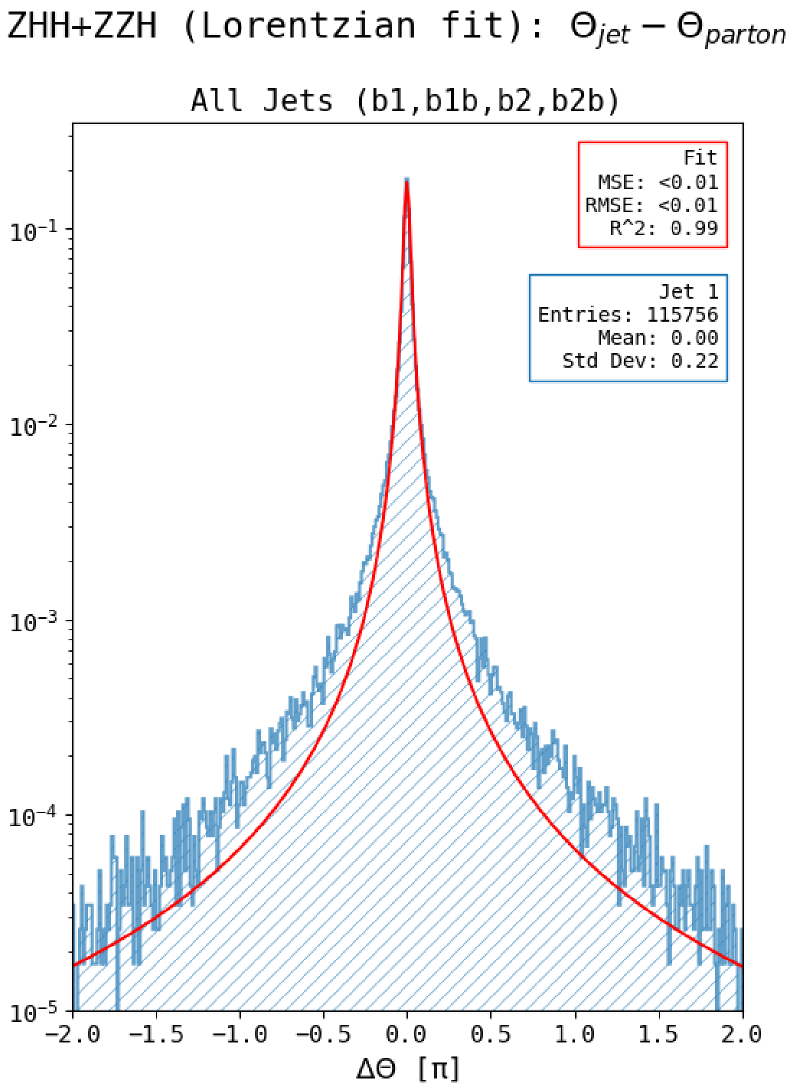
- on generator level: integration reduces to evaluation of matrix elements
- discriminator: $D_{bkg}(y) = \left(1 + \frac{P_{ZZH}(y)}{P_{ZHH}(y)}\right); 0 \leq D_{bkg} \leq 1$
- perfect separation of ZHH and ZZH event data, as expected



Detector transfer functions (I)

- PDF for energies/angles between reconstructed and parton-level particles
- „conventional approach“: fitting transfer functions manually

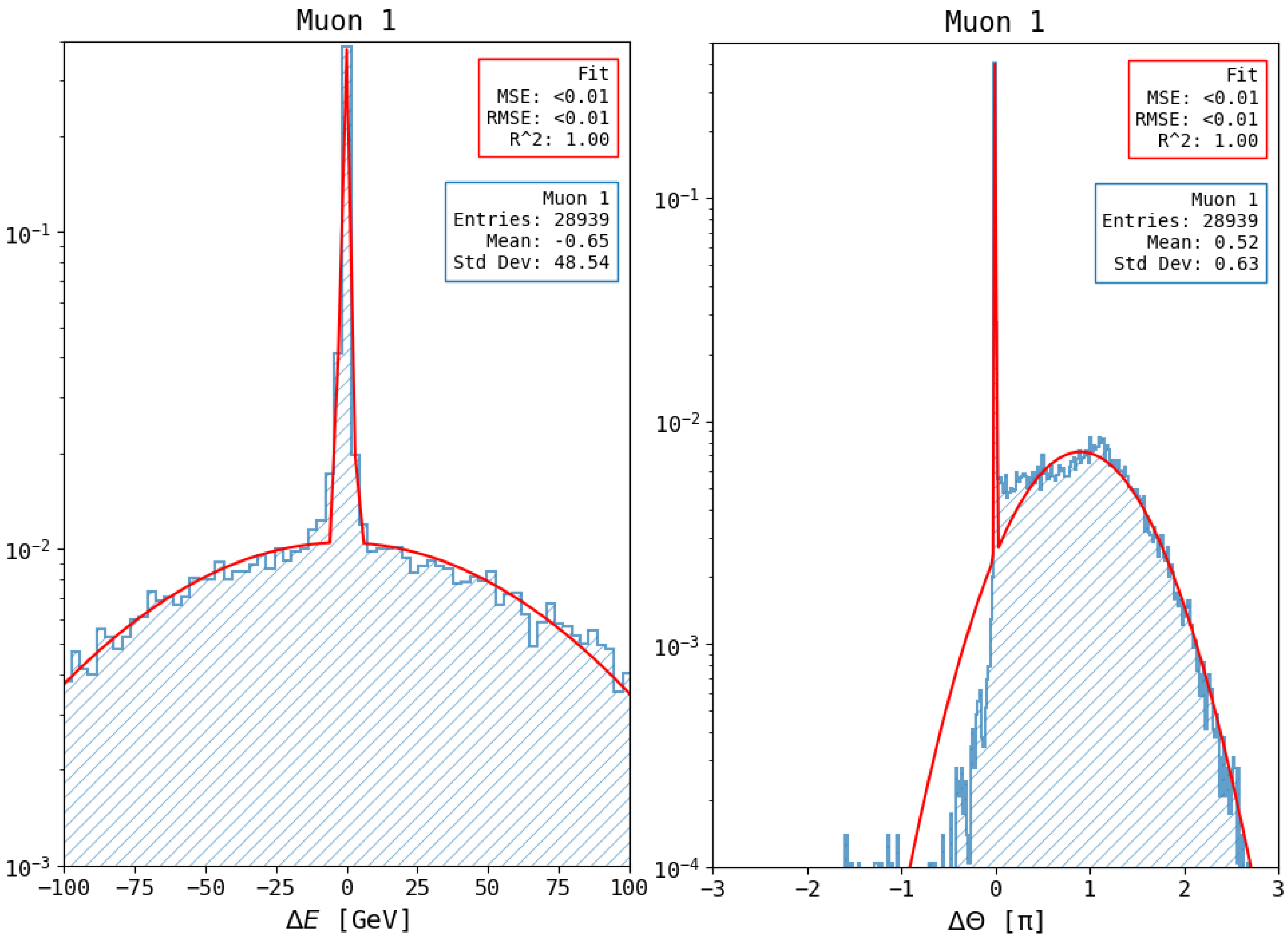




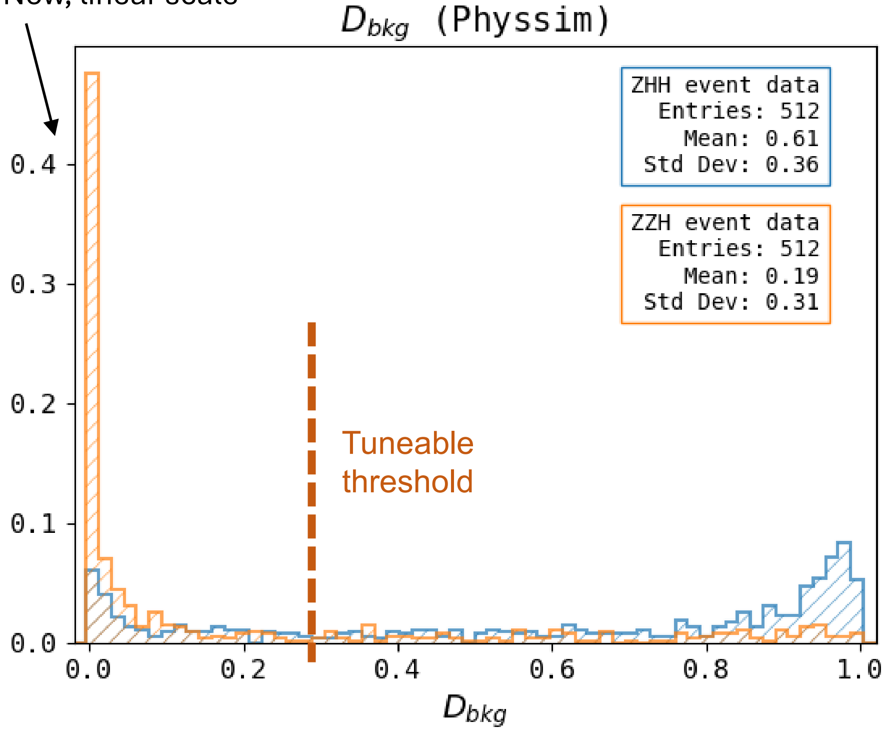
Detector transfer functions (II)

- For well-measured quantities (e.g. leptons): δ-function

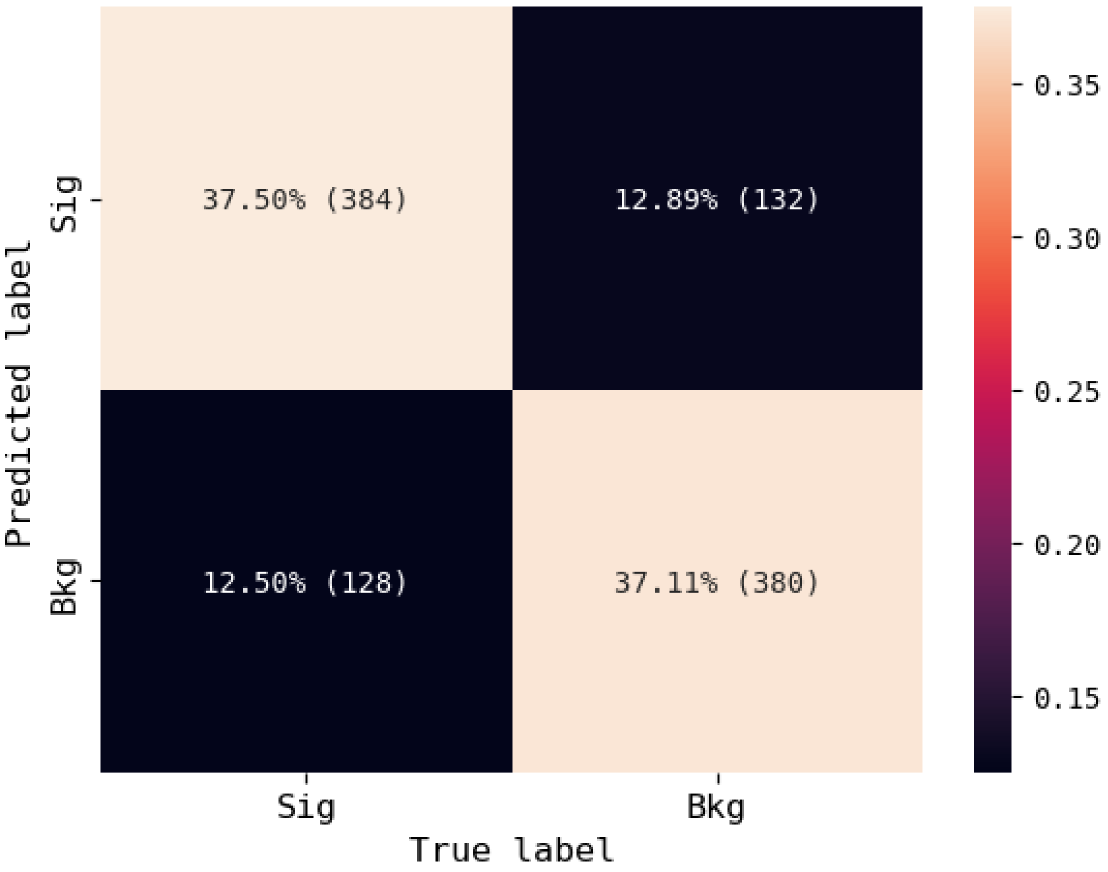
$$W_{lep}\left(E_p, E_{reco}\right) = \delta\left(E_p - E_{reco}\right)$$



Results: Process discrimination


Now, linear scale

Tuneable threshold



- Limitations:
  - Cheated jet-matching
  - ISR still to be addressed
  - Computationally intensive: integration time $o(\mathrm{min})$ per event

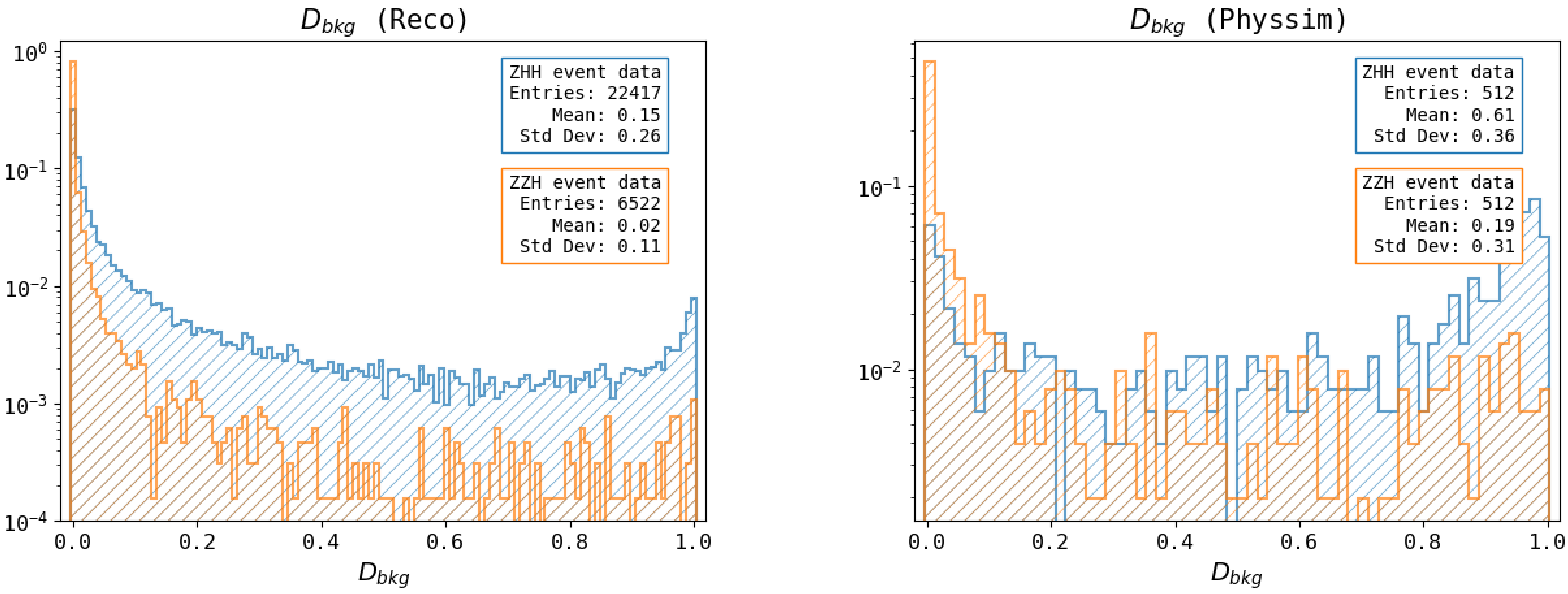
Outlook

- ML-based approaches for increasing speed and performance
  - invertible neural network (INNs) for transfer functions $W_i(\boldsymbol{x}|\boldsymbol{y})$
  - Graph Neural Networks (GNNs) for jet clustering (major source of error in analysis)

- Promising additions to the conventional approach:
  - (weighted) average over jet-matching combinatorics
  - further consistency checks, using Whizard LO matrix elements
  - addressing ISR

Backup

- Distributions before and after including transfer functions

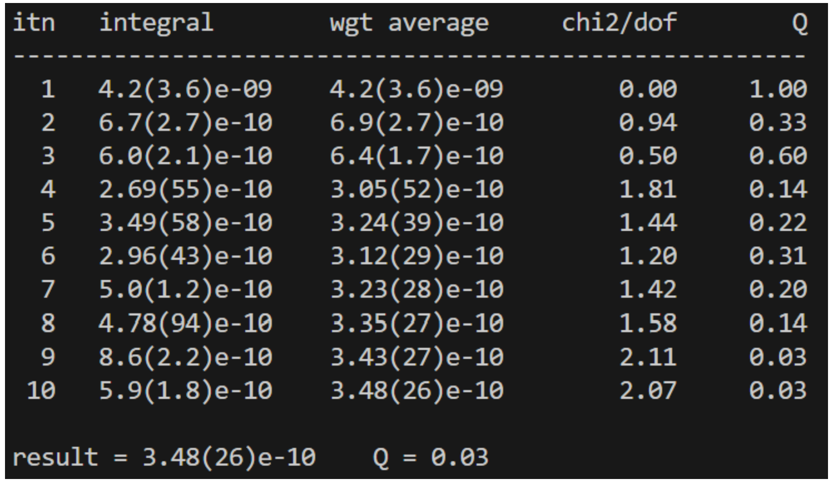


Backup: Monte-Carlo phase space integration

$$P_i\,(\boldsymbol{y}\,|\,\boldsymbol{a}) = \frac{1}{\sigma_i(\boldsymbol{a})\cdot A_i(\boldsymbol{a})}\int W_i(\boldsymbol{y}\,|\,\boldsymbol{x},\boldsymbol{a})|M_i(\boldsymbol{x},\boldsymbol{a})|^2 T_i(\boldsymbol{x},\boldsymbol{a})\,d\boldsymbol{\Phi}_n$$

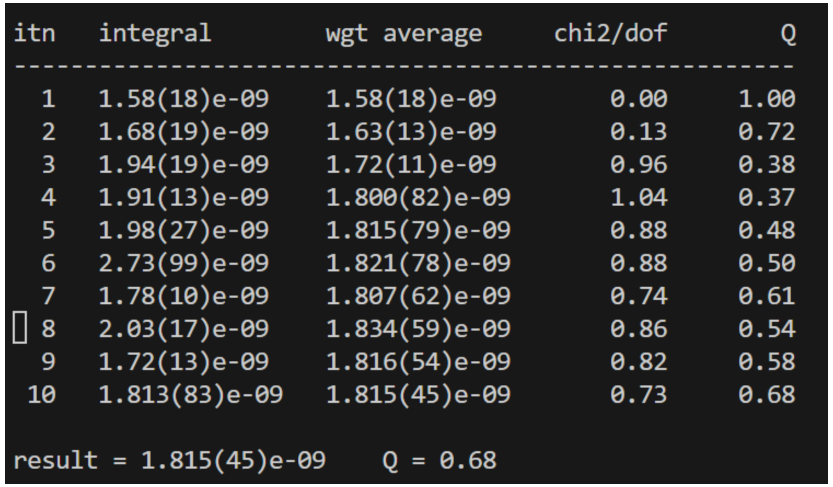
$$d\boldsymbol{\Phi}_n = \prod_i^{\mu^-,\mu^+,b_1,\overline{b_1},b_2,\overline{b_2}} \frac{d^3\boldsymbol{p_i}}{(2\pi)^3 2E_i}$$

- leptons well measured ➔ no integration for μ^-, μ^+
- conservation of four momentum and narrow-width-approximation ➔ reduction of integration to 7 dimensions
- integration variables: $\Theta_{b1}, \phi_{b1}, \rho_{b1}, \theta_{b1b}, \phi_{b1b}, \rho_{b2}, \Theta_{b2}$
- with VEGAS+ and integrand in C++, computation time 1-3 minutes per process (including setup of integration grid)
- „accept-and-reject“ MC



```
itn   integral        wgt average     chi2/dof        Q
-------------------------------------------------------
  1   4.2(3.6)e-09    4.2(3.6)e-09        0.00     1.00
  2   6.7(2.7)e-10    6.9(2.7)e-10        0.94     0.33
  3   6.0(2.1)e-10    6.4(1.7)e-10        0.50     0.60
  4   2.69(55)e-10    3.05(52)e-10        1.81     0.14
  5   3.49(58)e-10    3.24(39)e-10        1.44     0.22
  6   2.96(43)e-10    3.12(29)e-10        1.20     0.31
  7   5.0(1.2)e-10    3.23(28)e-10        1.42     0.20
  8   4.78(94)e-10    3.35(27)e-10        1.58     0.14
  9   8.6(2.2)e-10    3.43(27)e-10        2.11     0.03
 10   5.9(1.8)e-10    3.48(26)e-10        2.07     0.03

result = 3.48(26)e-10     Q = 0.03
```



```
itn   integral        wgt average     chi2/dof        Q
-------------------------------------------------------
  1   1.58(18)e-09    1.58(18)e-09        0.00     1.00
  2   1.68(19)e-09    1.63(13)e-09        0.13     0.72
  3   1.94(19)e-09    1.72(11)e-09        0.96     0.38
  4   1.91(13)e-09    1.800(82)e-09       1.04     0.37
  5   1.98(27)e-09    1.815(79)e-09       0.88     0.48
  6   2.73(99)e-09    1.821(78)e-09       0.88     0.50
  7   1.78(10)e-09    1.807(62)e-09       0.74     0.61
▯ 8   2.03(17)e-09    1.834(59)e-09       0.86     0.54
  9   1.72(13)e-09    1.816(54)e-09       0.82     0.58
 10   1.813(83)e-09   1.815(45)e-09       0.73     0.68

result = 1.815(45)e-09     Q = 0.68
```

MEM results for example ZHH (top) and ZZH (bottom) event