

$\tilde{\tau}$ searches at future e^+e^- colliders

Mikael Berggren¹, Terasa Núñez¹, Jenny List¹

¹DESY, Hamburg

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Motivation for $\tilde{\tau}$ searches

For SUSY searches it is a Good Idea (TM):

- To search for well motivated and maximally difficult NLSPs
- Since, if one can find this, then one can find any other NLSP

The $\tilde{\tau}$, the scalar super-partner of τ -lepton, satisfies both conditions.

- Well motivated:
 - Due to mixing, likely to be the lightest stermion.
 - Can do co-annihilation.
 - Least constrained from data.
- Difficult:
 - Due to mixing, has lower cross-section than other sleptons and squarks
 - Decays partially invisibly
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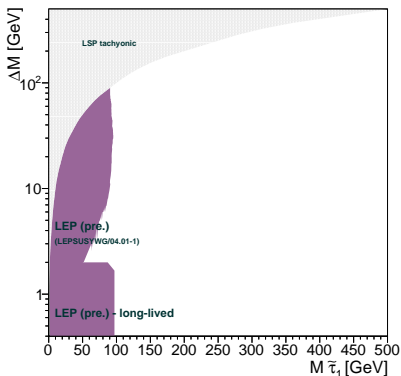
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The $\tilde{\tau}$...

- Two weak hypercharge eigenstates ($\tilde{\tau}_R, \tilde{\tau}_L$), not mass degenerate
- Mixing yields to the physical states ($\tilde{\tau}_1, \tilde{\tau}_2$), the lightest one being likely to be the lightest sfermion (stronger trilinear couplings)
- With assumed R-parity conservation:
 - Pair produced in s-channel via Z^0/γ exchange. Low σ since $\tilde{\tau}$ -mixing suppresses coupling to the Z^0 .
 - Decay to LSP and τ , implying more difficult signal identification than the other sfermions

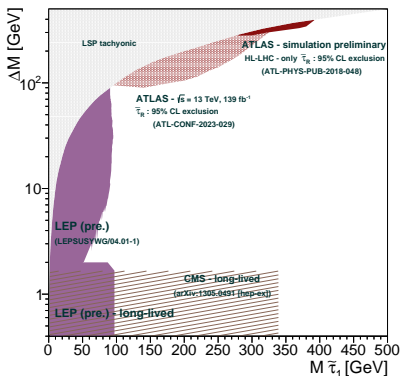
Limits at LEP and LHC/HL-LHC

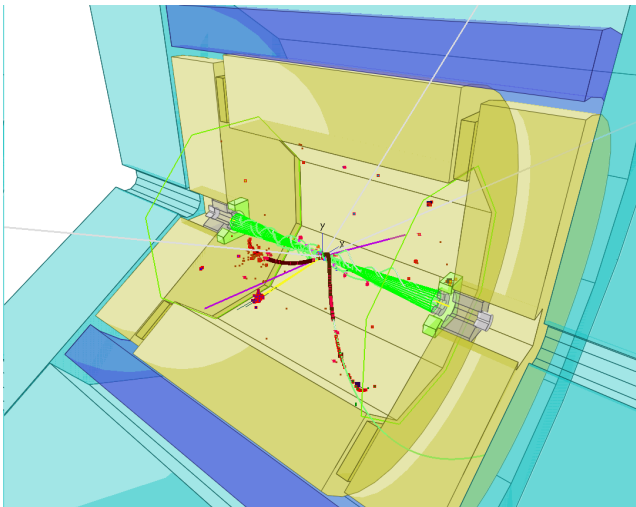
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 - PDG: Best published limit (DELPHI) 81.9 GeV (any mixing if $\Delta M > 15$ GeV), 26.3 for any mixing and ΔM
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- LHC : ATLAS model-dependent (only for $\tilde{\tau}_R$), excludes only very high ΔM . No discovery potential..
- HiLumi: exclude somewhat higher $\tilde{\tau}_R$ masses for very high ΔM . No discovery potential..

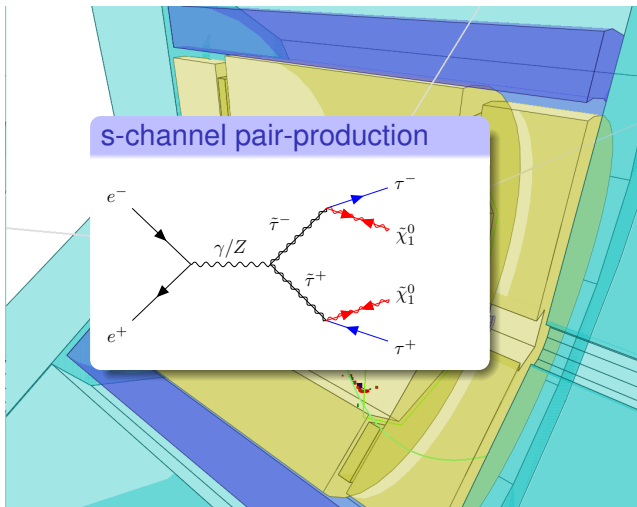


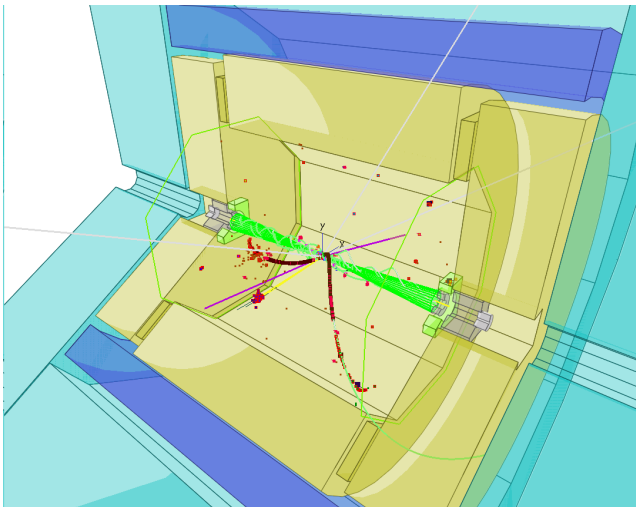
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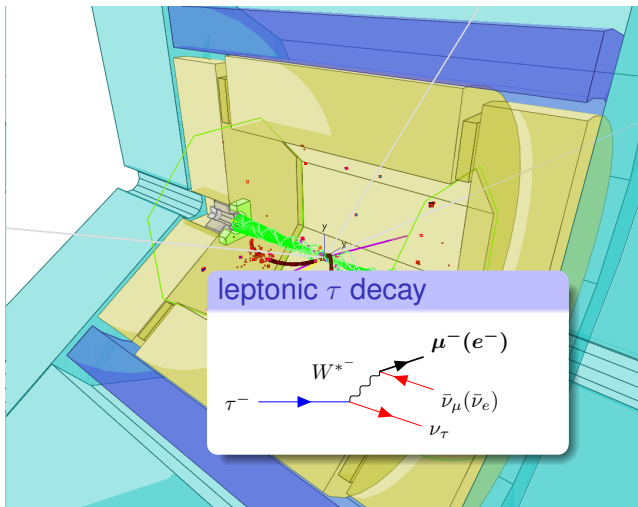
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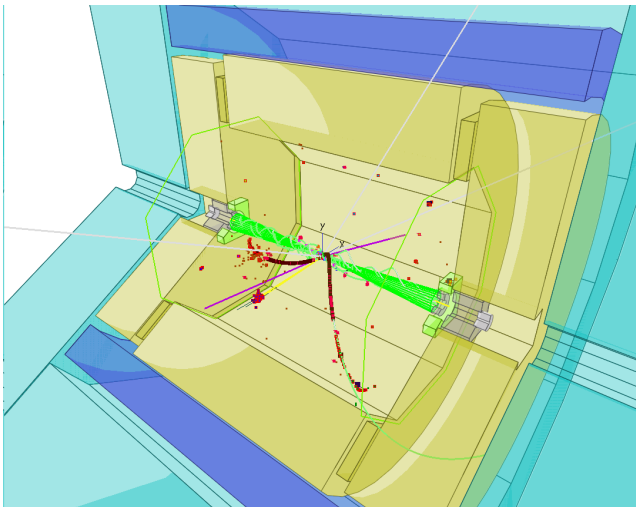


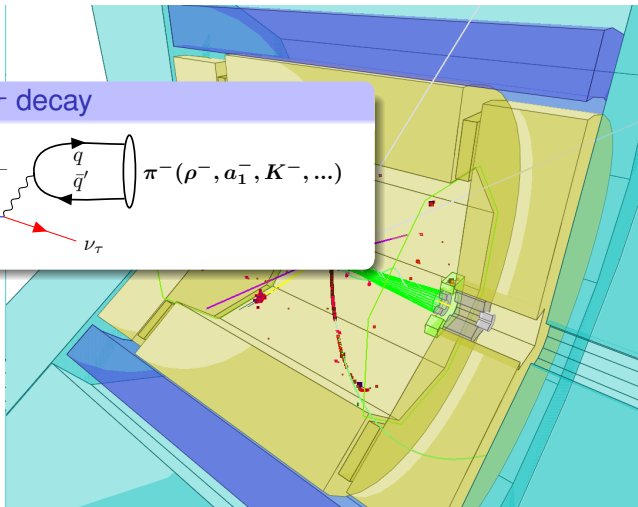
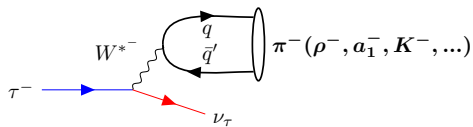
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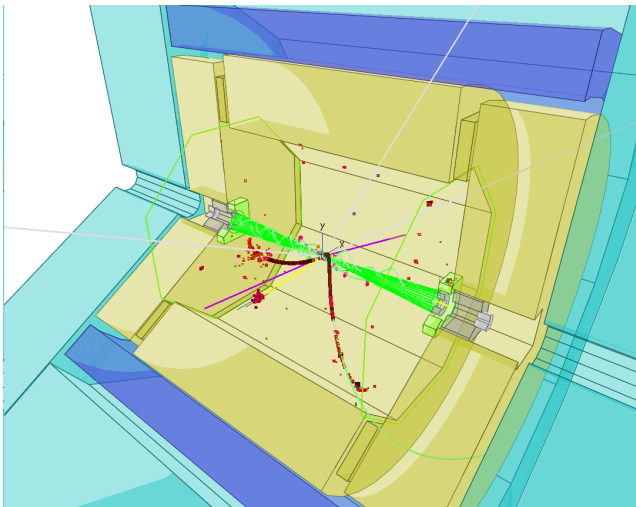
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Signature

- Large missing energy and momentum
- Large fraction of detected activity in central detector (isotropic production of scalar particles)
- Large angle between the two τ -lepton directions
- Unbalanced transverse momentum
- Zero forward-backward asymmetry

$\tilde{\tau}$ properties at e^+e^- colliders: Backgrounds

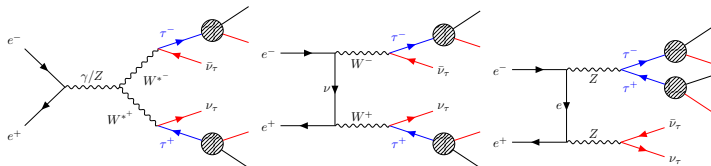
SM processes with **real** or **fake** missing energy

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Irreducible

- 4-fermion production with two of the fermions being neutrinos and two τ 's

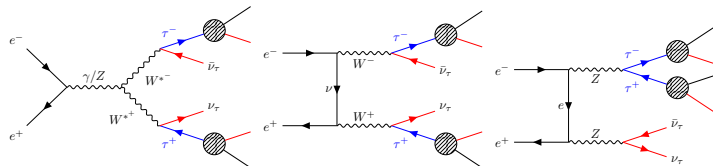


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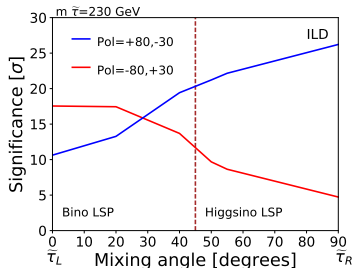


Almost Irreducible

- $e^+e^- \rightarrow \tau\tau$, $ZZ \rightarrow \nu\nu ll$, $WW \rightarrow l\nu l\nu$ ($l = e$ or μ)
- $e^+e^- \rightarrow \tau\tau + ISR$, $e^+e^- \rightarrow \tau\tau ee$, $\gamma\gamma \rightarrow \tau\tau$
- Mis-identification of τ 's or of missing momentum

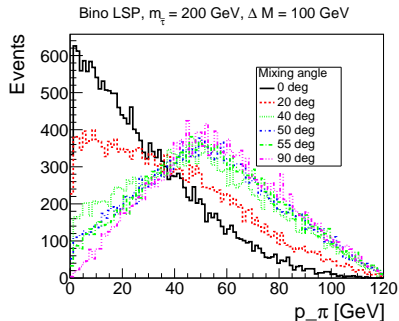
$\tilde{\tau}$ properties at e^+e^- colliders : impact of mixing and LSP nature

- Production cross-section depends on mixing.
- Visibility depends on the τ polarisation, and τ polarisation depends on both $\tilde{\tau}$ and neutralino nature.
- So, to get the worst case, the combination of low cross-section and low visibility should be found.



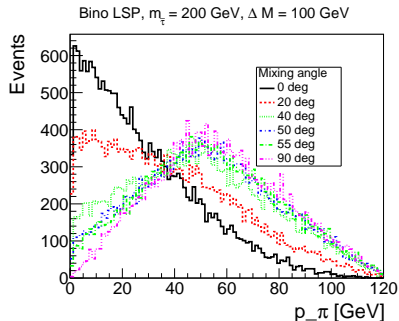
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$\tilde{\tau}$ properties at e^+e^- colliders: Impact of mixing and LSP nature

- At ILC, both beams are polarised, and same luminosity will be collected for LR and RL beams. so:
- Use Likelihood-ratio statistic to weight both polarisations.
- Then, the sensitivity becomes \sim uniform wrt. mixing angles, with a slight minimum at $\sim 55^\circ$

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Neyman-Pearson's lemma applied to a counting experiment

- Use Likelihood weight bo

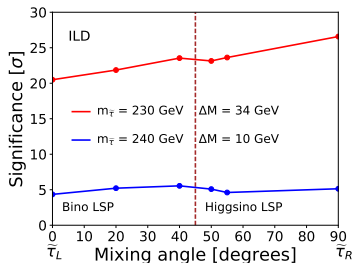
$$N_\sigma = \frac{\sum_{i=1}^{n_{\text{samp}}} s_i \ln(1 + s_i/b_i)}{\sqrt{\sum_{i=1}^{n_{\text{samp}}} n_i [\ln(1 + s_i/b_i)]^2}}$$

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(n_i is either $s_i + b_i$ (exclusion), or b_i (discovery))

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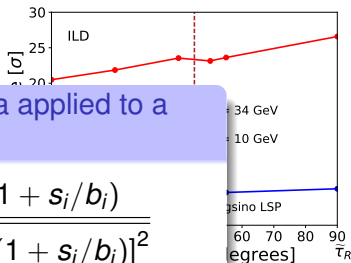
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(n_i is either $s_i + b_i$ (exclusion), or b_i (discovery))

Bookmark this formula !



ILD full simulation analysis: MC samples

- Use the **IDR** 500 GeV FullSim samples
- Covering the full SM background with all $e^+e^-/e^{+/-}\gamma/\gamma\gamma$ processes ($> 10^7$ events)
- Beam-spectrum and pairs background from **GuineaPig**, low P_T hadrons from **Barklow generator**.
- Signal
 - Spectrum obtained with **Spheno**.
 - Generated with **Whizard**
 - Simulated with **SGV**, with pairs and low P_T hadrons **extracted from full-sim**
 - 10000 events per point and polarisation,
 - 1867 mass-points, 37×10^6 events.

ILD full simulation analysis: Event selection

Properties $\tilde{\tau}$ -events “must” have

- Missing energy: $E_{miss} > 2 \times M_{LSP}$ GeV
- Visible mass: $M_{vis} < 2 \times (M_{\tilde{\tau}} - M_{LSP})$ GeV
- Momentum of all jets: $p_{jet} < 70\% E_{beam}$ (or $M_{\tilde{\tau}}/M_{LSP}$ dependent)

Well-known initial stat and hermeticity !

- Two well identified τ 's and little other activity
- Maximum jet momentum:

$$P_{max} = \frac{\sqrt{s}}{4} \left(1 - \left(\frac{M_{LSP}}{M_{\tilde{\tau}}} \right)^2 \right) \left(1 + \sqrt{1 - \frac{4M_{\tilde{\tau}}^2}{s}} \right)$$

Clean final state with no pile-up.

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Well-known is Above 95 % signal efficiency after these

- Two well known cuts (excluding for the τ -identification)
- Maximum jet momentum:

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ILD full simulation analysis: Event selection

Properties $\tilde{\tau}$'s “might” have, but background “rarely” has

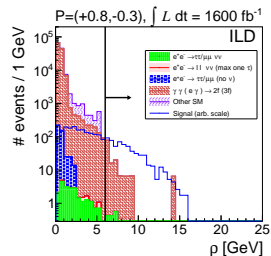
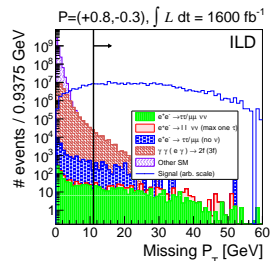
- Missing P_T
- Large acoplanarity
- Large P_T wrt. thrust-axis (ρ)
- High angles to beam

properties of irreducible sources of background

- Charge asymmetry ($q_{jet} \cos \theta_{jet}$)
- Difference between visible mass and Z mass

Properties that background often “does not” have

- Low energy in small angles
- Low energy of isolated neutral clusters



ILD full simulation analysis: Beam-induced backgrounds

e^+e^- beams are accompanied by real and virtual photon interactions between these produce:

- Low p_T hadrons
 - At ILC500 $\langle N \rangle = 1.05/BX$, CLIC380(3000) $\langle N \rangle = 0.17(3.1)/BX$, FCCee $\langle N \rangle = 0/BX$
 - Low p_T hadrons are “physics”: the **total** number collected scale with $\int \mathcal{L}$
- e^+e^- pairs
 - At ILC, 10^5 pairs per bunch crossing, but only ~ 10 will hit any tracking detector.
 - Absent at FCCee

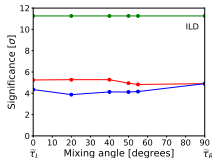
$\gamma\gamma$ interactions are independent of the e^+e^- process, but can happen simultaneously to it (**overlay-on-physics** events) or not (**overlay-only** events)

ILD full simulation analysis: Beam-induced backgrounds

Overlay-on-physics events: Not an issue at FCCee, due to low per-BX luminosity.

Green: No overlay, Red, Blue: with overlay with or w/o mitigation. $M_{\tilde{\tau}}=240$ GeV.

- $\Delta M = 3$ GeV
- $\Delta M = 10$ GeV
- Larger effect for low ΔM , hardly any for $\Delta M > 10$ GeV.



Overlay-only events: Similar for ILC and FCCee.

- Need reduction-factor $\sim 10^{-10}$, which can be achieved.
- Some slight effect at $\Delta M = 2$, completely negligible wrt. other backgrounds at $\Delta M = 10$.

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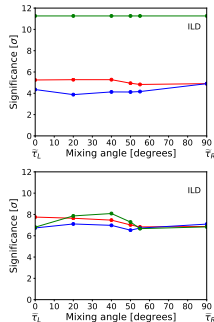
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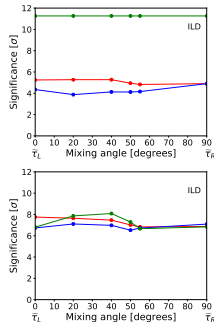


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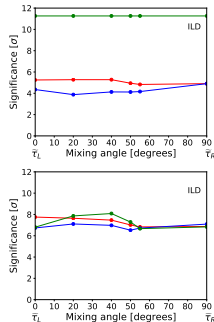
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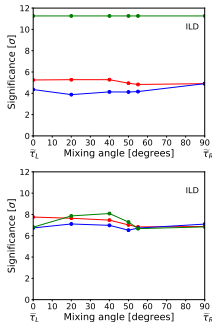
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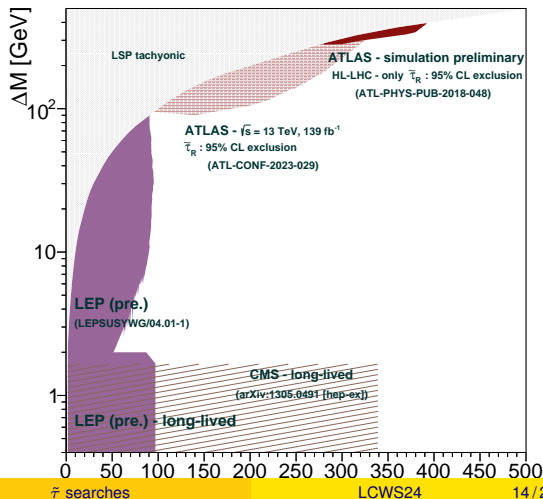
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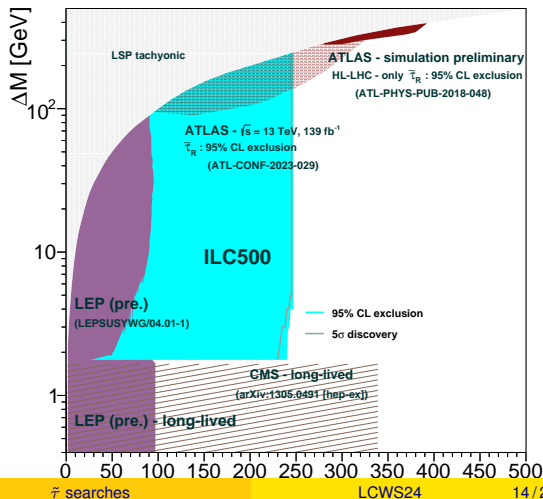
ILD full simulation analysis: Results

- Current model-independent limits for $\Delta M > \tau$ mass come from LEP
- Final result of our study [arXiv:2105.08616](https://arxiv.org/abs/2105.08616)
- At ILC discovery and exclusion are almost the same.
- Extra treat: Extrapolations to 250 GeV and 1 TeV



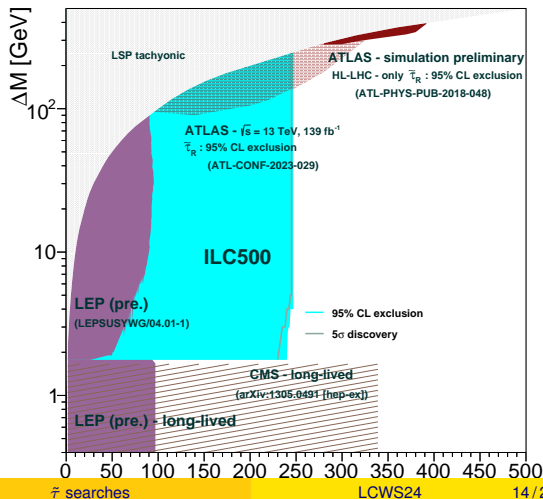
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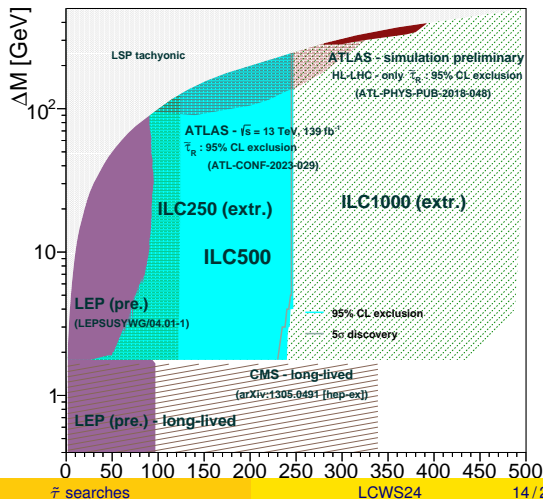
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WIP: Impact of specific ILD/ILC features: Energy, triggerless operation

Energy, the main advantage for any linear option, a **no-brainer**:

- increase in centre-of-mass energy covers much more parameter space, up to **close to kinematic limit**

Triggerless operation:

- Big advantage when searching for unexpected signatures

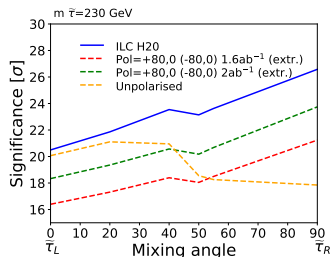
Possible at linear colliders due to low collision frequency, not possible at circular colliders

WIP: Impact of specific ILD/ILC features: Polarisation

Polarisation:

- Combination different polarisation samples allows for **equal sensitivity** to all mixing angles
- Polarisation provides **higher sensitivity**: Likelihood ratio weighting.
- Both beams polarised: **Effective luminosity** for s-channel processes increased, +24 % for ILC wrt. FCCee.

Clear edge for ILC - CLIC/C3 only e^- polarisation, FCCee has no polarisation. CepC studies if polarisation *might* be possible.

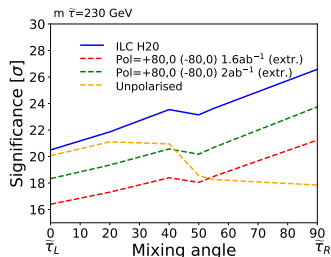


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WIP: Impact of specific ILD/ILC features: Luminosity, Beam-induced backgrounds

Luminosity, the strong points for FCCee and CepC.

- But: higher luminosity gives only very **little improvement**
 - Ex. 2 to 5 (10) ab^{-1} at 250 GeV for $\Delta M = 2$ GeV changes excl. limit on $M_{\tilde{\tau}}$ from 122 to 117 (117) GeV, negligible for $\Delta M = 10$ GeV

Beam-induced backgrounds:

- **Overlay-on-physics:** Due to low per-BX-luminosity this is **not an issue for the circular colliders.**
- **Overlay-only:** to first order, similar for both options (goes with total luminosity)
- The details enter: Smaller beam-spot, triggerless operation, thinner beam-pipe and vertex detector, polarisation, all makes the linear options more powerful

WIP: Impact of specific ILD/ILC features: Hermeticity

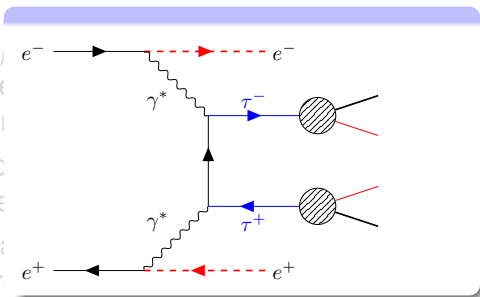
Hermeticity: The issue is can you see the beam-remnant $e^{+/-}$ in $\gamma\gamma$ processes? If not, false missing P_T will be seen ...

- ILD at ILC: hermetic to 6 mrad - Any detector at FCCee; hermetic to 50 mrad.

- **Very bad** for $\gamma\gamma \rightarrow \mu\mu$ for ϵ beam-remnant

- ... but less so if $\mu\mu$ is from the near

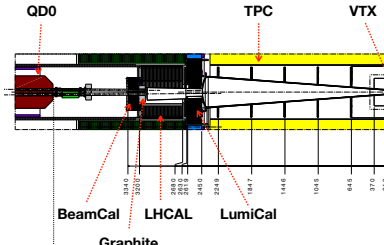
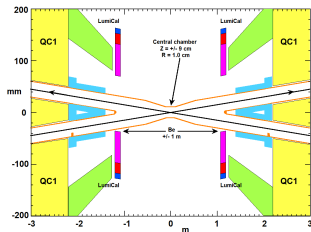
- However, ρ vs ν see the difference are back-to-back, or not.



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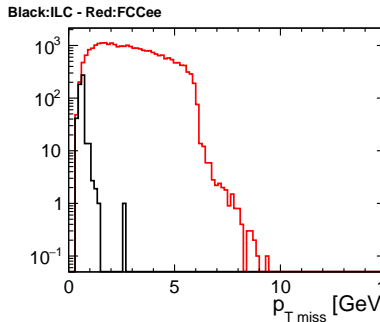
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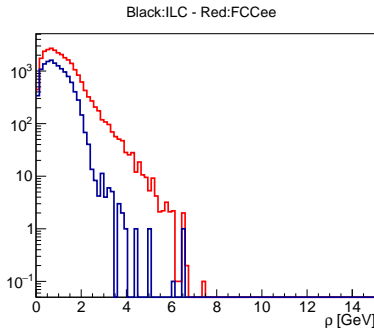
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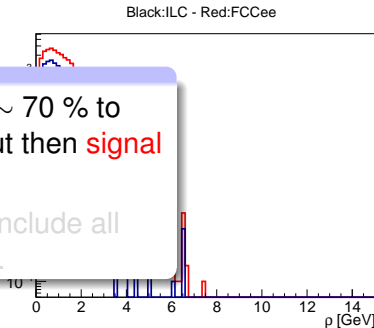
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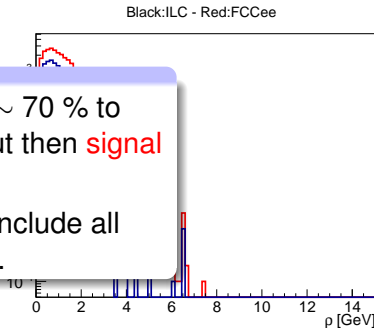
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How to extrapolate $E_{CMS} = 500$ GeV to 240 GeV?

Like this, for expected efficiencies:

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- Away from resonances, the angular distributions do not change with E_{CM} , so that transverse quantities - or projected ones in any direction - scales linearly with E_{CM} .
- Now for the signal:

$$P_{T \max} = P_{\max} = \frac{E_{beam}}{2} \left[1 - \left(\frac{M_{lsp}}{M_{\tilde{\tau}}} \right)^2 \right] \left[1 + \sqrt{1 - \left(\frac{M_{\tilde{\tau}}}{E_{beam}} \right)^2} \right]$$

If one scales both $M_{\tilde{\tau}}$ and M_{lsp} by E_{beam} , both brackets remain unchanged, so that $P_{T \max}$ scales E_{beam} , just like the background.
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Which means, for the ILC-500 to FCCee-250 comparison

- $\sigma \propto 1/s \Rightarrow$ both S and B are 4 times higher at 250 GeV
- If S/B is the same, S/\sqrt{B} is twice better at 250, **if** the efficiency is the same.
- **But** we lose 4/5 of the signal (but background is the same) at FCCee, so $S_{FCCee,250} = 0.8 \times S_{ILC,500}$; B remains 4 times higher.
- So S/\sqrt{B} at FCCee at 250 would be 0.4 of that at ILC at 500 (where it would be for 2 times higher SUSY masses).
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Conclusions

- Even after HL-LHC $\tilde{\tau}$ -LSP mass plane will remain almost completely unexplored
- Future electron-positron colliders are ideally suited for $\tilde{\tau}$ searches
- $\tilde{\tau}$ mixing and LSP nature influence production cross-sections and decay kinematics \Rightarrow picked “worst scenario” for actual analysis
- Polarised beams: combination of data-taking with different signs enables equal sensitivity to all mixing angles
- Beam-induced backgrounds at Linear Colliders can be mitigated up to small residual impact of ~ 1 GeV on highest reachable mass for lowest ΔM
- Higher centre-of-mass energies cover much more parameter space, higher luminosity gives only very little improvement, ex. increase of ILC250 luminosity from 2 to 10 ab^{-1} affects the $\tilde{\tau}$ mass limit only by 5 GeV
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BACKUP

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ILD full simulation analysis: Beam-induced backgrounds

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Identify a set of independent cuts: total rejection factor as the product of the factors obtained with either.

- Achieved rejection factor factor: $\sim 8.2 \times 10^{-11}$ for $\Delta M = 2$; 1.8×10^{-10} for $\Delta M = 10$.
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