A New Method For Measuring Higgs Mass

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Abstract. The Higgs mass as one of the fundamental parameters in the Standard Model has been already measured with a precision of 110 MeV with the data collected so far at the LHC. However in some cases of looking for small deviations from the SM, current precision or projection of the Higgs mass measurement at the LHC or HL-LHC may not be enough. One prominent example is for the SM prediction of the Higgs partial decay width $H \to WW^*$ or $H \to ZZ^*$, in which the Higgs mass uncertainty becomes one of the leading sources of parametric theory error. It is expected that at future e+e- colliders the Higgs mass precision can be significantly improved by the "recoil mass method", at least statistically. This research proposes a new method which may complement to the recoil mass method in terms of systematic errors. The new method employs the signal channel of Higgs decaying to a pair of fermions, in particular τ leptons, or 2 quarks $b\bar{b}$ and makes use of transverse momentum conservations alone instead of the 4-momentum conservation in the recoil mass method. The key experimental observables will be the momentum directions of tau leptons or b-jets without any input from energy measurement, and the momentum directions can possibly be measured by reconstructing the decaying vertex of the tau leptons or B-hadrons. This new method can in principle be applied at lepton colliders and the LHC as well. This method is studied by performing realistic detector simulation and physics analysis with the ILC frameworks based on the ILD. In the case of $H \to b\bar{b}$ without any background the statistical precision is found to be comparable to the expectation by recoil mass method, thus potentially very useful. The $H \to \tau^-\tau^+$ channel is rather statistically limited and worth of further investigations.

1 Introduction

This work is performed in the context of the ILC (International Linear Collider) project, a future 20 km long linear e^+e^- collider that would be built in Japan [1]. Upon its construction, it is expected to operate at a center-of-mass energy of $\sqrt{s} = 250$ GeV in the initial stage (ILC250), with upgrades planned up to around 500 GeV and 1 TeV. An integrated luminosity of 2000 fb^{-1} is assumed for this study focusing on ILC250. For simplicity the beam polarizations are assumed to be $P(e^-, e^+) = (-0.8, +0.3)$.

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1.1 Physics Motivations

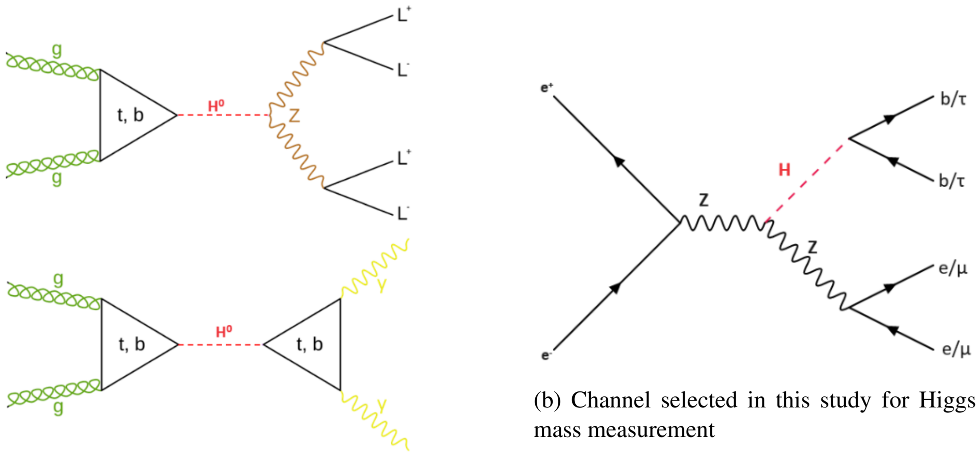
The Higgs boson is a particle predicted by the Standard Model (SM) [2], which was discovered at the LHC in 2012. The mass of this particle, about 125 GeV, is a crucial parameter in the study of physics beyond the SM. The couplings of reactions $H \to ZZ^*$ and $H \to WW^*$ are used to study deviations between different models and the Higgs mass serves as one dominant sources of parametric theory uncertainty in the prediction of corresponding partial widths [3]:

$$\frac{\Delta\Gamma(H \to ZZ^*)}{\Gamma(H \to ZZ^*)} = 16 \cdot \frac{\Delta m_H}{m_H}$$
$$\frac{\Delta\Gamma(H \to WW^*)}{\Gamma(H \to WW^*)} = 14 \cdot \frac{\Delta m_H}{m_H},$$

Where Γ corresponds to the partial widths, m_H is the Higgs mass, and Δ stands for corresponding uncertainty. Thus, in order to achieve a relative precision of $0.1 - 0.3\%$ for the Higgs couplings to Z or W, a precision of $15 - 50$ MeV is needed fo m_H. The current precision on m_H is 0.11 GeV [4] by combining 4 leptons and 2 photons channels where the Higgs mass is reconstructed directly using the measured 4 vector of final state particles (Fig. 1a).

1.2 Energy Losses and Motivation Of A New Method

The standard method for measuring m_H at future e^+e^- collides is based on the well-known recoil mass technique, which can offer a statistical precision of about 15 MeV at the ILC250 [5]. A key element in that method is the employment of 4-momentum conservation which relies on the full momenta of initial states. However various phenomena can cause the particles to lose energy and increase the uncertainty about the initial state when the particles collide. One of the main effects is Beamsstrahlung [2], where particles emit photons due to the beam-beam



(a) Decay Channels selected in LHC for Higgs mass measurement

(b) Channel selected in this study for Higgs mass measurement

Figure 1: Feynman diagrams for selected processes that are used for measuing m_H at the LHC and ILC.

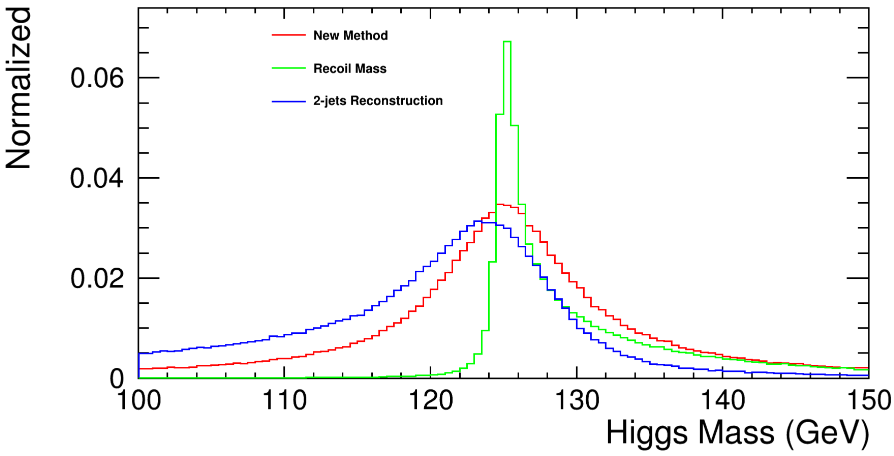


Figure 2: Comparison between methods for measuring Higgs Mass at the ILC 250 GeV. The Recoil Mass method is the standard method using 4-momentum conservation, 2-jets reconstruction is a direct method using 4-momenta of Higgs decay products as what used at the LHC, and the New Method is the one explained in this study.

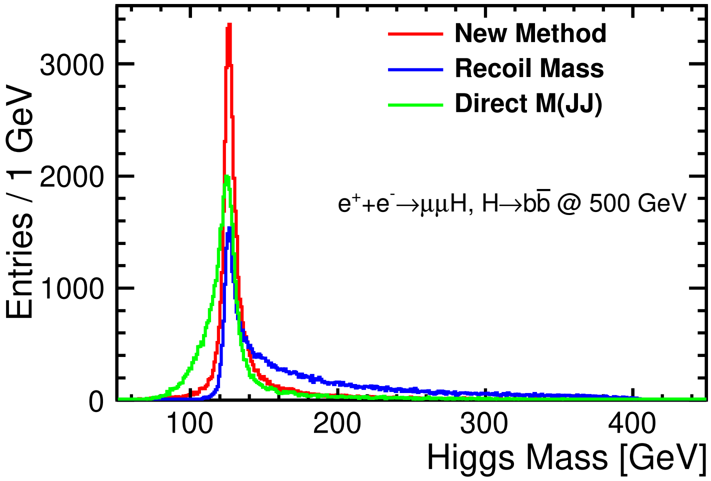


Figure 3: Reconstructed Higgs Mass for the three methods introduced above at the ILC $\sqrt{s}$ = 500 GeV [6].

interactions. There are other types of radiations, ISR (Initial State Radiation) and FSR (Final State Radiation), which are emitted by initial and final state particles in the hard interaction. When photons are radiated in the beam direction which can not be directly detected, some information in the reaction is lost which is a source of systematic errors in the measurements.

2 A New Method For Measuring Higgs Mass

2.1 Method

The signal studied here is $e^+e^- \to ZH$, $Z \to l^+l^-$ and $H \to b\bar{b}$, then a comparison with the mode $H \to \tau^+\tau^-$ will be made (fig.1b). Let θ, ϕ, and p respectively denote the polar angles, azimuthal angles and magnitude of momenta of the particles, assigning indices $i = 1, 2$ to the quarks b and $\bar{b}$ with p_x, p_y being the projection of the momentum of H on x and y:

$$\begin{aligned} p_1 \sin\theta_1 \cos\phi_1 + p_2 \sin\theta_2 \cos\phi_2 &= p_x, \\ p_1 \sin\theta_1 \sin\phi_1 + p_2 \sin\theta_2 \sin\phi_2 &= p_y. \end{aligned} \quad (1)$$

Experimentally, θ_i and ϕ_i are directly measured from the jets direction and p_x, p_y are obtained with the following method by imposing only transverse-momentum conservation:

$$\begin{aligned} p_x &= -p_x^{\ell\ell} + \sqrt{s} \sin\frac{\alpha}{2}, \\ p_y &= -p_y^{\ell\ell}, \end{aligned} \quad (2)$$

where $\alpha = 0.014$ radian is the crossing angle between the 2 beams,$p_x^{\ell\ell}$ and $p_y^{\ell\ell}$ are the momenta of the Z boson reconstructed from the 2 leptons. With equations (2) and (1):

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \frac{p_t}{\sin\phi_{12}} \begin{pmatrix} \frac{\sin(\phi_2 - \phi)}{\sin\phi_2} \\ \frac{\sin(\phi_1 - \phi)}{\sin\theta_1 \sin\theta_2} \end{pmatrix}, \quad (3)$$

where $\phi_{12} = \phi_1 - \phi_2$, $p_t = \sqrt{p_x^2 + p_y^2}$ and ϕ without an index being the azimuthal angle of the transverse momentum (p_x, p_y). Thus the magnitudes of two jet momenta can be determined based on the measured jet angles and transverse momentum conservation. Once p_1 and p_2 are determined, the energies of the jets coming from the b and $\bar{b}$ quarks are obtained by $E_i = \sqrt{p_i^2 + m_i^2}, (i = 1, 2)$, and m_1, m_2 are the masses of the jets measured from their four-momenta. In the case where $m_i/p_i \ll 1$, with the mass of a b quark being $m_b = 4.183 \pm 0.007$ GeV [7], the mass of the Higgs boson is approximated as:

$$\begin{aligned} m_H^2 \approx{}& (1 + \frac{p_2}{p_1}) m_1^2 + (1 - \frac{p_1}{p_2}) m_2^2 \\ &+ 2p_t^2 \frac{\sin(\phi - \phi_2)\sin(\phi_1 - \phi)}{\sin\theta_1 \sin\theta_2 \sin^2\phi} (1 - \sin\theta_1 \sin\theta_2 \cos\phi - \cos\theta_1 \cos\theta_2). \end{aligned} \quad (4)$$

2.2 Advantages of this method

By using only the conservation of transverse-momentum without utilizing the longitudinal component, this method is not affected by uncertainties in beam calibration, energy precision, Beamstrahlung, or ISR, which in particular become more significant at higher energies $\sqrt{s}$. In addition rather than using the four-momentum of the jets to measure the invariant mass and thus obtain the Higgs boson mass, the major observables of this method are the jet directions, so jet energy resolution impacts the precision less given that jet energy only appears in the jet mass measurement. A preview of how the new method performs in this study for $\sqrt{s} = 250$ GeV can be found in 2, as well as 3 from earlier study for $\sqrt{s} = 500$ GeV [6].

Moreover, the nature of the boson originating the two jets is independent in this method. Hence, the mass of the Z boson can also be well reconstructed for background processes such as $e^+e^- \to ZZ$. This can potentially help improve significantly the di-jet mass separation between signal and background.

3 Simulation Framework

The study of this method and its application within the framework of experiments at the ILC was conducted using simulations and tools from ILCSoft [8]. Event generation was done with WHIZARD 2.8.5 [9] for both signal and background. Detector simulation was performed using GEANT4 based on the ILD (International Large Detector) model [10], which is one of the two proposed detector concepts for the ILC. Event reconstruction was performed using Marlin [11]. The PandoraPFA algorithm [12] was used for particle flow analysis to reconstruct final state particles.

The signal and background event samples used in this study have been centrally produced by ILD MC Production Group (technically tagged as mc-2020 samples). This study was conducted assuming the center-of-mass energy of $\sqrt{s} = 250$ GeV with an integrated luminosity of $L = 2000\ \mathrm{fb}^{-1}$. Two sets of beam polarizations are investigated: $P(e^-, e^+) = (-0.8, 0.3)$ or $P(e^-, e^+) = (0.8, -0.3)$, labeled LR and RL respectively, where $P = \frac{N_R - N_L}{N_R + N_L}$ with N_L and N_R representing the number of left- or right-polarized particles.

4 Event Selection and Analysis

To isolate the signal corresponding to collisions leading to the process $e^+e^- \to ZH$, $Z \to \mu^+\mu^-$ and the two decay modes of the Higgs boson ($H \to b\bar{b}$ and $H \to \tau^+\tau^-$), a pre-selection is first applied based on the basic signal characteristics of number of leptons and jets, followed by cuts to improve the signal selection.

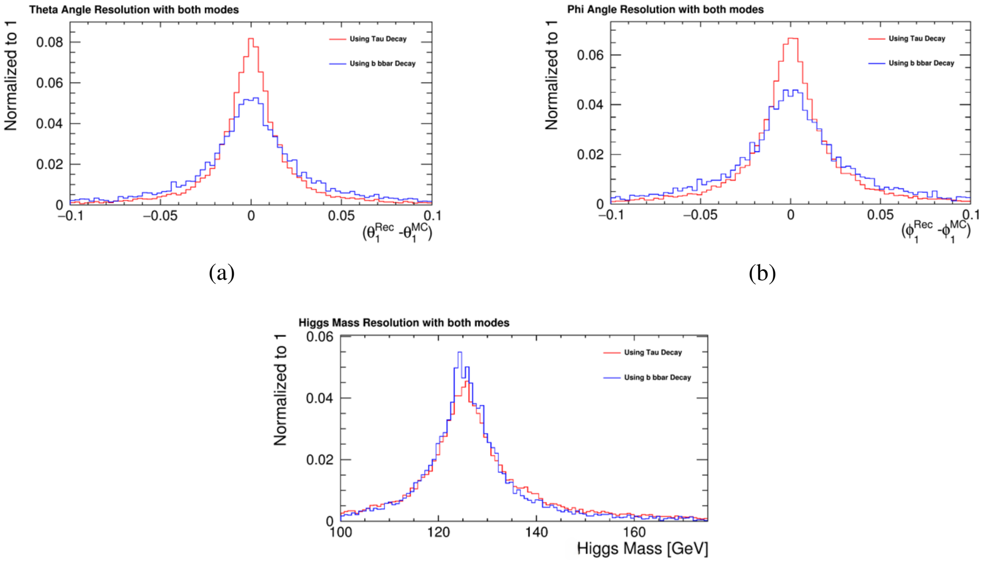
4.1 Pre-selection

In the pre-selection, an algorithm called "IsolatedLeptonTagging" [6] is employed to identify at least 2 isolated leptons with opposite charges. The lepton pair must have an invariant mass m_{ll} close to the mass of the Z boson $m_Z = 91.18$ GeV from which they should originate. If multiple pairs are detected, the one with the mass closest to m_Z is retained. A loose criterion is applied: $|m_{ll} - m_Z| < 40$ GeV. For simplification in this study, events of the type $\gamma\gamma \to$hadrons with low p_T are removed using MC truth information. This should be done by an algorithm in a more realistic analysis, nevertheless at ILC250 the expected impact from the overlay of those $\gamma\gamma \to$hadrons is small. After the identification of the lepton pair, the remaining PFOs are then passed to LCFIPlus [13] for the reconstruction and flavor-tagging of the 2 jets. Durham algorithm [14] is used for the jet-clustering. Note up to here the two signal channels are reconstructed in a completely identical procedure.

With the 2 leptons and 2 jets reconstructed, investigated following are the performance of the corresponding jet angles, which are needed in this new method to reconstruct the Higgs mass as shown in Eq. 2-4.

4.2 Performance of Detector Resolutions for Jet Angles

By comparing to their MC truth information, the resolution for the angles θ and ϕ in each channel of $H \to b\bar{b}$ and $H \to \tau^+\tau^-$ is shown in Fig. 4a and Fig. 4b, for the beam polarization $P = (-0.8, 0.3)$ at ILC250. The reconstructed Higgs boson mass is also shown in Fig. 4c. Simple Gaussian fits were performed using ROOT6 [15] for those figures to obtain a rough estimation of the resolution for the jet angles as well as the Higgs mass ($\sigma(m_H)$) by the new method, summarized in Tab. 1.



(a)

(b)

(c)

Figure 4: Performance of the measured jet angles for one of the 2 jets (the one with larger momentum) in comparison with their MC truth information: θ in (a), ϕ in (b); reconstructed Higgs mass based on the new method in (c), for both decay modes at ILC 250 GeV and $P(e^-, e^+)$=(-0.8,0.3).

Resolution	θ_1 [degree]	ϕ_1 [degree]	$\Sigma(m_H)$ [GeV]
$H \to b\bar{b}$	0.80 ± 0.03	0.92 ± 0.02	4.17 ± 0.16
$H \to \tau^+\tau^-$	0.67 ± 0.01	0.73 ± 0.01	4.51 ± 0.14

Table 1: Resolution of the jet angles as well as the width of the reconstructed Higgs mass in two signal channels. The uncertainties are attributed to the MC statistics.

A naive estimation of the final uncertainty on m_H can be given as $\Delta m_H = \frac{\sigma(m_H)}{\sqrt{N}}$, where $\sigma(m_H)$ is the resolution in Tab. 1 and N is the number of expected events for the corresponding signal channel. As an example, assuming 100% signal efficiency and 2000 fb^{-1} with $P(e^-, e^+) = (-0.8, +0.3)$, we would obtain $\Delta m_H \sim 20$ MeV for $H \to b\bar{b}$ and $\Delta m_H \sim 100$ MeV for $H \to \tau^+\tau^-$. The effect of different beam polarizations was checked and no difference was found in terms of the resolution for the reconstructed Higgs mass, as shown in Fig. 5.

4.3 Final Selection

For each retained event, cuts are applied and optimized to maximize the signal significance defined as $S/\sqrt{S+B}$ where S and B are for numbers of signal and background events. For the $H \to b\bar{b}$ channel:

- Cut 0: The lepton type in the pair candidate must be muons with a stricter mass cut: $|m_{ll} - m_Z| < 10 GeV$.

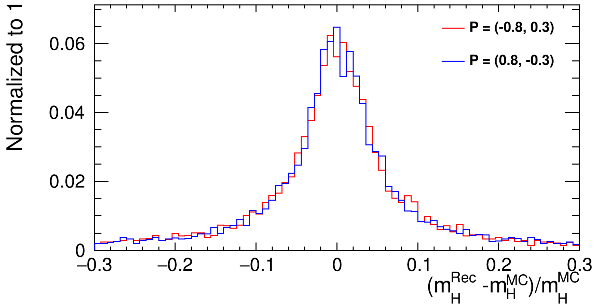


Figure 5: m_H Resolution for both polarizations studied at $\sqrt{s} = 250$ GeV with the new method for $H \to b\bar{b}$ channel.

- Cut 1: Number of charged PFOs in each jet must be superior to 3
- Cut 2: Total visible energy which is the sum of lepton pair energy and jet energies, must be higher than 200 GeV.
- Cut 3: For each b-likeness > 0.66, where b-likeness is the BDT output from flavor-tagging algorithm LCFI+.
- Cut 4: The angle of the lepton pair (forming the Z boson) must verify $|cos(\theta_{ll})| < 0.9$.
- Cut 5: The reconstructed Higgs mass must be close to nominal Higgs mass: $m_H^{new} \subset (110, 150)\ GeV$.

The cuts are similar for the $H \to \tau^+\tau^-$ channel except the cuts related to jet multiplicity and jet flavor tagging:

- Cut 0: Same.
- Cut 1: The number of charged PFOs in each jet must be less than or equal to 3 because the final states are: $\tau \to 1$ charged particle or $\tau \to 3$ charged particles.
- Cut 2: Same.
- Cut 3: b-likeness < 0.66.
- Cut 4: Same.
- Cut 5: The number of charged PFOs in each jet must be non-zero.
- Cut 6: The Recoil Mass against the μ-pair must be between 110 and 150 GeV.
- Cut 7: The reconstructed Higgs mass must be between 110 and 150 GeV.

The new cuts for $H \to \tau\tau$ filter the leptonic 2-fermion background processes that are still present after Cut 1 has been changed. Tables 2 and 3 summarize the results for each cut applied for the $b\bar{b}$ and $\tau\tau$ channels, respectively. The final efficiency and signal significance are 60% (45%) and 71σ (11σ) for $H \to b\bar{b}$ ($H \to \tau^+\tau^+$) channel. The background is dominated by 4-fermion semi-leptonic processes mainly from $e^+e^- \to ZZ$. The contributions from other Higgs decay modes are negligible in both channels. The distributions of the reconstructed Higgs mass for signal and background processes after all cuts are shown in Fig. 6. The preliminary results for the Higgs mass from a simple Gaussian fit are:

$$m_H^{b\bar{b}} = 125.28, \quad \Delta m_H^{b\bar{b}} = 20\,\text{MeV}$$
$$m_H^{\tau\tau} = 125.31, \quad \Delta m_H^{\tau\tau} = 71\,\text{MeV}$$

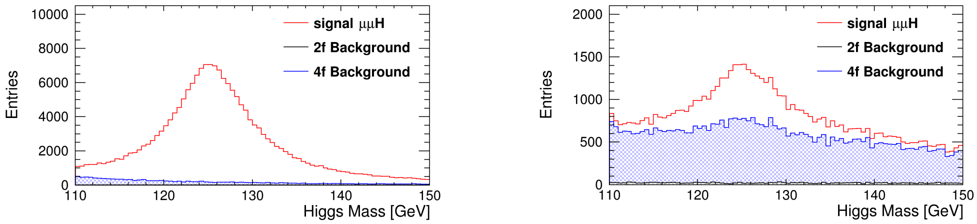


Figure 6: The reconstructed Higgs mass using the new method based on full simulations including full 2-fermion and 4-fermion backgrounds after all cuts for $H \to b\bar{b}$ (left) and $H \to \tau^+\tau^-$ (right) channels.

Process	2f_l	2f_h	4f_l	4f_sl	4f_h	BG	llh	Signal	σ
Events	2.6×10^7	4.6×10^8	3.2×10^7	3.8×10^7	3.4×10^7	5.9×10^8	2.1×10^4	1.2×10^4	
Cut0	1.5×10^6	1.2×10^4	3.3×10^6	8.2×10^5	2.7×10^5	5.6×10^6	1.9×10^4	1.1×10^4	8.23
Cut1	1.0×10^6	21	8.2×10^4	1.6×10^5	0	1.3×10^6	1.7×10^4	1.0×10^4	15.4
Cut2	2.40	1.5×10^4	3.2×10^3	1.2×10^5	0	1.2×10^5	1.4×10^4	9.5×10^3	39.0
Cut3	2.40	1.5×10^4	3.2×10^3	1.2×10^5	0	1.2×10^5	1.4×10^4	9.5×10^3	39.0
Cut4	0	733	219	2.9×10^5	0	2.9×10^5	9.2×10^3	9.0×10^3	47.4
Cut5	0	733	0	2.3×10^4	0	2.3×10^3	8.4×10^3	8.2×10^3	48.0
Cut6	0	0	0	2514	0	2514	7281	7085	74.3

Table 2: Cuts Table for $H \to b\bar{b}$ signal channel: number of signal and background events as well as signal significance after each cut assuming an integrated luminosity of 2000 fb^{-1} and $P(e^-, e^+) = (-0.8, +0.3)$. *llh* standards for all Higgs decay events. 2f or 4f standards for processes with 2 or 4 fermions. "_l" standards for leptonic, "_h" standards for hadronic, and "_sl" standards semi-leptonic.

4.4 Possible Improvements with Decay Vertex Reconstruction

The study using the new method can still be improved, especially for the $H \to \tau^+\tau^-$ channel. By reconstructing the possible τ or B-hadron decay vertex in the case of a decay into 3 charged particles for example, the precision could be improved compared to the case studied here, since the direction and thus θ and ϕ are obtained directly from the decay vertex. Figure 7 shows the results when using this improvement compared to other methods like Recoil Mass Method or the direct di-jet mass reconstruction, where for τ channel this was done only in the case where both leptons decayed to 3 charged particles as an illustration.

The new method when using vertices is more precise for $H \to \tau^+\tau^-$ with a sharp peak with a width of 2 GeV. This improves the previous results by a factor of 2. However, because a decay vertex must be reconstructed, the signal efficiency drops significantly since $\text{BR}(\tau \to 3 - prong)$= 15%. The final results may get limited available statistics. The same method using decay vertex for τ channel may become very useful for the m_H measurement at the HL-LHC where much larger statistics of $H \to \tau^+\tau^-$ events is expected. For $H \to b\bar{b}$ channel the resolution turns out to become slightly worse, indicating that for b-jet the angle information based on full jet constitutes is more precise than just using decay vertex of B-hadrons.

Process	2f_l	2f_h	4f_l	4f_sl	4f_h	BG	llh	Signal	σ
Events	2.6×10^{7}	4.6×10^{8}	3.2×10^{7}	3.8×10^{7}	3.4×10^{7}	5.9×10^{8}	2.1×10^{4}	1.3×10^{3}	
Cut0	1.5×10^{6}	1.6×10^{4}	3.3×10^{6}	8.2×10^{5}	271	5.6×10^{6}	1.9×10^{4}	1.2×10^{3}	0.518
Cut1	1.0×10^{6}	31	8.2×10^{4}	1.6×10^{5}	0	1.3×10^{6}	1.7×10^{4}	1.1×10^{3}	0.97
Cut2	1.0×10^{6}	1	8.1×10^{4}	2.1×10^{3}	0	1.1×10^{6}	1.6×10^{3}	1.1×10^{3}	1.00
Cut3	1.0×10^{6}	1	8.1×10^{4}	2.1×10^{3}	0	1.1×10^{6}	1.6×10^{3}	1.1×10^{3}	1.00
Cut4	1.0×10^{6}	0	4.4×10^{4}	1.1×10^{3}	0	1.1×10^{6}	1.5×10^{3}	966	1.83
Cut5	2.4×10^{5}	0	2.7×10^{4}	1.1×10^{3}	0	2.6×10^{5}	1.4×10^{3}	929	4.52
Cut6	1.3×10^{4}	0	4.1×10^{3}	136	0	1.7×10^{4}	1.1×10^{3}	741	10.0
Cut7	460	0	2294	30	0	2780	648	606	11.1

Table 3: Cuts Table for $H \to \tau^+\tau^-$ signal channel: number of signal and background events as well as signal significance after each cut assuming an integrated luminosity of 2000 fb^{-1} and $P(e^-, e^+) = (-0.8, +0.3)$. *llh* standards for all Higgs decay events. 2f or 4f standards for processes with 2 or 4 fermions. "_l" standards for leptonic, "_h" standards for hadronic, and "_sl" standards semi-leptonic.

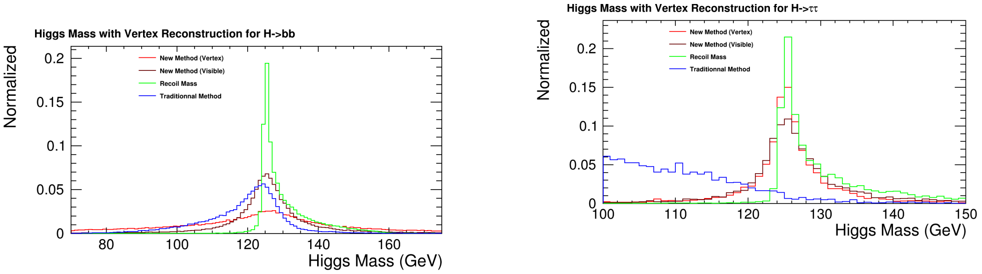


Figure 7: Higgs Mass measured with the new method using Decay Vertex (red), compared to the measurement using the Recoil Mass method (green) or the direct di-jet mass method (blue), for $H \to b\bar{b}$ (left) and $H \to \tau^+\tau-$ (right) channel.

5 Conclusion

A new method is developed for the precise measurement of the Higgs mass at future lepton colliders. This method takes advantage of only transverse-momentum conservation, insensitive to any uncertainty in the longitudinal components of the initial states, thus complementary to the classic recoil mass method. The key observables are jet angles instead of jet energies. The new method is studied with the full detector simulation analysis based the ILD. A sensitivity of about 20 MeV (100 MeV) can be achieved by using $e^+e^- \to ZH, Z \to \mu^+\mu^-$, $H \to b\bar{b}$ ($H \to \tau^+\tau^-$) channel at the ILC 250 GeV with an integrated luminosity of 2000 fb^{-1}. The results are still very preliminary and can be improved by improved algorithms for jet angle measurements (for instance using τ decay vertex), and by employing more signal channels (for instance $Z \to e^+e^-$ or $Z \to q\bar{q}$).

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