Recoil Mass — Version 2

— from a single cross section to a line-shape

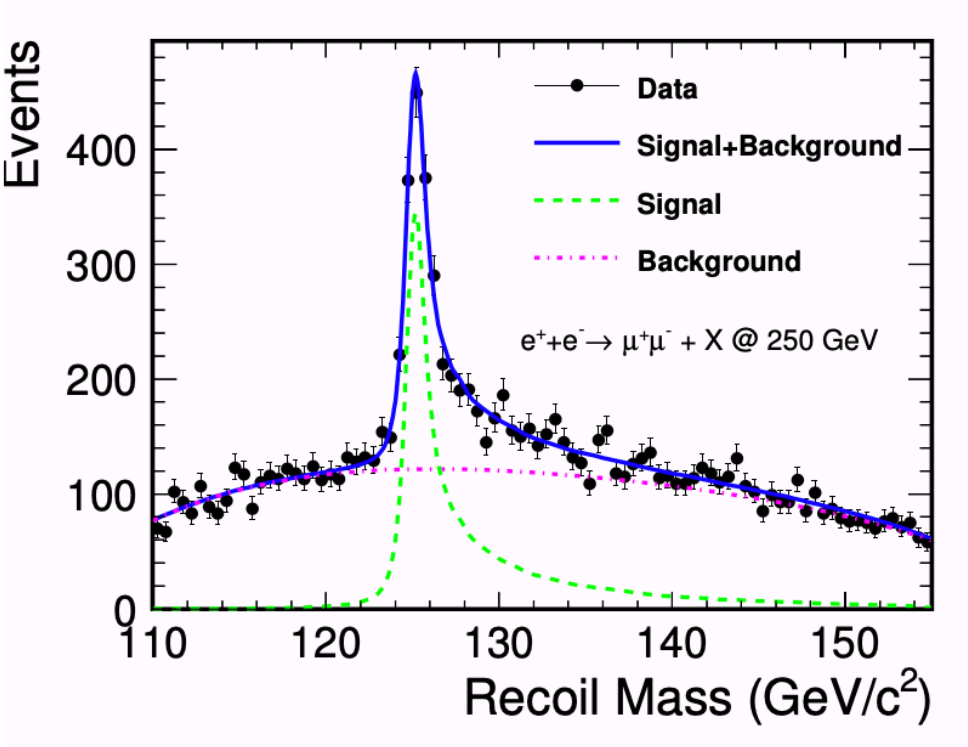
work in progress

Junping Tian (U. Tokyo)

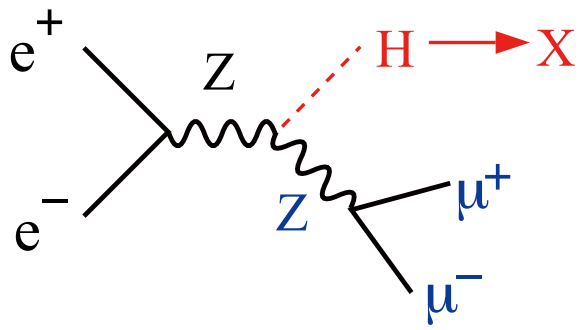
LCWS 2026, November 11, 2026 @ KEK, Tsukuba

Recoil Mass: flagship measurement at e+e- Higgs Factory

[for Z->ll, Yan et al, arXiv:1604.07524; for Z->qq, Thomson, arXiv:1509.02853]



$\Delta m_H = 14\mathrm{MeV}$



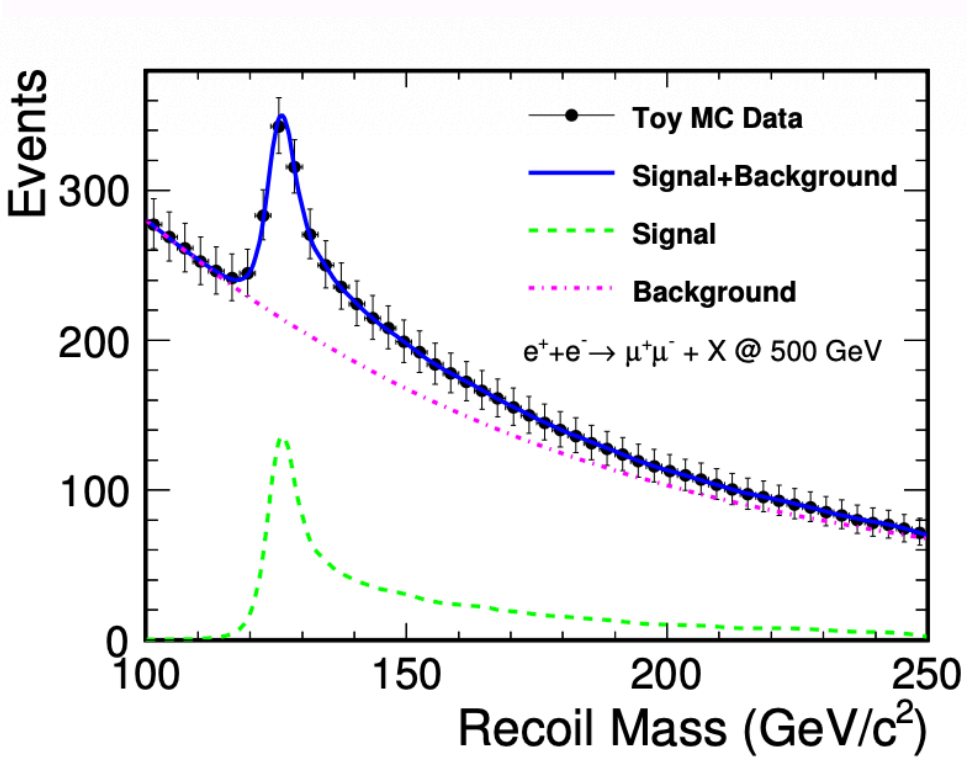
$$M^2_{rec} = \left(\sqrt{s} - E_{\ell\ell}\right)^2 - \left|\vec{p}_{\ell\ell}\right|^2$$

- well defined initial states at e+e-
- recoil mass technique —> tag Z only
- Higgs is tagged without looking into H decay
- **absolute cross section** of e^+e^- —> ZH

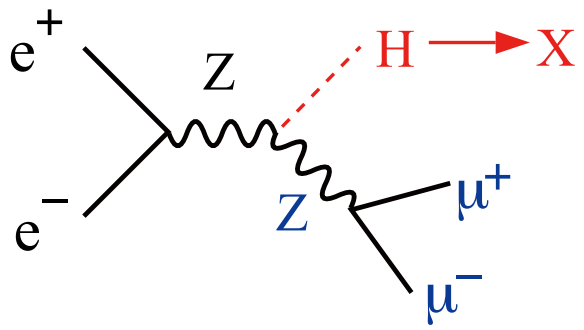
long tail: energy lost to beamstrahlung and Initial State Radiation (ISR)

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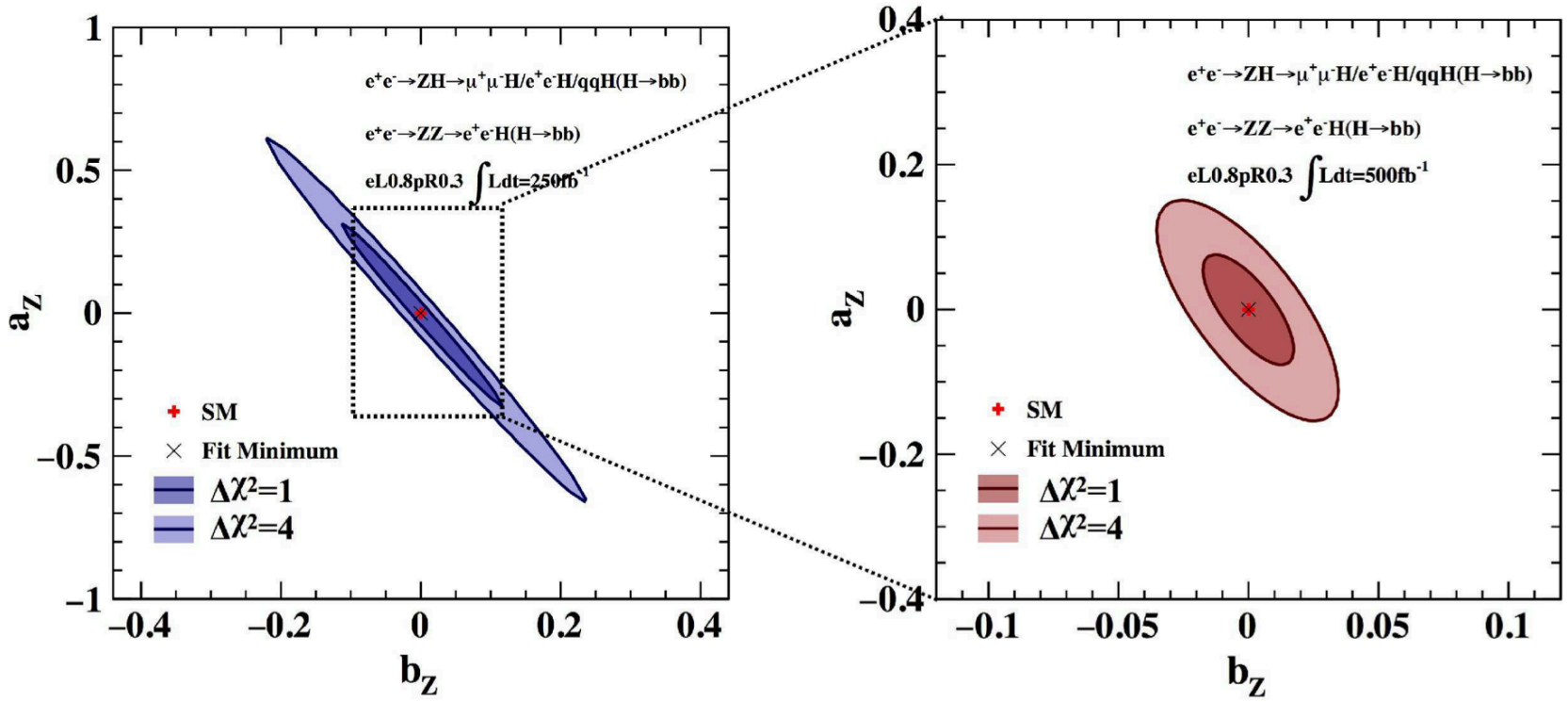
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Anomalous HZZ Coupling

[Ogawa et al, arXiv:1712.09772]

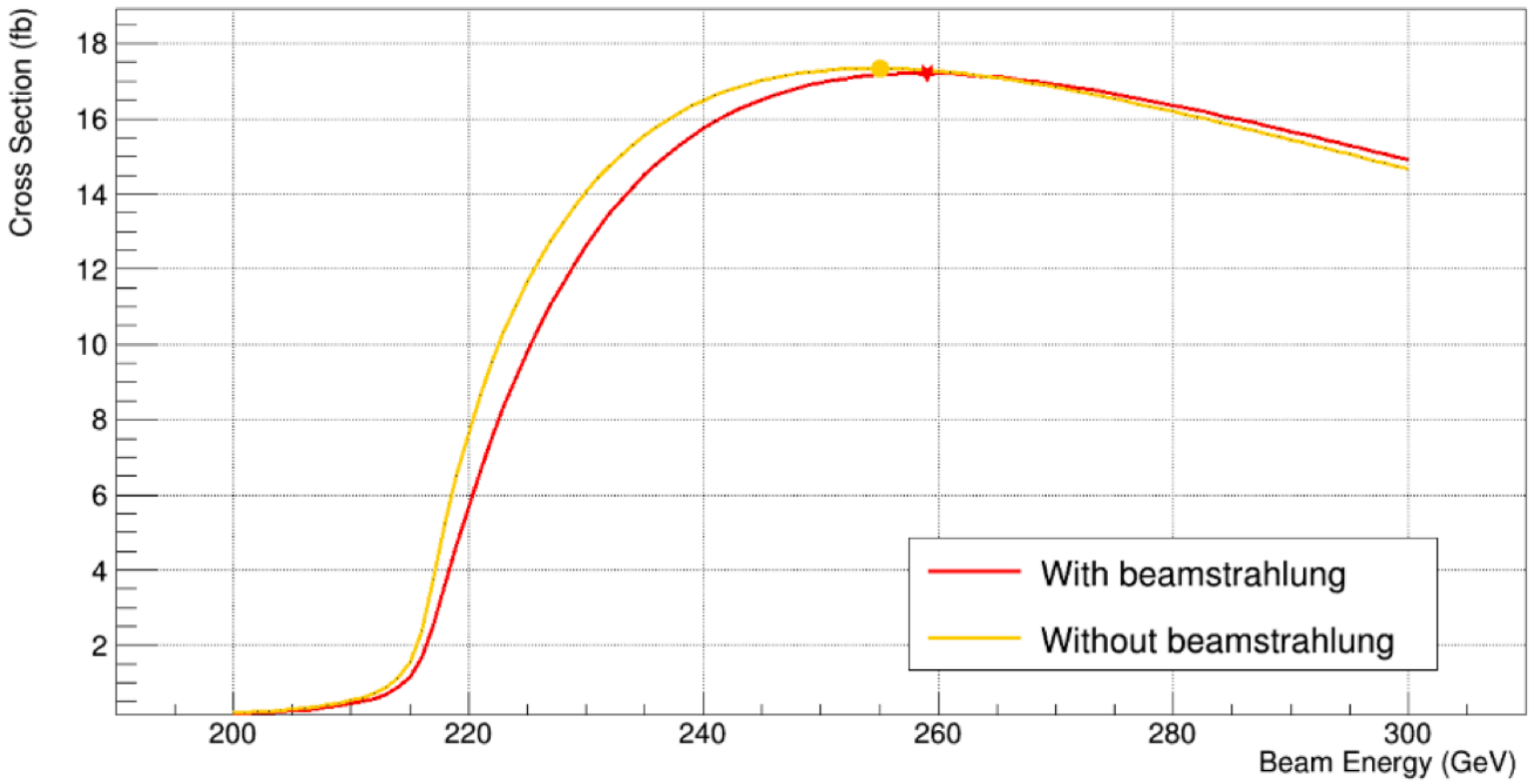
$$L_{hZZ} = M_Z^2(\frac{1}{v} + \frac{a}{\Lambda})hZ_\mu Z^\mu + \frac{b}{2\Lambda}hZ_{\mu\nu}Z^{\mu\nu} + \frac{\tilde{b}}{2\Lambda}hZ_{\mu\nu}\tilde{Z}_{\mu\nu}$$



correlation ~ 0.9998: degeneracy, lifted by angular meas.; Q^2 dependent

Optimal Collision Energy

[Maria et al, EPJ Web Conf. 315 (2024) 01002]

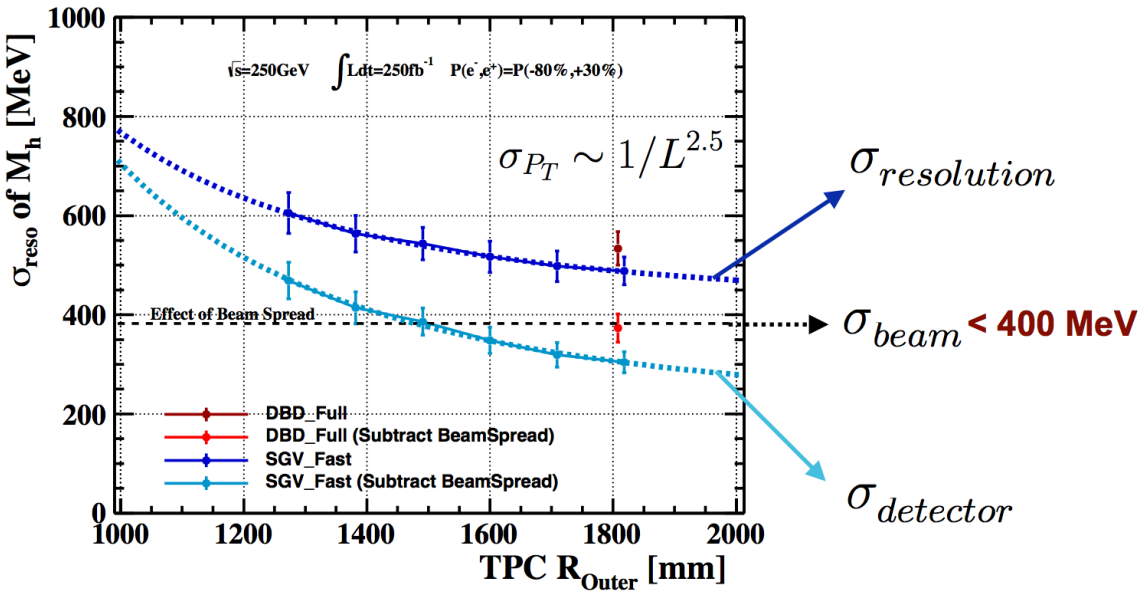


proposed to scan $\sqrt{s}$ in order to lift the degeneracy

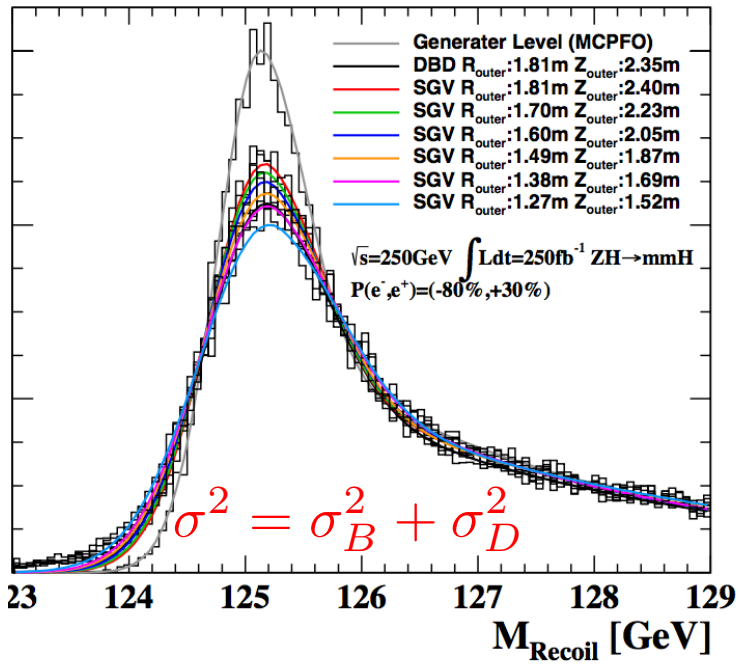
Higgs Mass vs Detector / Machine Resolution

$$M_{rec}^2 = \left(\sqrt{s} - E_{\ell\ell}\right)^2 - |\vec{p}_{\ell\ell}|^2$$

σ_B σ_D



[T. Ogawa @ 7th ILD Opt. Meeting]



Higgs Mass is now better known than any of other observables in the e+e- -> ZH

new idea: use the Higgs mass as a constraint

m_H is known to <0.1 GeV (LHC), better than the recoil-mass resolution

the 4-unknown in the Higgs recoil system can be reduced to 3

this allows to resolve 1-unknown from energy loss (beamstrahlung / ISR)

$$m_H^2 = (p_{e^+e^-} - p_\gamma - p_{\ell\ell})^2$$

assume one “effective” massless photon along the beam axis, energy unknown

become a linear equation in the photon energy:

$$E_\gamma^{\pm} = \frac{M_{rec}^2 - m_H^2}{2\left(\sqrt{s} - E_{\ell\ell} \pm p_{z,\ell\ell}\right)}$$

$$s' = s - 2\sqrt{s}\,E_\gamma$$

± : photon along the electron or the positron beam

still only the two leptons → model independence is kept

the real hard interaction √s’ is determined event-by-event

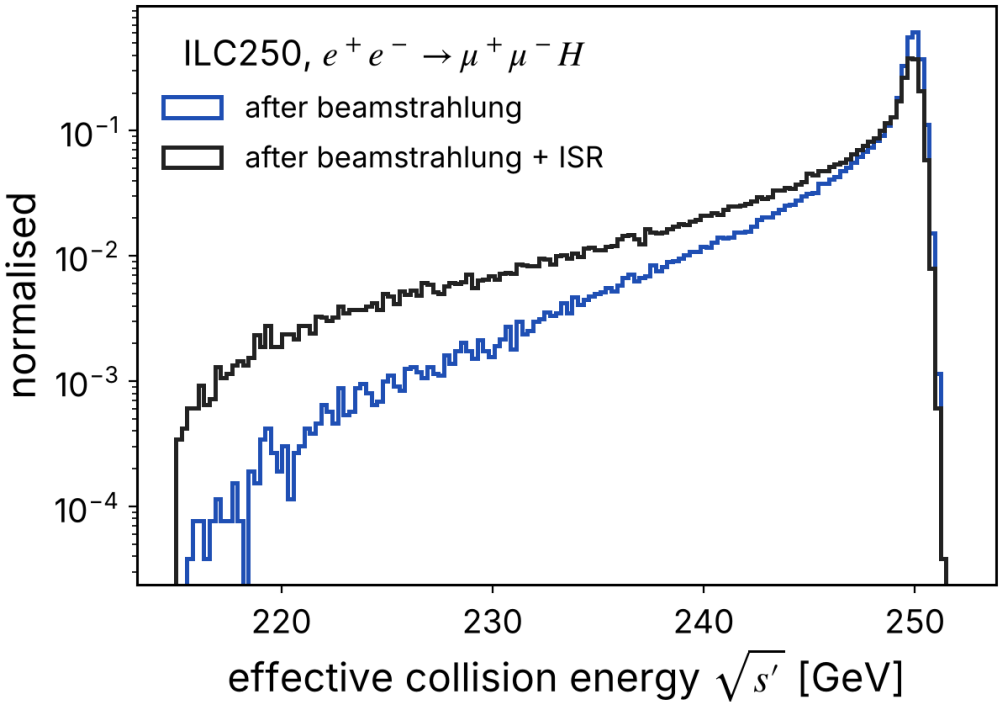
simulation setup

generator	WHIZARD 3.1.5, e+e- → μ+μ-H, ISR + beam spectrum
beam spectra	ILC250 (SetA), ILC280 (test), ILC500 (TDR), FCC-ee 240
detector	DELPHES 3.5: ILD (ILCgen card), IDEA for FCC-ee
muons	tracks + photons within $\Delta R < 0.2$ (FSR recovery)
statistics	100k events per energy; signal only (no background yet)

truth $\sqrt{s'}$: invariant mass of the e+e- pair after beamstrahlung and ISR

analysis is over-simplified, mainly for proof of principle

ILC initial state



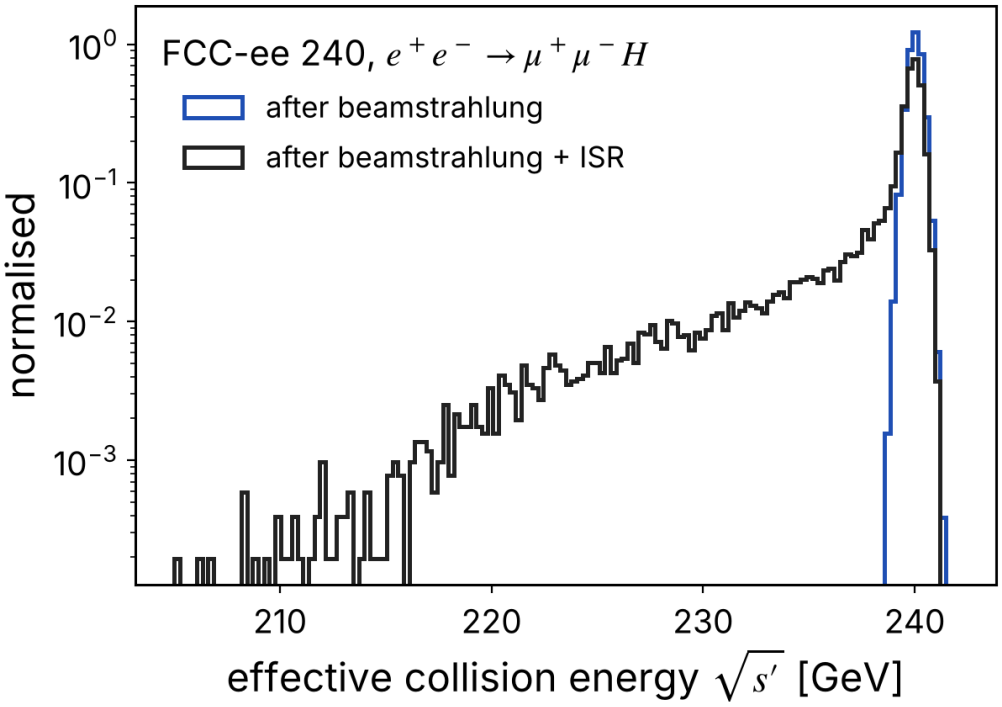
ILC250

32% of events lose > 5 GeV

larger at higher energy:
8.7 GeV at 280, 48 GeV at 500

recoil mass assumes nominal $\sqrt{s}$ → information in the tail is diluted

FCC-ee 240: beamstrahlung much smaller, but the ISR tail remains



FCC-ee 240

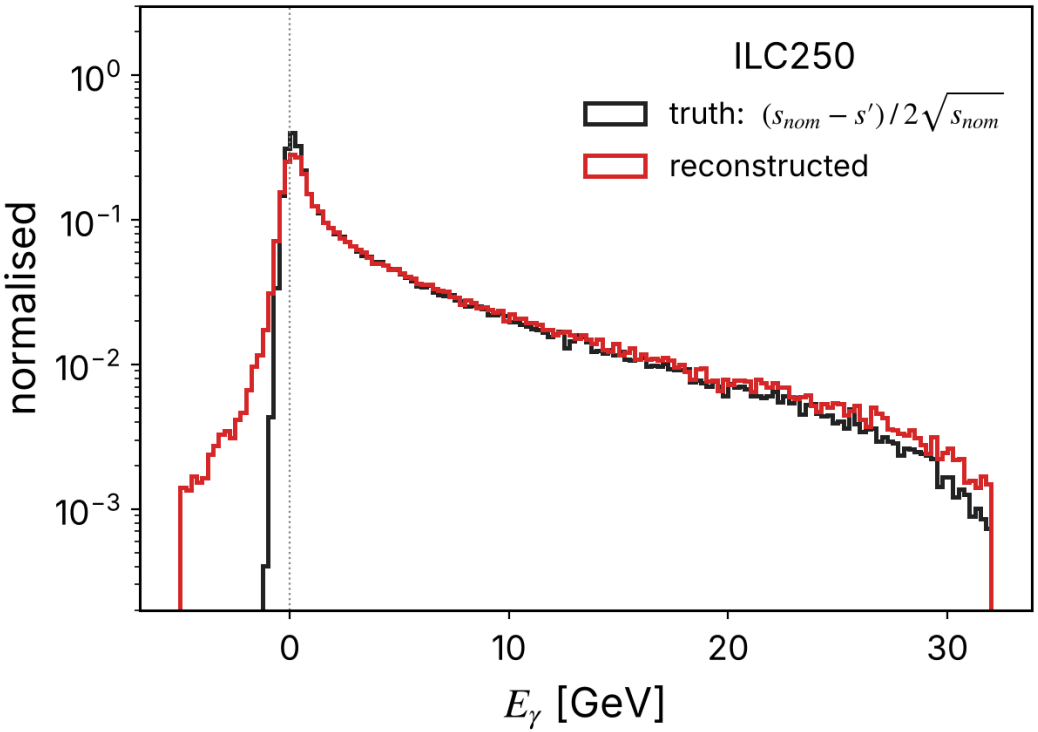
28% of events lose > 1 GeV
13% lose > 5 GeV

beamstrahlung is small;
the tail is ISR

circular or linear: the same loss to calibrate, the same method applies

preliminary result: reconstructed photon energy

$$E_\gamma^{\pm} = \frac{M_{rec}^2 - m_H^2}{2\left(\sqrt{s} - E_{\ell\ell} \pm p_{z,\ell\ell}\right)}$$



$E_\gamma < 0$ in truth

beam energy above nominal
(beam energy spread)
12% of events, down to −1 GeV

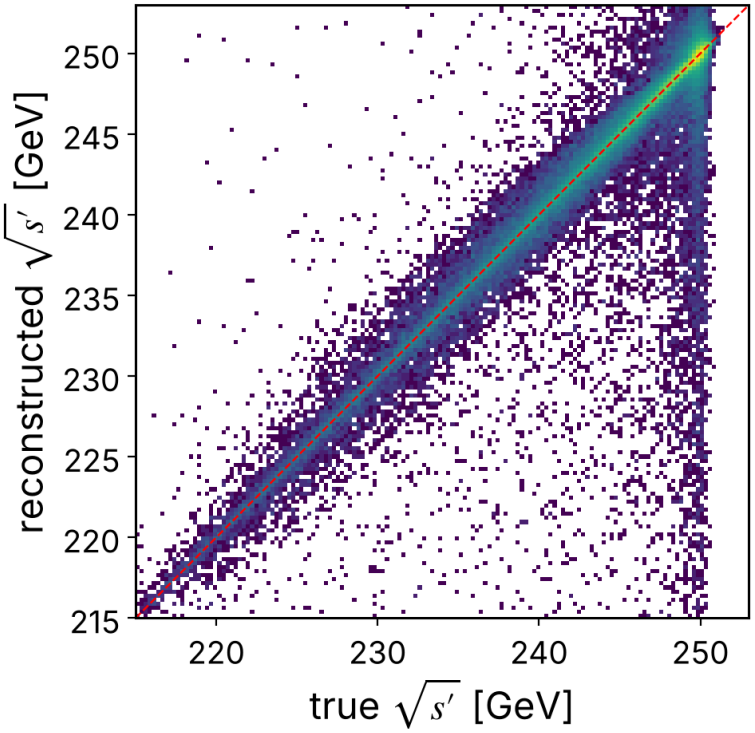
$E_\gamma < 0$ reconstructed

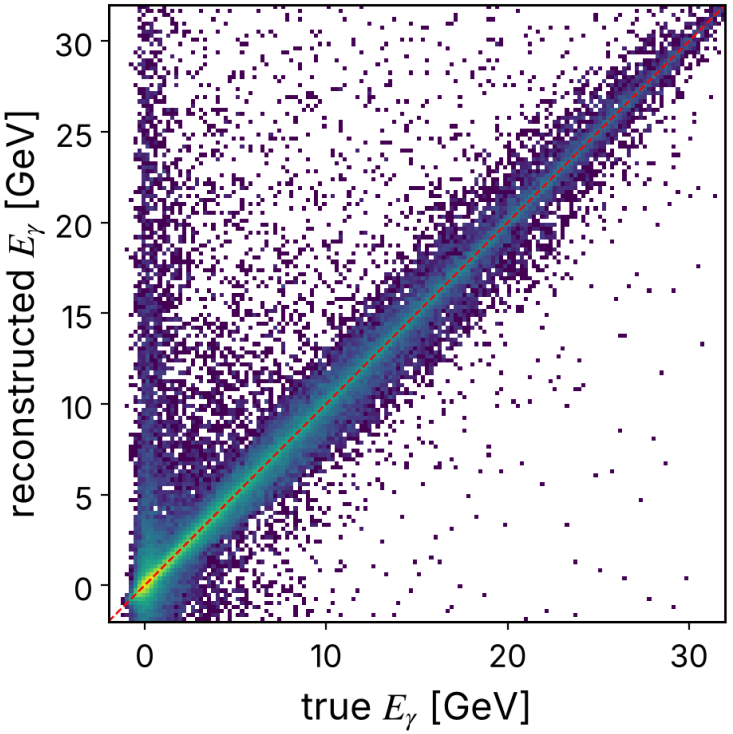
recoil mass below m_H
(resolution, no radiation)
15% of events

the reconstructed spectrum matches well the truth from 0 to 30 GeV

potential improvement: kinematic fitting with proper beam / tracking resolution

Reconstructed: collision energy and radiated energy, event by event

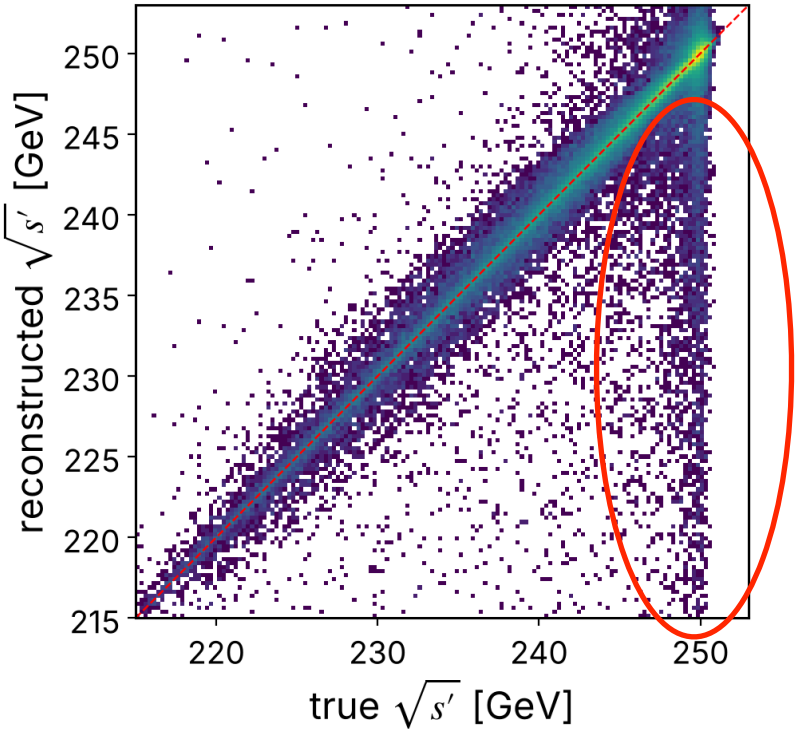


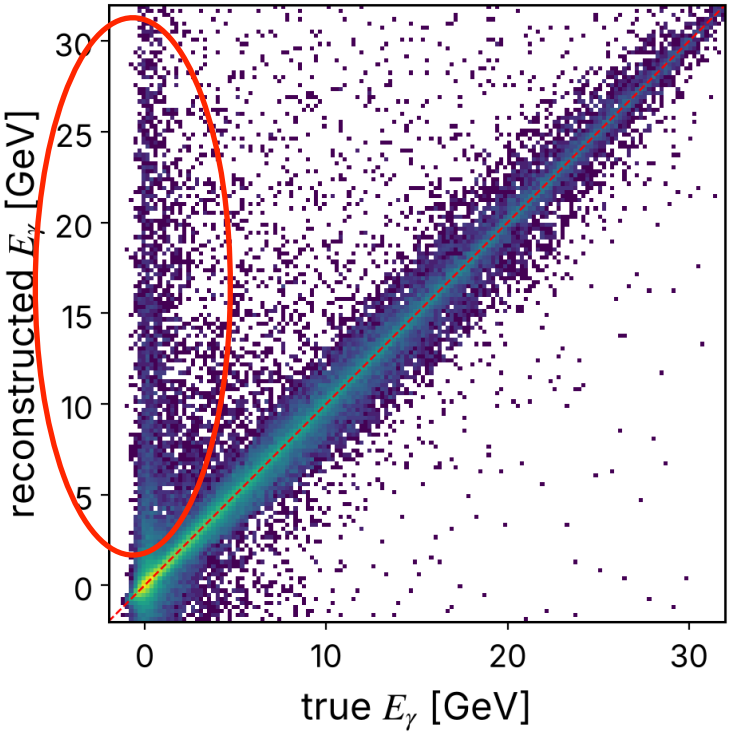


ILC250

(note: correct photon direction assumed)

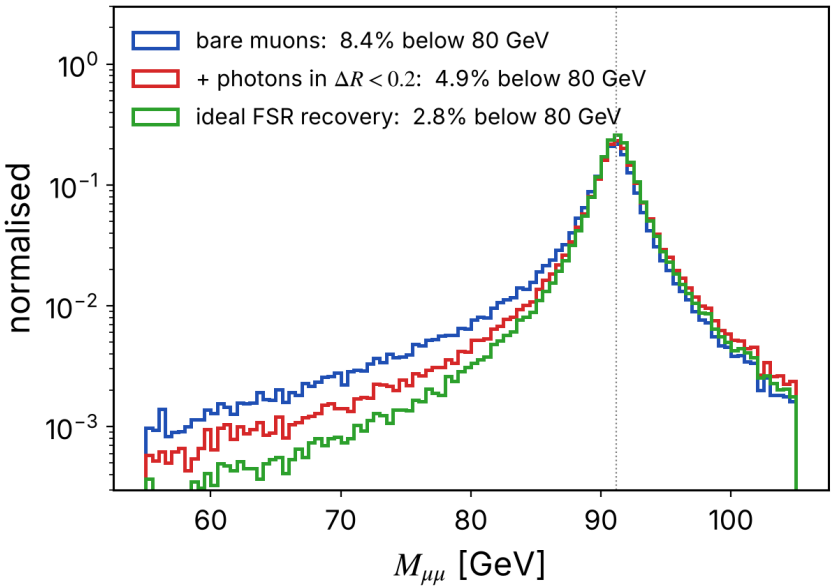
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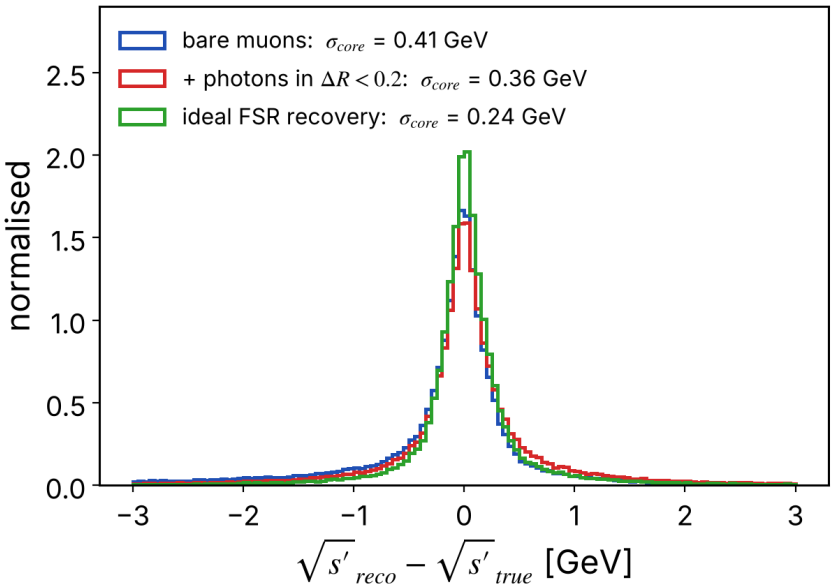


over-calibration: Final State Radiation (FSR) from di-muon

FSR recovery matters



dimuon mass



collision-energy residual

ILC250: a simple cone recovers about half of the loss; ideal recovery gives 0.24 GeV

(close to the beam energy spread, ~0.27 GeV)

room for a better FSR-recovery algorithm

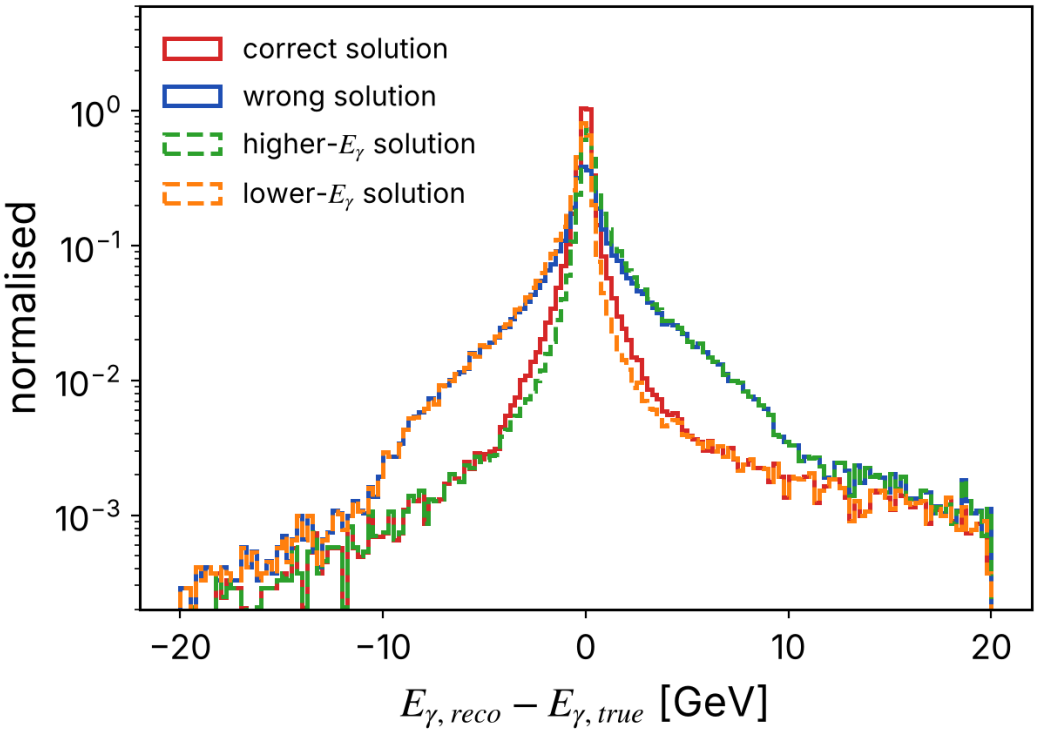
resolution vs. what has to be recovered

	spread of $\sqrt{s'}$	resolution	ideal FSR recovery
FCC-ee 240	2.0	0.27	0.19
ILC250	5.3	0.36	0.24
ILC280	8.7	0.50	0.35
ILC500	48	1.5	1.2

all in GeV; spread = half-width containing 68% of events; resolution = core width

higher energy: worse resolution, but a much longer line shape to measure

an open issue: which beam radiated?



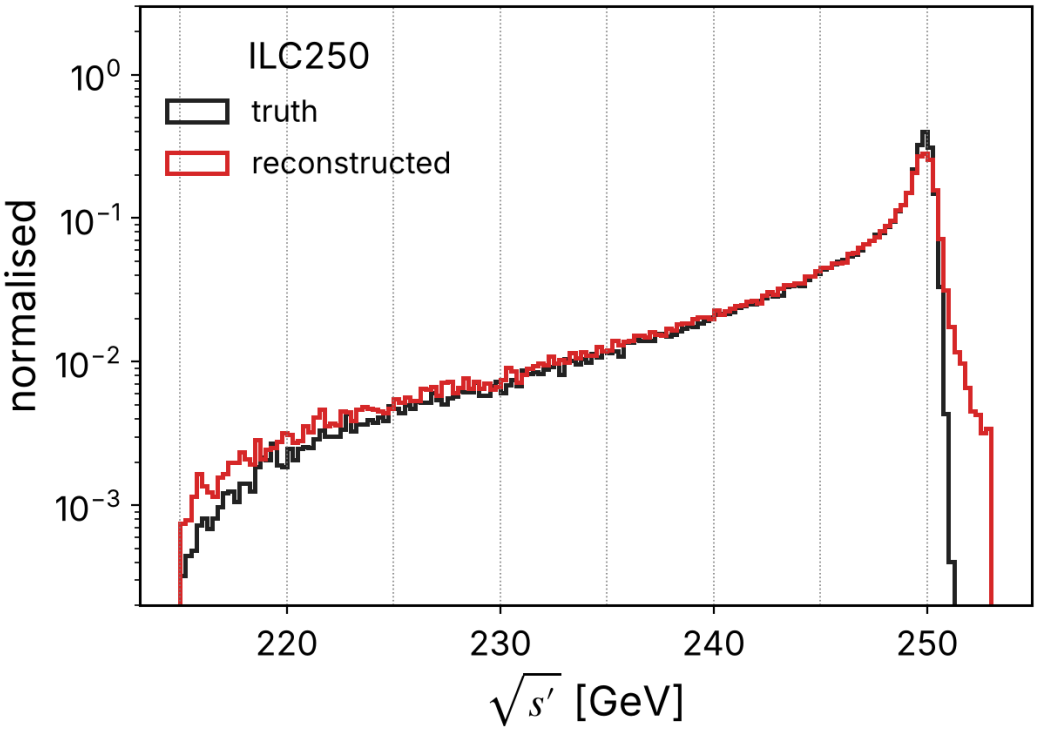
they differ by > 3 GeV in $\sqrt{s'}$ for a quarter of the events

simple rules pick correctly in ≲ 60% of events

blind choice → one-sided tail, known from simulation

in the following: correct solution assumed (the target for a real treatment)

reconstructed collision-energy spectrum



ILC250

reconstructed spectrum
follows the truth
from threshold to 250 GeV

count signal events
in bins of $\sqrt{s'}$
(dotted lines: 5 GeV bins)

note: counts alone are not
a cross section

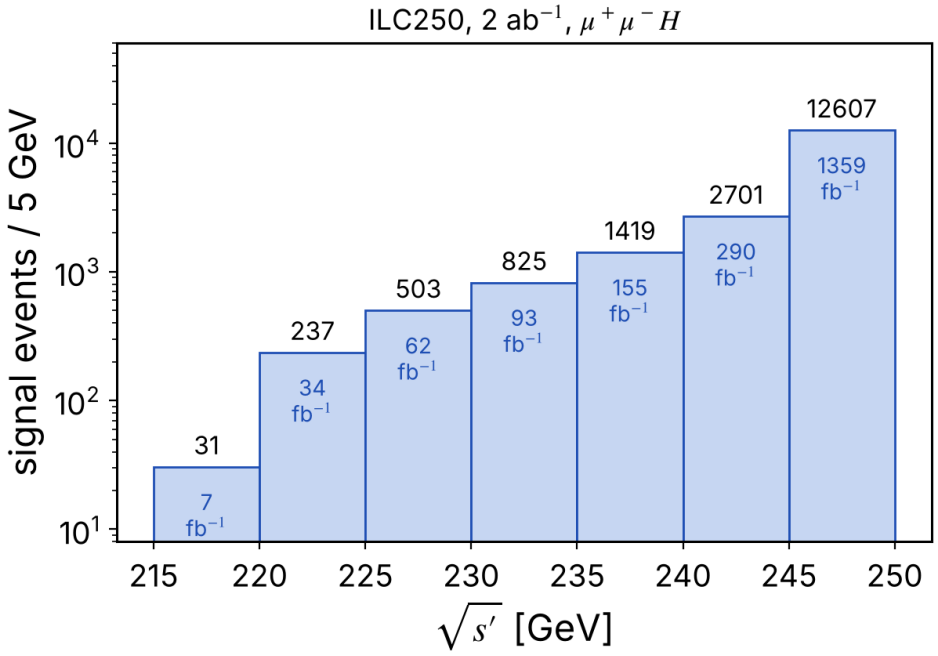
events per bin ÷ luminosity in that bin = cross section per bin → line shape

the luminosity spectrum (beamstrahlung ⊗ ISR) is needed as an input

one data set = an energy scan

$$N_i = \varepsilon_i \cdot L f_i \cdot \sigma_{ZH}\left(\sqrt{s'_i}\right)$$

f_i : fraction of luminosity at $\sqrt{s'_i}$ (beamstrahlung ⊗ ISR; measurable with e+e- → μ+μ-(γ))



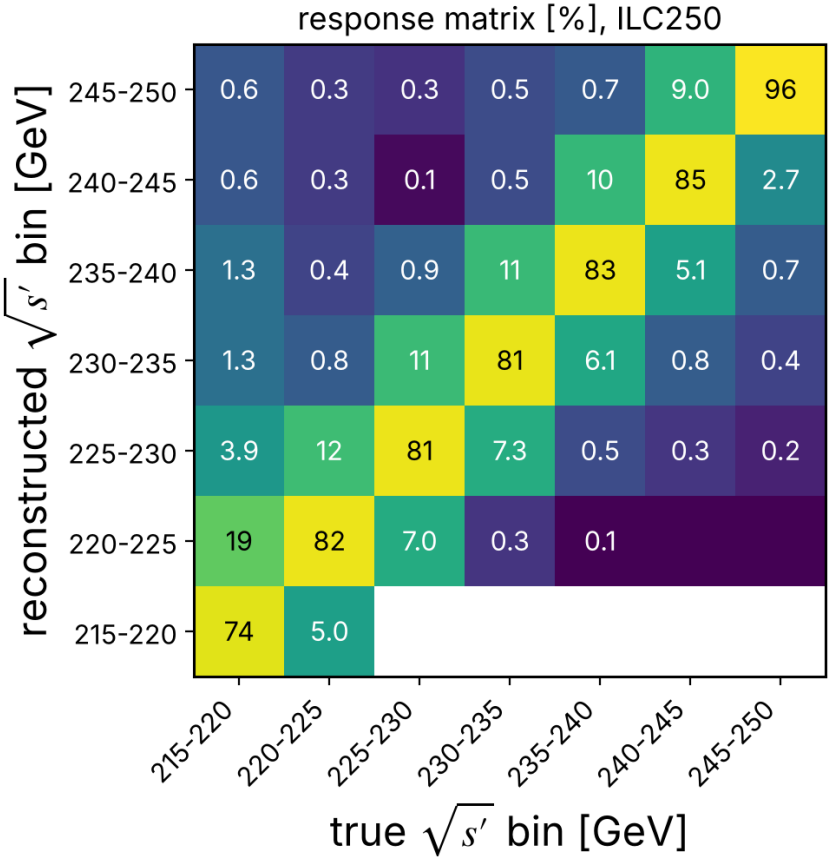
ILC250, 2 ab^{-1}

1/3 of the luminosity is below 245 GeV

equivalent to a scan with 290, 155, 93, 62, 34 fb^{-1} in 5 GeV steps

normalisation: Yan et al. (σ = 10.4 fb, ε = 88%)

from reconstructed to true √s': migration matrix



$$N^{reco} = R\, N^{true}$$

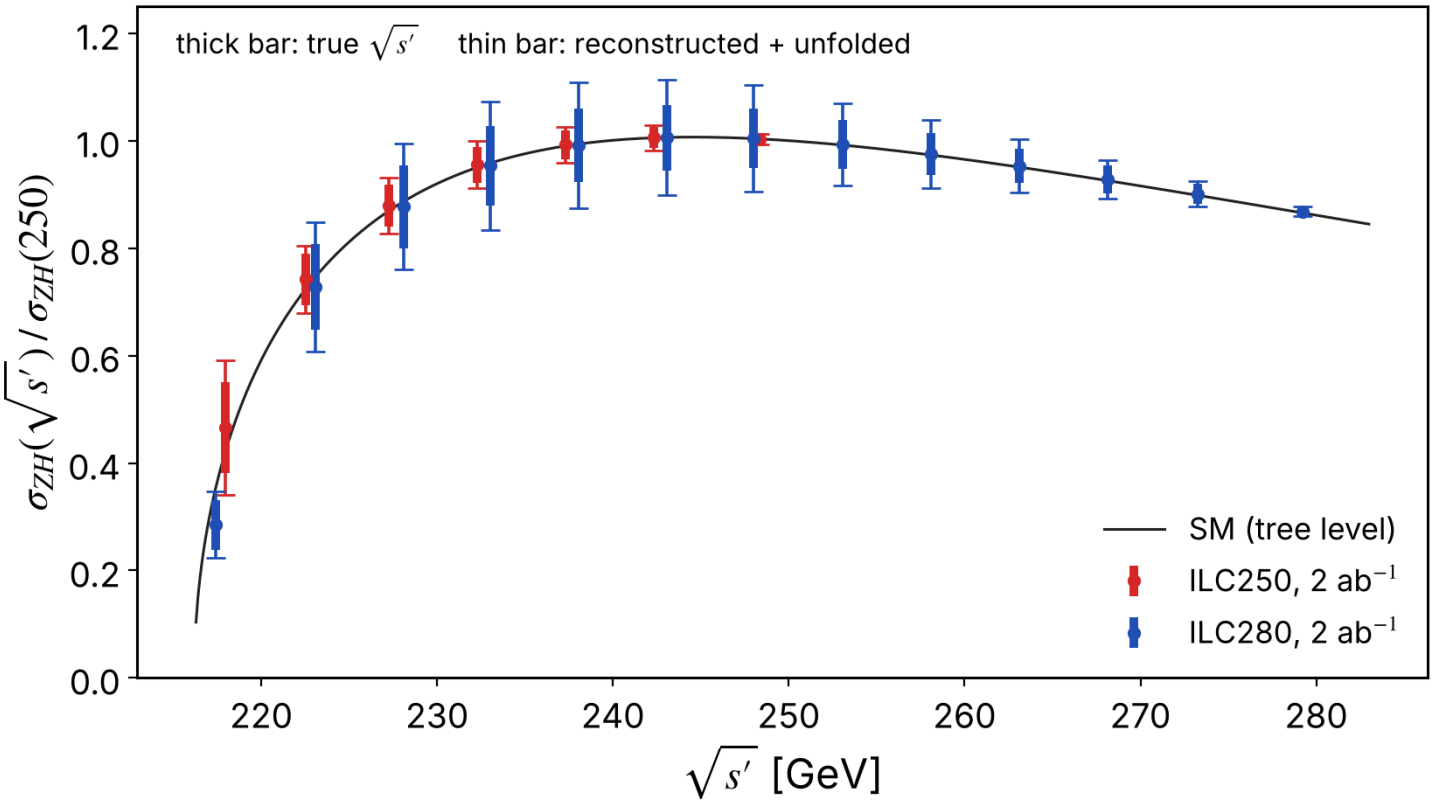
> 80% of events in their true bin (ILC250)

migration mostly to neighbouring bins

far tails: unrecovered FSR, mis-paired muons

cuts: $M_{\mu\mu} \in [73,120]$, $p_T \in [10,70]$, $M_{rec} \in [110,155]$ GeV

ZH line shape: expected statistical precision



ILC250: 2 – 6% per 5 GeV bin for 225 – 245 GeV

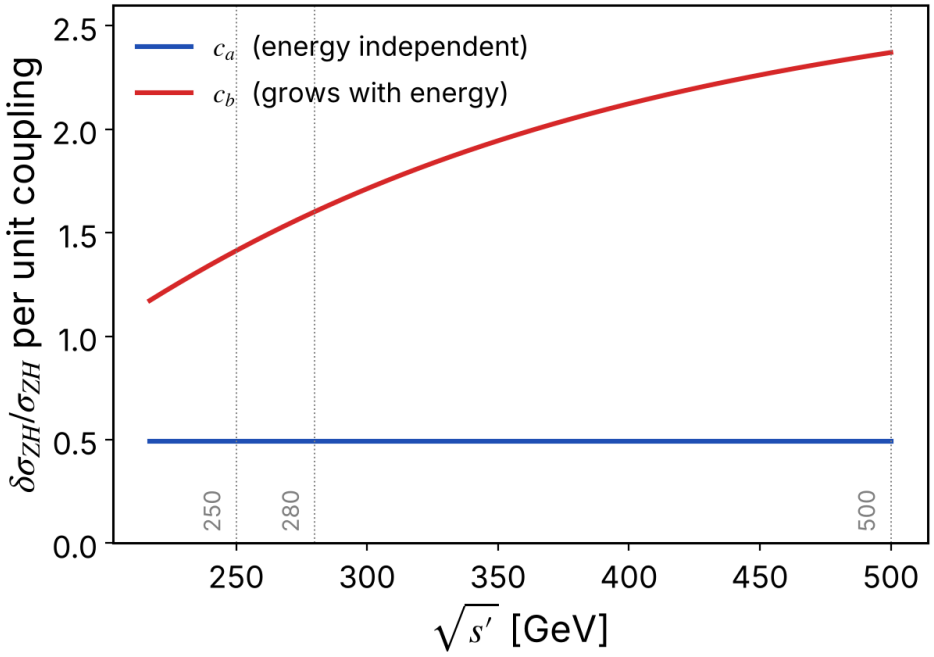
ILC280: rise, maximum and fall of σ_{ZH} in one data set

caution: μ+μ-H only, signal only, statistical errors; just for proof of principle

example application: anomalous ZZH couplings

$$\mathcal{L}_{ZZH} = M_Z^2 \left(\frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H + \frac{b_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H$$

(Ogawa et al; Λ = 1 TeV)



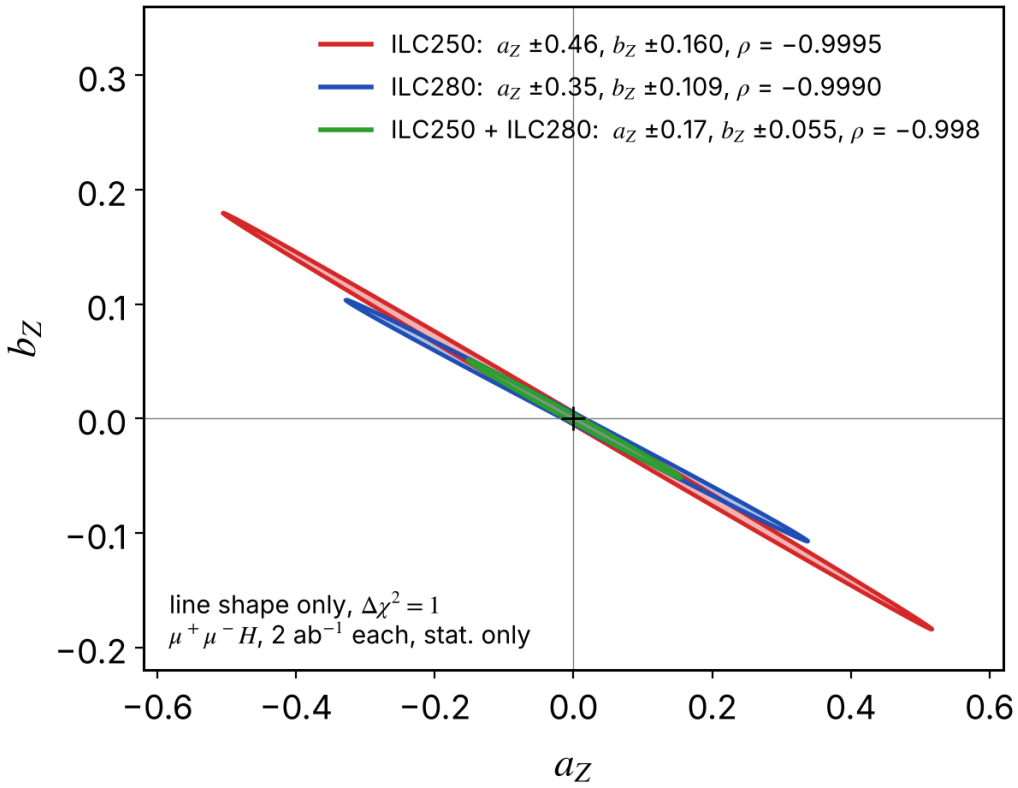
$$\frac{\delta\sigma_{ZH}}{\sigma_{ZH}} = c_a\, a_Z + c_b(s')\, b_Z$$

a_Z : same structure as the SM
→ same effect at all energies

b_Z : momentum dependent
→ effect grows with energy

a single total cross section cannot separate them; a line shape can

a_Z vs. b_Z from the line shape alone



the band from the total rate closes into an ellipse

but the lever arm is short:
c_b/c_a changes by 18% (220–250)
and 34% (220–280)

correlation stays at −0.999

improvement is potentially significant

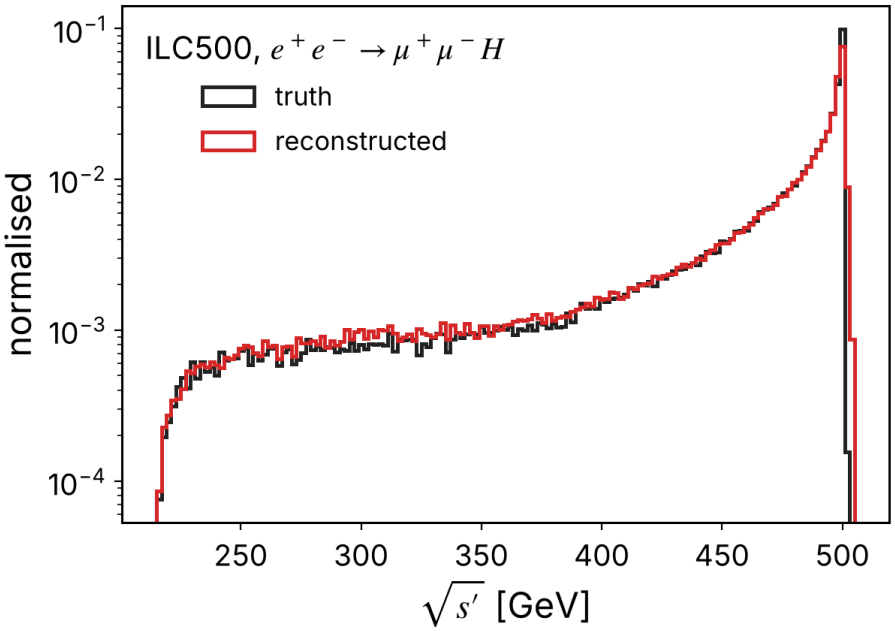
a proof of principle; ideas about new nominal energy points still ongoing

what helps more: a longer lever arm

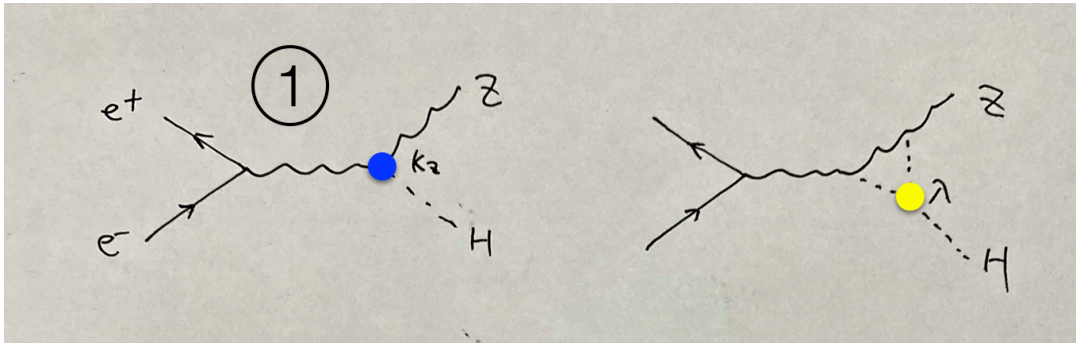
two total rates	a_Z	b_Z
250 + 280	±0.17	±0.057
230 + 280	±0.094	±0.032
230 + 300	±0.074	±0.024

2 ab^{-1} each, μ+μ-H, nominal energies

ILC500 line shape:
216 → 500 GeV in one data set
c_b/c_a : 2.4 → 4.8

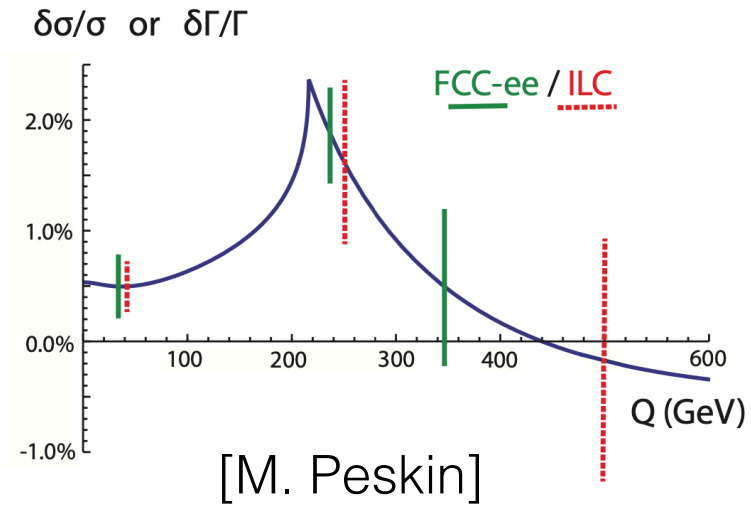


another example: How to discriminate λ_{hhh} with HZZ coupling



[McCullough, '13]

$$\delta_\sigma^{240} = 100\,(2\delta_Z + 0.014\delta_h)\,\%$$



[M. Peskin]

- **"easy" solution: lift degeneracy by multiple √s**

summary

started version 2 of recoil mass analysis

known m_H → radiated energy and $\sqrt{s'}$ of every ZH event, from the two leptons only

a fixed-energy run becomes a scan: $\sigma_{ZH}(\sqrt{s'})$ to a few % per 5 GeV bin with 2 ab^{-1}

first application anomalous HZZ (a_Z, b_Z): degeneracy closes, but wider energy range better

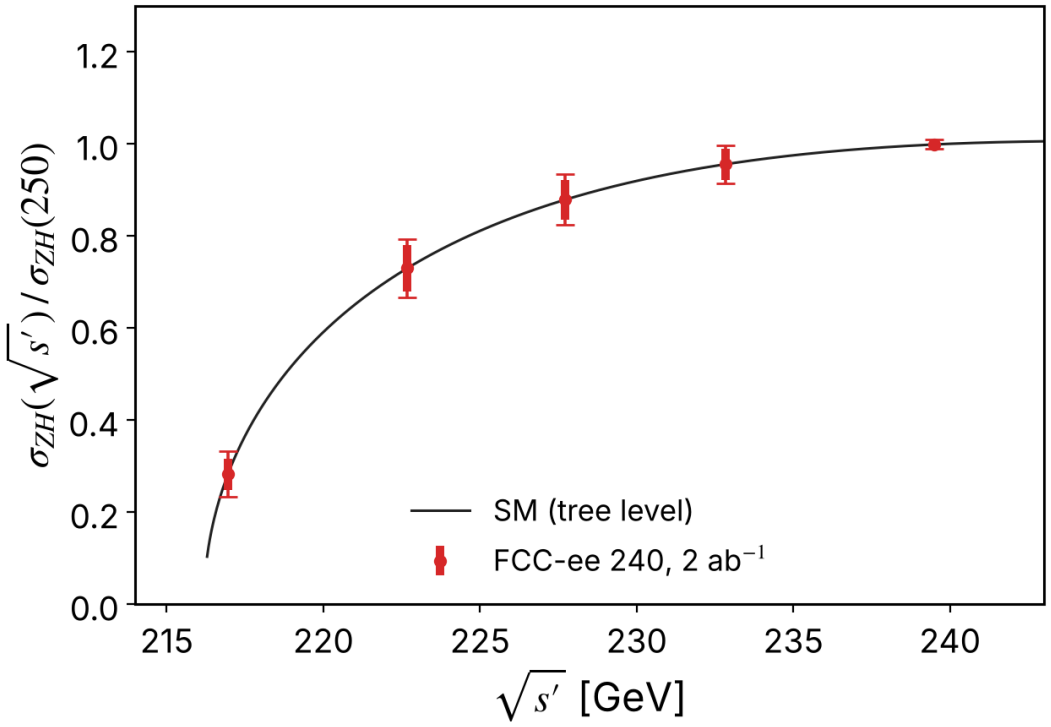
to do

photon direction (two solutions) · better FSR recovery · more BSM applications
luminosity spectrum per $\sqrt{s'}$ bin · background · other Z decay channels

comments / ideas / volunteers are very welcome

backup

line shape at FCC-ee 240 (a test)



5 GeV bins, 11.9k signal events

error per bin (reconstructed):

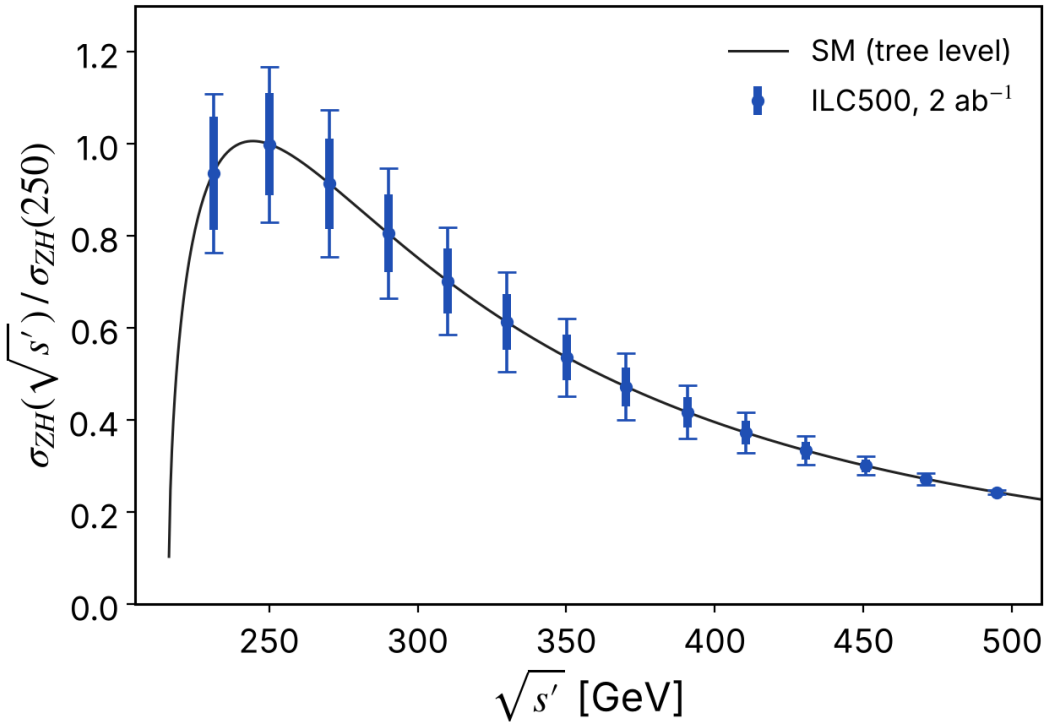
18% 215–220
8.6% 220–225
6.3% 225–230
4.3% 230–235
1.0% 235–240

10.8 ab^{-1} : all errors × 0.43

87% of events reconstructed in their true bin; 15% of the luminosity below 235 GeV

σ = 6.76 fb (unpolarised) and ε = 88% assumed; 20k-event sample; μ+μ-H, signal only

line shape at ILC500



20 GeV bins, 6.1k signal events

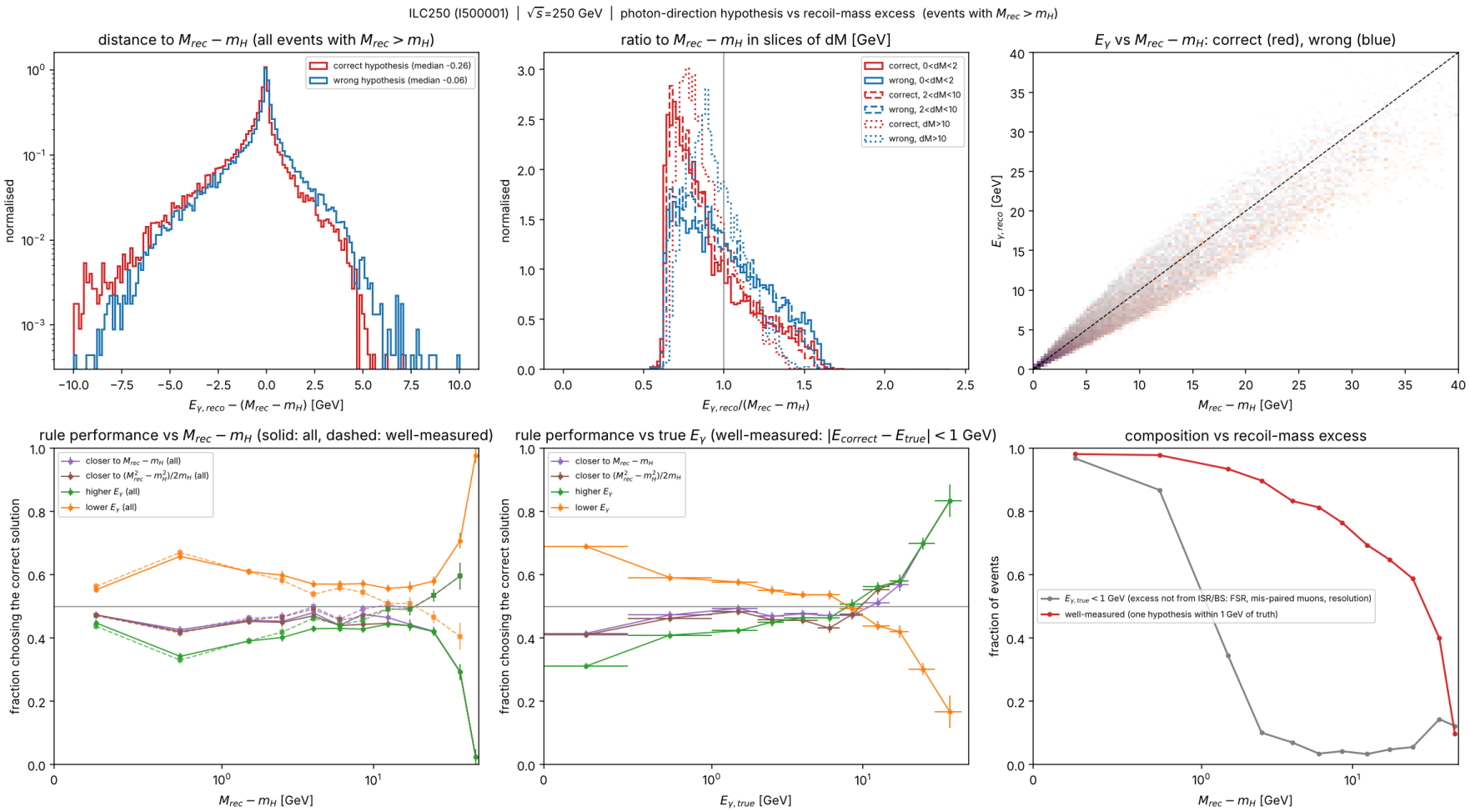
error per bin (reconstructed):
17–18% 220–340
12–16% 340–420
4–9% 420–480
1.8% 480–500

a_Z ±0.28, b_Z ±0.060
from this line shape alone

220 → 500 GeV in one data set; 35% of the luminosity below 480 GeV

σ = 3.45 fb (Yan et al.) and ε = 88% assumed; μ+μ-H, signal only; lowest bin (5 events) not shown

photon direction vs. recoil-mass excess



both solutions scale with Mrec – mH ; simple rules stay near 50 – 60% except at large true Eγ

line shape: statistical error per 5 GeV bin

√s′ bin [GeV]	ILC250 true	ILC250 reco	ILC280 true	ILC280 reco
215–220	18	27	16	22
220–225	6.5	8.4	11	17
225–230	4.5	5.9	8.8	13
230–235	3.5	4.6	7.7	13
235–240	2.7	3.4	6.9	12
240–245	1.9	2.3	6.1	11
245–250	0.9	0.9	5.5	9.8
250–255			4.6	7.7
255–260			4.1	6.5
260–265			3.4	5.2
265–270			2.7	3.9
270–275			2.1	2.6
275–280			1.0	1.0

in %; 2 ab^{-1}, μ+μ-H, signal only; top bin of each machine includes events above nominal energy